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Research article

A strategy for hepatitis diagnosis by using spherical q-linear Diophantine fuzzy Dombi aggregation information and the VIKOR method

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Abstract: Hepatitis is an infectious disease typified by inflammation in internal organ tissues, and it is caused by infection or inflammation of the liver. Hepatitis is often feared as a fatal illness, especially in developing countries, mostly due to contaminated water, poor sanitation, and risky blood transfusion practices. Although viruses are typically blamed, other potential causes of this kind of liver infection include autoimmune disorders, toxins, medicines, opioids, and alcohol. Viral hepatitis may be diagnosed using a variety of methods, including a physical exam, liver surgery (biopsy), imaging investigations like an ultrasound or CT scan, blood tests, a viral serology panel, a DNA test, and viral antibody testing. Our study proposes a new decision-support system for hepatitis diagnosis based on spherical q-linear Diophantine fuzzy sets (Sq-LDFS). Sq-LDFS form the generalized structure of all existing notions of fuzzy sets. Furthermore, a list of novel Einstein aggregation operators is developed under Sq-LDF information. Also, an improved VIKOR method is presented to address the uncertainty in analyzing the viral hepatitis categories demonstration. Interesting and useful properties of the proposed operators are given. The core of this research is the proposed algorithm based on the proposed Einstein aggregation operators and improved VIKOR approach to address uncertain information in decision support problems. Finally, a hepatitis diagnosis case study is examined to show how the suggested approach works in practice. Additionally, a comparison is provided to demonstrate the superiority and efficacy of the suggested decision technique.

Keywords: Sq-LDFS; Dombi aggregation operators; VIKOR technique; hepatitis diagnosis; decision-making
Mathematics Subject Classification: 03E72, 47S40

1. Introduction

Extensive decision making is a very challenging task, particularly when one must face ever-changing circumstances and informative data. In multi-criteria decision making (MCDM), one decision maker makes a decision in the presence of distinct principles. Everyday issues require consideration of the complex normative system, which has an impact on both the solutions and the ultimate consequences.

The fuzzy set (FS), invented by Zadeh [1], is the path-finder theory in ambiguous thoughts that has significantly influenced the traditional decision-making theory. The technique has been utilized to address misleading and false information in challenging decision-making processes. However, FS modeling parameters can only handle ambiguous and fuzzy data if there are two or more sources of uncertainty present simultaneously. Therefore, several FS versions and generalizations have been proposed. The intuitionistic FS (IFS) is one of the most significant and well-known generalization forms [2]. The selection is limited to the satisfaction and dissatisfaction classes because IFS is formed by the truth-membership grade (TM-grade) and falsity-membership grade (FM-grade), which fulfill the requirement that their sum is limited to one; $0 \leq \mathcal{T} + \mathcal{F} \leq 1$. It fails when $0 \leq 0.5 + 0.6 > 1$. Eradicating this issue, Yager [3] specified the Pythagorean FS (PyFS) principle, which states that the square sum of the TM- and FM-grades must not surpass 1; $0 \leq (\mathcal{T})^2 + (\mathcal{F})^2 \leq 1$. In light of this circumstance, $0.5^2 + 0.6^2 = 0.25 + 0.36 = 0.61 < 1$. IFS and PyFS are insufficient for modelling in the case of $0.7^2 + 0.9^2 = 0.49 + 0.81 = 1.3 > 1$. Yager [4] asserted once more that, in some circumstances, the sum of the squares of the TM-grade and FM-grade in the PyFS structure may be greater than 1. He therefore, investigated the q-rung orthopair FS (q-ROFS), which is made up of the TM-grade and FM-grade and has the condition that the sum of the qth powers of the TM-grade and FM-grade is less than 1. For $q = 3$, we get $0.7^3 + 0.7^3 = 0.343 + 0.343 = 0.686 < 1$, utilizing the q-ROFS condition. The above mentioned uncertainty sets have been great in dealing with the subject of much research and practical application. These sets do, however, have certain rigorous requirements for membership levels. Once more, if we choose the information in the form of $(1, 0.3)$, we may obtain $1^q + 0.3^q \geq 1$ for any value of q by utilizing the q-ROFS condition. Then, we get $0.7^3 + 0.9^3 = 0.343 + 0.512 = 0.855 < 1$. Improving this, Riaz and Hashmi put forward a new theory.

Riaz and Hashmi [5] developed the unique framework of the linear Diophantine fuzzy set (LDFS) in order to loosen these limits. By designating control factors (CFs) that correspond to the TM-grades and FM-grades, the LDFS expands IFS and PyFS. The multi-attribute decision making (MADM) paradigm, where decision-makers may freely pick the grades, is the most suitable framework. The LDFS is under the situation that $0 \leq \lambda\mathcal{T} + \eta\mathcal{F} \leq 1$ with $0 \leq \lambda + \eta \leq 1$. In this case, if we select CFs $\lambda = 0.2$ and $\eta = 0.3$, the LDFS technique yields $(0.2)(1) + (0.3)(0.3) = 0.29 < 1$ and $0.2 + 0.3 = 0.5 < 1$, respectively. However, in this case as well, the total of the CFs in LDFS is occasionally more than 1, i.e., $\lambda + \eta > 1$. Almagrabi et al. [6] devised the flexible theory of the q-rung LDFS (q-LDFS) with the inclusion of CFs in the structure of the q-ROFS in this direction. They introduced the q-LDFS, in which $0 \leq \lambda^q\mathcal{T} + \eta^q\mathcal{F} \leq 1$ and $0 \leq \lambda^q + \eta^q \leq 1$. This removed the constraint. This method is more effective and adaptable than previous approaches. This novel concept of a spherical fuzzy set (SFS), which generalizes the FS, IFS and picture FS (PFS), has been presented by Ashraf and colleagues [7, 8] and only partially extends PFS; for instance, if $\mathcal{T}, \mathcal{F}$ and $\mathcal{I}$ are assumed to be 0.6, 0.7 and 0.8, respectively, even squaring is insufficient, since $0.36 + 0.49 + 0.64 = 1.49 > 1$. Research is now focused on making spherical linear Diophantine fuzzy (SLDF) judgments. In decision-making models, it is essential to establish appropriate information representations and procedures. It is reported that spherical MADM difficulties are significantly impacted by the SLDF aggregation method. In order to relax strict restrictions, Riaz [9] suggested the concept of SLDF sets (SLDFSs) with reference or control parameters. The concept of SLDFSs with parameterizations is very helpful for modelling uncertainty in MCDM. The SLDFS-based aggregation operators are an effective tool for the MCDM approach.

A comprehensive class of fuzzy operators was launched by Dombi [10]. Due to the fact that the Dombi T-operators are based on operational parameters, they are more versatile than other algebraic operations. In this context Jana et al. [11] introduced the intuitionistic fuzzy Dombi hybrid average operator and the intuitionistic fuzzy Dombi hybrid geometric operator to deal with this data. Operators benefit from being adaptable in the working parameters. Jana et al. [12] also described a technique for MADM based on intuitionistic Dombi operators and their use in evaluating mutual funds. Li et al. [13] provided multiple attribute group decision making methods based on intuitionistic fuzzy Hamy mean, Dombi Hamy mean, and Dombi dual Hamy mean operators. Senapati et al. [14] developed a powerful decision-support system based on Aczel-Alsina aggregation operators and power operators in an intuitionistic fuzzy setting. For the developed decision support model's superiority, they set criteria weights using the Shannon entropy-based power weighted technique.

In order to produce acceptable methods of assessing emerging innovation in commercialization, Jana et al. [15] employed Pythagorean fuzzy Dombi hybrid weighted geometric operators in the MADM problem. Akram et al. [16] presented the weighted geometric operator and Pythagorean Dombi fuzzy weighted arithmetic averaging for analytical textile technologists. By merging the Dombi t-norm, t-conorm and power operators, Liu et al. [17] proposed some improved Pythagorean fuzzy Dombi power aggregation operators used in application for multiple-attribute decision making. Akram et al. [18] suggested Dombi operations to create a few aggregation operators, including the 2-tuple linguistic-rung picture fuzzy information. The fuzzy 2-tuple linguistic-rung picture with Dombi weighted averaging operator with Dombi weighted geometry is used to handle a large range of triplets that are allowed. Senapati [19] introduced the Aczel-Alsina t-norm and the t-conorm in order to depict the picture-fuzzy conditions and propose a few novel picture fuzzy number techniques. To deal with large amounts of data, he offered an array of aggregation operators. The main goal of Jana et al. [20] was to demonstrate a few q-rung orthopair fuzzy Dombi aggregation operations under Pythagorean fuzzy data for evaluating the different priorities of the choices throughout the decision-making process. Ashraf proposed the SFS, and Ashraf et al. [21] applied the Dombi operational rules to SFSs to make them more applicable in decision-making. There are various restrictions in the current Dombi operational rules for SFSs and Dombi weighted aggregation operators. To tackle uncertainty in MCDM, Aldring et al. [22] presented an LDFS with the inclusion of reference parameters, which paves the way for an advanced approach. With the aid of Dombi mean aggregation operators, an MCDM approach has been developed in this article by using LDFSs.

Ashraf [23] presented the SFS to described the list of aggregation operators using algebraic t-norms and t-co-norms. Liu et al. [24] introduced linguistic SFSs and discussed their application in decision making. Also, presented the linguistic spherical fuzzy weighted averaging operator to aggregate the uncertain information. Rafiq [25] considered the positive, neutral, negative, and refusal grades in SFSs to examine the family of innovative similarity measures under SFSs and the cosine function. Ashraf et al. [26] explored the family of six spherical fuzzy Dombi weighted aggregation operators and discussed their application in real word decision aid problems. Furthermore, Khan et al. [27] developed the improved Dombi aggregation operators under spherical fuzzy environment. Guner [28] seeks to show how SFSs may be extended in order to accommodate aggregation operators. In the beginning, he introduces Einstein sum, product, and scalar multiplication for extended SFSs based on triangle norm and conorm of Einstein. A few aggregation operators created in order to expand the idea of SFSs. As an extension of PFS and PyFSs, the SFS has much importance. Ashraf [29] works on SFS using the well-known sine trigonometric function. He presents several strong sine trigonometric (ST) operation rules for spherical fuzzy (SF) sets, taking into account these characteristics and the significance of SF sets. To aggregate the SF data following these rules, we design some novel aggregation operators (AOs), including geometric and ST-weighted averaging operators. Qiyas [30] described some trustworthy sine trigonometric rules for SFSs. He developed new average and geometric aggregation operations to aggregate the spherical fuzzy numbers following these principles. He created to score and accuracy functions for comparison. Also introduced the method for combining the aggregation operators based on entropy concepts as well. The ultimate focus of Chinram [31] was to define SFS in the context of Yager norms. In this study, he developed new operational rules for spherical fuzzy environments based on the Yager t-norm and t-conorm (SFE). He also created a list of innovative aggregation operators under SFE and ranked the aggregation operators by score function based on these Yager operational principles. At the beginning of his study, Riaz [32] introduced a few new SFS operations, including the Aczel-Alsina product, sum, exponent, and scalar multiplication. A family of aggregation operations was created, and scoring functions were established. He noticed that moment to be the most advantageous for his notion. Qiyas [33] defined spherical uncertain linguistic sets and creates several SULN operational laws. The authors also constructed spherical uncertain linguistic Hamacher averaging and geometric aggregation operators by extending these operating principles to the aggregation operator. At the same time Wei [34] established a few Hamacher power aggregation operators under the SFSs by combining power operators and Hamacher operators, such as the spherical fuzzy Hamacher power average operator and the spherical fuzzy Hamacher power geometric operator. Wei also found a weight vector by the entropy of the SFS to find aggregation operators.

By utilizing the aggregation operators and scoring function of an SFS, Kutlu [35] seek to expand the traditional VlseKriterijumska Optimizacija I Kompromisno Resenje (VIKOR) approach to the spherical fuzzy VIKOR (SF-VIKOR) method. Kutlu [36] advocated using the article Kutlu Gündodu and Kahraman [35] and utilizing their defuzzification, aggregation, and arithmetic standards. His goal is to solve waste management issues using the SF-VIKOR approach. Akram [37] provides four novel aggregation operators namely the complex spherical fuzzy weighted average/ geometric operators as fundamental operations that act as helping instruments. Then, employing the principles of the VIKOR method, he presents a multi-skilled and high-potential multi-criteria group decision-making (MCGDM) tool called the complex spherical fuzzy VIKOR (CSF-VIKOR) method. Akram [38] once again uses these fundamental processes, which act as aids in the creation of four additional CSFNSFW aggregating operators, to determine rankings utilizing the score and accuracy functions. He presents the complex spherical fuzzy N soft VIKOR (CSFNSF-VIKOR) approach as a multi-skilled and highly-potential multi-criteria group decision-making (MCGDM) tool for comparison. Based on two aggregation operators (SWAM, SWGM) and two ideal solutions, Sharaf's [39] suggested spherical fuzzy VIKOR (SFVIKOR) by computed in four distinct methods. The two best options accommodate the level of reluctance while avoiding the scoring function. Results compared by using SFVIKOR with the four implementations to those obtained using the spherical fuzzy approach of order preference by the resemblance to an ideal outcome (SFTOPSIS). Ayyildiz [40] suggested for the first time in the literature of fuzzy techniques, spherical fuzzy VIKOR extended with spherical fuzzy AHP produced the best results for resolving an important Covid 19 problem. Chen [41] uses a T-SFS to develop the VIKOR approach, which forms desired T-SF features and develops T-SF score functions and T-SF(VIKOR). To demonstrate the effectiveness of the suggested notion, Qiyas et al. [42] used the linguistic picture fuzzy weighted averaging/geometric operators to tackle the uncertainty in decision aid problems. They compared these to sine trigonometric spherical VIKOR and TOPSIS. Nguyen [43] uses spherical fuzzy numbers to locate spherical AHP-VIKOR in order to demonstrate an efficient method. The SFSs are used by Sharaf [44] to find the spherical VIKOR and TOPSIS approach.

It is important to extend q-LDFS to Sq-LDFS in three-dimensional space in order to remove this restriction and draw inspiration from the notion of q-LDFS in three-dimensional space. In Sq-LDFS, we present the qth power of CFs which, with the aid of the qth power of CFs in three dimensions, covers a large space of membership grades and the space of the current structure. LDFS and q-LDFS are both two-dimensional special instances. For example, Sq-LDFS changes to SLDFS in the Sq-LDFS environment if q = 1, and Sq-LDFS is reduced to SFS if each CF equals 1.

Motivation

The IFS, PyFS, q-ROFS, SFS, LDFS, and their extensions describe the Dombi operating laws and Dombi aggregation operators. However, q-LDFS and SLDFS do not describe these concepts. Dombi operational rules are a crucial family of aggregation operators derived from $\frac{1}{1+x}$ operator because they are more versatile than conventional T-operators. This drives researchers to create Dombi Sq-LDF aggregation operators that can effectively manage inconsistent, reluctant, and imprecise data in a complex work frame where the basis is a crisp value.

Practical strategies that produce the closest ideal outcome can be determined using mutual methods. Opricovic (1998) developed the VIKOR technique as a MADM tactic for resolving discrete choice problems with unbalanced and opposing criteria. This article presents a comprehensive literature survey on various fuzzy sets, with a focus on fuzzy VIKOR variations. While there are many sets in three dimensions (such as SFSs, SLDFSs, T-SFSs, complex SFSs, etc.) made up of MG, IG, and NMG, the aim of this article is to introduce the three-dimensional VIKOR method. Our method is more effective because it uses control parameters from the spherical q-linear Diophantine fuzzy sets in the spherical fuzzy VIKOR technique that can choose the grades independently.

The rest part of this article is outlined as follows: In successive part briefly review some fundamental Sq-LDFSs ideas and its related operational laws and properties. The 3^{rd} section consists on primal definitions of the ordered weighted and Sq-LD fuzzy Dombi weighted averaging operators. The 4^{th} Section proposes order weighted and Sq-LD fuzzy Dombi weighted and geometric operators with a few score functions. In 5^{th} section we addressed issues involving Sq-LD fuzzy MADM by utilizing these operators. In 6^{th} Section we provides an example to illustrate mathematical analysis of patient of viral hepatitis types. The 7^{th} section describe the TODIM method with an example to illustrate problem. The comparison analysis describe in 8^{th} section. There are some comments made on the paper in 9^{th} section.

2. Preliminaries

This section briefly covered the preliminary knowledge of FS, IFS, PyFS, LDFS, q-LDFS and SFS.

Definition 1. *[2] Suppose the fixed set $\aleph$ is universe of discourse. The fuzzy set C over $\aleph$ can be described as*

$$C = \{\eth, [\mathcal{T}_C(\eth)]; \eth \in \aleph\},$$

where $\mathcal{T}_C(\eth) \in [0,1]$ is truth-membership grade (TM-grade) of $\mathcal{T}$ in $\aleph$.

Definition 2. *[1] Suppose the fixed set $\aleph$ is universe of discourse. The intuitionistic fuzzy set (IFS) $\mathcal{H}$ over $\aleph$ can be described as*

$$\mathcal{H} = \{\eth, [\mathcal{T}_{\mathcal{H}}(\eth), \mathcal{F}_{\mathcal{H}}(\eth)]; \eth \in \aleph\},$$

where $\mathcal{T}_C(\eth)$ is TM-grade and $\mathcal{F}_C(\eth)$ is FM-grade of $\mathcal{H}$ in $\aleph$ with condition that $0 \leq \mathcal{T}_{\mathcal{H}}(\eth) + \mathcal{F}_{\mathcal{H}}(\eth) \leq 1$.

Definition 3. *[3] A Pythagorean fuzzy set (PyFS) $\mathfrak{P}$ in universal set $\aleph$ is defined as*

$$\mathfrak{P} = \{\eth, [\mathcal{T}_{\mathfrak{P}}(\eth), \mathcal{F}_{\mathfrak{P}}(\eth)]; \eth \in \aleph\},$$

where the TM-grade and FM-grade are described by $\mathcal{T}_{\mathcal{H}} : \aleph \to [0,1]$, $\mathcal{T}_{\mathcal{H}} : \aleph \to [0,1]$ orderly with limitation that $0 \leq (\mathcal{T}_{\mathfrak{P}}(\eth))^2 + (\mathcal{F}_{\mathfrak{P}}(\eth))^2 \leq 1$.

Definition 4. *[5] An LDFS $\mathfrak{L}$ over a reference set $\mathfrak{M}$ is defined as*

$$\mathfrak{L} = \{\eth, [\mathcal{T}_{\mathfrak{L}}(\eth), \mathcal{F}_{\mathfrak{L}}(\eth)], [\lambda(\eth), \eta(\eth)]; \eth \in \mathfrak{M}\},$$

where the $\mathcal{T}_{\mathfrak{L}}(\eth)$, $\mathcal{F}_{\mathfrak{L}}(\eth)$, $\lambda(\eth)$ and $\eta(\eth) \in [0,1]$ are the appropriate reference parameters. These functions fall within the range $0 \leq \lambda\mathcal{T}_{\mathfrak{L}}(\eth) + \eta\mathcal{F}_{\mathfrak{L}}(\eth) \leq 1;\ \forall\ \eth \in \mathfrak{M}$ with $0 \leq \lambda + \eta \leq 1$.

Definition 5. *[37] A q-LDFS $\mathfrak{A}$ for a fixed universe of discourse $\mathfrak{E}$ is describe as*

$$\mathfrak{A} = \{\eth, [\mathcal{T}_{\mathfrak{A}}(\eth), \mathcal{F}_{\mathfrak{A}}(\eth)], [\lambda(\eth), \eta(\eth)]; \eth \in \mathfrak{E}\},$$

where TM-grade and FM-grade are denoted by $\mathcal{T}_{\mathfrak{A}}(\eth) : \mathfrak{E} \to [0,1], \mathcal{F}_{\mathfrak{A}}(\eth) : \mathfrak{E} \to [0,1]$ and CFs are denoted as $\lambda : \mathfrak{E} \to [0,1]$ and $\eta : \mathfrak{E} \to [0,1]$ sequencely with constraints $0 \leq \lambda^{\mathfrak{q}}\mathcal{T}_{\mathfrak{A}}(\eth) + \eta^{\mathfrak{q}}\mathcal{F}_{\mathfrak{A}}(\eth) \leq 1$ and $0 \leq \lambda^{\mathfrak{q}} + \eta^{\mathfrak{q}} \leq 1$. If we set $\mathfrak{q} = 1$ it violate by second constrain in some cases when the sum of CFs is out of range but it fulfill the first constraint.

Definition 6. *[8] Let $\mathfrak{Y}$ be the universe for discourse. A spherical fuzzy set under $\mathfrak{S}$ is defined as*

$$\mathfrak{S} = \{\eth, [\mathcal{T}_{\mathfrak{S}}(\eth), \mathcal{F}_{\mathfrak{S}}(\eth), \mathcal{I}_{\mathfrak{S}}(\eth)]; \eth \in \mathfrak{Y}\},$$

where $\mathcal{T}_{\mathfrak{S}} : \mathfrak{S} \to [0,1], : \mathfrak{S} \to [0,1], \mathfrak{I}_{\mathfrak{S}} : \mathfrak{V} \to [0,1]$ are representations of TM-grade, FM-grade and IM-grade respectively also the CFs λ, η and $\beta \in [0,1]$ with the limit $0 \leq (\mathcal{T}_{\mathfrak{S}}(\eth))^2 + (\mathcal{F}_{\mathfrak{S}}(\eth))^2 + (\mathcal{I}_{\mathfrak{S}}(\eth))^2 \leq 1$. The SFS has the quality to cover the data in 3-dimensional space.

Definition 7. *[6] An SLDFS $\mathfrak{V}$ for a fixed universal set $\mathfrak{X}$ can interpret as*

$$\mathfrak{V} = \{\eth, [\mathcal{T}_{\mathfrak{V}}(\eth), \mathcal{F}_{\mathfrak{V}}(\eth), \mathcal{I}_{\mathfrak{V}}(\eth)], [\lambda(\eth), \eta(\eth), \xi(\eth)]; \eth \in \mathfrak{X}\}$$

where all reference parameters $\mathcal{T}_{\mathfrak{V}}, \mathcal{F}_{\mathfrak{V}}, \mathcal{I}_{\mathfrak{V}}, \lambda, \eta$ and $\xi \in [0,1]$ with a boundary conditions $0 \leq \lambda\mathcal{T}_{\mathfrak{V}}(\eth) + \eta\mathcal{F}_{\mathfrak{V}}(\eth) + \xi\mathcal{I}_{\mathfrak{V}}(\eth) \leq 1$ and $0 \leq \lambda + \eta + \xi \leq 1$.

