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Research article

Stochastic dynamics of the fractal-fractional Ebola epidemic model combining a fear and environmental spreading mechanism

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Abstract: Recent Ebola virus disease infections have been limited to human-to-human contact as well as the intricate linkages between the habitat, people and socioeconomic variables. The mechanisms of infection propagation can also occur as a consequence of variations in individual actions brought on by dread. This work studies the evolution of the Ebola virus disease by combining fear and environmental spread using a compartmental framework considering stochastic manipulation and a newly defined non-local fractal-fractional (F-F) derivative depending on the generalized Mittag-Leffler kernel. To determine the incidence of infection and person-to-person dissemination, we developed a fear-dependent interaction rate function. We begin by outlining several fundamental characteristics of the system, such as its fundamental reproducing value and equilibrium. Moreover, we examine the existence-uniqueness of non-negative solutions for the given randomized process. The ergodicity and stationary distribution of the infection are then demonstrated, along with the basic criteria for its eradication. Additionally, it has been studied how the suggested framework behaves under the F-F complexities of the Atangana-Baleanu derivative of fractional-order ρ and fractal-dimension τ. The developed scheme has also undergone phenomenological research in addition to the combination of nonlinear characterization by using the fixed point concept. The projected findings are demonstrated through numerical simulations. This research is anticipated to substantially increase the scientific underpinnings for understanding the patterns of infectious illnesses across the globe.

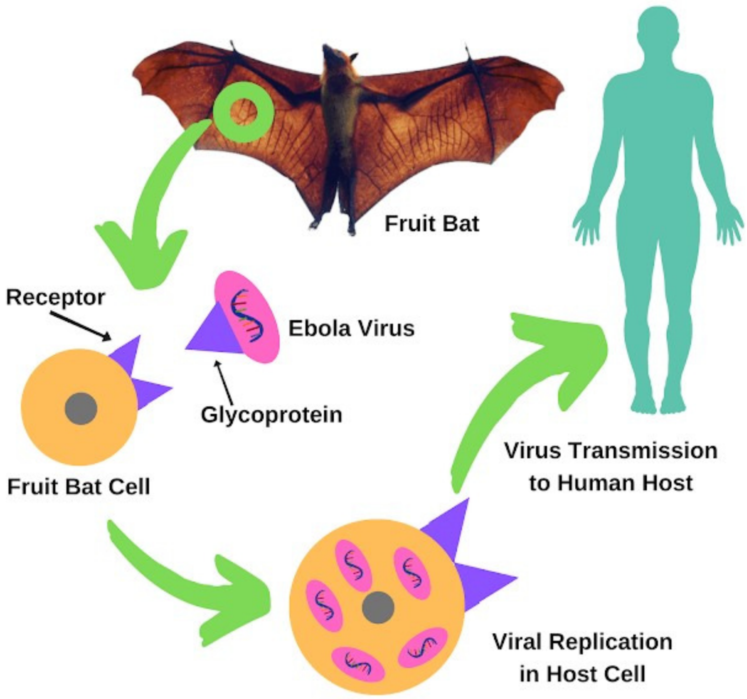
Keywords: Ebola virus disease; fractal-fractional differential operators; extinction; qualitative analysis; stochastic analysis
Mathematics Subject Classification: 46S40, 47H10, 54H25

1. Introduction

Ebola virus (EV) is a filovirus that is a member of the class *Filoviridae* in both humans and nonhuman primates. People who are at threat of contamination can potentially catch EV disease from their surroundings by interacting or getting into direct contact with items that are contaminated with pathogens [1]. Once a person has contracted the sickness, it can take between 2 and 21 days for symptoms to appear EV, the virus can persist for between 4 and 10 days [2]. Migraine, starvation, fatigue, sore muscles or ligaments, respiration problems, nausea, dysentery, gastrointestinal discomfort, unexplained hemorrhage or any unexpected unexplained mortality are common indications of infection in people [3]. Afflicted individuals may either pass away right away or survive the infection and recuperate with therapy, depending on the strength of their antibodies. People who survive develop immunity to the viral genotype they contracted over a decade ago [4]. Nevertheless, recombinant transcription polymerase chain response (RT-PCR) screening for the EV for more than 36 weeks indicates that in a relatively small proportion of patients, certain bodily functions (semen, ocular and neurological network secretions, foetal membranes, breastfeeding) may screen positive [5].

Recently, a predictive SEIHR (susceptible-exposed-infected-hospitalized-recovered) framework has been implemented in numerous numerical modeling experiments to explain the propagation and prevention of EV disease [6], it distinguishes between elevated (such as healthcare personnel) and minimal-risk populations. They determined the best propagation speed by using a simple least-squares estimator and the successful interaction probability in an assessment of the reproductive factor, $\mathbb{R}_0$. For logistic regression from the epidemic in the Congo and Uganda, SEIR mechanisms have been applied in certain publications [7]. The framework in the cited source was expanded by [8] to include two more categories for the untreated and un-buried EV victims. The impact of anti-EV epidemic prevention strategies, including advertisements, increasing hospitalization, appropriate interment of EV disease victims and the availability and implementation of preventative equipment in residences, were taken into account in other research studies [9, 10]. For more information on epidemics and related models [11, 12].

Residents of any afflicted region surely dread EV disease because of its significant mortality incidence. According to a 2016 World Health Organization advisory, both EV patient contamination and mortality have the potential to affect the diseased people and their associated relations [13]. Anxiety can cause locals to be extra conservative and take preventative precautions regarding the infection, including preventing close interaction with contaminated people, consuming animal flesh and attempting to avoid communal areas including markets, institutions, churches and cemeteries of EV victims (see Figure 1). People-to-human pathogenic connection levels may decrease significantly as a result of these behavioral changes brought on by EV disease concern. The preceding mathematical frameworks and other earlier EV estimates did not take into consideration the important implications this has on infection behavior and progression. Furthermore, most studies fail to consider the polluted environment caused by pathogen-infested materials (for example, infected injections used in outpatient clinics; bedsheets contaminated by infectious people's faces, saliva, vomit or perspiration) [14, 15]. Fractional calculus, which has a wide range of applications in molecular responses, heat transfer, mechanical design, nanoscale evolution, laser scanning and architecture classification, neuroscience, photoelectric operations, viscoelasticity, machine learning and many other domains [16–18], was recently developed as a result of research into Lévy inertia, chaotic behavior and the financial sector. The conventional differential and integral formulations are unable to capture heterogeneity. However, these fractional derivative/integral formulations have been recognized as powerful computing tools. Additionally, it is noteworthy that when representing natural and physical events, fractal-fractional (F-F) techniques preserve multiple kinds of variabilities and behave in diverse ways. It seems that there are undoubtedly many real-world problems, but neither fractal nor fractional approaches are able to accurately reproduce them on an interpersonal basis. Researchers came to the conclusion that, in order to mimic extremely complicated formations, they urgently needed novel computational procedures. Despite the assertion, including others, that there is hardly anything at all novel or transformative, it is difficult to believe that the combination of two existing concepts may produce an innovative technique. The first implementation of a revolutionary differential formulation was designed in [19] in order to accommodate more complexity. This scholastic statement might be understood as the outcome of the combination of the fractal differentiation of a fractional derivative of a specific projection. Evidently, there are three potential readings and it all depends on the operating system. The idea was contested and extended to a range of issues, involving turbulent equilibria, outbreaks and propagation, among many others [20–22], and the overwhelming proportion of the featured publications yielded extremely outstanding modeled projections.



(a)

Figure 1. EV transmission cycle. [23]

In the past, numerical models for complicated processes in pharmaceutical, biological, mechanical and data analysis were traditionally created by using the resolutions of integro, linear and nonlinear differential equations (DEs). The underpinning for fidelity evaluation and further research into the pertinent dynamical systems is the predominance and specificity of mathematical methodologies. The inquiry scope has also been extended to structurally intricate equations and time-latency systems [24] that contain the hereditary trait of dynamic interplay in physics and technology. Around 1960, the strategies of common Ito-Doob version randomized DEs [25], randomized partial DEs [26], randomized fractional DEs [27] and randomized fractional PDEs [28] in esoteric settings were defined as the cornerstone of Ito-Doob form stochastic integral equations. This was done for apparent supercomputing reasons. The regulated effects of randomized environmental pressures are described by the Wiener process [29]. This has been thoroughly investigated, utilizing local martingale formulae [30]. Based on the aforementioned contextualized production of innovative model development work and the implementation of multiple responsive and/or randomized perturbation characteristics of interplay in the numerical prediction highlighted by DEs, We understand that researchers are increasingly pursuing advances in fundamental constituent ideologies that devote an initial estimate's comprehension to its commensurately efficient numerical consideration. Furthermore, a confluence of traditional analytical simulation and randomized methodologies was subsequently used to construct randomized dynamic structures for monetary records in [31]. In an effort to adapt this structure to quite enormously delicate strategies in the scientific world operating under intangible responsive and external randomized intervention, we need to notify the evolving computational algorithms by intentionally completing innovative, straightforwardly referenced standards or attributes to design variables. Atangana and Araz [32] pioneered the utilization of randomized and numerical methods to simulate and predict the transmission of COVID-19 across Africa and Europe in 2021. Alkahtani and Alzaid [33] later considered the randomized quantitative framework of chikungunya transmission, including the fractional calculus. Cui et al. [34] proposed a novel concept of F-F and stochastic evaluation of norovirus spread and vaccination impacts. Rashid et al. [35] performed a detailed assessment of the stochastic F-F tuberculosis model considering the Mittag-Leffler kernel and random densities.

Numerous numerical simulations have successfully been implemented to research how to more efficiently supervise interventions against re-emerging and developing serious infections, such as quarantine and vaccination [36–40]. Implementation strategies primarily intend to reduce such impacts by reducing spreading or lessening aggravation. Immunization and antigens have emerged as the main methods for controlling several chronic illnesses. If there are vaccinations and monoclonal antibodies for these infections, a category of vaccine recipients that is at best somewhat resistant should be involved when developing the framework. However, confinement is the primary preventive method for an outbreak of disease like the EV, in which no vaccine prevention is available. In the particular instance of the EV pathogen, the scenario of the French caregiver who was diagnosed with the disease is evidence of the potential of clinical assistance. However, this therapeutic care is not accessible for the poor nations, including Sierra Leone, Liberia, and Guinea, which lack the potential to safeguard individuals against the pathogen. Consequently, quarantine is a key strategy for reducing infection within those underdeveloped nations.

This study constructs a generalized epizootic framework for EV that is represented by the nonlinear system represented below in order to account for these mathematical and biological concerns through the use of a novel approach known as the F-F derivative operator. For the sake of clarity, the Atangana-Baleanu derivative notion is properly considered alongside the Brownian motion. The deterministic and stochastic reproductive values of the model's solutions were all identified and examined. The system's research suggests that a stochastic threshold technique to reduce the pandemic risk is needed. In the eradication of the virus, we additionally assess the stochastic framework by applying ergodicity and stationary distribution. The numerical investigation of the suggested system is evaluated by considering the F-F Atangana-Baleanu formulation featuring white noise. To bolster our theoretical findings, we provide simulated outcomes. In a nutshell, we provide various insights in the conclusion.

The remainder of this work is presented. Section 2 briefly discusses some of the attributes and interpretations employed in this investigation. Section 3 describes the model's conception and representation using the F-F derivative. Also, Section 3 examines the solution's well-posedness, and equilibrium conditions for the F-F EV model. In Section 4, the qualitative aspects of the stochastic model are investigated. In Section 5, the numerical scheme is derived. Furthermore, the numerical simulation and graphical results are presented. Finally, the conclusion can be found in Section 6.

2. Preliminaries

Before advancing on to the formal description, it is imperative to study certain fundamental F-F operator concepts. Take into account the parameters provided in [19] as well as the functional $\mathbf{v}(\zeta)$, which is continuous and fractal differentiable on $[c, d]$ with fractal-dimension τ and fractional-order ρ.

Definition 2.1. [19] The F-F operator of $\mathbf{v}(\zeta)$ involving the index law kernel from the perspective of Riemann–Liouville (RL) can be described as follows for $\zeta \in [0, 1]$:

$$^{FFP}\mathbf{D}_{0,\zeta}^{\rho,\tau}(\mathbf{v}(\zeta)) = \frac{1}{\Gamma(\chi-\rho)}\frac{d}{d\zeta^{\tau}}\int_{0}^{\zeta}(\zeta-\mathbf{w})^{\chi-\rho-1}\mathbf{v}(\mathbf{w})d\mathbf{w}, \tag{2.1}$$

where $\frac{d\mathbf{v}(\mathbf{w})}{d\mathbf{w}^{\tau}} = \lim\limits_{\zeta\mapsto\varkappa}\frac{\mathbf{v}(\zeta)-\mathbf{v}(\varkappa)}{\zeta^{\tau}-\varkappa^{\tau}}$ and $\chi - 1 < \rho, \tau \leq \chi \in \mathbb{N}$.

Definition 2.2. [19] The F-F operator of $\mathbf{v}(\zeta)$ involving the exponential decay kernel in terms of RL can be described as follows for $\rho \in [0, 1]$:

$$^{FFE}\mathbf{D}_{0,\zeta}^{\rho,\tau}(\mathbf{v}(\zeta)) = \frac{\mathbb{M}(\rho)}{1-\rho}\frac{d}{d\zeta^{\tau}}\int_{0}^{\zeta}\exp\Big(-\frac{\rho}{1-\rho}(\zeta-\varkappa)\Big)\mathbf{v}(\varkappa)d\varkappa, \tag{2.2}$$

such that $\mathbb{M}(0) = \mathbb{M}(1) = 1$, with $\rho > 0,\ \tau \leq \chi \in \mathbb{N}$.

Definition 2.3. [19] The F-F operator of $\mathbf{v}(\zeta)$ involving the generalized Mittag-Leffler kernel from the perspective of RL can be described as follows for $\rho \in [0, 1]$:

$$^{FFM}\mathbf{D}_{0,\zeta}^{\rho,\tau}(\mathbf{v}(\zeta)) = \frac{\mathbf{ABC}(\rho)}{1-\rho}\frac{d}{d\zeta^{\tau}}\int_{0}^{\zeta}E_{\rho}\Big(-\frac{\rho}{1-\rho}(\zeta-\varkappa)\Big)\mathbf{v}(\varkappa)d\varkappa, \tag{2.3}$$

such that $\mathbf{ABC}(\rho) = 1 - \rho + \frac{\rho}{\Gamma(\rho)}$, with $\rho > 0,\ \tau \leq 1 \in \mathbb{N}$.

