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Review

A review on portfolio optimization models for Islamic finance

Doong Toong Lim[1,*], Khang Wen Goh[2,*] and Yee Wai Sim[3]

[1] Faculty of Computing and Engineering, Quest International University, Perak, Malaysia
[2] Faculty of Data Science and Information Technology, INTI International University, Nilai, Malaysia
[3] Faculty of Innovation and Technology, Taylor's University, Subang Jaya, Malaysia

* **Correspondence:** Email: doongtoong.lim@qiu.edu.my, khangwen.goh@newinti.edu.my.

Abstract: The era of modern portfolio theory began with the revolutionary approach by Harry Markowitz in 1952. However, several drawbacks of the model have rendered it impractical to be used in reality. Thus, various modifications have been done to refine the classical model, including concerns about risk measures, trading practices and computational efficiency. On the other hand, Islamic finance is proven to be a viable alternative to the conventional system following its outstanding performance during the financial crisis in 2008. This emerging sector has gained a lot of attention from investors and economists due to its significantly increasing impact on today's economy, corresponding to globalization and a demand for a sustainable investment strategy. A comprehensive literature review of the notable conventional and Islamic models is done to aid future research and development of portfolio optimization, particularly for Islamic investment. Additionally, the study provides a concisely detailed overview of the principles of Islamic finance to prepare for the future development of an Islamic finance model. Generally, this study outlines the comprehensive features of portfolio optimization models over the decades, with an attempt to classify and categorize the advantages and drawbacks of the existing models. The trend of portfolio optimization modelling can be captured by gathering and recording the problems and solutions of the reviewed models.

Keywords: portfolio optimization; Shariah-compliant portfolio; portfolio selection; Islamic finance; diversification
Mathematics Subject Classification: 90C90, 91G10, 91G70

1. Introduction

In this era of globalization, the awareness of investing is gradually increasing as its importance as a source of income has surged following the advancement of technology. Portfolio investment is highly associated with the concept of diversification, whereby proper management of a portfolio comprising of different assets can result in a lower risk level as compared to investing in the respective individual assets. Portfolio investment differs a lot from individual stock investment, mainly due to its involvement of diversification. It allows investors to adjust their preferred risk and return levels, providing a relatively flexible investment. Generally, portfolio investment refers to investing in a collection of financial assets with the expectation of gain either in terms of return or value appreciation over time, subject to different level of risk tolerance. With the vast number of choices following the different combinations of assets, the complexity of selecting the desired portfolio is significantly increased. In contrast to investment in stocks alone that only requires the choice of assets to be invested, portfolio requires the consideration of assets proportion to form a desired portfolio. Moreover, the characteristics of the individual assets in a portfolio can differ a lot from the characteristics of the portfolio as a whole [1].

Before 1952, diversification was merely a concept in the world of finance. Though its importance was clearly known between economists, no mathematical reasonings or precisions were provided to justify its significance. At that moment, the risk associated with an investment was rather undefined and unquantified, whereby the relationship between risk and return was treated casually [2]. The investment strategy used before 1952 can be generalized as a betting strategy, whereby investors tend to expect a high return from the single asset without considering the associated risk. Clearly, this strategy does not signify any superiority in diversification. In 1952, Harry Markowitz introduced a revolutionary analytical construct of the relationship between risk and return in investment, thereby initiated the era of modern portfolio theory (MPT) [3]. The Mean-Variance (MV) model developed by Markowitz proposes a trade-off between risk and return, whereby a balance between these two elements is emphasized. Also, risk was defined by Markowitz as the performance of the portfolio. While diversification was inferred to a large number of securities contained in the portfolio prior to the era of MPT, the MV model considers diversification for a wide range of portfolio risk and return, further examining the covariances between securities. This ground-breaking approach had become a milestone for portfolio management, earning Markowitz the renowned Nobel Prize in 1990.

On the other hand, Islamic finance has evolved to be prominent in today's world, serving as one of the fastest growing segments [4]. Muslims held a large proportion in the world population by having Islam as the second largest religion in the world. Holding nearly a quarter of the world population, the growth of Islamic finance has geared up by approximately 20% annual growth in the last few years [5]. The rise of Islamic finance is due to its distinctive characteristics against unethical activities, interest payment, excessive risk and gambling, thereby successfully attracted the attention from investors including non-Muslims. Precisely, the importance of Islamic finance arose to the global financial system after the global financial crisis in 2008. The Islamic finance industry thrived during the financial crisis unlike the economies of other nations, by recording an assets growth of 29% in 2008 [4]. The observable stability and performance of the Islamic finance industry had drawn a lot of attention from policymakers and investors around the world. Also, there was an observable increased interest of investors to invest in Islamic funds after crisis due to its superiority in diversification and stabilization [6]. Correspondingly, Islamic finance is booming as a core component in the global financial system.

From the Islamic perspective, the ownership of everything belongs to Allah SWT, while humans hold the responsibility as a trustee or custodian (*khalifa*) by conducting life in accordance with Allah SWT. In addition, Islam embraces all aspects of life and concerns about social and spiritual aspects of His creations. Therefore, religious Muslims are recognized to integrate their beliefs into their daily dealings [7]. The distinctive principles of the Islamic finance do not aim to refrain Muslims from investing. Instead, Allah SWT encourages the acquisition and accumulation of wealth to achieve success in this world and the hereafter (*al-falah*), within the economic framework as prescribed by *Shariah*:

"He it is Who made the earth smooth for you, therefore go about in the spacious sides thereof, and eat of His sustenance, and to Him is the return after death." (Qur'ān, 67:15)

The benefits of employing the Islamic economy, include the reduction of economic disparity, expansion of market, promotion of transparency, prevention of economic crises and the enhancement of economic development [8]. The high potential of Islamic finance in attracting investment is particularly substantial for developing countries by diversifying their economies that are of high dependence on natural resources, apart from building up their development strategies [9].

The current form of Islamic finance is contestably imitating the conventional finance to survive particularly in this modern economic world. The operations of the Islamic banking and finance system are based on the second principle that associates with alterations to fit into the current situation [10]. Similarly, the modifications of the ideal *Shariah* rulings are acknowledged to be due to the relatively low acceptability level of the public towards a new form of market. However, these constraints on the Islamic products had restricted the devout of Muslims who seek to adhere truthfully and wholeheartedly to the commands of Allah SWT. The unfulfilled provisions in Islamic finance have rendered Muslims a reduced engagement in trading due to the lack of confidence. While trade is generally dealt among the Western and Islamic countries, it is crucial for all participants in the economy to recognize and have a better understanding on the beliefs and convictions of Muslims for better trade and socio economy [11].

This study aims to review both the conventional and Islamic portfolio optimization models while providing corresponding insight into the principles and guidelines for Islamic finance. The remainder of this paper is structured as follows: Section 2 devotes to the prominent *Shariah* principles in Islamic finance; Section 3 presents the notable developments in conventional portfolio optimization models, followed by Section 4, which explains the development of the Islamic portfolio optimization model sought in the literature; Section 5 focuses on the recent developments of portfolio optimization; Section 6 concerns about the dynamic relationship between markets/assets and hedging properties; lastly, the paper ends with conclusions.

2. Shariah principles for Islamic finance

In the Islamic world, all aspects of life have to be carried out in accordance with *Shariah* or the Islamic law. *Shariah* is the legal framework of Islam derived from Qur'ān and the teachings of Sunnah that has to be strictly followed by Muslims in their decision-making process [12]. Qur'ān plays a significant role in regulating the economy of the *ummah* (community) as *Shariah* is Allah SWT-centric. While the primitive rulings of *Shariah* are clearly defined, some rulings that involve daily dealings are often subject to a variety of interpretations. This is mainly due to the imperfectness of humans in understanding the flawless *Shariah*, thus, the derivation of the *Shariah* rulings is a perpetual work. In other words, Islamic economic regulations can evolve over time [13].

On the other hand, *fiqh* refers to the normative interpretation of *Shariah*, where the guidelines and regulations are derived by Islamic jurists, and is flexible and changeable with development, also referred to as the Islamic jurisprudence. The claim that an Islamic product is *Shariah*-compliant suggests that the product is perfect and immutable, which is rather unrealistic. While the *Shariah*-compliant products are ought to be *fiqh*-compliant, the simplification of the category into *Shariah*-compliant is only to better distinguish an Islamic product from a conventional product [14]. Moreover, the constitutional principle of "Everything which is not forbidden is allowed" pertains in the Islamic finance.

Apart from the practicability of commercial transactions, it is imperative for Islamic finance to take into consideration social and moral issues, such as social justice, equity, and fairness [14]. The distinctive characteristic of Islamic finance is the reason of its progressive viability in the global financial system. The fundamental principles of Islamic finance are the prohibition of *riba*, *gharar* and *maysir* [15]. This section focuses on the current practices of the Islamic stock market, whereby the trading rules and regulations are outlined.

2.1. Prohibition of riba

Commonly known as interest or usury, *riba* is a broad concept mainly referring to increases, excess, surplus or growth. As the term was not precisely defined leading to a wide range of interpretations, Iqbal and Mirakhor [16] concluded *riba* as the action of charging interest or a predetermined payment in excess of the principal amount. The concept of an unequal exchange was seen as a characteristic of the capitalist society, strongly condemn by the anti-capitalist Muslim author, Haque [17]. While prohibiting interest, Islam encourages profits earning. The reason behind this rationale is that profits are determined ex post, resulting from the successful creation of wealth through entrepreneurship. In contrast, interest is determined ex ante, representing an accrued cost regardless of business performance, and no earnings could be obtained in the case of business losses [16]. Therefore, a guaranteed return without considering the performance of the investment would also be treated as *riba* [18]. This is because a guaranteed return may indicate an effortless gain and would lead to ungratefulness and selfishness in human beings. *Riba* was deemed to diminish humanity, whereby it evokes the greediness of human to receive more interest without an end.