3. Spherical q-linear Diophantine fuzzy sets (Sq-LDFSs)

When the sum of CFs for certain cases exceeds their boundary, the SLDFS cannot meet its conditions. Our motive is to cover this gap with a new theory. This section will introduce the Sq-LDF set and their basic operational rules.

Definition 8. *Let $\mathfrak{H}$ be a finite universal set. Then the following is a representation of an Sq-LDFS $\mathfrak{I}$ on $\mathfrak{H}$ as*

$$\mathfrak{I} = \{\eth, [\mathcal{T}_{\mathfrak{I}}(\eth), \mathcal{F}_{\mathfrak{I}}(\eth), \mathcal{I}_{\mathfrak{I}}(\eth)], [\lambda(\eth), \eta(\eth), \xi(\eth)]; \eth \in \mathfrak{H}\},$$

where reality grade ($\mathcal{T}_{\mathfrak{I}}$), falsity grade ($\mathcal{F}_{\mathfrak{I}}$), abstinence grade ($\mathcal{I}_{\mathfrak{I}}$) and control factors λ, η and ξ $\in [0,1]$ *with necessary conditions* $0 \leq \lambda^{\mathrm{q}}\mathcal{T}_{\mathfrak{I}}(\eth) + \eta^{\mathrm{q}}\mathcal{F}_{\mathfrak{I}}(\eth) + \xi^{\mathrm{q}}\mathcal{I}_{\mathfrak{I}}(\eth) \leq 1$ *with,* $0 \leq \lambda^{\mathrm{q}} + \eta^{\mathrm{q}} + \xi^{\mathrm{q}} \leq 1$. *The rejection component is defined as follows:* $\Psi R_{\mathfrak{I}}=[1 - (\lambda^{\mathrm{q}}\mathcal{T}_{\mathfrak{I}}(\eth) + \eta^{\mathrm{q}}\mathcal{F}_{\mathfrak{I}}(\eth) + \xi^{\mathrm{q}}\mathcal{I}_{\mathfrak{I}}(\eth))]^{\frac{1}{\mathrm{q}}}$, *Ψ appears as a CFs about their related grades.*

The assemblage $\omega = \{[\mathcal{T}_{\mathfrak{I}}(\eth), \mathcal{F}_{\mathfrak{I}}(\eth), \mathcal{I}_{\mathfrak{I}}(\eth)], [\lambda(\eth), \eta(\eth), \xi(\eth)]\}$ *of Sq-LDFN including* $0 \leq \lambda^{\mathrm{q}}\mathcal{T}_{\mathfrak{I}}(\eth) + \eta^{\mathrm{q}}\mathcal{F}_{\mathfrak{I}}(\eth) + \xi^{\mathrm{q}}\mathcal{I}_{\mathfrak{I}}(\eth) \leq 1$ *and* $0 \leq \lambda^{\mathrm{q}} + \eta^{\mathrm{q}} + \xi^{\mathrm{q}} \leq 1$.

Remark 1. *For proceeding definition*

(1) If $\mathrm{q} = 1$*, then Sq-LDFN givesback SLDFN.*

(2) If $\mathcal{I} = 0 = \xi$*, then* $S\,\mathrm{q}$*-LDFN turns to* q*-LDFN.*

Furthermore, we illustrate the definitions of absolute Sq-LDFS and null Sq-LDFS.

Definition 9. $\mathfrak{R}_{\mathfrak{I}} = \{\eth, [\mathcal{T}_{\mathfrak{I}}(\eth), \mathcal{I}_{\mathfrak{I}}(\eth), \mathcal{F}_{\mathfrak{I}}(\eth)], [\lambda(\eth), \eta(\eth), \xi(\eth)]; \eth \in \mathfrak{H}\}$ *over $\mathfrak{H}$ be absolute* $S\,\mathrm{q}-$ *LDFS described by Φ if* $\mathcal{T}_{\mathfrak{I}}(\eth) = 1 = \lambda$ *and* $\mathcal{F}_{\mathfrak{I}}(\eth) = \mathcal{I}_{\mathfrak{I}}(\eth) = 0 = \eta = \xi$, $\forall\ \eth \in \mathfrak{H}$, *i.e,* $\Phi = \{(1,0,0), (1,0,0)\}$.

Definition 10. $£_{\mathfrak{I}} = \{\eth, [\mathcal{T}_{\mathfrak{I}}(\eth), \mathcal{I}_{\mathfrak{I}}(\eth), \mathcal{F}_{\mathfrak{I}}(\eth)], [\lambda(\eth), \eta(\eth), \xi(\eth)]; \eth \in \mathfrak{H}\}$ *over $\mathfrak{H}$ be null Sq-LDFS described by £ if* $\mathcal{T}_{\mathfrak{I}}(\eth) = \mathcal{I}_{\mathfrak{I}}(\eth) = 0 = \lambda = \eta$ *and* $\mathcal{F}_{\mathfrak{I}}(\eth) = 1 = \xi(\eth)$, $\forall\ \eth \in \mathfrak{H}$, *i.e,* $£ = \{(0,0,1), (0,0,1)\}$.

4. Dombi operational rules on Sq-LDF elements

Dombi approved operations include the Dombi product and Dombi sum, which are particular varieties of triangle norm and conorm described in the following description. Assume that f and g are any two real numbers. The following statement defines Dombi TN and Dombi TCN:

$$Dom(f,g) = \frac{1}{1 + \left\{\left(\frac{1-f}{f}\right)^{\partial} + \left(\frac{1-g}{g}\right)^{\partial}\right\}^{\frac{1}{\partial}}}$$

$$Dom(f,g)^{\complement} = 1 - \frac{1}{1 + \left\{\left(\frac{f}{1-f}\right)^{\partial} + \left(\frac{g}{1-g}\right)^{\partial}\right\}^{\frac{1}{\partial}}}$$

where $\partial \geq 1$ and $(f,g) \in [0,1] \times [0,1]$.

Definition 11. *Let* $\mho_1 = \{x, \langle \mathcal{T}_{d_1}, \mathcal{I}_{d_1}, \mathcal{F}_{d_1} \rangle, \langle \lambda_{d_1}, \eta_{d_1}, \xi_{d_1} \rangle\}$,

$\mho_2 = \{x, \langle \mathcal{T}_{d_2}, \mathcal{I}_{d_2}, \mathcal{F}_{d_2} \rangle, \langle \lambda_{d_2}, \eta_{d_2}, \xi_{d_2} \rangle\}$ *be two set of* $\left(\acute{S}q - LDFS\right)$ *over* $\aleph$ *and* $\aleph > 0,\ q \geq 1$, *then*

(1) $\mho_1^c = \{\langle \mathcal{F}_{d_1}, \mathcal{I}_{d_1}, \mathcal{T}_{d_1} \rangle, \langle \xi_{d_1}, \eta_{d_1}, \lambda_{d_1} \rangle\}$

(2) $\mho_1 = \mho_2 \Leftrightarrow \mathcal{T}_{d_1} = \eth_{d_2}, \mathcal{I}_{d_1} = \mathcal{I}_{d_2}, \mathcal{F}_{d_2} = \mathcal{F}_{d_2}, \lambda_{d_1} = \lambda_{d_2}, \eta_{d_1} = \eta_{d_2}, \xi_{d_1} = \xi_{d_2}$

(3) $\mho_1 \subseteq \mho_2 \Leftrightarrow \mathcal{T}_{d_1} \leq \eth_{d_2}, \mathcal{I}_{d_1} \geq \mathcal{I}_{d_2}, \mathcal{F}_{d_1} \geq \mathcal{F}_{d_2}, \lambda_{d_1} \leq \lambda_{d_2}, \eta_{d_1} \geq \eta_{d_2}, \xi_{d_1} \geq \xi_{d_2}$

(4) $$\mho_1 \cup \mho_2 = \left\{ \begin{array}{c} \sup(\mathcal{T}_{d_1}, \mathcal{T}_{d_2}), \inf(\mathcal{I}_{d_1}, \mathcal{I}_{d_2}), \inf(\mathcal{F}_{d_1}, \mathcal{F}_{d_2}), \\ \sup(\lambda_{d_1}, \lambda_{d_2}), \inf(\eta_{d_1}, \eta_{d_2}), \inf(\xi_{d_1}, \xi_{d_2}) \end{array} \right\}$$

(5) $$\mho_1 \cap \mho_2 = \left\{ \begin{array}{c} \inf(\mathcal{T}_{d_1}, \mathcal{T}_{d_2}), \inf(\mathcal{I}_{d_1}, \mathcal{I}_{d_2}), \sup(\mathcal{F}_{d_1}, \mathcal{F}_{d_2}), \\ \inf(\lambda_{d_1}, \lambda_{d_2}), \inf(\eta_{d_1}, \eta_{d_2}), \sup(\xi_{d_1}, \xi_{d_2}) \end{array} \right\}$$

(6) $$\mho_1 \oplus \mho_2 = \left[\begin{array}{c} \left(\begin{array}{c} \sqrt[q]{1 - \frac{1}{1+\left[\left(\frac{\mathcal{T}^q_{d_1}}{1-\mathcal{T}^q_{d_1}}\right)^{\hbar} + \left(\frac{\mathcal{T}^q_{d_2}}{1-\mathcal{T}^q_{d_2}}\right)^{\hbar}\right]^{\frac{1}{\hbar}}}}, \frac{1}{\sqrt[q]{1+\left[\left(\frac{1-\mathcal{I}^q_{d_1}}{\mathcal{I}^q_{d_1}}\right)^{\hbar} + \left(\frac{1-\mathcal{I}^q_{d_2}}{\mathcal{I}^q_{d_2}}\right)^{\hbar}\right]^{\frac{1}{\hbar}}}}, \\ \frac{1}{\sqrt[q]{1+\left[\left(\frac{1-\mathcal{F}^q_{d_1}}{\mathcal{F}^q_{d_1}}\right)^{\hbar} + \left(\frac{1-\mathcal{F}^q_{d_2}}{\mathcal{F}^q_{d_2}}\right)^{\hbar}\right]^{\frac{1}{\hbar}}}} \end{array} \right), \\ \left(\begin{array}{c} \sqrt[q]{1 - \frac{1}{1+\left[\left(\frac{\lambda^q_{d_1}}{1-\lambda^q_{d_1}}\right)^{\hbar} + \left(\frac{\lambda^q_{d_2}}{1-\lambda^q_{d_2}}\right)^{\hbar}\right]^{\frac{1}{\hbar}}}}, \frac{1}{\sqrt[q]{1+\left[\left(\frac{1-\eta^q_{d_1}}{\eta^q_{d_1}}\right)^{\hbar} + \left(\frac{1-\eta^q_{d_2}}{\eta^q_{d_2}}\right)^{\hbar}\right]^{\frac{1}{\hbar}}}}, \\ \frac{1}{\sqrt[q]{1+\left[\left(\frac{1-\xi^q_{d_1}}{\xi^q_{d_1}}\right)^{\hbar} + \left(\frac{1-\xi^q_{d_2}}{\xi^q_{d_2}}\right)^{\hbar}\right]^{\frac{1}{\hbar}}}} \end{array} \right) \end{array} \right]$$

(7) $$\mho_1 \otimes \mho_2 = \left[\begin{array}{c} \left(\begin{array}{c} \frac{1}{\sqrt[q]{1+\left[\left(\frac{1-\mathcal{T}^q_{d_1}}{\mathcal{T}^q_{d_1}}\right)^{\hbar} + \left(\frac{1-\mathcal{T}^q_{d_2}}{\mathcal{T}^q_{d_2}}\right)^{\hbar}\right]^{\frac{1}{\hbar}}}}, \frac{1}{\sqrt[q]{1+\left[\left(\frac{1-\mathcal{I}^q_{d_1}}{\mathcal{I}^q_{d_1}}\right)^{\hbar} + \left(\frac{1-\mathcal{I}^q_{d_2}}{\mathcal{I}^q_{d_2}}\right)^{\hbar}\right]^{\frac{1}{\hbar}}}}, \\ \sqrt[q]{1 - \frac{1}{1+\left[\left(\frac{\mathcal{F}^q_{d_1}}{1-\mathcal{F}^q_{d_1}}\right)^{\hbar} + \left(\frac{\mathcal{F}^q_{d_2}}{1-\mathcal{F}^q_{d_2}}\right)^{\hbar}\right]^{\frac{1}{\hbar}}}} \end{array} \right), \\ \left(\begin{array}{c} \frac{1}{\sqrt[q]{1+\left[\left(\frac{1-\lambda^q_{d_1}}{\lambda^q_{d_1}}\right)^{\hbar} + \left(\frac{1-\lambda^q_{d_2}}{\lambda^q_{d_2}}\right)^{\hbar}\right]^{\frac{1}{\hbar}}}}, \frac{1}{\sqrt[q]{1+\left[\left(\frac{1-\eta^q_{d_1}}{\eta^q_{d_1}}\right)^{\hbar} + \left(\frac{1-\eta^q_{d_2}}{\eta^q_{d_2}}\right)^{\hbar}\right]^{\frac{1}{\hbar}}}}, \\ \sqrt[q]{1 - \frac{1}{1+\left[\left(\frac{\xi^q_{d_1}}{1-\xi^q_{d_1}}\right)^{\hbar} + \left(\frac{\xi^q_{d_2}}{1-\xi^q_{d_2}}\right)^{\hbar}\right]^{\frac{1}{\hbar}}}} \end{array} \right) \end{array} \right]$$

(8) $$\aleph\mho_1 = \left[\begin{array}{c} \left(\sqrt[q]{1 - \frac{1}{1+\left[\aleph\left(\frac{\mathcal{T}^q_{d_1}}{1-\mathcal{T}^q_{d_1}}\right)^{\hbar}\right]^{\frac{1}{\hbar}}}}, \frac{1}{\sqrt[q]{1+\left[\aleph\left(\frac{1-\mathcal{I}^q_{d_1}}{\mathcal{I}^q_{d_1}}\right)^{\hbar}\right]^{\frac{1}{\hbar}}}}, \frac{1}{\sqrt[q]{1+\left[\aleph\left(\frac{1-\mathcal{F}^q_{d_1}}{\mathcal{F}^q_{d_1}}\right)^{\hbar}\right]^{\frac{1}{\hbar}}}} \right), \\ \left(\sqrt[q]{1 - \frac{1}{1+\left[\aleph\left(\frac{\lambda^q_{d_1}}{1-\lambda^q_{d_1}}\right)^{\hbar}\right]^{\frac{1}{\hbar}}}}, \frac{1}{\sqrt[q]{1+\left[\aleph\left(\frac{1-\eta^q_{d_1}}{\eta^q_{d_1}}\right)^{\hbar}\right]^{\frac{1}{\hbar}}}}, \frac{1}{\sqrt[q]{1+\left[\aleph\left(\frac{1-\xi^q_{d_1}}{\xi^q_{d_1}}\right)^{\hbar}\right]^{\frac{1}{\hbar}}}} \right) \end{array} \right]$$

$$
(9)\ \mho_1^{\aleph} = \left[\begin{array}{c}\left(\dfrac{1}{\sqrt[q]{1+\left[\aleph\left(\frac{1-\mathcal{T}^q_{d_1}}{\mathcal{T}^q_{d_1}}\right)^{\hbar}\right]^{\frac{1}{\hbar}}}}, \dfrac{1}{\sqrt[q]{1+\left[\aleph\left(\frac{1-\mathcal{I}^q_{d_1}}{\mathcal{I}^q_{d_1}}\right)^{\hbar}\right]^{\frac{1}{\hbar}}}}, \sqrt[q]{1-\dfrac{1}{1+\left[\aleph\left(\frac{\mathcal{F}^q_{d_1}}{1-\mathcal{F}^q_{d_1}}\right)^{\hbar}\right]^{\frac{1}{\hbar}}}}\right), \\ \left(\dfrac{1}{\sqrt[q]{1+\left[\aleph\left(\frac{1-\lambda^q_{d_1}}{\lambda^q_{d_1}}\right)^{\hbar}\right]^{\frac{1}{\hbar}}}}, \dfrac{1}{\sqrt[q]{1+\left[\aleph\left(\frac{1-\eta^q_{d_1}}{\eta^q_{d_1}}\right)^{\hbar}\right]^{\frac{1}{\hbar}}}}, \sqrt[q]{1-\dfrac{1}{1+\left[\aleph\left(\frac{\xi^q_{d_1}}{1-\xi^q_{d_1}}\right)^{\hbar}\right]^{\frac{1}{\hbar}}}}\right)\end{array}\right]
$$

Proposition 1. *Let $\mho_1, \mho_2$ and $\mho_3$ be three Sq-LDFNs. Then the following properties are held:*

(1) $\mho_1 \cup \mho_2 = \mho_2 \cup \mho_1$

(2) $\mho_1 \cap \mho_2 = \mho_2 \cap \mho_1$

(3) $\mho_1 \cup (\mho_2 \cap \mho_3) = (\mho_1 \cup \mho_2) \cap (\mho_1 \cup \mho_3)$

(4) $\mho_1 \cap (\mho_2 \cup \mho_3) = (\mho_1 \cap \mho_2) \cup (\mho_1 \cap \mho_3)$

(5) $(\mho_1 \cap \mho_2)^c = \mho_1^c \cup \mho_2^c$

(6) $(\mho_1 \cap \mho_2)^c = \mho_1^c \cup \mho_2^c$

(7) $\mho_1 \oplus \mho_2 = \mho_2 \oplus \mho_1$

(8) $\mho_1 \otimes \mho_2 = \mho_2 \otimes \mho_1$

(9) $\aleph(\mho_1 \oplus \mho_2) = \aleph\mho_1 \oplus \aleph\mho_2$

(10) $(\mho_1 \otimes \mho_2)^{\aleph} = \mho_1^{\aleph} \otimes \mho_2^{\aleph}$.

Proof. Equalities (1)–(6) are obvious. Now we justify equality (7).
(**7**) $\mho_1 \oplus \mho_2 = \mho_2 \oplus \mho_1$

By taking the left-hand side of the above equality, we prove the right-hand side of equality.

$$
\begin{aligned}
&\mho_1 \oplus \mho_2 \\
&= \left[\begin{array}{c}\left(\begin{array}{c}\sqrt[q]{1-\dfrac{1}{1+\left[\left(\frac{\mathcal{T}^q_{d_1}}{1-\mathcal{T}^q_{d_1}}\right)^{\hbar}+\left(\frac{\mathcal{T}^q_{d_2}}{1-\mathcal{T}^q_{d_2}}\right)^{\hbar}\right]^{\frac{1}{\hbar}}}}, \dfrac{1}{\sqrt[q]{1+\left[\left(\frac{1-\mathcal{I}^q_{d_1}}{\mathcal{I}^q_{d_1}}\right)^{\hbar}+\left(\frac{1-\mathcal{I}^q_{d_2}}{\mathcal{I}^q_{d_2}}\right)^{\hbar}\right]^{\frac{1}{\hbar}}}}, \\ \dfrac{1}{\sqrt[q]{1+\left[\left(\frac{1-\mathcal{F}^q_{d_1}}{\mathcal{F}^q_{d_1}}\right)^{\hbar}+\left(\frac{1-\mathcal{F}^q_{d_2}}{\mathcal{F}^q_{d_2}}\right)^{\hbar}\right]^{\frac{1}{\hbar}}}}\end{array}\right), \\ \left(\begin{array}{c}\sqrt[q]{1-\dfrac{1}{1+\left[\left(\frac{\lambda^q_{d_1}}{1-\lambda^q_{d_1}}\right)^{\hbar}+\left(\frac{\lambda^q_{d_2}}{1-\lambda^q_{d_2}}\right)^{\hbar}\right]^{\frac{1}{\hbar}}}}, \dfrac{1}{\sqrt[q]{1+\left[\left(\frac{1-\eta^q_{d_1}}{\eta^q_{d_1}}\right)^{\hbar}+\left(\frac{1-\eta^q_{d_2}}{\eta^q_{d_2}}\right)^{\hbar}\right]^{\frac{1}{\hbar}}}}, \\ \dfrac{1}{\sqrt[q]{1+\left[\left(\frac{1-\xi^q_{d_1}}{\xi^q_{d_1}}\right)^{\hbar}+\left(\frac{1-\xi^q_{d_2}}{\xi^q_{d_2}}\right)^{\hbar}\right]^{\frac{1}{\hbar}}}}\end{array}\right)\end{array}\right]
\end{aligned}
$$

$$
= \left[\begin{array}{c}\left(\begin{array}{c}\sqrt[q]{1-\frac{1}{1+\left[\left(\frac{\mathcal{T}^{q}_{\mathrm{d}_2}}{1-\mathcal{T}^{q}_{\mathrm{d}_2}}\right)^{\hbar}+\left(\frac{\mathcal{T}^{q}_{\mathrm{d}_1}}{1-\mathcal{T}^{q}_{\mathrm{d}_1}}\right)^{\hbar}\right]^{\frac{1}{\hbar}}}}, \frac{1}{\sqrt[q]{1+\left[\left(\frac{1-\mathcal{I}^{q}_{\mathrm{d}_2}}{\mathcal{I}^{q}_{\mathrm{d}_2}}\right)^{\hbar}+\left(\frac{1-\mathcal{I}^{q}_{\mathrm{d}_1}}{\mathcal{I}^{q}_{\mathrm{d}_1}}\right)^{\hbar}\right]^{\frac{1}{\hbar}}}},\\ \frac{1}{\sqrt[q]{1+\left[\left(\frac{1-\mathcal{F}^{q}_{\mathrm{d}_2}}{\mathcal{F}^{q}_{\mathrm{d}_2}}\right)^{\hbar}+\left(\frac{1-\mathcal{F}^{q}_{\mathrm{d}_1}}{\mathcal{F}^{q}_{\mathrm{d}_1}}\right)^{\hbar}\right]^{\frac{1}{\hbar}}}}\end{array}\right),\\ \left(\begin{array}{c}\sqrt[q]{1-\frac{1}{1+\left[\left(\frac{\lambda^{q}_{\mathrm{d}_2}}{1-\lambda^{q}_{\mathrm{d}_2}}\right)^{\hbar}+\left(\frac{\lambda^{q}_{\mathrm{d}_1}}{1-\lambda^{q}_{\mathrm{d}_1}}\right)^{\hbar}\right]^{\frac{1}{\hbar}}}}, \frac{1}{\sqrt[q]{1+\left[\left(\frac{1-\eta^{q}_{\mathrm{d}_2}}{\eta^{q}_{\mathrm{d}_2}}\right)^{\hbar}+\left(\frac{1-\eta^{q}_{\mathrm{d}_1}}{\eta^{q}_{\mathrm{d}_1}}\right)^{\hbar}\right]^{\frac{1}{\hbar}}}},\\ \frac{1}{\sqrt[q]{1+\left[\left(\frac{1-\xi^{q}_{\mathrm{d}_2}}{\xi^{q}_{\mathrm{d}_2}}\right)^{\hbar}+\left(\frac{1-\xi^{q}_{\mathrm{d}_1}}{\xi^{q}_{\mathrm{d}_1}}\right)^{\hbar}\right]^{\frac{1}{\hbar}}}}\end{array}\right)\end{array}\right]
$$

$$
= \mho_2 \oplus \mho_1.
$$

Hence, it proved.

