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Research article

Integrating TOPSIS and ELECTRE-I methods with cubic m-polar fuzzy sets and its application to the diagnosis of psychiatric disorders

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Abstract: Many real-world decision-making issues frequently involve competing sets of criteria, uncertainty, and inaccurate information. Some of these require the involvement of a group of decision-makers, where it is necessary to reduce the various available individual preferences to a single collective preference. To enhance the effectiveness of multi-criteria decisions, multi-criteria decision-making is a popular decision-making technique that makes the procedure more precise, reasonable, and efficient. The "Technique for Order Preference by Similarity to Ideal Solution (TOPSIS)" and "Elimination and Choice Transforming Reality (ELECTRE)" are prominent ranking methods and widely used in the multi-criteria decision-making to solve complicated decision-making problems. In this study, two m-polar fuzzy set-based ranking methods are proposed by extending the ELECTRE-I and TOPSIS approaches equipped with cubic m-polar fuzzy (CmPF) sets, where the experts provide assessment results on feasible alternatives through a CmPF decision matrix. The first proposed method, CmPF-TOPSIS, focuses on the alternative that is closest to a CmPF positive ideal solution and farthest away from the CmPF negative ideal solution. The Euclidean and normalized Euclidean distances are used to determine the proximity of an alternative to ideal solutions. In contrast, the second developed method is CmPF-ELECTRE-I which uses an outranking directed decision graph to determine the optimal alternative, which entirely depends on the CmPF concordance and discordance sets. Furthermore, a practical case study is carried out in the diagnosis of impulse

control disorders to illustrate the feasibility and applicability of the proposed methods. Finally, a comparative analysis is performed to demonstrate the veracity, superiority, and effectiveness of the proposed methods.

Keywords: C*m*PF set; multi-criteria decision-making; C*m*PF-ELECTRE-I approach; C*m*PF-TOPSIS approach; comparative analysis
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1. Introduction

Today, when decision-makers (DMs) deal with daily-life problems caused by indefinite and vague information without the proper tools, they lead to imprecise reasoning and inexact solutions. Consequently, it is extremely difficult for DMs to make sensible and rational decisions when dealing with such problems. As a result, addressing vagueness and uncertainties has become especially important for these problems and difficulties. Zadeh [1] proposed a novel fuzzy set model as an extension of the crisp set theory. It was a significant accomplishment and a watershed moment in the evolution of uncertainty theories. To address the different daily-life decision problems having vague and uncertain information, several extensions of fuzzy sets have been introduced by researchers worldwide, including intuitionistic fuzzy sets (IFSs) [2], and Pythagorean fuzzy sets (PFSs) [3]. However, a new extension of the fuzzy set that is based on 'm' membership degrees for each alternative of the universe was suggested by Chen et al. [4] and named m-polar fuzzy sets (mPFSs). An mPFS on a set $\mathfrak{S}$ is a mapping from $\mathfrak{S}$ to 'm' times the cross product of closed unit intervals. The motivation behind the invention of mPFS is the fact that multi-polar information occurs in almost every domain because data sets of different real-world problems sometimes contain multiple characters and agents.

With another perspective, Jun et al. [5] generalized Atanassov's IFS and introduced a new model called the cubic set (CS), which deals with two parts of information, one of which is an interval-valued fuzzy (IVF) set represents membership grades and the other is a non-membership function. However, CS theory fails to deal with m-polar fuzzy information in CS formats. To overcome this shortcoming and complexity in CS theory, Riaz and Hashmi [6] introduced the concept of a cubic m-polar fuzzy set (or CmPFS). This innovative idea of the CmPF set is a generalized structure of CS and mPFS, and it has the ability to explore both m-polar data and cubic information accumulatively. Therefore, one can easily observe that CS and mPFS are special cases of CmPFS.

Decision-making is a technique for making choices involving the identification of decisions, gathering information, and resolving problems to select the best alternative. Additionally, multi-criteria decision-making (MCDM) is a general term for methods that provides a quantitative and systematic approach to support decision-making problems involving multiple criteria and alternatives [7]. MCDM is a widely used decision methodology that aims to help the decision-makers in performing the decision process more explicitly, logically, and efficiently. To efficiently address multiple-criteria decision-making (MCDM) problems, the TOPSIS is a widely adapted and prominent ranking technique in MCDM. The TOPSIS approach works on a fundamental principle: select the best alternative that is closest to a positive ideal solution (PIS) and farthest away from the negative ideal solution (NIS). In 1981, Hwang and Yoon [8] were the first researchers who introduced a crisp version of the TOPSIS approach to cope with real-life MCDM issues. However, it is very rare in real-life decision-making to deals with crisp and precise data, most of the time, the data is imprecise and vague. To tackle the complexities and uncertainty of real-life MCDM problems containing uncertainties, Chen [9] merged the theories of TOPSIS and fuzzy sets and proposed the fuzzy version of the TOPSIS approach. Later, Amir [10] used the fuzzy TOPSIS method for project selection for oil field development. Chakraborty [11] provided a comparative analysis between existing TOPSIS and modified TOPSIS methods. In addition, Boran et al. [12] introduced the generalized version of the TOPSIS approach based on IFSs. Bilgili et al. [13] used intuitionistic fuzzy TOPSIS method for the evaluation of renewable energy alternatives for sustainable development in Turkey. Zhang and Xu [14] extended the TOPSIS method under the Pythagorean fuzzy environment. Further, Akram et al. [15] implemented the Pythagorean fuzzy TOPSIS method for the evaluation of risk in failure modes and effect analysis. Adeel et al. [16] proposed the m-polar fuzzy linguistic TOPSIS approach for MCGDM problems involving multi-polar information. Chen et al. [17] introduced a proportional interval type-2 hesitant fuzzy TOPSIS approach based on Hamacher aggregation operators and optimization models. Arora and Naithani [18] used the TOPSIS approach to compute exponential divergence measures for Pythagorean fuzzy sets. Farrokhizadeh et al. [19] proposed interval-valued spherical fuzzy TOPSIS method based on similarity measure and also introduced spherical fuzzy maximum deviation methodology for finding unknown criteria weights. Recently, Ali et al. [20] used m-polar fuzzy aggregation operators for multi-criteria decision-making problems. Bairagi [21] used the extended TOPSIS method under subjective and objective factors for the selection of robotic systems. For other related notations, terminologies, and applications, the readers are referred to [22–28].

Outranking is an MCDM technique in which alternatives are systematically compared to one another on each criterion. The comparison between the alternatives leads to numerical results that show the concordance and/or the discordance between them. In 1966, Benayoun [29] was the first who suggested the crisp version of an outranking approach called ELECTRE. Roy [30] proposed the generalized version of ELECTRE called the ELECTRE-I approach. Since then, several other versions of ELECTRE method are developed by the researchers (i.e., ELECTRE-II [31], ELECTRE-III [32], et cetera). Nowadays, in the group of outranking approaches, the ELECTRE method and its variants such as ELECTRE I, II, III, and IV play a key role in different real-world disciplines. The primary goal of ELECTRE is to make the best use of the outranking relationships. For more details and further members of the ELECTRE family, see [33]. To address fuzzy outranking issues, Hatami-Marbini and Tavana [34] created the extension of the ELECTRE-I method under a fuzzy context. Later, Rouyendegh and Erkan [35] applied the fuzzy ELECTRE technique for academic staff selection. Wu and Chen [36] introduced the extended ELECTRE-I method based on IFSs for dealing with MCDM problems, which requires both membership and nonmembership information. Kirisci et al. [37] introduced Fermatean fuzzy ELECTRE approach and employed it for the selection of most suitable biomedical material. Akram et al. [38] proposed m-polar fuzzy ELECTRE-I approach to deal with MCDM scenarios. Jagtap et al. [39] developed m-polar fuzzy ELECTRE-I algorithm for the rank assessment of robots. Further, Adeel et al. [40] presented the m-polar fuzzy linguistic ELECTRE-I method for linguistic group decision-making problems. For other related notions of ELECTRE methods and cubical fuzzy systems, the readers may refer to [41–45].

1.1. *Motivations*

Our inspiration to extend the TOPSIS and ELECTRE-I methods under C*m*PF information is based on the following reasons.

- The C*m*PF set offers a wide range of applications as it combines the benefits of CS and *m*-polar fuzzy sets. However, cubic methods, especially the C*m*PF MCDM method, remain a challenge for us in the multi-polar fuzzy case, which we have addressed in this paper.
- Existing strategies to solve the MCDM problem are confined to deal with *m*-polar ambiguous information. These techniques are incapable to account for the cubic nature of *m*-polar fuzzy data. As a result, information may be lost, leading to undesirable outcomes. However, the newly proposed methods can overcome current technological restrictions.
- The limited literature on C*m*PFS is a major incentive for our research as there are no preexisting decision techniques, based on TOPSIS or ELECTRE-I methods based on C*m*PF data. Therefore, C*m*PF-TOPSIS and C*m*PF- ELECTRE-I models are developed to address this research gap.

1.2. *Study contributions*

The major contributions of this article are:

- The most important contribution of this work is the development of two novel hybrid MCDM techniques for effectively and precisely manipulating C*m*PF information, which are C*m*PF-TOPSIS and C*m*PF ELECTRE-I.
- Two flowcharts are presented to better understand the developed approaches, which completely demonstrate the step-by-step methodology of both developed algorithms under C*m*PF-TOPSIS and C*m*PF ELECTRE-I.
- A practical case study was carried out in the diagnosis of psychiatric disorders to illustrate the feasibility and applicability of the initiated MCDM approaches.
- Finally, a comparative analysis is performed to demonstrate the veracity, superiority, and effectiveness of the developed methods.

The remaining contents of this article are provided as follows: In Section 2, we review some basic terminologies and fundamental properties of the hybrid C*m*PF model along with examples. In Section 3, we propose the C*m*PF-TOPSIS approach and provide a numerical application in medical-diagnosis of the impulse control disorders supported by a developed algorithm. In Section 4, we develop an algorithm for the initiated ELECTRE-I method under the C*m*PF environment and apply it to a similar MCDM problem as provided in Section 3 (that is, psychiatric diagnosis of impulse control disorders). In Section 5, we provide a comparative study between proposed and existing techniques. In the end, Section 6 gives the concluding remarks and future directions.

2. Preliminaries

In this section, we review some basic definitions and operations of the hybrid model, namely, C*m*PFSs. Throughout the paper, we use $\mathfrak{S}$ as a universal set.

Definition 2.1. *[4] An m-polar fuzzy set (or mPFS) $\mathcal{M}_{\mathfrak{P}}$ on a universe $\mathfrak{S}$ is a mapping, from $\mathfrak{S}$ to $[0,1]^m$, that assigns m-independent fuzzy membership values to each element of $\mathfrak{S}$, mathematically, we can write it as:*

$$\begin{aligned}\mathcal{M}_{\mathfrak{P}} &= \{\langle \tau, \{\eta^1(\tau), \eta^2(\tau), \eta^3(\tau), \dots \eta^m(\tau)\}\rangle \mid \tau \in \mathfrak{S}, m \in \mathbb{N}\},\\ &= \{\langle \tau, (\eta^\alpha(\tau))_{\alpha=1}^m\rangle \mid \tau \in \mathfrak{S}, m \in \mathbb{N}\},\end{aligned}$$

where $(\eta^\alpha(\tau))_{\alpha=1}^m$ denotes the 'm' membership degrees of an element.

Definition 2.2. *[5] A cubic set (or CS) $\mathcal{C}$ on $\mathfrak{S}$ is an object which is given by*

$$\mathcal{C} = \{\langle \tau, \mathfrak{Q}(\tau), \eta(\tau)\rangle \mid \tau \in \mathfrak{S}\},$$

where $\mathfrak{Q} = [\mathfrak{Q}_l, \mathfrak{Q}_u]$ is an IVF set which serves as membership, and η represents the non-membership function.

Definition 2.3. *[6] A cubic m-polar fuzzy set (or CmPFS) $\mathcal{C}_{m\mathfrak{P}}$ on $\mathfrak{S}$ is an object which is given as:*

$$\mathcal{C}_{m\mathfrak{P}} = \{\langle \tau, \{\mathfrak{Q}^1(\tau), \mathfrak{Q}^2(\tau), \dots, \mathfrak{Q}^m(\tau)\}, \{\eta^1(\tau), \eta^2(\tau), \dots, \eta^m(\tau)\}\rangle \mid \tau \in \mathfrak{S}\},$$

$$= \{\langle \tau, \mathfrak{Q}^\alpha(\tau), \eta^\alpha(\tau)\rangle_{\alpha=1}^m \mid \tau \in \mathfrak{S}\},$$

where $\mathfrak{Q}^\alpha = [\mathfrak{Q}_l^\alpha, \mathfrak{Q}_u^\alpha]$ are IVF sets serves as membership, and η^α are non-membership functions. For convenience, we can write cubic m-polar fuzzy number (CmPFN) as $\langle \mathfrak{Q}^\alpha(\tau), \eta^\alpha(\tau)\rangle_{\alpha=1}^m$.

Example 2.1. *Let $\mathfrak{S} = \{\tau_1, \tau_2, \tau_3, \tau_4\}$ ba a universe. Then the C3PFS $\mathcal{C}_{3\mathfrak{P}}$ over $\mathfrak{S}$ is given below:*

$$\begin{aligned}\mathcal{C}_{3\mathfrak{P}} =&\{\langle \tau_1, \{[0.32, 0.56], [0.55, 0.70], [0.35, 0.45]\}, \{0.66, 0.75, 0.45\}\rangle,\\ &\langle \tau_2, \{[0.22, 0.46], [0.30, 0.35], [0.45, 0.60]\}, \{0.45, 0.70, 0.25\}\rangle,\\ &\langle \tau_3, \{[0.45, 0.55], [0.45, 0.65], [0.75, 0.80]\}, \{0.70, 0.43, 0.40\}\rangle,\\ &\langle \tau_4, \{[0.30, 0.60], [0.55, 0.74], [0.66, 0.79]\}, \{0.78, 0.55, 0.65\}\rangle\}.\end{aligned}$$

Definition 2.4. *[6] A CmPFS $\mathcal{C}_{m\mathfrak{P}} = \{\langle \tau, \mathfrak{Q}^\alpha(\tau), \eta^\alpha(\tau)\rangle_{\alpha=1}^m \mid \tau \in \mathfrak{S}\}$ on $\mathfrak{S}$ is said to be internal cubic m-polar fuzzy set (or ICmPFS) if $\eta^\alpha(\tau) \in [\mathfrak{Q}_l^\alpha(\tau), \mathfrak{Q}_u^\alpha(\tau)]$. Similarly, a CmPFS $\mathcal{C}_{m\mathfrak{P}}$ on $\mathfrak{S}$ is said to be an external cubic m-polar fuzzy set (or ECmPFS) if $\eta^\alpha(\tau) \notin [\mathfrak{Q}_l^\alpha(\tau), \mathfrak{Q}_u^\alpha(\tau)]$.*

2.1. Operations on CmPFSs

In this sub-section, we discuss the operations of CmPFSs along with numerical examples.

Definition 2.5. *[6] Let $\mathcal{C}_{m\mathfrak{P}}^1 = \{\langle \tau, \mathfrak{Q}^\alpha(\tau), \eta^\alpha(\tau)\rangle_{\alpha=1}^m \mid \tau \in \mathfrak{S}\}$ and $\mathcal{C}_{m\mathfrak{P}}^2 = \{\langle \tau, \mathfrak{R}^\alpha(\tau), \zeta^\alpha(\tau)\rangle_{\alpha=1}^m \mid \tau \in \mathfrak{S}\}$ be the CmPFSs, and $\kappa > 0$ be any real number. Then, the operations on these sets under $\mathcal{P}$-ordering are defined as follows:*

(1) Equality: $\mathcal{C}_{m\mathfrak{P}}^1 = \mathcal{C}_{m\mathfrak{P}}^2$ if and only if $\mathfrak{Q}^\alpha(\tau) = \mathfrak{R}^\alpha(\tau)$ and $\eta^\alpha(\tau) = \zeta^\alpha(\tau)$, $\quad \forall\ \alpha = 1, 2, 3, \dots, m$ and $\tau \in \mathfrak{S}$.

(2) Complement: $(\mathcal{C}_{m\mathfrak{P}}^1)^c = \{\langle \tau, [1 - \mathfrak{Q}_u^\alpha(\tau), 1 - \mathfrak{Q}_l^\alpha(\tau)], 1 - \eta^\alpha(\tau)\rangle_{\alpha=1}^m \mid \tau \in \mathfrak{S}\}$.

(3) *Subset:* $\mathfrak{C}^1_{m\mathfrak{P}} \subseteq_{\mathcal{P}} \mathfrak{C}^2_{m\mathfrak{P}}$ *if and only if* $\mathfrak{Q}^\alpha(\tau) \subseteq \mathfrak{R}^\alpha(\tau)$ *and* $\eta^\alpha(\tau) \leq \zeta^\alpha(\tau)$, $\forall\ \alpha = 1, 2, \ldots, m$ *and* $\tau \in \mathfrak{S}$.

(4) *Union:* $\mathfrak{C}^1_{m\mathfrak{P}} \bigcup_{\mathcal{P}} \mathfrak{C}^2_{m\mathfrak{P}} = \{\langle \tau, \mathfrak{Q}^\alpha(\tau) \vee \mathfrak{R}^\alpha(\tau), \eta^\alpha(\tau) \vee \zeta^\alpha(\tau) \rangle_{\alpha=1}^m \mid \tau \in \mathfrak{S}\}$, *where* $\mathfrak{Q}^\alpha(\tau) \vee \mathfrak{R}^\alpha(\tau) = [\max\{\mathfrak{Q}^\alpha_l(\tau), \mathfrak{R}^\alpha_l(\tau)\}, \max\{\mathfrak{Q}^\alpha_u(\tau), \mathfrak{R}^\alpha_u(\tau),\}]$ *and* $\eta^\alpha(\tau) \vee \zeta^\alpha(\tau) = \max\{\eta^\alpha(\tau), \zeta^\alpha(\tau)\}$, $\forall\ \alpha = 1, 2, \ldots, m$.

(5) *Intersection:* $\mathfrak{C}^1_{m\mathfrak{P}} \bigcap_{\mathcal{P}} \mathfrak{C}^2_{m\mathfrak{P}} = \{\langle \tau, \mathfrak{Q}^\alpha(\tau) \wedge \mathfrak{R}^\alpha(\tau), \eta^\alpha(\tau) \wedge \zeta^\alpha(\tau) \rangle_{\alpha=1}^m \mid \tau \in \mathfrak{S}\}$, *where* $\mathfrak{Q}^\alpha(\tau) \wedge \mathfrak{R}^\alpha(\tau) = [\min\{\mathfrak{Q}^\alpha_l(\tau), \mathfrak{R}^\alpha_l(\tau)\}, \min\{\mathfrak{Q}^\alpha_u(\tau), \mathfrak{R}^\alpha_u(\tau),\}]$ *and* $\eta^\alpha(\tau) \wedge \zeta^\alpha(\tau) = \min\{\eta^\alpha(\tau), \zeta^\alpha(\tau)\}$, $\forall\ \alpha = 1, 2, \ldots, m$.

(6) *Ring sum:* $\mathfrak{C}^1_{m\mathfrak{P}} \bigoplus_{\mathcal{P}} \mathfrak{C}^2_{m\mathfrak{P}} = \{\langle \tau, [\mathfrak{Q}^\alpha_l(\tau) + \mathfrak{R}^\alpha_l(\tau) - \mathfrak{Q}^\alpha_l(\tau) \cdot \mathfrak{R}^\alpha_l(\tau), \mathfrak{Q}^\alpha_u(\tau) + \mathfrak{R}^\alpha_u(\tau) - \mathfrak{Q}^\alpha_u(\tau) \cdot \mathfrak{R}^\alpha_u(\tau)], \eta^\alpha(\tau) + \zeta^\alpha(\tau) - \eta^\alpha(\tau) \cdot \zeta^\alpha(\tau) \rangle_{\alpha=1}^m \mid \tau \in \mathfrak{S}\}$.

(7) *Ring product:* $\mathfrak{C}^1_{m\mathfrak{P}} \bigotimes_{\mathcal{P}} \mathfrak{C}^2_{m\mathfrak{P}} = \{\langle \tau, [\mathfrak{Q}^\alpha_l(\tau) \cdot \mathfrak{R}^\alpha_l(\tau), \mathfrak{Q}^\alpha_u(\tau) \cdot \mathfrak{R}^\alpha_u(\tau)], \eta^\alpha(\tau) \cdot \zeta^\alpha(\tau) \rangle_{\alpha=1}^m \mid \tau \in \mathfrak{S}\}$.

(8) *κ-scalar power:* $(\mathfrak{C}^1_{m\mathfrak{P}})^\kappa = \{\langle \tau, [(\mathfrak{Q}^\alpha_l(\tau))^\kappa, (\mathfrak{Q}^\alpha_u(\tau))^\kappa], (\eta^\alpha(\tau))^\kappa \rangle_{\alpha=1}^m \mid \tau \in \mathfrak{S}\}$.

(9) *κ-scalar product:* $\kappa \cdot (\mathfrak{C}^1_{m\mathfrak{P}}) = \{\langle \tau, [1 - (1 - \mathfrak{Q}^\alpha_l(\tau))^\kappa, 1 - (1 - \mathfrak{Q}^\alpha_u(\tau))^\kappa], 1 - (1 - \eta^\alpha(\tau))^\kappa \rangle_{\alpha=1}^m \mid \tau \in \mathfrak{S}\}$.