Definition 2.4. [19] The respective F-F integral formulae of (2.1) is described as:

$$ {}^{FFP}\mathbb{J}^{\rho}_{0,\zeta}(\mathbf{v}(\zeta)) = \frac{\tau}{\Gamma(\rho)} \int\limits_{0}^{\zeta} (\zeta - \varkappa)^{\rho-1} \varkappa^{\tau-1} \mathbf{v}(\varkappa) d\varkappa. \tag{2.4} $$

Definition 2.5. [19] The respective F-F integral formulae of (2.2) be described as:

$$ {}^{FFE}\mathbb{J}^{\rho}_{0,\zeta}(\mathbf{v}(\zeta)) = \frac{\rho\tau}{\mathbb{M}(\rho)} \int\limits_{0}^{\zeta} \varkappa^{\tau-1} \mathbf{v}(\varkappa) d\varkappa + \frac{\tau(1-\rho)\zeta^{\tau-1}\mathbf{v}(\zeta)}{\mathbb{M}(\rho)}. \tag{2.5} $$

Definition 2.6. [19] The respective F-F integral formulae of (2.3) is described as:

$$ {}^{FFM}\mathbb{J}^{\rho}_{0,\zeta}(\mathbf{v}(\zeta)) = \frac{\rho\tau}{\mathbf{ABC}(\rho)} \int\limits_{0}^{\zeta} \varkappa^{\tau-1} (\zeta - \varkappa)^{\rho-1} \mathbf{v}(\varkappa) d\varkappa + \frac{\wp(1-\rho)\zeta^{\wp-1}\mathbf{v}(\zeta)}{\mathbf{ABC}(\rho)}. \tag{2.6} $$

Definition 2.7. [18] Let $\mathbf{v} \in H^1(\mathbf{c}, \mathbf{d})$, $\mathbf{c} < \mathbf{d}$ and the Atangana-Baleanu fractional derivative operator is described as:

$$ {}^{ABC}_{\mathbf{c}}\mathbf{D}^{\rho}_{\zeta}(\mathbf{v}(\zeta)) = \frac{\mathbf{ABC}(\rho)}{1-\rho} \int\limits_{\mathbf{c}}^{\zeta} \mathbf{v}'(\varkappa) E_{\rho}\Big(- \frac{\rho(\zeta - \varkappa)^{\rho}}{1-\rho} \Big) d\varkappa, \ \ \rho \in [0, 1], \tag{2.7} $$

where $\mathbf{ABC}(\rho)$ represents the normalization function.

Definition 2.8. [41] The Gaussian hypergeometric function ${}_2\mathbf{F}_1$ is characterized as:

$$ {}_2\mathbf{F}_1(\mathbf{u}_1, \mathbf{u}_2; \mathbf{u}_3, \mathbf{u}_4) = \frac{1}{\mathbb{B}(\mathbf{u}_2, \mathbf{u}_3 - \mathbf{u}_2)} \int\limits_{1}^{1} \zeta^{\mathbf{u}_2 - 1} (1-\zeta)^{\mathbf{u}_3 - \mathbf{u}_2 - 1} (1 - \mathbf{u}_4\zeta)^{-\mathbf{u}_1} d\zeta, \ \ (\mathbf{u}_3 > \mathbf{u}_2 > 0, |\mathbf{u}_1| < 1), \tag{2.8} $$

where $\mathbb{B}(\mathbf{u}_1, \mathbf{u}_2) = \frac{\Gamma(\mathbf{u}_1)\Gamma(\mathbf{u}_2)}{\Gamma(\mathbf{u}_1 + \mathbf{u}_2)}$ and $\Gamma(\mathbf{u}_1) = \int\limits_{0}^{\infty} \exp(-\zeta)\zeta^{\mathbf{u}_1} d\zeta$ is the gamma function.

3. Model conception and depiction

Here, we suggest a mathematical approach comprising six different cohorts, including a section for environmental infections ($\mathcal{P}$), susceptibility ($\mathcal{S}$), infectious ($\mathcal{I}$), hospitalized ($\mathcal{H}$), restored ($\mathcal{R}$)EV deceased ($\mathcal{D}$), respectively. People getting into contact with substances that are infected with the infection are thought to be the two main ways that EV disease is spread (individual to individual interaction or participant to infection interaction). It is believed that anxiety, which is equivalent to the number of fatalities, will lower the risk of spread in the case of an EV spread in a population. This presumption is based on the idea that people remain increasingly apprehensive when EV disease-related mortality increases in a region. The communication ratio declined as a result of this. We use the factor that estimates the effect of dread ϵ_1 per the deceased to represent the sense of paranoia. The quantity $\epsilon_1 > 0$, where $0 < \epsilon_1 \leq \tilde{\epsilon}_1$ represents a complete lack of anxiety and $\epsilon_1 = 0$ displays escalating amounts of dread up to a high of $\tilde{\epsilon}_1$. It is reasonable to assume that higher feelings of anxiety will cause the meaningful interaction probability to decline, such that $\sigma = \frac{\gamma_1 \mathcal{P}}{\kappa + \mathcal{P}} + \frac{\gamma_2(1+\vartheta_1 \mathcal{H} + \vartheta_2 \mathcal{D})}{1+\epsilon_1 \mathcal{D}}$, where κ denotes the virus intensity at which there is a 50% chance of transmission, also known as the 1/2-saturation factor. So, b_1 and b_2 represent the actual interaction patterns that result in an outbreak between vulnerable people and environmental infections between sensitive people and infectious people, respectively. A steady stream of vulnerable individuals is recruited by conception or immigration π. The incidence of EV pathogen infection in vulnerable people is σ. The contagious individuals can recover at the level η_1, be hospitalized at the level η_2 or die at the level η_3. We assume that people who are hospitalized are similarly contagious but have significantly less pathogenicity ϑ_1 than those who are dead and contagious ϑ_2 and thus have $0 < \vartheta_1 < 1$. Hospitalized patients can either recover at a rate of ω_1 or die from EV disease at a rate of ω_2 and their corpses are disposed of at a rate of ϱ. Additionally, infections in the environment degrade at a rate of ν and contaminated people and the deceased corpses of EV victims release infections into the culture at rates of δ_1 and δ_2, respectively. We presume that individuals pass away naturally at a pace of ς. Figure 2 displays the schematic representation of the system.

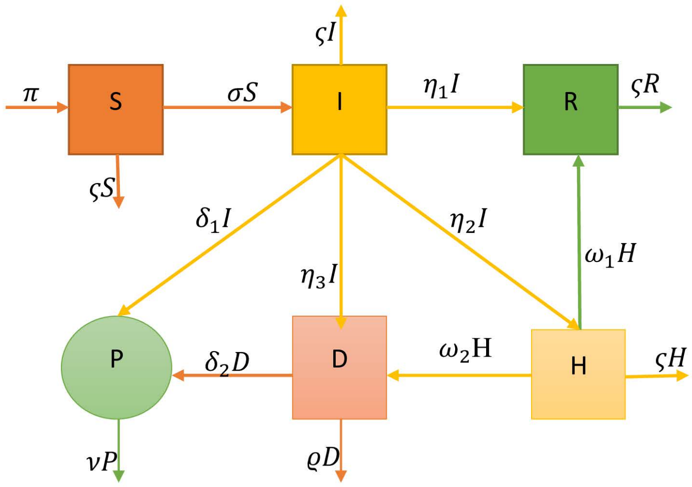


Figure 2. Schematic diagram for EV model.

The preceding auxiliary criteria are also predicated on the framework interpretation: the sentient community is the primary recipient group taken into account in this approach. Wildlife is not seen as a component of the ecosystem, which seems reinforced by the reality that various creatures, including monkeys, chimpanzees and bonnet macaque, which seem to be pathogen carriers, reside in woods farthest from domestic areas and have limited interactions with individuals. The atmosphere has a holding capability because it is a solid object by necessity. The use of a dominating dynamical system in the intensity of transmission results from applying the nonlinear and saturation nature of the growth in the number of infections in the environment as the incidence of developing disease. The transfer of contagious people from the pathogenic to the hospitalized section as well as from the inpatient to the deceased category is unaffected by dread.

This seems to be attributable to the fact that, regardless of their health, individuals constantly bring ailing relatives to the clinic. Additionally, an EV diagnosis is typically made after a patient receives a definitive clinical diagnosis in a laboratory. The individuals present are kept in secure settings and cared for by experts wearing hazard gear designed for handling EV victims and the corpses of EV victims. As a result, the treated patients' nervousness about developing the infection seems to be very weak and considered minimal. We suppose that the level at which sick people excrete germs into the environment is insignificant because of their comparatively reduced infectiousness as well.

The succeeding framework of DEs results from the schematic flow and the system suppositions:

$$\begin{cases} \dot{\mathcal{S}}(\zeta) = \pi - (\sigma + \varsigma)\mathcal{S}(\zeta), \\ \dot{\mathcal{I}}(\zeta) = \sigma\mathcal{S} - (\varsigma + \eta_1 + \eta_2 + \eta_3)\mathcal{I}(\zeta), \\ \dot{\mathcal{H}}(\zeta) = \eta_2\mathcal{I} - (\varsigma + \omega_1 + \omega_2)\mathcal{H}(\zeta), \\ \dot{\mathcal{D}}(\zeta) = \eta_3\mathcal{I} + \omega_2\mathcal{H} - \varrho\mathcal{D}(\zeta), \\ \dot{\mathcal{P}}(\zeta) = \delta_1\mathcal{I} + \delta_2\mathcal{D} - \nu\mathcal{P}(\zeta), \end{cases} \tag{3.1}$$

which are subject to the initial conditions (ICs) $\mathcal{S}(0) > 0,\ \mathcal{I}(0) > 0,\ \mathcal{H}(0) > 0,\ \mathcal{D}(0) > 0,\ \mathcal{P}(0) > 0$ *and* $\forall\ \zeta > 0$ and the restored group is viewed as being unnecessary.

To the best of our knowledge, no comprehensive analyses have been conducted on the implications of the F-F model of the EV. This operator has the ability to comprehend nature in a better way. In order to comprehend the complexities of the model, we present the F-F version of the EV epidemic model, which has not been studied yet. This is the main motivation for this study. Thus, in this article, we will adhere to the methodology described in [42] and take into account the F-F framework of the EV in the generalized Mittag-Leffler kernel context:

$$\begin{cases} {}_0^{\mathbf{FF}}\mathbf{D}_{\zeta}^{\rho,\tau}\mathcal{S}(\zeta) = \pi - (\sigma + \varsigma)\mathcal{S}, \\ {}_0^{\mathbf{FF}}\mathbf{D}_{\zeta}^{\rho,\tau}\mathcal{I}(\zeta) = \sigma\mathcal{S} - (\varsigma + \eta_1 + \eta_2 + \eta_3)\mathcal{I}, \\ {}_0^{\mathbf{FF}}\mathbf{D}_{\zeta}^{\rho,\tau}\mathcal{H}(\zeta) = \eta_2\mathcal{I} - (\varsigma + \omega_1 + \omega_2)\mathcal{H}, \\ {}_0^{\mathbf{FF}}\mathbf{D}_{\zeta}^{\rho,\tau}\mathcal{D}(\zeta) = \eta_3\mathcal{I} + \omega_2\mathcal{H} - \varrho\mathcal{D}, \\ {}_0^{\mathbf{FF}}\mathbf{D}_{\zeta}^{\rho,\tau}\mathcal{P}(\zeta) = \delta_1\mathcal{I} + \delta_2\mathcal{D} - \nu\mathcal{P}, \end{cases} \tag{3.2}$$

where the F-F operator is the convolution of the generalized Mittag-Leffler kernel and the fractal derivative. The fractional-order ρ and the fractal-dimension τ are the orders of these operators. The innovative operators' objective is to seek out non-local challenges in nature that exhibit fractal behavior. F-F derivatives are used to describe long-term relationships, macro- and microscaled manifestations, discontinuous differential concerns and irregular physical phenomena. Atangana and Qureshi [43] described a notion in which they predicted chaotic behavior of the modified Lu Chen attractor, modified Chua chaotic attractor, Lu Chen attractor and Chen attractor using F-F operators. Rashid et al. [44] expounded upon the F-F model for the prediction of the oscillatory and complex behavior of the human liver with a non-singular kernel. For more details on the F-F operator, see [42, 45].

In this research, we propose a stochastic viral mathematical framework for EV epidemics that is developed using the five stochastic DEs presented as follows:

$$\begin{cases} d\mathcal{S}(\zeta) = (\pi - (\sigma + \varsigma)\mathcal{S}(\zeta))d\zeta + \wp_1\mathcal{S}(\zeta)\mathfrak{B}_1(\zeta), \\ d\mathcal{I}(\zeta) = (\sigma\mathcal{S} - (\varsigma + \eta_1 + \eta_2 + \eta_3)\mathcal{I}(\zeta))d\zeta + \wp_2\mathcal{I}(\zeta)\mathfrak{B}_2(\zeta), \\ d\mathcal{H}(\zeta) = (\eta_2\mathcal{I} - (\varsigma + \omega_1 + \omega_2)\mathcal{H}(\zeta))d\zeta + \wp_3\mathcal{H}(\zeta)\mathfrak{B}_3(\zeta), \\ d\mathcal{D}(\zeta) = (\eta_3\mathcal{I} + \omega_2\mathcal{H} - \varrho\mathcal{D}(\zeta))d\zeta + \wp_4\mathcal{D}(\zeta)\mathfrak{B}_4(\zeta), \\ d\mathcal{P}(\zeta) = (\delta_1\mathcal{I} + \delta_2\mathcal{D} - \nu\mathcal{P}(\zeta))d\zeta + \wp_5\mathcal{P}(\zeta)\mathfrak{B}_5(\zeta), \end{cases} \tag{3.3}$$

where $\mathfrak{B}_\ell,\ \ell = 1, ..., 5$ represents real-valued standard Brownian motion described on a complete probability space $(\Omega, \mathcal{A}, P)$ fulfilling the requirements specified in [46]; similarly, $\wp_\ell,\ \ell = 1, ..., 5$ represents the strengths of standard Gaussain white noises.

3.1. Properties of the deterministic case

To demonstrate that the EV model provided by (3.1) is epidemiologically meaningful, we must demonstrate that the model's corresponding state variables remain non-negative. It can also be said that the EV model solution with non-negative initial conditions will stay non-negative for every time greater than zero. We have the following lemma.

Lemma 3.1. *Consider the initial data* $\mho(0) \geq 0$, *where*

$$\mho(\xi) = (\mathcal{S}(\zeta), \mathcal{I}(\zeta), \mathcal{H}(\zeta), \mathcal{D}(\zeta), \mathcal{P}(\zeta)).$$

Then the solutions of the system presented by (3.1) *are non-negative for every time* $\zeta > 0$. *Further,*

$$\lim_{\zeta \mapsto \infty} \mathcal{N}(\zeta) = \frac{\pi}{\varsigma}$$

with $\mathcal{N}(\zeta) = \mathcal{S}(\zeta) + \mathcal{I}(\zeta) + \mathcal{H}(\zeta) + \mathcal{D}(\zeta) + \mathcal{P}(\zeta)$.