(**8**) To prove equality (8), we use the left side to prove the right side of equality.

$$
\mho_1 \otimes \mho_2 = \mho_2 \otimes \mho_1
$$

$$
\mho_1 \otimes \mho_2
$$

$$
= \left[\begin{array}{c}\left(\begin{array}{c}\frac{1}{\sqrt[q]{1+\left[\left(\frac{1-\mathcal{T}^{q}_{\mathrm{d}_1}}{\mathcal{T}^{q}_{\mathrm{d}_1}}\right)^{\hbar}+\left(\frac{1-\mathcal{T}^{q}_{\mathrm{d}_2}}{\mathcal{T}^{q}_{\mathrm{d}_2}}\right)^{\hbar}\right]^{\frac{1}{\hbar}}}}, \frac{1}{\sqrt[q]{1+\left[\left(\frac{1-\mathcal{I}^{q}_{\mathrm{d}_1}}{\mathcal{I}^{q}_{\mathrm{d}_1}}\right)^{\hbar}+\left(\frac{1-\mathcal{I}^{q}_{\mathrm{d}_2}}{\mathcal{I}^{q}_{\mathrm{d}_2}}\right)^{\hbar}\right]^{\frac{1}{\hbar}}}},\\ \sqrt[q]{1-\frac{1}{1+\left[\left(\frac{\mathcal{F}^{q}_{\mathrm{d}_1}}{1-\mathcal{F}^{q}_{\mathrm{d}_1}}\right)^{\hbar}+\left(\frac{\mathcal{F}^{q}_{\mathrm{d}_2}}{1-\mathcal{F}^{q}_{\mathrm{d}_2}}\right)^{\hbar}\right]^{\frac{1}{\hbar}}}}\end{array}\right),\\ \left(\begin{array}{c}\frac{1}{\sqrt[q]{1+\left[\left(\frac{1-\lambda^{q}_{\mathrm{d}_1}}{\lambda^{q}_{\mathrm{d}_1}}\right)^{\hbar}+\left(\frac{1-\lambda^{q}_{\mathrm{d}_2}}{\lambda^{q}_{\mathrm{d}_2}}\right)^{\hbar}\right]^{\frac{1}{\hbar}}}}, \frac{1}{\sqrt[q]{1+\left[\left(\frac{1-\eta^{q}_{\mathrm{d}_1}}{\eta^{q}_{\mathrm{d}_1}}\right)^{\hbar}+\left(\frac{1-\eta^{q}_{\mathrm{d}_2}}{\eta^{q}_{\mathrm{d}_2}}\right)^{\hbar}\right]^{\frac{1}{\hbar}}}},\\ \sqrt[q]{1-\frac{1}{1+\left[\left(\frac{\xi^{q}_{\mathrm{d}_1}}{1-\xi^{q}_{\mathrm{d}_1}}\right)^{\hbar}+\left(\frac{\xi^{q}_{\mathrm{d}_2}}{1-\xi^{q}_{\mathrm{d}_2}}\right)^{\hbar}\right]^{\frac{1}{\hbar}}}}\end{array}\right)\end{array}\right]
$$

$$
= \left[\begin{array}{c}\left(\begin{array}{c}\frac{1}{\sqrt[q]{1+\left[\left(\frac{1-\mathcal{T}^{q}_{\mathrm{d}_2}}{\mathcal{T}^{q}_{\mathrm{d}_2}}\right)^{\hbar}+\left(\frac{1-\mathcal{T}^{q}_{\mathrm{d}_1}}{\mathcal{T}^{q}_{\mathrm{d}_1}}\right)^{\hbar}\right]^{\frac{1}{\hbar}}}}, \frac{1}{\sqrt[q]{1+\left[\left(\frac{1-\mathcal{I}^{q}_{\mathrm{d}_2}}{\mathcal{I}^{q}_{\mathrm{d}_2}}\right)^{\hbar}+\left(\frac{1-\mathcal{I}^{q}_{\mathrm{d}_1}}{\mathcal{I}^{q}_{\mathrm{d}_1}}\right)^{\hbar}\right]^{\frac{1}{\hbar}}}},\\ \sqrt[q]{1-\frac{1}{1+\left[\left(\frac{\mathcal{F}^{q}_{\mathrm{d}_2}}{1-\mathcal{F}^{q}_{\mathrm{d}_2}}\right)^{\hbar}+\left(\frac{\mathcal{F}^{q}_{\mathrm{d}_1}}{1-\mathcal{F}^{q}_{\mathrm{d}_1}}\right)^{\hbar}\right]^{\frac{1}{\hbar}}}}\end{array}\right),\\ \left(\begin{array}{c}\frac{1}{\sqrt[q]{1+\left[\left(\frac{1-\lambda^{q}_{\mathrm{d}_2}}{\lambda^{q}_{\mathrm{d}_2}}\right)^{\hbar}+\left(\frac{1-\lambda^{q}_{\mathrm{d}_1}}{\lambda^{q}_{\mathrm{d}_1}}\right)^{\hbar}\right]^{\frac{1}{\hbar}}}}, \frac{1}{\sqrt[q]{1+\left[\left(\frac{1-\eta^{q}_{\mathrm{d}_2}}{\eta^{q}_{\mathrm{d}_2}}\right)^{\hbar}+\left(\frac{1-\eta^{q}_{\mathrm{d}_1}}{\eta^{q}_{\mathrm{d}_1}}\right)^{\hbar}\right]^{\frac{1}{\hbar}}}},\\ \sqrt[q]{1-\frac{1}{1+\left[\left(\frac{\xi^{q}_{\mathrm{d}_2}}{1-\xi^{q}_{\mathrm{d}_2}}\right)^{\hbar}+\left(\frac{\xi^{q}_{\mathrm{d}_1}}{1-\xi^{q}_{\mathrm{d}_1}}\right)^{\hbar}\right]^{\frac{1}{\hbar}}}}\end{array}\right)\end{array}\right]
$$

$$= \mho_2 \otimes \mho_1.$$

(**9**) By integrating points (8) and (6) of Definition 11, we have

$$\aleph\left(\mho_1 \oplus \mho_2\right) = \aleph\mho_1 \oplus \aleph\mho_2$$

$$\aleph\left(\mho_1 \oplus \mho_2\right)$$

$$= \left[\begin{array}{c} \left(\begin{array}{c} \sqrt{1 - \frac{1}{1+\aleph\left[\left(\frac{\mathcal{T}^q_{\mathrm{d}_1}}{1-\mathcal{T}^q_{\mathrm{d}_1}}\right)^{\hbar}+\left(\frac{\mathcal{T}^q_{\mathrm{d}_2}}{1-\mathcal{T}^q_{\mathrm{d}_2}}\right)^{\hbar}\right]^{\frac{1}{\hbar}}}}, \frac{1}{\sqrt[q]{1+\aleph\left[\left(\frac{1-\mathcal{I}^q_{\mathrm{d}_1}}{\mathcal{I}^q_{\mathrm{d}_1}}\right)^{\hbar}+\left(\frac{1-\mathcal{I}^q_{\mathrm{d}_2}}{\mathcal{I}^q_{\mathrm{d}_2}}\right)^{\hbar}\right]^{\frac{1}{\hbar}}}}, \\ \frac{1}{\sqrt[q]{1+\aleph\left[\left(\frac{1-\mathcal{F}^q_{\mathrm{d}_1}}{\mathcal{F}^q_{\mathrm{d}_1}}\right)^{\hbar}+\left(\frac{1-\mathcal{F}^q_{\mathrm{d}_2}}{\mathcal{F}^q_{\mathrm{d}_2}}\right)^{\hbar}\right]^{\frac{1}{\hbar}}}} \end{array}\right), \\ \left(\begin{array}{c} \sqrt[q]{1 - \frac{1}{1+\aleph\left[\left(\frac{\lambda^q_{\mathrm{d}_1}}{1-\lambda^q_{\mathrm{d}_1}}\right)^{\hbar}+\left(\frac{\lambda^q_{\mathrm{d}_2}}{1-\lambda^q_{\mathrm{d}_2}}\right)^{\hbar}\right]^{\frac{1}{\hbar}}}}, \frac{1}{\sqrt[q]{1+\aleph\left[\left(\frac{1-\eta^q_{\mathrm{d}_1}}{\eta^q_{\mathrm{d}_1}}\right)^{\hbar}+\left(\frac{1-\eta^q_{\mathrm{d}_2}}{\eta^q_{\mathrm{d}_2}}\right)^{\hbar}\right]^{\frac{1}{\hbar}}}}, \\ \frac{1}{\sqrt[q]{1+\aleph\left[\left(\frac{1-\xi^q_{\mathrm{d}_1}}{\xi^q_{\mathrm{d}_1}}\right)^{\hbar}+\left(\frac{1-\xi^q_{\mathrm{d}_2}}{\xi^q_{\mathrm{d}_2}}\right)^{\hbar}\right]^{\frac{1}{\hbar}}}} \end{array}\right) \end{array}\right]$$

we may retrieve it by utilizing the equation's right side. Let

$$\aleph\mho_1 = \left[\begin{array}{c} \left(\sqrt[q]{1 - \frac{1}{1+\left[\aleph\left(\frac{\mathcal{T}^q_{\mathrm{d}_1}}{1-\mathcal{T}^q_{\mathrm{d}_1}}\right)^{\hbar}\right]^{\frac{1}{\hbar}}}}, \frac{1}{\sqrt[q]{1+\left[\aleph\left(\frac{1-\mathcal{I}^q_{\mathrm{d}_1}}{\mathcal{I}^q_{\mathrm{d}_1}}\right)^{\hbar}\right]^{\frac{1}{\hbar}}}}, \frac{1}{\sqrt[q]{1+\left[\aleph\left(\frac{1-\mathcal{F}^q_{\mathrm{d}_1}}{\mathcal{F}^q_{\mathrm{d}_1}}\right)^{\hbar}\right]^{\frac{1}{\hbar}}}}\right), \\ \left(\sqrt[q]{1 - \frac{1}{1+\left[\aleph\left(\frac{\lambda^q_{\mathrm{d}_1}}{1-\lambda^q_{\mathrm{d}_1}}\right)^{\hbar}\right]^{\frac{1}{\hbar}}}}, \frac{1}{\sqrt[q]{1+\left[\aleph\left(\frac{1-\eta^q_{\mathrm{d}_1}}{\eta^q_{\mathrm{d}_1}}\right)^{\hbar}\right]^{\frac{1}{\hbar}}}}, \frac{1}{\sqrt[q]{1+\left[\aleph\left(\frac{1-\xi^q_{\mathrm{d}_1}}{\xi^q_{\mathrm{d}_1}}\right)^{\hbar}\right]^{\frac{1}{\hbar}}}}\right) \end{array}\right]$$

$$\aleph\mho_2 = \left[\begin{array}{c} \left(\sqrt[q]{1 - \frac{1}{1+\left[\aleph\left(\frac{\mathcal{T}^q_{\mathrm{d}_2}}{1-\mathcal{T}^q_{\mathrm{d}_2}}\right)^{\hbar}\right]^{\frac{1}{\hbar}}}}, \frac{1}{\sqrt[q]{1+\left[\aleph\left(\frac{1-\mathcal{I}^q_{\mathrm{d}_2}}{\mathcal{I}^q_{\mathrm{d}_2}}\right)^{\hbar}\right]^{\frac{1}{\hbar}}}}, \frac{1}{\sqrt[q]{1+\left[\aleph\left(\frac{1-\mathcal{F}^q_{\mathrm{d}_2}}{\mathcal{F}^q_{\mathrm{d}_2}}\right)^{\hbar}\right]^{\frac{1}{\hbar}}}}\right), \\ \left(\sqrt[q]{1 - \frac{1}{1+\left[\aleph\left(\frac{\lambda^q_{\mathrm{d}_2}}{1-\lambda^q_{\mathrm{d}_2}}\right)^{\hbar}\right]^{\frac{1}{\hbar}}}}, \frac{1}{\sqrt[q]{1+\left[\aleph\left(\frac{1-\eta^q_{\mathrm{d}_2}}{\eta^q_{\mathrm{d}_2}}\right)^{\hbar}\right]^{\frac{1}{\hbar}}}}, \frac{1}{\sqrt[q]{1+\left[\aleph\left(\frac{1-\xi^q_{\mathrm{d}_2}}{\xi^q_{\mathrm{d}_2}}\right)^{\hbar}\right]^{\frac{1}{\hbar}}}}\right) \end{array}\right]$$

$$\aleph\mho_1 \oplus \aleph\mho_2$$

$$
= \left[\begin{array}{c} \left(\begin{array}{c} \sqrt[q]{1 - \frac{1}{1+\aleph\left[\left(\frac{\mathcal{T}^q_{\mathrm{d}_1}}{1-\mathcal{T}^q_{\mathrm{d}_1}}\right)^{\hbar} + \left(\frac{\mathcal{T}^q_{\mathrm{d}_2}}{1-\mathcal{T}^q_{\mathrm{d}_2}}\right)^{\hbar}\right]^{\frac{1}{\hbar}}}}, \frac{1}{\sqrt[q]{1+\aleph\left[\left(\frac{1-\mathcal{I}^q_{\mathrm{d}_1}}{\mathcal{I}^q_{\mathrm{d}_1}}\right)^{\hbar} + \left(\frac{1-\mathcal{I}^q_{\mathrm{d}_2}}{\mathcal{I}^q_{\mathrm{d}_2}}\right)^{\hbar}\right]^{\frac{1}{\hbar}}}}, \\ \frac{1}{\sqrt[q]{1+\aleph\left[\left(\frac{1-\mathcal{F}^q_{\mathrm{d}_1}}{\mathcal{F}^q_{\mathrm{d}_1}}\right)^{\hbar} + \left(\frac{1-\mathcal{F}^q_{\mathrm{d}_2}}{\mathcal{F}^q_{\mathrm{d}_2}}\right)^{\hbar}\right]^{\frac{1}{\hbar}}}} \end{array} \right), \\ \left(\begin{array}{c} \sqrt[q]{1 - \frac{1}{1+\aleph\left[\left(\frac{\lambda^q_{\mathrm{d}_1}}{1-\lambda^q_{\mathrm{d}_1}}\right)^{\hbar} + \left(\frac{\lambda^q_{\mathrm{d}_2}}{1-\lambda^q_{\mathrm{d}_2}}\right)^{\hbar}\right]^{\frac{1}{\hbar}}}}, \frac{1}{\sqrt[q]{1+\aleph\left[\left(\frac{1-\eta^q_{\mathrm{d}_1}}{\eta^q_{\mathrm{d}_1}}\right)^{\hbar} + \left(\frac{1-\eta^q_{\mathrm{d}_2}}{\eta^q_{\mathrm{d}_2}}\right)^{\hbar}\right]^{\frac{1}{\hbar}}}}, \\ \frac{1}{\sqrt[q]{1+\aleph\left[\left(\frac{1-\xi^q_{\mathrm{d}_1}}{\xi^q_{\mathrm{d}_1}}\right)^{\hbar} + \left(\frac{1-\xi^q_{\mathrm{d}_2}}{\xi^q_{\mathrm{d}_2}}\right)^{\hbar}\right]^{\frac{1}{\hbar}}}} \end{array} \right) \end{array} \right]
$$

(10) $(\mho_1 \otimes \mho_2)^{\aleph} = \mho_1^{\aleph} \otimes \mho_2^{\aleph}$.
By integrating points (7) and (9) of Definition 11, we have

$$
\begin{aligned}
&(\mho_1 \otimes \mho_2)^{\aleph} \\
= &\left(\left[\begin{array}{c} \left(\begin{array}{c} \frac{1}{\sqrt[q]{1+\left[\left(\frac{1-\mathcal{T}^q_{\mathrm{d}_1}}{\mathcal{T}^q_{\mathrm{d}_1}}\right)^{\hbar} + \left(\frac{1-\mathcal{T}^q_{\mathrm{d}_2}}{\mathcal{T}^q_{\mathrm{d}_2}}\right)^{\hbar}\right]^{\frac{1}{\hbar}}}}, \frac{1}{\sqrt[q]{1+\left[\left(\frac{1-\mathcal{I}^q_{\mathrm{d}_1}}{\mathcal{I}^q_{\mathrm{d}_1}}\right)^{\hbar} + \left(\frac{1-\mathcal{I}^q_{\mathrm{d}_2}}{\mathcal{I}^q_{\mathrm{d}_2}}\right)^{\hbar}\right]^{\frac{1}{\hbar}}}}, \\ \sqrt[q]{1 - \frac{1}{1+\left[\left(\frac{\mathcal{F}^q_{\mathrm{d}_1}}{1-\mathcal{F}^q_{\mathrm{d}_1}}\right)^{\hbar} + \left(\frac{\mathcal{F}^q_{\mathrm{d}_2}}{1-\mathcal{F}^q_{\mathrm{d}_2}}\right)^{\hbar}\right]^{\frac{1}{\hbar}}}} \end{array} \right), \\ \left(\begin{array}{c} \frac{1}{\sqrt[q]{1+\left[\left(\frac{1-\lambda^q_{\mathrm{d}_1}}{\lambda^q_{\mathrm{d}_1}}\right)^{\hbar} + \left(\frac{1-\lambda^q_{\mathrm{d}_2}}{\lambda^q_{\mathrm{d}_2}}\right)^{\hbar}\right]^{\frac{1}{\hbar}}}}, \frac{1}{\sqrt[q]{1+\left[\left(\frac{1-\eta^q_{\mathrm{d}_1}}{\eta^q_{\mathrm{d}_1}}\right)^{\hbar} + \left(\frac{1-\eta^q_{\mathrm{d}_2}}{\eta^q_{\mathrm{d}_2}}\right)^{\hbar}\right]^{\frac{1}{\hbar}}}}, \\ \sqrt[q]{1 - \frac{1}{1+\left[\left(\frac{\xi^q_{\mathrm{d}_1}}{1-\xi^q_{\mathrm{d}_1}}\right)^{\hbar} + \left(\frac{\xi^q_{\mathrm{d}_2}}{1-\xi^q_{\mathrm{d}_2}}\right)^{\hbar}\right]^{\frac{1}{\hbar}}}} \end{array} \right) \end{array} \right] \right)^{\aleph}
\end{aligned}
$$

$$
= \left[\begin{array}{c} \left(\begin{array}{c} \frac{1}{\sqrt[q]{1+\aleph\left[\left(\frac{1-\mathcal{T}^q_{\mathrm{d}_1}}{\mathcal{T}^q_{\mathrm{d}_1}}\right)^{\hbar} + \left(\frac{1-\mathcal{T}^q_{\mathrm{d}_2}}{\mathcal{T}^q_{\mathrm{d}_2}}\right)^{\hbar}\right]^{\frac{1}{\hbar}}}}, \frac{1}{\sqrt[q]{1+\aleph\left[\left(\frac{1-\mathcal{I}^q_{\mathrm{d}_1}}{\mathcal{I}^q_{\mathrm{d}_1}}\right)^{\hbar} + \left(\frac{1-\mathcal{I}^q_{\mathrm{d}_2}}{\mathcal{I}^q_{\mathrm{d}_2}}\right)^{\hbar}\right]^{\frac{1}{\hbar}}}}, \\ \sqrt[q]{1 - \frac{1}{1+\aleph\left[\left(\frac{\mathcal{F}^q_{\mathrm{d}_1}}{1-\mathcal{F}^q_{\mathrm{d}_1}}\right)^{\hbar} + \left(\frac{\mathcal{F}^q_{\mathrm{d}_2}}{1-\mathcal{F}^q_{\mathrm{d}_2}}\right)^{\hbar}\right]^{\frac{1}{\hbar}}}} \end{array} \right), \\ \left(\begin{array}{c} \frac{1}{\sqrt[q]{1+\aleph\left[\left(\frac{1-\lambda^q_{\mathrm{d}_1}}{\lambda^q_{\mathrm{d}_1}}\right)^{\hbar} + \left(\frac{1-\lambda^q_{\mathrm{d}_2}}{\lambda^q_{\mathrm{d}_2}}\right)^{\hbar}\right]^{\frac{1}{\hbar}}}}, \frac{1}{\sqrt[q]{1+\aleph\left[\left(\frac{1-\eta^q_{\mathrm{d}_1}}{\eta^q_{\mathrm{d}_1}}\right)^{\hbar} + \left(\frac{1-\eta^q_{\mathrm{d}_2}}{\eta^q_{\mathrm{d}_2}}\right)^{\hbar}\right]^{\frac{1}{\hbar}}}}, \\ \sqrt[q]{1 - \frac{1}{1+\aleph\left[\left(\frac{\xi^q_{\mathrm{d}_1}}{1-\xi^q_{\mathrm{d}_1}}\right)^{\hbar} + \left(\frac{\xi^q_{\mathrm{d}_2}}{1-\xi^q_{\mathrm{d}_2}}\right)^{\hbar}\right]^{\frac{1}{\hbar}}}} \end{array} \right) \end{array} \right].
$$