Example 2.2. *Consider* $\mathfrak{C}^1_{3\mathfrak{P}}$ *and* $\mathfrak{C}^2_{3\mathfrak{P}}$ *be two C3PFSs over the universe* $\mathfrak{S} = \{\tau_1, \tau_2, \tau_3\}$, *which are given below:*

$$\begin{aligned}
\mathfrak{C}^1_{3\mathfrak{P}} =& \{\langle \tau_1, \{[0.32, 0.56], [0.55, 0.70], [0.35, 0.45]\}, \{0.66, 0.75, 0.45\}\rangle,\\
&\langle \tau_2, \{[0.22, 0.46], [0.30, 0.35], [0.45, 0.60]\}, \{0.45, 0.70, 0.25\}\rangle,\\
&\langle \tau_3, \{[0.45, 0.55], [0.45, 0.65], [0.75, 0.80]\}, \{0.70, 0.43, 0.40\}\rangle\},\\
\mathfrak{C}^2_{3\mathfrak{P}} =& \{\langle \tau_1, \{[0.45, 0.55], [0.32, 0.60], [0.55, 0.70]\}, \{0.77, 0.70, 0.55\}\rangle,\\
&\langle \tau_2, \{[0.20, 0.35], [0.40, 0.55], [0.40, 0.65]\}, \{0.35, 0.50, 0.45\}\rangle,\\
&\langle \tau_3, \{[0.25, 0.65], [0.55, 0.60], [0.65, 0.85]\}, \{0.30, 0.55, 0.60\}\rangle\}.
\end{aligned}$$

Then, $\mathcal{P}$-Ordering operations on these C3PFSs are evaluated as:

$$\begin{aligned}
(\mathfrak{C}^1_{3\mathfrak{P}})^c =& \{\langle \tau_1, \{[0.44, 0.68], [0.30, 0.45], [0.55, 0.65]\}, \{0.34, 0.25, 0.55\}\rangle,\\
&\langle \tau_2, \{[0.54, 0.78], [0.65, 0.70], [0.40, 0.55]\}, \{0.55, 0.30, 0.75\}\rangle,\\
&\langle \tau_3, \{[0.45, 0.55], [0.35, 0.55], [0.20, 0.25]\}, \{0.30, 0.57, 0.60\}\rangle\}.\\
\mathfrak{C}^1_{3\mathfrak{P}} \bigcup_{\mathcal{P}} \mathfrak{C}^2_{3\mathfrak{P}} =& \{\langle \tau_1, \{[0.45, 0.56], [0.55, 0.70], [0.55, 0.70]\}, \{0.77, 0.75, 0.55\}\rangle,\\
&\langle \tau_2, \{[0.22, 0.46], [0.40, 0.55], [0.45, 0.65]\}, \{0.45, 0.70, 0.45\}\rangle,\\
&\langle \tau_3, \{[0.45, 0.65], [0.55, 0.65], [0.75, 0.85]\}, \{0.70, 0.55, 0.60\}\rangle\}.\\
\mathfrak{C}^1_{3\mathfrak{P}} \bigcap_{\mathcal{P}} \mathfrak{C}^2_{3\mathfrak{P}} =& \{\langle \tau_1, \{[0.32, 0.55], [0.32, 0.60], [0.35, 0.45]\}, \{0.66, 0.70, 0.45\}\rangle,\\
&\langle \tau_2, \{[0.20, 0.35], [0.30, 0.35], [0.40, 0.60]\}, \{0.35, 0.50, 0.25\}\rangle,\\
&\langle \tau_3, \{[0.25, 0.55], [0.45, 0.60], [0.65, 0.80]\}, \{0.30, 0.43, 0.40\}\rangle\}.\\
\mathfrak{C}^1_{3\mathfrak{P}} \bigoplus_{\mathcal{P}} \mathfrak{C}^2_{3\mathfrak{P}} =& \{\langle \tau_1, \{[0.63, 0.80], [0.69, 0.88], [0.71, 0.84]\}, \{0.92, 0.93, 0.75\}\rangle,
\end{aligned}$$

$$\langle\tau_2,\{[0.38,0.65],[0.58,0.71],[0.67,0.86]\},\{0.64,0.85,0.59\}\rangle,$$
$$\langle\tau_3,\{[0.59,0.84],[0.75,0.86],[0.91,0.97]\},\{0.79,0.74,0.76\}\rangle\}.$$
$$\mathfrak{C}^1_{3\mathfrak{P}}\bigotimes_{\mathcal{P}}\mathfrak{C}^2_{3\mathfrak{P}}=\{\langle\tau_1,\{[0.14,0.31],[0.18,0.42],[0.19,0.32]\},\{0.51,0.53,0.25\}\rangle,$$
$$\langle\tau_2,\{[0.04,0.16],[0.12,0.19],[0.18,0.39]\},\{0.16,0.35,0.11\}\rangle,$$
$$\langle\tau_3,\{[0.11,0.36],[0.25,0.39],[0.49,0.68]\},\{0.21,0.24,0.24\}\rangle\}.$$

For $\kappa = 0.3$, we have:

$$(\mathfrak{C}^1_{3\mathfrak{P}})^{0.3}=\{\langle\tau_1,\{[0.71,0.84],[0.84,0.90],[0.73,0.79]\},\{0.88,0.92,0.79\}\rangle,$$
$$\langle\tau_2,\{[0.63,0.79],[0.70,0.73],[0.79,0.86]\},\{0.79,0.90,0.66\}\rangle,$$
$$\langle\tau_3,\{[0.79,0.84],[0.79,0.88],[0.92,0.94]\},\{0.90,0.78,0.76\}\rangle\}.$$
$$0.3\cdot\mathfrak{C}^1_{3\mathfrak{P}}=\{\langle\tau_1,\{[0.54,0.81],[0.80,0.91],[0.58,0.70]\},\{0.88,0.94,0.70\}\rangle,$$
$$\langle\tau_2,\{[0.39,0.71],[0.51,0.58],[0.70,0.84]\},\{0.70,0.91,0.44\}\rangle,$$
$$\langle\tau_3,\{[0.70,0.80],[0.70,0.88],[0.94,0.96]\},\{0.91,0.68,0.64\}\rangle\}.$$

Definition 2.6. *[6] Let $\mathfrak{C}^1_{m\mathfrak{P}}=\{\langle\tau,\mathfrak{Q}^\alpha(\tau),\eta^\alpha(\tau)\rangle_{\alpha=1}^m\mid\tau\in\mathfrak{S}\}$ and $\mathfrak{C}^2_{m\mathfrak{P}}=\{\langle\tau,\mathfrak{R}^\alpha(\tau),\zeta^\alpha(\tau)\rangle_{\alpha=1}^m\mid\tau\in\mathfrak{S}\}$ be the CmPFSs and $\kappa>0$ be any real number. Then, the operations on these sets under $\mathcal{R}$-ordering are defined as follows:*

(1) Equality: $\mathfrak{C}^1_{m\mathfrak{P}}=\mathfrak{C}^2_{m\mathfrak{P}}$ if and only if $\mathfrak{Q}^\alpha(\tau)=\mathfrak{R}^\alpha(\tau)$ and $\eta^\alpha(\tau)=\zeta^\alpha(\tau)$, $\forall\ \alpha=1,2,3,\ldots,m$ and $\tau\in\mathfrak{S}$.

(2) Complement: $(\mathfrak{C}^1_{m\mathfrak{P}})^c=\{\langle\tau,[1-\mathfrak{Q}^\alpha_u(\tau),1-\mathfrak{Q}^\alpha_l(\tau)],1-\eta^\alpha(\tau)\rangle_{\alpha=1}^m\mid\tau\in\mathfrak{S}\}$.

(3) Subset: $\mathfrak{C}^1_{m\mathfrak{P}}\subseteq_{\mathcal{P}}\mathfrak{C}^2_{m\mathfrak{P}}$ if and only if $\mathfrak{Q}^\alpha(\tau)\subseteq\mathfrak{R}^\alpha(\tau)$ and $\eta^\alpha(\tau)\geq\zeta^\alpha(\tau)$, $\forall\ \alpha=1,2,\ldots,m$ and $\tau\in\mathfrak{S}$.

(4) Union: $\mathfrak{C}^1_{m\mathfrak{P}}\bigcup_{\mathcal{R}}\mathfrak{C}^2_{m\mathfrak{P}}=\{\langle\tau,\mathfrak{Q}^\alpha(\tau)\vee\mathfrak{R}^\alpha(\tau),\eta^\alpha(\tau)\wedge\zeta^\alpha(\tau)\rangle_{\alpha=1}^m\mid\tau\in\mathfrak{S}\}$, where $\mathfrak{Q}^\alpha(\tau)\vee\mathfrak{R}^\alpha(\tau)=[\max\{\mathfrak{Q}^\alpha_l(\tau),\mathfrak{R}^\alpha_l(\tau)\},\max\{\mathfrak{Q}^\alpha_u(\tau),\mathfrak{R}^\alpha_u(\tau),\}]$ and $\eta^\alpha(\tau)\wedge\zeta^\alpha(\tau)=\min\{\eta^\alpha(\tau),\zeta^\alpha(\tau)\}$, $\forall\ \alpha=1,2,\ldots,m$.

(5) Intersection: $\mathfrak{C}^1_{m\mathfrak{P}}\bigcap_{\mathcal{R}}\mathfrak{C}^2_{m\mathfrak{P}}=\{\langle\tau,\mathfrak{Q}^\alpha(\tau)\wedge\mathfrak{R}^\alpha(\tau),\eta^\alpha(\tau)\vee\zeta^\alpha(\tau)\rangle_{\alpha=1}^m\mid\tau\in\mathfrak{S}\}$, where $\mathfrak{Q}^\alpha(\tau)\wedge\mathfrak{R}^\alpha(\tau)=[\min\{\mathfrak{Q}^\alpha_l(\tau),\mathfrak{R}^\alpha_l(\tau)\},\min\{\mathfrak{Q}^\alpha_u(\tau),\mathfrak{R}^\alpha_u(\tau),\}]$ and $\eta^\alpha(\tau)\vee\zeta^\alpha(\tau)=\max\{\eta^\alpha(\tau),\zeta^\alpha(\tau)\}$, $\forall\ \alpha=1,2,\ldots,m$.

(6) Ring sum: $\mathfrak{C}^1_{m\mathfrak{P}}\bigoplus_{\mathcal{R}}\mathfrak{C}^2_{m\mathfrak{P}}=\{\langle\tau,[\mathfrak{Q}^\alpha_l(\tau)+\mathfrak{R}^\alpha_l(\tau)-\mathfrak{Q}^\alpha_l(\tau)\cdot\mathfrak{R}^\alpha_l(\tau),\mathfrak{Q}^\alpha_u(\tau)+\mathfrak{R}^\alpha_u(\tau)-\mathfrak{Q}^\alpha_u(\tau)\cdot\mathfrak{R}^\alpha_u(\tau)],\eta^\alpha(\tau)\cdot\zeta^\alpha(\tau)\rangle_{\alpha=1}^m\mid\tau\in\mathfrak{S}\}$.

(7) Ring product $\mathfrak{C}^1_{m\mathfrak{P}}\bigotimes_{\mathcal{R}}\mathfrak{C}^2_{m\mathfrak{P}}=\{\langle\tau,[\mathfrak{Q}^\alpha_l(\tau)\cdot\mathfrak{R}^\alpha_l(\tau),\mathfrak{Q}^\alpha_u(\tau)\cdot\mathfrak{R}^\alpha_u(\tau)],\eta^\alpha(\tau)+\zeta^\alpha(\tau)-\eta^\alpha(\tau)\cdot\zeta^\alpha(\tau)\rangle_{\alpha=1}^m\mid\tau\in\mathfrak{S}\}$.

(8) κ-scalar power: $(\mathfrak{C}^1_{m\mathfrak{P}})^\kappa=\{\langle\tau,[(\mathfrak{Q}^\alpha_l(\tau))^\kappa,(\mathfrak{Q}^\alpha_u(\tau))^\kappa],1-(1-\eta^\alpha(\tau))^\kappa\rangle_{\alpha=1}^m\mid\tau\in\mathfrak{S}\}$.

(9) κ-scalar product: $\kappa\cdot(\mathfrak{C}^1_{m\mathfrak{P}})=\{\langle\tau,[1-(1-\mathfrak{Q}^\alpha_l(\tau))^\kappa,1-(1-\mathfrak{Q}^\alpha_u(\tau))^\kappa],(\eta^\alpha(\tau))^\kappa\rangle_{\alpha=1}^m\mid\tau\in\mathfrak{S}\}$.

Example 2.3. *Reconsider C3PFSs $\mathfrak{C}^1_{3\mathfrak{P}}$ and $\mathfrak{C}^2_{3\mathfrak{P}}$ as provided in Example 2.2. Then, $\mathcal{R}$-ordering operations on these C3PFSs are evaluated as below:*

$$\begin{aligned}
\mathfrak{C}^1_{m\mathfrak{P}} \bigcup_{\mathcal{R}} \mathfrak{C}^2_{m\mathfrak{P}} =&\{\langle \tau_1, \{[0.45, 0.56], [0.55, 0.70], [0.55, 0.70]\}, \{0.66, 0.70, 0.45\}\rangle,\\
&\langle \tau_2, \{[0.22, 0.46], [0.40, 0.55], [0.45, 0.65]\}, \{0.35, 0.50, 0.25\}\rangle,\\
&\langle \tau_3, \{[0.45, 0.65], [0.55, 0.65], [0.75, 0.85]\}, \{0.30, 0.43, 0.40\}\rangle\}.\\
\mathfrak{C}^1_{m\mathfrak{P}} \bigcap_{\mathcal{R}} \mathfrak{C}^2_{m\mathfrak{P}} =&\{\langle \tau_1, \{[0.32, 0.55], [0.32, 0.60], [0.35, 0.45]\}, \{0.66, 0.75, 0.55\}\rangle,\\
&\langle \tau_2, \{[0.20, 0.35], [0.30, 0.35], [0.40, 0.60]\}, \{0.45, 0.70, 0.45\}\rangle,\\
&\langle \tau_3, \{[0.25, 0.55], [0.45, 0.60], [0.65, 0.80]\}, \{0.70, 0.55, 0.60\}\rangle\}.\\
\mathfrak{C}^1_{m\mathfrak{P}} \bigoplus_{\mathcal{R}} \mathfrak{C}^2_{m\mathfrak{P}} =&\{\langle \tau_1, \{[0.63, 0.80], [0.69, 0.88], [0.71, 0.84]\}, \{0.51, 0.53, 0.25\}\rangle,\\
&\langle \tau_2, \{[0.38, 0.65], [0.58, 0.71], [0.67, 0.86]\}, \{0.16, 0.35, 0.11\}\rangle,\\
&\langle \tau_3, \{[0.59, 0.84], [0.75, 0.86], [0.91, 0.97]\}, \{0.21, 0.24, 0.24\}\rangle\}.\\
\mathfrak{C}^1_{m\mathfrak{P}} \bigotimes_{\mathcal{R}} \mathfrak{C}^2_{m\mathfrak{P}} =&\{\langle \tau_1, \{[0.14, 0.31], [0.18, 0.42], [0.19, 0.32]\}, \{0.92, 0.93, 0.75\}\rangle,\\
&\langle \tau_2, \{[0.04, 0.16], [0.12, 0.19], [0.18, 0.39]\}, \{0.64, 0.85, 0.59\}\rangle,\\
&\langle \tau_3, \{[0.11, 0.36], [0.25, 0.39], [0.49, 0.68]\}, \{0.79, 0.74, 0.76\}\rangle\}.
\end{aligned}$$

For $\kappa = 0.3$, we have:

$$\begin{aligned}
(\mathfrak{C}^1_{m\mathfrak{P}})^{0.3} =&\{\langle \tau_1, \{[0.71, 0.84], [0.84, 0.90], [0.73, 0.79]\}, \{0.88, 0.94, 0.70\}\rangle,\\
&\langle \tau_2, \{[0.63, 0.79], [0.70, 0.73], [0.79, 0.86]\}, \{0.70, 0.91, 0.44\}\rangle,\\
&\langle \tau_3, \{[0.79, 0.84], [0.79, 0.88], [0.92, 0.94]\}, \{0.91, 0.68, 0.64\}\rangle\}.\\
0.3 \cdot \mathfrak{C}^1_{m\mathfrak{P}} =&\{\langle \tau_1, \{[0.54, 0.81], [0.80, 0.91], [0.58, 0.70]\}, \{0.88, 0.92, 0.79\}\rangle,\\
&\langle \tau_2, \{[0.39, 0.71], [0.51, 0.58], [0.70, 0.84]\}, \{0.79, 0.90, 0.66\}\rangle,\\
&\langle \tau_3, \{[0.70, 0.80], [0.70, 0.88], [0.94, 0.96]\}, \{0.90, 0.78, 0.76\}\rangle\}.
\end{aligned}$$

Definition 2.7. *Let $\Psi(\mathfrak{S})$ denotes the set of all CmPFSs. For CmPFSs $\mathfrak{C}^1_{m\mathfrak{P}}$ and $\mathfrak{C}^2_{m\mathfrak{P}}$, the distance measure is a real-valued function d from $\Psi(\mathfrak{S})\times\Psi(\mathfrak{S}) \to [0, 1]$, that satisfying the following conditions:*

- *C1:* $0 \le d(\mathfrak{C}^1_{m\mathfrak{P}}, \mathfrak{C}^2_{m\mathfrak{P}}) \le 1$*;*
- *C2:* $d(\mathfrak{C}^1_{m\mathfrak{P}}, \mathfrak{C}^2_{m\mathfrak{P}}) = 0$ *if and only if* $\mathfrak{C}^1_{m\mathfrak{P}} = \mathfrak{C}^2_{m\mathfrak{P}}$*;*
- *C3:* $d(\mathfrak{C}^1_{m\mathfrak{P}}, \mathfrak{C}^2_{m\mathfrak{P}}) = d(\mathfrak{C}^2_{m\mathfrak{P}}, \mathfrak{C}^1_{m\mathfrak{P}})$*;*
- *C4: For* $\mathfrak{C}^3_{m\mathfrak{P}} \in \Psi(\mathfrak{S})$*, if* $\mathfrak{C}^1_{m\mathfrak{P}} \subseteq \mathfrak{C}^2_{m\mathfrak{P}} \subseteq \mathfrak{C}^3_{m\mathfrak{P}}$*, then* $d(\mathfrak{C}^1_{m\mathfrak{P}}, \mathfrak{C}^2_{m\mathfrak{P}}) \le d(\mathfrak{C}^1_{m\mathfrak{P}}, \mathfrak{C}^3_{m\mathfrak{P}})$ *and* $d(\mathfrak{C}^1_{m\mathfrak{P}}, \mathfrak{C}^2_{m\mathfrak{P}}) \le d(\mathfrak{C}^2_{m\mathfrak{P}}, \mathfrak{C}^3_{m\mathfrak{P}})$

For CmPFSs $\mathfrak{C}^1_{m\mathfrak{P}} = \{\langle \tau, \mathfrak{Q}^\alpha(\tau), \eta^\alpha(\tau)\rangle^m_{\alpha=1} \mid \tau \in \mathfrak{S}\}$ *and* $\mathfrak{C}^2_{m\mathfrak{P}} = \{\langle \tau, \mathfrak{R}^\alpha(\tau), \zeta^\alpha(\tau)\rangle^m_{\alpha=1} \mid \tau \in \mathfrak{S}\}$ *over the universe* $\mathfrak{S} = \{\tau_1, \tau_2, \cdots, \tau_n\}$*, we define some distances between these CmPFSs as follows:*

(1) Hamming distance:

$$d_{\mathcal{H}}(\mathcal{C}^1_{m\mathfrak{P}}, \mathcal{C}^2_{m\mathfrak{P}}) = \frac{1}{3m}\Bigg[\sum_{\beta=1}^{n}\sum_{\alpha=1}^{m}\Big(|\mathfrak{Q}^\alpha_l(\tau_\beta) - \mathfrak{R}^\alpha_l(\tau_\beta)| + |\mathfrak{Q}^\alpha_u(\tau_\beta) - \mathfrak{R}^\alpha_u(\tau_\beta)| + |\eta^\alpha(\tau_\beta) - \zeta^\alpha(\tau_\beta)|\Big)\Bigg].$$

(2) Normalized Hamming distance:

$$d_{\mathcal{NH}}(\mathcal{C}^1_{m\mathfrak{P}}, \mathcal{C}^2_{m\mathfrak{P}}) = \frac{1}{3mn}\Bigg[\sum_{\beta=1}^{n}\sum_{\alpha=1}^{m}\Big(|\mathfrak{Q}^\alpha_l(\tau_\beta) - \mathfrak{R}^\alpha_l(\tau_\beta)| + |\mathfrak{Q}^\alpha_u(\tau_\beta) - \mathfrak{R}^\alpha_u(\tau_\beta)| + |\eta^\alpha(\tau_\beta) - \zeta^\alpha(\tau_\beta)|\Big)\Bigg].$$

(3) Euclidean distance:

$$d_{\mathcal{E}}(\mathcal{C}^1_{m\mathfrak{P}}, \mathcal{C}^2_{m\mathfrak{P}}) = \sqrt{\frac{1}{3m}\Bigg[\sum_{\beta=1}^{n}\sum_{\alpha=1}^{m}\Big((\mathfrak{Q}^\alpha_l(\tau_\beta) - \mathfrak{R}^\alpha_l(\tau_\beta))^2 + (\mathfrak{Q}^\alpha_u(\tau_\beta) - \mathfrak{R}^\alpha_u(\tau_\beta))^2 + (\eta^\alpha(\tau_\beta) - \zeta^\alpha(\tau_\beta))^2\Big)\Bigg]}.$$

(4) Normalized Euclidean distance:

$$d_{\mathcal{NE}}(\mathcal{C}^1_{m\mathfrak{P}}, \mathcal{C}^2_{m\mathfrak{P}}) = \sqrt{\frac{1}{3mn}\Bigg[\sum_{\beta=1}^{n}\sum_{\alpha=1}^{m}((\mathfrak{Q}^\alpha_l(\tau_\beta) - \mathfrak{R}^\alpha_l(\tau_\beta))^2 + (\mathfrak{Q}^\alpha_u(\tau_\beta) - \mathfrak{R}^\alpha_u(\tau_\beta))^2 + (\eta^\alpha(\tau_\beta) - \zeta^\alpha(\tau_\beta))^2)\Bigg]}.$$

From the above distance measures, we obtained the following results.

Theorem 2.1. *The distances $d_{\mathcal{H}}$ and $d_{\mathcal{NH}}$ between CmPFSs $\mathcal{C}^1_{m\mathfrak{P}}$ and $\mathcal{C}^2_{m\mathfrak{P}}$ satisfy the conditions C1–C4, as provided in Definition 2.7.*

Proof. From CmPFSs $\mathcal{C}^1_{m\mathfrak{P}}$ and $\mathcal{C}^2_{m\mathfrak{P}}$, we have

C1: Since $0 \le \mathfrak{Q}^1_l(\tau_\beta), \cdots, \mathfrak{Q}^m_l(\tau_\beta) \le 1$, $0 \le \mathfrak{Q}^1_u(\tau_\beta), \cdots, \mathfrak{Q}^m_u(\tau_\beta) \le 1$, and $0 \le \eta^1(\tau_\beta), \cdots, \eta^m(\tau_\beta) \le 1$ also $0 \le \mathfrak{R}^1_l(\tau_\beta), \cdots, \mathfrak{R}^m_l(\tau_\beta) \le 1$, $0 \le \mathfrak{R}^1_u(\tau_\beta), \cdots, \mathfrak{R}^m_u(\tau_\beta) \le 1$, and $0 \le \zeta^1(\tau_\beta), \cdots, \zeta^m(\tau_\beta) \le 1$. This implies that $0 \le |\mathfrak{Q}^1_l(\tau_\beta) - \mathfrak{R}^1_l(\tau_\beta)| \le 1, \cdots, 0 \le |\mathfrak{Q}^m_l(\tau_\beta) - \mathfrak{R}^m_l(\tau_\beta)| \le 1, 0 \le |\mathfrak{Q}^1_u(\tau_\beta) - \mathfrak{R}^1_u(\tau_\beta)| \le 1, \cdots, 0 \le |\mathfrak{Q}^m_u(\tau_\beta) - \mathfrak{R}^m_u(\tau_\beta)| \le 1$ and $0 \le |\eta^1(\tau_\beta) - \zeta^1(\tau_\beta)| \le 1, \cdots, 0 \le |\eta^m(\tau_\beta) - \zeta^m(\tau_\beta)| \le 1$. Therefore, we get

$$0 \le \begin{bmatrix} |\mathfrak{Q}^1_l(\tau_\beta) - \mathfrak{R}^1_l(\tau_\beta)| + \cdots + |\mathfrak{Q}^m_l(\tau_\beta) - \mathfrak{R}^m_l(\tau_\beta)| + |\mathfrak{Q}^1_u(\tau_\beta) - \mathfrak{R}^1_u(\tau_\beta)| + \cdots \\ +|\mathfrak{Q}^m_u(\tau_\beta) - \mathfrak{R}^m_u(\tau_\beta)| + |\eta^1(\tau_\beta) - \zeta^1(\tau_\beta)| + \cdots + |\eta^m(\tau_\beta) - \zeta^m(\tau_\beta)| \end{bmatrix} \le m + m + m$$

$$0 \le \left[\sum_{\alpha=1}^{m}|\mathfrak{Q}^\alpha_l(\tau_\beta) - \mathfrak{R}^\alpha_l(\tau_\beta)| + |\mathfrak{Q}^\alpha_u(\tau_\beta) - \mathfrak{R}^\alpha_u(\tau_\beta)| + |\eta^\alpha(\tau_\beta) - \zeta^\alpha(\tau_\beta)|\right] \le 3m.$$

This implies that $0 \le d_{\mathcal{H}}(\mathcal{C}^1_{m\mathfrak{P}}, \mathcal{C}^2_{m\mathfrak{P}}) \le 1$.

