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Research article

Comparison measures for Pythagorean m-polar fuzzy sets and their applications to robotics and movie recommender system

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Abstract: The perception of comparison measures is vitally significant in more or less every scientific field. They have many practical implementations in areas such as medicine, molecular biology, management, meteorology, etc. In this article, novel similarity, distance, and correlation comparison measures for Pythagorean m-polar fuzzy sets are proposed. The leading qualities of these comparison measures are investigated. The numerical examples are provided to demonstrate their formulation. In PmFSs, elements are allowed to duplicate finitely, which supports the usage of the measures put forward in here-and-now situations where we ponder time and again to reach some decision. The three algorithms are proposed to discuss the applications of comparison measures for PmFSs in robotics and movie recommender systems.

Keywords: comparison measure; Pythagorean m-polar fuzzy sets; robotics and movie recommender system
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1. Introduction

Mathematical rationale and set theory are believed to be the pillars of modern mathematics. Indeed, together they constitute the language bridging in nearly every branch of science. In actual fact, the prompt expansion and growth of science has set in motion an imperative requisite for the advancement of mathematical modeling based upon modern set theories. While describing a (crisp) set, a characteristic function is attached to that set. Taking the impact of uncertainty factor into account, Zadeh [1] gave the conception of fuzzy sets by attaching a membership function with each member of the classical set. Later, Zadeh [2] proposed similarity relations and fuzzy orderings. Zadeh [3] presented applications to approximate reasoning by floating the idea of a linguistic variable.

Following the footsteps of Zadeh, numerous theories and approaches treating uncertainty, imprecision and vagueness have been proposed so far. Atanassov [4,5] originated a modernistic class of sets: intuitionistic fuzzy sets (IFSs), as extension of fuzzy sets. Feng et al. [6] unveiled Lexicographic orders of IF values and established relationships between them. Pythagorean fuzzy set (PFS), initiated by Yager [7–9] is further expansion of IFSs. Peng et al. [10, 11] further studied results for PFSs and corresponding soft sets with applications. Guleria and Bajaj [12] coined matrix representation of PFSSs. Naeem et al. [13, 14] studied Pythagorean fuzzy soft multi-criteria group decision making (MCGDM) methods. Naeem et al. [15] originated Pythagorean m-polar fuzzy sets (PmFSs) and their utilizations. In PmFSs, contrary to PFSs, elements may appear multiple times with the possibility of identical or different membership grades. Naeem et al. [16] studied the topological structure on PmFSs and in [17] explored many interesting features of PmFSs. Riaz et al. [18] established weighted aggregation operators for PmFSs.

Since IFSs and PFSs are widely used in numerous fields like decision making, market prediction, pattern recognition, forecasting, business and commerce analysis, medical diagnosis, speech recognition, logic programming etc., so comparison measures of these sets perform a significant part in contemporary research areas. Many researchers, like [19–22] etc., worked on choice making techniques inclusive of comparison measures. Akram et al. [23] discussed the urban quality of life through the MULTIMOORA method with 2-tuple linguistic Fermatean fuzzy sets. Liu et al. [24] focused on the variation coefficient similarity measures and their applications to medical diagnosis and pattern recognition. An aggregate operator-based approach to cancer therapy assessment had been developed by Kausar et al. [25]. Pan et al. [26] proposed a quaternion model of PFSs and its distance measure. Akram et al. [27] extended the ELECTRE method for m-polar fuzzy N-soft sets and discussed their applications in the selection of rehabilitation centers. Khan et al. [28] defined the divergence measures for circular IFSs and discussed their applications to pattern recognition, multi-period medical diagnosis and MCDM problems. The ELECTRE-I method for hesitant PFSs and their applications to risk evaluation were focused [29]. Akram et al. [30] proposed complex PFSs and their applications to MCDM problems.

The role of multipolar statistics is gaining momentum, especially in making large scale decisions related to capital investment, therapeutic analysis and pattern recognition etc. In making far reaching decisions, we have to think time and again to reach at some conclusion.

The purpose of this article is to investigate some comparison measures for PmFSs in order to quantify uncertain information. The aims include studying the basic properties and numerical examples of the proposed comparison measures. Also, to discuss the applications of comparison measures for

PmFSs in robotics and movie recommender systems.

To attain the purpose, we propose similarity, distance, and correlation measures along with their leading characteristics. The mathematical results of the examples support the notion that the developed comparison measures are well suited to multipolar statistics. Three algorithms based on similarity, distance and correlation measures are discussed to configure robotics and the movie recommender system.

The remnant part of the article is sorted out as pursues. In Section 2, we discuss several fundamentals that are necessary to comprehend the next ideas. In Section 3, we propose three comparison measures with associated properties: a similarity measure, a distance measure and a correlation measure. We define the entropy measure for PmFS with the assistance of similarity measure in the same section. Section 4 consists of three algorithms based on the suggested comparison measures and applied them on artificial intelligence and demonstrated that all the measures yield the same optimal choice. Section 5 deals with another practical usage of the suggested comparison measures in the machine learning technique of movie recommending system. We conclude the paper in Section 6.

2. Preliminaries

In this section, we discuss some fundamentals that are necessary to comprehend the next ideas. The definitions of fuzzy sets, PFSs, PmFSs and their basic operations are mentioned.

Definition 2.1. *[1] Presume that $X \neq \phi$ is a universe. A fuzzy set A in X is given as*

$$A = \{< \omega, \mu_A(\omega) >: \omega \in X\},$$

where $\mu_A : X \to [0, 1]$ is the membership function of A.

Definition 2.2. *[7] A Pythagorean fuzzy set, abbreviated as PFS, is a family of the form*

$$P = \{ < \omega, \mu_P(\omega), \nu_P(\omega) >: \omega \in X\},$$

where $\mu_P(\omega), \nu_P(\omega) \in [0, 1]$ such that $0 \leq \mu_P^2(\omega) + \nu_P^2(\omega) \leq 1$. These maps are called correspondingly the association and non-association grades of $\omega \in X$ to the set P.

Definition 2.3. *[15] A Pythagorean m-polar fuzzy set (PmFS) P is typified by the maps $\mu_P^{(i)}$ (affiliation grades) and $\nu_P^{(i)}$ (dissociation degrees) giving members of X to [0, 1] such that $0 \leq \left(\mu_P^{(i)}(\omega)\right)^2 + \left(\nu_P^{(i)}(\omega)\right)^2 \leq 1$, for all i.*

A PmFS is commonly expressed in different ways as

$$\begin{aligned} P &= \left\{\langle \omega, \left((\mu_P^{(1)}(\omega), \nu_P^{(1)}(\omega)), \cdots, (\mu_P^{(m)}(\omega), \nu_P^{(m)}(\omega))\right)\rangle\right\} \\ &= \left\{\frac{\omega}{\left((\mu_P^{(1)}(\omega), \nu_P^{(1)}(\omega)), \cdots, (\mu_P^{(m)}(\omega), \nu_P^{(m)}(\omega))\right)}\right\} \\ &= \left\{\frac{\omega}{\left((\mu_P^{(i)}(\omega), \nu_P^{(i)}(\omega))\right)}\right\}_{i=1}^{m}. \end{aligned}$$

The tabulator array of P is produced by Table 1 if the size of X is r.

Table 1. Tabulatory array of P.