Moreover, *riba* leads to "money makes money". In the Islamic world, money is treated as an exchange medium with no intrinsic value, whereby money acts as a standard of value or a unit of measurement. Thus, investments must be based on an underlying asset. Incurrence of an excess payment over the principal amount for a deferred payment indicates a charge against time alone. This situation can specify a scenario where money is exchanged for money. Furthermore, returns are to be earned by risk-taking and must be based on the asset performance, whereby the passage of time alone does not satisfy the ruling [5]. Based on the Hadith, the exchange of *ribawi* items is only permissible when held on spot and in equal amount, whereby *ribawi* items refers to substances sold by weight and measure, taking currency into account. Nonetheless, the prohibition of *riba* protects investors from unjust transactions. As Islamic finance employs the idea of risk-sharing, it is important to ensure that no single party gains over the losses of the other. In view of the prohibition of *riba* and its reasonings, Islamic finance allows only transactions with direct participation in asset performance while entitling investors to obtain correspondent return that is state-contingent based on a pre-agreed date and amount corresponding to the investment performance [19]. In a different manner, an investor is only allowed for a pro-rata profit that is truly earned by the fund, whereby the loss suffered by the fund is also borne by the investor [20].

Overall, the modern theory revolving around *riba* categorized the prohibition into two broad categories, namely *riba al-nasia* and *riba al-fadl* [17]. *Riba al-nasia* regards *riba* as the deferment caused by deferred transaction with the inclusion of interest payment in the form of a predetermined payment. Whereas *riba al-fadl* refers to an excess or additional amount paid in the direct exchange of commodities. In addition, *riba al-jahiliyyah* is classified by many scholars as a form of *riba al-nasia*, while a separate category for the others. *Riba al-jahiliyyah* refers to a penalty imposed on a default that was not stipulated in advance. Nonetheless, there exists two different views on *riba*, namely the modern and conservative views. The modernists extended several arguments on the actual indication of *riba* in the Qur'ān, with the attempt of yielding a more liberal regulation in the economic dealings. Despite the efforts, the arguments were strongly resolved by ample of *Shariah* references [21].

2.2. Prohibition of gharar

Gharar can be referred to as risk, uncertainty, hazard and ambiguity. Various definitions of *gharar* relate to a doubtful transaction, whereby the existence or information of the transacted item is uncertain, unknown, incomplete or inaccurate [12]. One of the core rationales behind Islamic finance is to condemn injustice between the contracting parties, thus, no single party is allowed to have advantages over the other. The concept of risk-sharing is employed rather than risk-transfer as in conventional finance. All transactions in Islamic finance must be transparent with full disclosure of necessary information to avoid or reduce the risk of information asymmetry and moral hazard [5]. However, it is known that some degree of uncertainty is present in the reality, thereby separating *gharar* into two categories, making this prohibition not absolute. *Gharar Fahish* refers to major *gharar* that is intolerable and is strictly prohibited in Islamic finance. On the other hand, *Gharar Yasir* refers to minor *gharar*, whereby slight uncertainty including gift or bequest, and inevitable transaction for the public can be tolerated, thus, would not deem a contract invalid. In addition, *gharar* is associated with the factor of ownership. A transaction without complete asset ownership is deemed as *gharar* since it violates the basis of right and involves uncertainty, mainly due to the concerns of possible factors that could render the asset undelivered, and to prevent any dispute between the transacting parties.

As the sale of items not owned is deemed as *gharar*, short selling is forbidden in Islamic finance. Also, the fact that investors short sell with the expectation of a price drop in asset is regarded as speculation (*maysir*). Nonetheless, the return that is gained from lending the asset can be deemed as *riba*. Although the primary features of short selling seemed clearly prohibited according to Islamic principles, the permissibility of short selling is highly argumentative. To enhance market competitiveness and efficiency, the *Shariah* Advisory Council (SAC) has resolved that Regulated Short Selling (RSS) is *Shariah*-compliant with the inclusion of a strict securities borrowing and lending (SBL) regulations during SAC's 96th meeting on 18th April 2006 [22]. According to a reported tradition, Prophet Muhammad SAW once involved in a forward contract where the exchange of commodities was not on spot [23]. Thus, several Islamic jurists have allowed this type of transactions, while the SAC has legalized it using MGII that gives a similar effect as the RSS during its 177th meeting on 20th June 2017, subjected to three conditions, such that the contract has to be structured based on the bilateral binding promise concept, the promise is only entitled to the actual loss suffered in the event where the promisor failed to execute the promise and that the MGII and its counter-value has to be delivered on the pre-agreed date [24].

Moreover, *qabd* often refers to possession, receive and accumulation [25]. According to the Hanafis, *qabd* is a subsidiary condition but not a mandatory requirement for sale, whereby it is Islamically lawful to postpone the delivery if the transaction does not involve any *ribawi* items [22]. On the other hand, Ibn Taymitah provides a more liberal definition for *qabd* for the precise meaning of *qabd* is subject to indefinite prevailing customs. According to Ibn Taymiyah, Ibn al-Qayyim, and Imam al-Shafii, the Hadith focuses on the seller's ability to deliver rather than the possession at the point of sale [26]. Although the element of *gharar* could be eliminated by adopting the respective interpretations, the element of *riba* from the return gained from lending the asset could hardly be resolved.

2.3. Prohibition of maysir

Maysir can be referred to as gambling, games of chance or speculation. *Shariah* prohibits *maysir* as it involves high risk and inequity, whereby winners win on the expense of others [12]. Huge losses may be suffered by the losers and may eventually lead them to bankruptcy. The course of transactions that involve *maysir* brings no societal benefits and are associated with extremely adverse outcomes. In this regard, transactions involving *maysir* such as derivatives that act as a zero-sum game without involving any real economic activity are forbidden in Islamic finance [15].

2.4. Haram and unethical activities

Several activities besides interest-bearing, highly uncertain, and speculative activities are deemed impermissible in *Shariah*. Islam prohibits activities involving alcohol, pork, tobacco and other activities that are deemed unethical, such as prostitution. The stated activities are deemed as *haram*, whereby an income sourced from them is unlawful according to *Shariah*. Therefore, Muslims are not allowed to invest in businesses which primary operations are based on *haram* activities.

2.5. Purification

Zakat is defined as purification or growth, serving as one of the five pillars of Islam [27]. It refers to an obligation of Muslims to donate a certain proportion of wealth to the poor and needy annually, provided that a minimum level of wealth known as *nisab* is met. Nowadays, businesses are progressed to be multifaceted and intertwined resulting in a multisource of income which may include forbidden sources. In the opinion of Nagaoka [28], the newly introduced Islamic finance products do not ideally match with the Islamic law since they are merely a result of patchwork screenings by the SAC. These justifications had raised questions regarding the *Shariah* compliancy especially for those who wish to adhere fully to *Shariah*. Accordingly, *zakat* deems to purify the tainted income.

Apart from income purification, *zakat* holds moral and social aspects in Islamic finance [29]. In addition to its nature to purify haram aspects and human salvation, such as greediness and envy, *zakat* is derived from the rationale of Islam, namely, to condemn injustice and inequality. The redistribution of income helps to prevent a morbid accumulation of wealth in the rich while playing its part in reducing poverty by helping the less fortunate. Overall, the advantages of *zakat* include the elimination of poverty, purification of soul, expression of sincerity of faith and the establishment of social justice and security through redistributing wealth, so to reduce inequality and to reconcile the gap between the poor and the wealthy [30]. In regard to the global financial system, the application of *zakat* can help to promote a healthy wealth circulation, thus, providing a more efficient market. Moreover, the non-payment of *zakat* is forbidden in Islam, whereby Allah SWT strongly condemns those who ignore *zakat* and regards them as sinful. The severe consequences of the non-payment of *zakat* are specified in the Hadith [31]:

"Whoever is made wealthy by Allah and does not pay the *zakat* of his wealth, then on the Day of Resurrection his wealth will be made like a bald-headed poisonous male snake with two black spots over the eyes. The snake will encircle his neck and bite his cheeks and say, "I am your wealth, I am your treasure"".

Zakat for wealth and possessions are classified into several categories, whereby this study will focus on *zakat* for stocks. The *zakat* rate is 2.5% calculated on the market value of share and is payable once in a lunar year [23]. Different calculations are prescribed to different holding types, such that *zakat* for share that is continuously owned until the end of one lunar year is 2.5% of the lowest share price, while *zakat* for share without continuous possession for the whole year is 2.5% of the selling price after deducting the purchasing cost [29]. Furthermore, different investment natures are subject to different *zakat* calculation at individual level [23]. For short term investors who hope for an appreciation in value and is ready to sell the share in an immediate future, *zakat* is 2.5% on the full market value each lunar year. In contrast, the *zakat* calculation for long term investors who aim for dividend gain is based on the dividend amount each year, along with a one-time *zakat* payment to be made on the full selling price at the time of transaction. A slightly different calculation is exercised in Bursa Malaysia [32], whereby the *zakat* calculation for a short-term investor is 2.5% on the disposal gain, whereas for a long-term investor is 2.5% on the value of shares plus dividend, both after the deduction of the transaction costs.

3. Conventional portfolio optimization models

In 1952, an asset allocation theory introduced by Harry Markowitz initiated the era of MPT. Generally, the model assembles portfolio selection from various assets by considering the investor's risk tolerance or desired return. Despite the innovative strategy introduced, there appear to be discrepancies in the suitability of the risk measure used by Markowitz, resulting in the development of different models and risk measures in the field. There have been numerous studies to investigate the most well-suited framework for portfolio selection. The notable portfolio optimization models are presented in Table 1. According to the advanced search engine in Google Scholar, four high-frequency models reported in the table, in addition to a few direct extensions from the standard model, will be discussed in the following subsections.

Table 1. Notable conventional portfolio optimization models.