C2: Assume that $d_{\mathcal{H}}(\mathcal{C}^1_{m\mathfrak{P}}, \mathcal{C}^2_{m\mathfrak{P}}) = 0$, which implies that

$$\frac{1}{3m}\left[\sum_{\beta=1}^{n}\sum_{\alpha=1}^{m}|\mathfrak{Q}^\alpha_l(\tau_\beta) - \mathfrak{R}^\alpha_l(\tau_\beta)| + |\mathfrak{Q}^\alpha_u(\tau_\beta) - \mathfrak{R}^\alpha_u(\tau_\beta)| + |\eta^\alpha(\tau_\beta) - \zeta^\alpha(\tau_\beta)|\right] = 0,$$

$$\sum_{\beta=1}^{n}\begin{bmatrix}|\mathfrak{Q}_l^1(\tau_\beta)-\mathfrak{R}_l^1(\tau_\beta)|+\cdots+|\mathfrak{Q}_l^m(\tau_\beta)-\mathfrak{R}_l^m(\tau_\beta)|+|\mathfrak{Q}_u^1(\tau_\beta)-\mathfrak{R}_u^1(\tau_\beta)|+\cdots\\+|\mathfrak{Q}_u^m(\tau_\beta)-\mathfrak{R}_u^m(\tau_\beta)|+|\eta^1(\tau_\beta)-\zeta^1(\tau_\beta)|+\cdots+|\eta^m(\tau_\beta)-\zeta^m(\tau_\beta)|\end{bmatrix}=0,$$

if and only if for all β $|\mathfrak{Q}_l^1(\tau_\beta)-\mathfrak{R}_l^1(\tau_\beta)|=0,\cdots,|\mathfrak{Q}_l^m(\tau_\beta)-\mathfrak{R}_l^m(\tau_\beta)|=0,$ $|\mathfrak{Q}_u^1(\tau_\beta)-\mathfrak{R}_u^1(\tau_\beta)|=0,\cdots,|\mathfrak{Q}_u^m(\tau_\beta)-\mathfrak{R}_u^m(\tau_\beta)|=0,|\eta^1(\tau_\beta)-\zeta^1(\tau_\beta)|=0,\cdots,|\eta^m(\tau_\beta)-\zeta^m(\tau_\beta)|=0,$ which is equivalent to
$\mathfrak{Q}_l^1(\tau_\beta)=\mathfrak{R}_l^1(\tau_\beta),\cdots,\mathfrak{Q}_l^m(\tau_\beta)=\mathfrak{R}_l^m(\tau_\beta),\mathfrak{Q}_u^1(\tau_\beta)=\mathfrak{R}_u^1(\tau_\beta),\cdots,\mathfrak{Q}_u^m(\tau_\beta)=\mathfrak{R}_u^m(\tau_\beta),\eta^1(\tau_\beta)=\zeta^1(\tau_\beta),\cdots,\eta^m(\tau_\beta)=\zeta^m(\tau_\beta).$
Thus, $d_{\mathcal{H}}(\mathfrak{C}^1_{m\mathfrak{P}},\mathfrak{C}^2_{m\mathfrak{P}})=0$, implies that $\mathfrak{C}^1_{m\mathfrak{P}}=\mathfrak{C}^2_{m\mathfrak{P}}$.

C3: We know that

$$\begin{aligned}d_{\mathcal{H}}(\mathfrak{C}^1_{m\mathfrak{P}},\mathfrak{C}^2_{m\mathfrak{P}})&=\frac{1}{3m}\Bigg[\sum_{\beta=1}^{n}\sum_{\alpha=1}^{m}\Big(|\mathfrak{Q}_l^\alpha(\tau_\beta)-\mathfrak{R}_l^\alpha(\tau_\beta)|+|\mathfrak{Q}_u^\alpha(\tau_\beta)-\mathfrak{R}_u^\alpha(\tau_\beta)|+|\eta^\alpha(\tau_\beta)-\zeta^\alpha(\tau_\beta)|\Big)\Bigg]\\&=\frac{1}{3m}\Bigg[\sum_{\beta=1}^{n}\sum_{\alpha=1}^{m}\Big(|\mathfrak{R}_l^\alpha(\tau_\beta)-\mathfrak{Q}_l^\alpha(\tau_\beta)|+|\mathfrak{R}_u^\alpha(\tau_\beta)-\mathfrak{Q}_u^\alpha(\tau_\beta)|+|\zeta^\alpha(\tau_\beta)-\eta^\alpha(\tau_\beta)|\Big)\Bigg]\\&=d_{\mathcal{H}}(\mathfrak{C}^2_{m\mathfrak{P}},\mathfrak{C}^1_{m\mathfrak{P}}).\end{aligned}$$

Thus, $d_{\mathcal{H}}(\mathfrak{C}^1_{m\mathfrak{P}},\mathfrak{C}^2_{m\mathfrak{P}})=d_{\mathcal{H}}(\mathfrak{C}^2_{m\mathfrak{P}},\mathfrak{C}^1_{m\mathfrak{P}})$.

C4: For $\mathfrak{C}^3_{m\mathfrak{P}}=\{\langle\tau,\mathfrak{L}^\alpha(\tau),\mu^\alpha(\tau)\rangle_{\alpha=1}^m\mid\tau\in\mathfrak{S}\}$, if $\mathfrak{C}^1_{m\mathfrak{P}}\subseteq\mathfrak{C}^2_{m\mathfrak{P}}\subseteq\mathfrak{C}^3_{m\mathfrak{P}}$ then $[\mathfrak{Q}_l^\alpha(\tau_\beta),\mathfrak{Q}_u^\alpha(\tau_\beta)]\subseteq[\mathfrak{R}_l^\alpha(\tau_\beta),$ $\mathfrak{R}_u^\alpha(\tau_\beta)]\subseteq[\mathfrak{L}_l^\alpha(\tau_\beta),\mathfrak{L}_u^\alpha(\tau_\beta)]$ and $\eta^\alpha(\tau_\beta)\le\zeta^\alpha(\tau_\beta)\le\mu^\alpha(\tau_\beta)$. Therefore, $|\mathfrak{Q}_l^1(\tau_\beta)-\mathfrak{R}_l^1(\tau_\beta)|\le|\mathfrak{Q}_l^1(\tau_\beta)-\mathfrak{L}_l^1(\tau_\beta)|$, $|\mathfrak{Q}_u^1(\tau_\beta)-\mathfrak{R}_u^1(\tau_\beta)|\le|\mathfrak{Q}_u^1(\tau_\beta)-\mathfrak{L}_u^1(\tau_\beta)|$ and $|\eta^\alpha(\tau_\beta)-\zeta^\alpha(\tau_\beta)|\le|\eta^\alpha(\tau_\beta)-\mu^\alpha(\tau_\beta)|$, then we have

$$\begin{aligned}d_{\mathcal{H}}(\mathfrak{C}^1_{m\mathfrak{P}},\mathfrak{C}^3_{m\mathfrak{P}})&=\frac{1}{3m}\Bigg[\sum_{\beta=1}^{n}\sum_{\alpha=1}^{m}\Big(|\mathfrak{Q}_l^\alpha(\tau_\beta)-\mathfrak{L}_l^\alpha(\tau_\beta)|+|\mathfrak{Q}_u^\alpha(\tau_\beta)-\mathfrak{L}_u^\alpha(\tau_\beta)|+|\eta^\alpha(\tau_\beta)-\mu^\alpha(\tau_\beta)|\Big)\Bigg]\\&\ge\frac{1}{3m}\Bigg[\sum_{\beta=1}^{n}\sum_{\alpha=1}^{m}\Big(|\mathfrak{Q}_l^\alpha(\tau_\beta)-\mathfrak{R}_l^\alpha(\tau_\beta)|+|\mathfrak{Q}_u^\alpha(\tau_\beta)-\mathfrak{R}_u^\alpha(\tau_\beta)|+|\eta^\alpha(\tau_\beta)-\zeta^\alpha(\tau_\beta)|\Big)\Bigg]\\d_{\mathcal{H}}(\mathfrak{C}^1_{m\mathfrak{P}},\mathfrak{C}^3_{m\mathfrak{P}})&\ge d_{\mathcal{H}}(\mathfrak{C}^1_{m\mathfrak{P}},\mathfrak{C}^2_{m\mathfrak{P}}).\end{aligned}$$

Similarly, $d_{\mathcal{H}}(\mathfrak{C}^1_{m\mathfrak{P}},\mathfrak{C}^3_{m\mathfrak{P}})\ge d_{\mathcal{H}}(\mathfrak{C}^2_{m\mathfrak{P}},\mathfrak{C}^3_{m\mathfrak{P}})$.

All the conditions are satisfied. Hence, $d_{\mathcal{H}}$ is an accurate distance measure between $\mathfrak{C}^1_{m\mathfrak{P}}$ and $\mathfrak{C}^2_{m\mathfrak{P}}$. Similarly, the proof of $d_{\mathcal{NH}}$ is straightforward on the same lines. □

Theorem 2.2. *The distances $d_{\mathcal{E}}$ and $d_{\mathcal{NE}}$ between CmPFSs $\mathfrak{C}^1_{m\mathfrak{P}}$ and $\mathfrak{C}^2_{m\mathfrak{P}}$ satisfies the conditions C1–C4, as described in Definition 2.7.*

Proof. From CmPFSs $\mathfrak{C}^1_{m\mathfrak{P}}$ and $\mathfrak{C}^2_{m\mathfrak{P}}$, we have

C1: Since $0\le\mathfrak{Q}_l^1(\tau_\beta),\cdots,\mathfrak{Q}_l^m(\tau_\beta)\le1$, $0\le\mathfrak{Q}_u^1(\tau_\beta),\cdots,\mathfrak{Q}_u^m(\tau_\beta)\le1$, and $0\le\eta^1(\tau_\beta),\cdots,\eta^m(\tau_\beta)\le1$ also $0\le\mathfrak{R}_l^1(\tau_\beta),\cdots,\mathfrak{R}_l^m(\tau_\beta)\le1$, $0\le\mathfrak{R}_u^1(\tau_\beta),\cdots,\mathfrak{R}_u^m(\tau_\beta)\le1$, and $0\le\zeta^1(\tau_\beta),\cdots,\zeta^m(\tau_\beta)\le1$. This implies that $0\le(\mathfrak{Q}_l^1(\tau_\beta)-\mathfrak{R}_l^1(\tau_\beta))^2\le1,\cdots,0\le(\mathfrak{Q}_l^m(\tau_\beta)-\mathfrak{R}_l^m(\tau_\beta))^2\le1$, $0\le(\mathfrak{Q}_u^1(\tau_\beta)-$

$\mathfrak{R}_u^1(\tau_\beta))^2 \leq 1, \cdots, 0 \leq (\mathfrak{Q}_u^m(\tau_\beta) - \mathfrak{R}_u^m(\tau_\beta)^2 \leq 1$ and $0 \leq (\eta^1(\tau_\beta) - \zeta^1(\tau_\beta))^2 \leq 1, \cdots, 0 \leq (\eta^m(\tau_\beta) - \zeta^m(\tau_\beta))^2 \leq 1$. Therefore, we get

$$0 \leq \begin{bmatrix} (\mathfrak{Q}_l^1(\tau_\beta) - \mathfrak{R}_l^1(\tau_\beta))^2 + \cdots + (\mathfrak{Q}_l^m(\tau_\beta) - \mathfrak{R}_l^m(\tau_\beta|)^2 + (\mathfrak{Q}_u^1(\tau_\beta) - \mathfrak{R}_u^1(\tau_\beta))^2 + \cdots \\ +(\mathfrak{Q}_u^m(\tau_\beta) - \mathfrak{R}_u^m(\tau_\beta))^2 + (\eta^1(\tau_\beta) - \zeta^1(\tau_\beta))^2 + \cdots + (\eta^m(\tau_\beta) - \zeta^m(\tau_\beta))^2 \end{bmatrix} \leq m + m + m$$

$$0 \leq \left[\sum_{\alpha=1}^{m} (\mathfrak{Q}_l^\alpha(\tau_\beta) - \mathfrak{R}_l^\alpha(\tau_\beta))^2 + (\mathfrak{Q}_u^\alpha(\tau_\beta) - \mathfrak{R}_u^\alpha(\tau_\beta))^2 + (\eta^\alpha(\tau_\beta) - \zeta^\alpha(\tau_\beta))^2 \right] \leq 3m.$$

This implies that $0 \leq d_{\mathcal{E}}(\mathcal{C}_{m\mathfrak{P}}^1, \mathcal{C}_{m\mathfrak{P}}^2) \leq 1$.

C2: Assume that $d_{\mathcal{E}}(\mathcal{C}_{m\mathfrak{P}}^1, \mathcal{C}_{m\mathfrak{P}}^2) = 0$, which implies that

$$\left[\frac{1}{3m} \left[\sum_{\beta=1}^{n} \sum_{\alpha=1}^{m} (\mathfrak{Q}_l^\alpha(\tau_\beta) - \mathfrak{R}_l^\alpha(\tau_\beta))^2 + (\mathfrak{Q}_u^\alpha(\tau_\beta) - \mathfrak{R}_u^\alpha(\tau_\beta))^2 + (\eta^\alpha(\tau_\beta) - \zeta^\alpha(\tau_\beta))^2 \right] \right]^{\frac{1}{2}} = 0,$$

$$\sum_{\beta=1}^{n} \begin{bmatrix} (\mathfrak{Q}_l^1(\tau_\beta) - \mathfrak{R}_l^1(\tau_\beta))^2 + \cdots + (\mathfrak{Q}_l^m(\tau_\beta) - \mathfrak{R}_l^m(\tau_\beta))^2 + (\mathfrak{Q}_u^1(\tau_\beta) - \mathfrak{R}_u^1(\tau_\beta))^2 + \cdots \\ +(\mathfrak{Q}_u^m(\tau_\beta) - \mathfrak{R}_u^m(\tau_\beta)| + (\eta^1(\tau_\beta) - \zeta^1(\tau_\beta))^2 + \cdots + (\eta^m(\tau_\beta) - \zeta^m(\tau_\beta))^2 \end{bmatrix} = 0,$$

if and only for all β
$(\mathfrak{Q}_l^1(\tau_\beta) - \mathfrak{R}_l^1(\tau_\beta))^2 = 0, \cdots, (\mathfrak{Q}_l^m(\tau_\beta) - \mathfrak{R}_l^m(\tau_\beta))^2 = 0, (\mathfrak{Q}_u^1(\tau_\beta) - \mathfrak{R}_u^1(\tau_\beta))^2 = 0, \cdots,$ $(\mathfrak{Q}_u^m(\tau_\beta) - \mathfrak{R}_u^m(\tau_\beta))^2 = 0, (\eta^1(\tau_\beta) - \zeta^1(\tau_\beta))^2 = 0, \cdots, (\eta^m(\tau_\beta) - \zeta^m(\tau_\beta))^2 = 0$, which is equivalent to $\mathfrak{Q}_l^1(\tau_\beta) = \mathfrak{R}_l^1(\tau_\beta), \cdots, \mathfrak{Q}_l^m(\tau_\beta) = \mathfrak{R}_l^m(\tau_\beta), \mathfrak{Q}_u^1(\tau_\beta) = \mathfrak{R}_u^1(\tau_\beta), \cdots, \mathfrak{Q}_u^m(\tau_\beta) = \mathfrak{R}_u^m(\tau_\beta), \eta^1(\tau_\beta) = \zeta^1(\tau_\beta), \cdots, \eta^m(\tau_\beta) = \zeta^m(\tau_\beta)$.
Thus, $d_{\mathcal{E}}(\mathcal{C}_{m\mathfrak{P}}^1, \mathcal{C}_{m\mathfrak{P}}^2) = 0$, implies that $\mathcal{C}_{m\mathfrak{P}}^1 = \mathcal{C}_{m\mathfrak{P}}^2$.

C3: We know that

$$\begin{aligned} d_{\mathcal{E}}(\mathcal{C}_{m\mathfrak{P}}^1, \mathcal{C}_{m\mathfrak{P}}^2) &= \left[\frac{1}{3m} \left[\sum_{\beta=1}^{n} \sum_{\alpha=1}^{m} \left((\mathfrak{Q}_l^\alpha(\tau_\beta) - \mathfrak{R}_l^\alpha(\tau_\beta))^2 + (\mathfrak{Q}_u^\alpha(\tau_\beta) - \mathfrak{R}_u^\alpha(\tau_\beta))^2 + (\eta^\alpha(\tau_\beta) - \zeta^\alpha(\tau_\beta))^2 \right) \right] \right]^{\frac{1}{2}} \\ &= \left[\frac{1}{3m} \left[\sum_{\beta=1}^{n} \sum_{\alpha=1}^{m} \left((\mathfrak{R}_l^\alpha(\tau_\beta) - \mathfrak{Q}_l^\alpha(\tau_\beta))^2 + (\mathfrak{R}_u^\alpha(\tau_\beta) - \mathfrak{Q}_u^\alpha(\tau_\beta))^2 + (\zeta^\alpha(\tau_\beta) - \eta^\alpha(\tau_\beta))^2 \right) \right] \right]^{\frac{1}{2}} \\ &= d_{\mathcal{E}}(\mathcal{C}_{m\mathfrak{P}}^2, \mathcal{C}_{m\mathfrak{P}}^1). \end{aligned}$$

Thus, $d_{\mathcal{E}}(\mathcal{C}_{m\mathfrak{P}}^1, \mathcal{C}_{m\mathfrak{P}}^2) = d_{\mathcal{E}}(\mathcal{C}_{m\mathfrak{P}}^2, \mathcal{C}_{m\mathfrak{P}}^1)$.

C4: For $\mathcal{C}_{m\mathfrak{P}}^3 = \{\langle \tau, \mathfrak{L}^\alpha(\tau), \mu^\alpha(\tau) \rangle_{\alpha=1}^m \mid \tau \in \mathfrak{S}\}$, if $\mathcal{C}_{m\mathfrak{P}}^1 \subseteq \mathcal{C}_{m\mathfrak{P}}^2 \subseteq \mathcal{C}_{m\mathfrak{P}}^3$ then $[\mathfrak{Q}_l^\alpha(\tau_\beta), \mathfrak{Q}_u^\alpha(\tau_\beta)] \subseteq [\mathfrak{R}_l^\alpha(\tau_\beta), \mathfrak{R}_u^\alpha(\tau_\beta)] \subseteq [\mathfrak{L}_l^\alpha(\tau_\beta), \mathfrak{L}_u^\alpha(\tau_\beta)]$ and $\eta^\alpha(\tau_\beta) \leq \zeta^\alpha(\tau_\beta) \leq \mu^\alpha(\tau_\beta)$. Therefore, $(\mathfrak{Q}_l^1(\tau_\beta) - \mathfrak{R}_l^1(\tau_\beta))^2 \leq (\mathfrak{Q}_l^1(\tau_\beta) - \mathfrak{L}_l^1(\tau_\beta))^2$, $(\mathfrak{Q}_u^1(\tau_\beta) - \mathfrak{R}_u^1(\tau_\beta))^2 \leq (\mathfrak{Q}_u^1(\tau_\beta) - \mathfrak{L}_u^1(\tau_\beta))^2$ and $(\eta^\alpha(\tau_\beta) - \zeta^\alpha(\tau_\beta))^2 \leq (\eta^\alpha(\tau_\beta) - \mu^\alpha(\tau_\beta))^2$, then we have

$$d_{\mathcal{E}}(\mathcal{C}_{m\mathfrak{P}}^1, \mathcal{C}_{m\mathfrak{P}}^3) = \left[\frac{1}{3m} \left[\sum_{\beta=1}^{n} \sum_{\alpha=1}^{m} \left((\mathfrak{Q}_l^\alpha(\tau_\beta) - \mathfrak{L}_l^\alpha(\tau_\beta))^2 + (\mathfrak{Q}_u^\alpha(\tau_\beta) - \mathfrak{L}_u^\alpha(\tau_\beta))^2 + (\eta^\alpha(\tau_\beta) - \mu^\alpha(\tau_\beta))^2 \right) \right] \right]^{\frac{1}{2}}$$

$$\geq \left[\frac{1}{3m}\left[\sum_{\beta=1}^{n}\sum_{\alpha=1}^{m}\left((\mathfrak{Q}_l^{\alpha}(\tau_\beta)-\mathfrak{R}_l^{\alpha}(\tau_\beta))^2+(\mathfrak{Q}_u^{\alpha}(\tau_\beta)-\mathfrak{R}_u^{\alpha}(\tau_\beta))^2+(\eta^{\alpha}(\tau_\beta)-\zeta^{\alpha}(\tau_\beta))^2\right)\right]\right]^{\frac{1}{2}}$$

$$d_{\mathcal{E}}(\mathfrak{C}^1_{m\mathfrak{P}},\mathfrak{C}^3_{m\mathfrak{P}}) \geq d_{\mathcal{E}}(\mathfrak{C}^1_{m\mathfrak{P}},\mathfrak{C}^2_{m\mathfrak{P}}).$$

Similarly, $d_{\mathcal{E}}(\mathfrak{C}^1_{m\mathfrak{P}},\mathfrak{C}^3_{m\mathfrak{P}}) \geq d_{\mathcal{E}}(\mathfrak{C}^2_{m\mathfrak{P}},\mathfrak{C}^3_{m\mathfrak{P}})$.

All the conditions are satisfied. Hence, $d_{\mathcal{E}}$ is an accurate distance measure between $\mathfrak{C}^1_{m\mathfrak{P}}$ and $\mathfrak{C}^2_{m\mathfrak{P}}$. Similarly, the proof of $d_{\mathcal{NE}}$ is trivial on the same lines and also a valid distance measure. □

3. Cubic *m*-polar fuzzy TOPSIS method

In this section, we merge the hybrid C*m*PFS model with the TOPSIS method, and propose a new algorithm for the initiated C*m*PF-TOPSIS approach. Then, we implement it to solve a challenging real-life decision-making problem. Let us assume that a decision-maker has to choose one of 's' possible alternatives with respect to 't' criteria. According to the decision-maker, the rating of each alternative $\aleph_i(i = 1, 2, 3, \cdots, s)$ with respect to each criterion $\mathcal{A}_j(j = 1, 2, 3, \cdots, t)$ is a C*m*PF value that is denoted by τ_{ij}. Also, let $\mathcal{W} = [\beth_1, \beth_2, \cdots, \beth_t]$ be the vector of criteria weights which describe the positiveness of each criterion according to the given information. The algorithm for C*m*PF-TOPSIS approach is provided as below:

Algorithm 1: (Technique for preference ranking order between alternatives)

1). Input:

(i) Each alternative is evaluated with respect to t-criteria. So, all the values which are precise to the alternatives regarding each criterion form a decision matrix as:

$$\mathfrak{S} = [\tau_{ij}]_{s\times t} = \begin{bmatrix} \tau_{11} & \tau_{12} & \cdots & \tau_{1(t-1)} & \tau_{1t} \\ \tau_{21} & \tau_{22} & \cdots & \tau_{2(t-1)} & \tau_{2t} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \tau_{(s-1)1} & \tau_{(s-1)2} & \cdots & \tau_{(s-1)(t-1)} & \tau_{(s-1)t} \\ \tau_{s1} & \tau_{s2} & \cdots & \tau_{s(t-1)} & \tau_{st} \end{bmatrix}. \tag{3.1}$$

In this matrix each entry $\tau_{ij} = \langle\, \mathfrak{Q}^{\alpha}_{ij}, \eta^{\alpha}_{ij}\rangle^{m}_{\alpha=1}$, where $\mathfrak{Q}^{\alpha}_{ij} = [\mathfrak{Q}^{\alpha}_{lij}, \mathfrak{Q}^{\alpha}_{uij}]$, $i = 1, 2, \cdots, s$ and $j = 1, 2, \cdots, t$.

(ii) The weights $\mathcal{W} = [\beth_1, \beth_2, \cdots, \beth_t]^T$ are adopted for each criterion by the decision-maker, which satisfy the normalized condition, that is, $\sum_{j=1}^{t} \beth_j = 1$ where $\beth_j$ is the weight of j-th criterion where $j = 1, 2, \cdots, t$.