In addition,

$$\mho_1^{\aleph}=\left[\begin{array}{c}\left(\dfrac{1}{\sqrt[q]{1+\left[\aleph\left(\frac{1-\mathcal{T}^q_{\mathrm{d}_1}}{\mathcal{T}^q_{\mathrm{d}_1}}\right)^{\hbar}\right]^{\frac{1}{\hbar}}}},\dfrac{1}{\sqrt[q]{1+\left[\aleph\left(\frac{1-\mathcal{I}^q_{\mathrm{d}_1}}{\mathcal{I}^q_{\mathrm{d}_1}}\right)^{\hbar}\right]^{\frac{1}{\hbar}}}},\sqrt[q]{1-\dfrac{1}{1+\left[\aleph\left(\frac{\mathcal{F}^q_{\mathrm{d}_1}}{1-\mathcal{F}^q_{\mathrm{d}_1}}\right)^{\hbar}\right]^{\frac{1}{\hbar}}}}\right),\\ \left(\dfrac{1}{\sqrt[q]{1+\left[\aleph\left(\frac{1-\lambda^q_{\mathrm{d}_1}}{\lambda^q_{\mathrm{d}_1}}\right)^{\hbar}\right]^{\frac{1}{\hbar}}}},\dfrac{1}{\sqrt[q]{1+\left[\aleph\left(\frac{1-\eta^q_{\mathrm{d}_1}}{\eta^q_{\mathrm{d}_1}}\right)^{\hbar}\right]^{\frac{1}{\hbar}}}},\sqrt[q]{1-\dfrac{1}{1+\left[\aleph\left(\frac{\xi^q_{\mathrm{d}_1}}{1-\xi^q_{\mathrm{d}_1}}\right)^{\hbar}\right]^{\frac{1}{\hbar}}}}\right)\end{array}\right],$$

$$\mho_2^{\aleph}=\left[\begin{array}{c}\left(\dfrac{1}{\sqrt[q]{1+\left[\aleph\left(\frac{1-\mathcal{T}^q_{\mathrm{d}_2}}{\mathcal{T}^q_{\mathrm{d}_2}}\right)^{\hbar}\right]^{\frac{1}{\hbar}}}},\dfrac{1}{\sqrt[q]{1+\left[\aleph\left(\frac{1-\mathcal{I}^q_{\mathrm{d}_2}}{\mathcal{I}^q_{\mathrm{d}_2}}\right)^{\hbar}\right]^{\frac{1}{\hbar}}}},\sqrt[q]{1-\dfrac{1}{1+\left[\aleph\left(\frac{\mathcal{F}^q_{\mathrm{d}_2}}{1-\mathcal{F}^q_{\mathrm{d}_2}}\right)^{\hbar}\right]^{\frac{1}{\hbar}}}}\right),\\ \left(\dfrac{1}{\sqrt[q]{1+\left[\aleph\left(\frac{1-\lambda^q_{\mathrm{d}_2}}{\lambda^q_{\mathrm{d}_2}}\right)^{\hbar}\right]^{\frac{1}{\hbar}}}},\dfrac{1}{\sqrt[q]{1+\left[\aleph\left(\frac{1-\eta^q_{\mathrm{d}_2}}{\eta^q_{\mathrm{d}_2}}\right)^{\hbar}\right]^{\frac{1}{\hbar}}}},\sqrt[q]{1-\dfrac{1}{1+\left[\aleph\left(\frac{\xi^q_{\mathrm{d}_2}}{1-\xi^q_{\mathrm{d}_2}}\right)^{\hbar}\right]^{\frac{1}{\hbar}}}}\right)\end{array}\right]$$

$$\begin{aligned}&\mho_1^{\aleph}\otimes\mho_2^{\aleph}\\&=\left[\begin{array}{c}\left(\begin{array}{c}\dfrac{1}{\sqrt[q]{1+\aleph\left[\left(\frac{1-\mathcal{T}^q_{\mathrm{d}_1}}{\mathcal{T}^q_{\mathrm{d}_1}}\right)^{\hbar}+\left(\frac{1-\mathcal{T}^q_{\mathrm{d}_2}}{\mathcal{T}^q_{\mathrm{d}_2}}\right)^{\hbar}\right]^{\frac{1}{\hbar}}}},\dfrac{1}{\sqrt[q]{1+\aleph\left[\left(\frac{1-\mathcal{I}^q_{\mathrm{d}_1}}{\mathcal{I}^q_{\mathrm{d}_1}}\right)^{\hbar}+\left(\frac{1-\mathcal{I}^q_{\mathrm{d}_2}}{\mathcal{I}^q_{\mathrm{d}_2}}\right)^{\hbar}\right]^{\frac{1}{\hbar}}}},\\ \sqrt[q]{1-\dfrac{1}{1+\aleph\left[\left(\frac{\mathcal{F}^q_{\mathrm{d}_1}}{1-\mathcal{F}^q_{\mathrm{d}_1}}\right)^{\hbar}+\left(\frac{\mathcal{F}^q_{\mathrm{d}_2}}{1-\mathcal{F}^q_{\mathrm{d}_2}}\right)^{\hbar}\right]^{\frac{1}{\hbar}}}}\end{array}\right),\\ \left(\begin{array}{c}\dfrac{1}{\sqrt[q]{1+\aleph\left[\left(\frac{1-\lambda^q_{\mathrm{d}_1}}{\lambda^q_{\mathrm{d}_1}}\right)^{\hbar}+\left(\frac{1-\lambda^q_{\mathrm{d}_2}}{\lambda^q_{\mathrm{d}_2}}\right)^{\hbar}\right]^{\frac{1}{\hbar}}}},\dfrac{1}{\sqrt[q]{1+\aleph\left[\left(\frac{1-\eta^q_{\mathrm{d}_1}}{\eta^q_{\mathrm{d}_1}}\right)^{\hbar}+\left(\frac{1-\eta^q_{\mathrm{d}_2}}{\eta^q_{\mathrm{d}_2}}\right)^{\hbar}\right]^{\frac{1}{\hbar}}}},\\ \sqrt[q]{1-\dfrac{1}{1+\aleph\left[\left(\frac{\xi^q_{\mathrm{d}_1}}{1-\xi^q_{\mathrm{d}_1}}\right)^{\hbar}+\left(\frac{\xi^q_{\mathrm{d}_2}}{1-\xi^q_{\mathrm{d}_2}}\right)^{\hbar}\right]^{\frac{1}{\hbar}}}}\end{array}\right)\end{array}\right]\end{aligned}$$

□

5. Spherical q-linear Diophantine aggregation operators

The main objective of this section is to examine the abstract ideas of Sq-LDFDWG and SqLDFDOWG operating rules based on the principles derived from Sq-LDFNs. In addition, this section explores the various kinds of Score Functions.

5.1. Sq-LDFDWG operators

Definition 12. *Let* $\mho_t = \{x, \langle {}^t\mathcal{T}_d, {}^t\mathcal{I}_d, {}^t\mathcal{F}_d \rangle, \langle {}^t\lambda_d, {}^t\eta_d, {}^t\xi_d \rangle : t = 1, 2, ..., \mathfrak{n}\}$ *be an assemblage of* Sq-*LDFNs over* $\aleph > 0,\ q \geq 1$. *The Sq-LDFDWG operator is a transformation Sq-LDFDWG: Sq-LDFNs*$^n(\aleph) \rightarrow$*Sq-LDFNs*$^n(\aleph)$, *defined as Sq-LDFDWG*$(\mho_1, \mho_2, ..., \mho_\mathfrak{n}) = \mho_1^{\sigma_1} \otimes \mho_2^{\sigma_2} \otimes, ..., \otimes \mho_\mathfrak{n}^{\sigma_\mathfrak{n}}$, *where* $\sigma = (\sigma_1, \sigma_2, ..., \sigma_\mathfrak{n})$ *represents the weight vector of* $\mho_t\ (t = 1, 2, ..., \mathfrak{n})$, $0 \leq \sigma_t \leq 1$ *and* $\sum_{t=1}^{\mathfrak{n}} \sigma_t = 1$.

Theorem 1. *Suppose that* $\mho_t = \{x, \langle {}^t\mathcal{T}_d, {}^t\mathcal{I}_d, {}^t\mathcal{F}_d \rangle, \langle {}^t\lambda_d, {}^t\eta_d, {}^t\xi_d \rangle : t = 1, 2, ..., \mathfrak{n}\}$ *be an assemblage of* $Sq-LDFNs$ *over* $\aleph > 0,\ q \geq 1$. *Let the weight vector of* $\mho_t$ *be* $\sigma = (\sigma_1, \sigma_2, ..., \sigma_\mathfrak{n})$, *then* Sq-$LDFDWG(\mho_1, \mho_2, ..., \mho_\mathfrak{n}) =$

$$\left[\begin{array}{c}\left(\begin{array}{c}\sqrt[q]{1-\frac{1}{1+\left[\sum_{t=1}^{\mathfrak{n}}\sigma_t\left(\frac{\mathcal{T}^q_{d_t}}{1-\mathcal{T}^q_{d_t}}\right)^{\sigma}\right]^{\frac{1}{\sigma}}}}, \sqrt[q]{1-\frac{1}{1+\left[\sum_{t=1}^{\mathfrak{n}}\sigma_t\left(\frac{\mathcal{I}^q_{d_t}}{1-\mathcal{I}^q_{d_t}}\right)^{\sigma}\right]^{\frac{1}{\sigma}}}},\\ \frac{1}{\sqrt[q]{1+\left[\sum_{t=1}^{\mathfrak{n}}\sigma_t\left(\frac{1-\mathcal{F}^q_{d_t}}{\mathcal{F}^q_{d_t}}\right)^{\sigma}\right]^{\frac{1}{\sigma}}}}\end{array}\right),\\ \left(\begin{array}{c}\sqrt[q]{1-\frac{1}{1+\left[\sum_{t=1}^{\mathfrak{n}}\sigma_t\left(\frac{\lambda^q_{d_t}}{1-\lambda^q_{d_t}}\right)^{\sigma}\right]^{\frac{1}{\sigma}}}}, \sqrt[q]{1-\frac{1}{1+\left[\sum_{t=1}^{\mathfrak{n}}\sigma_t\left(\frac{\eta^q_{d_t}}{1-\eta^q_{d_t}}\right)^{\sigma}\right]^{\frac{1}{\sigma}}}},\\ \frac{1}{\sqrt[q]{1+\left[\sum_{t=1}^{\mathfrak{n}}\sigma_t\left(\frac{1-\xi^q_{d_t}}{\xi^q_{d_t}}\right)^{\sigma}\right]^{\frac{1}{\sigma}}}}\end{array}\right)\end{array}\right]. \tag{5.1}$$

Proof. To demonstrate this theorem, we shall apply the mathematical induction approach. For $t = 2$, since

$$\mho_1^{\sigma_1} = \left[\begin{array}{c}\left(\sqrt[q]{1-\frac{1}{1+\left[\sigma_1\left(\frac{\mathcal{T}^q_{d_1}}{1-\mathcal{T}^q_{d_1}}\right)^{\hbar}\right]^{\frac{1}{\hbar}}}}, \sqrt[q]{1-\frac{1}{1+\left[\sigma_1\left(\frac{\mathcal{I}^q_{d_1}}{1-\mathcal{I}^q_{d_1}}\right)^{\hbar}\right]^{\frac{1}{\hbar}}}}, \frac{1}{\sqrt[q]{1+\left[\sigma_1\left(\frac{1-\mathcal{F}^q_{d_1}}{\mathcal{F}^q_{d_1}}\right)^{\hbar}\right]^{\frac{1}{\hbar}}}}\right),\\ \left(\sqrt[q]{1-\frac{1}{1+\left[\sigma_1\left(\frac{\lambda^q_{d_1}}{1-\lambda^q_{d_1}}\right)^{\hbar}\right]^{\frac{1}{\hbar}}}}, \sqrt[q]{1-\frac{1}{1+\left[\sigma_1\left(\frac{\eta^q_{d_1}}{1-\eta^q_{d_1}}\right)^{\hbar}\right]^{\frac{1}{\hbar}}}}, \frac{1}{\sqrt[q]{1+\left[\sigma_1\left(\frac{1-\xi^q_{d_1}}{\xi^q_{d_1}}\right)^{\hbar}\right]^{\frac{1}{\hbar}}}}\right)\end{array}\right],$$

$$\mho_2^{\sigma_2} = \left[\begin{array}{c}\left(\sqrt[q]{1-\frac{1}{1+\left[\sigma_2\left(\frac{\mathcal{T}^q_{d_2}}{1-\mathcal{T}^q_{d_2}}\right)^{\hbar}\right]^{\frac{1}{\hbar}}}}, \sqrt[q]{1-\frac{1}{1+\left[\sigma_2\left(\frac{\mathcal{I}^q_{d_2}}{1-\mathcal{I}^q_{d_2}}\right)^{\hbar}\right]^{\frac{1}{\hbar}}}}, \frac{1}{\sqrt[q]{1+\left[\sigma_2\left(\frac{1-\mathcal{F}^q_{d_2}}{\mathcal{F}^q_{d_2}}\right)^{\hbar}\right]^{\frac{1}{\hbar}}}}\right),\\ \left(\sqrt[q]{1-\frac{1}{1+\left[\sigma_2\left(\frac{\lambda^q_{d_2}}{1-\lambda^q_{d_2}}\right)^{\hbar}\right]^{\frac{1}{\hbar}}}}, \sqrt[q]{1-\frac{1}{1+\left[\sigma_2\left(\frac{I^q_{d_2}}{1-I^q_{d_2}}\right)^{\hbar}\right]^{\frac{1}{\hbar}}}}, \frac{1}{\sqrt[q]{1+\left[\sigma_2\left(\frac{1-\xi^q_{d_2}}{\xi^q_{d_2}}\right)^{\hbar}\right]^{\frac{1}{\hbar}}}}\right)\end{array}\right].$$

Then Sq-LDFDWG$(\mho_1, \mho_2)$ is

$$\mho_1^{\sigma_1} \otimes \mho_2^{\sigma_2}$$

$$= \left[\begin{array}{c} \left(\sqrt[q]{1 - \frac{1}{1+\left[\sigma_1\left(\frac{\mathcal{T}^q_{\mathrm{d}_1}}{1-\mathcal{T}^q_{\mathrm{d}_1}}\right)^{\hbar}+\sigma_2\left(\frac{\mathcal{T}^q_{\mathrm{d}_2}}{1-\mathcal{T}^q_{\mathrm{d}_2}}\right)^{\hbar}\right]^{\frac{1}{\hbar}}}}, \sqrt[q]{1 - \frac{1}{1+\left[\sigma_1\left(\frac{\mathcal{I}^q_{\mathrm{d}_1}}{1-\mathcal{I}^q_{\mathrm{d}_1}}\right)^{\hbar}+\sigma_2\left(\frac{\mathcal{I}^q_{\mathrm{d}_2}}{1-\mathcal{I}^q_{\mathrm{d}_2}}\right)^{\hbar}\right]^{\frac{1}{\hbar}}}}, \frac{1}{\sqrt[q]{1+\left[\sigma_1\left(\frac{1-\mathcal{F}^q_{\mathrm{d}_1}}{\mathcal{F}^q_{\mathrm{d}_1}}\right)^{\hbar}+\sigma_2\left(\frac{1-\mathcal{F}^q_{\mathrm{d}_2}}{\mathcal{F}^q_{\mathrm{d}_2}}\right)^{\hbar}\right]^{\frac{1}{\hbar}}}} \right), \\ \left(\sqrt[q]{1 - \frac{1}{1+\left[\sigma_1\left(\frac{\lambda^q_{\mathrm{d}_1}}{1-\lambda^q_{\mathrm{d}_1}}\right)^{\hbar}+\sigma_2\left(\frac{\lambda^q_{\mathrm{d}_2}}{1-\lambda^q_{\mathrm{d}_2}}\right)^{\hbar}\right]^{\frac{1}{\hbar}}}}, \sqrt[q]{1 - \frac{1}{1+\left[\sigma_1\left(\frac{\eta^q_{\mathrm{d}_1}}{1-\eta^q_{\mathrm{d}_1}}\right)^{\hbar}+\sigma_2\left(\frac{\eta^q_{\mathrm{d}_2}}{1-\eta^q_{\mathrm{d}_2}}\right)^{\hbar}\right]^{\frac{1}{\hbar}}}}, \frac{1}{\sqrt[q]{1+\left[\sigma_1\left(\frac{1-\xi^q_{\mathrm{d}_1}}{\xi^q_{\mathrm{d}_1}}\right)^{\hbar}+\sigma_2\left(\frac{1-\xi^q_{\mathrm{d}_2}}{\xi^q_{\mathrm{d}_2}}\right)^{\hbar}\right]^{\frac{1}{\hbar}}}} \right) \end{array} \right]$$

$$= \left[\begin{array}{c} \left(\sqrt[q]{1 - \frac{1}{1+\left[\sum_{t=1}^{2}\sigma_t\left(\frac{\mathcal{T}^q_{\mathrm{d}_t}}{1-\mathcal{T}^q_{\mathrm{d}_t}}\right)^{\sigma}\right]^{\frac{1}{\sigma}}}}, \sqrt[q]{1 - \frac{1}{1+\left[\sum_{t=1}^{2}\sigma_t\left(\frac{\mathcal{I}^q_{\mathrm{d}_t}}{1-\mathcal{I}^q_{\mathrm{d}_t}}\right)^{\sigma}\right]^{\frac{1}{\sigma}}}}, \frac{1}{\sqrt[q]{1+\left[\sum_{t=1}^{2}\sigma_t\left(\frac{1-\mathcal{F}^q_{\mathrm{d}_t}}{\mathcal{F}^q_{\mathrm{d}_t}}\right)^{\sigma}\right]^{\frac{1}{\sigma}}}} \right), \\ \left(\sqrt[q]{1 - \frac{1}{1+\left[\sum_{t=1}^{2}\sigma_t\left(\frac{\lambda^q_{\mathrm{d}_t}}{1-\lambda^q_{\mathrm{d}_t}}\right)^{\sigma}\right]^{\frac{1}{\sigma}}}}, \sqrt[q]{1 - \frac{1}{1+\left[\sum_{t=1}^{2}\sigma_t\left(\frac{\eta^q_{\mathrm{d}_t}}{1-\eta^q_{\mathrm{d}_t}}\right)^{\sigma}\right]^{\frac{1}{\sigma}}}}, \frac{1}{\sqrt[q]{1+\left[\sum_{t=1}^{2}\sigma_t\left(\frac{1-\xi^q_{\mathrm{d}_t}}{\xi^q_{\mathrm{d}_t}}\right)^{\sigma}\right]^{\frac{1}{\sigma}}}} \right) \end{array} \right].$$

Obviously, Theorem 1 holds for $t = 2$. Suppose that Theorem 1 is hold for $t = k$, then Sq-$LDFDWG(\mho_1, \mho_2, ..., \mho_t) =$

$$\left[\begin{array}{c} \left(\sqrt[q]{1 - \frac{1}{1+\left[\sum_{t=1}^{k}\sigma_t\left(\frac{\mathcal{T}^q_{\mathrm{d}_t}}{1-\mathcal{T}^q_{\mathrm{d}_t}}\right)^{\sigma}\right]^{\frac{1}{\sigma}}}}, \sqrt[q]{1 - \frac{1}{1+\left[\sum_{t=1}^{k}\sigma_t\left(\frac{\mathcal{I}^q_{\mathrm{d}_t}}{1-\mathcal{I}^q_{\mathrm{d}_t}}\right)^{\sigma}\right]^{\frac{1}{\sigma}}}}, \frac{1}{\sqrt[q]{1+\left[\sum_{t=1}^{k}\sigma_t\left(\frac{1-\mathcal{F}^q_{\mathrm{d}_t}}{\mathcal{F}^q_{\mathrm{d}_t}}\right)^{\sigma}\right]^{\frac{1}{\sigma}}}} \right), \\ \left(\sqrt[q]{1 - \frac{1}{1+\left[\sum_{t=1}^{k}\sigma_t\left(\frac{\lambda^q_{\mathrm{d}_t}}{1-\lambda^q_{\mathrm{d}_t}}\right)^{\sigma}\right]^{\frac{1}{\sigma}}}}, \sqrt[q]{1 - \frac{1}{1+\left[\sum_{t=1}^{k}\sigma_t\left(\frac{\eta^q_{\mathrm{d}_t}}{1-\eta^q_{\mathrm{d}_t}}\right)^{\sigma}\right]^{\frac{1}{\sigma}}}}, \frac{1}{\sqrt[q]{1+\left[\sum_{t=1}^{k}\sigma_t\left(\frac{1-\xi^q_{\mathrm{d}_t}}{\xi^q_{\mathrm{d}_t}}\right)^{\sigma}\right]^{\frac{1}{\sigma}}}} \right) \end{array} \right].$$

