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*Research article*

## Fuzzy topological analysis of pizza graph

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**Abstract:** Fuzzy topological indices are getting attention these days due to their vast applications in daily life. In crisp case, topological indices are beneficial in chemical graph theory but as far as fuzzy graph theory is concerned, fuzzy topological indices are useful in identifying human trafficking, and multi-criteria decision-making environments. In this paper, we have computed the fuzzy topological indices such as the first and second fuzzy Zagreb indices, Randic index, and Harmonic index for the  $Pz_n$  pizza graph. We have found generalized results for the above-mentioned structure.

**Keywords:** pizza graph; fuzzy graph; topological indices

**Mathematics Subject Classification:** 05C15

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### 1. Introduction

Engineers and technologists are using modern tools now a day to resolve daily life problems. Graph theory is playing a vital role in various fields few of them are listed below. Graph theory is often used for the study of algorithms like, Dijkstra's Algorithm, Kruskal's Algorithm, Prims's Algorithm, defined for the flow of computation for finding the shortest path in a network or a road represented in a communication network system to study the data in computer science. Engineers have used graph theory to design the circuit connection among nodes which are termed as topologies. Some of these topologies are termed as bridge, series, parallel and star topologies. In chemistry and physics, graph theory is also used to study the molecular structures of ionic and covalent bonds also. Moreover, the 3D structure of complicated compounds in organic chemistry can also be studied easily with the help of graph theory relation between carbon and hydrogen can be represented with the help of edges and carbon and hydrogen can be treated as a vertex set. Statistical physics not only uses the graphs

as local connections among interacting parts of the system but also use to study the dynamics of the physical process on that particular system. To express micro-scale channels of porous media graphs are used, in which the edges represent the smaller channels connections among the pores and the vertex set represents the pores. Graph theory is frequently used in computer networking to study the relationships among interconnected computers for a network system. For example, in a security network system vertex coloring algorithm is used to allocate different colors for different frequencies for any GSM mobile phone networks. Some real-world problems along with their solution using graph theory are listed below.

Fuzzy topological indices are playing a pivotal role in different areas of sciences. In recent research, these indices are used to identify cybercrimes. Ahmad et al. [1] discussed the fuzzy topological indices and also found some bounds for different products of graphs and also use one of the index as an application of cybercrimes. The same fuzzy environment is also used in algebraic graph theory [4, 5, 18]. Rajeshwari et al. [19] established the fuzzy topological indices for bi-polar fuzzy graphs. In 2023, Islam et al. [11] presented the first Zagreb index for fuzzy graphs and discussed its application too. Islam et al. [12] also discussed the second Zagreb index and its use in mathematical chemistry. In [10], a new index F-index already has been investigated and improved.

Facebook had more than 2.9 billion active users in January 2022. Most of the revenue comes from advertising as a social media platform. Having a large number of users, advertising agency will find it very expensive to play their advertisement within the reach of all the users as a major theme is to target the people that may be interested to use your product. How can one find such an audience who is interested to use your product?

Graph theory has resolved the problem through social network graph by allocating a vertex to each user and two vertices are connected with each other with an edge if they are friends on Facebook, therefore undirected graphs are formed and the massive graph appears at the beginning very difficult to handle now becomes easier to find patterns in it. Numerous algorithms like hierarchical clustering algorithms or Karger's algorithm for minimum cut methods can be used. The formation of subgraph from the original graph is actually the identification of peoples connected on Facebook as friends, these group of people (connected vertices) are termed as communities having common interests, like specific brands, artists, or even political associations. By identifying such communities, the advantage of floating an advertisement become easier to attract the people having common interest, detection of such communities can be also used for different purposes other than advertisement.

Google search engine navigates through the World Wide Web without any problem when a search is made for a specific set of words. The engine finds all the websites that relates with the search after millions of relating finds how does the ranking is made regarding the search being most popular should appear as first one?

Graph theory has resolved this issue of search engine by creating a web graph in which websites are treated as vertices and directed edges are used among the websites to represent the hyperlink between them. The said directed graph shows the relation/ link among the websites. Moreover, weights can be assigned to the vertices as priority, importance or authenticity of the results and website.

For classification regarding the popularity of the websites, various algorithms can be used PageRank is one of them used by google. The engine allocates probabilities to introduce a hyperlink and iteratively adds them to form the probability distribution. The said distribution characterizes the likelihood of a user randomly arriving at a specific website afterword's the list of websites has been

ordered by the engine according to the distribution as a result highest one is shown. Nowadays, there are more advanced algorithms used for this problem but the main source is still graph theory.

Fuzzy logic another field of mathematics is extensively used by engineers like mechanical, electrical, civil, aerospace, chemical, biomedical, agricultural, environmental, computer, geological, mechatronics and industrial as well as mathematicians, natural scientists (physics, biology, earth science and chemistry), computer software developers, medical researchers, social scientists (management, economics, psychology and political science), business analysts, and public policy analysts in research areas and development of new algorithms in the field of science and technology.

Fuzzy logic theory is conceived on the idea of relative graded membership, as inspired by the procedures of human cognition and perception, whereas Lotfi A. Zadeh in 1965 published his well-known research paper on fuzzy sets [25]. Information gained through computational perception and cognition, i.e., imprecise, uncertain, partially accurate, vague, or without sharp limitations can be dealt through fuzzy logic. Advanced computing techniques based on fuzzy logic can also be effectively used for the development of intelligent systems for identification, decision-making, optimization, control and pattern recognition. Moreover, it provides an effective way to conflict resolution and better assessment of opportunities and multiple criteria. For further study one can read [7–13].