P	(1)	(2)	$\cdots$	(m)
ω_1	$(\mu_P^{(1)}(\omega_1), \nu_P^{(1)}(\omega_1))$	$(\mu_P^{(2)}(\omega_1), \nu_P^{(2)}(\omega_1))$	$\cdots$	$(\mu_P^{(m)}(\omega_1), \nu_P^{(m)}(\omega_1))$
ω_2	$(\mu_P^{(1)}(\omega_2), \nu_P^{(1)}(\omega_2))$	$(\mu_P^{(2)}(\omega_2), \nu_P^{(2)}(\omega_2))$	$\cdots$	$(\mu_P^{(m)}(\omega_2), \nu_P^{(m)}(\omega_2))$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
ω_r	$(\mu_P^{(1)}(\omega_r), \nu_P^{(1)}(\omega_r))$	$(\mu_P^{(2)}(\omega_r), \nu_P^{(2)}(\omega_r))$	$\cdots$	$(\mu_P^{(m)}(\omega_r), \nu_P^{(m)}(\omega_r))$

The corresponding matrix format is

$$P = \begin{pmatrix} (\mu_P^{(1)}(\omega_1), \nu_P^{(1)}(\omega_1)) & (\mu_P^{(2)}(\omega_1), \nu_P^{(2)}(\omega_1)) & \cdots & (\mu_P^{(m)}(\omega_1), \nu_P^{(m)}(\omega_1)) \\ (\mu_P^{(1)}(\omega_2), \nu_P^{(1)}(\omega_2)) & (\mu_P^{(2)}(\omega_2), \nu_P^{(2)}(\omega_2)) & \cdots & (\mu_P^{(m)}(\omega_2), \nu_P^{(m)}(\omega_2)) \\ \vdots & \vdots & \ddots & \vdots \\ (\mu_P^{(1)}(\omega_r), \nu_P^{(1)}(\omega_r)) & (\mu_P^{(2)}(\omega_r), \nu_P^{(2)}(\omega_r)) & \cdots & (\mu_P^{(m)}(\omega_r), \nu_P^{(m)}(\omega_r)) \end{pmatrix}.$$

This matrix of size $r \times m$ is titled as PmF matrix. The aggregate of all PmFSs defined over X is designated as $PmFS(X)$.

Definition 2.4. *[15] A PmFS P_1 is called a subset of the PmFS P_2, written $P_1 \subseteq P_2$ if $\mu_{P_1}^{(i)}(\omega) \leq \mu_{P_2}^{(i)}(\omega)$ and $\nu_{P_1}^{(i)}(\omega) \geq \nu_{P_2}^{(i)}(\omega)$, for all $\omega \in X$ and all i.*

P_1 and P_2 are said to be equal if and only if $P_1 \subseteq P_2 \subseteq P_1$.

3. Comparison measures for PmFS

The similarity, distance and correlation measures are the three comparison metrics for PmFS that are presented in this section. We present illustrative instances for these metrics and prove their necessary properties for them.

3.1. Similarity measure for PmFSs

This section is devoted to outlining a PmFSs similarity metric and some of its key features.

Definition 3.1. *A measure Sim, given by the mapping $Sim : PmFS(X) \times PmFS(X) \to [0, 1]$, is called a similarity measure if the following axioms hold:*

(1) $Sim(P_1, P_2) \in [0, 1]$;
(2) $Sim(P_1, P_2) = 1 \Leftrightarrow P_1 = P_2$;
(3) $Sim(P_1, P_2) = Sim(P_2, P_1)$;
(4) If $P_1 \subseteq P_2 \subseteq P_3$, then $Sim(P_1, P_3) \leq Sim(P_1, P_2)$ and $Sim(P_1, P_3) \leq Sim(P_2, P_3)$,

where $P_1, P_2, P_3 \in PmFS(X)$.

Definition 3.2. *Let $X = \{\omega_i : i = 1, \cdots, n\}$. The mapping between PmFS $P_1 = \left\{ \frac{\omega_i}{(\mu_{P_1}^{(j)}(\omega_i), \nu_{P_1}^{(j)}(\omega_i))} : \omega_i \in X \right\}$ and $P_2 = \left\{ \frac{\omega_i}{(\mu_{P_2}^{(j)}(\omega_i), \nu_{P_2}^{(j)}(\omega_i))} : \omega_i \in X \right\}$ comprising membership and non-membership functions defined over the finite universe $X = \{\omega_i : i = 1, 2, \cdots, n\}$ may be defined as*

$$Sim(P_1, P_2) = \frac{< P_1, P_2 >}{||P_1|| ||P_2||},$$

where

$$< P_1, P_2 > = \sum_{j=1}^{m} \Big\{ \sum_{i=1}^{n} (\mu_{P_1}^{(j)}(\omega_i)\mu_{P_2}^{(j)}(\omega_i) + \nu_{P_1}^{(j)}(\omega_i)\nu_{P_2}^{(j)}(\omega_i)) \Big\},$$

$$||P_1|| = \sqrt{\sum_{j=1}^{m} \Big\{ \sum_{i=1}^{n} \Big((\mu_{P_1}^{(j)}(\omega_i))^2 + (\nu_{P_1}^{(j)}(\omega_i))^2 \Big) \Big\}},$$

$$||P_2|| = \sqrt{\sum_{j=1}^{m} \Big\{ \sum_{i=1}^{n} \Big((\mu_{P_2}^{(j)}(\omega_i))^2 + (\nu_{P_2}^{(j)}(\omega_i))^2 \Big) \Big\}}.$$

Proposition 3.1. *The metric in Definition 3.2 complies with the following criteria:*

(1) $0 \leq Sim(P_1, P_2) \leq 1$.
(2) $Sim(P_1, P_2) = 1 \Leftrightarrow P_1 = P_2$.
(3) $Sim(P_1, P_2) = Sim(P_2, P_1)$.

Proof. The first and third requirements are clear-cut. Assume $Sim(P_1, P_2)$ has a value of 1. Following that, we have for all conceivable values of i and j,

$$\frac{< P_1, P_2 >}{||P_1||||P_2||} = 1,$$

which in turn yields $\mu_{P_1}^{(j)}(\omega_i) = \mu_{P_2}^{(j)}(\omega_i)$ and $\nu_{P_1}^{(j)}(\omega_i) = \nu_{P_2}^{(j)}(\omega_i)$. Hence, $P_1 = P_2$.

Conversely, assume that $P_1 = P_2$. Thus, for all i and j, we have $\mu_{P_1}^{(j)}(\omega_i) = \mu_{P_2}^{(j)}(\omega_i)$ and $\nu_{P_1}^{(j)}(\omega_i) = \nu_{P_2}^{(j)}(\omega_i)$. Thus,

$$\begin{aligned} < P_1, P_2 > &= \sum_{j=1}^{m} \Big\{ \sum_{i=1}^{n} (\mu_{P_1}^{(j)}(\omega_i)\mu_{P_2}^{(j)}(\omega_i) + \nu_{P_1}^{(j)}(\omega_i)\nu_{P_2}^{(j)}(\omega_i)) \Big\} \\ &= \sum_{j=1}^{m} \Big\{ \sum_{i=1}^{n} \Big((\mu_{P_1}^{(j)}(\omega_i))^2 + (\nu_{P_1}^{(j)}(\omega_i))^2 \Big) \Big\} \\ &= ||P_1||^2 = ||P_1||||P_2||, \end{aligned}$$

therefore,

$$Sim(P_1, P_2) = \frac{< P_1, P_2 >}{||P_1||||P_2||} = 1.$$

Proposition 3.2. *If P_1, P_2 and P_3 are PmFSs defined over X such that $P_1 \subseteq P_2 \subseteq P_3$, then, $Sim(P_1, P_3) \leq Sim(P_1, P_2)$ and $Sim(P_1, P_3) \leq Sim(P_2, P_3)$.*

Proof. Straightforward.

Proposition 3.3. *The metric in Definition 3.2 is a similarity measure for PmFSs.*

Proof. The proof is straightforward by Propositions 3.1 and 3.2.