2). The C*m*PF weighted decision matrix $\mathfrak{V}$ is constructed by multiplying the columns of C*m*PF decision matrix $\mathfrak{S}$ with the associated weights $\beth_j \in [0, 1]$.

$$\mathfrak{V} = [\varkappa_{ij}]_{s\times t} = \begin{bmatrix} \varkappa_{11} & \varkappa_{12} & \cdots & \varkappa_{1t} \\ \varkappa_{21} & \varkappa_{22} & \cdots & \varkappa_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ \varkappa_{s1} & \varkappa_{s2} & \cdots & \varkappa_{st} \end{bmatrix}, \tag{3.2}$$

where each entry $\varkappa_{ij} = \langle \Re^{\alpha}_{ij}, \zeta^{\alpha}_{ij} \rangle_{\alpha=1}^{m}$ is calculated as:

$$\begin{aligned}
\varkappa_{ij} &= \sqsupseteq_j \cdot \tau_{ij} \\
&= \langle [1 - (1 - \mathfrak{Q}^{\alpha}_{lij})^{\sqsupseteq_j}, 1 - (1 - \mathfrak{Q}^{\alpha}_{uij})^{\sqsupseteq_j}], (\eta^{\alpha}_{ij})^{\sqsupseteq_j} \rangle_{\alpha=1}^{m} \\
&= \langle [\Re^{\alpha}_{lij}, \Re^{\alpha}_{uij}], \zeta^{\alpha}_{ij} \rangle_{\alpha=1}^{m} = \langle \Re^{\alpha}_{ij}, \zeta^{\alpha}_{ij} \rangle_{\alpha=1}^{m}
\end{aligned}$$

3). This step determines the CmPF positive ideal solution (CmPFPIS) $\mathcal{F}^+$ and CmPF negative ideal solution (CmPFNIS) $\mathcal{F}^-$ as follows:

$$\begin{aligned}
\mathcal{F}^+ =&\{\langle \{[\mathfrak{Q}^{1+}_{l1}, \mathfrak{Q}^{1+}_{u1}], [\mathfrak{Q}^{2+}_{l1}, \mathfrak{Q}^{2+}_{u1}], \ldots, [\mathfrak{Q}^{m+}_{l1}, \mathfrak{Q}^{m+}_{u1}]\}, \{\eta^{1+}_1, \eta^{2+}_1, \ldots, \eta^{m+}_1\} \rangle, \\
&\langle \{[\mathfrak{Q}^{1+}_{l2}, \mathfrak{Q}^{1+}_{u2}], [\mathfrak{Q}^{2+}_{l2}, \mathfrak{Q}^{2+}_{u2}], \ldots, [\mathfrak{Q}^{m+}_{l2}, \mathfrak{Q}^{m+}_{u2}]\}, \{\eta^{1+}_2, \eta^{2+}_2, \ldots, \eta^{m+}_2\} \rangle, \\
&\cdots, \langle \{[\mathfrak{Q}^{1+}_{lj}, \mathfrak{Q}^{1+}_{uj}], [\mathfrak{Q}^{2+}_{lj}, \mathfrak{Q}^{2+}_{uj}], \ldots, [\mathfrak{Q}^{m+}_{lj}, \mathfrak{Q}^{m+}_{uj}]\}, \{\eta^{1+}_j, \eta^{2+}_j, \ldots, \eta^{m+}_j\} \rangle\}, \\
\mathcal{F}^+ =&\{\langle \{[\mathfrak{Q}^{\alpha+}_{lj}, \mathfrak{Q}^{\alpha+}_{uj}], \eta^{\alpha+}_j \rangle_{\alpha=1}^{m}, {}_{j=1}^{t}\}.
\end{aligned} \tag{3.3}$$

where $[\mathfrak{Q}^{\alpha+}_{lj}, \mathfrak{Q}^{\alpha+}_{uj}] = [\max_i \Re^{\alpha}_{lij}, \max_i \Re^{\alpha}_{uij}]$, $\eta^{\alpha+}_j = \max_i \zeta^{\alpha}_{ij}$.

$$\begin{aligned}
\mathcal{F}^- =&\{\langle \{[\mathfrak{Q}^{1-}_{l1}, \mathfrak{Q}^{1-}_{u1}], [\mathfrak{Q}^{2-}_{l1}, \mathfrak{Q}^{2-}_{u1}], \ldots, [\mathfrak{Q}^{m-}_{l1}, \mathfrak{Q}^{m-}_{u1}]\}, \{\eta^{1-}_1, \eta^{2-}_1, \ldots, \eta^{m-}_1 \rangle\}, \\
&\langle \{[\mathfrak{Q}^{1-}_{l2}, \mathfrak{Q}^{1-}_{u2}], [\mathfrak{Q}^{2-}_{l2}, \mathfrak{Q}^{2-}_{u2}], \ldots, [\mathfrak{Q}^{m-}_{l2}, \mathfrak{Q}^{m-}_{u2}]\}, \{\eta^{1-}_2, \eta^{2-}_2, \ldots, \eta^{m-}_2\} \rangle, \\
&\cdots, \langle \{[\mathfrak{Q}^{1-}_{lj}, \mathfrak{Q}^{1-}_{uj}], [\mathfrak{Q}^{2-}_{lj}, \mathfrak{Q}^{2-}_{uj}], \ldots, [\mathfrak{Q}^{m-}_{lj}, \mathfrak{Q}^{m-}_{uj}]\}, \{\eta^{1-}_j, \eta^{2-}_j, \ldots, \eta^{m-}_j\} \rangle\}, \\
\mathcal{F}^- =&\{\langle \{[\mathfrak{Q}^{\alpha-}_{lj}, \mathfrak{Q}^{\alpha-}_{uj}], \eta^{\alpha-}_j \rangle_{\alpha=1}^{m}, {}_{j=1}^{t}\}.
\end{aligned} \tag{3.4}$$

where $[\mathfrak{Q}^{\alpha-}_{lj}, \mathfrak{Q}^{\alpha-}_{uj}] = [\min_i \Re^{\alpha}_{lij}, \min_i \Re^{\alpha}_{uij}]$, $\eta^{\alpha-}_j = \min_i \zeta^{\alpha}_{ij}$ and $(i = 1, 2, \cdots, s)$.

4). Euclidean distance $d_{\mathcal{E}}$ of each alternative $\aleph_i$ from $\mathcal{F}^+$ and $\mathcal{F}^-$ is determined by the following formulas:

$$d_{\mathcal{E}}(\aleph_i \mathcal{F}^+) = \sqrt{\frac{1}{3m} \sum_{j=1}^{t} \sum_{\alpha=1}^{m} \left((\Re^{\alpha}_{lij} - \mathfrak{Q}^{\alpha+}_{lj})^2 + (\Re^{\alpha}_{uij} - \mathfrak{Q}^{\alpha+}_{uj})^2 + (\zeta^{\alpha}_{ij} - \eta^{\alpha+}_j)^2\right)} \tag{3.5}$$

and

$$d_{\mathcal{E}}(\aleph_i, \mathcal{F}^-) = \sqrt{\frac{1}{3m} \sum_{j=1}^{t} \sum_{\alpha=1}^{m} \left((\Re^{\alpha}_{lij} - \mathfrak{Q}^{\alpha-}_{lj})^2 + (\Re^{\alpha}_{uij} - \mathfrak{Q}^{\alpha-}_{uj})^2 + (\zeta^{\alpha}_{ij} - \eta^{\alpha-}_j)^2\right)}. \tag{3.6}$$

5). The relative closeness of each alternative to CmPFPIS $\mathcal{F}^+$ is calculated as follows:

$$\varrho(\aleph_i) = \frac{d_{\mathcal{E}}(\aleph_i, \mathcal{F}^-)}{d_{\mathcal{E}}(\aleph_i, \mathcal{F}^+) + d_{\mathcal{E}}(\aleph_i, \mathcal{F}^-)}, \quad i = 1, 2, \cdots, s. \tag{3.7}$$

6). Output:
The alternative with highest closeness degree is the best one.
For better understanding, the Algorithm 1 is summarized in Figure 1.

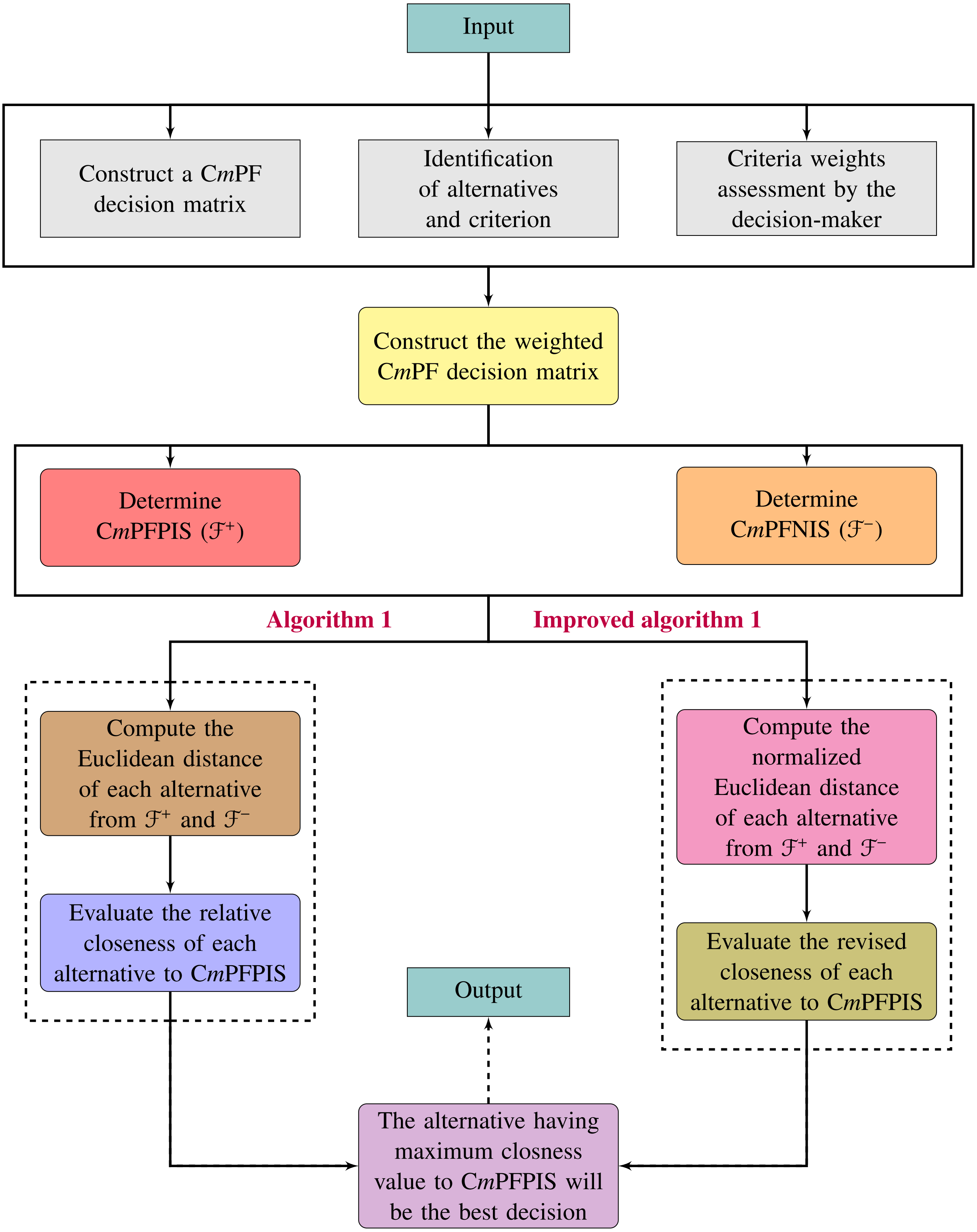

Figure 1. Graphical structure of Algorithm 1 and improved Algorithm 1.

Remark 3.1. *In practical life decision-making problems, some times the Euclidean distance of each* *alternative from positive ideal solution (PIS) and negative ideal solution (NIS) gives inaccurate numeric values.*

Remark 3.2. *The relative closeness index $\varrho(\aleph_i)$ may be used to determine the preference ranking of alternatives as well as the ideal option. However, HadiVencheh and Mirjaberi [28] suggested that in some cases, it may be impossible to find an ideal alternative, that is, closest to the PIS and farthest from the NIS. To compensate for this deficiency, they devised the updated closeness index.*

So, to overcome these shortcomings in Algorithm 1, we provided improved Algorithm 1 that focuses on normalized Euclidean distance and updated closeness index.

Improved Algorithm 1:

In this algorithm, steps 1–3 is similar as Algorithm 1. So, we start from step 4.

4). The normalized Euclidean distance $d_{\mathcal{NE}}$ of each alternative α_i from $\mathcal{F}^+$ and $\mathcal{F}^-$ is computed by the following formulas:

$$d_{\mathcal{NE}}(\aleph_i, \mathcal{F}^+) = \sqrt{\frac{1}{3mt}\sum_{j=1}^{t}\sum_{\alpha=1}^{m}((\Re^{\alpha}_{lij} - \mathfrak{Q}^{\alpha+}_{lj})^2 + (\Re^{\alpha}_{uij} - \mathfrak{Q}^{\alpha+}_{uj})^2 + (\zeta^{\alpha}_{ij} - \eta^{\alpha+}_{j})^2)}, \tag{3.8}$$

and

$$d_{\mathcal{NE}}(\aleph_i, \mathcal{F}^-) = \sqrt{\frac{1}{3mt}\sum_{j=1}^{t}\sum_{\alpha=1}^{m}((\Re^{\alpha}_{lij} - \mathfrak{Q}^{\alpha-}_{lj})^2 + (\Re^{\alpha}_{uij} - \mathfrak{Q}^{\alpha-}_{uj})^2 + (\zeta^{\alpha}_{ij} - \eta^{\alpha-}_{j})^2)}. \tag{3.9}$$

5). The updated closeness index for our developed model is calculated using the following formula:

$$\wp(\aleph_i) = \frac{d_{\mathcal{NE}}(\aleph_i, \mathcal{F}^-)}{\max_i d_{\mathcal{NE}}(\aleph_i, \mathcal{F}^-)} - \frac{d_{\mathcal{NE}}(\aleph_i, \mathcal{F}^+)}{\min_i d_{\mathcal{NE}}(\aleph_i, \mathcal{F}^+)}, \quad i = 1, 2, \cdots, s. \tag{3.10}$$

6). After the calculation of revised closeness index $\wp(\aleph_i)$, rank the alternatives in descending order and the alternative with maximum revised closeness degree will be the suitable optimal alternative.
For better understanding, the improved Algorithm 1 is displayed in Figure 1.

3.1. Numerical application for CmPF-TOPSIS approach

3.1.1. Diagnosis of impulse control disorders

Impulse control disorders (ICDs) are the types of behavioral disorders that cause someone to fall into impulsive thinking. These disorders are often linked to chemical imbalances or structural changes in a region of the brain called the prefrontal cortex. The prefrontal cortex plays an important role in

decision-making processes. These ICDs as a group of mental health disorders involve problems with self-control. People suffering from ICDs may find it difficult to resist the temptation to perform a certain action. In many cases, these urges are related to acting out in some way, through aggressive, dishonest, rule-breaking, or unsafe behavior.

The American Psychiatric Association (APA) rearranged and regrouped a variety of mental health conditions in its most recent revision of the Diagnostic and Statistical Manual of Mental Disorders, Fifth Edition (DSM-5) [46]. From the several types of ICDs, DSM-5 [46] listed six major types of ICDs, which are given as

- Oppositional defiant disorder (ODD),
- Intermittent explosive disorder (IED),
- Conduct disorder (CD),
- Antisocial personality disorder (ASPD),
- Kleptomania,
- Pyromania.

All these ICDs involve problems with self-control in terms of behavior and emotions. People may not realize how common ICDs are in children and adults, but this group of conditions affects many people every year. According to DSM-5 [46], the current annual prevalence rate of these ICDs is estimated about 3.3% of the United States (US) population meets the criteria for ODD, and up to 4% of people meet the criteria for CD. IED is the most common impulse disorder, as around 7% of the US population meet the diagnostic criteria for IED at some point during their lifetime. Antisocial personality disorder (ASPD) rates are different in males and females, according to DSM-5 [46], up to 3.5% of Americans have symptoms of ASPD. Pyromania and kleptomania are rarer, with prevalence estimates hovering at around 1% of the population. These estimates are approximate, and also the rates of these ICDs are different in children, males, and females. The summary of the statistics of these ICDs is shown in Figure 2.

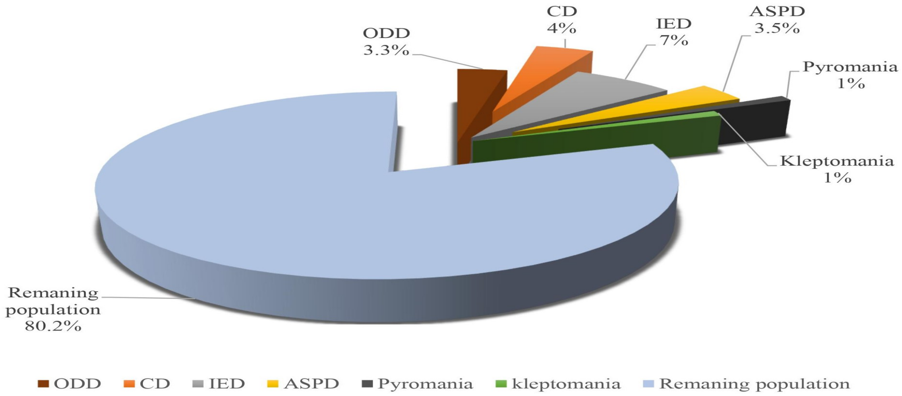

Figure 2. Annual rate of ICDs symptoms in the US population.

Assume that two patients $\{\mathcal{P}_1, \mathcal{P}_2\}$ have a mental problem and go to a Psychiatrist (decision maker) for a checkup. The patients have several common symptoms like irritability, criminal behavior, argumentativeness, vindictiveness, mistreating children or animals, etc. After a complete checkup of the mental condition of the patients, the Psychiatrist ensures that the patients may have one of the aforementioned ICDs.

So, let $\aleph = \{\aleph_i : i = 1, 2, \ldots, 6\}$ be the set of ICDs (alternatives) where $\aleph_1$ = ODD, $\aleph_2$ = IED, $\aleph_3$ = CD, $\aleph_4$ = ASPD, $\aleph_5$ = kleptomania, and $\aleph_6$ = pyromania. Let $S = \{S_j : j = 1, 2, \ldots, 8\}$ be the set of major common symptoms (criteria) in patients which are organized by Psychiatrist after examination. The detail description of symptoms in patients is explained in Figure 3.

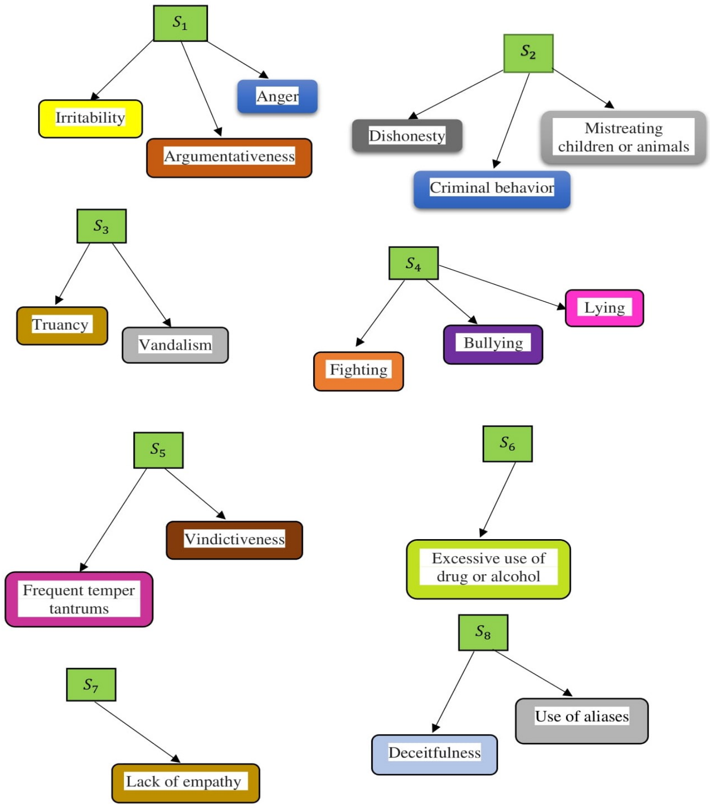

Figure 3. Detailed description of symptoms.

The appropriate rating of each ICD is given by the Psychiatrist with respect to the corresponding symptoms in the form of C3PF value. Now we have to assess the accurate disease, by using Algorithm 1 (CmPF-TOPSIS method) as follows:

1). Input:

(i) The C3PF decision matrix according to the explored problem is provided as:

$$\mathfrak{S} = \begin{array}{c} \\ \aleph_1 \\ \aleph_2 \\ \aleph_3 \\ \aleph_4 \\ \aleph_5 \\ \aleph_6 \end{array} \begin{array}{c} \begin{array}{cccccccc} S_1 & S_2 & S_3 & S_4 & S_5 & S_6 & S_7 & S_8 \end{array} \\ \begin{pmatrix} \tau_{11} & \tau_{12} & \tau_{13} & \tau_{14} & \tau_{15} & \tau_{16} & \tau_{17} & \tau_{18} \\ \tau_{21} & \tau_{22} & \tau_{23} & \tau_{24} & \tau_{25} & \tau_{26} & \tau_{27} & \tau_{28} \\ \tau_{31} & \tau_{32} & \tau_{33} & \tau_{34} & \tau_{35} & \tau_{36} & \tau_{37} & \tau_{38} \\ \tau_{41} & \tau_{42} & \tau_{43} & \tau_{44} & \tau_{45} & \tau_{46} & \tau_{37} & \tau_{38} \\ \tau_{51} & \tau_{52} & \tau_{53} & \tau_{54} & \tau_{55} & \tau_{56} & \tau_{57} & \tau_{58} \\ \tau_{61} & \tau_{62} & \tau_{63} & \tau_{64} & \tau_{65} & \tau_{66} & \tau_{67} & \tau_{68} \end{pmatrix} \end{array},$$

where the entries τ_{ij} according to the judgments of decision-maker for $\mathcal{P}_1$ and $\mathcal{P}_2$ are expressed in Tables 1 and 2, respectively.

(ii) Weights assessments by the decision-maker that satisfying the normalized condition $\sum_{j=1}^{8} \beth_j = 1$ are given as:

$$\mathcal{W} = \begin{bmatrix} \beth_1 & \beth_2 & \beth_3 & \beth_4 & \beth_5 & \beth_6 & \beth_7 & \beth_8 \end{bmatrix}^T = \begin{bmatrix} 0.1 & 0.2 & 0.15 & 0.1 & 0.15 & 0.1 & 0.1 & 0.1 \end{bmatrix}^T.$$

Table 1. C3PF decision matrix for '$\mathcal{P}_1$'.