Author(s)	Model/ Framework	Problem Type	Risk Measure or Equivalent	Key Feature
Markowitz [3]	Mean-Variance Model	Quadratic	Variance	Risk (variance)-return (mean) trade-off
Markowitz [33]	Mean-Semi-Variance Model	Quadratic	Semi-Variance	Downside risk
Konno and Yamazaki [34]	Mean-Absolute Deviation Model	Linear	Absolute deviation	Linear program
KonnoShirakawa and Yamazaki [35]	Mean-Absolute Deviation-Skewness Model	Linear	Absolute deviation	Linear program with the consideration of skewness
Konno and Suzuki [36]	Mean-Variance-Skewness Model	Non-linear	Variance	Consideration of skewness
Jorion [37]	Value-at-Risk	Non-linear	Value-at-Risk	Downside risk
Young [38]	Minimax	Linear	Minimum return	Linear program
Young [38]	Maximin	Linear	Minimum return	Linear program
Rockafellar and Uryasev [39]	Conditional Value-at-Risk	Linear	Conditional Value-at-Risk	Tail risk

3.1. Mean-variance (MV) model

MV model is the first portfolio selection model pioneered by Harry Markowitz in 1952 [3]. It is a quadratic model, taking mean or expected return as a measure of return, and variance and covariance as the risk measures. The model is formulated as:

Minimize

$$\sum_{i=1}^{n}\sum_{j=1}^{n} x_i x_j \sigma_{ij} \tag{1}$$

Subject to

$$\sum_{i=1}^{n} x_i r_i \geq R \tag{2}$$

$$\sum_{i=1}^{n} x_i = 1 \tag{3}$$

$$0 \leq x_i \leq 1 \quad , \quad i = 1, \ldots, n, \tag{4}$$

where x_i indicates the amount invested in *i*th security; r_i is the return of the *i*th security in the next period; σ_{ij} is the covariance between r_i and r_j; *R* is the desired level of return; *n* is the number of securities. For r_{it} representing the return of the *i*th security at time *t*, the covariance, σ_{ij}, is given as:

$$\sigma_{ij} = \frac{1}{T}\sum_{t=1}^{T}(r_{it} - \bar{r}_i)(r_{jt} - \bar{r}_j). \tag{5}$$

The MV model is a single-objective optimization model that aims to minimize portfolio risk subject to three linear constraints, solved by a quadratic programming method. The model prohibits short-sell with the inclusion of long-only constraints, whereby each asset is not allowed to hold negative weightage in the portfolio, in addition to the limitation of total asset weights of 1. The latter constraint also implies total investment from the available funds.

Every combination of securities can be plotted against a return-risk graph, whereby portfolios with the best combination of risk and return, known as the efficient portfolios, can be found on a boundary known as the efficient frontier. The set of portfolios lying on the efficient frontier can be selected flexibly according to investor preferences, such that each efficient portfolio represents a portfolio with the highest possible level of return with the associated risk level. The MV model is based on a few key assumptions:

1) Investors are risk averse, indicating that, given two securities with the same expected return, the security with a lower variance will be preferred.
2) All investors consist of the same one-period investment horizon.
3) Investors are aware of their desired risk and return level.
4) Rate of return follows a multi-variate normal distribution.
5) Investor utility is a quadratic function.

3.1.1. Limitations of the MV model

Numerous critics were received responding to the assumptions of the MV model, regarding them as impractical. First, investors have different risk preferences, including risk-seeking investors who work contrary to risk averse-investors. Second, contrary to the assumption that it is a one-period problem indicating no opportunity for portfolio composition adjustment during the horizon, the composition of a portfolio with risky assets varies over time due to the random outcomes of the asset returns. Hence, portfolio composition adjustments are crucial so that an efficient portfolio can be present for each subperiod in the horizon. Moreover, investors may inconsistently change their risk and return preferences, constituting different investment horizons depending on their investment decisions.

Next, not all investors are aware of their risk and return level, making it unrealistic to obtain an optimal selection for all investors without prior information about their preferences [40]. According to Konno and Suzuki [36], neither assumption 4 or 5 holds, whereby real stock data are not multivariate normally distributed, nor do the investors have quadratic utility model as most of the investors do not purchase “efficient” portfolio. According to Konno and Yamazaki [34], the MV model was not widely used by practitioners, especially in the case of optimizing large-scale portfolios, despite its theoretical reputation. Several drawbacks of the MV model were listed by the authors:

- Computational burden: To solve a large-scale quadratic programming model with a dense covariance matrix is rather tedious, and can be hardly solvable on a real time basis. As $n(n+1)/2$ constant(s) are required to build a model, solving such a dense quadratic programming model is very computationally demanding when n is large.
- Investors' perspective on risk: The use of variance or standard deviation as the risk measure did not convince many practitioners as investors' risk preference is non-symmetrical around the mean. Typically, investors are only concerned about negative or small returns, which can be classified as the downside risk. Standard deviation that measures both the positive and negative gains might contradict with investors' perspective since a positive deviation from the mean is generally preferred.
- Distribution of returns: r_i is not normally distributed or symmetrical. Hence, apart from the first and second moments of the distribution, namely mean and variance, a third moment of the distribution can be added. Security returns in the real financial world are not normally distributed but, rather, asymmetrically distributed [41]. The skewed distribution has rendered variance to be an ineffective risk measure by underestimating risk when an amount of unfavorable risk is penalized by the favorable upside deviation.
- Transaction/ management cost: The MV model suggests the purchase of a large number of different stocks to achieve diversification. However, purchasing and managing such a portfolio is often expensive, as well as computationally difficult [42]. Moreover, portfolio revision that exists in most applications of portfolio optimization involves transaction cost. Also, it was found that an MV model subject to transaction cost achieved superior performance when compared to the original MV model [43].

Another major drawback of the MV model includes its ignorance on the uncertainty associated with the input parameters, namely estimation error. While the inputs, namely mean, variance, and correlations are calculated from the historical data, the resulted optimal portfolio could only represent an approximation of a true optimal portfolio since the selection was made ex-ante without considering the associated estimation error [44]. This factor has rendered the computation of the efficient frontier unpractical. Besides, Pedersen and Satchell [45] concluded the basic properties for financial risk measures, including non-negativity, homogeneity, sub-additivity and shift-invariance. As a result, variance does not satisfy the proposed properties. As claimed by Cox [46] who analyzed the appropriateness of using variance as a risk measure, the author observed that other risk measures, such as the probabilities of different consequences, were introduced more frequently to users than to variance due to their flexibility. On the other hand, Bagheri [47] argued on the necessary consideration of multiple goals into the investment decisions since investors practically tend to consider several conflicting goals, particularly to maximize returns and minimize risk, as well as to optimize liquidity. Given these known limitations and numerous growing alternatives, variance is still popularly used as a risk measure due to its simplicity and convenience [48].

3.1.2. Direct extensions from the MV model

The MV model introduced by Markowitz presented the mean-risk concept for portfolio optimization, in which different risk measures are adopted in place of the variance for various objectives. The work of Markowitz has stimulated a body of literature in portfolio optimization using

mean-risk analysis as the base model. The mean-risk analysis is generalized according to two principles. The first formulation aims to minimize risk with a lower bound on the expected return, whereas the second formulation selects a portfolio with the maximum expected return with an upper limit on the portfolio risk. The two mean-risk models are expressed as:
Formulation 1:
Minimize

$$\rho(r_p) \tag{6}$$

Subject to

$$\sum_{i=1}^{n} w_i = 1 \tag{7}$$

$$\sum_{i=1}^{n} w_i r_i \geq R_* \tag{8}$$

$$w \geq 0 \tag{9}$$

Formulation 2:
Maximize

$$\sum_{i=1}^{n} w_i r_i \tag{10}$$

Subject to

$$\sum_{i=1}^{n} w_i = 1 \tag{11}$$

$$\rho(r_p) \leq R^* \tag{12}$$

$$w \geq 0, \tag{13}$$

where $\rho(r_p)$ is the risk measure, w_i is the weight of asset *i*, r_i denotes the random return of asset *i*, R_* is the lower bound on the portfolio return, and $\boldsymbol{R}^*$ refers to the upper limit on the portfolio risk. The standard mean-risk concept remains the principal of the MV model, such that the problems consider long-only constraints that limit the sum or weights to one while the weightage of each asset must be non-negative, in addition to an equality constraint that requires the investor to use up the total available funds.

Multi-objective portfolio optimization

The risk-return trade-off principle depicts a higher return following a higher risk level and vice versa. However, the MV model could not provide an optimal selection for all investors without prior information about the investment objectives since investors are made up of different risk tolerance, leading to an unrealistic assumption. Therefore, studies considered extending the single-objective MV model into a multi-objective MV model [49]. While it may seem ideal to produce a multi-objective optimization problem, often to simultaneously maximize return and minimize risk, it is in fact too idealistic and impractical to attempt a set of feasible solutions by achieving both objectives at once. The definition of the efficient frontier that presents a set of feasible solutions, according to their respective risk or return levels, shows the computationally impractical desire to reach two extreme objectives at one single point. Besides, the key concept of the mean-risk model is to offer a balanced combination of portfolio risk and return instead of simply selecting the best of both. The explanations are certified following the prior studies that mostly reduce the initial entitled multi-objective functions to a single-objective optimization problem [50,51].

A study by RomanDarby-Dowman and Mitra [52] proposed a multi-objective portfolio optimization model that maximizes portfolio return while minimizing variance and CVaR. To achieve the objectives, the authors suggested two ways to transform the multi-objective problem into a single-objective problem. First, the multiple objective functions are reduced to a single-objective function, whereby the extra variable is transformed into constraints by exerting lower limits. This method is similar to the concept of the mean-risk analysis. Second, scalarization by taking the weighted sum of the objective functions could be used for simplicity [53]. However, the later approach was explored to carry several drawbacks, such as the complex interpretation of the respective weights. The authors concluded that the first method is more meaningful for portfolio optimization.