Example 3.1. Consider the P*m*FSs

$$P_1 = \left\{\frac{\omega_1}{(0.71, 0.29), (0.69, 0.53)}, \frac{\omega_2}{(0.53, 0.41), (0.16, 0.91)}\right\},$$
$$P_2 = \left\{\frac{\omega_1}{(0.54, 0.37), (0.17, 0.12)}, \frac{\omega_2}{(0.48, 0.47), (0.51, 0.51)}\right\}$$

defined over the crisp set $X = \{\omega_1, \omega_2\}$. Then,

$$\begin{aligned} < P_1, P_2 > &= \sum_{j=1}^{2}\Big\{\sum_{i=1}^{2}(\mu_{P_1}^{(j)}(\omega_i)\mu_{P_2}^{(j)}(\omega_i) + \nu_{P_1}^{(j)}(\omega_i)\nu_{P_2}^{(j)}(\omega_i))\Big\} \\ &= (0.71)(0.54) + (0.29)(0.37) + (0.69)(0.17) + \cdots + (0.91)(0.51) \\ &= 1.6644, \end{aligned}$$

$$\|P_1\| = \sqrt{\sum_{j=1}^{2}\Big\{\sum_{i=1}^{2}\Big((\mu_{P_1}^{(j)}(\omega_i))^2 + (\nu_{P_1}^{(j)}(\omega_i))^2\Big)\Big\}} = \sqrt{(0.71)^2 + (0.29)^2 + \cdots + (0.91)^2} = 1.6272,$$
$$\|P_2\| = \sqrt{\sum_{j=1}^{2}\Big\{\sum_{i=1}^{2}\Big((\mu_{P_2}^{(j)}(\omega_i))^2 + (\nu_{P_2}^{(j)}(\omega_i))^2\Big)\Big\}} = \sqrt{(0.54)^2 + (0.37)^2 + \cdots + (0.51)^2} = 1.2014,$$

and hence

$$Sim(P_1, P_2) = \frac{< P_1, P_2 >}{\|P_1\|\|P_2\|} = 0.8514.$$

Definition 3.3. *The angle* θ_{P_1,P_2} *defined by* $\theta_{P_1,P_2} = \arccos\left(\frac{<P_1,P_2>}{\|P_1\|\|P_2\|}\right)$ *is called the angle of similarity between the PmFSs* P_1 *and* P_2*.*

Example 3.2. For P_1 and P_2, cited in Example 3.1, we have

$$\theta_{P_1,P_2} = \arccos\left(\frac{< P_1, P_2 >}{\|P_1\|\|P_2\|}\right) = \arccos(0.8514) = 31°38'.$$

Remark 3.1. *By multipolarizing the supplied PFS in accordance with the given PmFS, we first convert the given PFS to PmFS in order to compare the two. The idea is demonstrated by the next example.*

Example 3.3. Consider the P*m*FS

$$P_1 = \left\{\frac{\omega_1}{(0.53, 0.40), (0.21, 0.23)}, \frac{\omega_2}{(0.35, 0.47), (0.99, 0.03)}, \frac{\omega_3}{(0.08, 0.38), (0.57, 0.61)}\right\}$$

and a PFS

$$P = \left\{\frac{\omega_1}{(0.81, 0.29)}, \frac{\omega_2}{(0.61, 0.34)}, \frac{\omega_3}{(0.60, 0.12)}\right\}$$

defined over $X = \{\omega_1, \omega_2, \omega_3\}$. We convert the PFS P to a P2FS (and refer to the resulting set as P_2) to make it compatible with P_1 as the unit of comparison.

$$P_2 = \left\{\frac{\omega_1}{(0.81, 0.29), (0.81, 0.29)}, \frac{\omega_2}{(0.61, 0.34), (0.61, 0.34)}, \frac{\omega_3}{(0.60, 0.12), (0.60, 0.12)}\right\}.$$

Now,

$$
\begin{aligned}
< P_1, P_2 > &= \sum_{j=1}^{2}\Big\{\sum_{i=1}^{3}(\mu_{P_1}^{(j)}(\omega_i)\mu_{P_2}^{(j)}(\omega_i) + \nu_{P_1}^{(j)}(\omega_i)\nu_{P_2}^{(j)}(\omega_i))\Big\}\\
&= (0.53)(0.81) + (0.40)(0.29) + (0.21)(0.81) + \cdots + (0.57)(0.60)\\
&= 2.2783,
\end{aligned}
$$

$$
\|P_1\| = \sqrt{\sum_{j=1}^{2}\Big\{\sum_{i=1}^{3}\Big((\mu_{P_1}^{(j)}(\omega_i))^2 + (\nu_{P_1}^{(j)}(\omega_i))^2\Big)\Big\}} = \sqrt{(0.53)^2 + (0.40)^2 + \cdots + (0.61)^2} = 1.6462,
$$

$$
\|P_2\| = \sqrt{\sum_{j=1}^{2}\Big\{\sum_{i=1}^{3}\Big((\mu_{P_2}^{(j)}(\omega_i))^2 + (\nu_{P_2}^{(j)}(\omega_i))^2\Big)\Big\}} = \sqrt{(0.81)^2 + (0.29)^2 + \cdots + (0.12)^2} = 1.7901,
$$

and hence

$$
Sim(P_1, P_2) = \frac{< P_1, P_2 >}{\|P_1\|\|P_2\|} = 0.7731.
$$

Proposition 3.4. *P is a crisp set $\Leftrightarrow Sim(P, P^c) = 1$.*

Proof. The evidence is provided by the fact that a crisp set possesses the form

$$
P = \left\{\frac{\omega_\alpha}{(0,1)}, \frac{\omega_\beta}{(1,0)}\right\}
$$

for all acceptable α and β values.

Example 3.4. Consider the P*m*FSs

$$
P_1 = \begin{pmatrix}(0.52, 0.31) & (0.47, 0.12) & (0.36, 0.71)\\ (0.16, 0.45) & (0.28, 0.14) & (0.39, 0.60)\\ (0.02, 0.99) & (0.87, 0.26) & (0.83, 0.01)\end{pmatrix},
$$

$$
P_2 = \begin{pmatrix}(0.57, 0.25) & (0.52, 0.12) & (0.40, 0.54)\\ (0.25, 0.41) & (0.33, 0.09) & (0.40, 0.48)\\ (0.14, 0.58) & (0.88, 0.21) & (0.85, 0.01)\end{pmatrix},
$$

and

$$
P_3 = \begin{pmatrix}(0.61, 0.20) & (0.57, 0.09) & (0.56, 0.31)\\ (0.34, 0.36) & (0.54, 0.01) & (0.55, 0.22)\\ (0.17, 0.52) & (0.89, 0.18) & (0.89, 0.01)\end{pmatrix},
$$

so that $P_1 \subseteq P_2 \subseteq P_3$. Here,

$$
\begin{aligned}
Sim(P_1, P_2) &= 0.9741,\\
Sim(P_2, P_3) &= 0.9687,\\
Sim(P_1, P_3) &= 0.9109.
\end{aligned}
$$

As a result, Proposition 3.2's findings are corroborated.

Definition 3.4. *Two PmFSs P_1 and P_2 are said to be λ-similar, written $P_1 \approx^{\lambda} P_2$, if and only if $Sim(P_1, P_2) \geq \lambda$ for some $0 < \lambda < 1$.*

We observe from Example 3.4 that $Sim(P_1, P_2) \geq 0.95$ and $Sim(P_2, P_3) \geq 0.95$, but $Sim(P_1, P_3) \ngeq 0.95$, i.e., $P_1 \approx^{0.95} P_2$ and $P_2 \approx^{0.95} P_3$, but $P_1 \approx^{0.95} P_3$ is untrue. It implies that the transitive property of a relation does not follow the being λ-similar.

Proposition 3.5. *Being λ-similar is a relation, not an equivalence relation.*

Definition 3.5. *The similarity between a PmFS and its complement gives entropy of PmFS, that is, $E(P) = Sim(P, P^c)$. It is directed from definition that $E(P) = E(P^c)$.*

Definition 3.6. *P_1 is referred to as being less fuzzy than P_2 for two PmFSs, P_1 and P_2, if and only if $E(P_1) \leq E(P_2)$.*

Example 3.5. For P_1 and P_2 given in Example 3.4, we have $E(P_1) = 0.4697$ and $E(P_2) = 0.5425$, so P_1 is less fuzzy than P_2.