	S_1	S_2
$\aleph_1$	$\langle\{[0.7,0.9],[0.4,0.5],[0.2,0.3]\},\{0.7,0.6,0.3\}\rangle$	$\langle\{[0.3,0.4],[0.5,0.6],[0.5,0.7]\},\{0.6,0.2,0.8\}\rangle$
$\aleph_2$	$\langle\{[0.1,0.3],[0.2,0.5],[0.4,0.5]\},\{0.3,0.4,0.6\}\rangle$	$\langle\{[0.1,0.2],[0.4,0.5],[0.1,0.3]\},\{0.3,0.4,0.5\}\rangle$
$\aleph_3$	$\langle\{[0.2,0.5],[0.1,0.3],[0.5,0.6]\},\{0.2,0.3,0.5\}\rangle$	$\langle\{[0.2,0.3],[0.0,0.1],[0.8,0.9]\},\{0.4,0.1,0.3\}\rangle$
$\aleph_4$	$\langle\{[0.3,0.4],[0.4,0.6],[0.1,0.2]\},\{0.1,0.5,0.2\}\rangle$	$\langle\{[0.3,0.7],[0.5,0.7],[0.3,0.4]\},\{0.2,0.5,0.4\}\rangle$
$\aleph_5$	$\langle\{[0.1,0.4],[0.2,0.5],[0.1,0.3]\},\{0.7,0.3,0.4\}\rangle$	$\langle\{[0.4,0.5],[0.3,0.5],[0.0,0.2]\},\{0.5,0.4,0.3\}\rangle$
$\aleph_6$	$\langle\{[0.6,0.7],[0.3,0.4],[0.4,0.5]\},\{0.1,0.8,0.6\}\rangle$	$\langle\{[0.5,0.6],[0.2,0.3],[0.2,0.3]\},\{0.2,0.7,0.6\}\rangle$
	S_3	S_4
$\aleph_1$	$\langle\{[0.4,0.6],[0.2,0.3],[0.1,0.3]\},\{0.3,0.5,0.6\}\rangle$	$\langle\{[0.2,0.3],[0.2,0.5],[0.2,0.4]\},\{0.2,0.3,0.7\}\rangle$
$\aleph_2$	$\langle\{[0.1,0.4],[0.5,0.6],[0.7,0.8]\},\{0.7,0.3,0.2\}\rangle$	$\langle\{[0.1,0.2],[0.3,0.4],[0.1,0.3]\},\{0.1,0.4,0.5\}\rangle$
$\aleph_3$	$\langle\{[0.3,0.5],[0.2,0.4],[0.5,0.6]\},\{0.6,0.4,0.1\}\rangle$	$\langle\{[0.3,0.4],[0.1,0.3],[0.3,0.5]\},\{0.3,0.5,0.6\}\rangle$
$\aleph_4$	$\langle\{[0.5,0.7],[0.1,0.4],[0.3,0.4]\},\{0.1,0.2,0.4\}\rangle$	$\langle\{[0.4,0.6],[0.0,0.2],[0.0,0.1]\},\{0.6,0.3,0.2\}\rangle$
$\aleph_5$	$\langle\{[0.2,0.3],[0.4,0.7],[0.2,0.3]\},\{0.5,0.3,0.4\}\rangle$	$\langle\{[0.5,0.7],[0.2,0.4],[0.7,0.8]\},\{0.2,0.2,0.0\}\rangle$
$\aleph_6$	$\langle\{[0.0,0.1],[0.1,0.3],[0.4,0.5]\},\{0.0,0.4,0.3\}\rangle$	$\langle\{[0.3,0.5],[0.0,0.2],[0.3,0.4]\},\{0.1,0.3,0.2\}\rangle$
	S_5	S_6
$\aleph_1$	$\langle\{[0.2,0.3],[0.2,0.4],[0.2,0.3]\},\{0.3,0.6,0.5\}\rangle$	$\langle\{[0.7,0.9],[0.4,0.5],[0.2,0.3]\},\{0.7,0.6,0.3\}\rangle$
$\aleph_2$	$\langle\{[0.4,0.5],[0.5,0.6],[0.1,0.2]\},\{0.3,0.5,0.8\}\rangle$	$\langle\{[0.4,0.6],[0.3,0.6],[0.3,0.4]\},\{0.4,0.1,0.2\}\rangle$
$\aleph_3$	$\langle\{[0.1,0.2],[0.3,0.5],[0.2,0.3]\},\{0.5,0.4,0.7\}\rangle$	$\langle\{[0.5,0.7],[0.2,0.4],[0.1,0.2]\},\{0.1,0.3,0.4\}\rangle$
$\aleph_4$	$\langle\{[0.5,0.6],[0.2,0.3],[0.5,0.6]\},\{0.2,0.7,0.6\}\rangle$	$\langle\{[0.0,0.1],[0.1,0.3],[0.4,0.5]\},\{0.0,0.4,0.3\}\rangle$
$\aleph_5$	$\langle\{[0.3,0.5],[0.0,0.2],[0.3,0.4]\},\{0.1,0.3,0.3\}\rangle$	$\langle\{[0.2,0.5],[0.1,0.2],[0.5,0.6]\},\{0.2,0.1,0.5\}\rangle$
$\aleph_6$	$\langle\{[0.5,0.7],[0.1,0.4],[0.4,0.5]\},\{0.4,0.2,0.4\}\rangle$	$\langle\{[0.4,0.6],[0.2,0.3],[0.1,0.3]\},\{0.3,0.5,0.4\}\rangle$
	S_7	S_8
$\aleph_1$	$\langle\{[0.1,0.3],[0.2,0.4],[0.3,0.5]\},\{0.2,0.3,0.1\}\rangle$	$\langle\{[0.2,0.5],[0.1,0.3],[0.3,0.4]\},\{0.1,0.4,0.6\}\rangle$
$\aleph_2$	$\langle\{[0.3,0.4],[0.5,0.6],[0.1,0.3]\},\{0.0,0.2,0.1\}\rangle$	$\langle\{[0.1,0.2],[0.3,0.5],[0.4,0.6]\},\{0.0,0.2,0.3\}\rangle$
$\aleph_3$	$\langle\{[0.2,0.5],[0.3,0.5],[0.2,0.4]\},\{0.1,0.4,0.3\}\rangle$	$\langle\{[0.0,0.3],[0.2,0.4],[0.1,0.3]\},\{0.5,0.3,0.4\}\rangle$
$\aleph_4$	$\langle\{[0.6,0.7],[0.1,0.3],[0.4,0.6]\},\{0.3,0.5,0.2\}\rangle$	$\langle\{[0.2,0.4],[0.1,0.2],[0.0,0.2]\},\{0.2,0.0,0.1\}\rangle$
$\aleph_5$	$\langle\{[0.4,0.6],[0.0,0.2],[0.5,0.7]\},\{0.4,0.4,0.2\}\rangle$	$\langle\{[0.3,0.6],[0.0,0.1],[0.2,0.5]\},\{0.3,0.1,0.2\}\rangle$
$\aleph_6$	$\langle\{[0.1,0.4],[0.2,0.5],[0.1,0.2]\},\{0.6,0.3,0.4\}\rangle$	$\langle\{[0.5,0.7],[0.2,0.3],[0.0,0.1]\},\{0.4,0.2,0.5\}\rangle$

Table 2. C3PF decision matrix for patient '$\mathcal{P}_2$'.

	S_1	S_2
$\aleph_1$	$\langle\{[0.5, 0.7], [0.5, 0.6], [0.2, 0.4]\}, \{0.6, 0.4, 0.1\}\rangle$	$\langle\{[0.1, 0.3], [0.5, 0.8], [0.3, 0.4]\}, \{0.7, 0.3, 0.2\}\rangle$
$\aleph_2$	$\langle\{[0.1, 0.4], [0.2, 0.3], [0.6, 0.7]\}, \{0.3, 0.5, 0.2\}\rangle$	$\langle\{[0.3, 0.6], [0.2, 0.5], [0.0, 0.1]\}, \{0.4, 0.2, 0.4\}\rangle$
$\aleph_3$	$\langle\{[0.2, 0.3], [0.4, 0.7], [0.5, 0.6]\}, \{0.5, 0.3, 0.4\}\rangle$	$\langle\{[0.2, 0.5], [0.4, 0.6], [0.8, 0.9]\}, \{0.3, 0.1, 0.5\}\rangle$
$\aleph_4$	$\langle\{[0.3, 0.5], [0.2, 0.4], [0.1, 0.2]\}, \{0.4, 0.6, 0.5\}\rangle$	$\langle\{[0.3, 0.6], [0.3, 0.7], [0.1, 0.3]\}, \{0.5, 0.4, 0.0\}\rangle$
$\aleph_5$	$\langle\{[0.4, 0.6], [0.1, 0.3], [0.4, 0.5]\}, \{0.2, 0.5, 0.6\}\rangle$	$\langle\{[0.4, 0.7], [0.2, 0.4], [0.2, 0.5]\}, \{0.4, 0.5, 0.3\}\rangle$
$\aleph_6$	$\langle\{[0.6, 0.8], [0.4, 0.5], [0.3, 0.7]\}, \{0.8, 0.7, 0.1\}\rangle$	$\langle\{[0.2, 0.4], [0.1, 0.3], [0.0, 0.2]\}, \{0.2, 0.4, 0.6\}\rangle$
	S_3	S_4
$\aleph_1$	$\langle\{[0.3, 0.4], [0.4, 0.6], [0.1, 0.2]\}, \{0.1, 0.5, 0.2\}\rangle$	$\langle\{[0.0, 0.3], [0.3, 0.7], [0.2, 0.4]\}, \{0.2, 0.3, 0.1\}\rangle$
$\aleph_2$	$\langle\{[0.2, 0.3], [0.3, 0.5], [0.2, 0.4]\}, \{0.4, 0.8, 0.6\}\rangle$	$\langle\{[0.1, 0.4], [0.2, 0.5], [0.1, 0.2]\}, \{0.5, 0.4, 0.6\}\rangle$
$\aleph_3$	$\langle\{[0.7, 0.9], [0.2, 0.3], [0.4, 0.5]\}, \{0.7, 0.6, 0.3\}\rangle$	$\langle\{[0.6, 0.7], [0.1, 0.3], [0.4, 0.6]\}, \{0.6, 0.5, 0.7\}\rangle$
$\aleph_4$	$\langle\{[0.2, 0.5], [0.1, 0.4], [0.5, 0.6]\}, \{0.1, 0.3, 0.5\}\rangle$	$\langle\{[0.2, 0.5], [0.2, 0.4], [0.3, 0.5]\}, \{0.3, 0.6, 0.4\}\rangle$
$\aleph_5$	$\langle\{[0.6, 0.7], [0.4, 0.7], [0.1, 0.3]\}, \{0.7, 0.5, 0.4\}\rangle$	$\langle\{[0.4, 0.6], [0.0, 0.2], [0.5, 0.7]\}, \{0.4, 0.3, 0.2\}\rangle$
$\aleph_6$	$\langle\{[0.1, 0.3], [0.4, 0.5], [0.3, 0.7]\}, \{0.3, 0.4, 0.6\}\rangle$	$\langle\{[0.3, 0.4], [0.4, 0.6], [0.6, 0.8]\}, \{0.0, 0.2, 0.1\}\rangle$
	S_5	S_6
$\aleph_1$	$\langle\{[0.1, 0.2], [0.1, 0.3], [0.4, 0.6]\}, \{0.3, 0.5, 0.2\}\rangle$	$\langle\{[0.2, 0.4], [0.1, 0.2], [0.3, 0.5]\}, \{0.2, 0.4, 0.6\}\rangle$
$\aleph_2$	$\langle\{[0.4, 0.5], [0.2, 0.3], [0.1, 0.2]\}, \{0.1, 0.4, 0.5\}\rangle$	$\langle\{[0.3, 0.5], [0.2, 0.4], [0.1, 0.4]\}, \{0.1, 0.0, 0.2\}\rangle$
$\aleph_3$	$\langle\{[0.2, 0.3], [0.5, 0.6], [0.2, 0.3]\}, \{0.5, 0.6, 0.7\}\rangle$	$\langle\{[0.4, 0.7], [0.3, 0.5], [0.2, 0.6]\}, \{0.7, 0.5, 0.3\}\rangle$
$\aleph_4$	$\langle\{[0.3, 0.4], [0.2, 0.4], [0.3, 0.5]\}, \{0.2, 0.3, 0.6\}\rangle$	$\langle\{[0.6, 0.8], [0.4, 0.6], [0.0, 0.3]\}, \{0.5, 0.3, 0.4\}\rangle$
$\aleph_5$	$\langle\{[0.0, 0.1], [0.4, 0.5], [0.0, 0.4]\}, \{0.4, 0.2, 0.4\}\rangle$	$\langle\{[0.5, 0.6], [0.1, 0.3], [0.1, 0.2]\}, \{0.3, 0.2, 0.6\}\rangle$
$\aleph_6$	$\langle\{[0.5, 0.7], [0.1, 0.4], [0.4, 0.5]\}, \{0.0, 0.1, 0.3\}\rangle$	$\langle\{[0.1, 0.5], [0.0, 0.1], [0.1, 0.3]\}, \{0.4, 0.1, 0.3\}\rangle$
	S_7	S_8
$\aleph_1$	$\langle\{[0.3, 0.5], [0.4, 0.6], [0.2, 0.4]\}, \{0.2, 0.4, 0.5\}\rangle$	$\langle\{[0.1, 0.3], [0.2, 0.4], [0.1, 0.4]\}, \{0.2, 0.4, 0.5\}\rangle$
$\aleph_2$	$\langle\{[0.1, 0.4], [0.2, 0.4], [0.0, 0.3]\}, \{0.2, 0.3, 0.1\}\rangle$	$\langle\{[0.0, 0.2], [0.4, 0.6], [0.4, 0.5]\}, \{0.3, 0.1, 0.3\}\rangle$
$\aleph_3$	$\langle\{[0.2, 0.6], [0.1, 0.5], [0.3, 0.5]\}, \{0.4, 0.2, 0.3\}\rangle$	$\langle\{[0.2, 0.4], [0.4, 0.5], [0.3, 0.7]\}, \{0.5, 0.7, 0.4\}\rangle$
$\aleph_4$	$\langle\{[0.0, 0.2], [0.3, 0.7], [0.4, 0.6]\}, \{0.2, 0.5, 0.4\}\rangle$	$\langle\{[0.3, 0.5], [0.2, 0.3], [0.0, 0.2]\}, \{0.4, 0.3, 0.1\}\rangle$
$\aleph_5$	$\langle\{[0.4, 0.5], [0.0, 0.2], [0.5, 0.7]\}, \{0.3, 0.7, 0.0\}\rangle$	$\langle\{[0.4, 0.6], [0.1, 0.2], [0.2, 0.3]\}, \{0.6, 0.1, 0.2\}\rangle$
$\aleph_6$	$\langle\{[0.5, 0.7], [0.2, 0.3], [0.1, 0.3]\}, \{0.1, 0.3, 0.6\}\rangle$	$\langle\{[0.6, 0.7], [0.0, 0.1], [0.5, 0.6]\}, \{0.6, 0.2, 0.0\}\rangle$

2). The weighted C3PF decision-matrix $\mathfrak{V}$ is constructed as:

$$\mathfrak{V} = [\varkappa_{ij}]_{5\times 6} = \begin{bmatrix} \varkappa_{11} & \varkappa_{12} & \cdots & \varkappa_{16} \\ \varkappa_{21} & \varkappa_{22} & \cdots & \varkappa_{26} \\ \vdots & \vdots & \ddots & \vdots \\ \varkappa_{51} & \varkappa_{52} & \cdots & \varkappa_{56} \end{bmatrix},$$

where the entries $\varkappa_{ij} = \sqsupseteq_j \cdot \tau_{ij}$ for both patients is calculated by using Definition 2.6 and expressed in Tables 3 and 4.

3). The C3PFPIS ($\mathcal{F}^+$) and C3PFNIS ($\mathcal{F}^-$) are computed and displayed in Table 5.

Table 3. Weighted C3PF decision matrix for patient '$\mathcal{P}_1$'.

	S_1	S_2
$\aleph_1$	$\langle\{[0.11, 0.21], [0.05, 0.07], [0.02, 0.04]\}, \{0.96, 0.95, 0.89\}\rangle$	$\langle\{[0.07, 0.10], [0.13, 0.17], [0.13, 0.21]\}, \{0.90, 0.72, 0.96\}\rangle$
$\aleph_2$	$\langle\{[0.01, 0.04], [0.02, 0.07], [0.05, 0.07]\}, \{0.89, 0.91, 0.95\}\rangle$	$\langle\{[0.02, 0.04], [0.10, 0.13], [0.02, 0.07]\}, \{0.79, 0.83, 0.87\}\rangle$
$\aleph_3$	$\langle\{[0.02, 0.07], [0.01, 0.04], [0.07, 0.09]\}, \{0.85, 0.89, 0.93\}\rangle$	$\langle\{[0.04, 0.07], [0.00, 0.02], [0.28, 0.37]\}, \{0.83, 0.63, 0.79\}\rangle$
$\aleph_4$	$\langle\{[0.04, 0.05], [0.05, 0.09], [0.01, 0.02]\}, \{0.79, 0.93, 0.85\}\rangle$	$\langle\{[0.07, 0.21], [0.13, 0.21], [0.07, 0.10]\}, \{0.72, 0.87, 0.83\}\rangle$
$\aleph_5$	$\langle\{[0.01, 0.05], [0.02, 0.07], [0.01, 0.04]\}, \{0.96, 0.89, 0.91\}\rangle$	$\langle\{[0.10, 0.13], [0.07, 0.13], [0.00, 0.04]\}, \{0.87, 0.83, 0.79\}\rangle$
$\aleph_6$	$\langle\{[0.09, 0.11], [0.04, 0.05], [0.05, 0.07]\}, \{0.79, 0.98, 0.95\}\rangle$	$\langle\{[0.13, 0.17], [0.04, 0.07], [0.04, 0.07]\}, \{0.72, 0.93, 0.90\}\rangle$
	S_3	S_4
$\aleph_1$	$\langle\{[0.07, 0.13], [0.03, 0.05], [0.02, 0.05]\}, \{0.83, 0.90, 0.93\}\rangle$	$\langle\{[0.02, 0.04], [0.02, 0.07], [0.02, 0.05]\}, \{0.85, 0.89, 0.96\}\rangle$
$\aleph_2$	$\langle\{[0.02, 0.07], [0.10, 0.13], [0.17, 0.21]\}, \{0.95, 0.83, 0.79\}\rangle$	$\langle\{[0.01, 0.02], [0.04, 0.05], [0.01, 0.04]\}, \{0.79, 0.91, 0.93\}\rangle$
$\aleph_3$	$\langle\{[0.05, 0.10], [0.03, 0.07], [0.10, 0.13]\}, \{0.93, 0.87, 0.71\}\rangle$	$\langle\{[0.04, 0.05], [0.01, 0.04], [0.04, 0.07]\}, \{0.89, 0.93, 0.95\}\rangle$
$\aleph_4$	$\langle\{[0.10, 0.17], [0.02, 0.07], [0.05, 0.07]\}, \{0.71, 0.79, 0.87\}\rangle$	$\langle\{[0.05, 0.09], [0.00, 0.02], [0.00, 0.01]\}, \{0.95, 0.89, 0.85\}\rangle$
$\aleph_5$	$\langle\{[0.03, 0.05], [0.07, 0.17], [0.03, 0.05]\}, \{0.90, 0.83, 0.79\}\rangle$	$\langle\{[0.07, 0.11], [0.02, 0.05], [0.11, 0.15]\}, \{0.85, 0.85, 0.00\}\rangle$
$\aleph_6$	$\langle\{[0.00, 0.02], [0.02, 0.05], [0.07, 0.10]\}, \{0.00, 0.87, 0.83\}\rangle$	$\langle\{[0.04, 0.07], [0.00, 0.02], [0.04, 0.05]\}, \{0.79, 0.89, 0.85\}\rangle$
	S_5	S_6
$\aleph_1$	$\langle\{[0.03, 0.05], [0.03, 0.07], [0.03, 0.05]\}, \{0.83, 0.93, 0.90\}\rangle$	$\langle\{[0.11, 0.21], [0.05, 0.07], [0.02, 0.04]\}, \{0.96, 0.95, 0.89\}\rangle$
$\aleph_2$	$\langle\{[0.07, 0.10], [0.10, 0.13], [0.02, 0.03]\}, \{0.83, 0.90, 0.97\}\rangle$	$\langle\{[0.05, 0.09], [0.04, 0.09], [0.04, 0.05]\}, \{0.91, 0.79, 0.85\}\rangle$
$\aleph_3$	$\langle\{[0.02, 0.03], [0.05, 0.10], [0.03, 0.05]\}, \{0.90, 0.87, 0.95\}\rangle$	$\langle\{[0.07, 0.11], [0.02, 0.05], [0.01, 0.02]\}, \{0.79, 0.89, 0.91\}\rangle$
$\aleph_4$	$\langle\{[0.10, 0.13], [0.03, 0.05], [0.10, 0.13]\}, \{0.79, 0.95, 0.93\}\rangle$	$\langle\{[0.00, 0.01], [0.01, 0.04], [0.05, 0.07]\}, \{0.00, 0.91, 0.89\}\rangle$
$\aleph_5$	$\langle\{[0.05, 0.10], [0.00, 0.03], [0.05, 0.07]\}, \{0.71, 0.83, 0.83\}\rangle$	$\langle\{[0.02, 0.07], [0.01, 0.02], [0.07, 0.09]\}, \{0.85, 0.79, 0.93\}\rangle$
$\aleph_6$	$\langle\{[0.10, 0.17], [0.02, 0.07], [0.07, 0.10]\}, \{0.87, 0.79, 0.87\}\rangle$	$\langle\{[0.05, 0.09], [0.02, 0.04], [0.01, 0.04]\}, \{0.89, 0.93, 0.91\}\rangle$
	S_7	S_8
$\aleph_1$	$\langle\{[0.01, 0.04], [0.02, 0.05], [0.04, 0.07]\}, \{0.85, 0.89, 0.79\}\rangle$	$\langle\{[0.02, 0.07], [0.01, 0.04], [0.04, 0.05]\}, \{0.79, 0.91, 0.95\}\rangle$
$\aleph_2$	$\langle\{[0.04, 0.05], [0.07, 0.09], [0.01, 0.04]\}, \{0.00, 0.85, 0.79\}\rangle$	$\langle\{[0.01, 0.02], [0.04, 0.07], [0.05, 0.09]\}, \{0.00, 0.85, 0.89\}\rangle$
$\aleph_3$	$\langle\{[0.02, 0.07], [0.04, 0.07], [0.02, 0.05]\}, \{0.71, 0.91, 0.89\}\rangle$	$\langle\{[0.00, 0.04], [0.02, 0.05], [0.01, 0.04]\}, \{0.93, 0.89, 0.91\}\rangle$
$\aleph_4$	$\langle\{[0.09, 0.11], [0.01, 0.04], [0.05, 0.09]\}, \{0.89, 0.93, 0.85\}\rangle$	$\langle\{[0.02, 0.05], [0.01, 0.02], [0.00, 0.02]\}, \{0.85, 0.00, 0.79\}\rangle$
$\aleph_5$	$\langle\{[0.05, 0.09], [0.00, 0.02], [0.07, 0.11]\}, \{0.91, 0.91, 0.85\}\rangle$	$\langle\{[0.04, 0.09], [0.00, 0.01], [0.02, 0.07]\}, \{0.89, 0.79, 0.85\}\rangle$
$\aleph_6$	$\langle\{[0.01, 0.05], [0.02, 0.07], [0.01, 0.02]\}, \{0.95, 0.89, 0.91\}\rangle$	$\langle\{[0.07, 0.11], [0.02, 0.04], [0.00, 0.01]\}, \{0.91, 0.85, 0.93\}\rangle$

Table 4. Weighted C3PF decision matrix for patient '$\mathcal{P}_2$'.