Zadeh's [24] landmark published an article "fuzzy sets" and later on Rosenfeld [20] introduced the fuzzy graphs regarding some relations, properties of tree graph, path graph and several other graphs. Several properties of a fuzzy graph and crisp graph are similar as fuzzy graph is the generalization of the crisp graph. A crisp graph is a one-to-one correspondence from  $f : VUE \rightarrow N$ .

The topological molecular conformation index is a non-empirical value that affects the size of the molecular structure and its sequence of branches. In this sense, it can be viewed as a fractional topological index, which describes individual molecular structures with a real number and is examined as a descriptor of the molecule.

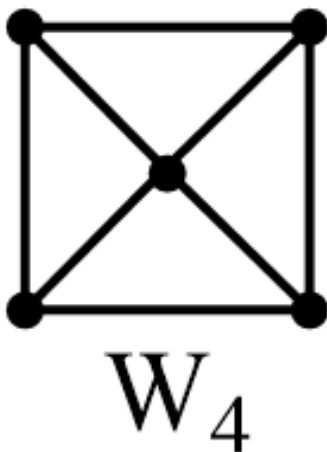
There are several topological indicators, several of which are applied in chemistry nowadays. The structural characteristics of the diagrams used to calculate this may be graded. For example, hosoya index, randic communication index, zagreb community index etc. The wiener index, which depends on the relative distance of the topological tops in the graph, is the most known and most used topological index mathematicians and chemists study in this field. The interest in this subject is therefore being sought worldwide for topological graphic indices.

In 1973, Kaufmann [14] introduced the first definition of a fuzzy graph using the Zadeh's [24] fuzzy relations. In 1975, a more detailed description is credited to Rosenfeld [20] who introduced the fuzzy graph theory by considering fuzzy relations on fuzzy sets. He established some relations regarding the properties of path graph, trees and various graphs. The notion of fuzzy cut, nodes and fuzzy bridges was introduced by Bhattacharya in [6]. The generalization of a crisp graph is a fuzzy graph. Hence, there exist many similar properties between them and also deviate in several places. In crisp graph  $G = (V, E)$ , where  $V$  and  $E$  are the sets of vertices and edges respectively, a bijective function  $f : V \cup E \rightarrow N$  that allocates a natural number to every vertex and (or) edge is known as labeling.

Numerous definitions and basic concepts are discussed in [2, 3]. Zeeshan et al. [23] computed first and second fuzzy zagreb indices of linear and multiacyclic hydrocarbons. This paper deals with the first fuzzy zagreb Index, second fuzzy zagreb index, randic fuzzy zagreb index and harmonic fuzzy zagreb index of flower graph  $f_{m \times r}$ .

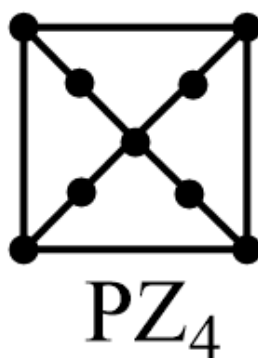
## 2. Materials and methods

A **wheel graph**  $w_n$  for  $n \geq 4$  can be obtained if we insert a vertex  $v$  inside a  $C_{n-1}$  cycle graph and join each vertex of cycle graph with that vertex  $v$ .  $w_4$  wheel graph is shown in Figure 1.



**Figure 1.**  $w_4$  wheel graph.

A **pizza graph**  $Pz_n$  for  $n \geq 3$  can be obtained when each spoke of  $W_n$  is subdivided one time and is denoted by  $Pz_n$ . The central vertex is labeled as  $b$  while the neighboring vertices of  $b$  are  $c_k$  where  $1 \leq k \leq n$  and the other neighboring vertices of  $c_k$  are the vertices of cycle graph  $d_k$  where  $1 \leq k \leq n$ . One can easily understand the vertices representation  $c_k$ ,  $d_k$  and  $b$  through Figure 2 where  $Pz_4$  pizza graph is shown.



**Figure 2.**  $Pz_4$  pizza graph.

## 3. Preliminaries

Some basic definitions are discussed in this section.

**Graph** [21]

A (crisp) graph  $G = (V, E)$  consists of two sets called the vertices  $V$  and the edges  $E$ .

#### Vertices [21]

The elements of  $V = \{v_1, v_2, \dots, v_n\}$  are called vertices or nodes or points.

#### Edges [21]

the elements of  $E = \{e_1, e_2, \dots, e_m\}$  are called edges or arcs or lines.

#### Fuzzy Graph [21]

An undirected finite graph  $H$ , without multiple edges or loops is termed as fuzzy graph having finite set of edges and vertices  $E(H)$  and  $V(H)$  respectively if the following properties are satisfied i.e.,

$$\sigma : V(H) \rightarrow [0, 1]$$

and

$$\mu : V(H) \times V(H) \rightarrow [0, 1]$$

such that  $\mu(v_1v_2) \leq \sigma(v_1) \wedge \sigma(v_2)$  for every pair of vertices  $v_1, v_2 \in V(H)$ .

Example of fuzzy graph is given in Figure 3.

#### Degree of a Fuzzy Graph [21]

The degree of vertex of a fuzzy graph  $H$  can be calculated by adding the weights of all corresponding edges to that vertex  $v$  mathematically it is represented as;

$$d(v) = \sum_{n=1}^j \mu(vu), \quad \text{where } v \neq u.$$

#### Order of a Fuzzy Graph [21]

The order  $O(H)$  of a fuzzy graph  $H$  can be calculated by adding the weight of all the vertices, mathematically it is represented as;

$$O(H) = \sum \sigma(v).$$

#### Size of a Fuzzy Graph [21]

The size  $S(H)$  of a fuzzy graph  $H$  can be calculated by adding the weights of all the edges, mathematically it is represented as;

$$S(H) = \sum \mu(vu),$$

for all vertices  $u, v \in V(H)$  and  $v \neq u$ .