3.2. Distance measure for PmFSs

A distance measure for P*m*FSs and some of its key features are presented in this subsection. Some numerical examples are provided for illustration.

Definition 3.7. *Let $P_1, P_2 \in PmFS(X)$. If all of the following conditions are satisfied, a measure D_m provided by the mapping $D_m : PmFS(X) \times PmFS(X) \to [0, 1]$ is referred to as a distance measure, if the following axioms hold:*

(1) $D_m(P_1, P_2) \in [0, 1]$.
(2) $D_m(P_1, P_2) = 0 \Leftrightarrow P_1 = P_2$.
(3) $D_m(P_1, P_2) = D_m(P_2, P_1)$.
(4) If P_1, P_2 and P_3 be three PmFSs defined over X, then $D_m(P_1, P_2) + D_m(P_2, P_3) \geq D_m(P_1, P_3)$.

Definition 3.8. *The mapping between PmFSs $P_1 = \left\{\frac{\omega_i}{(\mu^{(j)}_{P_1}(\omega_i), \nu^{(j)}_{P_1}(\omega_i))} : \omega_i \in X\right\}$ and $P_2 = \left\{\frac{\omega_i}{(\mu^{(j)}_{P_2}(\omega_i), \nu^{(j)}_{P_2}(\omega_i))} : \omega_i \in X\right\}$ comprising membership and non-membership functions defined over the finite universe $X = \{\omega_i : i = 1, 2, \cdots, n\}$ may be defined as*

$$D_m(P_1, P_2) = \frac{1}{2mn} \sum_{j=1}^{m} \left\{ \sum_{i=1}^{n} \left\{ \left| \left(\mu^{(j)}_{P_1}(\omega_i)\right)^2 - \left(\mu^{(j)}_{P_2}(\omega_i)\right)^2 \right| + \left| \left(\nu^{(j)}_{P_1}(\omega_i)\right)^2 - \left(\nu^{(j)}_{P_2}(\omega_i)\right)^2 \right| \right\} \right\}.$$

Proposition 3.6. *The specifications in Definition 3.7 are satisfied by the mapping D_m provided in Definition 3.8.*

Proof. Straightforward.

Proposition 3.7. *The metric in Definition 3.7 is a distance measure for PmFSs.*

Proof. The proof comes just after Proposition 3.6.

Example 3.6. As seen in Example 3.1, the distance between PmFSs is

$$D_m(P_1,P_2)=\frac{1}{2(2)(2)}\sum_{j=1}^{2}\Big\{\sum_{i=1}^{2}\Big\{\Big|\Big(\mu_{P_1}^{(j)}(\omega_i)\Big)^2-\Big(\mu_{P_2}^{(j)}(\omega_i)\Big)^2\Big|+\Big|\Big(\nu_{P_1}^{(j)}(\omega_i)\Big)^2-\Big(\nu_{P_2}^{(j)}(\omega_i)\Big)^2\Big|\Big\}\Big\}$$
$$=\frac{1}{8}\Big\{\Big|0.71^2-0.54^2\Big|+\Big|0.29^2-0.37^2\Big|+\cdots+\Big|0.91^2-0.51^2\Big|\Big\}$$
$$=0.2346.$$

Remark 3.2. *For finding the distance measure between a PFS and a PmFS, we proceed like Example 3.3.*

Proposition 3.8. *P is a crisp set $\Leftrightarrow D_m(P,P^c)=1$.*

Proof. The demonstration follows from the fact that a crisp set has the shape

$$P=\left\{\frac{\omega_\alpha}{(0,1)},\frac{\omega_\beta}{(1,0)}\right\}$$

for all acceptable α and β values.

Proposition 3.9. *If P_1, P_2 and P_3 are PmFSs defined over X such that $P_1\subseteq P_2\subseteq P_3$, then $D_m(P_1,P_2)\leq D_m(P_1,P_3)$ and $D_m(P_2,P_3)\leq D_m(P_1,P_3)$.*

Proof. Since $P_1\subseteq P_2\subseteq P_3$, so $\mu_{P_1}^{(j)}(\omega_i)\leq\mu_{P_2}^{(j)}(\omega_i)\leq\mu_{P_3}^{(j)}(\omega_i)$ and $\nu_{P_1}^{(j)}(\omega_i)\geq\nu_{P_2}^{(j)}(\omega_i)\geq\nu_{P_3}^{(j)}(\omega_i)$ for all i and j.

Now, by definition

$$D_m(P_1,P_2)=\frac{1}{2mn}\sum_{j=1}^{m}\Big\{\sum_{i=1}^{n}\Big\{\Big|\Big(\mu_{P_1}^{(j)}(\omega_i)\Big)^2-\Big(\mu_{P_2}^{(j)}(\omega_i)\Big)^2\Big|+\Big|\Big(\nu_{P_1}^{(j)}(\omega_i)\Big)^2-\Big(\nu_{P_2}^{(j)}(\omega_i)\Big)^2\Big|\Big\}\Big\}$$
$$=\frac{1}{2mn}\sum_{j=1}^{m}\Big\{\sum_{i=1}^{n}\Big\{\Big(\mu_{P_2}^{(j)}(\omega_i)\Big)^2-\Big(\mu_{P_1}^{(j)}(\omega_i)\Big)^2+\Big(\nu_{P_1}^{(j)}(\omega_i)\Big)^2-\Big(\nu_{P_2}^{(j)}(\omega_i)\Big)^2\Big\}\Big\},$$
$$D_m(P_1,P_3)=\frac{1}{2mn}\sum_{j=1}^{m}\Big\{\sum_{i=1}^{n}\Big\{\Big|\Big(\mu_{P_1}^{(j)}(\omega_i)\Big)^2-\Big(\mu_{P_3}^{(j)}(\omega_i)\Big)^2\Big|+\Big|\Big(\nu_{P_1}^{(j)}(\omega_i)\Big)^2-\Big(\nu_{P_3}^{(j)}(\omega_i)\Big)^2\Big|\Big\}\Big\}$$
$$=\frac{1}{2mn}\sum_{j=1}^{m}\Big\{\sum_{i=1}^{n}\Big\{\Big(\mu_{P_3}^{(j)}(\omega_i)\Big)^2-\Big(\mu_{P_1}^{(j)}(\omega_i)\Big)^2+\Big(\nu_{P_1}^{(j)}(\omega_i)\Big)^2-\Big(\nu_{P_3}^{(j)}(\omega_i)\Big)^2\Big\}\Big\}.$$

On subtraction, we have

$$D_m(P_1,P_3)-D_m(P_1,P_2)=\frac{1}{2mn}\sum_{j=1}^{m}\Big\{\sum_{i=1}^{n}\Big\{\Big(\mu_{P_3}^{(j)}(\omega_i)\Big)^2-\Big(\mu_{P_2}^{(j)}(\omega_i)\Big)^2+\Big(\nu_{P_2}^{(j)}(\omega_i)\Big)^2-\Big(\nu_{P_3}^{(j)}(\omega_i)\Big)^2\Big\}\Big\}\geq 0$$
$$\Rightarrow D_m(P_1,P_2)\leq D_m(P_1,P_3).$$

Analogously, the other result might also be proven.

Example 3.7. The P*m*FSs mentioned in Example 3.4 have

$$D_m(P_1, P_2) = 0.0760,$$
$$D_m(P_2, P_3) = 0.0700,$$
$$D_m(P_1, P_3) = 0.1461.$$

As a result, Proposition 3.9's findings are corroborated.

3.3. Correlation measure for PmFSs

The correlation measure for P*m*FSs is suggested in this subsection, along with some of its key features.