Proof. Consider $\zeta_1 = \sup\{\zeta > 0 : \mho(\zeta) > 0\}$. So, $\zeta_1 > 0$. The first equation of the EV system (3.1) leads to the following

$$\frac{d\mathcal{S}}{d\zeta} = \pi - (\sigma + \varsigma)\mathcal{S}, \tag{3.4}$$

with $\theta_\varsigma = \sigma$; then, (3.4) reduces to

$$\frac{d\mathcal{S}}{d\zeta} = \pi - \theta_\varsigma\mathcal{S} - \varsigma\mathcal{S}. \tag{3.5}$$

Thus, (3.5) can be expressed further as follows:

$$\frac{d}{d\zeta}\Big\{\mathcal{S}(\zeta)\exp\Big(\varsigma\zeta + \int_0^\zeta \theta_\varsigma(\phi)d\phi\Big)\Big\} = \theta_\varsigma \exp\Big(\varsigma\zeta + \int_0^\zeta \theta_\varsigma(\phi)d\phi\Big).$$

Therefore, we have

$$\mathcal{S}(\zeta_1)\exp\Big(\varsigma\zeta_1 + \int_0^{\zeta_1} \theta_\varsigma(\phi)d\phi\Big) - \mathcal{U}(0) = \theta_\varsigma \exp\Big(\varsigma\mathbf{y} + \int_0^{\mathbf{y}} \theta_\varsigma(\psi)d\psi\Big)d\mathbf{y}.$$

It follows that

$$\mathcal{S}(\zeta_1) = \mathcal{S}(0)\exp\Big\{-\Big(\varsigma\zeta_1 + \int\limits_0^{\zeta_1}\theta_\varsigma(\phi)d\phi\Big)\Big\} + \exp\Big\{-\Big(\varsigma\zeta_1 + \int\limits_0^{\zeta_1}\theta_\varsigma(\phi)d\phi\Big)\Big\}$$
$$\times\int\limits_0^{\zeta_1}\theta_\varsigma\exp\Big(\varsigma\mathbf{y} + \int\limits_0^{\mathbf{y}}\theta_\varsigma(\psi)d\psi\Big)d\mathbf{y} > 0.$$

Similar steps can be followed for the rest of the equations of the EV system (3.1), i.e., $\mho(\zeta) > 0$ for every $\zeta > 0$. Note that $0 < \mathcal{S}(0) \leq \mathcal{N}(\zeta),\ 0 < \mathcal{I}(0) \leq \mathcal{N}(\zeta),\ 0 < \mathcal{H}(0) \leq \mathcal{N}(\zeta),\ 0 < \mathcal{D}(0) \leq \mathcal{N}(\zeta)$ and $0 < \mathcal{P}(0) \leq \mathcal{N}(\zeta)$. Now, summing the EV model (3.1) compartments leads to the following:

$$\frac{d\mathcal{N}}{d\zeta} = \pi - \varsigma\mathcal{N}(\zeta).$$

Thus,

$$\lim_{\zeta\mapsto\infty}\mathcal{N}(\zeta) \leq \frac{\pi}{\varsigma},$$

which is the required claim. □

3.2. Positivity of the proposed model

For the positivity of the model solution, let us use the following set $\mathbb{R}^5_+$:

$$\mathbb{R}^5_+ = \Big\{\eta \in \mathbb{R}^5 : \eta \geq 0 \ and\ \eta(\zeta) = (\mathcal{S}(\zeta), \mathcal{I}(\zeta), \mathcal{H}(\zeta), \mathcal{D}(\zeta), \mathcal{P}(\zeta))^T\Big\}.$$

Theorem 3.1. *Suppose that the solution $\eta(\zeta)$ of the proposed F-F EV model* (3.2) *exists and belongs to $\mathbb{R}^5_+$. Moreover, the solution will be non-negative.*

Proof. Taking into account (3.2), we observed that

$$\begin{cases} {}^{\mathbf{FF}}_{0}\mathbf{D}^{\rho,\tau}_{\zeta}\mathcal{S}(\zeta)\Big|_{\mathcal{S}=0} = \pi \geq 0, \\ {}^{\mathbf{FF}}_{0}\mathbf{D}^{\rho,\tau}_{\zeta}\mathcal{I}(\zeta)\Big|_{\mathcal{I}=0} = \sigma\mathcal{S} \geq 0, \\ {}^{\mathbf{FF}}_{0}\mathbf{D}^{\rho,\tau}_{\zeta}\mathcal{H}(\zeta)\Big|_{\mathcal{H}=0} = \eta_2\mathcal{I} \geq 0, \\ {}^{\mathbf{FF}}_{0}\mathbf{D}^{\rho,\tau}_{\zeta}\mathcal{D}(\zeta)\Big|_{\mathcal{D}=0} = \eta_3\mathcal{I} + \omega_2\mathcal{H} \geq 0, \\ {}^{\mathbf{FF}}_{0}\mathbf{D}^{\rho,\tau}_{\zeta}\mathcal{P}(\zeta)\Big|_{\mathcal{P}=0} = \delta_1\mathcal{I} + \delta_2\mathcal{D} \geq 0. \end{cases} \tag{3.6}$$

As a result, we conclude that the solution will remain in $\mathbb{R}^5_+$ for all $\zeta \geq 0$. The aforesaid system of equations given by (3.2) can be used to calculate the total dynamics of the individuals:

$${}^{\mathbf{FF}}_{0}\mathbf{D}^{\rho,\tau}_{\zeta}\mathcal{N}(\zeta) = \pi - \varsigma\mathcal{N}(\zeta). \tag{3.7}$$

Implementing the Laplace transform, we get

$$\mathcal{N}(\zeta) = E_{\rho,1}(-\varsigma\zeta^{\rho})\mathcal{N}(0) + \pi\Gamma(p+1)\zeta^{p+\rho-1}E_{\rho,p+\rho}(-\varsigma\zeta^{\rho}), \tag{3.8}$$

where $E_{\rho,\varsigma}$ is called the Mittag-Leffler function. In view of [28], $E_{\rho,\varsigma}$ has asymptotic behavior, thus, we have $\lim_{\zeta\mapsto\infty} \mathcal{N}(\zeta) \leq \frac{\pi}{\varsigma}$. The feasible region for EV model (3.2) is structured as

$$\Upsilon := \Big\{(\mathcal{S}, \mathcal{I}, \mathcal{H}, \mathcal{D}, \mathcal{P}) \in \mathbb{R}^5_+ : 0 \leq \mathcal{S} + \mathcal{I} + \mathcal{H} + \mathcal{D} + \mathcal{P} \leq \frac{\pi}{\varsigma}\Big\}. \tag{3.9}$$

□

3.3. Disease-free equilibrium

In this subsection, we elaborate the bounded region and the system's equilbria provide us with useful information about the model's trajectory in time ζ. The system has two equlibria, i.e., the disease-free equilibrium (DFE) and the endemic equilibrium (EE).

First, we need to analyze the existence of the equilibrium of (3.2). The interior and boundary of Υ in $\mathbb{R}^5_+$ can be denoted as Υ^0 and $\partial\Upsilon$, respectively. The DFE appears when the number of infectious is zero in (3.2). Regardless of any values of the parameters in (3.2), the DFE $\Phi_0(\mathcal{S}, \mathcal{I}, \mathcal{H}, \mathcal{D}, \mathcal{P}) = \left(\frac{\pi}{\varsigma}, 0, 0, 0, 0\right)$ always exists.

Applying the next-generation matrix approach, we calculate the fundamental reproductive value $\mathbb{R}^D_0$ and the related Jacobian matrices **F** and **V** evaluated at the DFE are taken into account by taking into consideration both the acquired pathogens and the transference matrices [47] as follows:

$$\mathbf{F} = \begin{bmatrix} \frac{\pi\gamma_2}{\varsigma} & \frac{\pi\gamma_2\vartheta_1}{\varsigma} & \frac{\pi\gamma_2\vartheta_2}{\varsigma} & \frac{\pi\gamma_2}{\varsigma\kappa} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \ \mathbf{V} = \begin{bmatrix} \varsigma + \eta_1 + \eta_2 + \eta_3 & 0 & 0 & 0 \\ -\eta_2 & \varsigma + \omega_1 + \omega_2 & 0 & 0 \\ -\eta_3 & -\omega_2 & \varrho & 0 \\ -\delta_1 & -\delta_2 & -\wp_3 & \nu \end{bmatrix}. \tag{3.10}$$

The reproductive value is the spectral radius of the next generation matrix, i.e., $\mathbf{FV}^{-1}$; hence, we have

$$\mathbb{R}^D_0 = \frac{\pi}{\varsigma}\Big(\frac{\nu\kappa\gamma_2(\varrho(\varsigma + \omega_1 + \omega_2) + \varrho\vartheta_1\eta_2 + \vartheta_2(\omega_2\eta_2 + \eta_3(\varsigma + \omega_1 + \omega_2))) + \gamma_1(\delta_2\omega_2\eta_2 + (\varrho\delta_2 + \delta_2\eta_3)(\varsigma + \omega_1 + \omega_2))}{\varrho(\varsigma + \omega_1 + \omega_2)(\varsigma + \eta_1 + \eta_2 + \eta_3)}\Big).$$

An EE $\Phi_1^* = (\mathcal{S}^*, \mathcal{I}^*, \mathcal{H}^*, \mathcal{D}^*, \mathcal{P}^*)$ satisfies $\mathcal{S}^*, \mathcal{I}^*, \mathcal{H}^*, \mathcal{D}^*, \mathcal{P}^* > 0$. To obtain the EE $\Phi_1^* = (\mathcal{S}^*, \mathcal{I}^*, \mathcal{H}^*, \mathcal{D}^*, \mathcal{P}^*)$, we set the left hand side of System (3.2) equal to zero; then,

$$\begin{cases} \pi - (\sigma + \varsigma)\mathcal{S} = 0, \\ \sigma\mathcal{S} - (\varsigma + \eta_1 + \eta_2 + \eta_3)\mathcal{I} = 0, \\ \eta_2\mathcal{I} - (\varsigma + \omega_1 + \omega_2)\mathcal{H} = 0, \\ \eta_3\mathcal{I} + \omega_2\mathcal{H} - \varrho\mathcal{D} = 0, \\ \delta_1\mathcal{I} + \delta_2\mathcal{D} - \nu\mathcal{P} = 0. \end{cases} \tag{3.11}$$

From (3.11), we can deduce that a unique Φ_1^* exists with

$$\mathcal{S}^* = \frac{(\varsigma + \eta_1 + \eta_2 + \eta_3)(1 + \epsilon\theta_2\mathcal{I}^*)(\kappa + \mathcal{I}^*\theta_3)}{\gamma_2\psi(\kappa + \theta_3\mathcal{I}^*) + \gamma_1\theta_3(1 + \epsilon\mathcal{I}^*\theta_2)},$$

$$\mathcal{I}^* = \frac{-b_1 \pm \sqrt{b_1^2 - 4b_2b_0}}{2b_2},$$

$$\mathcal{H}^* = \theta_1\mathcal{I}^*,$$
$$\mathcal{D}^* = \theta_2\mathcal{I}^*,$$
$$\mathcal{P}^* = \theta_3\mathcal{I}^*,$$

where

$$\theta_1 = \frac{\eta_2}{\varsigma+\omega_1+\omega_2},\ \theta_2 = \frac{\eta_3+\omega_2\theta_1}{\varrho},\ \theta_3 = \frac{\delta_1+\delta_2\theta_2}{\nu},\ \psi = \gamma_2(1+\vartheta_1\theta_1+\vartheta_2\theta_2).$$

Also, we have

$$b_0 = \kappa\varsigma(\varsigma+\eta_1+\eta_2+\eta_3)(1-\mathbb{R}_0^D),$$
$$b_1 = (\varsigma+\eta_1+\eta_2+\eta_3)\Big[\kappa\gamma_2(1+\vartheta_1\theta_1+\vartheta_2\theta_2)+\kappa\epsilon\varsigma+(\gamma_1+\varsigma)\theta_3\Big]-\pi\Big[\gamma_2(1+\vartheta_1\theta_1+\vartheta_2\theta_2)+\gamma_1\epsilon\theta_2\Big],$$
$$b_2 = \gamma_2\theta_3(\varsigma+\eta_1+\eta_2+\eta_3)(1+\vartheta_1\theta_1+\vartheta_2\theta_2)+\epsilon(\varsigma+\eta_1+\eta_2+\eta_3)\theta_2\theta_3(\varsigma+\gamma_1)>0.$$

Lemma 3.2. *If there is an EE Φ_1 is locally and globally asymptotically stable, then $\mathbb{R}_0^D < 1$; otherwise, it is unstable if $\mathbb{R}_0^D > 1$.*

Proof. We omitted the lemma's explanation here because it is straightforward. □

4. Existence-uniqueness of non-negative solution

We propose the accompanying result to explain the existence-uniqueness of the stochastic framework of (3.3).

Theorem 4.1. *Suppose there is a unique solution of the stochastic model* (3.3) *for $\zeta \geq 0$ with ICs $(\mathcal{S}(0),\mathcal{I}(0),\mathcal{H}(0),\mathcal{D}(0),\mathcal{P}(0)) \in \mathbb{R}_+^5$. In addition, the solution will stay in $\mathbb{R}_+^5$ with a probability of* 1, *i.e., $(\mathcal{S}(0),\mathcal{I}(0),\mathcal{H}(0),\mathcal{D}(0),\mathcal{P}(0)) \in \mathbb{R}_+^5 \forall\zeta \geq 0$ almost surely (a. s).*

Proof. The system's coefficients supposed for the initial values settings $(\mathcal{S}(\zeta),\mathcal{I}(\zeta),\mathcal{H}(\zeta),\mathcal{D}(\zeta),\mathcal{P}(\zeta)) \in \mathbb{R}_+^5$ are continuous and locally lipschtz. Consequently, the system $(\mathcal{S}(\zeta),\mathcal{I}(\zeta),\mathcal{H}(\zeta),\mathcal{D}(\zeta),\mathcal{P}(\zeta))$ has only one solution for $\zeta \in [0,\varphi_\varepsilon)$. For the explosive period φ_ε is thoroughly examined in [48]. In order to show the solution's diverse nature, we must prove that $\varphi_\varepsilon = \infty$ (a.s). Assume that we do have a somewhat large positive number $\Bbbk_0$ such that every state's ICs fall inside the given interval $[\Bbbk_0, \frac{1}{\Bbbk_0}]$. Choose $\Bbbk \geq \Bbbk_0$ as the terminal duration specification for each non-negative integer:

$$\varphi_{\Bbbk} = \inf\Big\{\zeta \in [0,\varphi_\varepsilon) : \min\{\mathcal{S}(\zeta),\mathcal{I}(\zeta),\mathcal{H}(\zeta),\mathcal{D}(\zeta),\mathcal{P}(\zeta)\} \leq \frac{1}{\Bbbk}\ or\ \max\{\mathcal{S}(\zeta),\mathcal{I}(\zeta),\mathcal{H}(\zeta),\mathcal{D}(\zeta),\mathcal{P}(\zeta)\} \geq \Bbbk\Big\}.$$

Throughout this investigation, we will employ $\inf\phi = \infty$, whilst ϕ refers to an empty set. The idea of $\Bbbk$ compels us to claim that it increases as $\Bbbk$ approaches ∞. Fixing $\varphi_\infty = \lim_{\Bbbk\mapsto\infty}\varphi_\varepsilon \geq \varphi_\infty$ (a.s). After verifying that $\varphi_\infty = \infty$ (a.s), we argue that $\varphi_\varepsilon = \infty$ and thus $(\mathcal{S}(\zeta),\mathcal{I}(\zeta),\mathcal{H}(\zeta),\mathcal{D}(\zeta),\mathcal{P}(\zeta))$ stayed in $\mathbb{R}_+^5$ a.s. So, we will verify that $\varphi_\infty = \infty$ (a.s). For this, we suppose two non-negative fixed values $\varepsilon \in (0,1)$ and $\mathbf{T}$ must exist such that

$$P\{\mathbf{T} \geq \varphi_\infty\} > \epsilon. \tag{4.1}$$

So, the integer $\Bbbk_1 \geq \Bbbk_0$ exists in the subsequent version:

$$P\{\mathbf{T} \geq \varphi_{\Bbbk}\} \geq \epsilon,\ \Bbbk_1 \leq \Bbbk.$$

Thus, we shall investigate a mapping $\mathcal{J} : \mathbb{R}^5_+ \mapsto \mathbb{R}_+$ in the following manner:

$$\mathcal{J}(\mathcal{S}, \mathcal{I}, \mathcal{H}, \mathcal{D}, \mathcal{P}) = \mathcal{S} + \mathcal{I} + \mathcal{H} + \mathcal{D} + \mathcal{P} - 5 - (\ln \mathcal{S} + \ln \mathcal{I} + \ln \mathcal{H} + \ln \mathcal{D} + \ln \mathcal{P}). \tag{4.2}$$

$\mathcal{J}$ is a positive function, which should be noticed and may be confirmed by the argument that $0 \leq \mathbf{u}_1 - \ln \mathbf{u}_1 - 1,\ \forall \mathbf{u}_1 > 0$. Suppose the arbitrary terms $\Bbbk_0 \leq \Bbbk$ and $\mathbf{T} > 0$.