Kalathian et al. [15] introduce the **fuzzy zagreb indices** of following types.

#### First zagreb index

The first zagreb index is denoted by  $M(H)$  and defined as

$$M(H) = \sum_{k=1}^q \sigma(u_k)[du_k]^2.$$

### Second zagreb index

The second zagreb index is denoted by  $M^*(H)$  and defined as

$$M^*(H) = \frac{1}{2} \sum_{kl \in E(G)} [\sigma(u_k)(du_k)\sigma(v_l)(dv_l)].$$

### Randic index

The randic index of fuzzy graph  $H$  is defined as

$$R(H) = \frac{1}{2} \sum_{kl \in E(G)} [\sigma(u_k)(du_k)\sigma(v_l)(dv_l)]^{-1/2}.$$

### Harmonic index

The harmonic index of fuzzy graph  $H$  is defined as

$$H(H) = \frac{1}{2} \sum_{kl \in E(G)} \left[ \frac{1}{\sigma(u_k)(du_k) + \sigma(v_l)(dv_l)} \right].$$

### Example

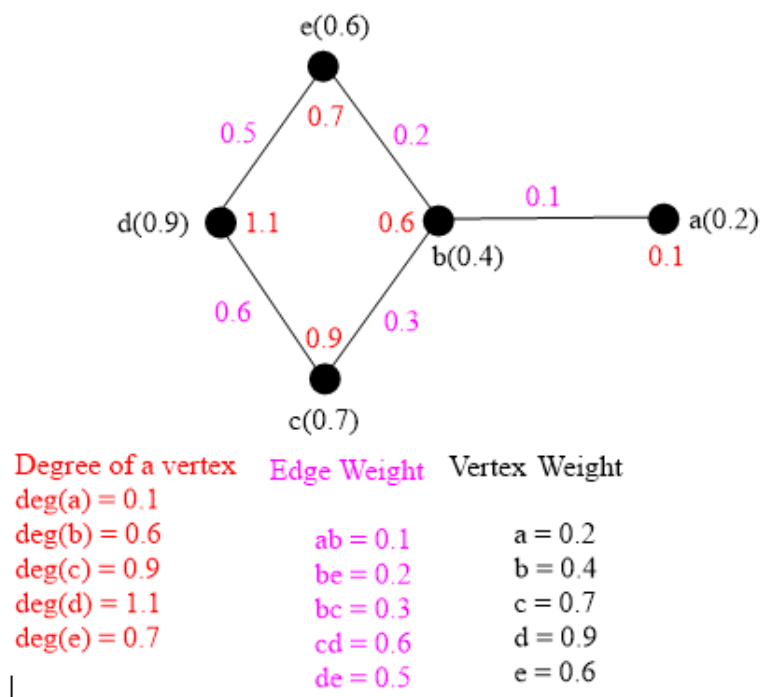


Figure 3. Example.

$$M(H) = \sum_{k=1}^5 \sigma(u_k)[du_k]^2$$

$$M(H) = (0.2)(0.1) + (0.4)(0.6) + (0.7)(0.9) + (0.9)(1.1) + (0.6)(1.1)$$

$$M(H) = 2.54$$

$$M^*(H) = \frac{1}{2} \sum_{kl \in E(G)} [\sigma(u_k)(du_k)\sigma(v_l)(dv_l)]$$

$$M^*(H) = \frac{1}{2} \left( (0.2)(0.1)(0.6)(0.4) + (0.4)(0.6)(0.6)(0.7) + (0.7)(0.9)(0.4)(0.6) \right. \\ \left. + (0.7)(0.9)(0.9)(1.1) + (0.6)(1.1)(0.9)(0.7) \right)$$

$$M^*(H) = 0.6482$$

$$R(H) = \frac{1}{2} \sum_{kl \in E(G)} [\sigma(u_k)(du_k)\sigma(v_l)(dv_l)]^{-1/2}$$

$$R(H) = \frac{1}{2} \left( ((0.2)(0.1)(0.6)(0.4))^{-\frac{1}{2}} + ((0.4)(0.6)(0.6)(0.7))^{-\frac{1}{2}} + ((0.7)(0.9)(0.4)(0.6))^{-\frac{1}{2}} \right. \\ \left. + ((0.7)(0.9)(0.9)(1.1))^{-\frac{1}{2}} + ((0.6)(1.1)(0.9)(0.7))^{-\frac{1}{2}} \right)$$

$$R(H) = 1.1294$$

$$H(H) = \frac{1}{2} \sum_{kl \in E(G)} \left[ \frac{1}{\sigma(u_k)(du_k) + \sigma(v_l)(dv_l)} \right]$$

$$M(H) = \left( \frac{1}{(0.2)(0.1) + (0.6)(0.4)} \right) + \left( \frac{1}{(0.4)(0.6) + (0.6)(0.7)} \right) + \left( \frac{1}{(0.7)(0.9) + (0.4)(0.6)} \right) \\ + \left( \frac{1}{(0.7)(0.9) + (0.9)(1.1)} \right) + \left( \frac{1}{(1.1)(0.9) + (0.6)(0.7)} \right)$$

$$M(H) = 3.9436.$$

#### 4. Application of fuzzy graphs

Social networking, the telecom sector and other fields of networking like traffic signals, air networks and many more are using fuzzy graphs frequently. Moreover, to optimize the real world problems in the field of mathematics, economics, statistic etc. fuzzy graphs are used. Fuzzy networks are also used as a reliable model to study natural flows like, flow of data between different routers through internet or flow of people traveling to different countries. Mordeson and Mathew have discussed a few applications of fuzzy graphs in detail in [16, 22].