Now, we need to prove that it is valid for $t = k + 1$, then we have

$$\left[\begin{array}{c} \left(\sqrt[q]{1 - \frac{1}{1+\left[\sum_{t=1}^{k}\sigma_t\left(\frac{\mathcal{T}^q_{\mathrm{d}_t}}{1-\mathcal{T}^q_{\mathrm{d}_t}}\right)^{\sigma}\right]^{\frac{1}{\sigma}}}}, \sqrt[q]{1 - \frac{1}{1+\left[\sum_{t=1}^{k}\sigma_t\left(\frac{\mathcal{I}^q_{\mathrm{d}_t}}{1-\mathcal{I}^q_{\mathrm{d}_t}}\right)^{\sigma}\right]^{\frac{1}{\sigma}}}}, \frac{1}{\sqrt[q]{1+\left[\sum_{t=1}^{k}\sigma_t\left(\frac{1-\mathcal{F}^q_{\mathrm{d}_t}}{\mathcal{F}^q_{\mathrm{d}_t}}\right)^{\sigma}\right]^{\frac{1}{\sigma}}}} \right), \\ \left(\sqrt[q]{1 - \frac{1}{1+\left[\sum_{t=1}^{k}\sigma_t\left(\frac{\lambda^q_{\mathrm{d}_t}}{1-\lambda^q_{\mathrm{d}_t}}\right)^{\sigma}\right]^{\frac{1}{\sigma}}}}, \sqrt[q]{1 - \frac{1}{1+\left[\sum_{t=1}^{k}\sigma_t\left(\frac{\eta^q_{\mathrm{d}_t}}{1-\eta^q_{\mathrm{d}_t}}\right)^{\sigma}\right]^{\frac{1}{\sigma}}}}, \frac{1}{\sqrt[q]{1+\left[\sum_{t=1}^{k}\sigma_t\left(\frac{1-\xi^q_{\mathrm{d}_t}}{\xi^q_{\mathrm{d}_t}}\right)^{\sigma}\right]^{\frac{1}{\sigma}}}} \right) \end{array} \right]$$

$$\otimes\left[\begin{array}{l}\left(\sqrt[q]{1-\frac{1}{1+\left[\sigma_t\left(\frac{\mathcal{T}^q_{\mathrm{d}\,\mathfrak{n}+g}}{1-\mathcal{T}^q_{\mathrm{d}\,\mathfrak{n}+g}}\right)^{\sigma}\right]^{\frac{1}{\sigma}}}},\sqrt[q]{1-\frac{1}{1+\left[\sigma_t\left(\frac{\mathcal{I}^q_{\mathrm{d}\,\mathfrak{n}+g}}{1-\mathcal{I}^q_{\mathrm{d}\,\mathfrak{n}+g}}\right)^{\sigma}\right]^{\frac{1}{\sigma}}}},\frac{1}{\sqrt[q]{1+\left[\sigma_{\mathfrak{n}+g}\left(\frac{1-\mathcal{F}^q_{\mathrm{d}\,\mathfrak{n}+g}}{\mathcal{F}^q_{\mathrm{d}\,\mathfrak{n}+g}}\right)^{\sigma}\right]^{\frac{1}{\sigma}}}}\right),\\ \left(\sqrt[q]{1-\frac{1}{1+\left[\sigma_{\mathfrak{n}+g}\left(\frac{\lambda^q_{\mathrm{d}\,\mathfrak{n}+g}}{1-\lambda^q_{\mathrm{d}\,\mathfrak{n}+g}}\right)^{\sigma}\right]^{\frac{1}{\sigma}}}},\sqrt[q]{1-\frac{1}{1+\left[\sigma_{\mathfrak{n}+g}\left(\frac{\eta^q_{\mathrm{d}\,\mathfrak{n}+g}}{1-\eta^q_{\mathrm{d}\,\mathfrak{n}+g}}\right)^{\sigma}\right]^{\frac{1}{\sigma}}}},\frac{1}{\sqrt[q]{1+\left[\sigma_{\mathfrak{n}+g}\left(\frac{1-\xi^q_{\mathrm{d}\,\mathfrak{n}+g}}{\xi^q_{\mathrm{d}\,\mathfrak{n}+g}}\right)^{\sigma}\right]^{\frac{1}{\sigma}}}}\right)\end{array}\right]$$

$$=\left[\begin{array}{l}\left(\sqrt[q]{1-\frac{1}{1+\left[\sum_{t=1}^{k+1}\sigma_t\left(\frac{\mathcal{T}^q_{\mathrm{d}_t}}{1-\mathcal{T}^q_{\mathrm{d}_t}}\right)^{\sigma}\right]^{\frac{1}{\sigma}}}},\sqrt[q]{1-\frac{1}{1+\left[\sum_{t=1}^{k+1}\sigma_t\left(\frac{\mathcal{I}^q_{\mathrm{d}_t}}{1-\mathcal{I}^q_{\mathrm{d}_t}}\right)^{\sigma}\right]^{\frac{1}{\sigma}}}},\frac{1}{\sqrt[q]{1+\left[\sum_{t=1}^{k+1}\sigma_t\left(\frac{1-\mathcal{F}^q_{\mathrm{d}_t}}{\mathcal{F}^q_{\mathrm{d}_t}}\right)^{\sigma}\right]^{\frac{1}{\sigma}}}}\right),\\ \left(\sqrt[q]{1-\frac{1}{1+\left[\sum_{t=1}^{k+1}\sigma_t\left(\frac{\lambda^q_{\mathrm{d}_t}}{1-\lambda^q_{\mathrm{d}_t}}\right)^{\sigma}\right]^{\frac{1}{\sigma}}}},\sqrt[q]{1-\frac{1}{1+\left[\sum_{t=1}^{k+1}\sigma_t\left(\frac{\eta^q_{\mathrm{d}_t}}{1-\eta^q_{\mathrm{d}_t}}\right)^{\sigma}\right]^{\frac{1}{\sigma}}}},\frac{1}{\sqrt[q]{1+\left[\sum_{t=1}^{k+1}\sigma_t\left(\frac{1-\xi^q_{\mathrm{d}_t}}{\xi^q_{\mathrm{d}_t}}\right)^{\sigma}\right]^{\frac{1}{\sigma}}}}\right)\end{array}\right].$$

Hence proved that the statement is valid for all t. □

Properties: Sq-LDFDWG contented the properties that are detailed below:
1. (*Idempotency*) : Let $\mho_t = \{x, \langle \mathcal{T}_{\mathrm{d}_t}, \mathcal{I}_{\mathrm{d}_t}, \mathcal{F}_{\mathrm{d}_t}\rangle, \langle \lambda_{\mathrm{d}_t}, \eta_{\mathrm{d}_t}, \xi_{\mathrm{d}_t}\rangle \,| t \in n\}$ be a number belongs to family of Sq-LDFEs that are all equal $\mho_t = \mho \;\; \forall\, t$, then

$$Sq - LDFDWG_{\theta}(\mho_1, \mho_2, ..., \mho_n) = \mho. \tag{5.2}$$

Proof. As $\mho_k = \{x, \langle \mathcal{T}_{\mathrm{d}_t}, \mathcal{I}_{\mathrm{d}_t}, \mathcal{F}_{\mathrm{d}_t}\rangle, \langle \lambda_{\mathrm{d}_t}, \eta_{\mathrm{d}_t}, \xi_{\mathrm{d}_t}\rangle \,| t \in n\}$. Then by above equation,
$Sq - LDFDWG_{\theta}(\mho_1, \mho_2, ..., \mho_n) = \otimes_{t=1}^{n}[\mho_t^{\sigma_t}]$

$$=\left[\begin{array}{l}\left(\sqrt[q]{1-\frac{1}{1+\left[\sum_{t=1}^{\mathfrak{n}}\sigma_t\left(\frac{\mathcal{T}^q_{\mathrm{d}_t}}{1-\mathcal{T}^q_{\mathrm{d}_t}}\right)^{\sigma}\right]^{\frac{1}{\sigma}}}},\sqrt[q]{1-\frac{1}{1+\left[\sum_{t=1}^{\mathfrak{n}}\sigma_t\left(\frac{\mathcal{I}^q_{\mathrm{d}_t}}{1-\mathcal{I}^q_{\mathrm{d}_t}}\right)^{\sigma}\right]^{\frac{1}{\sigma}}}},\frac{1}{\sqrt[q]{1+\left[\sum_{t=1}^{\mathfrak{n}}\sigma_t\left(\frac{1-\mathcal{F}^q_{\mathrm{d}_t}}{\mathcal{F}^q_{\mathrm{d}_t}}\right)^{\sigma}\right]^{\frac{1}{\sigma}}}}\right),\\ \left(\sqrt[q]{1-\frac{1}{1+\left[\sum_{t=1}^{\mathfrak{n}}\sigma_t\left(\frac{\lambda^q_{\mathrm{d}_t}}{1-\lambda^q_{\mathrm{d}_t}}\right)^{\sigma}\right]^{\frac{1}{\sigma}}}},\sqrt[q]{1-\frac{1}{1+\left[\sum_{t=1}^{\mathfrak{n}}\sigma_t\left(\frac{\eta^q_{\mathrm{d}_t}}{1-\eta^q_{\mathrm{d}_t}}\right)^{\sigma}\right]^{\frac{1}{\sigma}}}},\frac{1}{\sqrt[q]{1+\left[\sum_{t=1}^{\mathfrak{n}}\sigma_t\left(\frac{1-\xi^q_{\mathrm{d}_t}}{\xi^q_{\mathrm{d}_t}}\right)^{\sigma}\right]^{\frac{1}{\sigma}}}}\right)\end{array}\right]$$

$$=\left[\begin{array}{l}\left(\sqrt[q]{1-\frac{1}{1+\left[\left(\frac{\mathcal{T}^q_{\mathrm{d}}}{1-\mathcal{T}^q_{\mathrm{d}}}\right)^{\sigma}\right]^{\frac{1}{\sigma}}}},\sqrt[q]{1-\frac{1}{1+\left[\left(\frac{\mathcal{I}^q_{\mathrm{d}}}{1-\mathcal{I}^q_{\mathrm{d}}}\right)^{\sigma}\right]^{\frac{1}{\sigma}}}},\frac{1}{\sqrt[q]{1+\left[\left(\frac{1-\mathcal{F}^q_{\mathrm{d}}}{\mathcal{F}^q_{\mathrm{d}}}\right)^{\sigma}\right]^{\frac{1}{\sigma}}}}\right),\\ \left(\sqrt[q]{1-\frac{1}{1+\left[\left(\frac{\lambda^q_{\mathrm{d}}}{1-\lambda^q_{\mathrm{d}}}\right)^{\sigma}\right]^{\frac{1}{\sigma}}}},\sqrt[q]{1-\frac{1}{1+\left[\left(\frac{\eta^q_{\mathrm{d}}}{1-\eta^q_{\mathrm{d}}}\right)^{\sigma}\right]^{\frac{1}{\sigma}}}},\frac{1}{\sqrt[q]{1+\left[\left(\frac{1-\xi^q_{\mathrm{d}}}{\xi^q_{\mathrm{d}}}\right)^{\sigma}\right]^{\frac{1}{\sigma}}}}\right)\end{array}\right]$$

$$= \left[\begin{array}{c}\left(\sqrt[q]{1-\frac{1}{1+\left(\frac{\mathcal{T}_{\mathrm{d}}^{q}}{1-\mathcal{T}_{\mathrm{d}}^{q}}\right)}}, \sqrt[q]{1-\frac{1}{1+\left(\frac{\mathcal{I}_{\mathrm{d}}^{q}}{1-\mathcal{I}_{\mathrm{d}}^{q}}\right)}}, \frac{1}{\sqrt[q]{1+\left(\frac{1-\mathcal{F}_{\mathrm{d}}^{q}}{\mathcal{F}_{\mathrm{d}}^{q}}\right)}}\right), \\ \left(\sqrt[q]{1-\frac{1}{1+\left(\frac{\lambda_{\mathrm{d}}^{q}}{1-\lambda_{\mathrm{d}}^{q}}\right)}}, \sqrt[q]{1-\frac{1}{1+\left(\frac{\eta_{\mathrm{d}}^{q}}{1-\eta_{\mathrm{d}}^{q}}\right)}}, \frac{1}{\sqrt[q]{1+\left(\frac{1-\xi_{\mathrm{d}}^{q}}{\xi_{\mathrm{d}}^{q}}\right)}}\right)\end{array}\right]$$

$$= \langle \mathcal{T}_{\mathrm{d}}, \mathcal{I}_{\mathrm{d}}, \mathcal{F}_{\mathrm{d}} \rangle, \langle \lambda_{\mathrm{d}}, \eta_{\mathrm{d}}, \xi_{\mathrm{d}} \rangle = \mho.$$

Thus, the $Sq\text{-}LDFDWG_{\theta}(\mho_1, \mho_2, ..., \mho_n) = \mho$ holds.
$Sq\text{-}LDFDWG_{\theta}(\mho_1, \mho_2, ..., \mho_n) = \otimes_{t=1}^{n} [\mho_t^{\sigma_t}]$

$$= \left[\begin{array}{c}\left(\sqrt[q]{1-\frac{1}{1+\left[\sum_{t=1}^{n}\sigma_t\left(\frac{\mathcal{T}_{\mathrm{d}_t}^{q}}{1-\mathcal{T}_{\mathrm{d}_t}^{q}}\right)^{\sigma}\right]^{\frac{1}{\sigma}}}}, \sqrt[q]{1-\frac{1}{1+\left[\sum_{t=1}^{n}\sigma_t\left(\frac{\mathcal{I}_{\mathrm{d}_t}^{q}}{1-\mathcal{I}_{\mathrm{d}_t}^{q}}\right)^{\sigma}\right]^{\frac{1}{\sigma}}}}, \frac{1}{\sqrt[q]{1+\left[\sum_{t=1}^{n}\sigma_t\left(\frac{1-\mathcal{F}_{\mathrm{d}_t}^{q}}{\mathcal{F}_{\mathrm{d}_t}^{q}}\right)^{\sigma}\right]^{\frac{1}{\sigma}}}}\right), \\ \left(\sqrt[q]{1-\frac{1}{1+\left[\sum_{t=1}^{n}\sigma_t\left(\frac{\lambda_{\mathrm{d}_t}^{q}}{1-\lambda_{\mathrm{d}_t}^{q}}\right)^{\sigma}\right]^{\frac{1}{\sigma}}}}, \sqrt[q]{1-\frac{1}{1+\left[\sum_{t=1}^{n}\sigma_t\left(\frac{\eta_{\mathrm{d}_t}^{q}}{1-\eta_{\mathrm{d}_t}^{q}}\right)^{\sigma}\right]^{\frac{1}{\sigma}}}}, \frac{1}{\sqrt[q]{1+\left[\sum_{t=1}^{n}\sigma_t\left(\frac{1-\xi_{\mathrm{d}_t}^{q}}{\xi_{\mathrm{d}_t}^{q}}\right)^{\sigma}\right]^{\frac{1}{\sigma}}}}\right)\end{array}\right]$$

$$= \left[\begin{array}{c}\left(\sqrt[q]{1-\frac{1}{1+\left[\left(\frac{\mathcal{T}_{\mathrm{d}}^{q}}{1-\mathcal{T}_{\mathrm{d}}^{q}}\right)^{\sigma}\right]^{\frac{1}{\sigma}}}}, \sqrt[q]{1-\frac{1}{1+\left[\left(\frac{\mathcal{I}_{\mathrm{d}}^{q}}{1-\mathcal{I}_{\mathrm{d}}^{q}}\right)^{\sigma}\right]^{\frac{1}{\sigma}}}}, \frac{1}{\sqrt[q]{1+\left[\left(\frac{1-\mathcal{F}_{\mathrm{d}}^{q}}{\mathcal{F}_{\mathrm{d}}^{q}}\right)^{\sigma}\right]^{\frac{1}{\sigma}}}}\right), \\ \left(\sqrt[q]{1-\frac{1}{1+\left[\left(\frac{\lambda_{\mathrm{d}}^{q}}{1-\lambda_{\mathrm{d}}^{q}}\right)^{\sigma}\right]^{\frac{1}{\sigma}}}}, \sqrt[q]{1-\frac{1}{1+\left[\left(\frac{\eta_{\mathrm{d}}^{q}}{1-\eta_{\mathrm{d}}^{q}}\right)^{\sigma}\right]^{\frac{1}{\sigma}}}}, \frac{1}{\sqrt[q]{1+\left[\left(\frac{1-\xi_{\mathrm{d}}^{q}}{\xi_{\mathrm{d}}^{q}}\right)^{\sigma}\right]^{\frac{1}{\sigma}}}}\right)\end{array}\right]$$

$$= \left[\begin{array}{c}\left(\sqrt[q]{1-\frac{1}{1+\left(\frac{\mathcal{T}_{\mathrm{d}}^{q}}{1-\mathcal{T}_{\mathrm{d}}^{q}}\right)}}, \sqrt[q]{1-\frac{1}{1+\left(\frac{\mathcal{I}_{\mathrm{d}}^{q}}{1-\mathcal{I}_{\mathrm{d}}^{q}}\right)}}, \frac{1}{\sqrt[q]{1+\left(\frac{1-\mathcal{F}_{\mathrm{d}}^{q}}{\mathcal{F}_{\mathrm{d}}^{q}}\right)}}\right), \\ \left(\sqrt[q]{1-\frac{1}{1+\left(\frac{\lambda_{\mathrm{d}}^{q}}{1-\lambda_{\mathrm{d}}^{q}}\right)}}, \sqrt[q]{1-\frac{1}{1+\left(\frac{\eta_{\mathrm{d}}^{q}}{1-\eta_{\mathrm{d}}^{q}}\right)}}, \frac{1}{\sqrt[q]{1+\left(\frac{1-\xi_{\mathrm{d}}^{q}}{\xi_{\mathrm{d}}^{q}}\right)}}\right)\end{array}\right]$$

$$= \langle \mathcal{T}_{\mathrm{d}}, \mathcal{I}_{\mathrm{d}}, \mathcal{F}_{\mathrm{d}} \rangle, \langle \lambda_{\mathrm{d}}, \eta_{\mathrm{d}}, \xi_{\mathrm{d}} \rangle = \mho.$$

Thus, the $Sq\text{-}LDFDWG_{\theta}(\mho_1, \mho_2, ..., \mho_n) = \mho$ holds. □

2. (*Boundness*) Suppose $\mho_t = \{x, \langle \mathcal{T}_{\mathrm{d}_t}, \mathcal{I}_{\mathrm{d}_t}, \mathcal{F}_{\mathrm{d}_t} \rangle, \langle \lambda_{\mathrm{d}_t}, \eta_{\mathrm{d}_t}, \xi_{\mathrm{d}_t} \rangle | t \in n\}$ be a number belongs to the family of Sq-LDFEs. Then $\mho^- \leq Sq - LDFDWG_{\theta}(\mho_1, \mho_2, ..., \mho_n) \leq \mho^+$.

Proof. Suppose $\mho^- = \min(\mho_1, \mho_2, ..., \mho_n) = \left\{\left\langle \mathcal{T}_{\mathrm{d}}^-, \mathcal{I}_{\mathrm{d}}^-, \mathcal{F}_{\mathrm{d}}^- \right\rangle, \left\langle \lambda_{\mathrm{d}}^-, \eta_{\mathrm{d}}^-, \xi_{\mathrm{d}}^- \right\rangle\right\}$ and $\mho^+ = \max(\mho_1, \mho_2, ..., \mho_n) = \left\{\left\langle \mathcal{T}_{\mathrm{d}}^+, \mathcal{I}_{\mathrm{d}}^+, \mathcal{F}_{\mathrm{d}}^+ \right\rangle, \left\langle \lambda_{\mathrm{d}}^+, \eta_{\mathrm{d}}^+, \xi_{\mathrm{d}}^+ \right\rangle\right\}$, where $\mathcal{T}_{\mathrm{d}}^- = \min_t \left\{\mathcal{T}_{\mathrm{d}_t}^-\right\}, \mathcal{I}_{\mathrm{d}}^- = \min_t \left\{\mathcal{I}_{\mathrm{d}_t}^-\right\}, \mathcal{F}_{\mathrm{d}}^+ = \sup\left\{\mathcal{F}_{\mathrm{d}_t}^+\right\}$ with $\lambda_{\mathrm{d}}^- = \min_t \left\{\lambda_{\mathrm{d}_t}^-\right\}, \eta_{\mathrm{d}}^- = \min_t \left\{\eta_{\mathrm{d}_t}^-\right\}, \xi_{\mathrm{d}}^- = \min_t \left\{\xi_{\mathrm{d}_t}^-\right\}$ and $\mathcal{T}_{\mathrm{d}}^+ = \max_t \left\{\mathcal{T}_{\mathrm{d}_t}^+\right\}, \mathcal{I}_{\mathrm{d}}^+ =$

$\max_t\left\{\mathcal{I}^+_{\mathrm{d}_t}\right\}, \mathcal{F}^+_{\mathrm{d}} = \max_t\left\{\mathcal{F}^+_{\mathrm{d}_t}\right\}$ with $\lambda^+_{\mathrm{d}} = \max_t\left\{\lambda^+_{\mathrm{d}_t}\right\}, \eta^+_{\mathrm{d}} = \max_t\left\{\eta^+_{\mathrm{d}_t}\right\}, \xi^+_{\mathrm{d}} = \max_t\left\{\xi^+_{\mathrm{d}_t}\right\}$.
Therefore, we have the underlying inequalities,

$$\sqrt[q]{1-\frac{1}{1+\left[\sum_{t=1}^{\mathfrak{n}}\sigma_t\left(\frac{\mathcal{T}^{q-}_{\mathrm{d}_t}}{1-\mathcal{T}^{q-}_{\mathrm{d}_t}}\right)^{\sigma}\right]^{\frac{1}{\sigma}}}} \leq \sqrt[q]{1-\frac{1}{1+\left[\sum_{t=1}^{\mathfrak{n}}\sigma_t\left(\frac{\mathcal{T}^{q}_{\mathrm{d}_t}}{1-\mathcal{T}^{q}_{\mathrm{d}_t}}\right)^{\sigma}\right]^{\frac{1}{\sigma}}}}$$

$$\leq \sqrt[q]{1-\frac{1}{1+\left[\sum_{t=1}^{\mathfrak{n}}\sigma_t\left(\frac{\mathcal{T}^{q+}_{\mathrm{d}_t}}{1-\mathcal{T}^{q+}_{\mathrm{d}_t}}\right)^{\sigma}\right]^{\frac{1}{\sigma}}}}$$

$$\sqrt[q]{1-\frac{1}{1+\left[\sum_{t=1}^{\mathfrak{n}}\sigma_t\left(\frac{\mathcal{I}^{q-}_{\mathrm{d}_t}}{1-\mathcal{I}^{q-}_{\mathrm{d}_t}}\right)^{\sigma}\right]^{\frac{1}{\sigma}}}} \leq \sqrt[q]{1-\frac{1}{1+\left[\sum_{t=1}^{\mathfrak{n}}\sigma_t\left(\frac{\mathcal{I}^{q}_{\mathrm{d}_t}}{1-\mathcal{I}^{q}_{\mathrm{d}_t}}\right)^{\sigma}\right]^{\frac{1}{\sigma}}}}$$

$$\leq \sqrt[q]{1-\frac{1}{1+\left[\sum_{t=1}^{\mathfrak{n}}\sigma_t\left(\frac{\mathcal{I}^{q+}_{\mathrm{d}_t}}{1-\mathcal{I}^{q+}_{\mathrm{d}_t}}\right)^{\sigma}\right]^{\frac{1}{\sigma}}}}$$

$$\sqrt[q]{1-\frac{1}{1+\left[\sum_{t=1}^{\mathfrak{n}}\sigma_t\left(\frac{\mathcal{F}^{q-}_{\mathrm{d}_t}}{1-\mathcal{F}^{q-}_{\mathrm{d}_t}}\right)^{\sigma}\right]^{\frac{1}{\sigma}}}} \leq \sqrt[q]{1-\frac{1}{1+\left[\sum_{t=1}^{\mathfrak{n}}\sigma_t\left(\frac{\mathcal{F}^{q}_{\mathrm{d}_t}}{1-\mathcal{F}^{q}_{\mathrm{d}_t}}\right)^{\sigma}\right]^{\frac{1}{\sigma}}}}$$

$$\leq \sqrt[q]{1-\frac{1}{1+\left[\sum_{t=1}^{\mathfrak{n}}\sigma_t\left(\frac{\mathcal{F}^{q+}_{\mathrm{d}_t}}{1-\mathcal{F}^{q+}_{\mathrm{d}_t}}\right)^{\sigma}\right]^{\frac{1}{\sigma}}}}$$

$$\sqrt[q]{1-\frac{1}{1+\left[\sum_{t=1}^{\mathfrak{n}}\sigma_t\left(\frac{\lambda^{q-}_{\mathrm{d}_t}}{1-\lambda^{q-}_{\mathrm{d}_t}}\right)^{\sigma}\right]^{\frac{1}{\sigma}}}} \leq \sqrt[q]{1-\frac{1}{1+\left[\sum_{t=1}^{\mathfrak{n}}\sigma_t\left(\frac{\lambda^{q}_{\mathrm{d}_t}}{1-\lambda^{q}_{\mathrm{d}_t}}\right)^{\sigma}\right]^{\frac{1}{\sigma}}}}$$

$$\leq \sqrt[q]{1-\frac{1}{1+\left[\sum_{t=1}^{\mathfrak{n}}\sigma_t\left(\frac{\lambda^{q+}_{\mathrm{d}_t}}{1-\lambda^{q+}_{\mathrm{d}_t}}\right)^{\sigma}\right]^{\frac{1}{\sigma}}}}$$

$$\sqrt[q]{1-\frac{1}{1+\left[\sum_{t=1}^{\mathfrak{n}}\sigma_t\left(\frac{\eta^{q-}_{\mathrm{d}_t}}{1-\eta^{q-}_{\mathrm{d}_t}}\right)^{\sigma}\right]^{\frac{1}{\sigma}}}} \leq \sqrt[q]{1-\frac{1}{1+\left[\sum_{t=1}^{\mathfrak{n}}\sigma_t\left(\frac{\eta^{q}_{\mathrm{d}_t}}{1-\eta^{q}_{\mathrm{d}_t}}\right)^{\sigma}\right]^{\frac{1}{\sigma}}}}$$

$$\leq \sqrt[q]{1-\frac{1}{1+\left[\sum_{t=1}^{\mathfrak{n}}\sigma_t\left(\frac{\eta^{q+}_{\mathrm{d}_t}}{1-\eta^{q+}_{\mathrm{d}_t}}\right)^{\sigma}\right]^{\frac{1}{\sigma}}}}$$

$$\sqrt[q]{1-\frac{1}{1+\left[\sum_{t=1}^{\mathfrak{n}}\sigma_t\left(\frac{\xi_{\mathrm{d}_t}^{q-}}{1-\xi_{\mathrm{d}_t}^{q-}}\right)^{\sigma}\right]^{\frac{1}{\sigma}}}} \leq \sqrt[q]{1-\frac{1}{1+\left[\sum_{t=1}^{\mathfrak{n}}\sigma_t\left(\frac{\xi_{\mathrm{d}_t}^{q}}{1-\xi_{\mathrm{d}_t}^{q}}\right)^{\sigma}\right]^{\frac{1}{\sigma}}}}$$

$$\leq \sqrt[q]{1-\frac{1}{1+\left[\sum_{t=1}^{\mathfrak{n}}\sigma_t\left(\frac{\xi_{\mathrm{d}_t}^{q+}}{1-\xi_{\mathrm{d}_t}^{q+}}\right)^{\sigma}\right]^{\frac{1}{\sigma}}}}.$$