Employing Ito's technique to (4.2) yields,

$$\begin{aligned} d\mathcal{J}(\mathcal{S}, \mathcal{I}, \mathcal{H}, \mathcal{D}, \mathcal{P}) &= \mathcal{L}\mathcal{J}(\mathcal{S}, \mathcal{I}, \mathcal{H}, \mathcal{D}, \mathcal{P}) + \wp_1(\mathcal{S} - 1)d\mathfrak{B}_1(\zeta) + \wp_2(\mathcal{I} - 1)d\mathfrak{B}_2(\zeta) \\ &\quad + \wp_3(\mathcal{H} - 1)d\mathfrak{B}_3(\zeta) + \wp_4(\mathcal{D} - 1)d\mathfrak{B}_4(\zeta) + \wp_5(\mathcal{P} - 1)d\mathfrak{B}_5(\zeta). \end{aligned} \tag{4.3}$$

In view of (4.3), let us introduce the subsequent functional $\mathcal{L}\mathcal{J} : \mathbb{R}^5_+ \mapsto \mathbb{R}_+$ described as

$$\begin{aligned} \mathcal{L}\mathcal{J} &= \Big(1 - \frac{1}{\mathcal{S}}\Big)\Big(\pi - (\sigma + \varsigma)\mathcal{S}\Big) + \frac{\wp_1^2}{2} + \Big(1 - \frac{1}{\mathcal{I}}\Big)\Big(\sigma\mathcal{S} - (\varsigma + \eta_1 + \eta_2 + \eta_3)\mathcal{I}\Big) \\ &\quad + \frac{\wp_2^2}{2} + \Big(1 - \frac{1}{\mathcal{H}}\Big)\Big(\eta_2\mathcal{I} - (\varsigma + \omega_1 + \omega_2)\mathcal{H}\Big) + \frac{\wp_3^2}{2} + \Big(1 - \frac{1}{\mathcal{D}}\Big)\Big(\eta_3\mathcal{I} + \omega_2\mathcal{H} - \varrho\mathcal{D}\Big) \\ &\quad + \frac{\wp_4^2}{2} + \Big(1 - \frac{1}{\mathcal{P}}\Big)\Big(\delta_1\mathcal{I} + \delta_2\mathcal{D} - \nu\mathcal{P}\Big) + \frac{\wp_5^2}{2}. \end{aligned}$$

It follows that

$$\begin{aligned} \mathcal{L}\mathcal{J}(\mathcal{S}, \mathcal{I}, \mathcal{H}, \mathcal{D}, \mathcal{P}) &= \pi - d(\mathcal{S} + \mathcal{I} + \mathcal{H} + \mathcal{D} + \mathcal{P}) + (\sigma + \varsigma) + \frac{\pi}{\mathcal{S}} + \sigma\mathcal{S} - \sigma\frac{\mathcal{S}}{\mathcal{I}} + (\varsigma + \eta_1 + \eta_2 + \eta_3) \\ &\quad + \eta_2\mathcal{I} - \eta_2\frac{\mathcal{I}}{\mathcal{H}} - (\varsigma + \omega_1 + \omega_2) + \eta_3\mathcal{I} + \omega_2\mathcal{H} - \varrho\mathcal{D} - \eta_3\frac{\mathcal{I}}{\mathcal{D}} - \omega_2\frac{\mathcal{H}}{\mathcal{D}} \\ &\quad - \varrho + \delta_1\mathcal{I} + \delta_2\mathcal{D} - \nu\mathcal{P} - \delta_1\frac{\mathcal{I}}{\mathcal{P}} - \delta_2\frac{\mathcal{D}}{\mathcal{P}} - \nu + \frac{\wp_1^2 + \wp_2^2 + \wp_3^2 + \wp_4^2 + \wp_5^2}{2} \\ &\leq \pi + \sigma + \varsigma + \eta_1 + \eta_2 + \eta_3 - \omega_1 - \omega_2 - \varrho - \nu + \frac{\wp_1^2 + \wp_2^2 + \wp_3^2 + \wp_4^2 + \wp_5^2}{2} := \Omega. \end{aligned}$$

Therefore, we have

$$\begin{aligned} &\mathcal{U}\Big[\mathcal{J}(\mathcal{S}(\varphi_{\Bbbk} \wedge \mathbf{T}), \mathcal{I}(\varphi_{\Bbbk} \wedge \mathbf{T}), \mathcal{H}(\varphi_{\Bbbk} \wedge \mathbf{T}), \mathcal{D}(\varphi_{\Bbbk} \wedge \mathbf{T}), \mathcal{P}(\varphi_{\Bbbk} \wedge \mathbf{T}))\Big] \\ \leq &\mathcal{J}(\mathcal{S}(0), \mathcal{I}(0), \mathcal{H}(0), \mathcal{D}(0), \mathcal{P}(0)) + \mathcal{U}\Big\{\int_0^{\varphi_{\Bbbk} \wedge \mathbf{T}} \Omega d\zeta\Big\} \\ \leq &\mathcal{J}(\mathcal{S}(0), \mathcal{I}(0), \mathcal{H}(0), \mathcal{D}(0), \mathcal{P}(0)) + \Omega\mathbf{T}. \end{aligned} \tag{4.4}$$

Let $\Psi_{\Bbbk} = \{\varphi_{\Bbbk} \leq \mathbf{T}\}$ for $\Bbbk \geq \Bbbk_1$; (4.1) yields that $P(\psi_\kappa) \geq \epsilon$. Clearly, for every ω from $\Omega_{\Bbbk}$ there exists at least one $\mathcal{S}(\varphi_{\Bbbk}, \omega), \mathcal{I}(\varphi_{\Bbbk}, \omega), \mathcal{H}(\varphi_{\Bbbk}, \omega), \mathcal{D}(\varphi_{\Bbbk}, \omega)$ and $\mathcal{P}(\varphi_{\Bbbk}, \omega)$ that is equal to $\frac{1}{\Bbbk}$ or $\Bbbk$. Hence, $\mathcal{J}(\mathcal{S}(\varphi_{\Bbbk}), \mathcal{I}(\varphi_{\Bbbk}), \mathcal{H}(\varphi_{\Bbbk}), \mathcal{D}(\varphi_{\Bbbk}), \mathcal{P}(\varphi_{\Bbbk}))$ is no less than $\ln \Bbbk - 1 + \frac{1}{\Bbbk}$ or $\Bbbk - 1 - \ln \Bbbk$.

Consequently,

$$\mathcal{J}(\mathcal{S}(\varphi_{\Bbbk}), \mathcal{I}(\varphi_{\Bbbk}), \mathcal{H}(\varphi_{\Bbbk}), \mathcal{D}(\varphi_{\Bbbk}), \mathcal{P}(\varphi_{\Bbbk})) \geq \Big(\ln \Bbbk - 1 + \frac{1}{\Bbbk}\Big) \wedge \mathcal{U}(\Bbbk - 1 - \ln \Bbbk).$$

Using the fact of (4.1) and (4.4), we have

$$\begin{aligned}\mathcal{J}(\mathcal{S}(0), \mathcal{I}(0), \mathcal{H}(0), \mathcal{D}(0), \mathcal{P}(0)) + \Bbbk\mathbf{T} &\geq \mathcal{U}\Big[1_{\Psi(\omega)}\mathcal{J}(\mathcal{S}(\varphi_{\Bbbk}), \mathcal{I}(\varphi_{\Bbbk}), \mathcal{H}(\varphi_{\Bbbk}), \mathcal{D}(\varphi_{\Bbbk}), \mathcal{P}(\varphi_{\Bbbk}))\Big] \\ &\geq \epsilon\Big\{\Big(\ln \Bbbk - 1 + \frac{1}{\Bbbk}\Big) \wedge (\Bbbk - 1 - \ln \Bbbk)\Big\}.\end{aligned}$$

As seen, the indicator mapping of Ψ is $1_{\Psi(\omega)}$. Thus, applying limit $\Bbbk \mapsto \infty$ yields contradiction $\infty > \mathcal{J}(\mathcal{S}(0), \mathcal{I}(0), \mathcal{H}(0), \mathcal{D}(0), \mathcal{P}(0)) + \mathcal{M}\mathbf{T} = \infty$, showing that $\varphi_{\infty} = \infty$ a.s. □

4.1. Basic reproduction number for stochastic model

Taking into account the infected cohort of (3.3), we find the fundamental reproductive value for the stochastic scheme:

$$d\mathcal{I} = \Big[\sigma\mathcal{S}(\zeta) - (\varsigma + \eta_1 + \eta_2 + \eta_3)\mathcal{I}(\zeta)\Big] + \wp_2\mathcal{I}(\zeta)d\mathfrak{B}_2(\zeta).$$

Applying Ito's formula with twice differentiablity, we can find the stochastic reproductive value. For this let us choose, $\upsilon(\zeta, \mathcal{I}(\zeta)) = \ln(\mathcal{I}(\zeta))$; then, the well-noted Taylor expansion gives

$$\Upsilon(\zeta, \mathcal{I}(\zeta)) = \frac{\partial \Upsilon}{\partial \zeta}d\zeta + \frac{\partial \Upsilon}{\partial \mathcal{I}\zeta}d\mathcal{I}(\zeta) + \frac{1}{2}\frac{\partial^2 \Upsilon}{\partial \mathcal{I}^2(\zeta)}(d\mathcal{I}(\zeta))^2 + \frac{\partial^2 \Upsilon}{\partial \zeta \partial \mathcal{I}}d\zeta d\mathcal{I} + \frac{1}{2}\frac{\partial^2 \Upsilon}{\partial \zeta^2}(d\zeta)^2, \tag{4.5}$$

where $\frac{\partial \Upsilon}{\partial \zeta} = 0$, $\frac{\partial \Upsilon}{\partial \mathcal{I}\zeta} = \frac{1}{\mathcal{I}(\zeta)}$, $\frac{\partial^2 \Upsilon}{\partial \mathcal{I}^2(\zeta)} = -\frac{1}{\mathcal{I}^2(\zeta)}$, $\frac{\partial^2 \Upsilon}{\partial \zeta \partial \mathcal{I}} = 0$ *and* $\frac{\partial^2 \Upsilon}{\partial \zeta^2} = 0$.

Therefore, (4.5) reduces to

$$\begin{aligned}\Upsilon(\zeta, \mathcal{I}(\zeta)) &= \frac{1}{\mathcal{I}(\zeta)}d\mathcal{I}(\zeta) - \frac{1}{2\mathcal{I}^2(t)}(d\mathcal{I}(\zeta))^2 \\ &= \frac{1}{\mathcal{I}(\zeta)}\Big[\{\sigma\mathcal{S} - (\varsigma + \eta_1 + \eta_2 + \eta_3)\mathcal{I}\}d\zeta + \wp_2\mathcal{I}(\zeta)d\mathfrak{B}_2(\zeta)\Big] \\ &\quad - \frac{1}{2\mathcal{I}^2(\zeta)}\Big[\{\sigma\mathcal{S} - (\varsigma + \eta_1 + \eta_2 + \eta_3)\mathcal{I}\}d\zeta + \wp_2\mathcal{I}(\zeta)d\mathfrak{B}_2(\zeta)\Big]^2.\end{aligned}$$

Suppose $\mathcal{K}_1 = \sigma\mathcal{S}(\zeta) - (\varsigma + \eta_1 + \eta_2 + \eta_3)\mathcal{I}(\zeta)$ and $\mathcal{K}_2 = \wp_2\mathcal{I}(\zeta)$; then,

$$\begin{aligned}\Upsilon(\zeta, \mathcal{I}(\zeta)) &= \Big(\sigma\frac{\mathcal{S}}{\mathcal{I}} - (\varsigma + \eta_1 + \eta_2 + \eta_3)\Big)d\zeta + \wp_2 d\mathfrak{B}_2(\zeta) - \frac{1}{2\mathcal{I}^2(\zeta)}\Big[\mathcal{K}_1 d\zeta + \mathcal{K}_2 d\mathfrak{B}_2(\zeta)\Big]^2 \\ &= \Big(\sigma\frac{\mathcal{S}}{\mathcal{I}} - (\varsigma + \eta_1 + \eta_2 + \eta_3)\Big)d\zeta + \wp_2 d\mathfrak{B}_2(\zeta) \\ &\quad - \frac{1}{2\mathcal{I}^2(\zeta)}\{\mathcal{K}_1^2 d^2\zeta + 2\mathcal{K}_1\mathcal{K}_2 d\zeta d\mathfrak{B}_2(\zeta) + \mathcal{K}_2^2 d^2\mathfrak{B}_2(\zeta)\} \\ &= \Big(\sigma\frac{\mathcal{S}}{\mathcal{I}} - (\varsigma + \eta_1 + \eta_2 + \eta_3)\Big)d\zeta + \wp_2 d\mathfrak{B}_2(\zeta) - \frac{1}{2\mathcal{I}^2(\zeta)}\mathcal{K}_2^2 d^2\mathfrak{B}_2(\zeta).\end{aligned}$$

Employing the chain rule, we have

$$\begin{aligned} &d\zeta.d\zeta = 0,\\ &d\zeta.d\mathfrak{B}(\zeta) = 0,\\ &d\mathfrak{B}(\zeta).d\mathfrak{B}(\zeta) = d^2\mathfrak{B}(\zeta) = d\zeta. \end{aligned}$$

Therefore, we have

$$\begin{aligned} \Upsilon(\zeta, \mathcal{I}(\zeta)) &= \Big(\sigma\frac{\mathcal{S}}{\mathcal{I}} - (\varsigma + \eta_1 + \eta_2 + \eta_3)\Big)d\zeta + \wp_2 d\mathfrak{B}_2(\zeta) - \frac{1}{2\mathcal{I}^2(\zeta)}\wp_2^2\mathcal{I}^2(\zeta)d^2\mathfrak{B}_2(\zeta)\\ &= \Big(\sigma\frac{\mathcal{S}}{\mathcal{I}} - (\varsigma + \eta_1 + \eta_2 + \eta_3)\Big)d\zeta + \wp_2 d\mathfrak{B}_2(\zeta) - \frac{1}{2}\wp_2^2 d\zeta\\ &= \Big(\sigma\frac{\mathcal{S}}{\mathcal{I}} - \frac{1}{2}\wp_2^2 - (\varsigma + \eta_1 + \eta_2 + \eta_3)\Big)d\zeta + \wp_2 d\mathfrak{B}_2(\zeta). \end{aligned}$$

By using the next generation matrix, let $f_1 = \sigma\mathcal{S}/\mathcal{I} - \frac{1}{2}\wp_2^2$ and $\upsilon = \varsigma + \eta_1 + \eta_2 + \eta_3$.