Transaction costs

The exclusion of transaction costs in Markowitz's model was deemed impractical due to its significant impact on portfolio performance [54]. Aragon and Ferson [55] argued that the neglection of transaction costs might result in sub-optimal portfolios, whereby the actual returns may be lower than the market portfolio after costs. As is commonly known, the sale and purchase of assets incur various costs, such as in the event of portfolio revision because of changing expectations. Perold [56] suggested the application of turnover constraints to avoid unwanted price movement during the revision. Yoshimoto [43] argued that the turnover constraints might only reduce the transaction costs to an endurable level without contributing to the formation of an efficiently optimal portfolio. The author further claimed the incomparable movements between the transaction costs and securities return, thereby proposing a non-linear portfolio optimization model using the MV approach subject to transaction costs. The proposed model with transaction costs results to have a better portfolio performance than the standard MV model. Besides, NorhidayahHanim and Masitah [57] extended the MV and Maximin models by incorporating transaction cost with a rebalancing strategy, and concluded good portfolio performance against the market index.

Other trading concerns

According to Cornuejols and Tütüncü [50], the MV model might result in a portfolio that contains certain assets with absurdly large holdings due to estimation and model errors that prioritize high-return opportunities. Apparently, such portfolios are not well-diversified and are undesirable. A study by Green and Hollifield [58] explored that the unreasonably large weights of assets in an efficient portfolio are due to a single covariance dominating factor that causes high correlations between diversified portfolios resulting from a relatively naive strategy. The authors claimed that a smaller asset universe could create a better setting for diversification. A great deal of previous research into portfolio optimization has suggested adding an upper limit on the individual asset weightage [49,51,57,59]. Commonly known as the holding constraints, the limit on asset weights is expressed by the box-type constraint:

$$a \le w_i \le b \quad \text{for} \quad i = 1, \ldots, n, \tag{14}$$

where *a* and *b* refer to the lower and upper limits on the weight of asset *i*, respectively. On the other hand, Fabozzi, Focardi, Kolm et al. [51] suggested an alternative method to limit the portfolio concentration, carried out post-optimization, in which the resulting assets with smaller weights than a lower limit are eliminated manually after optimizing.

The holding constraint is often related to the cardinality constraint since a limitation on asset weightage would affect the portfolio size [60]. The addition of a cardinality constraint works to limit the portfolio size so that the model complexity can be reduced and result in better diversification performance. With δ_i representing the number of assets in a portfolio, the constraint is described as:

$$L_i \le \sum_{i=1}^{n} \delta_i \le U_i \,, \tag{15}$$

where L_i and U_i are the respective limits on number.

Furthermore, there appear to be discrepancies between the desirable number of assets in a portfolio. Evans and Archer [61] noticed a rapidly decreasing asymptotic relationship between the number of assets and portfolio risk level, implying that diversification might only perform up to a certain number of assets present in the portfolio, supported by the observation of reduced diversification benefits following an increase in the number of assets. Besides, an extremely large portfolio size is associated with high management and transaction costs, thereby increasing the surpass of marginal costs over marginal benefits that result in reduced diversification performance. Statman [62] argued that a well-diversified portfolio should hold at least 30 to 40 stocks. A different opinion by Adamiec and Cernauskas [63] claimed that most industrial professionals hold 20 to 30 stocks to reduce market risk. Although so, the authors concluded an uncertain optimal number of assets to be included in a portfolio for good diversification because of the inconsistent returns from different portfolio sizes, despite observing a significant risk reduction in a portfolio of 25 stocks. A study by Jimbo, Ngongo, Andjiga et al. [64] adopted CVaR while considering cardinality constraints using heuristic algorithms due to the complexity. The authors conducted the study by comparing portfolios with 3, 4, 8 and 8 assets.

3.2. Mean-semivariance (SV) model

Markowitz introduced the use of semi-variance in place of variance to better reflect investors' perceptions on risk [33]. The use of variance is controversial by concerning a dispersion of returns from $-\infty$ to ∞, capturing both the upside and downside risks. On the other hand, semi-variance intends to measure only movements of returns below the expected returns, capturing only the downside risk or the adverse deviations. According to Ballestero [65], semi-variance is suitable for investors who perceive risk as a danger of loss. The SV model is formed by replacing the objective function in the MV model with:

Maximize

$$\sum_{i=1}^{n}\sum_{j=1}^{n} x_i x_j S_{ij} \,, \tag{16}$$

where S_{ij} denotes semi-covariance, expressed as:

$$S_{ij} = \frac{1}{T}\sum_{k=1}^{K}(r_{it_k} - \bar{r}_i)(r_{jt_k} - \bar{r}_j) \tag{17}$$

3.2.1. Comparison to variance

Variance has its advantages when it comes to cost, conveniency, and familiarity. The semi-covariance matrix is asymmetric and endogenous, such that the term changes with a changing portfolio weight, requiring approximately two to four times more computing time than the MV model to derive the efficient sets [66]. Meanwhile, variance is more computationally friendly as it only requires mean, variance, and covariance to derive the efficient sets, rather than the entire joint distribution of returns as in a mean-semivariance model. Also, variance is a simple and well-known measure to most people in various fields. However, this superiority of variance does not rule out the plausibility of semi-variance.

Firstly, the computing cost for semi-variance is small. Moreover, the requirement for using the MV model would sometimes lead to a model similar or equivalent to the entire joint distribution required when using semi-variance. In fact, semi-variance tends to yield a better portfolio than variance, as variance perceives both extremely high and extremely low returns as equally undesirable, seeking to remove both the extremes, thereby violating the general preferences of investors [33]. In contrast, semi-variance that focuses on reducing loss can better reflect investors' preference. In other words, semi-variance measures the risk of loss rather than the volatility of portfolio returns, as in variance.

In addition, Pla-Santamaria and Bravo [67] demonstrated the difference between semi-variance and variance using an example of two lotteries. It is observed that two extremely different lotteries could yield the same expected return and variance, penalizing the function of the MV model to choose the best lottery base on these two elements. In contrast, the desired lottery can be chosen by minimizing the semi-variance. Thus, it is deduced that semi-variance is more efficient than variance for selection purposes. Although, there was no valid evidence of a significant difference in the approximation to expected utility between the two models [33]. Nonetheless, Markowitz based his analysis of semi-variance on variance [68].

3.3. Mean-absolute deviation (MAD) model

Konno [42] proposed a compound L_1 risk (absolute deviation) portfolio optimization model using piecewise linear risk functions to counter the limitations of the MV model. The proposed model can be solved by an easy linear program and is solvable on a real time basis, contrary to the "complex" quadratic program of the MV model.

3.3.1. Advantages of L_1 risk measure

Several advantages over the MV model were concluded:

- Computational ease: The associated problem for the L_1 risk model is a linear program that is a lot more computational friendly than a "complex" quadratic program. The speed of generating a L_1 risk-efficient frontier using an efficient parametric programming code can go up to 75 times faster than using variance [42]. In addition, the model can be easily updated with new data since it does not require the calculation of the covariance matrix [34].

- Equivalency and efficiency when compared to the classical model: Basically, the L_1 risk model is equivalent to the MV model if returns follow a multi-variate normal distribution. This property helps to assure its validity as an alternative risk measure. Moreover, the L_1 risk model is capable of solving problems with a large number of assets on a real time basis in contrary to the MV model.
- Better operationality: The L_1 risk model enables investors to incorporate their subjective perception against risk, for all values of ρ.
- Number of stocks: The resulted portfolio using L_1 risk model suggests significantly lesser stocks to be invested. This is because the portfolio is restricted to contain, at most, $2T+2$ assets regardless of n. Thus, the L_1 risk model can utilize T as a control variable of the number of assets invested. In contrast, the resulted portfolio derived from the MV model may contain a substantial number of stocks associated with the size of n, increasing the complexity and practicality of the model.

It is concluded that the MAD model somewhat generates similar optimal portfolios with similar performance to the MV model. Although, the advantages concluded above had proven the MAD model to be a good alternative to the MV model.

3.4. Value-at-Risk (VaR)

Jorion [37] introduced a new risk measure called the Value at Risk (VaR) to enhance the control of financial risks. VaR is well-known as a single, summary statistical risk measure that summarizes the worst expected loss of a risky asset or portfolio over a specified investment horizon within a predefined confidence level. In simple words, it helps to estimate the maximum possible loss of an investment with a given probability within a set time frame. The use of VaR helps to generate simple reporting conclusion, such as: “There is a 5% chance that the portfolio would experience losses of more than RM500,000 during the half-year time frame”. With VaR at a given tail probability α, where $\alpha \in (0,1)$, also known as VaR at confidence level $(1-\alpha)100\%$ expressed as the negative of the lower α-quantile of the return distribution,

$$VaR_{\alpha}(X) = -\inf_{x}\left[x \mid P(X \le x) \ge \alpha\right] = -F_X^{-1}(\alpha) \tag{18}$$

The expected loss summarized by VaR can be recognized as the exposure of a portfolio to market risk. In the mid-1990s, the Basel Committee on Banking Supervision (BCBS) approved the VaR measure as a desirable approach to evaluating capital reserves required to cover market risk, often requiring that the capital reserve be equivalent to the VaR value times a factor between 3 and 4. Evidently, authorities link VaR to the capital reserves for market risk [69]. However, a more recent study by Lockwood [70] observed a consistent failure of VaR in accurately predicting financial losses suffered in past financial crises, creating doubts about the persistent use of VaR.

Campbell, Huisman and Koedijk [71] developed an optimal portfolio selection model based on the VaR framework. The model was built by maximizing expected returns subject to the constraint that the expected maximum loss meets the VaR limits predefined. It was shown that the VaR model provides an almost identical result to the MV model when the returns are assumed to be normally distributed, since VaR is proportional to standard deviation under a normal distribution. The main difference of the VaR and MV models is the definition of risk, whereby VaR is based on the benchmark return while the MV model is based on variance or standard deviation. In addition, the standard deviation of the optimal portfolio may increase if a risk-averse investor chooses to employ VaR as the risk measure, subjected to the following rules: risk-free borrowing and lending are not allowed; risk-free lending is allowed but risk-free borrowing is not allowed; risk-free borrowing rate is higher than the risk-free lending rate [72].