Therefore,
$\mho^{-} \leq Sq-LDFDWG_{\theta}(\mho_1,\mho_2,...,\mho_n) \leq \mho^{+}$. ◻

3. (*Monotonicity*) Suppose $\mho_t = \{x,\langle\mathcal{T}_{\mathrm{d}_t},\mathcal{I}_{\mathrm{d}_t},\mathcal{F}_{\mathrm{d}_t}\rangle,\langle\lambda_{\mathrm{d}_t},\eta_{\mathrm{d}_t},\xi_{\mathrm{d}_t}\rangle|t\in n\}$ and $\mho_t^{\ell} = \left\{x,\left\langle\mathcal{T}^{\ell}_{\mathrm{d}_t},\mathcal{I}^{\ell}_{\mathrm{d}_t},\mathcal{F}^{\ell}_{\mathrm{d}_t}\right\rangle,\left\langle\lambda^{\ell}_{\mathrm{d}_t},\eta^{\ell}_{\mathrm{d}_t},\xi^{\ell}_{\mathrm{d}_t}\right\rangle|t\in n\right\}$ be two sets of Sq-LDFEs, if $\mho_t \leq \mho_t^{\ell}\ \forall\ t$, then
$Sq\text{-}LDFDWG_{\theta}(\mho_1,\mho_2,...,\mho_n) \leq Sq\text{-}LDFDWG_{\theta}\left(\mho_1^{\ell},\mho_2^{\ell},...,\mho_n^{\ell}\right)$.

Proof. Let $Sq\text{-}LDFDWG_{\theta}(\mho_1,\mho_2,...,\mho_n) = \{x,\langle\mathcal{T}_{\mathrm{d}_t},\mathcal{I}_{\mathrm{d}_t},\mathcal{F}_{\mathrm{d}_t}\rangle,\langle\lambda_{\mathrm{d}_t},\eta_{\mathrm{d}_t},\xi_{\mathrm{d}_t}\rangle|t\in n\}$,
$\mathcal{T} = \sqrt[q]{1-\frac{1}{1+\left[\sum_{t=1}^{\mathfrak{n}}\sigma_t\left(\frac{\mathcal{T}_{\mathrm{d}_t}^{q}}{1-\mathcal{T}_{\mathrm{d}_t}^{q}}\right)^{\sigma}\right]^{\frac{1}{\sigma}}}}$ and $\mathcal{T}^{\ell} = \sqrt[q]{1-\frac{1}{1+\left[\sum_{t=1}^{\mathfrak{n}}\sigma_t\left(\frac{\mathcal{T}_{\mathrm{d}_t}^{q\ell}}{1-\mathcal{T}_{\mathrm{d}_t}^{q\ell}}\right)^{\sigma}\right]^{\frac{1}{\sigma}}}}$
then

$$\mathcal{T}_{\mathrm{d}_t}^{q} \leq \mathcal{T}_{\mathrm{d}_t}^{\ell q}$$

$$\frac{\mathcal{T}_{\mathrm{d}_t}^{q}}{1-\mathcal{T}_{\mathrm{d}_t}^{q}} \leq \frac{\mathcal{T}_{\mathrm{d}_t}^{q\ell}}{1-\mathcal{T}_{\mathrm{d}_t}^{q\ell}}$$

$$\left(\frac{\mathcal{T}_{\mathrm{d}_t}^{q}}{1-\mathcal{T}_{\mathrm{d}_t}^{q}}\right)^{\sigma} \leq \left(\frac{\mathcal{T}_{\mathrm{d}_t}^{q\ell}}{1-\mathcal{T}_{\mathrm{d}_t}^{q\ell}}\right)^{\sigma}$$

$$\sum_{t=1}^{\mathfrak{n}}\left(\frac{\mathcal{T}_{\mathrm{d}_t}^{q}}{1-\mathcal{T}_{\mathrm{d}_t}^{q}}\right)^{\sigma} \leq \sum_{t=1}^{\mathfrak{n}}\left(\frac{\mathcal{T}_{\mathrm{d}_t}^{q\ell}}{1-\mathcal{T}_{\mathrm{d}_t}^{q\ell}}\right)^{\sigma}$$

$$\left[\sum_{t=1}^{\mathfrak{n}}\left(\frac{\mathcal{T}_{\mathrm{d}_t}^{q}}{1-\mathcal{T}_{\mathrm{d}_t}^{q}}\right)^{\sigma}\right]^{\frac{1}{\sigma}} \leq \left[\sum_{t=1}^{\mathfrak{n}}\left(\frac{\mathcal{T}_{\mathrm{d}_t}^{q\ell}}{1-\mathcal{T}_{\mathrm{d}_t}^{q\ell}}\right)^{\sigma}\right]^{\frac{1}{\sigma}}$$

$$1+\left[\sum_{t=1}^{\mathfrak{n}}\left(\frac{\mathcal{T}_{\mathrm{d}_t}^{q}}{1-\mathcal{T}_{\mathrm{d}_t}^{q}}\right)^{\sigma}\right]^{\frac{1}{\sigma}} \leq 1+\left[\sum_{t=1}^{\mathfrak{n}}\left(\frac{\mathcal{T}_{\mathrm{d}_t}^{q\ell}}{1-\mathcal{T}_{\mathrm{d}_t}^{q\ell}}\right)^{\sigma}\right]^{\frac{1}{\sigma}}$$

$$\frac{1}{1+\left[\sum_{t=1}^{\mathfrak{n}}\left(\frac{\mathcal{T}_{\mathrm{d}_t}^{q}}{1-\mathcal{T}_{\mathrm{d}_t}^{q}}\right)^{\sigma}\right]^{\frac{1}{\sigma}}} \leq \frac{1}{1+\left[\sum_{t=1}^{\mathfrak{n}}\left(\frac{\mathcal{T}_{\mathrm{d}_t}^{q\ell}}{1-\mathcal{T}_{\mathrm{d}_t}^{q\ell}}\right)^{\sigma}\right]^{\frac{1}{\sigma}}}$$

$$\sqrt[q]{1-\frac{1}{1+\left[\sum_{t=1}^{\mathfrak{n}}\left(\frac{\mathcal{T}_{\mathrm{d}_t}^{q}}{1-\mathcal{T}_{\mathrm{d}_t}^{q}}\right)^{\sigma}\right]^{\frac{1}{\sigma}}}} \leq \sqrt[q]{1-\frac{1}{1+\left[\sum_{t=1}^{\mathfrak{n}}\left(\frac{\mathcal{T}_{\mathrm{d}_t}^{q\ell}}{1-\mathcal{T}_{\mathrm{d}_t}^{q\ell}}\right)^{\sigma}\right]^{\frac{1}{\sigma}}}}.$$

Similar results can be obtained for other parameters. ◻

Definition 13. *Let* $\mho_t = \{x, \langle {}^t\mathcal{T}_{d\,\Theta}, {}^t\mathcal{I}_{d\,\Theta}, {}^t\mathcal{F}_{d\,\Theta}\rangle, \langle {}^t\lambda_{d\,\Theta}, {}^t\eta_{d\,\Theta}, {}^t\xi_{d\,\Theta}\rangle : t = 1, 2, ..., \mathfrak{n}\}$ *be an assemblage of* Sq*-LDFNs over* $\aleph > 0,\ q \geq 1$. *The Sq-LDFDOWG operator is a transformation Sq-LDFDWG: Sq-LDFNs*$^n(\aleph) \to$*Sq-LDFNs*$^n(\aleph)$, *defined as Sq-LDFDOWG*$(\mho_1, \mho_2, ..., \mho_\mathfrak{n}) = \prod_{\Theta=g}^{\mathfrak{n}} \mho_{\Theta(\xi)}^{\sigma_\Theta} = \mho_{1(\xi)}^{\sigma_1} \otimes \mho_{2(\xi)}^{\sigma_2} \otimes, ..., \otimes \mho_{\mathfrak{n}(\xi)}^{\sigma_\mathfrak{n}}$, *where* $\sigma = (\sigma_1, \sigma_2, ..., \sigma_\mathfrak{n})$ *represents the weight vector of* $\mho_t\,(t = 1, 2, ..., \mathfrak{n}), 0 \leq \sigma_t \leq 1$ *and* $\sum_{t=1}^{\mathfrak{n}} \sigma_t = 1, (\xi_1), (\xi_2), ...(\xi_\mathfrak{n})$ *representing the order pattern of* $\mathfrak{n} \in N$ *due to that* $\mho_{\Theta(\xi=1)} \geq \mho_{\Theta(\xi)},\ \forall\ \mathfrak{n} \in N$.

Theorem 2. *Suppose that* $\mho_t = \left\{ \begin{array}{c} x, \left\langle {}^t\mathcal{T}_{d\,(\xi)}, {}^t\mathcal{I}_{d\,(\xi)}, {}^t\mathcal{F}_{d\,(\xi)} \right\rangle, \left\langle {}^t\lambda_{d\,(\xi)}, {}^t\eta_{d\,(\xi)}, {}^t\xi_{d\,(\xi)} \right\rangle \\ : t = 1, 2, ..., \mathfrak{n} \end{array} \right\}$ *be an assemblage of* Sq*-LDFNs over* $\aleph > 0,\ q \geq 1$. *Let the weight vector of* $\mho_t$ *be* $\sigma = (\sigma_1, \sigma_2, ..., \sigma_\mathfrak{n})$, *then* Sq*-LDFDOWG*$(\mho_1, \mho_2, ..., \mho_\mathfrak{n}) =$

$$\left[\begin{array}{c} \left(\begin{array}{c} \sqrt[q]{1 - \frac{1}{1 + \left[\sum_{t=1}^{\mathfrak{n}} \sigma_{t(\xi)} \left(\frac{\mathcal{T}^q_{d\ t(\xi)}}{1 - \mathcal{T}^q_{d\ t(\xi)}} \right)^{\sigma} \right]^{\frac{1}{\sigma}}}}, \sqrt[q]{1 - \frac{1}{1 + \left[\sum_{t=1}^{\mathfrak{n}} \sigma_{t(\xi)} \left(\frac{\mathcal{I}^q_{d\ t(\xi)}}{1 - \mathcal{I}^q_{d\ t(\xi)}} \right)^{\sigma} \right]^{\frac{1}{\sigma}}}}, \\ \frac{1}{\sqrt[q]{1 + \left[\sum_{t=1}^{\mathfrak{n}} \sigma_{t(\xi)} \left(\frac{1 - \mathcal{F}^q_{d\ t(\xi)}}{\mathcal{F}^q_{d\ t(\xi)}} \right)^{\sigma} \right]^{\frac{1}{\sigma}}}} \end{array} \right), \\ \left(\begin{array}{c} \sqrt[q]{1 - \frac{1}{1 + \left[\sum_{t=1}^{\mathfrak{n}} \sigma_{t(\xi)} \left(\frac{\lambda^q_{d\ t(\xi)}}{1 - \lambda^q_{d\ t(\xi)}} \right)^{\sigma} \right]^{\frac{1}{\sigma}}}}, \sqrt[q]{1 - \frac{1}{1 + \left[\sum_{t=1}^{\mathfrak{n}} \sigma_{t(\xi)} \left(\frac{\eta^q_{d\ t(\xi)}}{1 - \eta^q_{d\ t(\xi)}} \right)^{\sigma} \right]^{\frac{1}{\sigma}}}}, \\ \frac{1}{\sqrt[q]{1 + \left[\sum_{t=1}^{\mathfrak{n}} \sigma_{t(\xi)} \left(\frac{1 - \xi^q_{d\ t(\xi)}}{\xi^q_{d\ t(\xi)}} \right)^{\sigma} \right]^{\frac{1}{\sigma}}}} \end{array} \right) \end{array} \right].$$

5.2. Score function

In this part, we define the SF, QSF and ESF.

Definition 14. $\mho = \{x, \langle \mathcal{T}_d, \mathcal{I}_d, \mathcal{F}_d \rangle, \langle \lambda_d, \eta_d, \xi_d \rangle\}$ *be a Sq-LDFN, then the score function* (SF) *over* $\mho$ *can be defined by the transformation* $\Phi : Sq - LDFN(x) \to [-1, 1]$ *and given by*

$$\Phi(\mho) = \frac{1}{2}\left[(\mathcal{T}_d - \mathcal{I}_d - \mathcal{F}_d) + \left(\lambda_d^q - \eta_d^q - \xi_d^q\right)\right]; q \geq 1 \tag{5.3}$$

where Sq*-LDFN*(x) *is an assemblage on Sq-LDFNs over* x.

Definition 15. *The quadratic score function* $(QSF)\ \chi$ *is defined by the transformation* $\chi : Sq$*-LDFN*$(x) \to [-1, 1]$ *and given as*

$$\chi(\mho) = \frac{1}{2}\left[\left(\mathcal{T}_d^2 - \mathcal{I}_d^2 - \mathcal{F}_d^2\right) + \left(\left(\lambda_d^q\right)^2 - \left(\eta_d^q\right)^2 - \left(\xi_d^q\right)^2\right)\right]; q \geq 1. \tag{5.4}$$

Definition 16. *Let* $\mho_1$ *and* $\mho_2$ *be two Sq-LDFNs. By utilising the above definitions we can make comparison of two Sq-LDFNs* $\mho_1$ *and* $\mho_2$ *as*

(1) If $\Phi_{\mho_1} < \Phi_{\mho_2}$, *then* $\mho_1 < \mho_2$.

(2) If $\Phi_{\mho_1} > \Phi_{\mho_1}$*,then* $\mho_1 > \mho_2$.

Next, we defined the definition of ESF.

Definition 17. *The transformation* $\varphi : Sq\text{-}LDFN \rightarrow [0,1]$ *represents the expectation score function* (ESF) *for the Sq-LDFN* $\mho$ *which can be described as*

$$\varphi(\mho) = \frac{1}{3}\left[\frac{1}{2}\left(\mathcal{T}_d - \mathcal{I}_d - \mathcal{F}_d + 2\right) + \frac{1}{2}\left(\lambda_d^q - \eta_d^q - \xi_d^q\right)\right]; q \geq 1. \tag{5.5}$$

6. Medical diagnostics in hepatitis patients with Sq-LDFS

In this section, we begin by discussing the different types of hepatitis, providing a brief but detailed description of this fatal disease and its symptoms. We then use the proposed VIKOR approach to identify the most affected individual. Additionally, we review some of the research history and practical examples related to hepatitis diagnosis.

A semantic-based method was used to predict and diagnose the hepatitis C virus by employing a fuzzy ontology and a fuzzy Bayesian network [45]. In a separate study, Naeem [46] utilized the TOPSIS (Technique for Order of Preference by Similarity to Ideal Solution), VIKOR (Vise Kriterijumska Optimizacija Kompromisno Resenje), and generalized aggregation operator models to analyze different types of viral hepatitis mathematically. In a study conducted between 2011 and 2020, 30 patients diagnosed with hepatitis B at the university hospital in Setif, Algeria were examined to explore several aspects of the disease [47]. Using a fuzzy approach, the study examined several parameters associated with the illness, including gender, diabetes, arterial hypertension, and body mass index. The results indicated that all three methods produced the same optimal solution. In a separate study, Liu [48] suggested utilizing classical machine learning and deep learning to diagnose various liver disorders using artificial intelligence (AI). In his study, Riaz [49] extends the VIKOR approach in the context of metrics based on bipolar fuzzy numbers (CNs) (BFNs). He first creates CNs of BFNs and metric spaces based on CNs, and discusses some intriguing properties of the suggested metric spaces. Second, he applies the VIKOR approach to address a multiple-attribute group decision-making (MAGDM) problem with information of the bipolar fuzzy kind, utilizing metrics based on CNs.

Case study

Hepatitis originates from the combination of the words “hepa,” which meant liver, and “titis,” which meant inflammation. Essentially, hepatitis is an inflammation of the liver, which is the primary organ responsible for storing energy, breaking down food, and expelling poisonous substances from the body. The main cause of hepatitis is a damaged liver, which can be caused by one of the five viruses commonly represented by the first five English letters, from A to E. Each of these viruses causes a distinct illness and has a different pattern of attenuation, making the epidemiology of hepatitis confusing. Some of these viruses can cause severe, chronic, or both types of hepatitis [50]. Please refer to Figure 1 for a visual representation of the stages of liver damage.

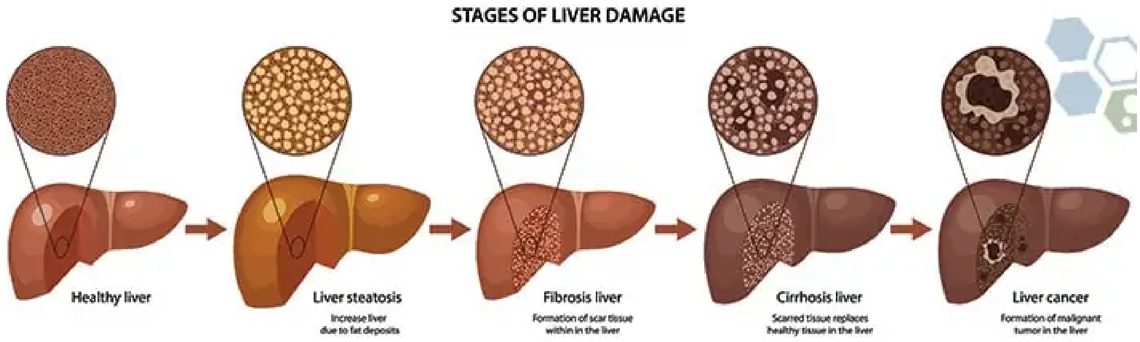

Figure 1. Five stages of liver damage.

A brief description of hepatitis is given below.
Hepatitis A: Hepatitis A is a highly contagious liver disease caused by the hepatitis A virus, which is one of several viruses that can cause inflammation and irritation and disrupt the liver's function. The virus is primarily transmitted through contaminated food or fluids or by direct contact with an infected person or object. In most cases, hepatitis A resolves on its own without causing long-term liver damage and does not require urgent medical attention. However, warning signs and symptoms of the disease can include fatigue, abdominal pain, loss of appetite, dark urine, sudden onset of nausea and vomiting, clay-colored bowel movements, low-grade fever, jaundice (yellowing of the skin and eyes), conjunctivitis, and severe itching.

Hepatitis B: Hepatitis B is a chronic infection caused by the hepatitis B virus (HBV) [50]. This illness usually lasts for more than six months and can increase the risk of developing liver cancer, liver failure, or cirrhosis, which is a condition that leaves a permanent scar on the liver. Although some patients may experience severe signs and symptoms, most individuals with hepatitis B recover well. Children and infants have a higher likelihood of developing chronic hepatitis B infection, which can cause minor to severe symptoms. While some symptoms may appear as early as two weeks after infection, they often manifest between one and four months after the infection. Some people, particularly children, may not exhibit any symptoms. HBV can spread through human contact, sharing of needles or syringes, unintended needle sticks, and mother-to-newborn infant contact. Symptoms of this type of virus may include fatigue and weakness, nausea and vomiting, stomach pain, lack of appetite, dark urine, fever, jaundice, and worsening joint pain. Figure 2 shows the chronic active virus, and Figure 3 shows the gross anatomy of the liver affected by Hepatitis B in the normal site.

Hepatitis C: It is an epidemiological infection that causes liver disorders and, occasionally, severe liver damage. Hepatitis C virus (HCV) typically spreads through contaminated blood. Nowadays, chronic HCV can usually be treated with oral drugs that are taken every day for two to six months. Because HCV infections typically have no outward symptoms and take years to manifest, over half of those infected are unaware that they have the illness. HCV may be found in a variety of unique forms, known as genotypes, around the world. Hepatitis C symptoms include ascites, spider angiomas, weariness, itchy skin, dark urine, hepatic encephalopathy, jaundice, easy bleeding, easy staining, swollen legs, low appetite, and ascites. Figure 4 shows the Liver effected by Hepatitis C.