	S_1	S_2
$\aleph_1$	$\langle\{[0.07, 0.11], [0.07, 0.09], [0.02, 0.05]\}, \{0.95, 0.91, 0.79\}\rangle$	$\langle\{[0.02, 0.07], [0.13, 0.28], [0.07, 0.10]\}, \{0.93, 0.79, 0.72\}\rangle$
$\aleph_2$	$\langle\{[0.01, 0.05], [0.02, 0.04], [0.09, 0.11]\}, \{0.89, 0.93, 0.85\}\rangle$	$\langle\{[0.07, 0.17], [0.04, 0.13], [0.00, 0.02]\}, \{0.83, 0.72, 0.83\}\rangle$
$\aleph_3$	$\langle\{[0.02, 0.04], [0.05, 0.11], [0.07, 0.09]\}, \{0.93, 0.89, 0.91\}\rangle$	$\langle\{[0.04, 0.13], [0.10, 0.17], [0.28, 0.37]\}, \{0.79, 0.63, 0.87\}\rangle$
$\aleph_4$	$\langle\{[0.04, 0.07], [0.02, 0.05], [0.01, 0.02]\}, \{0.91, 0.95, 0.93\}\rangle$	$\langle\{[0.07, 0.17], [0.07, 0.21], [0.02, 0.07]\}, \{0.87, 0.83, 0.00\}\rangle$
$\aleph_5$	$\langle\{[0.05, 0.09], [0.01, 0.04], [0.05, 0.07]\}, \{0.85, 0.93, 0.95\}\rangle$	$\langle\{[0.10, 0.21], [0.04, 0.10], [0.04, 0.13]\}, \{0.83, 0.87, 0.79\}\rangle$
$\aleph_6$	$\langle\{[0.09, 0.15], [0.05, 0.07], [0.04, 0.11]\}, \{0.98, 0.96, 0.79\}\rangle$	$\langle\{[0.04, 0.10], [0.02, 0.07], [0.00, 0.04]\}, \{0.72, 0.83, 0.90\}\rangle$
	S_3	S_4
$\aleph_1$	$\langle\{[0.05, 0.07], [0.07, 0.13], [0.02, 0.03]\}, \{0.71, 0.93, 0.79\}\rangle$	$\langle\{[0.00, 0.04], [0.04, 0.11], [0.02, 0.05]\}, \{0.85, 0.89, 0.79\}\rangle$
$\aleph_2$	$\langle\{[0.03, 0.05], [0.05, 0.10], [0.03, 0.07]\}, \{0.87, 0.97, 0.93\}\rangle$	$\langle\{[0.01, 0.05], [0.02, 0.07], [0.01, 0.02]\}, \{0.93, 0.91, 0.95\}\rangle$
$\aleph_3$	$\langle\{[0.17, 0.29], [0.03, 0.05], [0.07, 0.10]\}, \{0.95, 0.93, 0.83\}\rangle$	$\langle\{[0.09, 0.11], [0.01, 0.04], [0.05, 0.09]\}, \{0.95, 0.93, 0.96\}\rangle$
$\aleph_4$	$\langle\{[0.03, 0.10], [0.02, 0.07], [0.10, 0.13]\}, \{0.71, 0.83, 0.90\}\rangle$	$\langle\{[0.02, 0.07], [0.02, 0.05], [0.04, 0.07]\}, \{0.89, 0.95, 0.91\}\rangle$
$\aleph_5$	$\langle\{[0.13, 0.17], [0.07, 0.17], [0.02, 0.05]\}, \{0.95, 0.90, 0.87\}\rangle$	$\langle\{[0.05, 0.09], [0.00, 0.02], [0.07, 0.11]\}, \{0.91, 0.89, 0.85\}\rangle$
$\aleph_6$	$\langle\{[0.02, 0.05], [0.07, 0.10], [0.05, 0.17]\}, \{0.83, 0.87, 0.93\}\rangle$	$\langle\{[0.04, 0.05], [0.05, 0.09], [0.09, 0.15]\}, \{0.00, 0.85, 0.79\}\rangle$
	S_5	S_6
$\aleph_1$	$\langle\{[0.02, 0.03], [0.02, 0.05], [0.07, 0.13]\}, \{0.83, 0.90, 0.79\}\rangle$	$\langle\{[0.02, 0.05], [0.01, 0.02], [0.04, 0.07]\}, \{0.85, 0.91, 0.95\}\rangle$
$\aleph_2$	$\langle\{[0.07, 0.10], [0.03, 0.05], [0.02, 0.03]\}, \{0.71, 0.87, 0.90\}\rangle$	$\langle\{[0.04, 0.07], [0.02, 0.05], [0.01, 0.05]\}, \{0.79, 0.00, 0.85\}\rangle$
$\aleph_3$	$\langle\{[0.03, 0.05], [0.10, 0.13], [0.03, 0.05]\}, \{0.90, 0.93, 0.95\}\rangle$	$\langle\{[0.05, 0.11], [0.04, 0.07], [0.02, 0.09]\}, \{0.96, 0.93, 0.89\}\rangle$
$\aleph_4$	$\langle\{[0.05, 0.07], [0.03, 0.07], [0.05, 0.10]\}, \{0.79, 0.83, 0.93\}\rangle$	$\langle\{[0.09, 0.15], [0.05, 0.09], [0.00, 0.04]\}, \{0.93, 0.89, 0.91\}\rangle$
$\aleph_5$	$\langle\{[0.00, 0.02], [0.07, 0.10], [0.00, 0.07]\}, \{0.87, 0.79, 0.87\}\rangle$	$\langle\{[0.07, 0.09], [0.01, 0.04], [0.01, 0.02]\}, \{0.89, 0.85, 0.95\}\rangle$
$\aleph_6$	$\langle\{[0.10, 0.17], [0.02, 0.07], [0.07, 0.10]\}, \{0.00, 0.71, 0.83\}\rangle$	$\langle\{[0.01, 0.07], [0.00, 0.01], [0.01, 0.04]\}, \{0.91, 0.79, 0.81\}\rangle$
	S_7	S_8
$\aleph_1$	$\langle\{[0.04, 0.07], [0.05, 0.09], [0.02, 0.05]\}, \{0.85, 0.91, 0.93\}\rangle$	$\langle\{[0.01, 0.04], [0.02, 0.05], [0.01, 0.05]\}, \{0.85, 0.91, 0.93\}\rangle$
$\aleph_2$	$\langle\{[0.01, 0.05], [0.02, 0.05], [0.00, 0.04]\}, \{0.85, 0.89, 0.79\}\rangle$	$\langle\{[0.00, 0.02], [0.05, 0.09], [0.05, 0.07]\}, \{0.89, 0.79, 0.89\}\rangle$
$\aleph_3$	$\langle\{[0.02, 0.09], [0.01, 0.07], [0.04, 0.07]\}, \{0.91, 0.85, 0.89\}\rangle$	$\langle\{[0.02, 0.05], [0.05, 0.07], [0.04, 0.11]\}, \{0.93, 0.96, 0.91\}\rangle$
$\aleph_4$	$\langle\{[0.00, 0.02], [0.04, 0.11], [0.05, 0.09]\}, \{0.85, 0.93, 0.91\}\rangle$	$\langle\{[0.04, 0.07], [0.02, 0.04], [0.00, 0.02]\}, \{0.91, 0.89, 0.79\}\rangle$
$\aleph_5$	$\langle\{[0.05, 0.07], [0.00, 0.02], [0.07, 0.11]\}, \{0.89, 0.96, 0.00\}\rangle$	$\langle\{[0.05, 0.09], [0.01, 0.02], [0.02, 0.04]\}, \{0.95, 0.79, 0.85\}\rangle$
$\aleph_6$	$\langle\{[0.07, 0.11], [0.02, 0.04], [0.01, 0.04]\}, \{0.79, 0.89, 0.95\}\rangle$	$\langle\{[0.09, 0.11], [0.00, 0.01], [0.07, 0.09]\}, \{0.95, 0.85, 0.00\}\rangle$

Table 5. C3PFPIS and C3PFNIS.

	For $\mathcal{P}_1$	For $\mathcal{P}_2$
	$\mathcal{F}^+$	$\mathcal{F}^+$
S_1	$\langle\{[0.11, 0.21], [0.05, 0.09], [0.07, 0.09]\}, \{0.96, 0.98, 0.95\}\rangle$	$\langle\{[0.09, 0.15], [0.07, 0.11], [0.09, 0.11]\}, \{0.98, 0.96, 0.95\}\rangle$
S_2	$\langle\{[0.13, 0.21], [0.13, 0.21], [0.28, 0.37]\}, \{0.90, 0.93, 0.96\}\rangle$	$\langle\{[0.10, 0.21], [0.13, 0.28], [0.28, 0.37]\}, \{0.93, 0.87, 0.90\}\rangle$
S_3	$\langle\{[0.10, 0.17], [0.10, 0.17], [0.17, 0.21]\}, \{0.95, 0.90, 0.93\}\rangle$	$\langle\{[0.17, 0.29], [0.07, 0.17], [0.10, 0.17]\}, \{0.95, 0.97, 0.93\}\rangle$
S_4	$\langle\{[0.07, 0.11], [0.04, 0.07], [0.11, 0.15]\}, \{0.95, 0.93, 0.96\}\rangle$	$\langle\{[0.09, 0.11], [0.05, 0.11], [0.09, 0.15]\}, \{0.95, 0.95, 0.96\}\rangle$
S_5	$\langle\{[0.10, 0.17], [0.10, 0.13], [0.10, 0.13]\}, \{0.90, 0.95, 0.97\}\rangle$	$\langle\{[0.10, 0.17], [0.10, 0.13], [0.07, 0.13]\}, \{0.90, 0.93, 0.95\}\rangle$
S_6	$\langle\{[0.11, 0.21], [0.05, 0.09], [0.07, 0.09]\}, \{0.96, 0.95, 0.93\}\rangle$	$\langle\{[0.09, 0.15], [0.05, 0.09], [0.04, 0.09]\}, \{0.96, 0.93, 0.95\}\rangle$
S_7	$\langle\{[0.09, 0.11], [0.07, 0.09], [0.07, 0.11]\}, \{0.95, 0.93, 0.91\}\rangle$	$\langle\{[0.07, 0.11], [0.05, 0.11], [0.07, 0.11]\}, \{0.91, 0.96, 0.95\}\rangle$
S_8	$\langle\{[0.07, 0.11], [0.04, 0.07], [0.05, 0.09]\}, \{0.93, 0.91, 0.95\}\rangle$	$\langle\{[0.09, 0.11], [0.05, 0.09], [0.07, 0.11]\}, \{0.95, 0.96, 0.93\}\rangle$
	$\mathcal{F}^-$	$\mathcal{F}^-$
S_1	$\langle\{[0.01, 0.04], [0.01, 0.04], [0.01, 0.02]\}, \{0.79, 0.89, 0.85\}\rangle$	$\langle\{[0.01, 0.04], [0.01, 0.04], [0.01, 0.02]\}, \{0.85, 0.89, 0.79\}\rangle$
S_2	$\langle\{[0.02, 0.04], [0.00, 0.02], [0.00, 0.04]\}, \{0.72, 0.63, 0.79\}\rangle$	$\langle\{[0.02, 0.07], [0.02, 0.07], [0.00, 0.02]\}, \{0.72, 0.63, 0.00\}\rangle$
S_3	$\langle\{[0.00, 0.02], [0.02, 0.05], [0.02, 0.05]\}, \{0.00, 0.79, 0.71\}\rangle$	$\langle\{[0.02, 0.05], [0.02, 0.05], [0.02, 0.03]\}, \{0.71, 0.83, 0.79\}\rangle$
S_4	$\langle\{[0.01, 0.02], [0.00, 0.02], [0.00, 0.01]\}, \{0.79, 0.85, 0.00\}\rangle$	$\langle\{[0.00, 0.04], [0.00, 0.02], [0.01, 0.02]\}, \{0.00, 0.85, 0.79\}\rangle$
S_5	$\langle\{[0.02, 0.03], [0.00, 0.03], [0.02, 0.03]\}, \{0.71, 0.79, 0.83\}\rangle$	$\langle\{[0.00, 0.02], [0.02, 0.05], [0.00, 0.03]\}, \{0.00, 0.71, 0.79\}\rangle$
S_6	$\langle\{[0.00, 0.01], [0.01, 0.02], [0.01, 0.02]\}, \{0.00, 0.79, 0.85\}\rangle$	$\langle\{[0.01, 0.05], [0.00, 0.01], [0.00, 0.02]\}, \{0.79, 0.00, 0.81\}\rangle$
S_7	$\langle\{[0.01, 0.04], [0.00, 0.02], [0.01, 0.02]\}, \{0.00, 0.85, 0.79\}\rangle$	$\langle\{[0.00, 0.02], [0.00, 0.02], [0.00, 0.04]\}, \{0.79, 0.85, 0.00\}\rangle$
S_8	$\langle\{[0.00, 0.02], [0.00, 0.01], [0.00, 0.01]\}, \{0.00, 0.00, 0.79\}\rangle$	$\langle\{[0.00, 0.02], [0.00, 0.01], [0.00, 0.02]\}, \{0.85, 0.79, 0.00\}\rangle$

4). The Euclidean distance of each alternative from $\mathcal{F}^+$ and $\mathcal{F}^-$ are calculated using Eqs (3.5) and (3.6), and provided in Table 6.

Table 6. Distance from $\mathcal{F}^+$ and $\mathcal{F}^-$.

	For $\mathcal{P}_1$	For $\mathcal{P}_2$
$d_{\mathcal{E}}(\aleph_1, \mathcal{F}^+)$	0.2024	0.2614
$d_{\mathcal{E}}(\aleph_2, \mathcal{F}^+)$	0.5050	0.4154
$d_{\mathcal{E}}(\aleph_3, \mathcal{F}^+)$	0.2480	0.1646
$d_{\mathcal{E}}(\aleph_4, \mathcal{F}^+)$	0.5085	0.3813
$d_{\mathcal{E}}(\aleph_5, \mathcal{F}^+)$	0.4153	0.3875
$d_{\mathcal{E}}(\aleph_6, \mathcal{F}^+)$	0.3981	0.6023
$d_{\mathcal{E}}(\aleph_1, \mathcal{F}^-)$	0.7599	0.7303
$d_{\mathcal{E}}(\aleph_2, \mathcal{F}^-)$	0.6344	0.6423
$d_{\mathcal{E}}(\aleph_3, \mathcal{F}^-)$	0.7422	0.7921
$d_{\mathcal{E}}(\aleph_4, \mathcal{F}^-)$	0.5888	0.6664
$d_{\mathcal{E}}(\aleph_5, \mathcal{F}^-)$	0.6705	0.6702
$d_{\mathcal{E}}(\aleph_6, \mathcal{F}^-)$	0.6930	0.5414

5). The relative closeness of each disorder to $\mathcal{F}^+$ is calculated using Eq (3.7) and is given in Table 7.

Table 7. Relative closeness to $\mathcal{F}^+$.

	For $\mathcal{P}_1$	For $\mathcal{P}_2$
$\varrho(\aleph_1)$	0.7897	0.7364
$\varrho(\aleph_2)$	0.5568	0.6073
$\varrho(\aleph_3)$	0.7495	0.8280
$\varrho(\aleph_4)$	0.5366	0.6361
$\varrho(\aleph_5)$	0.6175	0.6336
$\varrho(\aleph_6)$	0.6351	0.4734

6). Output:
Finally, for patients $\mathcal{P}_1$ and $\mathcal{P}_2$ the ordering of ICDs in descending order according to their relative closeness to C3PFPIS is obtain as:

$$\aleph_1 > \aleph_3 > \aleph_6 > \aleph_5 > \aleph_2 > \aleph_4 \qquad \text{(for patient } \mathcal{P}_1\text{)},$$

and

$$\aleph_3 > \aleph_1 > \aleph_4 > \aleph_5 > \aleph_2 > \aleph_6 \qquad \text{(for patient } \mathcal{P}_2\text{)}.$$

Thus, according to the symptoms in patients Psychiatrist ensure that the patient $\mathcal{P}_1$ have oppositional defiant disorder (ODD) and patient $\mathcal{P}_2$ have conduct disorder (CD).

We now apply the improved Algorithm 1 of C*m*PF-TOPSIS approach to find the appropriate disease. We see that steps 1–3 in improved Algorithm 1 are similar as Algorithm 1. So, we start from step 4 as below:

4). Using Eqs (3.8) and (3.9), the normalized Euclidean distance of each alternative from $\mathcal{F}^+$ and $\mathcal{F}^-$ are computed below in Table 8.

Table 8. Normalized Euclidean distance from $\mathcal{F}^+$ and $\mathcal{F}^-$.

	For $\mathcal{P}_1$	For $\mathcal{P}_1$
$d_{\mathcal{NE}}(\aleph_1, \mathcal{F}^+)$	0.0716	0.0924
$d_{\mathcal{NE}}(\aleph_2, \mathcal{F}^+)$	0.1786	0.1469
$d_{\mathcal{NE}}(\aleph_3, \mathcal{F}^+)$	0.0877	0.0582
$d_{\mathcal{NE}}(\aleph_4, \mathcal{F}^+)$	0.1798	0.1350
$d_{\mathcal{NE}}(\aleph_5, \mathcal{F}^+)$	0.1470	0.1370
$d_{\mathcal{NE}}(\aleph_6, \mathcal{F}^+)$	0.1408	0.2130
$d_{\mathcal{NE}}(\aleph_1, \mathcal{F}^-)$	0.2687	0.2582
$d_{\mathcal{NE}}(\aleph_2, \mathcal{F}^-)$	0.2243	0.2271
$d_{\mathcal{NE}}(\aleph_3, \mathcal{F}^-)$	0.2624	0.2800
$d_{\mathcal{NE}}(\aleph_4, \mathcal{F}^-)$	0.2082	0.2356
$d_{\mathcal{NE}}(\aleph_5, \mathcal{F}^-)$	0.2370	0.2369
$d_{\mathcal{NE}}(\aleph_6, \mathcal{F}^-)$	0.2450	0.1914

5). The revised closeness of each ICD to $\mathcal{F}^+$ are calculated in Table 9 using Eq (3.10).

Table 9. Revised closeness to $\mathcal{F}^+$.

	For $\mathcal{P}_1$	For $\mathcal{P}_1$
$\wp(\aleph_1)$	0.00	−0.6655
$\wp(\aleph_2)$	−1.6597	−1.7130
$\wp(\aleph_3)$	−0.2483	0.00
$\wp(\aleph_4)$	−1.7363	−1.4782
$\wp(\aleph_5)$	−1.1710	−1.5079
$\wp(\aleph_6)$	−1.0547	−2.9762

6). Output:
By ordering the ICDs according to the revised closeness, we have

$$\aleph_1 > \aleph_3 > \aleph_6 > \aleph_5 > \aleph_2 > \aleph_4 \qquad \text{(for patient } \mathcal{P}_1\text{)},$$

and

$$\aleph_3 > \aleph_1 > \aleph_4 > \aleph_5 > \aleph_2 > \aleph_6 \qquad \text{(for patient } \mathcal{P}_2\text{)}.$$

So, the patient $\mathcal{P}_1$ is suffering from oppositional defiant disorder (ODD) and the patient $\mathcal{P}_2$ have conduct disorder (CD).

We see that the ordering relation between mental disorders by applying both Algorithm 1 and improved Algorithm 1 are same but the values are quite different. The comparison of these two algorithms of CmPF-TOPSIS approach by applying on the explored application with $m = 3$ are shown in Figure 4.

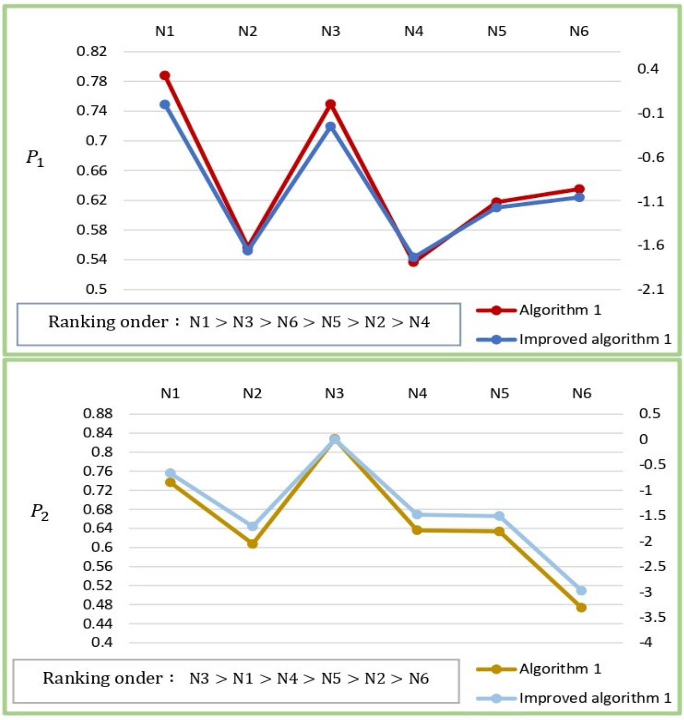

Figure 4. Comparison of Algorithm 1 and improved Algorithm 1 under the CmPF-TOPSIS approach.

4. Cubic m-polar fuzzy ELECTRE-I method

In this section, we develop an algorithm for hybrid model namely, CmPF-ELECTRE-I method and implemented it to solve the explored problem in the preceding section, to make a comparison with CmPF-TOPSIS method. Consider a MCDM problem where $\aleph_i(i = 1, 2, 3, \cdots, s)$ is the set of 's' alternatives evaluated under each attribute, $\mathcal{A}_j(j = 1, 2, 3, \cdots, t)$. Assume that the decision-maker has to select one from 's' possible alternatives with respect to 't' criteria. The steps of the launched algorithm for CmPF-ELECTRE-I approach are given as:

Algorithm 2: (Process for finding outranking relations between alternatives)

First two steps of CmPF-ELECTRE-I method are same to the Algorithm 1 (CmPF-TOPSIS method), it means steps 1 and 2 are already provided. So, we start with step 3 as follows:

3). Since the evaluation of objects are described with CmPFNs. So, the concordance set C_{xy} and discordance set D_{xy} are defined as:

$$C_{xy} = \{1 \leq j \leq t \mid \mathfrak{h}_{xj} \geq \mathfrak{h}_{yj}, \quad x \neq y; x, y = 1, 2, \cdots, s\}, \tag{4.1}$$

$$D_{xy} = \{1 \leq j \leq t \mid \mathfrak{h}_{xj} < \mathfrak{h}_{yj}, \quad x \neq y; x, y = 1, 2, \cdots, s\}, \tag{4.2}$$

where

$$\mathfrak{h}_{ij} = \Re^1_{lij} + \Re^1_{uij} + \Re^2_{lij} + \Re^2_{uij} + \ldots + \Re^m_{lij} + \Re^m_{uij} + \zeta^1_{ij} + \zeta^2_{ij} + \ldots + \zeta^m_{ij} \tag{4.3}$$

with $i = 1, 2, \cdots, s$ and $j = 1, 2, \cdots, t$.

4). The CmPF concordance indices $c_{xy}'s$ are obtained as:

$$c_{xy} = \sum_{j \in C_{xy}} \sqsupseteq_j. \tag{4.4}$$

So, the CmPF concordance matrix C is constructed as the following manner:

$$C = \begin{bmatrix} - & c_{12} & c_{13} & \cdots & c_{1s} \\ c_{21} & - & c_{23} & \cdots & c_{2s} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ c_{s1} & c_{s2} & c_{s3} & \cdots & - \end{bmatrix}. \tag{4.5}$$

5). The CmPF discordance indices $d_{xy}'s$ are computed as:

$$d_{xy} = \frac{\max_{j \in D_{xy}} \sqrt{\begin{array}{c} \frac{1}{3m}((\Re^1_{lxj} - \Re^1_{lyj})^2 + (\Re^1_{uxj} - \Re^1_{uyj})^2 + (\Re^2_{lxj} - \Re^2_{lyj})^2 \\ +(\Re^2_{uxj} - \Re^2_{uyj})^2 + \ldots + (\Re^m_{lxj} - \Re^m_{lyj})^2 + (\Re^m_{uxj} - \Re^m_{uyj})^2 \\ +(\zeta^1_{xj} - \zeta^1_{yj})^2 + (\zeta^2_{xj} - \zeta^2_{yj})^2 + \ldots + (\zeta^m_{xj} - \zeta^m_{yj})^2) \end{array}}}{\max_j \sqrt{\begin{array}{c} \frac{1}{3m}((\mathfrak{P}^1_{lxj} - \mathfrak{P}^1_{lyj})^2 + (\mathfrak{P}^1_{uxj} - \Re^1_{uyj})^2 + (\Re^2_{lxj} - \Re^2_{lyj})^2 \\ +(\Re^2_{uxj} - \Re^2_{uyj})^2 + \ldots + (\Re^m_{lxj} - \Re^m_{lyj})^2 + (\Re^m_{uxj} - \Re^m_{uyj})^2 \\ +(\zeta^1_{xj} - \zeta^1_{yj})^2 + (\zeta^2_{xj} - \zeta^2_{yj})^2 + \ldots + (\zeta^m_{xj} - \zeta^m_{yj})^2) \end{array}}}. \tag{4.6}$$

So, the CmPF discordance matrix D is constructed as the following manner:

$$D = \begin{bmatrix} - & d_{12} & d_{13} & \cdots & d_{1s} \\ d_{21} & - & d_{23} & \cdots & d_{2s} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ d_{s1} & d_{s2} & d_{s3} & \cdots & - \end{bmatrix}. \tag{4.7}$$

6). Now we need to calculate some threshold values, which are actually concordance and discordance levels. The CmPF concordance level ($\overline{c}$) and CmPF discordance level ($\overline{d}$) are respectively defined as the average of the CmPF concordance and CmPF discordance indices.

$$\overline{c} = \frac{1}{s(s-1)} \sum_{\substack{x=1 \\ x \neq y}}^{s} \sum_{\substack{y=1 \\ x \neq y}}^{s} c_{xy}, \tag{4.8}$$

$$\overline{d} = \frac{1}{s(s-1)} \sum_{\substack{x=1 \\ x \neq y}}^{s} \sum_{\substack{y=1 \\ x \neq y}}^{s} d_{xy}. \tag{4.9}$$

7). The CmPF concordance dominance matrix M according to the concordance level is constructed as:

$$M = \begin{bmatrix} - & m_{12} & m_{13} & \cdots & m_{1s} \\ m_{21} & - & m_{23} & \cdots & m_{2s} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ m_{s1} & m_{s2} & m_{s3} & \cdots & - \end{bmatrix}, \tag{4.10}$$

where the indices m_{xy} are defined as:

$$m_{xy} = \begin{cases} 1 & \text{if} \quad c_{xy} \geq \overline{c}, \\ 0 & \text{if} \quad c_{xy} < \overline{c}. \end{cases} \tag{4.11}$$

8). The CmPF discordance dominance matrix N according to the discordance level is constructed as:

$$N = \begin{bmatrix} - & n_{12} & n_{13} & \cdots & n_{1s} \\ n_{21} & - & n_{23} & \cdots & n_{2s} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ n_{s1} & n_{s2} & n_{s3} & \cdots & - \end{bmatrix}, \tag{4.12}$$

where the indices n_{xy} are defined as:

$$n_{xy} = \begin{cases} 1 & \text{if} \quad d_{xy} \leq \overline{d}, \\ 0 & \text{if} \quad d_{xy} > \overline{d}. \end{cases} \tag{4.13}$$

9). The CmPF aggregated dominance matrix (F) is constructed by performing the peer-to-peer multiplication of the entries of the matrices M and N.

$$F = \begin{bmatrix} - & f_{12} & f_{13} & \cdots & f_{1s} \\ f_{21} & - & f_{23} & \cdots & f_{2s} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ f_{s1} & f_{s2} & f_{s3} & \cdots & - \end{bmatrix}, \tag{4.14}$$

where the indices f_{xy} are calculated by

$$f_{xy} = m_{xy} \cdot n_{xy}. \tag{4.15}$$

10). Output:
Now, rank the alternatives according to the outranking values $f_{xy}'s$ of the matrix F. There is a directed edge from the alternative $\aleph_x$ to $\aleph_y$ if and only if $f_{xy} = 1$. Thus, we have the following three cases.