3.4.1. Advantages of VaR

VaR is a simple and useful summary measure of risk that had gained a lot of attention for its summarization ease. This intelligible risk measure facilitates the summarization of market risk in the form of potential money loss that can be easily understood by fund managers or investors. It serves as a comprehensive risk measure, as it enables the examination of market risk at several levels, including instrument, fund and portfolio levels. Its consistency also helps to compare risk measures across different securities. Moreover, VaR as a risk measure allows the analysis of risk-return trade-off at different confidence levels. This implies that VaR indirectly serves as the investor's risk aversion level since different confidence levels lead to different riskiness. Thus, VaR is useful for building a comparative generalized framework for portfolio selection [71].

3.4.2. Drawbacks of VaR

VaR was rendered a questionable risk metric due to several limitations. First, it is highly sample dependent, mostly depending on the chosen time horizon and confidence level. Different time horizon or confidence level used will result in different VaR values. Generally, a longer time horizon is associated with a less precise efficient frontier. Whereas a greater confidence level is accompanied by a greater portfolio VaR. Also, it is crucial to select a confidence level where the objective of VaR minimization is attainable [72]. This is because the minimum VaR portfolio may not exist with a sufficiently low confidence level while generating the VaR, whereby the mean-VaR efficient set may be empty. The choice of time horizon used holds a larger impact since the VaR using a t-day time horizon is approximately $\sqrt{t}$ times as large as the VaR calculated using a 1-day time horizon. Therefore, VaR values are not comparable across entities without appropriate adjustments for the holding period and confidence level [73]. Although it is concerned that sampling variation will lead to material changes in VaR, it was concluded that the discrepancy may also arise from the fundamental differences in methodologies that could possibly be solved with a good understanding on the methodologies used [37].

On the other hand, for VaR calculated using the variance-only approach, its assumption of normally distributed returns and totally independent returns over time might seem appropriate for return variances and correlations using low frequency data, due to a more acceptable extrapolation range [74]. However, returns are well-known as asymmetric. The assumption of normality can also result in inappropriate asset allocation for risk-averse investors who target a 99% confidence level. The greater the deviation from the normal distribution, the greater the underestimation of risk as the confidence levels increase [71]. Next, VaR does not satisfy the significant property of a risk measure, namely sub-additivity. Artzner [75] claimed that a desirable risk measure should satisfy the property of sub-additivity. Basically, this property indicates that a portfolio made up from asset A and asset B should be less risky than the individual asset A and B. This property makes sure that the principle of diversification holds, whereby a diversified portfolio should always yield lower risk than an undiversified portfolio. Violations of this property is claimed to be problematic for financial institutions, whereby there are possibilities of overestimating or underestimating risk. However, it was shown that the sub-additivity property holds under normal distribution.

Moreover, VaR serves only as an estimation of risk where the effect of estimation error was often unrecognized [37]. VaR acts as an exact measure only when the underlying distribution is measured with an infinite number of observations. However, financial data are only available for a limited time frame. Nonetheless, VaR does not examine the magnitude of loss when the loss exceeds the VaR threshold.

4. Islamic portfolio optimization models

Islamic investment is in a parallel universe as the Socially Responsible Investment (SRI) that refers to an investment style that considers social and environmental issues [76]. While having a parallel concept, Islamic investment differs from SRI that is based on social values, for being a faith-based or Morally Responsible Investment (MRI), whereby investments are carried out in consistent with the respective religious beliefs [77]. The argument that SRI involves the opportunity cost of various profitable stocks through screening was proven to be overconcerned as studies had concluded a relative identical performance between the SRI and conventional investments [49]. Also, Islamic finance consists of constraints that are identical to most ethical constraints apart from the prohibition of debt-related investments [78]. It was concluded that MRI and SRI generally result in a lower expense, since the respective investors have a lower tendency to speculate.

In this regard, Ballestero, Bravo, Pérez-Gladish et al. [79] developed a financial-ethical bi-criteria model that considers financial goal and ethical goal by employing a goal programming approach. The study was done in the aspect of SRI by focusing on the environmental responsibility, whereby an actual case of green investments was used. In addition, investors were categorized according to their level of ethical goal. The study concluded that a conventional MV model can yield a better performance than the bi-criteria model. Additionally, it was found that a high level of ethical aspiration involved more financial risk than a weak level of ethical aspiration. The result suggested a drawback on strong ecological investors that could be investigated further.

On the other hand, Derigs and Marzban [80] introduced a new paradigm for *Shariah*-compliant portfolio optimization by focusing on the application of sector screens and financial screens. The author argued that investors should focus on the *Shariah*-compliance of the selected portfolio as a whole and its return after purification rather than on single assets. The argument stood from the fact that investors are rather concerned about the compliance of the overall portfolio than the individual assets, which is similar to comparing the compliance of an investment made on a hotel that overall serves to be *Shariah*-compliant but consisting of minor non-compliant activities within. The strategy employed had prevented the significant reduction of asset universe subject to a pre-process, whereby investments are only allowed for *Shariah*-compliant asset classes, such as sukuk instead of bonds, and companies whose primary activities are not permissible were excluded beforehand. In other words, sector screening must be strictly followed. The proposed model was constructed based on the MV model. The resulted portfolios were found to have a comparable performance as the conventional portfolios as supported by the findings of Sandwick and Collazzo [78].

Masri [49] constructed the first *Shariah*-compliant portfolio optimization model by basing the construction on a multi-objective stochastic program which was an extension of the Markowitz's model, and the risk measure used was the Beta coefficient. Generally, the author intended to maximize portfolio return for a given probability of loss by taking *zakat* into consideration, treating it as a recourse cost. The approaches used in developing the model include goal programming approach, chance-constrained approach, and recourse approach, while the model was tested on the Bahrain

Islamic Market. The model was constructed based on four main principles, whereby the author reduced the asset universe to avoid unlawful income, excluded risk-free securities to avoid *riba*, included a 2.5% of total wealth above *nisab* to cover *zakat*, evaluated portfolio risk using Beta coefficient to indicate Islamic market return, and set a limit to the desired maximum return to avoid *Gharar Fahish*. Moreover, Masri employed the conventional CAPM for the evaluation of security returns after concluding the appropriateness of CAPM in Islamic portfolio. Overall, the author intended to maximize return in accordance with the deviations of *zakat* rate, while minimizing deviations from the benchmark risk as indicated by Beta coefficient 1. The model can be presented as follows:

Maximize

$$R-\sum_{s=1}^{2} p_s \varepsilon^{-}(w_s)-\lambda\left(\delta^{+}+\delta^{-}\right) \tag{19}$$

Subject to

$$\sum_{i=1}^{n} E(r_i)x_i-\phi^{-1}(1-\alpha)\sigma\left(\sum_{i=1}^{n} r_i x_i\right)\geq R \tag{20}$$

$$\sum_{i=1}^{n} r_i(w_1)x_i+\varepsilon^{-}(w_1)-\varepsilon^{+}(w_1)=Z \tag{21}$$

$$\sum_{i=1}^{n} r_i(w_2)x_i+\varepsilon^{-}(w_2)-\varepsilon^{+}(w_2)=Z \tag{22}$$

$$\sum_{i=1}^{n} \beta_i x_i+\delta^{-}-\delta^{+}=1 \tag{23}$$

$$\sum_{i=1}^{n} x_i=1 \tag{24}$$

$$0\leq x_i\leq u_i \quad , \quad i=1,\ldots,n \tag{25}$$

$$\delta^{-},\delta^{+},\varepsilon^{-}(w_1),\varepsilon^{+}(w_1),\varepsilon^{-}(w_2),\varepsilon^{+}(w_2)\geq 0, \tag{26}$$

where ε^{-} and ε^{+} are the negative and positive deviations of portfolio returns from *zakat* rate respectively; w_s represents the possible conditions of the Islamic market return, namely, bull market (w_1) and bear market (w_2), while p_s is the corresponding probabilities; λ is the weight that combines the two objectives outlined, namely maximizing returns considering *zakat* and minimizing the deviations of risk from the benchmark; δ^{-} and δ^{+} are the negative and positive deviations of the portfolio risk from the market risk; ϕ^{-1} refers to the inverse distribution function of a standard normal distribution; α is the acceptable probability of loss that ranges from 0 to 1; β_i is the beta coefficient of the *i*th security that signifies the corresponding systematic risk; *Z* is the *zakat* rate.

On the other hand, Bagheri [47] proposed a *Shariah*-compliant multi-goal program for portfolio optimization by using deterministic goal programming, and compared the performance of selection between conventional and Islamic portfolios. Goal programming was concluded to suit real-life portfolios that involve multiple conflicting goals in investment. Apart from the two main ideal goals for investors, namely, maximizing returns and minimizing risk, the author added two additional financial goals and one non-financial goal, which are the minimization of price/earnings (P/E) per share ratio and price/book value (P/B) per share ratio, as well as the inclusion of Islamic securities for ethical and moral purposes. The risk measure used in the study was covariance. The author further categorized investors into three different profiles known as Islamic, ethical and traditional investors. Bagheri observed a different preference on the impermissible income threshold to build a *Shariah*-compliant investment among different members, whereby most *Shariah* boards and Islamic indices employ a 5% threshold, which is different from the SAC of Malaysia that allows 5–25% impermissible income. By taking the different thresholds into consideration, the author adopted a threshold of 10–25% while categorizing the levels of *Shariah*-compliance into three, specifically 90%, 85% and 75%. The model assumed an equal weight for all goals while basing all values in the model on the expected values. The portfolio performance analysis that was conducted based on the Sharpe ratio concluded that the Islamic portfolio outperformed the conventional portfolio. Lastly, the author deduced a limitation of the proposed model that based the construction on deterministic variables, and suggested a stochastic program to yield more practical decisions.