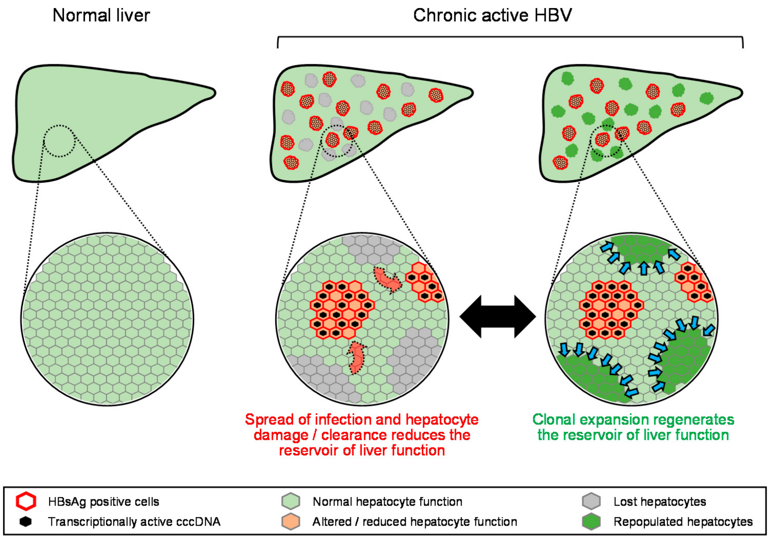

Figure 2. Chronic active virus.

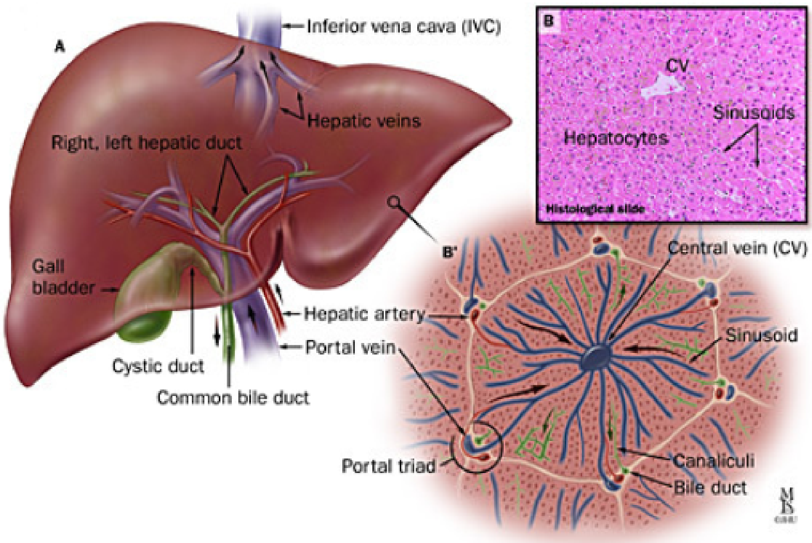

Figure 3. Hepatitis B gross anatomy in the normal state.

Hepatitis D: Hepatitis Delta Virus (HDV) is another name for it. This kind of virus causes the liver to enlarge, reducing liver function and leading to chronic liver conditions, the most serious of which include liver scarring and cancer. This kind of virus cannot be caught on its own, in contrast to other varieties. It spreads to those who have HBV infection already. HDV can be short-lived, severe, or chronic. Acute hepatitis D manifests suddenly and typically results in more severe symptoms. It could leave on its own. The disease is known as chronic hepatitis D if the infection persists for six months or longer. The long-term variant of this illness gradually gets worse over time. Before symptoms appear, the virus is likely present in the body for several months. Technical difficulties are more likely when chronic hepatitis D worsens. Many people who contract the virus later develop cirrhosis, or severe liver damage. Similar symptoms of both HBV and HDV include joint discomfort, weariness, lack of appetite, and stomach pain. Figure 5 shows the symptoms of different types of hepatitis.

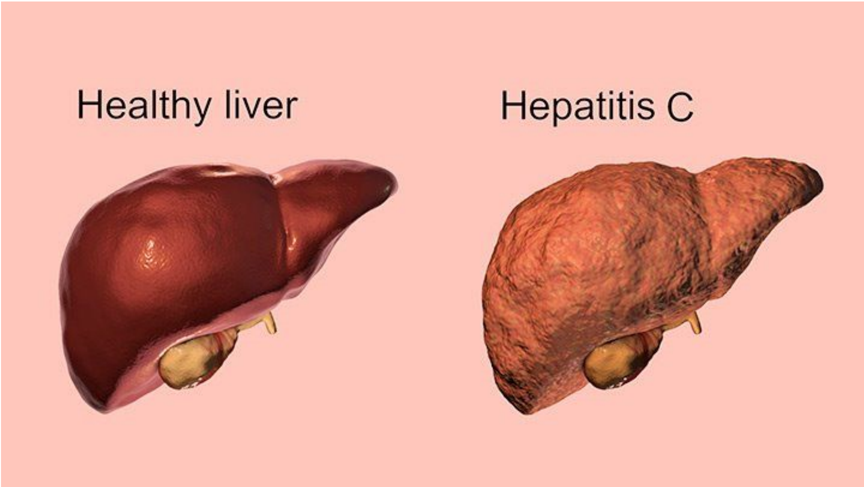

Figure 4. Liver effected by Hepatitis C.

Figure 5. Symptoms of Hepatitis A, B, C and D.

Hepatitis E: Hepatitis E spreads in a number of ways. The most popular methods are drinking low-quality beverages and eating under cooked, feces-tainted meat. Fecal waste from people or animals has the potential to contaminate water, which might ultimately lead to the spread of viruses. This kind predominates more in underdeveloped areas with poor water quality, especially in populated areas. Hepatitis E-infected pregnant women run the risk of becoming a host for the virus and passing it on

to their unborn children. Aside from these instances, it is uncommon for someone to spread this virus to other people. There may be a variation in the symptoms of this virus. For instance, some infected people exhibit no warning symptoms at all. The alternative possibility is that the symptoms might be so slight as to be hardly perceptible. Such circumstances are riskier and more hazardous. However, it has been shown that some infected people exhibit a variety of symptoms that typically manifest 15 to 60 days after viral contact. Poor appetite, exhaustion, vomiting, fever, upper abdomen discomfort, nausea, light or clay-colored stools, jaundice, and dark urine are all potential signs of hepatitis E.
We use the well-known technique "VIKOR" for selecting the unbeatable alternative from the perspective of a so-called compromise solution having the quality that the selected solution has that is closest to the idyllic solution and farthest from the worst solution, i.e., the negative ideal solution, in in order to make thorough, unanimous, and intelligent decisions.

In this section, we demonstrate how VIKOR may be used in a spherical q-linear Diophantine fuzzy environment. We will first modify VIKOR according to Sq-LDFSs and then use it to address a challenge from the life sciences.
First, we go through each step of the recommended plan, as shown in Table 1.

Table 1. Morphological evaluation terms for alternatives.

Morphological terms	Fuzzy weightages
Phase 0: Healthy liver (S_0)	0.10
Phase 1: The initiation of liver damage (S_1)	0.30
Phase 2: Moderately liver damage (S_2)	0.50
Phase 3: Significant liver damage (S_3)	0.70
Phase 4: Serious liver damage (S_4)	0.90

7. Medical diagnosis based on Sq-LDF VIKOR method

The word VIKOR derived from Serbian, is the abbreviated version of "Vlse Kriterijumska Optimizacija Kompromisno Resenje". We utilized this method for multi-attribute analysis. Opricovic (1998) created the VIKOR technique as a MADM strategy for dealing with discrete decision issues with incommensurable and conflicting criteria. In a decision-making situation with opposing qualities, it can identify compromise alternatives and assist the decision-makers in accepting a choice. The workable approach that comes closest to the ideal result is mutually acceptable (Opricovic, 2011). In the literature of MADM, extending VIKOR under various fuzzy circumstances constitutes a lucrative field. In Kutlu Gündogdu and Kahraman, a thorough literature review with an emphasis on fuzzy VIKOR variants can be found (2019). For instance, Wu et al. (2019) employed interval-valued IF numbers in the application of VIKOR and proved the utilization of the approach in the financial risk assessment of rural tourism projects, whereas Devi (2011) expanded VIKOR under the IFS environment for robot selection problem. PFS-based VIKOR propositions are identified by many researchers like T. Y. Chen (2018) for five different MADM problems consist of service quality assessment of domestic airlines, investment decisions regarding Internet stocks, etc., Rani et al. (2019) for the evaluation of renewable energy technologies in India, and Gul et al. (2019) in a problem regarding the safety risk assessment of mines. Krishan Kumar et al. (2020) proposed a q-ROFS version of VIKOR with unknown weight information and presented its application in solving the green supplier selection problem. As seen from the literature, there is a gap concerning the FFS-based extension of VIKOR in the literature, and the detailed algorithm below aims at fulfilling this gap (see Figure 6).

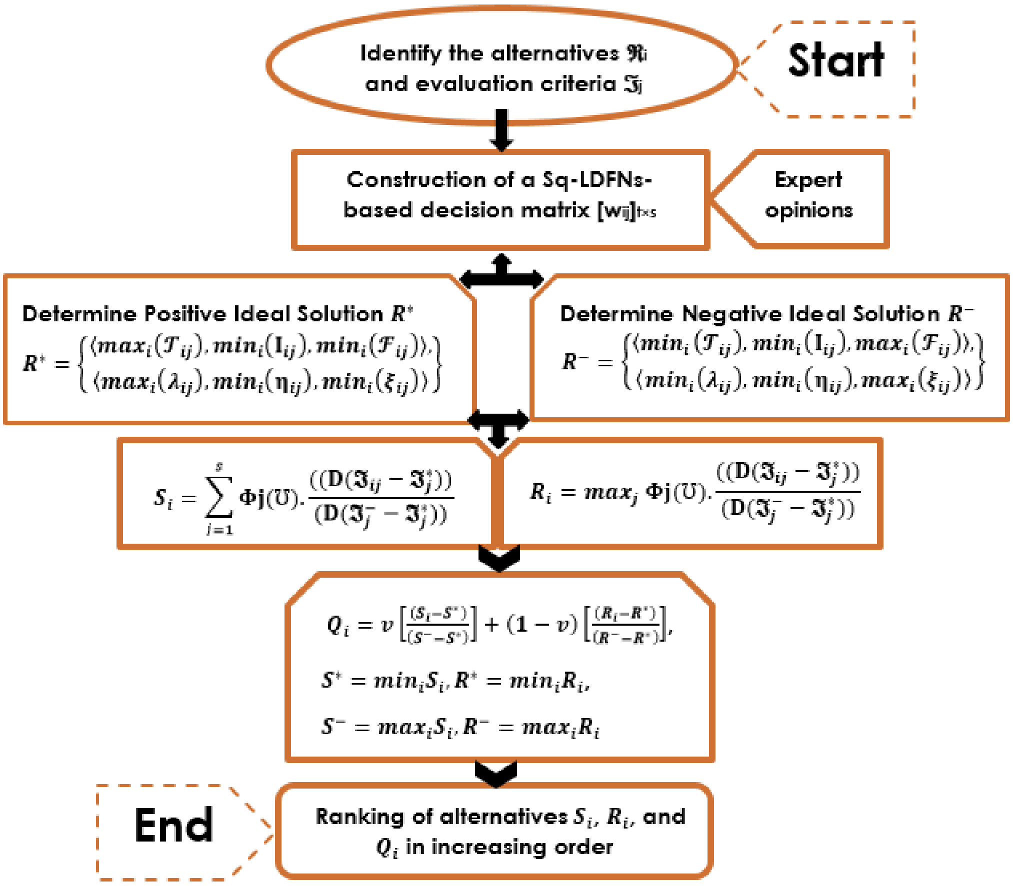

Figure 6. A modified version of the VIKOR.

Step 1: Diagnose the problem: The information for the decision matrix is obtained by a single expert, $\mathfrak{E}$, the selection of alternatives or attributes is represented by $\mathfrak{R} = \{\mathfrak{R}_1, \mathfrak{R}_2, ...\mathfrak{R}_i | i = 1, 2, ...s\}$, and the family of parameter criteria is represented by $\mathfrak{I} = \left\{\mathfrak{I}_1, \mathfrak{I}_2, ..., \mathfrak{I}_j | j = 1, 2, ...t\right\}$.

Step 2: Create a weighted criterion matrix if W represents the weight given by $\mathfrak{E}$ assigned to $\mathfrak{I}_j$ while taking the linguistic variables (LVs) in Table 2 into consideration.

$$W = \left[w_{ij}\right]_{t\times s} = \begin{pmatrix} w_{11} & w_{12} & \cdots & w_{1s} \\ w_{21} & w_{22} & \cdots & w_{1s} \\ w_{31} & w_{32} & \cdots & w_{1s} \\ \vdots & \vdots & \ddots & \vdots \\ w_{t1} & w_{t2} & \cdots & w_{ts} \end{pmatrix}.$$

Table 2. Śq-LDF decision matrix $\mho = \{x, \langle \mathcal{T}_{\rm d}, \mathcal{I}_{\rm d}, \mathcal{F}_{\rm d} \rangle, \langle \lambda_{\rm d}, \eta_{\rm d}, \xi_{\rm d} \rangle\}$.

A's	Qualities	Śq-LDFNS
$\Re_1$	$\Im_1, \Im_2,$ $\Im_3, \Im_4,$ $\Im_5$	$[(0.85, 0.24, 0.45), (0.25, .34, .18)], [(0.73, 0.31, 0.48), (0.34, 0.11, 0.23)],$ $[(0.63, 0.45, 0.38), (0.41, 0.28, 0.11)], [(0.81, 0.41, 0.32), (0.31, 0.23, 0.31)],$ $[(0.78, 0.17, 0.45), (0.33, 0.12, 0.27)]$
$\Re_2$	$\Im_1, \Im_2,$ $\Im_3, \Im_4,$ $\Im_5$	$[(0.77, 0.41, 0.52), (0.34, 0.21, 0.22)], [(0.82, 0.51, 0.43), (0.13, 0.25, 0.21)],$ $[(0.58, 0.43, 0.41), (0.31, 0.23, 0.15)], [(0.78, 0.45, 0.31), (0.51, 0.11, 0.18)],$ $[(0.83, 0.21, 0.43), (0.72, 0.13, 0.14)]$
$\Re_3$	$\Im_1, \Im_2,$ $\Im_3, \Im_4,$ $\Im_5$	$[(0.95, 0.41, 0.38), (0.41, 0.25, 0.18)], [(0.77, 0.62, 0.43), (0.31, 0.25, 0.21)],$ $[(0.86, 0.41, 0.38), (0.41, 0.23, 0.17)], [(0.89, 0.38, 0.46), (0.46, 0.32, 0.11)],$ $[(0.83, 0.21, 0.38), (0.51, 0.18, 0.17)]$
$\Re_4$	$\Im_1, \Im_2,$ $\Im_3, \Im_4,$ $\Im_5$	$[(0.82, 0.41, 0.38), (0.41, 0.21, 0.11)], [(0.91, 0.61, 0.53), (0.38, 0.21, 0.22)],$ $[(.73, .61, .48), (.25, .31, .18)], [(.83, .63, .47), (.38, .21, .17)],$ $[(0.76, 0.58, 0.43), (0.31, 0.23, 0.33)],$
$\Re_5$	$\Im_1, \Im_2,$ $\Im_3, \Im_4,$ $\Im_5$	$[(0.73, 0.61, 0.53), (0.41, 0.21, 0.18)], [(0.83, 0.51, 0.68), (0.31, 0.21, 0.15)],$ $[(0.73, 0.61, 0.58), (0.41, 0.23, 0.16)], [(0.81, 0.32, 0.58), (0.38, 0.31, 0.14)],$ $[(0.93, 0.21, 0.41), (0.41, 0.21, 0.13)]$

Step 3: In the VIKOR procedure, the aggregate normalized performance ratings are required. Any approach outlined in the literature can be used to determine the weight set, either objectively or subjectively. Calculating the aggregated performance ratings involves using the score function by utilizing Definition 13 and then normalizing it by making use of the below-mentioned formula

$$\overline{\Phi_j}(\mho) = \frac{\Phi_j(\mho)}{\sum_{j=1}^{s} \Phi_j(\mho)}.$$

The alternative is to keep on without defuzzifying the weights of the criteria. This technique is known as the full fuzzy approach. Our data is already in normalized form. So, we keep continuing the procedure by decision matrix.

Step 4: Estimate the spherical q-linear Diophantine fuzzy positive ideal solution (Sq-LDFPIS) and the spherical q-linear Diophantine fuzzy negative ideal solution (Sq-LDFNIS) based on the union and intersection of Sq-LDFNS.

To obtain the Sq-LDF-PIS decision matrix, property (4) is used.

$$R^* = \left\{ \begin{array}{c} \left\langle \max_i \left(\mathcal{T}_{{\rm d}\,ij}\right), \min_i \left(\mathcal{I}_{{\rm d}\,ij}\right), \min_i \left(\mathcal{F}_{{\rm d}\,ij}\right)\right\rangle, \left\langle \max_i \left(\lambda_{{\rm d}\,ij}\right), \min_i \left(\eta_{{\rm d}\,ij}\right), \min_i \left(\xi_{{\rm d}\,ij}\right)\right\rangle \\ |j = 1, 2, ..., s \end{array} \right\}$$

$$\overline{\Im}^* = \left\{ \begin{array}{c} \left(\Re_1, \left[\left(\mathcal{T}_1^*, \mathcal{I}_1^*, \mathcal{F}_1^*\right), \left(\lambda_1^*, \eta_1^*, \xi_1^*\right)\right]\right), \left(\left[\Re_2, \left(\mathcal{T}_2^*, \mathcal{I}_2^*, \mathcal{F}_2^*\right), \left(\lambda_2^*, \eta_2^*, \xi_2^*\right)\right]\right), ..., \\ \left(\left[\Re_s, \left(\mathcal{T}_t^*, \mathcal{I}_t^*, \mathcal{F}_t^*\right), \left(\lambda_t^*, \eta_t^*, \xi_t^*\right)\right]\right) \end{array} \right\}.$$

The Sq-LDF-NIS decision matrix is determined using property (5).

$$R^- = \left\{ \begin{array}{c} \left\langle \min_i \left(\mathcal{T}_{{\rm d}\,ij}\right), \min_i \left(\mathcal{I}_{{\rm d}\,ij}\right), \max_i \left(\mathcal{F}_{{\rm d}\,ij}\right)\right\rangle, \left\langle \min_i \left(\lambda_{{\rm d}\,ij}\right), \min_i \left(\eta_{{\rm d}\,ij}\right), \max_i \left(\xi_{{\rm d}\,ij}\right)\right\rangle \\ |j = 1, 2, ..., s \end{array} \right\}$$

$$\overline{\mathfrak{I}}^{-} = \left\{ \begin{array}{c} \left(\mathfrak{R}_1, \left[\left(\mathcal{T}_1^-, \mathcal{I}_1^-, \mathcal{F}_1^-\right), \left(\lambda_1^-, \eta_1^-, \xi_1^-\right)\right]\right), \left(\left[\mathfrak{R}_2, \left(\mathcal{T}_2^-, \mathcal{I}_2^-, \mathcal{F}_2^-\right), \left(\lambda_2^-, \eta_2^-, \xi_2^-\right)\right]\right), ..., \\ \left(\left[\mathfrak{R}_s, \left(\mathcal{T}_t^-, \mathcal{I}_t^-, \mathcal{F}_t^-\right), \left(\lambda_t^-, \eta_t^-, \xi_t^-\right)\right]\right) \end{array} \right\}.$$

Step 5: Calculate S_i and R_i values using partial fuzzy and full fuzzy techniques, as shown in Eqs (7.1) and (7.2), respectively.

$$S_i = \sum_{j=1}^{s} \overline{\Phi_j}(\mho) . \frac{D\left(\overline{\mathfrak{I}}_{ij} - \overline{\mathfrak{I}^*}_j\right)}{D\left(\overline{\mathfrak{I}}_j^- - \overline{\mathfrak{I}^*}_j\right)} \tag{7.1}$$

$$R_i = \max_j \overline{\Phi_j}(\mho) . \frac{D\left(\overline{\mathfrak{I}}_{ij} - \overline{\mathfrak{I}^*}_j\right)}{D\left(\overline{\mathfrak{I}}_j^- - \overline{\mathfrak{I}^*}_j\right)}. \tag{7.2}$$

Step 6: Calculate the distances using following Eqs (7.3) and (7.4).

$$D_{HD}\left(\overline{\mathfrak{I}}_{ij} - \overline{\mathfrak{I}^*}_j\right) = \frac{1}{6}\left[\begin{array}{c} \left|\left(\mathcal{T}_{ij}\right)^3 - \left(\mathcal{T}_j^*\right)^3\right| + \left|\left(\mathcal{I}_{ij}\right)^3 - \left(\mathcal{I}_j^*\right)^3\right| + \left|\left(\mathcal{F}_{ij}\right)^3 - \left(\mathcal{F}_j^*\right)^3\right| + \\ \left|\left(\lambda_{ij}\right)^3 - \left(\lambda_j^*\right)^3\right| + \left|\left(\eta_{ij}\right)^3 - \left(\eta_j^*\right)^3\right| + \left|\left(\xi_{ij}\right)^3 - \left(\xi_j^*\right)^3\right| \end{array} \right] \tag{7.3}$$

$$D_{HD}\left(\overline{\mathfrak{I}}_j^- - \overline{\mathfrak{I}^*}_j\right) = \frac{1}{6}\left[\begin{array}{c} \left|\left(\mathcal{T}_j^-\right)^3 - \left(\mathcal{T}_j^*\right)^3\right| + \left|\left(\mathcal{I}_j^-\right)^3 - \left(\mathcal{I}_j^*\right)^3\right| + \left|\left(\mathcal{F}_j^-\right)^3 - \left(\mathcal{F}_j^*\right)^3\right| + \\ \left|\left(\lambda_j^-\right)^3 - \left(\lambda_j^*\right)^3\right| + \left|\left(\eta_j^-\right)^3 - \left(\eta_j^*\right)^3\right| + \left|\left(\xi_j^-\right)^3 - \left(\xi_j^*\right)^3\right| \end{array} \right], \tag{7.4}$$

where S_i denotes the difference in evaluation between an alternative's best value, such that i, and R_i represents the difference in evaluation between an alternative's worst value.

Step 7: The average aggregation with maximum gaps (weighted distances) computed via Eq (7.5) where $S_i^* = \min_i S_i$, $S^- = \max_i S_i$ and $R_i^* = \min_i R_i$, $R^- = \max_i R_i$

$$Q_i = v\left[\frac{S_i - S^*}{S^- - S^*}\right] + (1 - v)\left[\frac{R_i - R^*}{R^- - R^*}\right]. \tag{7.5}$$

While "$(1 - v)$" is the weight of the individual regret, "v" is designated as a weight for the strategy of the largest group's utility. $v = 5$ could undermine these two tactics. Because the attribute (1of n) associated with R is also included in S, Opricovic (2011) updated v as $v = \frac{n+1}{2n}$ from $v + 0.5 * \frac{(n-1)}{n}$.

Step 8: Choose the most preferred alternative. The preferred answer with the least amount of Qi is then elected.

Si, Ri, and Qi are the alternatives ordered in increasing order. In ranking, the compromise option will have the lowest Q_i value.