Definition 3.9. *Take $P_1, P_2 \in PmFS(X)$. A measure $Cor(P_1, P_2)$, given by the mapping $Cor : PmFS(X) \times PmFS(X) \to [0, 1]$, is called a correlation measure if*

(1) $Cor(P_1, P_2) \in [0, 1]$.
(2) $Cor(P_1, P_2) = 1 \Leftrightarrow P_1 = P_2$.
(3) $Cor(P_1, P_2) = Cor(P_2, P_1)$.

Definition 3.10. *Let $X = \{\omega_i : i = 1, \cdots, n\}$. The mapping between PmFSs $P_1 = \left\{\frac{\omega_i}{(\mu^{(j)}_{P_1}(\omega_i), \nu^{(j)}_{P_1}(\omega_i))} : \omega_i \in X\right\}$ and $P_2 = \left\{\frac{\omega_i}{(\mu^{(j)}_{P_2}(\omega_i), \nu^{(j)}_{P_2}(\omega_i))} : \omega_i \in X\right\}$ comprising membership and non-membership functions defined over the finite universe $X = \{\omega_i : i = 1, 2, \cdots, n\}$ may be defined as*

$$Cor(P_1, P_2) = \frac{\eta(P_1, P_2)\{\eta(P_1, P_1) + \eta(P_2, P_2)\}}{2\eta(P_1, P_1)\eta(P_2, P_2)},$$

where

$$\eta(P_1, P_2) = \sum_{j=1}^{m}\Big\{\sum_{i=1}^{n} (\mu^{(j)}_{P_1}(\omega_i)\mu^{(j)}_{P_2}(\omega_i))^{\frac{3}{2}} + (\nu^{(j)}_{P_1}(\omega_i)\nu^{(j)}_{P_2}(\omega_i))^{\frac{3}{2}}\Big\},$$
$$\eta(P_1, P_1) = \sum_{j=1}^{m}\Big\{\sum_{i=1}^{n} (\mu^{(j)}_{P_1}(\omega_i))^3 + (\nu^{(j)}_{P_1}(\omega_i))^3\Big\},$$
$$\eta(P_2, P_2) = \sum_{j=1}^{m}\Big\{\sum_{i=1}^{n} (\mu^{(j)}_{P_2}(\omega_i))^3 + (\nu^{(j)}_{P_2}(\omega_i))^3\Big\}.$$

Proposition 3.10. *The mapping $Cor(P_1, P_2)$ specified in Definition 3.10 complies with Definition 3.9 specifications.*

Proof. Here, we demonstrate part (2). (1) and (3) come naturally from the definition. Assume that

$$\begin{aligned} Cor(P_1, P_2) = 1 \Rightarrow & \frac{\eta(P_1, P_2)\{\eta(P_1, P_1) + \eta(P_2, P_2)\}}{2\eta(P_1, P_1)\eta(P_2, P_2)} = 1 \\ \Rightarrow & \eta(P_1, P_2)\{\eta(P_1, P_1) + \eta(P_2, P_2)\} = 2\eta(P_1, P_1)\eta(P_2, P_2) \\ \Rightarrow & \eta(P_1, P_1)\{\eta(P_1, P_2) - \eta(P_2, P_2)\} + \eta(P_2, P_2)\{\eta(P_1, P_2) - \eta(P_1, P_1)\} = 0. \end{aligned}$$

In general, the final equation makes sense if

$$\eta(P_1, P_2) - \eta(P_2, P_2) = 0$$

and

$$\eta(P_1, P_2) - \eta(P_1, P_1) = 0,$$

i.e., if

$$\eta(P_1, P_2) = \eta(P_1, P_1) = \eta(P_2, P_2).$$

Now,

$$\begin{aligned}
&\eta(P_1, P_2) = \eta(P_1, P_1) \\
\Rightarrow &\sum_{j=1}^{m}\Big\{\sum_{i=1}^{n}(\mu_{P_1}^{(j)}(\omega_i)\mu_{P_2}^{(j)}(\omega_i))^{\frac{3}{2}} + (\nu_{P_1}^{(j)}(\omega_i)\nu_{P_2}^{(j)}(\omega_i))^{\frac{3}{2}}\Big\} = \sum_{j=1}^{m}\Big\{\sum_{i=1}^{n}(\mu_{P_1}^{(j)}(\omega_i))^3 + (\nu_{P_1}^{(j)}(\omega_i))^3\Big\} \\
\Rightarrow &(\mu_{P_1}^{(j)}(\omega_i)\mu_{P_2}^{(j)}(\omega_i))^{\frac{3}{2}} + (\nu_{P_1}^{(j)}(\omega_i)\nu_{P_2}^{(j)}(\omega_i))^{\frac{3}{2}} = (\mu_{P_1}^{(j)}(\omega_i))^3 + (\nu_{P_1}^{(j)}(\omega_i))^3 \\
\Rightarrow &(\mu_{P_1}^{(j)}(\omega_i))^{\frac{3}{2}}\Big\{\Big(\mu_{P_2}^{(j)}(\omega_i)\Big)^{\frac{3}{2}} - \Big(\mu_{P_1}^{(j)}(\omega_i)\Big)^{\frac{3}{2}}\Big\} + (\nu_{P_1}^{(j)}(\omega_i))^{\frac{3}{2}}\Big\{\Big(\nu_{P_2}^{(j)}(\omega_i)\Big)^{\frac{3}{2}} - \Big(\nu_{P_1}^{(j)}(\omega_i)\Big)^{\frac{3}{2}}\Big\} = 0 \\
\Rightarrow &\Big(\mu_{P_2}^{(j)}(\omega_i)\Big)^{\frac{3}{2}} - \Big(\mu_{P_1}^{(j)}(\omega_i)\Big)^{\frac{3}{2}} = 0 \ \& \ \Big(\nu_{P_2}^{(j)}(\omega_i)\Big)^{\frac{3}{2}} - \Big(\nu_{P_1}^{(j)}(\omega_i)\Big)^{\frac{3}{2}} = 0 \\
\Rightarrow &\mu_{P_2}^{(j)}(\omega_i) = \mu_{P_1}^{(j)}(\omega_i) \ \& \ \nu_{P_2}^{(j)}(\omega_i) = \nu_{P_1}^{(j)}(\omega_i) \\
\Rightarrow &P_1 = P_2.
\end{aligned}$$

Conversely, suppose that $P_1 = P_2$, then

$$\begin{aligned}
Cor(P_1, P_2) &= \frac{\eta(P_1, P_2)\{\eta(P_1, P_1) + \eta(P_2, P_2)\}}{2\eta(P_1, P_1)\eta(P_2, P_2)} \\
&= \frac{\eta(P_1, P_1)\{\eta(P_1, P_1) + \eta(P_1, P_1)\}}{2\eta(P_1, P_1)\eta(P_1, P_1)} \\
&= \frac{2\eta^2(P_1, P_1)}{2\eta^2(P_1, P_1)} = 1.
\end{aligned}$$

Proposition 3.11. *The metric in Definition 3.10 is a correlation measure for PmFSs.*

Proof. The proof comes just after Proposition 3.10.