(a). There exists a unique directed edge from $\aleph_x$ to $\aleph_y$.

(b). There exists a directed edge from $\aleph_x$ to $\aleph_y$ and $\aleph_y$ to $\aleph_x$.

(c). There does not exist any edge between $\aleph_x$ and $\aleph_y$.

In case (a), we say that $\aleph_x$ is dominant over $\aleph_y$. For case (b), we say that $\aleph_x$ and $\aleph_y$ are not-different and for case (c), we say that $\aleph_x$ and $\aleph_y$ are non-comparable. The flowchart of CmPF-ELECTRE-I method is shown by Figure 5.

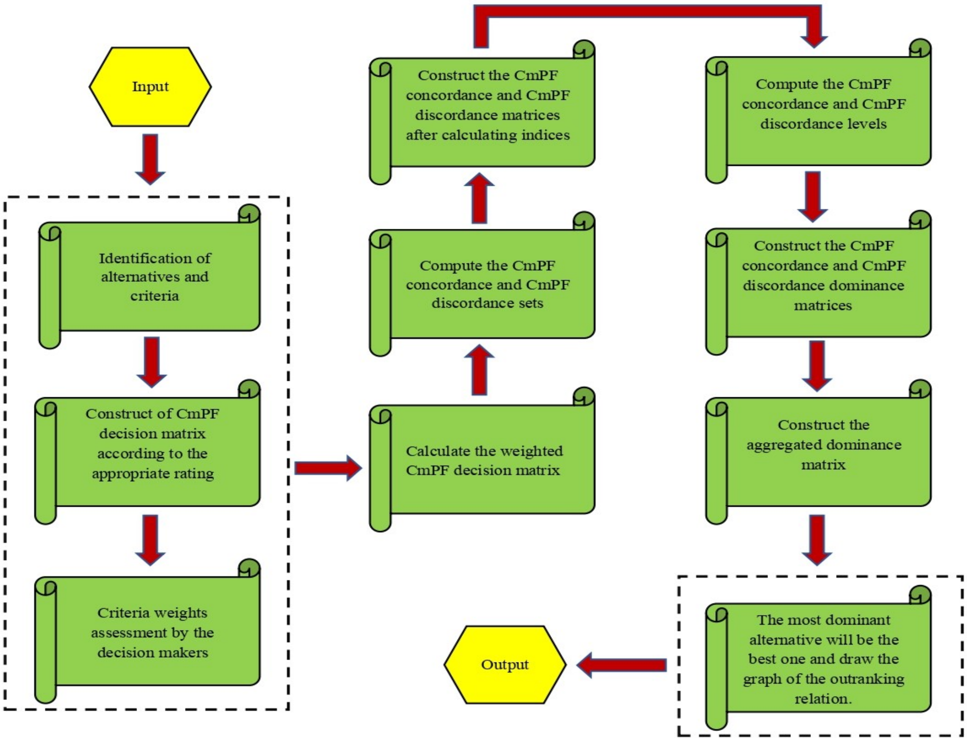

Figure 5. Flowchart of CmPF-ELECTRE-I approach.

4.1. Numerical application for CmPF-ELECTRE-I method

In this section we solved the presented MCDM application as in Section 3.1 using CmPF-ELECTRE-I method as below:
Steps 1 and 2 have already been done in Section 3.1. So, we start from step 3 as below:

3). For the evaluation of C3PF concordance set (C_{xy}) and C3PF discordance set (D_{xy}) the index matrices $\mathfrak{h}_{ij}^{\mathcal{P}_1}$ and $\mathfrak{h}_{ij}^{\mathcal{P}_2}$ are calculated below by using Eq (4.3).

$$\mathfrak{h}_{ij}^{\mathcal{P}_1} = \begin{bmatrix} 3.29 & 3.39 & 3.02 & 2.92 & 2.94 & 3.29 & 2.75 & 2.88 \\ 3.00 & 2.87 & 3.26 & 2.80 & 3.15 & 2.90 & 1.93 & 2.01 \\ 2.96 & 3.03 & 2.99 & 3.00 & 3.01 & 2.88 & 2.86 & 2.88 \\ 2.83 & 3.22 & 2.84 & 2.86 & 3.20 & 1.97 & 3.05 & 1.77 \\ 2.96 & 2.96 & 2.93 & 2.22 & 2.69 & 2.85 & 3.02 & 2.75 \\ 3.13 & 3.08 & 1.96 & 2.74 & 3.05 & 2.97 & 2.93 & 2.94 \end{bmatrix},$$

and

$$\mathfrak{h}_{ij}^{\mathcal{P}_2}=\begin{bmatrix}3.06 & 3.10 & 2.77 & 2.79 & 2.84 & 2.92 & 3.01 & 2.87\\ 2.99 & 2.82 & 3.11 & 2.98 & 2.79 & 1.88 & 2.70 & 2.84\\ 3.11 & 3.37 & 3.42 & 3.23 & 3.17 & 3.16 & 2.94 & 3.15\\ 3.00 & 2.31 & 2.89 & 3.01 & 2.93 & 3.14 & 3.00 & 2.77\\ 3.03 & 3.11 & 3.32 & 2.99 & 2.79 & 2.92 & 2.17 & 2.82\\ 3.23 & 2.73 & 3.09 & 2.10 & 2.06 & 2.73 & 2.91 & 2.17\end{bmatrix}.$$

Then, according to Eqs (4.1) and (4.2), the sets C_{xy} and D_{xy} are provided in Table 10.

Table 10. C3PF concordance and discordance sets.

					C3PF concordance sets		
For	y	1	2	3	4	5	6
	C_{1y}	-	{1, 2, 4, 6, 7, 8}	{1, 2, 3, 6, 8}	{1, 2, 3, 4, 6, 8}	{1, 2, 3, 4, 5, 6, 8}	{1, 2, 3, 4, 6}
	C_{2y}	{3, 5}	-	{1, 3, 5, 6}	{1, 3, 6, 8}	{1, 3, 4, 5, 6}	{3, 4, 5}
$\mathcal{P}_1$	C_{3y}	{4, 5, 7, 8}	{2, 4, 7, 8}	-	{1, 3, 4, 6, 8}	{1, 2, 3, 4, 5, 6, 8}	{3, 4}
	C_{4y}	{5, 7}	{2, 4, 5, 7}	{2, 5, 7}	-	{2, 4, 5, 7}	{2, 3, 4, 5, 7}
	C_{5y}	{7}	{2, 7, 8}	{1, 7}	{1, 3, 6, 8}	-	{3, 7}
	C_{6y}	{5, 7, 8}	{1, 2, 6, 7, 8}	{1, 2, 5, 6, 7, 8}	{1, 6, 8}	{1, 2, 4, 5, 6, 8}	-
	y	1	2	3	4	5	6
	C_{1y}	-	{1, 2, 5, 6, 7, 8}	{7}	{1, 2, 7, 8}	{1, 5, 6, 7, 8}	{2, 4, 5, 6, 7, 8}
	C_{2y}	{3, 4}	-	{}	{2, 3, 8}	{5, 7, 8}	{2, 3, 4, 5, 8}
$\mathcal{P}_2$	C_{3y}	{1, 2, 3, 4, 5, 6, 8}	{1, 2, 3, 4, 5, 6, 7, 8}	-	{1, 2, 3, 4, 5, 6, 8}	{1, 2, 3, 4, 5, 6, 7, 8}	{2, 3, 4, 5, 6, 7, 8}
	C_{4y}	{3, 4, 5, 6}	{1, 4, 5, 6, 7}	{7}	-	{4, 5, 6, 7}	{4, 5, 6, 7, 8}
	C_{5y}	{2, 3, 4, 6}	{1, 2, 3, 4, 5, 6}	{}	{1, 2, 3, 8}	-	{2, 3, 4, 5, 6, 8}
	C_{6y}	{1, 3}	{1, 6, 7}	{1}	{1, 2, 3}	{1, 7}	-
					C3PF discordance sets		
For	y	1	2	3	4	5	6
	D_{1y}	-	{3, 5}	{4, 5, 7}	{5, 7}	{7}	{5, 7, 8}
	D_{2y}	{1, 2, 4, 6, 7, 8}	-	{2, 4, 7, 8}	{2, 4, 5, 7}	{2, 7, 8}	{1, 2, 6, 7, 8}
$\mathcal{P}_1$	D_{3y}	{1, 2, 3, 6}	{1, 3, 5, 6}	-	{2, 5, 7}	{7}	{1, 2, 5, 6, 7, 8}
	D_{4y}	{1, 2, 3, 4, 6, 8}	{1, 3, 6, 8}	{1, 3, 4, 6, 8}	-	{1, 3, 6, 8}	{1, 6, 8}
	D_{5y}	{1, 2, 3, 4, 5, 6, 8}	{1, 2, 4, 5, 6}	{2, 3, 4, 5, 6, 8}	{2, 4, 5, 7}	-	{1, 2, 4, 5, 6, 8}
	D_{6y}	{1, 2, 3, 4, 6}	{3, 4, 5}	{3, 4}	{2, 3, 4, 5, 7}	{3, 7}	-
	y	1	2	3	4	5	6
	D_{1y}	-	{3, 4}	{1, 2, 3, 4, 5, 6, 8}	{3, 4, 5, 6}	{2, 3, 4}	{1, 3}
	D_{2y}	{1, 2, 5, 6, 7, 8}	-	{1, 2, 3, 4, 5, 6, 7, 8}	{1, 4, 5, 6, 7}	{1, 2, 3, 4, 6}	{1, 6, 7}
$\mathcal{P}_2$	D_{3y}	{7}	{}	-	{7}	{}	{1}
	D_{4y}	{1, 2, 7, 8}	{2, 3, 8}	{1, 2, 3, 4, 5, 6, 8}	-	{1, 2, 3, 8}	{1, 2, 3}
	D_{5y}	{1, 5, 7, 8}	{7, 8}	{1, 2, 3, 4, 5, 6, 7, 8}	{4, 5, 6, 7}	-	{1, 7}
	D_{6y}	{2, 4, 5, 6, 7, 8}	{2, 3, 4, 5, 8}	{2, 3, 4, 5, 6, 7, 8}	{4, 5, 6, 7, 8}	{2, 3, 4, 5, 6, 8}	-

4). The C3PF concordance matrices for both patients are constructed below using the formula of indices as defined in Eq (4.4).

$$C_{\mathcal{P}_1}=\begin{bmatrix}- & 0.70 & 0.65 & 0.75 & 0.90 & 0.65\\ 0.30 & - & 0.50 & 0.45 & 0.60 & 0.40\\ 0.45 & 0.50 & - & 0.55 & 0.90 & 0.25\\ 0.25 & 0.55 & 0.45 & - & 0.55 & 0.70\\ 0.10 & 0.40 & 0.20 & 0.45 & - & 0.25\\ 0.35 & 0.60 & 0.75 & 0.30 & 0.75 & -\end{bmatrix},\quad C_{\mathcal{P}_2}=\begin{bmatrix}- & 0.75 & 0.10 & 0.50 & 0.55 & 0.75\\ 0.25 & - & 0.00 & 0.45 & 0.35 & 0.70\\ 0.90 & 1.00 & - & 0.90 & 1.00 & 0.90\\ 0.50 & 0.55 & 0.10 & - & 0.45 & 0.55\\ 0.55 & 0.80 & 0.00 & 0.55 & - & 0.80\\ 0.25 & 0.30 & 0.10 & 0.45 & 0.20 & -\end{bmatrix}.$$

5). The C3PF discordance matrices for both patients are constructed below using the formula of indices as given by Eq (4.6).

$$D_{\mathcal{P}_1} = \begin{bmatrix} - & 0.3779 & 0.4988 & 0.1625 & 0.1278 & 0.2514 \\ 1 & - & 1 & 0.7471 & 0.9716 & 0.9889 \\ 1 & 0.1934 & - & 0.5670 & 0.2394 & 0.5645 \\ 1 & 1 & 1 & - & 0.9843 & 1 \\ 1 & 1 & 1 & 1 & - & 0.9449 \\ 1 & 1 & 1 & 0.8241 & 1 & - \end{bmatrix},$$

$$D_{\mathcal{P}_2} = \begin{bmatrix} - & 0.2448 & 1 & 0.2893 & 0.3186 & 0.2594 \\ 1 & - & 1 & 1 & 1 & 0.8361 \\ 0.2478 & 0 & - & 0.1392 & 0 & 0.2093 \\ 1 & 0.9284 & 1 & - & 0.8744 & 1 \\ 1 & 0.9294 & 1 & 1 & - & 1 \\ 1 & 1 & 1 & 0.9806 & 0.9536 & - \end{bmatrix}.$$

6). The C3PF concordance and C3PF discordance levels are computed as:

$$(\text{For patient } \mathcal{P}_1) \begin{cases} \bar{c} = & \frac{1}{s(s-1)} \sum_{\substack{x=1 \\ x \neq y}}^{s} \sum_{\substack{y=1 \\ x \neq y}}^{s} c_{xy} = \frac{1}{6(6-1)} \times 15.15 = 0.5050, \\ \bar{d} = & \frac{1}{s(s-1)} \sum_{\substack{x=1 \\ x \neq y}}^{s} \sum_{\substack{y=1 \\ x \neq y}}^{s} d_{xy} = \frac{1}{6(6-1)} \times 23.4436 = 0.7815, \end{cases}$$

and

$$(\text{For patient } \mathcal{P}_2) \begin{cases} \bar{c} = & \frac{1}{s(s-1)} \sum_{\substack{x=1 \\ x \neq y}}^{s} \sum_{\substack{y=1 \\ x \neq y}}^{s} c_{xy} = \frac{1}{6(6-1)} \times 15.20 = 0.5067, \\ \bar{d} = & \frac{1}{s(s-1)} \sum_{\substack{x=1 \\ x \neq y}}^{s} \sum_{\substack{y=1 \\ x \neq y}}^{s} d_{xy} = \frac{1}{6(6-1)} \times 22.2109 = 0.7404. \end{cases}$$

7). By using Eq (4.11), the C3PF concordance dominance matrices $M_{\mathcal{P}_1}$ and $M_{\mathcal{P}_2}$ are calculated as:

$$M_{\mathcal{P}_1} = \begin{bmatrix} - & 1 & 1 & 1 & 1 & 1 \\ 0 & - & 0 & 0 & 1 & 0 \\ 0 & 0 & - & 1 & 1 & 0 \\ 0 & 1 & 0 & - & 1 & 1 \\ 0 & 0 & 0 & 0 & - & 0 \\ 0 & 1 & 1 & 0 & 1 & - \end{bmatrix}, \quad M_{\mathcal{P}_2} = \begin{bmatrix} - & 1 & 0 & 0 & 1 & 1 \\ 0 & - & 0 & 0 & 0 & 1 \\ 1 & 1 & - & 1 & 1 & 1 \\ 0 & 1 & 0 & - & 0 & 1 \\ 1 & 1 & 0 & 1 & - & 1 \\ 0 & 0 & 0 & 0 & 0 & - \end{bmatrix}.$$

8). The C3PF discordance dominance matrices $N_{\mathcal{P}_1}$ and $N_{\mathcal{P}_2}$ are constructed below by using Eq (4.13).

$$N_{\mathcal{P}_1} = \begin{bmatrix} - & 1 & 1 & 1 & 1 & 1 \\ 0 & - & 0 & 1 & 0 & 0 \\ 0 & 1 & - & 1 & 1 & 1 \\ 0 & 0 & 0 & - & 0 & 0 \\ 0 & 0 & 0 & 0 & - & 0 \\ 0 & 0 & 0 & 0 & 0 & - \end{bmatrix}, \qquad N_{\mathcal{P}_2} = \begin{bmatrix} - & 1 & 0 & 1 & 1 & 1 \\ 0 & - & 0 & 0 & 0 & 0 \\ 1 & 1 & - & 1 & 1 & 1 \\ 0 & 0 & 0 & - & 0 & 0 \\ 0 & 0 & 0 & 0 & - & 0 \\ 0 & 0 & 0 & 0 & 0 & - \end{bmatrix}.$$

9). The C3PF aggregated dominance matrices $F_{\mathcal{P}_1}$ and $F_{\mathcal{P}_2}$ are constructed below by peer-to-peer multiplication of the entries of matrices M and N.

$$F_{\mathcal{P}_1} = \begin{bmatrix} - & 1 & 1 & 1 & 1 & 1 \\ 0 & - & 0 & 0 & 0 & 0 \\ 0 & 0 & - & 1 & 1 & 0 \\ 0 & 0 & 0 & - & 0 & 0 \\ 0 & 0 & 0 & 0 & - & 0 \\ 0 & 0 & 0 & 0 & 0 & - \end{bmatrix}, \qquad F_{\mathcal{P}_2} = \begin{bmatrix} - & 1 & 0 & 0 & 1 & 1 \\ 0 & - & 0 & 0 & 0 & 0 \\ 1 & 1 & - & 1 & 1 & 1 \\ 0 & 0 & 0 & - & 0 & 0 \\ 0 & 0 & 0 & 0 & - & 0 \\ 0 & 0 & 0 & 0 & 0 & - \end{bmatrix}.$$

10). Output:
The outranking relations between alternatives are shown in Figure 6. Thus, $\aleph_1$ and $\aleph_3$ are the favourable alternatives for the patients $\mathcal{P}_1$ and $\mathcal{P}_2$, respectively.

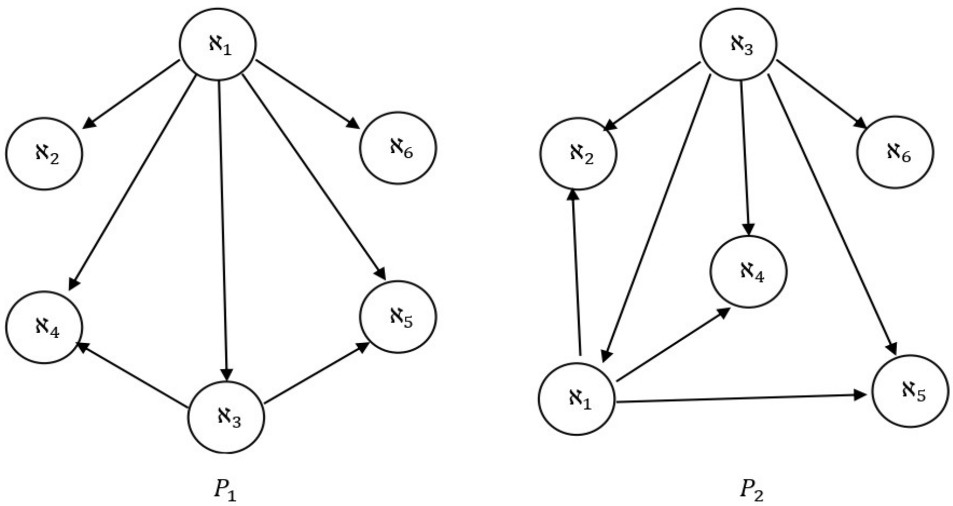

Figure 6. Outranking relations between alternatives.

5. Comparison analysis

5.1. Comparison between proposed techniques

In this subsection, we present a comparative study between our proposed C*m*PF-TOPSIS and C*m*PF-ELECTRE-I methods. The main function of the C*m*PF-TOPSIS method is to select an alternative, that is nearest to the CmPFPIS and far away from the CmPFNIS. On the other hand, in the CmPF-ELECTRE-I technique, the selection of favorable alternatives depends on the CmPF concordance and discordance sets, and this task is made by the means of an outranking directed decision graph. The CmPF-TOPSIS method provides a single alternative as a decision, but the CmPF-ELECTRE-I method sometimes gives two or more optimal alternatives at once by means of outranking relations. The comparison of proposed techniques by applying them to the presented numerical application is provided as:

- The comparison of CmPF-TOPSIS and CmPF-ELECTRE-I methods based on the proposed case study with $m = 3$ is described through a ranking comparison bar chart. This comparison shows patient $\mathcal{P}_1$ has the oppositional defiant disorder (ODD) and patient $\mathcal{P}_2$ is suffering from conduct disorder (CD). The desired ranking comparison bar chart is shown in Figure 7.

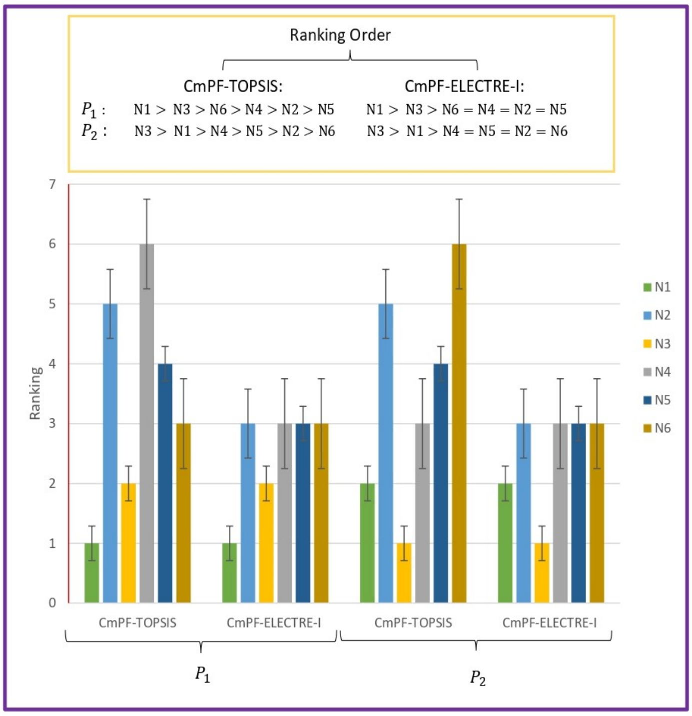

Figure 7. Ranking comparison of CmPF-TOPSIS and CmPF-ELECTRE-I approaches.

5.2. Comparison with existing methods

The superiority and effectiveness of our proposed methods based on CmPF-TOPSIS and CmPF-ELECTRE-I techniques are verified by implementing the mPF-TOPSIS method [16] and mPF-ELECTRE-I method [38] on the given case study.

- For patient $\mathcal{P}_1$, the results of mPF-TOPSIS [16] and mPF-ELECTRE-I [38] methods with $m = 3$ are displayed in Tables 11 and 12 respectively. From the calculations, one can easily observe that patient $\mathcal{P}_1$ is suffering from oppositional defiant disorder (ODD), which is similar to the optimal alternative computed by applying the proposed techniques.

Table 11. Results of mPF-TOPSIS method [16] for the patient $\mathcal{P}_1$.

	$\mathcal{P}_1$			
Alternatives	$d_{\mathcal{E}}(\aleph_i, \mathcal{F}^+)$	$d_{\mathcal{E}}(\aleph_i, \mathcal{F}^-)$	$\varrho(\aleph_i)$	Ranks
$\aleph_1$	0.0912	0.1339	0.5948	1
$\aleph_2$	0.1108	0.1049	0.4863	2
$\aleph_3$	0.1268	0.1012	0.4439	4
$\aleph_4$	0.1283	0.0866	0.4030	5
$\aleph_5$	0.1308	0.0858	0.3961	6
$\aleph_6$	0.1191	0.1122	0.4851	3

Table 12. Summery of mPF-ELECTRE-I method [38] for the patient $\mathcal{P}_1$.