Braiek, Bedoui and Belkacem [81] introduced a new methodology for Islamic portfolio optimization by employing CoVaR as the systemic risk measure. As proposed by Adrian and Brunnermeier [82], CoVaR offers a measurement of financial distress impact on the financial system by evaluating the VaR of the whole financial system conditional on financial institutions under financial distress, thereby allowing the capture of marginal contribution of a specific institution to the overall systemic risk so that the identification of risk level between institutions with an equal or unequal VaR could be done for better risk management. Generally, this measure of systemic risk evaluates the spillover effect of the financial institutions to the financial market, such that a coincidence of financial distress occurring in both the financial system and institutions indicates a high systemic risk in the corresponding institutions, as it implies that the institutions may contribute to systemic crisis. This systemic risk measure bases its spillover evaluation on the tail covariation between the financial system and institutions. The CoVaR was also claimed to be relatively general in studying the spillovers between institutions, therefore serving as a relatively simple measure for systemic risk in this regard. Particularly, the systemic risk contributed by a respective institution is indicated as $\Delta CoVaR$ that represents the difference between VaR of the institution conditional on the distressed financial system, and VaR of the institution conditional on the system in its benchmark state represented by the median of its return distribution.

The adoption of CoVaR was claimed to avoid the disadvantages of VaR. Beforehand, the author adopted ARMA-FIAPARCH and ARMA-FIGARCH model in evaluating the return distribution that is often composed of long-memory property, along with the assumption of being continuous. The authors performed risk management based on sectors classification rather than solely on the institutions due to the claim that risk management based on institutions may result in excessive risk. In addition, the evaluation of volatility and spillovers between sectors were claimed to be crucial as an increase in volatility might result in margin spiral among the market participants. The study was conducted on the Islamic Industry, whereby an Islamic sector and Islamic market analysis was carried out to evaluate the investment decisions in terms of portfolio management, concerning the

Dow Jones Islamic Market index and other sector indices classified based on the Industry Classification Benchmark (ICB). It was concluded that the mean-CoVaR outperformed the MV model in portfolio optimization, supporting the claim that portfolio selection is affected by systemic risk as well as the interdependence between sectors and market.

5. Recent developments of portfolio optimization

Li and Zhang [83] constructed a mean-variance-entropy model to solve portfolio optimization problems with uncertain returns by considering liquidity and diversification level. Similar to an MV model, the model concerns return as the expected value and risk as variance, in addition to an extension to address the diversification degree of the portfolio, namely entropy, which is also commonly used as a measure of risk or uncertainty. The proposed model includes liquidity and diversification as constraints. The authors denoted liquidity as turnover rates represented by uncertain variables due to its hard-to-forecast nature, expressed as Eq (28). On the other hand, entropy is expressed as Eq (29). Besides, the author introduced a risk aversion risk factor to convert the bi-objective model into the commonly seen single-objective model, resulting in the following problem:

Minimize

$$\left\{(1-\lambda)\frac{E_{\max}-E\left[\sum_{i=1}^{n}\xi_i x_i\right]}{E_{\max}}+\lambda\frac{V\left[\sum_{i=1}^{n}\xi_i x_i\right]-V_{\min}}{V_{\min}}\right\} \tag{27}$$

Subject to

$$E\left[\sum_{i=1}^{n}\gamma_i x_i\right]\geq k \tag{28}$$

$$-\sum_{i=1}^{n}x_i\ln(x_i+\varepsilon)\geq\beta \tag{29}$$

$$\sum_{i=1}^{n}x_i=1 \tag{30}$$

$$x_i\geq 0 \quad , \quad i=1,2,\ldots,n \tag{31}$$

$$\lambda\in(0,1), \tag{32}$$

where γ_i is the turnover rate of asset *i*, and *k* refers to a lower limit for turnover rate; β is the lower diversification limit and ε is white noise; $E_{\max}$ referring to the maximum portfolio return without concerning risk, and $V_{\min}$ is the minimum portfolio risk without taking into account the return. These two variables can be referred to as two auxiliary portfolio optimization models, which were then transformed into equivalent deterministic models for simplicity. The study concluded an impact of diversification degree and liquidity on optimizing portfolios.

While robust optimization can offer good out-of-sample performance, even when dealing with uncertain input parameters, Min, Dong, Liu et al. [84] explored the over-conservatism of its solution for most investors leading to unrealistic results when the solutions must strictly satisfy all designated constraints in all scenarios. Besides, the authors observed that the best-case of a robust optimization model serves as the least conservative method for investors compared to the worst-case, which refers to the most conservative one. The authors argued a demand for less conservative models from risk-seeking investors, thereby developing the hybrid robust MV and mean-VaR portfolio optimization models combining with machine learning algorithms to evaluate and forecast market movements, in addition to considering the best-case scenario using a trade-off parameter β to lower the conservative level of a classical robust model. The trade-off parameter β denotes the optimism level about the future through analyzing the historical pattern using machine algorithms, namely Long Short-Term Memory (LSTM) and eXtreme Gradient Boosting (XGBoost). Also, the cardinality constraint that limits the number of assets in the portfolio is incorporated. In conclusion, the designated methods may not yield the best portfolio performance, although the proposed algorithms could offer insights for future research. Additionally, the authors claimed that an efficient investment strategy could be formed by including forecasting information through machine learning for intelligent portfolio construction.

A study by Li, Uysal and Mulvey [85] constructed a multi-period portfolio optimization by employing Model Predictive Control (MPC) using Mean-Variance (MV) and risk parity frameworks. The authors further suggested that a hidden Markov model (HMM) could address the asymmetric market returns well, following its benefit in capturing stock price changes during the financial crises in 2008. Despite the higher effectiveness of a multi-period portfolio optimization model when compared to a single-period model, by taking into account future events while managing for the current event, the authors concluded the same drawbacks in addressing practical concerns in portfolio management, such as transaction costs. A multi-period deterministic optimization problem is obtained by turning the stochastic control problem using the MPC method. Although the MPC method results in a sub-optimal portfolio, it is proven to have similar solutions to the simulation studies with better computational efficiency as well as the inclusion of time-varying estimates of relevant parameters. A risk parity strategy assigns risk budgets to every asset's risk contribution, and then equalizes the risk budgets in the portfolio. The difference between a risk parity strategy and the MV model is that it offers a balanced risk concentration in a portfolio, in addition to low estimation errors, without requiring the expected return estimates as an input parameter. Thus, the resulting portfolios are prone to be low risk, having low but stable returns. The authors argued that this conservative form of investment could improve through leverage and target return constraints. The authors concluded an outperformance of a multi-period model to a single-period model, in addition to a better out-of-sample performance of the MV and risk parity frameworks when compared to a fix-mix benchmark.

6. Risk contagion and hedging tools

There has been a growing body of literature regarding green energy following the deterioration of the ecological environment. As an environmental protection emergence, this new sector puts in a huge effort for a sustainable mechanism. A series of recent studies investigated the hedging properties of crude oil and gold in playing the role of a "safe haven" for investment, in which gold is well-known as an inflationary hedge in addition to its strong resistance towards market uncertainties, including foreign exchange fluctuations. Moreover, gold is certified as a safe haven during crises [86]. Besides, the importance of volatility spillover effects between assets is well-known to be significant for portfolio diversification.

Dai, Zhu and Zhang [87] examined the dynamic relationships between WTI crude oil, gold and China's new energy stock market using the approach introduced by Diebold and Yilmaz [88], and Diebold and Yılmaz [89] based on the time-varying parameter vector autoregressive (TVP-VAR) model. The authors further investigated the optimal hedge ratios while solving the optimization problem involving the named assets using the DCC-GARCH-t-copula model. The selection of the financial commodity markets with the capital market was due to their low correlations that could result in a better diversification degree. The study shows strong relationships between the assets from 30 Dec 2014 to 24 May 2021, whereby drastic increments of interconnectedness are observed following key events such as the COVID-19 pandemic. Also, WTI crude oil and gold are found to be net risk receivers, while the remaining assets were all net risk transmitters. In optimizing the portfolio, the optimal hedge ratios between the markets indicated cheap hedging costs, in which the hedging costs could increase following major crises. The author further concluded a need to rebalance the portfolios following the observed strong time-varying characteristics in the examined factors.

Dai, Li and Yang [90] adopted the shrinkage method to allocate the MV portfolio. Following the importance of the future movements of stocks in portfolio management, this study aims to predict stock return volatility using shrinkage methods, the lasso, adaptive lasso, elastic net and ridge regression. Additionally, the authors supported the idea that the effectiveness of a predictive model is not sufficiently reflected by the statistical significance. Therefore, the proposed model intends to maximize investor utility while using the predicted volatility as a key factor in optimizing the portfolio in evaluating economic benefits. Finally, the prediction results are compared against the standard linear regression framework, whereby an outperformance is observed from the proposed methods, with elastic net recording the best performance. Despite being non-statistically significant, they possessed great economic value.

Dai and Zhu [91] investigated the risk spillovers across crude oil, gold, economic policy uncertainty and four Chinese financial sectors using four risk proxies. The authors explored the insufficiency of the first two order moments in examining spillovers due to the non-normal nature of the return distribution, where realized skewness and kurtosis are adopted as risk proxies in addition to return and volatility. This study uses the total spillover index (TSI) as a measurement of systemic risk. The author concluded different information and obvious characteristics fluctuations from analyzing dynamic spillovers based on the different risk proxies. Similar to the previous study, peak spillovers are observed during major crises. Nonetheless, the TSIs found strong explanatory ability on total return and volatility spillovers by term spread and credit spread as the macro state variables.

7. Conclusions

Over the decades, modifications and extensions against the MV model were done to enhance portfolio optimization modelling based on different schools of thought. In the conventional field, more emphasis was given to the type of risk measures and trading concerns. There have been numerous studies to investigate better risk measures for portfolio management. While most studies utilized the risk-reward concept pioneered by Markowitz, there is far less consensus on the best risk measure and framework for portfolio optimization, such that no single model was proven to outperform the other in all scenarios given the different objectives for different models.

Although extensive research has been carried out on the development of conventional portfolio optimization models, there is little discussion about the construction of an Islamic portfolio selection model. To date, publications that concentrate on Islamic portfolio optimization more frequently concern the screening methodology of *Shariah*-compliant assets. Much of the previous research in Islamic portfolio optimization tended to reduce the asset universe to *Shariah*-compliant securities, with selection done using the conventional models.