Practical numerical example

Step1: Consider $\mathfrak{R} = \{\mathfrak{R}_1, \mathfrak{R}_2, ...\mathfrak{R}_5\}$ be the set of patients under observation and $\mathfrak{I} = \{\mathfrak{I}_1, \mathfrak{I}_2, ..., \mathfrak{I}_5\}$ be the family of criteria, where

$\mathfrak{I}_1$ = jaundice

$\mathfrak{I}_2$ = abdominal pain and agitation

$\mathfrak{I}_3$ = stool with a pale or clay colour

$\mathfrak{I}_4$ = vomiting
$\mathfrak{I}_5$ = dark colored urine

Step 2: Sq-LDF matrices with the Sq-LDFN, the place representing patient grades row-wise and the criteria or symptoms column-wise.

$$W = \begin{matrix} \mathfrak{R}_1 \\ \mathfrak{R}_1 \\ \\ \vdots \\ \mathfrak{R}_l \end{matrix} \begin{bmatrix} \mathfrak{I}_1 & \mathfrak{I}_2 & \cdots & \mathfrak{I}_t \\ \left\{ \begin{matrix} (\mathcal{T}_{11}, \mathcal{I}_{11}, \mathcal{F}_{11}), \\ (\lambda_{11}, \eta_{11}, \xi_{11}) \end{matrix} \right\} & \left\{ \begin{matrix} (\mathcal{T}_{12}, \mathcal{I}_{12}, \mathcal{F}_{12}), \\ (\lambda_{12}, \eta_{12}, \xi_{12}) \end{matrix} \right\} & \cdots & \left\{ \begin{matrix} (\mathcal{T}_{1t}, \mathcal{I}_{1t}, \mathcal{F}_{1t}), \\ (\lambda_{1t}, \eta_{1t}, \xi_{1t}) \end{matrix} \right\} \\ \left\{ \begin{matrix} (\mathcal{T}_{21}, \mathcal{I}_{21}, \mathcal{F}_{21}), \\ (\lambda_{21}, \eta_{21}, \xi_{21}) \end{matrix} \right\} & \left\{ \begin{matrix} (\mathcal{T}_{22}, \mathcal{I}_{22}, \mathcal{F}_{22}), \\ (\lambda_{22}, \eta_{22}, \xi_{22}) \end{matrix} \right\} & \cdots & \left\{ \begin{matrix} (\mathcal{T}_{2t}, \mathcal{I}_{2t}, \mathcal{F}_{2t}), \\ (\lambda_{2t}, \eta_{2t}, \xi_{2t}) \end{matrix} \right\} \\ \vdots & \vdots & \ddots & \vdots \\ \left\{ \begin{matrix} (\mathcal{T}_{l1}, \mathcal{I}_{l1}, \mathcal{F}_{l1}), \\ (\lambda_{l1}, \eta_{l1}, \xi_{l1}) \end{matrix} \right\} & \left\{ \begin{matrix} (\mathcal{T}_{l2}, \mathcal{I}_{l2}, \mathcal{F}_{l2}), \\ (\lambda_{l2}, \eta_{l2}, \xi_{l2}) \end{matrix} \right\} & \cdots & \left\{ \begin{matrix} (\mathcal{T}_{lt}, \mathcal{I}_{lt}, \mathcal{F}_{lt}), \\ (\lambda_{lt}, \eta_{lt}, \xi_{lt}) \end{matrix} \right\} \end{bmatrix}$$

Step 3: w_{ij} represents the weight that team members assign to criteria $\mathfrak{I}_j$, assigned to the parameter j while by consideration the morphological terms already chosen in Table 1. In other words, the weight w_{ij} is assigned to the parameter $\mathfrak{I}_j$ by the specialist based on various tests and the patient's outward appearance. According to the expert, this weight determines the hepatitis virus's stage or phase.

Step 4: In the VIKOR procedure, this step consists of normalization of decision matrix, but our data is already in normalized form.

Step 5: Next, we find the Sq-LDF-PIS and Sq-LDF-NIS on behalf of properties (4) and (5), as under Sq-LDF-PIS

$= \{(0.85, 0.17, 0.32), (0.41, 0.11, 0.11)\}, \{(0.83, 0.21, 0.31), (0.72, 0.11, 0.14)\},$
$\{(0.95, 0.21, 0.38), (0.51, 0.18, 0.11)\}, \{(0.91, 0.41, 0.38), (0.41, 0.21, 0.11)\},$
$\{(0.93, 0.21, 0.41), (0.41, 0.21, 0.13)\}$

Sq-LDF-NIS

$= \{(0.63, 0.17, 0.48), (0.25, 0.11, 0.31)\}, \{(0.58, 0.21, 0.52), (0.13, 0.11, 0.22)\},$
$\{(0.77, 0.38, 0.46), (0.31, 0.18, 0.21)\}, \{(0.73, 0.41, 0.53), (0.25, 0.21, 0.33)\},$
$\{(0.73, 0.21, 0.68), (0.31, 0.21, 0.18)\}$

Step 6: The separations between each patient and PFSV-PIS and PFSV-NIS along with the related relative coefficients of closeness calculated by Eqs (7.3) and (7.4). As an example, the distance between the first alternative and the optimal solution is illustrated below for criteria $\mathfrak{I}_1$.

$$D_{HD}\left(\overline{\mathfrak{I}}_{ij} - \overline{\mathfrak{I}^*}_j\right) = \frac{1}{6}\left[\begin{matrix} \left|\left(\mathcal{T}_{ij}\right)^3 - \left(\mathcal{T}_j^*\right)^3\right| + \left|\left(\mathcal{I}_{ij}\right)^3 - \left(\mathcal{I}_j^*\right)^3\right| + \left|\left(\mathcal{F}_{ij}\right)^3 - \left(\mathcal{F}_j^*\right)^3\right| + \\ \left|\left(\lambda_{ij}\right)^3 - \left(\lambda_j^*\right)^3\right| + \left|\left(\eta_{ij}\right)^3 - \left(\eta_j^*\right)^3\right| + \left|\left(\xi_{ij}\right)^3 - \left(\xi_j^*\right)^3\right| \end{matrix} \right]$$

$$D_{HD}\left(\overline{\mathfrak{I}}_{11} - \overline{\mathfrak{I}^*}_1\right) = \frac{1}{6}\left[\begin{matrix} \left|(0.85)^3 - (0.85)^3\right| + \left|(0.24)^3 - (0.17)^3\right| + \\ \left|(0.45)^3 - (0.32)^3\right| + \left|(0.25)^3 - (0.41)^3\right| + \\ \left|(0.34)^3 - (0.11)^3\right| + \left|(0.18)^3 - (0.11)^3\right| \end{matrix} \right] = 0.0271.$$

The distances for the remaining four attributes are determined as $D_{HD}\left(\overline{\mathfrak{I}}_{12} - \overline{\mathfrak{I}^*}_2\right) = 0.1059, D_{HD}\left(\overline{\mathfrak{I}}_{13} - \overline{\mathfrak{I}^*}_3\right) = 0.0229.$

$D_{HD}\left(\overline{\mathfrak{I}}_{14} - \overline{\mathfrak{I}^*}_4\right) = 0337, D_{HD}\left(\overline{\mathfrak{I}}_{15} - \overline{\mathfrak{I}^*}_5\right) = 0.1194$ which can be write in a vector representation: $[0.0271, 0.1059, 0.0229, 0.0337, 0.1194]$. The list of remaining distances given as:

$[0.0614, 0.0976, 0.1286, 0.0459, 0.1070]$, $[0.0822, 0.1402, 0.0591, 0.1094, 0.1273]$, $[0.03750.0705, 0.0505, 0.0716, 0.0763]$, $[0.0416, 0.0084, 0.0482, 0.0903, 0.0000]$. The following equation generates the vector that represents the distance between negative and positive ideal solutions:

$$D_{HD}\left(\overline{\mathfrak{I}_j^-} - \overline{\mathfrak{I}^*}_j\right) = \frac{1}{6}\left[\begin{array}{c} \left|\left(\mathcal{T}_j^-\right)^3 - \left(\mathcal{T}_j^*\right)^3\right| + \left|\left(\mathcal{I}_j^-\right)^3 - \left(\mathcal{I}_j^*\right)^3\right| + \left|\left(\mathcal{F}_j^-\right)^3 - \left(\mathcal{F}_j^*\right)^3\right| + \\ \left|\left(\lambda_j^-\right)^3 - \left(\lambda_j^*\right)^3\right| + \left|\left(\eta_j^-\right)^3 - \left(\eta_j^*\right)^3\right| + \left|\left(\xi_j^-\right)^3 - \left(\xi_j^*\right)^3\right| \end{array}\right]$$

$$D_{HD}\left(\overline{\mathfrak{I}_j^-} - \overline{\mathfrak{I}^*}_j\right)_{j=1,2,\ldots,5} = [0.1391, 0.2091, 0.2088, 0.1217, 0.2379].$$

Step 7: The computation of Si and Ri values are as follows

$$S_1 = \underset{j}{sum}\left\{0.45 * \frac{0.0271}{0.1391}, 0.21 * \frac{0.1059}{0.2091}, 0.20 * \frac{0.0229}{0.2088}, 0.10 * \frac{0.0337}{0.1217}, 0.04 * \frac{0.1194}{0.2379}\right\} = 0.2640$$

The maximum gaps (weighted distances) S_i and R_i computed, where $S_i^* = \min_i S_i = 0.2635$, $S^- = \max_i S_i = 0.5746$ and $R_i^* = \min_i R_i = 0.0214$, $R^- = \max_i R_i = 0.2659$ by using below mentioned distances

$[0.0879, 0.1064, 0.0220, 0.0277, 0.0201]$, $[0.1986, 0.0980, 0.1232, 0.0377, 0.0180]$, $[0.2659, 0.1408, 0.0566, 0.0899, 0.0214]$, $[0.1214, 0.0708, 0.0483, 0.0588, 0.0128]$, $[0.1346, 0.0085, 0.0462, 0.0742, 0.0000]$.

Where Q_i calculated by using Eq (7.5). The relative similarity index for each of the patients is presented in Table 3.

Table 3. Each patient's relative similarity index.

Patients	$\mathfrak{R}_1$	$\mathfrak{R}_2$	$\mathfrak{R}_3$	$\mathfrak{R}_4$	$\mathfrak{R}_5$	$\mathfrak{R}j\,(Q)$ *Ranking*
S_i	0.2640	0.4755	0.5746	0.3121	0.2635	$5 < 1 < 4 < 2 < 3$
R_i	0.2659	0.1408	0.1232	0.0899	0.0214	$5 < 4 < 3 < 2 < 1$
Q_i	0.4011	0.6042	0.7666	0.2059	0.0000	$5 < 4 < 1 < 2 < 3$

Step 8: The decision-maker's best option is presented by $\mathfrak{R}_5$. The possibilities are ranked taking into account the Q_i values. Consequently, the patients' preferred order is : $\mathfrak{R}_5 < \mathfrak{R}_4 < \mathfrak{R}_2 < \mathfrak{R}_1 < \mathfrak{R}_3$. The graphic representation is given in Figure 7. Now, we use this model's alternatives and criteria of Table 2 in Sq-LDFWG and Sq-LDFOWG aggregation operators; after this, we apply the score function, quadratic score function and expectation score function to find the effectiveness of our proposed theory of spherical q-linear Diophantine fuzzy numbers with the Dombi operator.

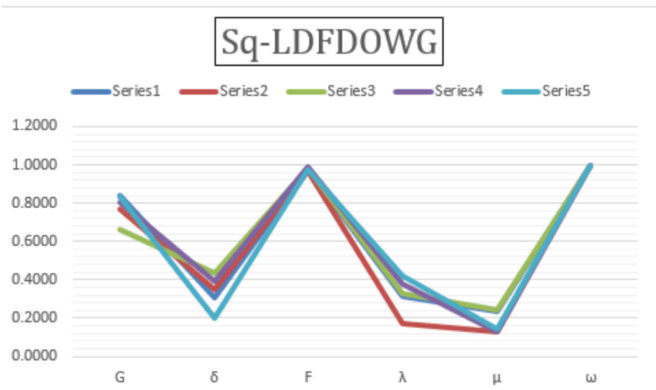

Figure 7. Sq-LDFDOWG aggregation operators algorithm.

8. Comparison-based evaluation and the effectiveness of the suggested idea

When employed and looked at in the context of decision support, VIKOR has the advantage of giving a ranking procedure for both positive and negative criteria. The VIKOR approach has gained a lot of traction in the decision-making field due to its ease of calculation and ability to deliver findings that are practically exact. This kind of issue is resolved via the VIKOR method of compromise. The utility weight [51] is a decision-making technique introduced by the VIKOR approach. When compared to TOPSIS and SAW, the utility weight is used more frequently, and its value can be altered in accordance with decision-makers' attitudes. In general, we assign a compromise attitude score of 0.5. Its value can be greater or lower than 0.5, depending primarily on the attitude of the majority. Lastly, an empirical instance is used to demonstrate the benefit of the VIKOR technique.

To assess how effectively the two proposed Sq-LDFG aggregation operations can deal with uncertain physical-world DMPs, they are compared with the operators already described in [9]. It's incredible how this concept encompasses the valuation spaces for IFSs, SFSs, LDFS, q-LFSs, and SLDFSs with CFs. Tables 4–6 present the findings of a study of two alternative ideas sq-LDFDWG and sq-LDFDOWG operating on hepatitis patients to diagnose the symptoms as sq-LDFNs using established methodologies and a planned concept. Tables 4–6 show that the expert judgements on hepatitis symptoms supported by the suggested approach are identical to those of the present procedures, which are expressive in and of themselves, shows support for the proposed method's validity and consistency.

Table 4. Sq-LDFDWG ratings.

	$[(\mathcal{T}_d, \mathcal{I}_d, \mathcal{F}_d), (\lambda_d, \eta_d, \xi_d)]$
$\mathfrak{R}_1$	$[(0.8215, 0.2718, 0.9736), (0.2803, 0.2402, 0.9988)]$
$\mathfrak{R}_2$	$[(0.7630, 0.3508, 0.9667), (0.1681, 0.1254, 0.9970)]$
$\mathfrak{R}_3$	$[(0.6389, 0.4426, 0.9793), (0.3281, 0.2500, 0.9994)]$
$\mathfrak{R}_4$	$[(0.8140, 0.4048, 0.9872), (0.3450, 0.1418, 0.9991)]$
$\mathfrak{R}_5$	$[(0.7973, 0.1863, 0.9744), (0.3562, 0.1306, 0.9984)]$

Table 5. Sq-LDFDOWG ratings.

	$[(\mathcal{T}_d, \mathcal{I}_d, \mathcal{F}_d), (\lambda_d, \eta_d, \xi_d)]$
$\mathfrak{R}_1$	$[(0.8405, 0.3036, 0.9784), (0.3120, 0.2349, 0.9991)]$
$\mathfrak{R}_2$	$[(0.7691, 0.3508, 0.9647), (0.1682, 0.1253, 0.9969)]$
$\mathfrak{R}_3$	$[(0.6647, 0.4313, 0.9793), (0.3288, 0.2401, 0.9992)]$
$\mathfrak{R}_4$	$[(0.8047, 0.3952, 0.9875), (0.3812, 0.1255, 0.9991)]$
$\mathfrak{R}_5$	$[(0.8296, 0.2033, 0.9759), (0.4174, 0.1395, 0.9990)]$

The rankings of Sq-LDFDWG and Sq-LDFDOWG are presented in descending order as given below:

Table 6. Ranking for Sq-LDFDWG and Sq-LDFOWG based on SF, QSF and ESF.

SF	−0.7060	−0.7713	−0.8807	−0.7685	−0.6579	$5 > 1 > 4 > 2 > 3$
SF	−0.7107	−0.7672	−0.8609	−0.7610	−0.6383	$5 > 1 > 4 > 2 > 3$
QSF	−0.6697	−0.7287	−0.8710	−0.7343	−0.6686	$5 > 1 > 2 > 4 > 3$
QSF	−0.6684	−0.7220	−0.8486	−0.7377	−0.6471	$5 > 1 > 2 > 4 > 3$
ESF	0.4313	0.4096	0.3731	0.4105	0.4474	$5 > 1 > 4 > 2 > 3$
ESF	0.4298	0.4109	0.3797	0.4130	0.4539	$5 > 1 > 4 > 2 > 3$

The graphic representations of SF, QSF and ESF are given in Figures 8–10.

Authenticity and flexibility: Our approach is adaptable and suitable for all kinds of input data. The recommended approach with the use of a newly proposed theory in a fuzzy field with the Dombi operator is effective in managing uncertainty. This technique also covers the fields of IFS, SFS, q-LDFS, LDFS, and SLDFS by including the qth degree of control factors. By increasing the qth degree of variables, more membership and non-membership spaces are formed, along with the physical layout. Our method may be used in several situations. It helps us select the best hepatitis symptoms. The proposed Sq-LDFS may be readily changed to provide the best numerical results. We saw the authenticity and flexibility of the results by comparing them with the VIKOR approach.

The score function's functionality: The score functions SF, QSF, and ESF that we mentioned before generalized them to utilize them in the MADM problem. We expect rather varied outcomes because each SF has its own observational and rating techniques. But Table 6 demonstrate how the rankings for the SF, ESF, and QSF differ a tad bit from one another. It is crucial to remember that the results from the two approaches Sq-LDFDWG with Sq-LDFDOWG and ranking in VIKOR are substantially the same for all scoring functions.

Aggregation flexibility with various inputs and outputs: Since the qth power of control variables widens the grade space and can vary according to the circumstances in MADM techniques, this strategy is significantly more versatile than others. It may also be applied to a variety of input and output circumstances. As our results show, the aggregation flexibility is good.

Comparing the suggested approach to the established technique: The fact that Sq-LDFSs maintains the qth computations makes it use up a lot more space when compared to IFSs, SFSs, q-LDFSs, LDFSs, and SLDFSs to Sq-LDFS. Although q-LDFSs has some limitations and cannot resolve the problem with three grades, [37] presented q-LDFSs with an additional qth degree. We design the concept of Sq-LDFSs and propose Sq-LDFSs to fill this research problem. The problems with MADM and the suggested technique are interconnected. Table 6, 3 compare aggregating operation values based on the Sq-LDFDWG, Sq-LDFDOWG, and VIKOR techniques.

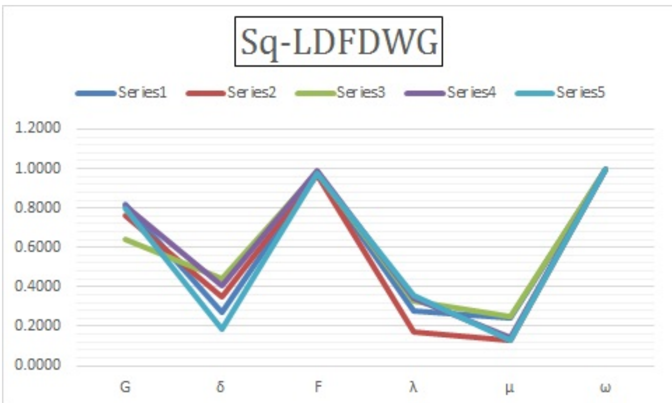

Figure 8. Evaluation of five hepatitis patients' symptoms as Sq-LDFNs by using SF, QSF and ESF.

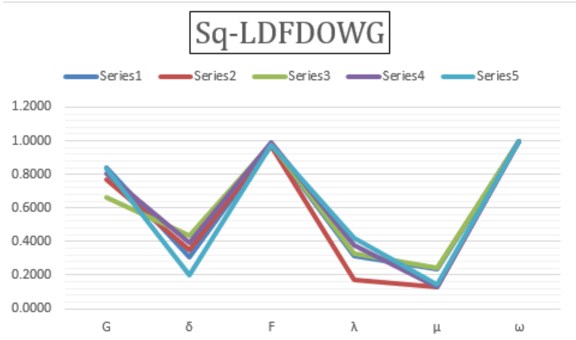

Figure 9. Evaluation of five hepatitis patients' symptoms as Sq-LDFNs by using SF, QSF and ESF.

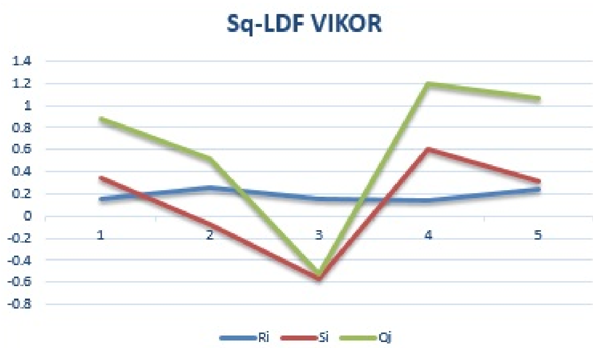

Figure 10. Evaluation of hepatitis patients by VIKOR approach.

9. Conclusions

In this article, we pioneered the notion of a spherical q-linear Diophantine fuzzy set. Then, we defined Dombi operations for Sq-LDF numbers based on the t-norm and t-conorm. We commented on some of the characteristics of these defined operations. Dombi aggregation operators are shown here in terms of the Sq-LDF Dombi weighted geometric operator. Subsequently, we defined the VIKOR technique and established a MADM technique under the Sq-LDF scenario. Ultimately, a realistic instance for the selection of the patient according to the hepatitis symptoms is illustrated, and a comparison analysis is given to verify the effectiveness and viability of the proposed technique. It has been observed that our suggested technique makes the information aggregation process more flexible because it involves parameters and makes fuzzy information readily articulated. As a result, the improved Dombi aggregation operators provide decision-makers with a novel, adaptable method to handle Sq-LDF MADM issues. Future research can apply our suggested approach to additional decision-making issues, risk assessment, and other medical diagnostics. To make the established operators provide more effective results, future work may also depend on building other scoring functions, ranking algorithms, and different types of operators in the context of sq-LDF.

We proposed the idea of a spherical q-linear Diophantine fuzzy set and examined its characteristics. We then employed this concept in several approaches, including Sq-LDFDWG, Sq-LDFDOWG, Sq-LDFDS-VIKOR, and the extended Sq-LDFS-AO method, to model uncertainties in the MCDM issue arising from the selection of hepatitis patients based on their illness symptoms. We utilized recommended algorithms and aggregation operators to rank a variety of hepatitis patients, taking into account the different varieties and phases of the disease. To enhance the clarity of the final rankings, we used statistical visuals and provided a three-rank comparison. We also defended the practicality of our approach, comparing the reported final grades using a statistical chart.

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Conflict of interest

The authors declare that they have no conflict of interest.

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