Correlations	C_{xy}	D_{xy}	c_{xy}	d_{xy}	m_{xy}	n_{xy}	f_{xy}	Outranking
$(\aleph_1, \aleph_2)$	$\{1,2,3,4,6,7,8\}$	$\{4\}$	0.85	0.5055	1	1	1	$\aleph_1 \to \aleph_2$
$(\aleph_1, \aleph_3)$	$\{1,2,3\}$	$\{4,5,7,8\}$	0.55	0.4747	1	1	1	$\aleph_1 \to \aleph_3$
$(\aleph_1, \aleph_4)$	$\{1,2,3,4,6,8\}$	$\{5,7\}$	0.75	0.4831	1	1	1	$\aleph_1 \to \aleph_4$
$(\aleph_1, \aleph_5)$	$\{1,2,3,4,5,6,8\}$	$\{7\}$	0.9	0.2331	1	1	1	$\aleph_1 \to \aleph_5$
$(\aleph_1, \aleph_6)$	$\{1,23,4,5,6,8\}$	$\{7\}$	0.9	0.3729	1	1	1	$\aleph_1 \to \aleph_6$
$(\aleph_2, \aleph_1)$	$\{5\}$	$\{1,2,3,4,6,7,8\}$	0.15	1	0	0	0	Incomparable
$(\aleph_2, \aleph_3)$	$\{21,2,3,5\}$	$\{4,6,7,8\}$	0.6	0.6944	1	1	1	$\aleph_2 \to \aleph_3$
$(\aleph_2, \aleph_4)$	$\{1,2,3,5,6,8\}$	$\{4,7\}$	0.8	0.6162	1	1	1	$\aleph_2 \to \aleph_4$
$(\aleph_2, \aleph_5)$	$\{2,3,4,5\}$	$\{1,6,7,8\}$	0.6	0.5332	1	1	1	$\aleph_2 \to \aleph_5$
$(\aleph_2, \aleph_6)$	$\{3,4,5\}$	$\{1,2,6,7,8\}$	0.4	0.6343	0	1	0	Incomparable
$(\aleph_3, \aleph_1)$	$\{4,5,7,8\}$	$\{1,2,3,6\}$	0.45	1	0	0	0	Incomparable
$(\aleph_3, \aleph_2)$	$\{4,5,6,7,8\}$	$\{1,2,3\}$	0.55	1	1	0	0	Incomparable
$(\aleph_3, \aleph_4)$	$\{1,3,4,5,6,8\}$	$\{2,7\}$	0.7	0.9906	1	0	0	Incomparable
$(\aleph_3, \aleph_5)$	$\{3,4,5,6,8\}$	$\{1,2,7\}$	0.6	0.7344	1	1	1	$\aleph_3 \to \aleph_5$
$(\aleph_3, \aleph_6)$	$\{3,4,5,8\}$	$\{1,2,6,7\}$	0.5	1	1	0	0	Incomparable
$(\aleph_4, \aleph_1)$	$\{5,7\}$	$\{1,2,3,4,6,8\}$	0.25	1	0	0	0	Incomparable
$(\aleph_4, \aleph_2)$	$\{4,6,7\}$	$\{1,2,3,5,8\}$	0.3	1	0	0	0	Incomparable
$(\aleph_4, \aleph_3)$	$\{2,7\}$	$\{1,3,4,5,6,8\}$	0.3	1	0	0	0	Incomparable
$(\aleph_4, \aleph_5)$	$\{4,5,7\}$	$\{1,2,3,6,8\}$	0.35	0.8982	0	0	0	Incomparable
$(\aleph_4, \aleph_6)$	$\{3,4,5\}$	$\{1,2,6,7,8\}$	0.4	0.6579	0	1	0	Incomparable
$(\aleph_5, \aleph_1)$	$\{7\}$	$\{1,2,3,4,5,6,8\}$	0.1	1	0	0	0	Incomparable
$(\aleph_5, \aleph_2)$	$\{1,2,6,7,8\}$	$\{3,4,5\}$	0.6	1	1	0	0	Incomparable
$(\aleph_5, \aleph_3)$	$\{1,2,6,7\}$	$\{3,4,5,8\}$	0.5	1	0	0	0	Incomparable
$(\aleph_5, \aleph_4)$	$\{1,2,3,6,7,8\}$	$\{4,5\}$	0.75	1	1	0	0	Incomparable
$(\aleph_5, \aleph_6)$	$\{3\}$	$\{1,2,4,5,6,7,8\}$	0.15	1	0	0	0	Incomparable
$(\aleph_6, \aleph_1)$	$\{7,8\}$	$\{1,2,3,4,5,6\}$	0.2	1	0	0	0	Incomparable
$(\aleph_6, \aleph_2)$	$\{1,2,6,7,8\}$	$\{3,4,5\}$	0.6	1	1	0	0	Incomparable
$(\aleph_6, \aleph_3)$	$\{1,2,6,7\}$	$\{3,4,5,8\}$	0.5	0.6782	0	1	0	Incomparable
$(\aleph_6, \aleph_4)$	$\{1,2,3,6,7,8\}$	$\{4,5\}$	0.75	1	1	0	0	Incomparable
$(\aleph_6, \aleph_5)$	$\{1,2,54,5,6,7,8\}$	$\{3\}$	0.7	0.75	1	1	1	$\aleph_6 \to \aleph_5$

- For patient $\mathcal{P}_2$, the results of mPF-TOPSIS [16] and mPF-ELECTRE-I [38] methods with $m = 3$ are respectively expressed in Tables 13 and 14. The calculations provided that the patient $\mathcal{P}_2$ is suffering from conduct disorder (CD), which is similar to the optimal alternative computed by applying the proposed techniques.

Table 13. Results of mPF-TOPSIS method [16] for the patient $\mathcal{P}_2$.

		$\mathcal{P}_2$		
Alternatives	$d_{\mathcal{E}}(\aleph_i, \mathcal{F}^{+})$	$d_{\mathcal{E}}(\aleph_i, \mathcal{F}^{-})$	$\varrho(\aleph_i)$	Ranks
$\aleph_1$	0.1234	0.1027	0.4542	4
$\aleph_2$	0.1167	0.1020	0.4664	3
$\aleph_3$	0.0895	0.1433	0.6155	1
$\aleph_4$	0.1226	0.0963	0.4399	5
$\aleph_5$	0.1097	0.1129	0.5072	2
$\aleph_6$	0.1374	0.1075	0.4390	6

Table 14. Summery of mPF-ELECTRE-I method [38] for the patient $\mathcal{P}_2$.

Correlations	C_{xy}	D_{xy}	c_{xy}	d_{xy}	m_{xy}	n_{xy}	f_{xy}	Outranking
$(\aleph_1, \aleph_2)$	$\{1,2,5,6,7,8\}$	$\{3,4\}$	0.75	1	1	0	0	Incomparable
$(\aleph_1, \aleph_3)$	$\{2,7\}$	$\{1,3,,4,5,6,8\}$	0.3	0.8585	0	0	0	Incomparable
$(\aleph_1, \aleph_4)$	$\{2,4,6,7,8\}$	$\{1,3,4,5\}$	0.6	1	1	0	0	Incomparable
$(\aleph_1, \aleph_5)$	$\{2,5,6,7,8\}$	$\{1,3,4\}$	0.65	1	1	0	0	Incomparable
$(\aleph_1, \aleph_6)$	$\{2,4,5,6,7,8\}$	$\{1,3\}$	0.75	0.7487	1	1	1	$\aleph_1 \to \aleph_6$
$(\aleph_2, \aleph_1)$	$\{3,4,5\}$	$\{1,2,6,7,8\}$	0.4	0.8554	0	0	0	Incomparable
$(\aleph_2, \aleph_3)$	$\{2,3\}$	$\{1,4,5,6,7,8\}$	0.35	1	0	0	0	Incomparable
$(\aleph_2, \aleph_4)$	$\{2,3,4\}$	$\{1,5,6,7,8\}$	0.45	0.6371	0	1	0	Incomparable
$(\aleph_2, \aleph_5)$	$\{3,4,5\}$	$\{1,2,6,7,8\}$	0.4	0.899	0	0	0	Incomparable
$(\aleph_2, \aleph_6)$	$\{3,4,5\}$	$\{1,2,6,7,8\}$	0.4	0.9434	0	0	0	Incomparable
$(\aleph_3, \aleph_1)$	$\{1,3,4,5,6,8\}$	$\{2,7\}$	0.7	1	1	0	0	Incomparable
$(\aleph_3, \aleph_2)$	$\{1,4,5,6,7,8\}$	$\{2,3\}$	0.65	0.8923	1	0	0	Incomparable
$(\aleph_3, \aleph_4)$	$\{2,3,4,5,6,8\}$	$\{1,7\}$	0.7	0.4129	1	1	1	$\aleph_3 \to \aleph_4$
$(\aleph_3, \aleph_5)$	$\{3,4,5,6,8\}$	$\{1,2,7\}$	0.6	1	1	0	0	Incomparable
$(\aleph_3, \aleph_6)$	$\{3,4,5,6,8\}$	$\{1,2,7\}$	0.6	0.5440	1	1	1	$\aleph_3 \to \aleph_6$
$(\aleph_4, \aleph_1)$	$\{1,3,4,5,6,7\}$	$\{2,4,8\}$	0.7	0.8715	1	0	0	Incomparable
$(\aleph_4, \aleph_2)$	$\{1,5,6,7,8\}$	$\{2,3,4\}$	0.55	1	0	0	0	Incomparable
$(\aleph_4, \aleph_3)$	$\{1,2,7\}$	$\{3,4,5,6,8\}$	0.4	0.8511	0	0	0	Incomparable
$(\aleph_4, \aleph_5)$	$\{1,4,5,6,7\}$	$\{2,3,8\}$	0.55	1	0	0	0	Incomparable
$(\aleph_4, \aleph_6)$	$\{4,5,6,7,8\}$	$\{1,2,3\}$	0.55	1	0	0	0	Incomparable
$(\aleph_5, \aleph_1)$	$\{1,2,3,4\}$	$\{6,7,8\}$	0.6	0.6241	1	1	1	$\aleph_5 \to \aleph_1$
$(\aleph_5, \aleph_2)$	$\{1,2,5,6,7,8\}$	$\{3,4\}$	0.75	1	1	0	0	Incomparable
$(\aleph_5, \aleph_3)$	$\{1,2,3,7\}$	$\{4,5,6,8\}$	0.55	0.8355	0	1	0	Incomparable
$(\aleph_5, \aleph_4)$	$\{2,3,8\}$	$\{1,4,5,6,7\}$	0.45	0.4775	0	1	0	Incomparable
$(\aleph_5, \aleph_6)$	$\{2,3,4,5,6,7,8\}$	$\{1\}$	0.90	1	1	0	0	Incomparable
$(\aleph_6, \aleph_1)$	$\{1,2,3\}$	$\{4,5,6,7,8\}$	0.45	0.5909	0	1	0	Incomparable
$(\aleph_6, \aleph_2)$	$\{1,2,6,7,8\}$	$\{3,4,5\}$	0,6	1	1	0	0	Incomparable
$(\aleph_6, \aleph_3)$	$\{1,2,7\}$	$\{3,4,5,6,8\}$	0.40	1	0	0	0	Incomparable
$(\aleph_6, \aleph_4)$	$\{1,2,3,8\}$	$\{4,5,6,7\}$	0.55	0.4606	0	1	0	Incomparable
$(\aleph_6, \aleph_5)$	$\{1,2,7\}$	$\{3,4,5,6,8\}$	0.4	0.8538	0	0	0	Incomparable

- The comparison of alternatives regarding rankings between proposed and existing techniques (see [16, 38]) are described in Table 15. For a better understanding, the comparison bar chart is shown in Figure 8. Thus, the similar optimal decision based on the proposed and existing techniques proved the sensitivity, reliability, and effectiveness of the proposed C*m*PF-TOPSIS and C*m*PF-ELECTRE-I methods.

Table 15. Comparison of proposed and existing methods regarding rankings.

	$\mathcal{P}_1$	
Models	Ranking order	Decision
*m*PF-TOPSIS [16]	$\aleph_1 > \aleph_2 > \aleph_6 > \aleph_3 > \aleph_4 > \aleph_5$	$\aleph_1$
*m*PF-ELECTRE-I [38]	$\aleph_1 > \aleph_2 > \aleph_6 = \aleph_3 > \aleph_4 = \aleph_5$	$\aleph_1$
C*m*PF-TOPSIS (proposed)	$\aleph_1 > \aleph_3 > \aleph_6 > \aleph_5 > \aleph_2 > \aleph_4$	$\aleph_1$
C*m*PF-ELECTRE-I (proposed)	$\aleph_1 > \aleph_3 > \aleph_6 = \aleph_5 = \aleph_2 = \aleph_4$	$\aleph_1$
	$\mathcal{P}_2$	
Models	Ranking order	Decision
*m*PF-TOPSIS [16]	$\aleph_3 > \aleph_5 > \aleph_2 > \aleph_1 > \aleph_4 > \aleph_6$	$\aleph_3$
*m*PF-ELECTRE-I [38]	$\aleph_3 > \aleph_5 = \aleph_1 > \aleph_2 = \aleph_4 = \aleph_6$	$\aleph_3$
C*m*PF-TOPSIS (proposed)	$\aleph_3 > \aleph_1 > \aleph_4 > \aleph_5 > \aleph_2 > \aleph_6$	$\aleph_3$
C*m*PF-ELECTRE-I (proposed)	$\aleph_3 > \aleph_1 > \aleph_4 = \aleph_5 = \aleph_2 = \aleph_6$	$\aleph_3$

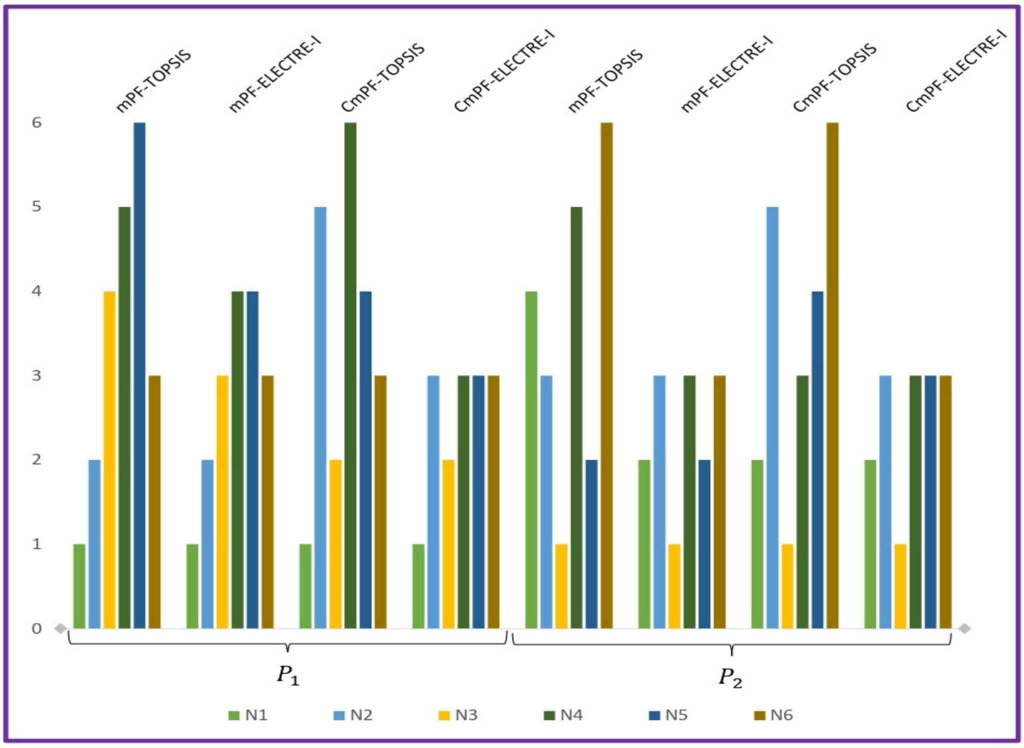

Figure 8. Comparison bar chart of the proposed and existing techniques concerning ranking.

5.3. Sensitivity analysis

In MCDM techniques, a sensitivity analysis is necessary to determine the stability of the solution when changes occur in the data or starting conditions. In this section, we present a sensitivity analysis of the proposed CmPF-TOPSIS and CmPF-ELECTRE-I methods for different values of m.

- Using the data of explored application in Section 3.1, the provided information in Tables 1 and 2 is respectively converted into one and two polar, that is, $m = 1$ and $m = 2$. We then applied the Algorithm 1 of CmPF-TOPSIS method for $m = 1$ and $m = 2$ to evaluate the performance of the alternatives and obtained the results for both patients, which are displayed in Tables 16 and 17, respectively.

Table 16. Results of CmPF-TOPSIS method for the patient $\mathcal{P}_1$ in case of $m = 1, 2, 3$.

	$\varrho(\aleph_i)$					
Alternatives	$m = 1$	Ranks	$m = 2$	Ranks	$m = 3$	Ranks
$\aleph_1$	0.7844	1	0.8177	1	0.7879	1
$\aleph_2$	0.4022	6	0.5324	5	0.5568	5
$\aleph_3$	0.7175	3	0.7501	3	0.7495	2
$\aleph_4$	0.4909	5	0.5153	6	0.5366	6
$\aleph_5$	0.7583	2	0.7831	2	0.6175	4
$\aleph_6$	0.5261	4	0.6330	4	0.6351	3

Table 17. Results of CmPF-TOPSIS method for the patient $\mathcal{P}_2$ in case of $m = 1, 2, 3$.

	$\varrho(\aleph_i)$					
Alternatives	$m = 1$	Ranks	$m = 2$	Ranks	$m = 3$	Ranks
$\aleph_1$	0.7062	5	0.7419	4	0.7364	2
$\aleph_2$	0.7095	4	0.5264	5	0.6073	5
$\aleph_3$	0.8283	1	0.7934	1	0.8280	1
$\aleph_4$	0.7377	3	0.7489	3	0.6361	3
$\aleph_5$	0.8088	2	0.7647	2	0.6336	4
$\aleph_6$	0.2129	6	0.3833	6	0.4734	6

By examining the results in Tables 16 and 17, one can easily observe that the best alternative remains consistent for both patients across different values of m. This finding suggests that the proposed method is sensitive and reliable and can provide consistent results even when the value of m is changed.

- In the same way as the previous analysis again repeating the process of the CmPF-ELECTRE-I method when $m = 1$ and $m = 2$, and obtained the results for both patients, which are described in Tables 18 and 19. From Tables 18 and 19, it can readily see that the final solution for both patients is stable across different values of m. This finding suggests that the developed approach is reliable.

Table 18. Results of CmPF-ELECTRE-I method for the patient $\mathcal{P}_1$ in case of $m = 1, 2, 3$.

			$m = 1$			
Alternatives	$\aleph_1$	$\aleph_2$	$\aleph_3$	$\aleph_4$	$\aleph_5$	$\aleph_6$
Submissive Alternatives	$\aleph_2$,$\aleph_4$,$\aleph_6$	–	$\aleph_2$	–	$\aleph_2$,$\aleph_4$,$\aleph_6$	–
Incomparable Alternatives	$\aleph_3$,$\aleph_5$	$\aleph_4$,$\aleph_6$	$\aleph_1$,$\aleph_4$,$\aleph_5$,$\aleph_6$	$\aleph_2$,$\aleph_3$,$\aleph_6$	$\aleph_1$,$\aleph_3$	$\aleph_2$,$\aleph_3$,$\aleph_4$
			$m = 2$			
Submissive Alternatives	$\aleph_2$,$\aleph_6$	–	–	–	$\aleph_2$	–
Incomparable Alternatives	$\aleph_3$,$\aleph_4$,$\aleph_5$	$\aleph_3$,$\aleph_4$,$\aleph_6$	$\aleph_1$,$\aleph_2$,$\aleph_4$,$\aleph_5$,$\aleph_6$	$\aleph_1$,$\aleph_2$,$\aleph_3$,$\aleph_5$,$\aleph_6$	$\aleph_1$,$\aleph_3$,$\aleph_4$,$\aleph_6$	$\aleph_2$,$\aleph_3$,$\aleph_4$,$\aleph_5$
			$m = 3$			
Submissive Alternatives	$\aleph_2$,$\aleph_3$,$\aleph_4$,$\aleph_5$,$\aleph_6$	–	$\aleph_4$,$\aleph_5$	–	–	–
Incomparable Alternatives	–	$\aleph_2$,$\aleph_3$,$\aleph_4$,$\aleph_5$,$\aleph_6$	$\aleph_2$,$\aleph_6$	$\aleph_2$,$\aleph_5$,$\aleph_6$	$\aleph_2$,$\aleph_4$,$\aleph_6$	$\aleph_1$,$\aleph_2$,$\aleph_3$,$\aleph_4$,$\aleph_5$

Table 19. Results of CmPF-ELECTRE-I method for the patient $\mathcal{P}_2$ in case of $m = 1, 2, 3$.

			$m = 1$			
Alternatives	$\aleph_1$	$\aleph_2$	$\aleph_3$	$\aleph_4$	$\aleph_5$	$\aleph_6$
Submissive Alternatives	–	$\aleph_6$	$\aleph_1$,$\aleph_2$,$\aleph_6$	$\aleph_1$,$\aleph_6$	$\aleph_1$,$\aleph_2$,$\aleph_6$	–
Incomparable Alternatives	$\aleph_2$,$\aleph_6$	$\aleph_1$,$\aleph_4$	$\aleph_4$,$\aleph_5$	$\aleph_2$,$\aleph_3$	$\aleph_3$,$\aleph_4$	$\aleph_1$
			$m = 2$			
Submissive Alternatives	$\aleph_6$	–	$\aleph_1$,$\aleph_2$,$\aleph_4$,$\aleph_6$	$\aleph_2$,$\aleph_6$	$\aleph_2$,$\aleph_6$	–
Incomparable Alternatives	$\aleph_2$,$\aleph_4$,$\aleph_5$	$\aleph_1$,$\aleph_6$	$\aleph_5$	$\aleph_1$,$\aleph_5$	$\aleph_1$,$\aleph_3$,$\aleph_4$	$\aleph_2$
			$m = 3$			
Submissive Alternatives	$\aleph_2$,$\aleph_5$	–	$\aleph_1$,$\aleph_2$,$\aleph_4$,$\aleph_5$,$\aleph_6$	–	–	–
Incomparable Alternatives	$\aleph_6$	$\aleph_4$,$\aleph_5$,$\aleph_6$	–	$\aleph_2$,$\aleph_5$,$\aleph_6$	$\aleph_2$,$\aleph_4$,$\aleph_6$	$\aleph_1$,$\aleph_2$,$\aleph_3$,$\aleph_4$,$\aleph_5$

Overall, the results of our analysis provide strong evidence that the CmPF-TOPSIS and the CmPF-ELECTRE-I methods can be applied to various real-life MCDM problems with different values of m. These findings have important implications for decision-makers who need to consider multiple conflicting criteria in their decision-making processes.

6. Conclusions and future directions

Many real-world decision-making problems emanate in a complex environment and involve conflicting systems of multiple criteria, uncertainty, and imprecise information. Several researchers have developed MCDM methods to solve these issues precisely. The existing MCDM approaches are an important part of the decision process for complex problems, and the theory of C*m*PSs is well-suited to handle the uncertain multi-polar data in multi-criteria decision problems because some decision problems have multi-polar information due to the involvement of multiple agents. The TOPSIS and ELECTRE-I approaches are widely used and well-established MCDM methods with a history of successful real-world applications under several uncertain theories. In this article, we have proposed a methodological and computational enhancement of the TOPSIS and ELECTRE-I MCDM methods for processing C*m*PF information. We have also proposed Hamming, normalized Hamming, Euclidean, and normalized Euclidean distance measures for C*m*PFSs, and verified their validity by corresponding results. Furthermore, we have illustrated our proposed techniques by solving the psychiatric problem of diagnosing ICDs. Both methods give similar results and verify their effectiveness and applicability by a comprehensive comparison with existing methods. The comparison results of psychiatric problems in the case of proposed and existing methods are given in Table 15, in which one can easily observe that $\aleph_1$ and $\aleph_3$ are the suitable options for the patients $\mathcal{P}_1$ and $\mathcal{P}_2$, respectively. All these arguments demonstrate that the C*m*PF-TOPSIS and C*m*PF-ELECTRE-I methods provide effective and comprehensive frameworks for solving MCDM problems.

There are some limitations of the proposed techniques, (i) The offered approaches only deal with C*m*PF information, and cannot account for decision makers' negative and reluctant preferences. (ii) The suggested techniques are challenging due to the cubic nature of multi-polar data and the evaluation of symbolic translation at each multi-calculated stage.

For future work, we will extend our work to other derivatives of the ELECTRE technique under C*m*PF information such as:

- Cubic *m*-polar fuzzy ELECTRE-II, III.
- ELECTRE-TRI under the cubic *m*-polar fuzzy framework.

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Conflict of interest

The authors declare no conflict of interest.

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