Example 3.8. The following formula is used to compute the correlation measure between the P*m*FSs in Example 3.1:

$$\begin{aligned}
\eta(P_1, P_2) &= \sum_{j=1}^{2}\Big\{\sum_{i=1}^{2}(\mu_{P_1}^{(j)}(\omega_i)\mu_{P_2}^{(j)}(\omega_i))^{\frac{3}{2}} + (\nu_{P_1}^{(j)}(\omega_i)\nu_{P_2}^{(j)}(\omega_i))^{\frac{3}{2}}\Big\} \\
&= (0.71 \times 0.54)^{\frac{3}{2}} + (0.29 \times 0.37)^{\frac{3}{2}} + \cdots + (0.91 \times 0.51)^{\frac{3}{2}} \\
&= 0.8811,
\end{aligned}$$

$$
\begin{aligned}
\eta(P_1, P_1) &= \sum_{j=1}^{2}\Big\{\sum_{i=1}^{2}(\mu_{P_1}^{(j)}(\omega_i))^3 + (\nu_{P_1}^{(j)}(\omega_i))^3\Big\} \\
&= (0.71)^3 + (0.29)^3 + \cdots + (0.91)^3 \\
&= 1.8352, \\
\eta(P_2, P_2) &= \sum_{j=1}^{2}\Big\{\sum_{i=1}^{2}(\mu_{P_2}^{(j)}(\omega_i))^3 + (\nu_{P_2}^{(j)}(\omega_i))^3\Big\} \\
&= (0.54)^3 + (0.37)^3 + \cdots + (0.51)^3 \\
&= 0.6945.
\end{aligned}
$$

Hence,

$$
\begin{aligned}
Cor(P_1, P_2) &= \frac{\eta(P_1, P_2)\{\eta(P_1, P_1) + \eta(P_2, P_2)\}}{2\eta(P_1, P_1)\eta(P_2, P_2)} \\
&= \frac{0.8811(1.8352 + 0.6945)}{2(1.8352)(0.6945)} \\
&= 0.8744.
\end{aligned}
$$

Remark 3.3. *We follow Example 3.3 to determine the correlation measure between a PFS and a PmFS.*

4. Application of proposed comparison measures in robotics

The human intellect mostly comprises usage of variables having values from fuzzy sets that led to the underpinning for perception of a morphological variable-a variable whose values are words instead of numbers. There are situations, mainly including decision-making problems (like sales analysis and trends, business, medical therapeutic analysis, marketing & advertising etc.), the representation merely using lingual variables by means of membership grades does not suffice. There is possibility of existence of a non-void complement. IFS may be successfully employed in this perspective as an appropriate technique. But in real life, there arise situations in which each element has different membership values and of course non-membership values too. In such state of affairs, P*m*FS is more adequate. Since P*m*FSs give users more flexibility in selecting the values for membership and non-membership degrees, so they are more appropriate than existing structures. We present P*m*FS based application of artificial intelligence for coping with such a situation.

Artificial intelligence (AI), also known as machine intelligence, is a wide-ranging term which means the usage of computer to model and/or duplicate intelligent performance. AI resembles with humanoid astuteness processes by means of machines, principally computer systems. Research in artificial intelligence emphases on development and examination of algorithms that learn and/or accomplish intelligent behavior with least human involvement. The techniques of artificial intelligence are successfully applied in robotics, military planning and logistics, e-commerce, voice recognition, speech recognition, medical diagnosis, computer vision and gaming etc.

Case study: Consider a multi-robot system comprising four patrolling robots deployed for surveillance in a large area, controlled wirelessly by a single controller (as presented in Figure 1). The total area is split into four equal parts and allocated to each robot. The robot perambulates in its designated territory. Each robot is equipped with necessary feeler/sensors/radars which send necessary information to the controller who makes decisions in view of radar readings. For example, if the temperature sensor value in some robot points out an unusual temperature, the controller can amend the commands that are conveyed to that particular robot e.g. the controller can give directions to that robot to turn in some particular direction like at an angle of 45°. Alike is the case with all other sensor readings.

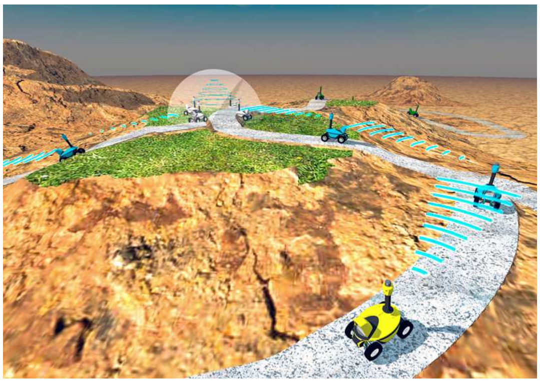

Figure 1. A multi-agent robotics system.

We initially propose Algorithm 1 as shown below before moving on to the suggested similarity measure's practical application. Algorithm 1 presents the MCDM method by which the controller decides whether the robot is near a fire, obstacle, bump, cliff, or vibration based on readings sent by the sensors on the robot to controllers. Similarity measures in Algorithm 1 basically find the similarity between the actual situation reading and the robot sensor's recorded reading.

Algorithm 1 Similarity measures based algorithm

S-1: Decide on the set of robots $R = \{R_1, R_2, \cdots, R_i\}$.

S-2: Choose the aggregate of situations $C = \{c_1, \cdots, c_j\}$ and the collection of sensors $S = \{s_1, s_2, \cdots, s_j\}$.

S-3: Drive tables of PFS of situations vs sensors & PmFS of robots vs situations.

S-4: Calculate the degree of similarity between robots and scenarios.

S-5: The robot and situations paired with the highest degree of resemblance is the best option.

S-6: Express the outcomes in layman's language.

Example 4.1. Let $R = \{R_i : i = 1, \cdots, 4\}$ be a system of four robots, $C = \{c_i : i = 1, \cdots, 5\}$, where

$$\begin{aligned} c_1 &= \text{Fire}, \\ c_2 &= \text{Stumbling block/Obstacle}, \\ c_3 &= \text{Collision/Bump}, \\ c_4 &= \text{Cliff}, \\ c_5 &= \text{Jolt/Vibration}, \end{aligned}$$

be the set of conditions/situations under consideration, and $S = \{s_i : i = 1, \cdots, 5\}$, where

$$\begin{aligned} s_1 &= \text{Temperature sensor}, \\ s_2 &= \text{Ultrasonic sensor}, \\ s_3 &= \text{Bump sensor}, \\ s_4 &= \text{Cliff sensor}, \\ s_5 &= \text{Accelerometer sensor}, \end{aligned}$$

be the family of sensors with which every robot is equipped. A single robot may be allocated dissimilar membership and non-membership values for the above mentioned sensor readings.

Obviously, taking decision on the basis of a single reading would be neither a wise nor justified decision. This is such a situation in which P*m*FSs come into picture. The sensor readings from each robot have to be uninterruptedly observed for a certain span of time, say for two minutes. For example, if the temperature device in some robot points toward an unusual temperature, it delivers a note to the controller for some appropriate direction. The controller has to ascertain whether that robot is in truth encountered an unusual temperature or not. For single-mindedness in taking appropriate decision, the controller keenly observes the temperature sensor readings for two minutes. Depending on the persistency of the readings, the controller ascertains the situation.

To comprehend P*m*FS theory, think about the situation where the robot R_1 experiences an unusual temperature, R_2 experiences a collision, R_3 faces a jolt and R_4 detects an obstacle. Thus, whenever the temperature sensor encounters an unusual temperature and the accelerometer sensor catches a jolt, signal is conveyed to the controller and the controller monitors the situation further to get ascertain and take necessary measures.

Table 2 renders sensor readings. Table 3 shows the sensor readings monitored, after every 40 seconds, for 2 minutes in the format $(\mu^{(j)}, \nu^{(j)})$, $j = 1, 2, 3$. In Table 4, the similarity measures of each robot to the situation under consideration are calculated and tabulated.