#### 5. Zagreb indices of $P_{Z_4}$ and $P_{Z_5}$ pizza graph

A  $P_{Z_4}$  pizza graph, consisting of 9 vertices and 12 edges is shown in Figure 2.

**Theorem 1.** Let  $H = P_{Z_m}$  be a fuzzy pizza graph for  $m = 4, 5$ , then the first fuzzy zagreb index of the pizza graph is

$M(H) = 1.32$  for  $m = 4$  and  $M(H) = 1.675$  for  $m = 5$ .

*Proof.*

Let  $H = P_{z_m}$  be a fuzzy pizza graph shown in Figure 2.

**Case I:** When  $m = 4$ .

The representation of the vertex set is given as under: Weight of the vertices  $c_i$ 's is 0.6 has a total count 4 and all having vertex degree 0.3, weight of the vertices  $d_i$ 's is 0.4 has a total count 4 and all having vertex degree 0.8 and the weight of the central vertex  $b$  is 0.5 having vertex degree 0.4.

$$\begin{aligned} M(H) &= \sum_{k=1}^q \sigma(u_k)[du_k]^2, \\ &= (0.4)[4(0.8)^2] + (0.6)[4(0.3)^2] + (0.5)[1(0.4)^2] \\ &= 1.024 + 0.216 + 0.08 \\ &= 1.32. \end{aligned}$$

**Case II:**  $m = 5$ .

The representation of the vertex set is given as under: Weight of the vertices  $c_i$ 's is 0.6 has a total count 5 and all having vertex degree 0.3, weight of the vertices  $d_i$ 's is 0.4 has a total count 5 and all having vertex degree 0.8 and the weight of the central vertex  $b$  is 0.5 having vertex degree 0.5.

$$\begin{aligned} M(H) &= \sum_{k=1}^q \sigma(u_k)[du_k]^2, \\ &= (0.4)[5(0.8)^2] + (0.6)[5(0.3)^2] + (0.5)[1(0.5)^2] \\ &= 1.28 + 0.27 + 0.125 \\ &= 1.675. \end{aligned}$$

**Theorem 2.** Let  $H = P_{z_m}$  be a fuzzy pizza graph for  $m = 4, 5$ , then the second fuzzy zagreb index of the pizza graph is  $M^*(H) = 0.392$  for  $m = 4$  and  $M^*(H) = 0.5125$  for  $m = 5$ .

*Proof.*

Let  $H = P_{z_m}$  be a fuzzy pizza graph shown in Figure 2.

**Case I:**  $m = 4$ .

The representation of the edge sets is given as under. The edge set  $(d_i, d_j)$  having vertex weight  $(0.4, 0.4)$  has four (04) edges all having degree type  $(0.8, 0.8)$ . The edge set  $(c_i, d_i)$  having vertex weight  $(0.6, 0.4)$  has four (04) edges all having degree type  $(0.3, 0.8)$ . The edge set  $(c_i, b)$  having vertex weight  $(0.6, 0.5)$  has four (04) edges all having degree type  $(0.3, 0.4)$ .

$$\begin{aligned} M^*(H) &= \frac{1}{2} \sum_{k \in E(G)} [\sigma(u_k)(du_k)\sigma(v_l)(dv_l)], \\ &= \frac{1}{2} [(4)(0.4)(0.8)(0.4)(0.8)] + \frac{1}{2} [(4)(0.4)(0.8)(0.6)(0.3)] \\ &+ \frac{1}{2} [(4)(0.6)(0.3)(0.5)(0.4)] \\ &= \frac{1}{2} [(0.4096 + 0.2304 + 0.144)] \end{aligned}$$



$$\begin{aligned}
&= \frac{1}{2}[0.784] \\
&= 0.392.
\end{aligned}$$

**Case II:**  $m = 5$ .

The representation of the edge sets is given as under. The edge set  $(d_i, d_j)$  having vertex weight  $(0.4, 0.4)$  has five (05) edges all having degree type  $(0.8, 0.8)$ . The edge set  $(c_i, d_i)$  having vertex weight  $(0.6, 0.4)$  has five (05) edges all having degree type  $(0.3, 0.8)$ . The edge set  $(c_i, b)$  having vertex weight  $(0.6, 0.5)$  has five (05) edges all having degree type  $(0.3, 0.5)$ .

$$\begin{aligned}
M^*(H) &= \frac{1}{2} \sum_{kl \in E(G)} [\sigma(u_k)(du_k)\sigma(v_l)(dv_l)], \\
&= \frac{1}{2}[(5)(0.4)(0.8)(0.4)(0.8)] + \frac{1}{2}[(5)(0.4)(0.8)(0.6)(0.3)] \\
&\quad + \frac{1}{2}[(5)(0.6)(0.3)(0.5)(0.5)] \\
&= \frac{1}{2}[(0.512 + 0.288 + 0.225)] \\
&= \frac{1}{2}[1.025] \\
&= 0.5125.
\end{aligned}$$

**Theorem 3.** Let  $H = Pz_m$  be a fuzzy pizza graph for  $m = 4, 5$ , then the randic fuzzy zagreb index of the pizza graph is

$R(H) = 25.1234$  for  $m = 4$  and  $R(H) = 30.01425$  for  $m = 5$ .

*Proof.*

Let  $H = Pz_m$  be a fuzzy pizza graph shown in Figure 2.