Collectively, the research area could have been more relevant if it had considered the model's efficiency and *Shariah* compliance in implementing Islamic finance. The construction of a new Islamic portfolio optimization model is highly encouraged, especially when most of the existing models were more towards conventional frameworks leading to a controversy about conventional mimicking. Additionally, a number of studies have proven Islamic finance to be closely linked to the SRI, supporting the SDGs. The development of a holistically *Shariah*-compliant portfolio optimization model can prop market expansion by creating a new supply line that is more resilient.

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Conflict of interest

We declare no conflicts of interest in this paper.

References

1. E. J. Elton, M. J. Gruber, S. J. Brown, *Modern portfolio theory and investment analysis*, 9 Eds., Wiley Global Education, 2013.
2. D. Peris, *Getting back to business: Why modern portfolio theory fails investors and how you can bring common sense to your portfolio*, McGraw-Hill Education, 2018.
3. H. Markowitz, Portfolio selection, *The Journal of Finance*, **7** (1952), 77–91. https://doi.org/10.2307/2975974
4. M. I. Tabash, R. S. Dhankar, The flow of Islamic finance and economic growth: an empirical evidence of Middle East, *Journal of Finance and Accounting*, **2** (2014), 11–19. https://doi.org/10.11648/j.jfa.20140201.12
5. M. Hussain, A. Shahmoradi, R. Turk, An overview of Islamic finance, *J. Int. Commer. Econ. Po.*, **7** (2016), 1650003. https://doi.org/10.1142/S1793993316500034
6. M. Arouri, H. Ben Ameur, N. Jawadi, F. Jawadi, W. Louhichi, Are Islamic finance innovations enough for investors to escape from a financial downturn? Further evidence from portfolio simulations, *Appl. Econ.*, **45** (2013), 3412–3420. https://doi.org/10.1080/00036846.2012.707776

7. M. M. Butt, E. C. de-Run, A. U-Din, D. Mutum, Religious symbolism in Islamic financial service advertisements, *J. Islamic Mark.*, **9** (2018), 384–401. https://doi.org/10.1108/JIMA-03-2017-0034
8. Imamuddin, A. Saeed, A. W. Arain, Principles of Islamic economics in the light of the Holy Quran and Sunnah, *International Journal of Humanities and Social Science Invention*, **5** (2016), 48–54.
9. D. Hoggarth, The rise of Islamic finance: post-colonial market-building in central Asia and Russia, *Int. Aff.*, **92** (2016), 115–136. https://doi.org/10.1111/1468-2346.12508
10. S. M. Yunus, Z. Kamaruddin, R. Embong, The concept of Islamic banking from the Islamic worldview, *International Journal of Academic Research in Business and Social Sciences*, **8** (2018), 539–550. http://doi.org/10.6007/IJARBSS/v8-i11/4928
11. I. Tahir, M. Brimble, Islamic investment behaviour, *Int. J. Islamic Middle*, **4** (2011), 116–130. https://doi.org/10.1108/17538391111144515
12. A. H. Gait, A. C. Worthington, A primer on Islamic finance: definitions, sources, principles and methods, University of Wollongong, School of Accounting and Finance Working Paper Series, 2007, No. 07/05.
13. O. M. Imad, M. S. A. Razimi, M. F. Osman, Investment in the light of Quran, Sunnah, and Islamic jurisprudence, *Journal of Global Business and Social Entrepreneurship*, **3** (2017), 1–11.
14. Journal of the Malaysian Judiciary July 2020, Judicial Appointments Commission, 2020. Available from: https://www.jac.gov.my/spk/images/stories/4_penerbitan/journal_malaysian_judiciary/julai2020.pdf.
15. M. A. Uddin, Principles of Islamic finance: prohibition of riba, gharar and maysir, 2015, preprint. https://doi.org/10.13140/RG.2.2.36029.20969
16. Z. Iqbal, A. Mirakhor, *An introduction to Islamic finance: theory and practice*, 2 Eds., John Wiley & Sons, 2011. https://doi/org/10.1002/9781118390474
17. H. Visser, *Islamic finance: principles and practice*, Edward Elgar Publishing, 2019.
18. M. Todorof, Shariah-compliant fintech in the banking industry, *ERA Forum*, **19** (2018), 1–17. https://doi.org/10.1007/s12027-018-0505-8
19. A. A. Jobst, Derivatives in Islamic finance, In: *International Conference on Islamic Capital Markets*, 2008, 97.
20. M. T. Usmani, Principles of Shariah governing Islamic investment funds, 2007.
21. A. U. F. Ahmad, M. K. Hassan, Riba and Islamic banking, *Journal of Islamic Economics, Banking and Finance*, **3** (2007), 1–33.
22. A. W. Dusuki, A. Abozaid, Fiqh issues in short selling as implemented in the Islamic capital market in Malaysia, *Journal of King Abdulaziz University: Islamic Economics*, **21** (2008), 63–78. https://doi.org/10.4197/islec.21-2.3
23. M. M. Alam, C. S. Akbar, S. M. Shahriar, M. M. Elahi, The Islamic Shariah principles for investment in stock market, *Qual. Res. Financ. Mark.*, **9** (2017), 132–146. https://doi.org/10.1108/QRFM-09-2016-0029
24. Bank Negara Malaysia, Regulated Short-Selling of Securities in the Wholesale of Money Market, 2017. Available from: https://www.bnm.gov.my/-/regulated-short-selling-of-securities-in-the-wholesale-money-market-1.

25. M. H. Rahman, Application of constructive possession (Qabd Hukmi) in Islamic banking products: Shariah analysis, *Turk. J. Islamic Econ.*, **7** (2020), 81–102. https://doi.org/10.26414/A075
26. R. Calder, Short-selling replication in Islamic finance: innovation and debate in Malaysia and beyond, In: *Current Issues in Islamic Banking and Finance*, World Scientific, 2010, 277–315. https://doi.org/10.1142/9789812833938_0014
27. E. R. Embong, R. Taha, M. N. M. Nor, Role of zakat to eradicate poverty in Malaysia, *Jurnal Pengurusan*, **39** (2013), 141–150. https://doi.org/10.17576/pengurusan-2013-39-13
28. S. Nagaoka, Resuscitation of the antique economic system or novel sustainable system?: Revitalization of the traditional Islamic economic institutions (Waqf and Zakat) in the Postmodern Era, *Kyoto Bulletin of Islamic Area Studies*, **7** (2014), 3–19. http://doi.org/10.14989/185839
29. P. Dhar, Zakat as a measure of social justice in Islamic finance: an accountant's overview, *Journal of Emerging Economies and Islamic Research*, **1** (2013), 64–74. https://doi.org/10.24191/jeeir.v1i1.9118
30. S. M. Ibrahim, The role of zakat in establishing social welfare and economic sustainability, *International Journal of Management and Commerce Innovations*, **3** (2015), 437–441. https://doi.org/10.5430/ijfr.v11n6p196
31. M. Clarke, *Handbook of research on development and religion*, Edward Elgar Publishing, 2013.
32. Bursa Malaysia, FAQs on Zakat on Shares, 2021. Available from: https://www.bursamalaysia.com/reference/faqs/islamic_market/faqs_on_zakat_on_shares.
33. H. Markowitz, *Portfolio selection: efficient diversification of investments*, Yale University Press, 1959.
34. H. Konno, H. Yamazaki, Mean-absolute deviation portfolio optimization model and its applications to Tokyo stock market, *Manage. Sci.*, **37** (1991), 519–531. https://doi.org/10.1287/mnsc.37.5.519
35. H. Konno, H. Shirakawa, H. Yamazaki, A mean-absolute deviation-skewness portfolio optimization model, *Ann. Oper. Res.*, **45** (1993), 205–220. https://doi.org/10.1007/BF02282050
36. H. Konno, K.-i. Suzuki, A mean-variance-skewness portfolio optimization model, *J. Oper. Res. Soc. Jpn.*, **38** (1995), 173–187. https://doi.org/10.15807/jorsj.38.173
37. P. Jorion, Risk^2: Measuring the risk in value at risk, *Financ. Anal. J.*, **52** (1996), 47–56. https://doi.org/10.2469/faj.v52.n6.2039
38. M. R. Young, A minimax portfolio selection rule with linear programming solution, *Manage. Sci.*, **44** (1998), 673–683. https://doi.org/10.1287/mnsc.44.5.673
39. R. T. Rockafellar, S. Uryasev, Optimization of conditional value-at-risk, *J. Risk*, **2** (2000), 21–42. https://doi.org/10.21314/JOR.2000.038
40. C. B. Kalayci, O. Ertenlice, M. A. Akbay, A comprehensive review of deterministic models and applications for mean-variance portfolio optimization, *Expert Syst. Appl.*, **125** (2019), 345–368. https://doi.org/10.1016/j.eswa.2019.02.011
41. V. Boasson, E. Boasson, Z. Zhou, Portfolio optimization in a mean-semivariance framework, *Investment Management and Financial Innovations*, **8** (2011), 58–68.
42. H. Konno, Piecewise linear risk function and portfolio optimization, *J. Oper. Res. Soc. Jpn.*, **33** (1990), 139–156. https://doi.org/10.15807/jorsj.33.139