Table 2. Pythagorean fuzzy set of situations vs sensors.

	s_1	s_2	s_3	s_4	s_5
c_1	$(0.88, 0.21)$	$(0.45, 0.84)$	$(0.32, 0.84)$	$(0.45, 0.71)$	$(0.32, 0.84)$
c_2	$(0.45, 0.84)$	$(0.89, 0.32)$	$(0.32, 0.84)$	$(0.32, 0.84)$	$(0.45, 0.71)$
c_3	$(0.32, 0.84)$	$(0.77, 0.55)$	$(0.95, 0.31)$	$(0.32, 0.84)$	$(0.32, 0.84)$
c_4	$(0.45, 0.71)$	$(0.45, 0.84)$	$(0.32, 0.84)$	$(0.84, 0.32)$	$(0.32, 0.84)$
c_5	$(0.71, 0.45)$	$(0.32, 0.84)$	$(0.45, 0.71)$	$(0.32, 0.84)$	$(0.89, 0.45)$

Table 3. PmFS of robots vs situations.

	s_1	s_2	s_3	s_4	s_5
R_1	$(0.84, 0.27)$ $(0.45, 0.43)$ $(0.31, 0.16)$	$(0.23, 0.86)$ $(0.23, 0.79)$ $(0.16, 0.80)$	$(0.45, 0.48)$ $(0.12, 0.91)$ $(0.08, 0.96)$	$(0.37, 0.48)$ $(0.29, 0.47)$ $(0.28, 0.48)$	$(0.46, 0.85)$ $(0.21, 0.84)$ $(0.13, 0.92)$
R_2	$(0.27, 0.95)$ $(0.26, 0.96)$ $(0.22, 0.97)$	$(0.31, 0.95)$ $(0.29, 0.92)$ $(0.13, 0.97)$	$(0.98, 0.14)$ $(0.86, 0.50)$ $(0.84, 0.54)$	$(0.22, 0.93)$ $(0.22, 0.93)$ $(0.15, 0.98)$	$(0.58, 0.80)$ $(0.49, 0.86)$ $(0.25, 0.96)$
R_3	$(0.77, 0.63)$ $(0.58, 0.80)$ $(0.54, 0.83)$	$(0.46, 0.86)$ $(0.44, 0.89)$ $(0.32, 0.92)$	$(0.86, 0.49)$ $(0.86, 0.46)$ $(0.80, 0.54)$	$(0.71, 0.68)$ $(0.59, 0.80)$ $(0.58, 0.79)$	$(0.98, 0.21)$ $(0.98, 0.18)$ $(0.97, 0.24)$
R_4	$(0.54, 0.81)$ $(0.41, 0.85)$ $(0.39, 0.91)$	$(0.99, 0.12)$ $(0.94, 0.30)$ $(0.67, 0.68)$	$(0.67, 0.60)$ $(0.47, 0.86)$ $(0.46, 0.73)$	$(0.51, 0.62)$ $(0.22, 0.84)$ $(0.20, 0.89)$	$(0.48, 0.83)$ $(0.45, 0.88)$ $(0.40, 0.86)$

Table 4. Similarity measures between robots and situations.

$Sim(R_i, c_j)$	c_1	c_2	c_3	c_4	c_5
R_1	**0.9231**	0.7939	0.7474	0.8828	0.8459
R_2	0.8071	0.8339	**0.9389**	0.8262	0.8615
R_3	0.8192	0.8197	0.8536	0.8327	**0.9477**
R_4	0.8636	**0.9746**	0.9354	0.8678	0.8504

The point having the maximum value of the similarity measures yields the location of the robot. Hence, the robot R_1 is near some fire spot, R_2 is bumped, R_3 experiences jolts/shocks and R_4 is near an obstacle.

Now, we use the suggested distance metric to resolve Example 4.1. But first, we provide the following proposal for Algorithm 2. Algorithm 2 uses a distance metric to reach a final decision. The distance between the actual situation measurements and the recorded measurements of the robot sensor is calculated. The less separation between any specific actual situation measurements, the closer the robot is.

Algorithm 2 Distance measures based algorithm

S-1: Steps 1–3 are same as in Algorithm 1.
S-4: Calculate the distance between robots & situations.
S-5: The robots & scenarios that are closest together are the best match.
S-6: Present the outcomes in layman's language.

Example 4.2. The calculated and tabulated distances between each robot and the situation under consideration are shown in Table 5.

Table 5. Distance measures between robots and situations.

$D_m(R_i, c_j)$	c_1	c_2	c_3	c_4	c_5
R_1	**0.1604**	0.3370	0.3795	0.2065	0.2758
R_2	0.3933	0.3715	**0.2154**	0.3805	0.3702
R_3	0.3892	0.4345	0.3633	0.3830	**0.2187**
R_4	0.3202	**0.1338**	0.2119	0.2999	0.3599

The point having the minimum value of the distance measures yields the accuracy of the robot. Hence, the robot R_1 is near some fire spot, R_2 is bumped, R_3 experiences jolts/shocks and R_4 is near an obstacle.

Finally, we use the proposed correlation measure to solve Example 4.1. As we did before, we start by suggesting the following Algorithm 3. Algorithm 3 uses a correlation measure to reach a final decision. The correlation between the actual situation measurements and the recorded measurements of the robot sensor is calculated. The closer they are, the greater the correlation between any specific actual situation measurements and the robot.

Algorithm 3 Correlation measures based algorithm

S-1: Steps 1–3 are same as in Algorithm 1.
S-4: Calculate the correlation between robots & situations.
S-5: The robots & scenarios pair with the highest correlation measure is the best option.
S-6: Present the outcomes in layman's language.

Example 4.3. Table 6 contains the computations and tabulations of the correlation measurements of each robot to the situation under consideration.

Table 6. Correlation measures between robots and situations.

$Cor(R_i, c_j)$	c_1	c_2	c_3	c_4	c_5
R_1	**0.9275**	0.7024	0.6379	0.8347	0.7795
R_2	0.7194	0.7512	**0.9159**	0.7465	0.7868
R_3	0.7060	0.6936	0.7613	0.7214	**0.9229**
R_4	0.7617	**0.9589**	0.9027	0.7816	0.7341

The point having the maximum value of the correlation measures yields the accuracy of the robot. Hence, the robot R_1 is near some fire spot, R_2 is bumped, R_3 experiences jolts/shocks and R_4 is near an obstacle.

The chart depicting the three comparison measures between R_i and c_j is exhibited in Figure 2.

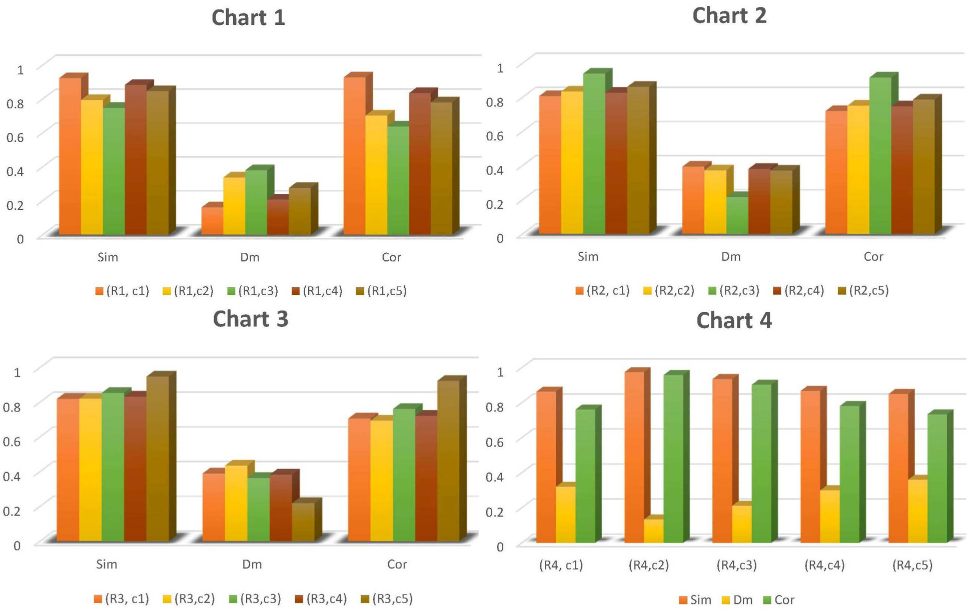

Figure 2. comparison measures between R_i and c_j.

We note that all three of the provided comparison measures produce the same result, confirming the validity of the offered measures.