Now f_1 and υ at DFE reduces to $f_1 = \sigma\frac{\pi}{\varsigma} - \frac{1}{2}\wp_2^2$ and $\upsilon^{-1} = \frac{1}{\varsigma+\eta_1+\eta_2+\eta_3}$.
Therefore,

$$f_1\upsilon^{-1} = \frac{\sigma\frac{\pi}{\varsigma} - \frac{1}{2}\wp_2^2}{\varsigma + \eta_1 + \eta_2 + \eta_3}.$$

Furthermore, the fundamental reproductive value for the stochastic system is

$$\mathbb{R}_0^s = \frac{\sigma\frac{\pi}{\varsigma} - \frac{1}{2}\wp_2^2}{\varsigma + \eta_1 + \eta_2 + \eta_3}.$$

4.2. Extinction

This subsection emphasizes the specifications for a disease's systemic (3.3) eradication. Before verifying the substantial discoveries, let us take a closer look at a vital concept.

Suppose

$$\langle \mathbf{X}(\zeta)\rangle = \frac{1}{\zeta}\int_0^{\zeta} \mathbf{x}(\mathbf{r})d\mathbf{r}. \tag{4.6}$$

Lemma 4.1. *[49] (**Strong law of large number**) Suppose there is a continuous and real-valued local martingale $\mathbf{W} = \{\mathbf{W}\}_{\zeta\geq 0}$ that disappears at $\zeta = 0$*

$$\begin{aligned} &\lim_{\zeta\mapsto\infty}\langle \mathbf{W}, \mathbf{W}\rangle_\zeta = \infty,\ a.s., \implies \lim_{\zeta\mapsto\infty}\frac{\langle \mathbf{W}_\zeta\rangle}{\langle \mathbf{W}, \mathbf{W}\rangle_\zeta} = 0, a.s.,\\ &\textit{and}\\ &\lim_{\zeta\mapsto\infty}\sup\frac{\langle \mathbf{W}, \mathbf{W}\rangle_\zeta}{\zeta} < 0,\ a.s, \implies \lim_{\zeta\mapsto\infty}\frac{\langle \mathbf{W}\rangle_\zeta}{\zeta} = 0, a.s. \end{aligned} \tag{4.7}$$

Lemma 4.2. *Consider the given ICs* $(\mathcal{S}(0), \mathcal{I}(0), \mathcal{H}(0), \mathcal{D}(0), \mathcal{P}(0)) \in \mathbb{R}^5_+$; *then, the solution* $(\mathcal{S}(\zeta), \mathcal{I}(\zeta), \mathcal{H}(\zeta), \mathcal{D}(\zeta), \mathcal{P}(\zeta))$ *for the model* (3.3) *has the subsequent characteristics:*

$$\begin{aligned}
&\lim_{\zeta\mapsto\infty} \frac{\mathcal{S}(\zeta)}{\zeta} = 0,\\
&\lim_{\zeta\mapsto\infty} \frac{\mathcal{I}(\zeta)}{\zeta} = 0,\\
&\lim_{\zeta\mapsto\infty} \frac{\mathcal{H}(\zeta)}{\zeta} = 0,\\
&\lim_{\zeta\mapsto\infty} \frac{\mathcal{D}(\zeta)}{\zeta} = 0,\\
&\lim_{\zeta\mapsto\infty} \frac{\mathcal{P}(\zeta)}{\zeta} = 0.
\end{aligned} \tag{4.8}$$

Moreover, when $\tilde{\mathbf{q}} > \frac{\wp_1^2 \vee \wp_2^2 \vee \wp_3^2 \vee \wp_4^2 \vee \wp_5^2}{2}$ *exists, then*

$$\begin{aligned}
&\lim_{\zeta\mapsto\infty} \frac{1}{\zeta} \int_0^\zeta \mathcal{S}(\mathbf{r}) d\mathfrak{B}_1(\mathbf{r}) = 0,\\
&\lim_{\zeta\mapsto\infty} \frac{1}{\zeta} \int_0^\zeta \mathcal{I}(\mathbf{r}) d\mathfrak{B}_2(\mathbf{r}) = 0,\\
&\lim_{\zeta\mapsto\infty} \frac{1}{\zeta} \int_0^\zeta \mathcal{H}(\mathbf{r}) d\mathfrak{B}_3(\mathbf{r}) = 0,\\
&\lim_{\zeta\mapsto\infty} \frac{1}{\zeta} \int_0^\zeta \mathcal{D}(\mathbf{r}) d\mathfrak{B}_1(\mathbf{r}) = 0,\\
&\lim_{\zeta\mapsto\infty} \frac{1}{\zeta} \int_0^\zeta \mathcal{P}(\mathbf{r}) d\mathfrak{B}_5(\mathbf{r}) = 0 \ \ a.s.
\end{aligned} \tag{4.9}$$

Proof. We can obtain the proof of Lemma 4.2 by following the work of [50]. □

Theorem 4.2. *For* $\mathbb{R}_0^s < 1$ *and* $\tilde{\mathbf{q}} > \frac{\wp_1^2 \vee \wp_2^2 \vee \wp_3^2 \vee \wp_4^2 \vee \wp_5^2}{2}$, *then the roots satisfying the framework* (3.3) *are presented as follows*

$$\lim_{\zeta\mapsto\infty} \frac{\ln\left(\sigma\mathcal{S}(\zeta) - (\varsigma + \eta_1 + \eta_2 + \eta_3)\mathcal{I}(\zeta)\right)}{\zeta} \leq \frac{\sigma^2 \frac{\wp_1^2}{2} \wedge (\varsigma + \eta_1 + \eta_2 + \eta_3)\frac{\wp_2^2}{2}}{2(\sigma - (\varsigma + \eta_1 + \eta_2 + \eta_3))^2}(\mathbb{R}_0^s - 1) < 0. \tag{4.10}$$

Proof. Let us introduce a differentiable mapping Υ as

$$\Upsilon = \ln\left(\sigma\mathcal{S}(\zeta) - (\varsigma + \eta_1 + \eta_2 + \eta_3)\mathcal{I}(\zeta)\right). \tag{4.11}$$

Applying Ito's technique and using the system (3.3), we have

$$
\begin{aligned}
d\Upsilon =& \Big\{\frac{\sigma(-(\sigma+\varsigma))}{\sigma\mathcal{S}(\zeta)-(\varsigma+\eta_1+\eta_2+\eta_3)\mathcal{I}(\zeta)} - \frac{\sigma^2\wp_1^2\mathcal{S}^2-(\varsigma+\eta_1+\eta_2+\eta_3)\mathcal{I}^2\wp_2^2}{2(\sigma\mathcal{S}(\zeta)-(\varsigma+\eta_1+\eta_2+\eta_3)\mathcal{I}(\zeta))^2}\Big\}d\zeta \\
&+ \frac{\sigma\wp_1\mathcal{S}}{\sigma\mathcal{S}(\zeta)-(\varsigma+\eta_1+\eta_2+\eta_3)\mathcal{I}(\zeta)}d\mathfrak{B}_1 - \frac{(\varsigma+\eta_1+\eta_2+\eta_3)\wp_2\mathcal{I}}{\sigma\mathcal{S}(\zeta)-(\varsigma+\eta_1+\eta_2+\eta_3)\mathcal{I}(\zeta)}d\mathfrak{B}_2 \\
\le& \Big\{\frac{-\sigma(\sigma+\varsigma)}{\sigma-(\varsigma+\eta_1+\eta_2+\eta_3)} - \frac{\sigma^2\frac{\wp_1^2}{2}\mathcal{S}^2+(\varsigma+\eta_1+\eta_2+\eta_3)\frac{\wp_2^2}{2}\mathcal{I}^2}{2(\sigma\mathcal{S}(\zeta)-(\varsigma+\eta_1+\eta_2+\eta_3)\mathcal{I}(\zeta))^2}\Big\}dt \\
&+ \frac{\sigma\wp_1\mathcal{S}}{\sigma\mathcal{S}(\zeta)-(\varsigma+\eta_1+\eta_2+\eta_3)\mathcal{I}(\zeta)}d\mathfrak{B}_1 - \frac{(\varsigma+\eta_1+\eta_2+\eta_3)\wp_2\mathcal{I}}{\sigma\mathcal{S}(\zeta)-(\varsigma+\eta_1+\eta_2+\eta_3)\mathcal{I}(\zeta)}d\mathfrak{B}_2 \\
=& \Big\{\frac{-\sigma(\sigma+\varsigma)}{\sigma-(\varsigma+\eta_1+\eta_2+\eta_3)} - \frac{\sigma^2\frac{\wp_1^2}{2}\wedge(\varsigma+\eta_1+\eta_2+\eta_3)\frac{\wp_2^2}{2}}{2(\sigma-(\varsigma+\eta_1+\eta_2+\eta_3))^2}\Big\}dt \\
&+ \frac{\sigma\wp_1\mathcal{S}}{\sigma\mathcal{S}(\zeta)-(\varsigma+\eta_1+\eta_2+\eta_3)\mathcal{I}(\zeta)}d\mathfrak{B}_1 - \frac{(\varsigma+\eta_1+\eta_2+\eta_3)\wp_2\mathcal{I}}{\sigma\mathcal{S}(\zeta)-(\varsigma+\eta_1+\eta_2+\eta_3)\mathcal{I}(\zeta)}d\mathfrak{B}_2.
\end{aligned} \tag{4.12}
$$

Now, by dividing (4.12) by ζ on both sides and integrating from 0 to ζ, we have

$$
\begin{aligned}
\frac{\ln(\sigma\mathcal{S}(\zeta)-(\varsigma+\eta_1+\eta_2+\eta_3)\mathcal{I}(\zeta))}{\zeta} \le& \frac{-\sigma(\sigma+\varsigma)}{\sigma-(\varsigma+\eta_1+\eta_2+\eta_3)} \\
&- \frac{\sigma^2\frac{\wp_1^2}{2}\wedge(\varsigma+\eta_1+\eta_2+\eta_3)\frac{\wp_2^2}{2}}{2(\sigma-(\varsigma+\eta_1+\eta_2+\eta_3))^2}\frac{\ln(\sigma\mathcal{S}(0)-(\varsigma+\eta_1+\eta_2+\eta_3)\mathcal{I}(0))}{\zeta} \\
&+ \frac{\sigma\wp_1}{\zeta}\int_0^\zeta \frac{\mathcal{S}(\mathbf{r})}{\sigma\mathcal{S}(\mathbf{r})-(\varsigma+\eta_1+\eta_2+\eta_3)\mathcal{I}(\mathbf{r})}d\mathfrak{B}_1 \\
&- \frac{(\varsigma+\eta_1+\eta_2+\eta_3)\wp_2}{\zeta}\int_0^\zeta \frac{\mathcal{I}(\mathbf{r})}{\sigma\mathcal{S}(\mathbf{r})-(\varsigma+\eta_1+\eta_2+\eta_3)\mathcal{I}(\mathbf{r})}d\mathfrak{B}_2.
\end{aligned} \tag{4.13}
$$

In view of Lemma 4.1, we have

$$
\begin{aligned}
\lim_{\zeta\mapsto\infty}\frac{\ln(\sigma\mathcal{S}(\zeta)-(\varsigma+\eta_1+\eta_2+\eta_3)\mathcal{I}(\zeta))}{\zeta} \le& \frac{-\sigma(\sigma+\varsigma)}{\sigma-(\varsigma+\eta_1+\eta_2+\eta_3)} - \frac{\sigma^2\frac{\wp_1^2}{2}\wedge(\varsigma+\eta_1+\eta_2+\eta_3)\frac{\wp_2^2}{2}}{2(\sigma-(\varsigma+\eta_1+\eta_2+\eta_3))^2} < 0 \\
\le& \frac{\sigma^2\frac{\wp_1^2}{2}\wedge(\varsigma+\eta_1+\eta_2+\eta_3)\frac{\wp_2^2}{2}}{2(\sigma-(\varsigma+\eta_1+\eta_2+\eta_3))^2}(\mathbb{R}_0^s-1) < 0, \ (a.s),
\end{aligned}
$$

which shows that

$$
\begin{aligned}
&\lim_{\zeta\mapsto\infty}\langle\mathcal{I}(\zeta)\rangle = 0, \\
&\lim_{\zeta\mapsto\infty}\langle\mathcal{H}(\zeta)\rangle = 0, \\
&\lim_{\zeta\mapsto\infty}\langle\mathcal{D}(\zeta)\rangle = 0,
\end{aligned}
$$

$$\lim_{\zeta\mapsto\infty}\langle\mathcal{P}(\zeta)\rangle = 0.$$

Also, on the first cohort of (3.3), performing integration from 0 to ζ we find

$$\frac{\mathcal{S}(\zeta)-\mathcal{S}(0)}{\zeta} = \pi - (\sigma+\varsigma)\langle\mathcal{S}\rangle + \frac{\wp_1}{\zeta}\int_0^{\zeta}\mathcal{S}(\mathbf{r})d\mathfrak{B}_1(\mathbf{r}).$$

By making the use of Lemma 4.1 and utilizing the expression $\sigma^* = \frac{\psi}{1+\epsilon\mathcal{I}^*+\theta_2} + \frac{\gamma_1\theta_3}{\kappa+\mathcal{I}^*\theta_3}\mathcal{I}^*$, we have

$$\lim_{\zeta\mapsto\infty}\langle\mathcal{I}(\zeta)\rangle = \frac{\pi}{\varsigma} = \mathcal{S}_0 \ (a.s).$$

Hence, the proof of Lemma 4.2 is completed. □

4.3. Ergodicity and stationary distribution

As we know, there is no EE in stochastic processes. The stability evaluation might therefore be performed to inquire into the disease's permanence. To combat infection prevalence, one should focus on the availability and distinctness of concepts for the stationary distribution. For this purpose, we shall employ Khasminskii acclaimed concept [51].