43. A. Yoshimoto, The mean-variance approach to portfolio optimization subject to transaction costs, *J. Oper. Res. Soc. Jpn.*, **39** (1996), 99–117. https://doi.org/10.15807/jorsj.39.99
44. P. Jorion, Portfolio optimization in practice, *Financ. Anal. J.*, **48** (1992), 68–74. https://doi.org/10.2469/faj.v48.n1.68
45. C. S. Pedersen, S. E. Satchell, Choosing the right measure of risk: a survey, In: *The current state of economic science*, 1998.
46. L. A. T. Cox, Why risk is not variance: an expository note, *Risk Anal.*, **28** (2008), 925–928. https://doi.org/10.1111/j.1539-6924.2008.01062.x
47. N. Bagheri, Deterministic goal programming approach for Islamic portfolio selection, *Oper. Res.*, **21** (2021), 1447–1459. https://doi.org/10.1007/s12351-019-00517-w
48. S. A. Bond, S. E. Satchell, Statistical properties of the sample semi-variance, *Applied Mathematical Finance*, **9** (2002), 219–239. https://doi.org/10.1080/13504860220000015850
49. H. Masri, A Shariah-compliant portfolio selection model, *J. Oper. Res. Soc.*, **69** (2018), 1568–1575. https://doi.org/10.1057/s41274-017-0223-6
50. G. Cornuejols, R. Tütüncü, *Optimization methods in finance*, Cambridge University Press, 2006.
51. F. J. Fabozzi, P. N. Kolm, D. A. Pachamanova, S. M. Focardi, *Robust portfolio optimization and management*, John Wiley & Sons, 2007.
52. D. Roman, K. Darby-Dowman, G. Mitra, Mean-risk models using two risk measures: a multi-objective approach, *Quant. Financ.*, **7** (2007), 443–458. https://doi.org/10.1080/14697680701448456
53. I. Usta, Y. M. Kantar, Mean-variance-skewness-entropy measures: a multi-objective approach for portfolio selection, *Entropy*, **13** (2011), 117–133. https://doi.org/10.3390/e13010117
54. R. D. Arnott, W. H. Wagner, The measurement and control of trading costs, *Financ. Anal. J.*, **46** (1990), 73–80. https://doi.org/10.2469/faj.v46.n6.73
55. G. O. Aragon, W. E. Ferson, Portfolio performance evaluation, *Foundations and Trends® in Finance*, **2** (2007), 83–190. http://doi.org/10.1561/0500000015
56. A. F. Perold, Large-scale portfolio optimization, *Manage. Sci.*, **30** (1984), 1143–1160. https://doi.org/10.1287/mnsc.30.10.1143
57. A. R. Norhidayah, K. Hanim, S. Masitah, Linear versus quadratic portfolio optimization model with transaction cost, *AIP Conference Proceedings*, **1602** (2014), 533. https://doi.org/10.1063/1.4882537
58. R. C. Green, B. Hollifield, When will mean-variance efficient portfolios be well diversified?, *The Journal of Finance*, **47** (1992), 1785–1809. https://doi.org/10.1111/j.1540-6261.1992.tb04683.x
59. P. Krokhmal, J. Palmquist, S. Uryasev, Portfolio optimization with conditional value-at-risk objective and constraints, *J. Risk*, **4** (2002), 43–68. https://doi.org/10.21314/JOR.2002.057
60. N. J. Jobst, M. D. Horniman, C. A. Lucas, G. Mitra, Computational aspects of alternative portfolio selection models in the presence of discrete asset choice constraints, *Quant. Financ.*, **1** (2001), 489–501. https://doi.org/10.1088/1469-7688/1/5/301
61. J .L. Evans, S. H. Archer, Diversification and the reduction of dispersion: an empirical analysis, *The Journal of Finance*, **23** (1968), 761–767. https://doi.org/10.2307/2325905
62. M. Statman, How many stocks make a diversified portfolio?, *J. Financ. Quant. Anal.*, **22** (1987), 353–363. https://doi.org/10.2307/2330969

63. L. Adamiec, D. Cernauskas, Optimal number of assets for reduction of market risk through diversification, *International Journal of Economics, Business and Management Research*, 3 (2019), 45–54.
64. H. C. Jimbo, I. S. Ngongo, N. G. Andjiga, T. Suzuki, C. A .Onana, Portfolio optimization under cardinality constraints: a comparative study, *Open Journal of Statistics*, **7** (2017), 731–742. https://doi.org/10.4236/ojs.2017.74051
65. E. Ballestero, Mean-semivariance efficient frontier: a downside risk model for portfolio selection, *Applied Mathematical Finance*, **12** (2005), 1–15. https://doi.org/10.1080/1350486042000254015
66. H. B. Salah, J. G. De Gooijer, A. Gannoun, M. Ribatet, Mean–variance and mean–semivariance portfolio selection: a multivariate nonparametric approach, *Financ. Mark. Portf. Manag.*, **32** (2018), 419–436. https://doi.org/10.1007/s11408-018-0317-4
67. D. Pla-Santamaria, M Bravo, Portfolio optimization based on downside risk: a mean-semivariance efficient frontier from Dow Jones blue chips, *Ann. Oper. Res.*, **205** (2013), 189–201. https://doi.org/10.1007/s10479-012-1243-x
68. W. F. Sharpe, Capital asset prices: a theory of market equilibrium under conditions of risk, *The Journal of Finance*, **19** (1964), 425–442. https://doi.org/10.1111/j.1540-6261.1964.tb02865.x
69. S. T. Rachev, F. J. Fabozzi, S. V. Stoyanov, Advanced stochastic models, risk assessment, and portfolio optimization: the ideal risk, uncertainty, and performance measures, John Wiley & Sons, 2008.
70. E. Lockwood, Predicting the unpredictable: value-at-risk, performativity, and the politics of financial uncertainty, *Rev. Int. Polit. Econ.*, **22** (2015), 719–756. https://doi.org/10.1080/09692290.2014.957233
71. R. Campbell, R. Huisman, K. Koedijk, Optimal portfolio selection in a value-at-risk framework, *J. Bank. Financ.*, **25** (2001), 1789–1804. https://doi.org/10.1016/S0378-4266(00)00160-6
72. G. J. Alexander, A. M. Baptista, Economic implications of using a mean-VaR model for portfolio selection: a comparison with mean-variance analysis, *J. Econ. Dyn. Control*, **26** (2002), 1159–1193. https://doi.org/10.1016/S0165-1889(01)00041-0
73. T. J. Linsmeier, N. D. Pearson, Value at risk, *Financ. Anal. J.*, **56** (2000), 47–67. https://doi.org/10.2469/faj.v56.n2.2343
74. B. Warwick, *The handbook of risk*, John Wiley & Sons, 2003.
75. P. Artzner, F. Delbaen, J.-M. Eber, D. Heath, Thinking coherently, *Risk*, **10** (1997), 68–71.
76. R. Sparkes, Socially responsible investment, *Handbook of finance. Volume 2: investment management and financial management*, 2008. https://doi.org/10.1002/9780470404324.hof002014
77. W. Ghoul, P. Karam, MRI and SRI mutual funds: a comparison of Christian, Islamic (morally responsible investing), and socially responsible investing (SRI) mutual funds, *The Journal of Investing*, **16** (2007), 96–102. https://doi.org/10.3905/joi.2007.686416
78. J. A. Sandwick, P. Collazzo, Modern portfolio theory with sharia: a comparative analysis, *J. Asset Manag.*, **22** (2021), 30–42. https://doi.org/10.1057/s41260-020-00187-w
79. E. Ballestero, M. Bravo, B. Pérez-Gladish, M. Arenas-Parra, D. Pla-Santamaria, Socially responsible investment: a multicriteria approach to portfolio selection combining ethical and financial objectives, *Eur. J. Oper. Res.*, **216** (2012), 487–494. https://doi.org/10.1016/j.ejor.2011.07.011

80. U. Derigs, S. Marzban, New strategies and a new paradigm for Shariah-compliant portfolio optimization, *J. Bank. Financ.*, **33** (2009), 1166–1176. https://doi.org/10.1016/j.jbankfin.2008.12.011
81. S. Braiek, R. Bedoui, L. Belkacem, Islamic portfolio optimization under systemic risk: Vine Copula-CoVaR based model, *Int. J. Financ. Econ.*, **27** (2022), 1321–1339. https://doi.org/10.1002/ijfe.2217
82. T. Adrian, M. K. Brunnermeier, CoVaR, NBER Working Paper, 2011, No. 17454. https://doi.org/10.3386/w17454
83. B. Li, R. Zhang, A new mean-variance-entropy model for uncertain portfolio optimization with liquidity and diversification, *Chaos Soliton. Fract.*, **146** (2021), 110842. https://doi.org/10.1016/j.chaos.2021.110842
84. L. Min, J. Dong, J. Liu, X. Gong, Robust mean-risk portfolio optimization using machine learning-based trade-off parameter, *Appl. Soft Comput.*, **113** (2021), 107948. https://doi.org/10.1016/j.asoc.2021.107948
85. X. Li, A. S. Uysal, J. M. Mulvey, Multi-period portfolio optimization using model predictive control with mean-variance and risk parity frameworks, *Eur. J. Oper. Res.*, **299** (2022), 1158–1176. https://doi.org/10.1016/j.ejor.2021.10.002
86. X. Wen, H. Cheng, Which is the safe haven for emerging stock markets, gold or the US dollar?, *Emerg. Mark. Rev.*, **35** (2018), 69–90. https://doi.org/10.1016/j.ememar.2017.12.006
87. Z. Dai, H. Zhu, X. Zhang, Dynamic spillover effects and portfolio strategies between crude oil, gold and Chinese stock markets related to new energy vehicle, *Energ. Econ.*, **109** (2022), 105959. https://doi.org/10.1016/j.eneco.2022.105959
88. F. X. Diebold, K. Yilmaz, Better to give than to receive: predictive directional measurement of volatility spillovers, *Int. J. Forecasting*, **28** (2012), 57–66. https://doi.org/10.1016/j.ijforecast.2011.02.006
89. F. X. Diebold, K. Yılmaz, On the network topology of variance decompositions: measuring the connectedness of financial firms, *J. Econometrics*, **182** (2014), 119–134. https://doi.org/10.1016/j.jeconom.2014.04.012
90. Z. Dai, T. Li, M. Yang, Forecasting stock return volatility: the role of shrinkage approaches in a data-rich environment, *J. Forecasting*, **41** (2022), 980–996. https://doi.org/10.1002/for.2841
91. Z. Dai, H. Zhu, Dynamic risk spillover among crude oil, economic policy uncertainty and Chinese financial sectors, *Int. Rev. Econ. Financ.*, **83** (2023), 421–450. https://doi.org/10.1016/j.iref.2022.09.005