5. Application of proposed comparison measures in building movie recommender system

A large class of machine learning processes that make pertinent recommendations to users are known as recommender systems. Netflix, Amazon, Youtube and other services operate on recommendation systems, which propose the following movie or product based on the viewer's prior activity (often referred to as content-based filtering) or based on the behaviors and preferences of other users who share your interests (called collaborative filtering). Similar to this, Facebook uses a recommended system to suggest Facebook individuals you might know offline based on your interests, activities, career, etc. The material or the individuals who access the content is what recommendation systems base their recommendations on. A basic overview of the operation of content-based filtering is shown in Figure 3.

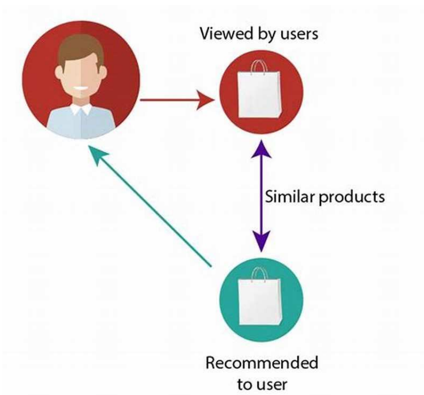

Figure 3. Working of content based filtering: medium.com.

In this section, we'll develop a machine learning technique that will recommend movies depending on a viewer's preference for a certain film, based on the provided comparison measurements.

There may be many features for lining up the recommended list, e.g., keywords, genres, title, language, production country, production group, runtime, cast, release date, popularity, vote count and director etc. Assume that among all of these features, the ones in which we are looking for similarities to make the following recommendation:

$$f_1 = \text{cast},$$
$$f_2 = \text{genres},$$
$$f_3 = \text{keywords, and}$$
$$f_4 = \text{vote count}.$$

A viewer who loves a romantic movie will most likely to watch another romantic movie. Another spectator might enjoy seeing his favorite actors in the film's cast. Others may adore films with high vote counts. Combining all of these traits, our four features that made the cut are sufficient to instruct our recommendation system.

After using the proposed comparison measures, the next stage is to print the similar movies utilizing the movie user likes–the last fragment of the project.

We elaborate the notion in the forthcoming example.

Example 5.1. Table 7 shows the readings about a particular viewer, monitored thrice, in the format $(\mu^{(j)}, \nu^{(j)})$, $j = 1, 2, 3$. M_1, M_2 and M_3 is the list of three movies.

Table 7. P*m*FS of movies vs features.

Movies	f_1	f_2	f_3	f_4
	$(0.59, 0.34)$	$(0.21, 0.78)$	$(0.67, 0.25)$	$(0.43, 0.08)$
M_1	$(0.68, 0.54)$	$(0.58, 0.56)$	$(0.28, 0.11)$	$(0.39, 0.34)$
	$(0.82, 0.15)$	$(0.37, 0.09)$	$(0.54, 0.16)$	$(0.28, 0.35)$
	$(0.46, 0.13)$	$(0.33, 0.54)$	$(0.73, 0.26)$	$(0.22, 0.53)$
M_2	$(0.56, 0.69)$	$(0.47, 0.11)$	$(0.72, 0.16)$	$(0.36, 0.11)$
	$(0.21, 0.90)$	$(0.20, 0.22)$	$(0.64, 0.42)$	$(0.51, 0.61)$
	$(0.48, 0.43)$	$(0.35, 0.14)$	$(0.66, 0.19)$	$(0.54, 0.26)$
M_3	$(0.50, 0.56)$	$(0.27, 0.19)$	$(0.82, 0.41)$	$(0.51, 0.38)$
	$(0.45, 0.68)$	$(0.39, 0.57)$	$(0.41, 0.27)$	$(0.24, 0.72)$

Table 8 shows the readings about a particular random movie stored in the database based upon the past activity of the user.

Table 8. PFS of a particular random movie.

Movies	f_1	f_2	f_3	f_4
M	$(1, 0)$	$(1, 0)$	$(1, 0)$	$(1, 0)$

Table 9 gives values of the proposed comparison measures between the movie M_α $(\alpha = 1, 2, 3)$ and the movie M taking into account the feature f_β $(\beta = 1, \cdots, 4)$.

Table 9. Comparison measures between M_α and M.

Comparison measure	(M_1, M)	(M_2, M)	(M_3, M)
Similarity measure (Sim)	**0.7586**	0.6703	0.7071
Distance measure (D_m)	**0.4362**	0.4897	0.4764
Correlation measure (Cor)	**0.9121**	0.7373	0.8408

Hence, the rankings so obtained are given in Table 10 and depicted in Figure 4.

Table 10. Rankings obtained by proposed Comparison measures.

Comparison measure	Ranking
Similarity measure (Sim)	$M_1 \succ M_3 \succ M_2$
Distance measure (D_m)	$M_1 \succ M_3 \succ M_2$
Correlation measure (Cor)	$M_1 \succ M_3 \succ M_2$

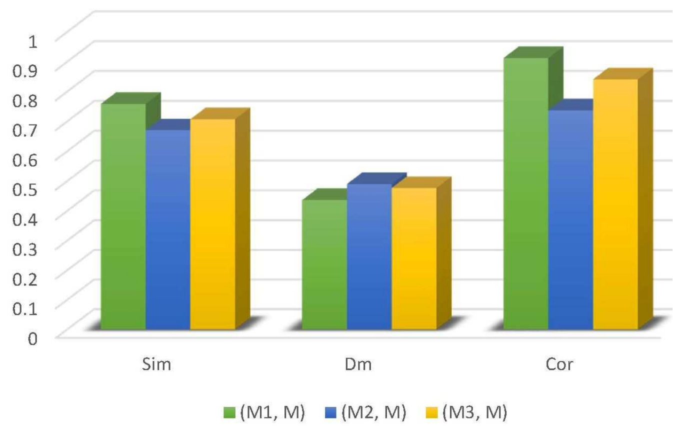

Figure 4. Rankings of M_α.

In view of above computations, it may be inferred that the movie M_1 best matches with the choice of the viewer. The second choice is M_3 and the third one is M_2.

Comparison analysis:

P*m*FSs were defined, and the TOPSIS method was proposed by Naeem et al. [15]. Naeem et al. [17] focused on P*m*FS relations and the extension principle for P*m*FSs. Riaz et al. [18] established weighted aggregation operators for P*m*FSs. In all previous studies, there was no work on similarity, distance, and correlation measures. The problem of judging the robotics from their sensor readings and movie recommendation systems cannot be solved by previous methods developed for PmFS. Thus, our method is superior in this regard because no previous method had the ability to solve these issues in P*m*FS environment. We also have the advantage of working in the more general P*m*FSs environment. This can be seen as: Robots send the message to the controller by means of sensors. It's not good to rely on a single reading. The robot sends multiple values after a suitable time interval. These multiple readings appear to be the P*m*FVs. In this situation, PFSs are ineffective.

6. Conclusions

In this paper, novel similarity, distance, and correlation measurements in a Pythagorean m-polar fuzzy environment are proposed along with some of their special characteristics. The suggested actions improve the methods for determining how similar two PmFSs are. The recommended metrics range from 0 to 1, eliminating the low similarity grade. The comparison measurements provided in this paper offer enormous potential for further research from an analytical beyond application standpoint. The idea may be skillfully used to manipulate uncertainty in a variety of real-world fields, most notably trade and business analysis, economics, voice recognition, coding theory, marketing, artificial intelligence, water management problems, image processing, transportation problems, speech recognition, agri-farming, robotics, pattern recognition, recruitment issues, forecasting and life sciences. In the future, we will extend the TOPSIS method [14], VIKOR method [21], MULTIMOORA method [23], ELECTRE method [27], divergence measures [28], and ELECTRE-I approach [29] for PmFSs.

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Conflict of interest

The authors have no conflicts of interest to declare.

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