Suppose there is a regular Markov technique $W_1(\zeta)$ in $\mathbb{C}^n_+$ which is stated as

$$dW_1(\zeta) = b_1(W_1)d\zeta + \sum_{\mathbf{s}}^{\ell}\lambda_\ell d\mathfrak{B}_\ell(\zeta).$$

Now, the diffusion matrix is defined as

$$\mathbb{A}(W_1) = [a_{ij}(w)],\ a_{ij}(w) = \sum_{\mathbf{s}=1}^{\ell}\lambda^i_{\mathbf{s}}(w)\lambda^{\mathbf{s}}_j(w).$$

Lemma 4.3. *[48] Suppose there is a unique stationary distribution process $W_1(\zeta)$. Also, assume that a bounded region has a regular boundary such that $V,\ \bar{V}\in\mathbb{C}^d\bar{V}$ closure $\bar{V}\in\mathbb{C}^d$ satisfies the following*
(1) *The lowest eigenvalue for $\mathbb{A}(\zeta)$ is bounded away from $(0,0)$ for the open region V having its neighborhood.*
(2) *For $w\in\mathbb{C}^dV$, the mean time is bounded and for every compact subset $C\subset\mathbb{C}^n$ i.e., $\sup_{w\in C}V^w\varphi<\infty$. When an integrable mapping $f_1(.)$ with the measure Θ, we have*

$$P\Big(\lim_{\mathbf{T}\mapsto\infty}\frac{1}{\mathbf{T}}\int_0^{\mathbf{T}} f_1(W_w(\zeta))d\zeta = \int_{\mathbb{C}^d} f_1(w)\Theta(dw)\Big) = 1,\quad \forall w\in\mathbb{C}^d. \tag{4.14}$$

Theorem 4.3. *Suppose that there is a model $(\mathcal{S}(\zeta),\mathcal{I}(\zeta),\mathcal{H}(\zeta),\mathcal{D}(\zeta),\mathcal{P}(\zeta))$ (3.3) that is ergodic with a unique stationary distribution. Since $\mathbb{R}^s_0>1$ and $\Theta(.)$ is utilized.*

Proof. In order to prove the second assumption of Lemma 4.3, we introduce a positive $\mathbb{R}^2$ mapping in such that $Z_1 : \mathbb{R}^5_+ \mapsto \mathbb{R}_+$. For this, we have

$$Z_1 = \mathcal{S} + \mathcal{I} + \mathcal{H} + \mathcal{D} + \mathcal{P} - q_1 \ln \mathcal{S} - q_2 \ln \mathcal{I} - q_3 \ln \mathcal{H}.$$

Consequently, it is necessary to determine the positive constants $q_1,\ q_2$ and q_3. Using Ito's technique and the specified approach (3.3), we obtain the findings shown below

$$\mathcal{L}(\mathcal{S} + \mathcal{I} + \mathcal{H} + \mathcal{D} + \mathcal{P}) = \Theta - d(\mathcal{S}(\zeta) + \mathcal{I}(\zeta) + \mathcal{H}(\zeta) + \mathcal{D}(\zeta) + \mathcal{P}(\zeta)).$$

It follows that

$$\begin{aligned}
\mathcal{L}(-\ln \mathcal{S}) &= -\frac{\pi}{\mathcal{S}} + (\sigma + \varsigma) + \frac{\wp_1^2}{2},\\
\mathcal{L}(-\ln \mathcal{I}) &= -\frac{\sigma}{\mathcal{I}} + (\varsigma + \eta_1 + \eta_2 + \eta_3) + \frac{\wp_2^2}{2},\\
\mathcal{L}(-\ln \mathcal{H}) &= -\frac{\eta_2}{\mathcal{H}} + (\varsigma + \omega_1 + \omega_2) + \frac{\wp_3^2}{2},\\
\mathcal{L}(-\ln \mathcal{D}) &= -\eta_3\frac{\mathcal{I}}{\mathcal{D}} - \omega_2\frac{\mathcal{H}}{\mathcal{D}} + \varrho + \frac{\wp_4^2}{2},\\
\mathcal{L}(-\ln \mathcal{P}) &= -\delta_1\frac{\mathcal{I}}{\mathcal{P}} - \delta_2\frac{\mathcal{D}}{\mathcal{P}} + \nu + \frac{\wp_5^2}{2}.
\end{aligned}$$

Now, we have

$$\begin{aligned}
\mathcal{L}Z_1 = & -d(\mathcal{S}(\zeta) + \mathcal{I}(\zeta) + \mathcal{H}(\zeta) + \mathcal{D}(\zeta) + \mathcal{P}(\zeta)) - q_1\Big(-\frac{\pi}{\mathcal{S}} + (\sigma + \varsigma) + \frac{\wp_1^2}{2}\Big)\\
& - q_2\Big(-\frac{\sigma}{\mathcal{I}} + (\varsigma + \eta_1 + \eta_2 + \eta_3) + \frac{\wp_2^2}{2}\Big) - q_3\Big(-\frac{\eta_2}{\mathcal{H}} + (\varsigma + \omega_1 + \omega_2) + \frac{\wp_3^2}{2}\Big).
\end{aligned}$$

This implies that

$$\begin{aligned}
\mathcal{L}Z_1 \le & -4\Big\{d(\mathcal{S}(\zeta) + \mathcal{I}(\zeta) + \mathcal{H}(\zeta) + \mathcal{D}(\zeta) + \mathcal{P}(\zeta)) \times \frac{q_1\pi}{\mathcal{S}} \times \frac{q_2\sigma}{\mathcal{I}} \times \frac{\eta_2}{\mathcal{H}}\Big\}^{1/4}\\
& - q_1\Big(\sigma + \varsigma + \frac{\wp_1^2}{2}\Big) - q_2\Big((\varsigma + \eta_1 + \eta_2 + \eta_3) + \frac{\wp_2^2}{2}\Big) - q_3\Big((\varsigma + \omega_1 + \omega_2) + \frac{\wp_3^2}{2}\Big).
\end{aligned}$$

Let

$$q_1\Big(\sigma + \varsigma + \frac{\wp_1^2}{2}\Big) = q_2\Big((\varsigma + \eta_1 + \eta_2 + \eta_3) + \frac{\wp_2^2}{2}\Big) = q_3\Big((\varsigma + \omega_1 + \omega_2) + \frac{\wp_3^2}{2}\Big) = \nabla.$$

Namely

$$\begin{aligned}
q_1 &= \frac{\nabla}{(\sigma + \varsigma + \frac{\wp_1^2}{2}},\\
q_2 &= \frac{\nabla}{(\varsigma + \eta_1 + \eta_2 + \eta_3) + \frac{\wp_2^2}{2}},
\end{aligned}$$

$$q_3 = \frac{\nabla}{(\varsigma + \omega_1 + \omega_2) + \frac{\wp_3^2}{2}}.$$

Thus, we have

$$\mathcal{L}Z_1 \leq -4\Big\{\Big(\frac{\nabla^4}{\Big((\sigma + \varsigma + \frac{\wp_1^2}{2}\Big)\Big((\varsigma + \eta_1 + \eta_2 + \eta_3) + \frac{\wp_2^2}{2}\Big)\Big((\varsigma + \omega_1 + \omega_2) + \frac{\wp_3^2}{2}\Big)}\Big) - \Theta\Big\}^{1/4} \leq -4\nabla[(\mathbb{C}_0^H)^{1/4} - 1].$$

Additionally, we find that

$$\begin{aligned} Z_2 &= q_4(\mathcal{S} + \mathcal{I} + \mathcal{H} + \mathcal{D} + \mathcal{P} - q_1 \ln \mathcal{S} - q_2\mathcal{I} - q_3\mathcal{H}) \\ &\quad - \ln \mathcal{S} - \ln \mathcal{I} - \ln \mathcal{H} + \mathcal{S}(\zeta) + \mathcal{I}(\zeta) + \mathcal{H}(\zeta) + \mathcal{D}(\zeta) + \mathcal{P}(\zeta) \\ &= (q_4 + 1)(\mathcal{S} + \mathcal{I} + \mathcal{H} + \mathcal{D} + \mathcal{P}) - (q_1q_4 + 1) \ln \mathcal{S} - q_2q_4 \ln \mathcal{I} - q_3q_4 \ln \mathcal{H} - \ln \mathcal{D} - \ln \mathcal{P}. \end{aligned}$$

Since $q_4 > 0$ is a constant, it helps to give an explanation of that

$$\lim_{(\mathcal{S},\mathcal{I},\mathcal{H},\mathcal{D},\mathcal{P})\in\mathbb{R}_+^5\setminus\mathbb{U}_{\Bbbk}} \inf Z_2(\mathcal{S}, \mathcal{I}, \mathcal{H}, \mathcal{D}, \mathcal{P}) = +\infty, \quad as\ \Bbbk \mapsto \infty. \tag{4.15}$$

Furthermore $\mathbb{U}_{\Bbbk} = (\frac{1}{\Bbbk}, \Bbbk) \times (\frac{1}{\Bbbk}, \Bbbk) \times (\frac{1}{\Bbbk}, \Bbbk)$. The next process is to demonstrate that $Z_2(\mathcal{S}, \mathcal{I}, \mathcal{H}, \mathcal{D}, \mathcal{P})$ has one minimum value $Z_2(\mathcal{S}_0, \mathcal{I}_0, \mathcal{H}_0, \mathcal{D}_,, \mathcal{P}_0)$.

The partial derivatives of $Z_2(\mathcal{S}, \mathcal{I}, \mathcal{H}, \mathcal{D}, \mathcal{P})$ with respect to $\mathcal{S}, \mathcal{I}, \mathcal{H}, \mathcal{D}$ *and* $\mathcal{P}$ presented as follows

$$\begin{aligned} \frac{\partial Z_2(\mathcal{S}, \mathcal{I}, \mathcal{H}, \mathcal{D}, \mathcal{P})}{\partial \mathcal{S}} &= 1 + q_4 - \frac{1}{\mathcal{S}}(1 + q_1q_4), \\ \frac{\partial Z_2(\mathcal{S}, \mathcal{I}, \mathcal{H}, \mathcal{D}, \mathcal{P})}{\partial \mathcal{I}} &= 1 + q_4 - \frac{1}{\mathcal{I}}q_2q_4, \\ \frac{\partial Z_2(\mathcal{S}, \mathcal{I}, \mathcal{H}, \mathcal{D}, \mathcal{P})}{\partial \mathcal{H}} &= 1 + q_4 - \frac{1}{\mathcal{H}}q_3q_4, \\ \frac{\partial Z_2(\mathcal{S}, \mathcal{I}, \mathcal{H}, \mathcal{D}, \mathcal{P})}{\partial \mathcal{D}} &= 1 + q_4 - \frac{1}{\mathcal{D}}, \\ \frac{\partial Z_2(\mathcal{S}, \mathcal{I}, \mathcal{H}, \mathcal{D}, \mathcal{P})}{\partial \mathcal{P}} &= 1 + q_4 - \frac{1}{\mathcal{P}}. \end{aligned}$$

It is simple to prove that Z_2 has a distinctive stagnation point:

$$(\mathcal{S}(0), \mathcal{I}(0), \mathcal{H}(0), \mathcal{D}(0), \mathcal{P}(0)) = \Big(\frac{1 + q_1q_4}{1 + q_4}, \frac{q_2q_4}{1 + q_4}, \frac{q_3q_4}{1 + q_4}, \frac{1}{1 + q_4}, \frac{1}{1 + q_4}\Big).$$

Also, the Hessian matrix of $Z_2(\mathcal{S}, \mathcal{I}, \mathcal{H}, \mathcal{D}, \mathcal{P})$ at $(\mathcal{S}(0), \mathcal{I}(0), \mathcal{H}(0), \mathcal{D}(0), \mathcal{P}(0))$ is

$$\mathbb{Q} = \begin{bmatrix} \frac{1+q_1q_4}{\mathcal{S}^2(0)} & 0 & 0 & 0 & 0 \\ 0 & \frac{q_2q_4}{\mathcal{I}^2(0)} & 0 & 0 & 0 \\ 0 & 0 & \frac{q_3q_4}{\mathcal{H}^2(0)} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{\mathcal{D}^2(0)} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{\mathcal{P}^2(0)} \end{bmatrix}.$$

It is evident that the aforesaid matrix is positive definite. Ultimately, $Z_2(\mathcal{S},\mathcal{I},\mathcal{H},\mathcal{D},\mathcal{P})$ has the least value $Z_2(\mathcal{S}(0),\mathcal{I}(0),\mathcal{H}(0),\mathcal{D}(0),\mathcal{P}(0))$. From (4.15) and in connection with the persistence of $Z_2(\mathcal{S},\mathcal{I},\mathcal{H},\mathcal{D},\mathcal{P})$, we may suggest that $Z_2(\mathcal{S},\mathcal{I},\mathcal{H},\mathcal{D},\mathcal{P})$ has a least value $Z_2(\mathcal{S}(0),\mathcal{I}(0),\mathcal{H}(0),\mathcal{D}(0),\mathcal{P}(0))$ that stays in $\mathbb{R}^5_+$.

Again, we use a positive functional $Z:\mathbb{R}^5_+\mapsto\mathbb{R}_+$ as follows:

$$Z(\mathcal{S},\mathcal{I},\mathcal{H},\mathcal{D},\mathcal{P})=Z_2(\mathcal{S},\mathcal{I},\mathcal{H},\mathcal{D},\mathcal{P})-Z_2(\mathcal{S}(0),\mathcal{I}(0),\mathcal{H}(0),\mathcal{D}(0),\mathcal{P}(0)).$$

Employing Ito's technique, the suggested framework reduces to

$$\begin{aligned}\mathcal{L}Z\leq{}& q_4\Big\{-4\nabla[(\mathbb{C}_0^H)^{1/4}-1]+q_1\frac{\pi}{\mathcal{S}}\Big\}-\frac{\pi}{\mathcal{S}}+(\sigma+\varsigma)+\frac{\wp_1^2}{2}-\frac{\eta_2}{\mathcal{H}}+(\varsigma+\omega_1+\omega_2)+\frac{\wp_3^2}{2}-\eta_3\frac{\mathcal{I}}{\mathcal{D}}\\&-\omega_2\frac{\mathcal{H}}{\mathcal{D}}+\varrho+\frac{\wp_4^2}{2}-d(\mathcal{S}(\zeta)+\mathcal{I}(\zeta)+\mathcal{H}(\zeta)+\mathcal{D}(\zeta)+\mathcal{P}(\zeta)).\end{aligned}$$

Accordingly, it is necessary to derive the hypothesis stated above as follows:

$$\begin{aligned}\mathcal{L}Z\leq{}& -q_4q_5+(1+q_1q_4)\frac{\pi}{\mathcal{S}}-\frac{\pi}{\mathcal{S}}+(\sigma+\varsigma)+\frac{\wp_1^2}{2}-\frac{\eta_2}{\mathcal{H}}+(\varsigma+\omega_1+\omega_2)+\frac{\wp_3^2}{2}-\eta_3\frac{\mathcal{I}}{\mathcal{D}}\\&-\omega_2\frac{\mathcal{H}}{\mathcal{D}}+\varrho+\frac{\wp_4^2}{2}-d(\mathcal{S}(\zeta)+\mathcal{I}(\zeta)+\mathcal{H}(\zeta)+\mathcal{D}(\zeta)+\mathcal{P}(\zeta)),\end{aligned}\tag{4.16}$$

where $q_5=4\nabla[(\mathbb{C}_0^H)^{1/4}-1]>0$.