**Case I:**  $m = 4$ .

The representation of the edge sets is given as under. The edge set  $(d_i, d_j)$  having vertex weight  $(0.4, 0.4)$  has four (04) edges all having degree type  $(0.8, 0.8)$ . The edge set  $(c_i, d_i)$  having vertex weight  $(0.6, 0.4)$  has four (04) edges all having degree type  $(0.3, 0.8)$ . The edge set  $(c_i, b)$  having vertex weight  $(0.6, 0.5)$  has four (04) edges all having degree type  $(0.3, 0.4)$ .

$$\begin{aligned}
R(H) &= \frac{1}{2} \sum_{kl \in E(G)} [\sigma(u_k)(du_k)\sigma(v_l)(dv_l)]^{-1/2}, \\
&= \frac{1}{2}[(4)[((0.4)(0.8)(0.4)(0.8))^{-1/2}] + \frac{1}{2}[(4)[(0.4)(0.8)(0.6)(0.3)]^{-1/2}] \\
&\quad + \frac{1}{2}[(4)[(0.6)(0.3)(0.5)(0.4)]^{-1/2}] \\
&= \frac{1}{2}[(4)(3.125) + (4)(4.1667) + (4)(5.270)] \\
&= \frac{1}{2}[50.2468] \\
&= 25.1234.
\end{aligned}$$

**Case II:**  $m = 5$ .

The representation of the edge sets is given as under. The edge set  $(d_i, d_j)$  having vertex weight  $(0.4, 0.4)$  has five (05) edges all having degree type  $(0.8, 0.8)$ . The edge set  $(c_i, d_i)$  having vertex weight  $(0.6, 0.4)$  has five (05) edges all having degree type  $(0.3, 0.8)$ . The edge set  $(c_i, b)$  having vertex weight  $(0.6, 0.5)$  has five (05) edges all having degree type  $(0.3, 0.5)$ .

$$\begin{aligned}
 R(H) &= \frac{1}{2} \sum_{kl \in E(G)} [\sigma(u_k)(du_k)\sigma(v_l)(dv_l)]^{-1/2}, \\
 &= \frac{1}{2} [(5)[((0.4)(0.8)(0.4)(0.8))^{-1/2}] + \frac{1}{2} [(5)[(0.4)(0.8)(0.6)(0.3)]^{-1/2}] \\
 &+ \frac{1}{2} [(5)[(0.6)(0.3)(0.5)(0.4)]^{-1/2}] \\
 &= \frac{1}{2} [(5)(3.125) + (5)(4.1667) + (5)(5.270)] \\
 &= \frac{1}{2} [60.01425] \\
 &= 30.01425.
 \end{aligned}$$

**Theorem 4.** Let  $H = Pz_m$  be a fuzzy pizza graph, then the harmonic fuzzy zagreb index of the pizza graph is

$$H(H) = 12.39 \text{ for } m = 4 \text{ and } H(H) = 14.721 \text{ for } m = 5.$$

*Proof.*

**Case I:**  $m = 4$ .

The representation of the edge sets is given as under. The edge set  $(d_i, d_j)$  having vertex weight  $(0.4, 0.4)$  has four (04) edges all having degree type  $(0.8, 0.8)$ . The edge set  $(c_i, d_i)$  having vertex weight  $(0.6, 0.4)$  has four (04) edges all having degree type  $(0.3, 0.8)$ . The edge set  $(c_i, b)$  having vertex weight  $(0.6, 0.5)$  has four (04) edges all having degree type  $(0.3, 0.4)$ .

$$\begin{aligned}
 H(H) &= \frac{1}{2} \sum_{kl \in E(G)} \left[ \frac{1}{\sigma(u_k)(du_k) + \sigma(v_l)(dv_l)} \right], \\
 &= \frac{1}{2} \left[ \frac{4}{(0.4)(0.8) + (0.4)(0.8)} + \frac{4}{(0.4)(0.8) + (0.6)(0.3)} \right. \\
 &+ \left. \frac{4}{(0.4)(0.8) + (0.6)(0.3)} \right] \\
 &= \frac{1}{2} \left[ \frac{4}{0.64} + \frac{4}{0.50} + \frac{4}{0.38} \right] \\
 &= \frac{1}{2} [24.78] \\
 &= 12.39.
 \end{aligned}$$

**Case II:**  $m = 5$ .

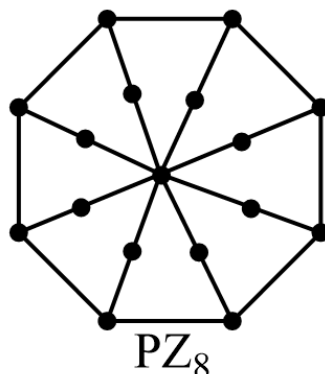
The representation of the edge sets is given as under. The edge set  $(d_i, d_j)$  having vertex weight  $(0.4, 0.4)$  has five (05) edges all having degree type  $(0.8, 0.8)$ . The edge set  $(c_i, d_i)$  having vertex weight  $(0.6, 0.4)$  has five (05) edges all having degree type  $(0.3, 0.8)$ . The edge set  $(c_i, b)$  having vertex weight

$(0.6, 0.5)$  has five (05) edges all having degree type  $(0.3, 0.5)$ .

$$\begin{aligned}
 H(H) &= \frac{1}{2} \sum_{kl \in E(G)} \left[ \frac{1}{\sigma(u_k)(du_k) + \sigma(v_l)(dv_l)} \right], \\
 &= \frac{1}{2} \left[ \frac{5}{(0.4)(0.8) + (0.4)(0.8)} + \frac{5}{(0.4)(0.8) + (0.6)(0.3)} \right. \\
 &\quad \left. + \frac{5}{(0.4)(0.8) + (0.6)(0.3)} \right] \\
 &= \frac{1}{2} \left[ \frac{5}{0.64} + \frac{5}{0.50} + \frac{5}{0.38} \right] \\
 &= \frac{1}{2} [29.721] \\
 &= 14.721.
 \end{aligned}$$

## 6. Zagrab indices of fuzzy pizza graph $Pz_m$

A  $Pz_8$  pizza graph is shown in Figure 4, consist of 17 vertices and 24 edges, similarly a  $Pz_m$  pizza graph consist of  $2m + 1$  vertices and  $3m$  edges.



**Figure 4.**  $Pz_8$  pizza graph.

**Theorem 5.** Let  $H = Pz_m$  be a fuzzy pizza graph, then the first fuzzy zagreb index of the pizza graph is  $M(H) = 0.31m + 0.005m^2$ .

*Proof.*

Let  $H = Pz_m$  be a fuzzy pizza graph shown in Figure 4. The representation of the vertex set is given as under: weight of the vertices  $c_i$ 's is 0.6 has a total count  $m$  and all having vertex degree 0.3, weight of the all vertices  $d_i$ 's is 0.4 has a total count  $m$  and has vertex degree 0.8 and the weight of the central vertex  $b$  is 0.5 having vertex degree  $0.1 \times m$

$$\begin{aligned}
 M(H) &= \sum_{k=1}^q \sigma(u_k)[du_k]^2, \\
 &= (0.4)[m(0.8)^2] + (0.6)[m(0.3)^2] + (0.5)[1(0.m)^2]
 \end{aligned}$$

$$\begin{aligned}
&= 0.256m + 0.054m + 0.005m^2 \\
&= 0.31m + 0.005m^2.
\end{aligned}$$

**Theorem 6.** Let  $H = Pz_m$  be a fuzzy pizza graph, then the second fuzzy zagreb index of the pizza graph is  $M^*(H) = (0.08m + 0.0045m^2)$ .

*Proof.*

Let  $H = Pz_m$  be a fuzzy pizza graph shown in Figure 4. The representation of the edge sets is given as under. The edge set  $(d_i, d_j)$  having vertex weight  $(0.4, 0.4)$  has  $m$  edges all having degree type  $(0.8, 0.8)$ . The edge set  $(c_i, d_i)$  having vertex weight  $(0.6, 0.4)$  has  $m$  edges all having degree type  $(0.3, 0.8)$ . The edge set  $(c_i, b)$  having vertex weight  $(0.6, 0.5)$  has  $m$  edges all having degree type  $(0.3, 0.1 \times m)$ .

$$\begin{aligned}
M^*(H) &= \frac{1}{2} \sum_{kl \in E(G)} [\sigma(u_k)(du_k)\sigma(v_l)(dv_l)], \\
&= \frac{1}{2} [(m)(0.4)(0.8)(0.4)(0.8)] + \frac{1}{2} [(m)(0.4)(0.8)(0.6)(0.3)] \\
&\quad + \frac{1}{2} [(m)(0.6)(0.3)(0.5)(0.1 \times m)] \\
&= \frac{1}{2} [(m)(0.1024)] + \frac{1}{2} [(m)(0.0576)] + \frac{1}{2} [(m^2)(0.009)] \\
&= \frac{1}{2} [0.16m + 0.009m^2] \\
&= 0.08m + 0.0045m^2.
\end{aligned}$$

**Theorem 7.** Let  $H = Pz_m$  be a fuzzy pizza graph, then the randic fuzzy zagreb index of the pizza graph is  $R(H) = (3.64585m + 5.2705\sqrt{m})$ .

*Proof.*

Let  $H = Pz_m$  be a fuzzy pizza graph shown in Figure 4. The representation of the edge sets is given as under. The edge set  $(d_i, d_j)$  having vertex weight  $(0.4, 0.4)$  has  $m$  edges all having degree type  $(0.8, 0.8)$ . The edge set  $(c_i, d_i)$  having vertex weight  $(0.6, 0.4)$  has  $m$  edges all having degree type  $(0.3, 0.8)$ . The edge set  $(c_i, b)$  having vertex weight  $(0.6, 0.5)$  has  $m$  edges all having degree type  $(0.3, 0.1 \times m)$ .

$$\begin{aligned}
R(H) &= \frac{1}{2} \sum_{kl \in E(G)} [\sigma(u_k)(du_k)\sigma(v_l)(dv_l)]^{-1/2}, \\
&= \frac{1}{2} [(m)[(0.4)(0.8)(0.4)(0.8)]^{-1/2}] + \frac{1}{2} [(m)[(0.4)(0.8)(0.6)(0.3)]^{-1/2}] \\
&\quad + \frac{1}{2} [(m)(0.6)(0.3)(0.5)(0.1 \times m)]^{-1/2}] \\
&= \frac{1}{2} [(m)(3.125) + (m)(4.1667) + (10.541)(\sqrt{m})] \\
&= \frac{1}{2} [(m)(7.2917) + (10.541)(\sqrt{m})] \\
&= 3.64585m + 5.2705\sqrt{m}.
\end{aligned}$$

**Theorem 8.** Let  $H = Pz_m$  be a fuzzy pizza graph, then the harmonic fuzzy zagreb index of the pizza graph is  $H(H) = (1.78125m + \frac{m}{0.36+0.1 \times m})$ .