The next goal is to construct the set,

$$\mathbf{D}=\Big\{\mathcal{S}\in(\epsilon_1,1/\epsilon_2),\mathcal{I}\in(\epsilon_1,1/\epsilon_2),\mathcal{H}\in(\epsilon_1,1/\epsilon_2),\mathcal{D}\in(\epsilon_1,1/\epsilon_2),\mathcal{P}\in(\epsilon_1,1/\epsilon_2)\Big\},$$

where $\epsilon_i,\ i=1,2$ is an arbitrary small constant. For the sake of simplicity, we will divide the full $\mathbb{R}^5_+\setminus\mathbf{D}$ into the aforementioned domain,

$$\begin{aligned}\mathbf{D}_1&=\Big\{(\mathcal{S},\mathcal{I},\mathcal{H},\mathcal{D},\mathcal{P})\in\mathbb{R}^5_+,\mathcal{S}\in(0,\epsilon_1]\Big\},\\
\mathbf{D}_2&=\Big\{(\mathcal{S},\mathcal{I},\mathcal{H},\mathcal{D},\mathcal{P})\in\mathbb{R}^5_+,\mathcal{I}\in(0,\epsilon_2],\mathcal{S}>\epsilon_2\Big\},\\
\mathbf{D}_3&=\Big\{(\mathcal{S},\mathcal{I},\mathcal{H},\mathcal{D},\mathcal{P})\in\mathbb{R}^5_+,\mathcal{H}\in(0,\epsilon_1],\mathcal{I}>\epsilon_2\Big\},\\
\mathbf{D}_4&=\Big\{(\mathcal{S},\mathcal{I},\mathcal{H},\mathcal{D},\mathcal{P})\in\mathbb{R}^5_+,\mathcal{D}\in(0,\epsilon_1],\mathcal{H}>\epsilon_2\Big\},\\
\mathbf{D}_5&=\Big\{(\mathcal{S},\mathcal{I},\mathcal{H},\mathcal{D},\mathcal{P})\in\mathbb{R}^5_+,\mathcal{P}\in(0,\epsilon_1],\mathcal{D}>\epsilon_2\Big\},\\
\mathbf{D}_6&=\Big\{(\mathcal{S},\mathcal{I},\mathcal{H},\mathcal{D},\mathcal{P})\in\mathbb{R}^5_+,\mathcal{S}\geq\frac{1}{\epsilon_2}\Big\},\\
\mathbf{D}_7&=\Big\{(\mathcal{S},\mathcal{I},\mathcal{H},\mathcal{D},\mathcal{P})\in\mathbb{R}^5_+,\mathcal{I}\geq\frac{1}{\epsilon_2}\Big\},\\
\mathbf{D}_8&=\Big\{(\mathcal{S},\mathcal{I},\mathcal{H},\mathcal{D},\mathcal{P})\in\mathbb{R}^5_+,\mathcal{H}\geq\frac{1}{\epsilon_2}\Big\},\\
\mathbf{D}_9&=\Big\{(\mathcal{S},\mathcal{I},\mathcal{H},\mathcal{D},\mathcal{P})\in\mathbb{R}^5_+,\mathcal{D}\geq\frac{1}{\epsilon_2}\Big\},\end{aligned}$$

$$\mathbf{D}_{10}=\Big\{(\mathcal{S},\mathcal{I},\mathcal{H},\mathcal{D},\mathcal{P})\in\mathbb{R}^5_+,\mathcal{P}\geq\frac{1}{\epsilon_2}\Big\}.$$

We will now demonstrate that $\mathcal{L}Z(\mathcal{S},\mathcal{I},\mathcal{H},\mathcal{D},\mathcal{P})<0$ on $\mathbb{R}^5_+$, which is equivalent to doing so for the 10 zones mentioned before.

Case I. For $(\mathcal{S},\mathcal{I},\mathcal{H},\mathcal{D},\mathcal{P})\in\mathbf{D}_1$, using (4.16) yields,

$$\begin{aligned}\mathcal{L}Z\leq&-q_4q_5+(1+q_1q_4)\frac{\pi}{\mathcal{S}}-\frac{\pi}{\mathcal{S}}+(\sigma+\varsigma)+\frac{\wp_1^2}{2}-\frac{\eta_2}{\mathcal{H}}+(\varsigma+\omega_1+\omega_2)+\frac{\wp_3^2}{2}-\eta_3\frac{\mathcal{I}}{\mathcal{D}}\\&-\omega_2\frac{\mathcal{H}}{\mathcal{D}}+\varrho+\frac{\wp_4^2}{2}-d\mathcal{N}\\\leq&(1+q_1q_4)\frac{\pi}{\epsilon_1}-\frac{\pi}{\epsilon_1}+(\sigma+\varsigma)+\frac{\wp_1^2}{2}-\frac{\eta_2}{\mathcal{H}}+(\varsigma+\omega_1+\omega_2)+\frac{\wp_3^2}{2}-\eta_3\frac{\mathcal{I}}{\mathcal{D}}\\&-\omega_2\frac{\mathcal{H}}{\mathcal{D}}+\varrho+\frac{\wp_4^2}{2}-d\mathcal{N}.\end{aligned}$$

Selecting $\epsilon_1>0$ produces $\mathcal{L}Z<-1,\ \forall(\mathcal{S},\mathcal{I},\mathcal{H},\mathcal{D},\mathcal{P})\in\mathbf{D}_1$.

Case II. For $(\mathcal{S},\mathcal{I},\mathcal{H},\mathcal{D},\mathcal{P})\in\mathbf{D}_2$, using (4.16) yields,

$$\begin{aligned}\mathcal{L}Z\leq&-q_4q_5+(1+q_1q_4)\frac{\pi}{\mathcal{S}}-\frac{\pi}{\mathcal{S}}+(\sigma+\varsigma)+\frac{\wp_1^2}{2}-\frac{\eta_2}{\mathcal{H}}+(\varsigma+\omega_1+\omega_2)+\frac{\wp_3^2}{2}-\eta_3\frac{\mathcal{I}}{\mathcal{D}}\\&-\omega_2\frac{\mathcal{H}}{\mathcal{D}}+\varrho+\frac{\wp_4^2}{2}-d\mathcal{N}\\\leq&-q_4q_5+(1+q_1q_4)\frac{\pi}{\mathcal{S}}-\frac{\pi}{\mathcal{S}}+(\sigma+\varsigma)+\frac{\wp_1^2}{2}-\frac{\eta_2}{\mathcal{H}}+(\varsigma+\omega_1+\omega_2)+\frac{\wp_3^2}{2}-\eta_3\frac{\mathcal{I}}{\mathcal{D}}\\&-\omega_2\frac{\mathcal{H}}{\mathcal{D}}+\varrho+\frac{\wp_4^2}{2}-d\epsilon_2.\end{aligned}$$

Selecting $\epsilon_2>0$ produces $\mathcal{L}Z<-1,\ \forall(\mathcal{S},\mathcal{I},\mathcal{H},\mathcal{D},\mathcal{P})\in\mathbf{D}_2$.

Case III. For $(\mathcal{S},\mathcal{I},\mathcal{H},\mathcal{D},\mathcal{P})\in\mathbf{D}_3$, using (4.16) yields,

$$\begin{aligned}\mathcal{L}Z\leq&-q_4q_5+(1+q_1q_4)\frac{\pi}{\mathcal{S}}-\frac{\pi}{\mathcal{S}}+(\sigma+\varsigma)+\frac{\wp_1^2}{2}-\frac{\eta_2}{\mathcal{H}}+(\varsigma+\omega_1+\omega_2)+\frac{\wp_3^2}{2}-\eta_3\frac{\mathcal{I}}{\mathcal{D}}\\&-\omega_2\frac{\mathcal{H}}{\mathcal{D}}+\varrho+\frac{\wp_4^2}{2}-d\mathcal{N}\\\leq&(1+q_1q_4)\frac{\pi}{\mathcal{S}}-\frac{\pi}{\mathcal{S}}+(\sigma+\varsigma)+\frac{\wp_1^2}{2}-\frac{\eta_2}{\mathcal{H}}+(\varsigma+\omega_1+\omega_2)+\frac{\wp_3^2}{2}-\eta_3\frac{\mathcal{I}}{\mathcal{D}}\\&-\omega_2\frac{\mathcal{H}}{\mathcal{D}}+\varrho+\frac{\wp_4^2}{2}-d\frac{\epsilon_2}{\epsilon_1}.\end{aligned}$$

Selecting $\epsilon_1,\epsilon_2>0$ produces $\mathcal{L}Z<-1,\ \forall(\mathcal{S},\mathcal{I},\mathcal{H},\mathcal{D},\mathcal{P})\in\mathbf{D}_3$.

Case IV. For $(\mathcal{S},\mathcal{I},\mathcal{H},\mathcal{D},\mathcal{P})\in\mathbf{D}_4$, using (4.16) yields,

$$
\begin{aligned}
\mathcal{L}Z \leq & -q_4q_5 + (1+q_1q_4)\frac{\pi}{\mathcal{S}} - \frac{\pi}{\mathcal{S}} + (\sigma+\varsigma) + \frac{\wp_1^2}{2} - \frac{\eta_2}{\mathcal{H}} + (\varsigma+\omega_1+\omega_2) + \frac{\wp_3^2}{2} - \eta_3\frac{\mathcal{I}}{\mathcal{D}} \\
& - \omega_2\frac{\mathcal{H}}{\mathcal{D}} + \varrho + \frac{\wp_4^2}{2} - d\mathcal{N} \\
\leq & (1+q_1q_4)\frac{\pi}{\mathcal{S}} - \frac{\pi}{\mathcal{S}} + (\sigma+\varsigma) + \frac{\wp_1^2}{2} - \frac{\eta_2}{\mathcal{H}} + (\varsigma+\omega_1+\omega_2) + \frac{\wp_3^2}{2} - \eta_3\frac{\mathcal{I}}{\mathcal{D}} \\
& - \omega_2\frac{\mathcal{H}}{\mathcal{D}} + \varrho + \frac{\wp_4^2}{2} - d\epsilon_1.
\end{aligned}
$$

Selecting $\epsilon_1 > 0$ produces $\mathcal{L}Z < -1,\ \forall(\mathcal{S},\mathcal{I},\mathcal{H},\mathcal{D},\mathcal{P}) \in \mathbf{D}_4$.

Case V. For $(\mathcal{S},\mathcal{I},\mathcal{H},\mathcal{D},\mathcal{P}) \in \mathbf{D}_5$, using (4.16) yields,

$$
\begin{aligned}
\mathcal{L}Z \leq & -q_4q_5 + (1+q_1q_4)\frac{\pi}{\mathcal{S}} - \frac{\pi}{\mathcal{S}} + (\sigma+\varsigma) + \frac{\wp_1^2}{2} - \frac{\eta_2}{\mathcal{H}} + (\varsigma+\omega_1+\omega_2) + \frac{\wp_3^2}{2} - \eta_3\frac{\mathcal{I}}{\mathcal{D}} \\
& - \omega_2\frac{\mathcal{H}}{\mathcal{D}} + \varrho + \frac{\wp_4^2}{2} - d\mathcal{N} \\
\leq & (1+q_1q_4)\frac{\pi}{\mathcal{S}} - \frac{\pi}{\mathcal{S}} + (\sigma+\varsigma) + \frac{\wp_1^2}{2} - \frac{\eta_2}{\mathcal{H}} + (\varsigma+\omega_1+\omega_2) + \frac{\wp_3^2}{2} - \eta_3\frac{\mathcal{I}}{\mathcal{D}} \\
& - \omega_2\frac{\mathcal{H}}{\mathcal{D}} + \varrho + \frac{\wp_4^2}{2} - d\epsilon_1.
\end{aligned}
$$

Selecting $\epsilon_1 > 0$ produces $\mathcal{L}Z < -1,\ \forall(\mathcal{S},\mathcal{I},\mathcal{H},\mathcal{D},\mathcal{P}) \in \mathbf{D}_5$.

Case VI. For $(\mathcal{S},\mathcal{I},\mathcal{H},\mathcal{D},\mathcal{P}) \in \mathbf{D}_6$, using (4.16) yields,

$$
\begin{aligned}
\mathcal{L}Z \leq & -q_4q_5 + (1+q_1q_4)\frac{\pi}{\mathcal{S}} - \frac{\pi}{\mathcal{S}} + (\sigma+\varsigma) + \frac{\wp_1^2}{2} - \frac{\eta_2}{\mathcal{H}} + (\varsigma+\omega_1+\omega_2) + \frac{\wp_3^2}{2} - \eta_3\frac{\mathcal{I}}{\mathcal{D}} \\
& - \omega_2\frac{\mathcal{H}}{\mathcal{D}} + \varrho + \frac{\wp_4^2}{2} - d\mathcal{N} \\
\leq & (1+q_1q_4)\frac{\pi}{\epsilon_1} - \frac{\pi}{\epsilon_1} + (\sigma+\varsigma) + \frac{\wp_1^2}{2} - \frac{\eta_2}{\mathcal{H}} + (\varsigma+\omega_1+\omega_2) + \frac{\wp_3^2}{2} - \eta_3\frac{\mathcal{I}}{\mathcal{D}} \\
& - \omega_2\frac{\mathcal{H}}{\mathcal{D}} + \varrho + \frac{\wp_4^2}{2} - \frac{d}{\epsilon_2}.
\end{aligned}
$$

Selecting $\epsilon_1, \epsilon_2 > 0$ produces $\mathcal{L}Z < -1,\ \forall(\mathcal{S},\mathcal{I},\mathcal{H},\mathcal{D},\mathcal{P}) \in \mathbf{D}_6$.

Case VIII. For $(\mathcal{S},\mathcal{I},\mathcal{H},\mathcal{D},\mathcal{P}) \in \mathbf{D}_8$, using (4.16) yields,

$$
\begin{aligned}
\mathcal{L}Z \leq & -q_4q_5 + (1+q_1q_4)\frac{\pi}{\mathcal{S}} - \frac{\pi}{\mathcal{S}} + (\sigma+\varsigma) + \frac{\wp_1^2}{2} - \frac{\eta_2}{\mathcal{H}} + (\varsigma+\omega_1+\omega_2) + \frac{\wp_3^2}{2} - \eta_3\frac{\mathcal{I}}{\mathcal{D}} \\
& - \omega_2\frac{\mathcal{H}}{\mathcal{D}} + \varrho + \frac{\wp_4^2}{2} - d\mathcal{N} \\
\leq & (1+q_1q_4)\frac{\pi}{\epsilon_1} - \frac{\pi}{\epsilon_1} + (\sigma+\varsigma) + \frac{\wp_1^2}{2} - \frac{\eta_2}{\mathcal{H}} + (\varsigma+\omega_1+\omega_2) + \frac{\wp_3^2}{2} - \eta_3\frac{\mathcal{I}}{\mathcal{D}} \\
& - \omega_2\frac{\mathcal{H}}{\mathcal{D}} + \varrho + \frac{\wp_4^2}{2} - \frac{d}{\epsilon_2}.
\end{aligned}
$$

Selecting $\epsilon_2 > 0$ produces $\mathcal{L}Z < -1,\ \forall(\mathcal{S},\mathcal{I},\mathcal{H},\mathcal{D},\mathcal{P}) \in \mathbf{D}_8$.