*Proof.*

Let  $H = Pz_m$  be a fuzzy pizza graph shown in Figure 4. The representation of the edge sets is given as under. The edge set  $(d_i, d_j)$  having vertex weight  $(0.4, 0.4)$  has  $m$  edges all having degree type  $(0.8, 0.8)$ . The edge set  $(c_i, d_i)$  having vertex weight  $(0.6, 0.4)$  has  $m$  edges all having degree type  $(0.3, 0.8)$ . The edge set  $(c_i, b)$  having vertex weight  $(0.6, 0.5)$  has  $m$  edges all having degree type  $(0.3, 0.1 \times m)$ .

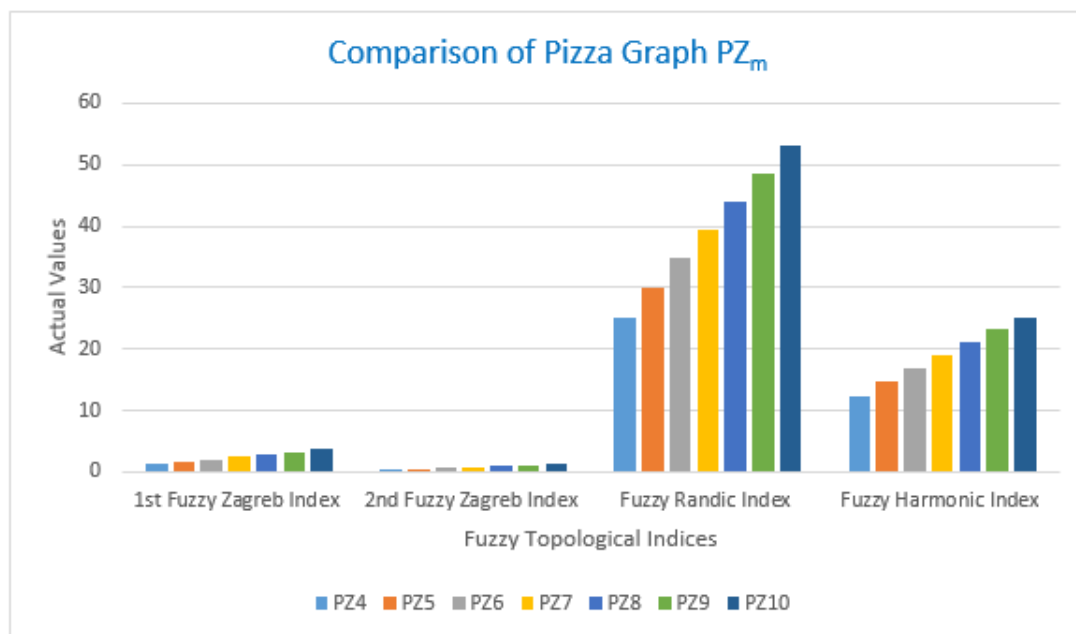
$$\begin{aligned}
 H(H) &= \frac{1}{2} \sum_{kl \in E(G)} \left[ \frac{1}{\sigma(u_k)(du_k) + \sigma(v_l)(dv_l)} \right], \\
 &= \frac{1}{2} \left[ \frac{m}{(0.4)(0.8) + (0.4)(0.8)} + \frac{m}{(0.4)(0.8) + (0.6)(0.3)} \right. \\
 &\quad \left. + \frac{m}{(0.6)(0.3) + (0.5)(0.1 \times m)} \right] \\
 &= \frac{1}{2} \left[ \frac{m}{0.64} + \frac{m}{0.50} + \frac{m}{0.18 + 0.05m} \right] \\
 &= \frac{1}{2} \left[ 3.5625m + \frac{m}{0.18 + 0.05m} \right] \\
 &= 1.78125m + \frac{m}{0.36 + 0.1 \times m}.
 \end{aligned}$$

## 7. Conclusions

In this paper, the first fuzzy zagreb index, second fuzzy zagreb index, randic fuzzy zagreb index and harmonic fuzzy zagreb index of  $H = Pz_m$  pizza graph has been discussed and it has been observed that all of these indexes can be calculated using the formula's defined in the preliminaries. We have also shown the comparison of  $H = Pz_m$  pizza graph in Table 1 and their graphs in Figure 5. This section is closed by raising the following open problem.

**Table 1.** Comparison of pizza graph  $PZ_m$ .

Pizza Graph $PZ_m$	1st Fuzzy Zagreb	2nd Fuzzy Zagreb	Fuzzy Randic	Fuzzy Harmonic
$PZ_4$	1.32	0.392	25.1234	12.39
$PZ_5$	1.675	0.5125	30.01425	14.721
$PZ_6$	2.04	0.642	34.7851	16.9375
$PZ_7$	2.415	0.7805	39.4654	19.0725
$PZ_8$	2.8	0.928	44.07403	21.14655
$PZ_9$	3.195	1.0845	48.6242	23.1741
$PZ_{10}$	3.6	1.25	53.1253	25.1654



**Figure 5.** Comparison of pizza graph.

## 8. Open problems

Open problems always urge the researcher to work on it or at least think about it. In our current paper, we have observed some problems that can be attempted using this approach we have adopted. Problems are mentioned below:

**Problem 1:** Fuzzy Zagreb indices of subdivided  $H = Pz_m$  be a pizza graph is still an open problem.

**Problem 2:** Fuzzy Zagreb indices of isomorphic copies of  $H = Pz_m$  be a pizza graph is still an open problem.

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## Conflict of interest

The authors declare that there are no conflicts of interest regarding the publication of this article.

## References

1. U. Ahmad, N. K. Khan, A. B. Saeid, Fuzzy topological indices with application to cybercrime problem, *Granular Comput.*, 2023, 1–14. <https://doi.org/10.1007/s41066-023-00365-2>

2. M. Akram, Bipolar fuzzy graphs, *Inform. Sci.*, **181** (2011), 5548–5564. <https://doi.org/10.1016/j.ins.2011.07.037>
3. M. Akram, W. A. Dudek, Interval-valued fuzzy graphs, *Comput. Math. Appl.*, **61** (2011), 289–299. <https://doi.org/10.1016/j.camwa.2010.11.004>
4. A. Al-Masarwah, m-polar fuzzy ideals of BCK/BCI-algebras, *J. King Saud Univ.-Sci.*, **31** (2019), 1220–1226. <https://doi.org/10.1016/j.jksus.2018.10.002>
5. A. Al-Masarwah, A. G. Ahmad, m-polar  $(\alpha, \beta)$ -fuzzy ideals in BCK/BCI-algebras, *Symmetry*, **11** (2019), 44. <https://doi.org/10.3390/sym11010044>
6. P. Bhattacharya, Some remarks on fuzzy graphs, *Pattern Recogn. Lett.*, **6** (1987), 297–302. [https://doi.org/10.1016/0167-8655\(87\)90012-2](https://doi.org/10.1016/0167-8655(87)90012-2)
7. M. Gulzar, D. Alghazzawi, M. H. Mateen, N. Kausar, A certain class of t-intuitionistic fuzzy subgroups, *IEEE Access*, **8** (2020), 163260–163268. <https://doi.org/10.1109/ACCESS.2020.3020366>
8. M. Gulzar, M. H. Mateen D. Alghazzawi, N. Kausar, A novel applications of complex intuitionistic fuzzy sets in group theory, *IEEE Access*, **8** (2020), 196075–196085. <https://doi.org/10.1109/ACCESS.2020.3034626>
9. M. Gulzar, M. H. Mateen, Y. M. Chu, D. Alghazzawi, G. Abbas, Generalized direct product of complex intuitionistic fuzzy subrings, *Int. J. Comput. Intell. Syst.*, **14** (2021), 582–593. <https://dx.doi.org/10.2991/ijcis.d.210106.001>
10. S. R. Islam, M. Pal, Further development of F-index for fuzzy graph and its application in Indian railway crime, *J. Appl. Math. Comput.*, 2022, 1–33. <https://doi.org/10.1007/s12190-022-01748-5>
11. S. R. Islam, M. Pal, First Zagreb index on a fuzzy graph and its application, *J. Intell. Fuzzy Syst.*, **40** (2021), 10575–10587. <https://dx.doi.org/10.3233/JIFS-201293>
12. S. R. Islam, M. Pal, Second Zagreb index for fuzzy graphs and its application in mathematical chemistry, *Iran. J. Fuzzy Syst.*, **20** (2023), 119–136. <https://doi.org/10.22111/IJFS.2023.7350>
13. N. Kausar, M. Munir, M. Gulzar, G. M. Addis, R. Anjum, Anti fuzzy bi-ideal on ordered AG-groupoids, *J. Indones. Math. Soc.*, **26** (2020), 299–318. <https://doi.org/10.22342/jims.26.3.862.299-318>
14. A. Kaufmann, *Introduction a la theorie des Sour-ensembles flous*, Masson et Cie, Paris, French, 1973.
15. S. Kalathian, S. Ramalingam, S. Raman, N. Srinivasan, Some topological indices in fuzzy graphs, *J. Intell. Fuzzy Syst.*, **39** (2020), 6033–6044. <https://doi.org/10.3233/JIFS-189077>
16. J. N. Mordeson, S. Mathew, Non-deterministic flow in fuzzy networks and its application in identification of human trafficking chains, *New Math. Nat. Comput.*, **3** (2017), 1–13. <https://doi.org/10.1142/S1793005717400087>
17. E. Mphako-Banda, Some polynomials of flower graphs, *Int. Math. Forum*, **2** (2007), 2511–2518. <https://doi.org/10.12988/imf.2007.07221>
18. M. A. Qamar, N. Hassan, *An approach to Q-neutrosophic soft rings*, Infinite Study, 2019.

19. M. Rajeshwari, R. Murugesan, M. Kaviyarasu, C. Subrahmanyam, Bipolar fuzzy graph on certain topological indices, *J. Algebr. Stat.*, **13** (2022), 2476–2481.
20. A. Rosenfeld, *Fuzzy graph*, In: Fuzzy sets and their Applications to cognitive and decision process, Academic Press, New York, 1975, 77–95.
21. M. Pal, S. Samanta, G. Ghorai, *Modern trends in fuzzy graph theory*, Springer, Berlin, 2020.
22. V. Obraztsov, V. V. Krasnoproshin, Management optimization problems on fuzzy graphs, *Invest. Eur. Direc. Econ. Empresa*, **1** (1995), 63–70.
23. Z. S. Mufti, E. Fatima, R. Anjum, F. Tchier, Q. Xin, M. M. Hossain, Computed first and second fuzzy Zagreb indices of linear and multiacyclic hydrocarbons, *J. Funct. Space.*, **2022** (2022). <https://doi.org/10.1155/2022/6281592>
24. L. A. Zadeh, Similarity relations and fuzzy orderings, *Inform. Sci.*, **3** (1971), 177–200. [https://doi.org/10.1016/S0020-0255\(71\)80005-1](https://doi.org/10.1016/S0020-0255(71)80005-1)
25. L. A. Zadeh, Fuzzy sets, *Inform. Control*, **8** (1965), 338–353. [https://doi.org/10.1016/S0019-9958\(65\)90241-X](https://doi.org/10.1016/S0019-9958(65)90241-X)



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