Case IX. For $(\mathcal{S},\mathcal{I},\mathcal{H},\mathcal{D},\mathcal{P}) \in \mathbf{D}_9$, using (4.16) yields,

$$\begin{aligned}\mathcal{L}Z &\le -q_4q_5 + (1+q_1q_4)\frac{\pi}{\mathcal{S}} - \frac{\pi}{\mathcal{S}} + (\sigma+\varsigma) + \frac{\wp_1^2}{2} - \frac{\eta_2}{\mathcal{H}} + (\varsigma+\omega_1+\omega_2) + \frac{\wp_3^2}{2} - \eta_3\frac{\mathcal{I}}{\mathcal{D}}\\ &- \omega_2\frac{\mathcal{H}}{\mathcal{D}} + \varrho + \frac{\wp_4^2}{2} - d\mathcal{N}\\ &\le -q_4q_5 + (1+q_1q_4)\frac{\pi}{\epsilon_1} - \frac{\pi}{\epsilon_1} + (\sigma+\varsigma) + \frac{\wp_1^2}{2} - \frac{\eta_2}{\mathcal{H}} + (\varsigma+\omega_1+\omega_2) + \frac{\wp_3^2}{2} - \eta_3\\ &- \omega_2\frac{\epsilon_1}{\epsilon_2} + \varrho + \frac{\wp_4^2}{2} - d\mathcal{N}.\end{aligned}$$

Selecting $\epsilon_1 > 0$ produces $\mathcal{L}Z < -1,\ \forall(\mathcal{S},\mathcal{I},\mathcal{H},\mathcal{D},\mathcal{P}) \in \mathbf{D}_9$.

Case IX. For $(\mathcal{S},\mathcal{I},\mathcal{H},\mathcal{D},\mathcal{P}) \in \mathbf{D}_{10}$, using (4.16) yields,

$$\begin{aligned}\mathcal{L}Z &\le -q_4q_5 + (1+q_1q_4)\frac{\pi}{\mathcal{S}} - \frac{\pi}{\mathcal{S}} + (\sigma+\varsigma) + \frac{\wp_1^2}{2} - \frac{\eta_2}{\mathcal{H}} + (\varsigma+\omega_1+\omega_2) + \frac{\wp_3^2}{2} - \eta_3\frac{\mathcal{I}}{\mathcal{D}}\\ &- \omega_2\frac{\mathcal{H}}{\mathcal{D}} + \varrho + \frac{\wp_4^2}{2} - d\mathcal{N}\\ &\le -q_4q_5 + (1+q_1q_4)\frac{\pi}{\epsilon_1} - \frac{\pi}{\epsilon_1} + (\sigma+\varsigma) + \frac{\wp_1^2}{2} - \frac{\eta_2}{\mathcal{H}} + (\varsigma+\omega_1+\omega_2) + \frac{\wp_3^2}{2} - \eta_3\frac{\mathcal{I}}{\mathcal{D}}\\ &- \omega_2\frac{\epsilon_1}{\epsilon_2} + \varrho + \frac{\wp_4^2}{2} - \frac{d}{\epsilon_2}.\end{aligned}$$

Selecting $\epsilon_1 > 0$ produces $\mathcal{L}Z < -1,\ \forall(\mathcal{S},\mathcal{I},\mathcal{H},\mathcal{D},\mathcal{P}) \in \mathbf{D}_{10}$.

As a result, we prove that a constant $\mathbb{Q} > 0$ is one that guarantees,

$$\mathcal{L}Z_1(\mathcal{S},\mathcal{I},\mathcal{H},\mathcal{D},\mathcal{P}) < -\mathbb{Q} < 0\ \ \forall(\mathcal{S},\mathcal{I},\mathcal{H},\mathcal{D},\mathcal{P}) \in \mathbb{R}_+^5 \setminus \mathbf{D}.$$

Thus,

$$\begin{aligned}dZ(\mathcal{S},\mathcal{I},\mathcal{H},\mathcal{D},\mathcal{P}) < -\mathbb{Q}d\zeta &+ [(q_4+1)\mathcal{S} - (1+q_1q_4)\wp_1]d\mathfrak{B}_1(\zeta) + [(q_4+1)\mathcal{I} - q_2q_4\wp_2]d\mathfrak{B}_2(\zeta)\\ &+ [(q_4+1)\mathcal{H} - q_3q_4\wp_3]d\mathfrak{B}_3(\zeta) + [(q_4+1)\mathcal{D} - \wp_4]d\mathfrak{B}_5(\zeta)\\ &+ [(q_4+1)\mathcal{P} - \wp_5]d\mathfrak{B}_5(\zeta).\end{aligned} \tag{4.17}$$

Suppose that $(\mathcal{S}(0),\mathcal{I}(0),\mathcal{H}(0),\mathcal{D}(0),\mathcal{P}(0)) = (\mathbf{u}_1,\mathbf{u}_2,\mathbf{u}_3,\mathbf{u}_4,\mathbf{u}_5) = \mathbf{u} \in \mathbb{R}_+^5 \setminus \mathbf{D}$ and $\varphi^{\mathbf{u}}$ is the time period for which a path beginning at $\mathbf{u}$ leads to the set $\mathbf{D}$

$$\varphi_n = \inf\{\zeta : |\mathbf{X}(\zeta)| = \mathfrak{n}\}\ \ \textit{and}\ \ \varphi^{(n)}(\zeta) = \min\{\varphi^{\mathbf{u}}, \zeta, \varphi_{\mathfrak{n}}\}.$$

The subsequent result can be obtained by integrating both sides of the variant (4.17) from 0 to $\varphi^{(n)}(\zeta)$, attempting to take expectation and applying Dynkin's computation:

$$\begin{aligned}&\mathbb{E}Z(\mathcal{S}(\varphi^{(n)}(\zeta)), \mathcal{I}(\varphi^{(n)}(\zeta)), \mathcal{H}(\varphi^{(n)}(\zeta)), \mathcal{D}(\varphi^{(n)}(\zeta)), \mathcal{P}(\varphi^{(n)}(\zeta)) - Z(\mathbf{u}))\\ &= \mathbb{E}\int_0^{\varphi^{(n)}(\zeta)} \mathcal{L}Z(\mathcal{S}(u_1),\mathcal{I}(u_1),\mathcal{H}(u_1),\mathcal{D}(u_1),\mathcal{P}(u_1))du_1 \le \mathbb{E}\int_0^{\varphi^{(n)}(\zeta)} -\mathbb{Q}du_1 = -\mathbb{Q}\mathbb{E}\varphi^{(n)}(\zeta).\end{aligned}$$

Since $Z(\mathbf{u})$ is a positive number,

$$\mathbb{E}\varphi^{(n)}(\zeta) \leq \frac{Z(\mathbf{u})}{\mathbb{Q}}.$$

Therefore, we have $P\{\varphi_\epsilon = \infty\}$ as an immediate consequence of Theorem 4.3.

Conversely, the framework defined in (3.3) can be characterized as regular. In light of this, if we choose $\zeta \mapsto \infty$ and $n \mapsto \infty$, we will almost surely get $\varphi^{(n)}(\zeta) \mapsto \varphi^{\mathbf{u}}$.

Consequently, utilizing Fatou's lemma, we achieve

$$\mathbb{E}\varphi^{(n)}(\zeta) \leq \frac{Z(\mathbf{u})}{\mathbb{Q}} < \infty.$$

Obviously, $\sup_{\mathbf{u}\in C} \mathbb{E}\varphi^{\mathbf{u}} < \infty$; here, $C \in \mathbb{R}^5_+$ is a compact subset. It validates Assumption **(2)** of Lemma 4.3.

Moreover, the system (3.3) diffusion matrix is

$$\mathbb{Q} = \begin{bmatrix} \wp_1^2\mathcal{S}^2 & 0 & 0 & 0 & 0 \\ 0 & \wp_2^2\mathcal{I}^2 & 0 & 0 & 0 \\ 0 & 0 & \wp_3^2\mathcal{H}^2 & 0 & 0 \\ 0 & 0 & 0 & \wp_4^2\mathcal{D}^2 & 0 \\ 0 & 0 & 0 & 0 & \wp_5^2\mathcal{P}^2 \end{bmatrix}.$$

Selecting $\mathcal{M} = \min_{(\mathcal{S},\mathcal{I},\mathcal{H},\mathcal{D},\mathcal{P})\in\bar{\mathbf{D}}\in\mathbb{R}^5_+} \{\wp_1^2\mathcal{S}^2, \wp_2^2\mathcal{I}^2, \wp_3^2\mathcal{H}^2, \wp_4^2\mathcal{D}^2, \wp_5^2\mathcal{P}^2\}$, we find that

$$\begin{aligned}\sum_{\iota,\ell=1}^{5} a_{\iota\ell}(\mathcal{S},\mathcal{I},\mathcal{H},\mathcal{D},\mathcal{P})_{d_\iota d_\ell} &= \wp_1^2\mathcal{S}^2 d_1^2 + \wp_2^2\mathcal{I}^2 d_2^2 + \wp_3^2\mathcal{H}^2 d_3^2 + \wp_4^2\mathcal{D}^2 d_4^2 + \wp_5^2\mathcal{P}^2 d_5^2 \\ &\geq \mathcal{M}|d|^2 \quad \forall(\mathcal{S},\mathcal{I},\mathcal{H},\mathcal{D},\mathcal{P}) \in \bar{\mathbf{D}},\end{aligned}$$

where $d = (d_1, d_2, d_3, d_4, d_5) \in \mathbb{R}^5_+$. This means that Assumption **(1)** of Lemma 4.3 is also valid.

According to the investigation that preceded before, Lemma 4.3 shows that the framework (3.3) is ergodic and has a single stationary distribution. □

5. Numerical scheme of the proposed model

In what follows, fractional calculus may be applied to a wider range of diverse situations than classical calculus, so it attracted the attention of many researchers. Fractional calculus has benefited from the prevalence and effectiveness of design, including reminiscence effects [16–18]. We have adapted the suggested EV disease framework to evaluate its sensitivity by using the system-available (5.1) and collected information on fractional operators, drawing inspiration from the research of Atangana and Araz [32]. We view the framework (3.3) as being of F-F derivative in the Atangana-Baleanu-Caputo context. This is because fractional order derivatives have an additional level of flexibility and many other traits responsible for the kernel utilized here, such as inheritance and the ability to describe the previous and also the current state. This formulation is non-local and involves the generalized Mittag-Leffler as a kernel. The accompanying F-F DEs illustrate the EV disease model:

$$\begin{cases} {}_0^{\mathbf{FFM}}\mathbf{D}_\zeta^{\rho,\tau}\mathcal{S}(\zeta) = \Big(\pi - (\sigma + \varsigma)\mathcal{S}\Big) + \wp_1\mathbf{G}_1(\zeta,\mathcal{S})\mathfrak{B}_1(\zeta), \\ {}_0^{\mathbf{FFM}}\mathbf{D}_\zeta^{\rho,\tau}\mathcal{I}(\zeta) = \Big(\sigma\mathcal{S} - (\varsigma + \eta_1 + \eta_2 + \eta_3)\mathcal{I}\Big) + \wp_2\mathbf{G}_2(\zeta,\mathcal{I})\mathfrak{B}_2(\zeta), \\ {}_0^{\mathbf{FFM}}\mathbf{D}_\zeta^{\rho,\tau}\mathcal{H}(\zeta) = \Big(\eta_2\mathcal{I} - (\varsigma + \omega_1 + \omega_2)\mathcal{H}\Big) + \wp_3\mathbf{G}_3(\zeta,\mathcal{H})\mathfrak{B}_3(\zeta), \\ {}_0^{\mathbf{FFM}}\mathbf{D}_\zeta^{\rho,\tau}\mathcal{D}(\zeta) = \Big(\eta_3\mathcal{I} + \omega_2\mathcal{H} - \varrho\mathcal{D}\Big) + \wp_4\mathbf{G}_4(\zeta,\mathcal{D})\mathfrak{B}_4(\zeta), \\ {}_0^{\mathbf{FFM}}\mathbf{D}_\zeta^{\rho,\tau}\mathcal{P}(\zeta) = \Big(\delta_1\mathcal{I} + \delta_2\mathcal{D} - \nu\mathcal{P}\Big) + \wp_5\mathbf{G}_5(\zeta,\mathcal{P})\mathfrak{B}_5(\zeta). \end{cases} \tag{5.1}$$

For $t_{\mathfrak{n}+1} = (\mathfrak{n}+1)\Delta\zeta$, the scheme (5.1) can be described by the appropriate form by using the F-F derivative operator in the Atangana-Baleanu-Caputo terminology:

$$\begin{aligned} \mathcal{S}_{\mathfrak{n}+1} &= \mathcal{S}_0 + \frac{(1-\rho)}{\mathbf{ABC}(\rho)}\tau\zeta_{\mathfrak{n}+1}^{\tau-1}\left[\begin{cases}\mathcal{S}^*(\zeta_{\mathfrak{n}+1},\mathcal{S}^p_{\mathfrak{n}+1}) \\ +\wp_1\mathbf{G}_1(\zeta_{\mathfrak{n}+1},\mathcal{S}^p_{\mathfrak{n}+1})(\mathfrak{B}_1(\zeta_{\mathfrak{n}+2}) - \mathfrak{B}_1(\zeta_{\mathfrak{n}+1}))\end{cases}\right] \\ &+ \frac{\rho\tau}{\mathbf{ABC}(\rho)\Gamma(\rho)}\sum_{\ell=0}^{\mathfrak{n}-1}\left[\begin{cases}\mathcal{S}^*(\zeta_{\ell+1},\mathcal{S}_{\ell+1})\mathfrak{I}^{\rho,\tau}_{1,\ell} \\ +\frac{\mathcal{S}^*(\zeta_{\ell+1},\mathcal{S}_{\ell+1}) - \mathcal{S}^*(\zeta_\ell,\mathcal{S}_\ell)}{\hbar}\mathfrak{I}^{\rho,\tau}_{2,\ell} \\ +\frac{\mathcal{S}^*(\zeta_{\ell+1},\mathcal{S}_{\ell+1}) - 2\mathcal{S}^*(\zeta_\ell,\mathcal{S}_\ell) + \mathcal{S}^*(\zeta_{\ell-1},\mathcal{S}_{\ell-1})}{\hbar}\mathfrak{I}^{\rho,\tau}_{3,\ell}\end{cases}\right] \\ &+ \frac{\rho\tau}{\mathbf{ABC}(\rho)\Gamma(\rho)}\sum_{\ell=0}^{\mathfrak{n}-1}\left[\begin{cases}\wp_1\mathbf{G}_1(\zeta_{\ell+1},\mathcal{S}_{\ell+1})(\mathfrak{B}_1(\zeta_{\ell+1}) - \mathfrak{B}_1(\zeta_\ell))\mathfrak{I}^{\rho,\tau}_{1,\ell} \\ +\Big\{\wp_1\mathbf{G}_1(\zeta_{\ell+1},\mathcal{S}_{\ell+1})(\mathfrak{B}_1(\zeta_{\ell+1}) - \mathfrak{B}_1(\zeta_\ell)) \\ -\wp_1\mathbf{G}_1(\zeta_\ell,\mathcal{S}_\ell)(\mathfrak{B}_1(\zeta_\ell) - \mathfrak{B}_1(\zeta_{\ell-1}))\Big\}\mathfrak{I}^{\rho,\tau}_{2,\ell} \\ +\Big\{\frac{\wp_1\mathbf{G}_1(\zeta_{\ell+1},\mathcal{S}_{\ell+1})(\mathfrak{B}_1(\zeta_{\ell+1}) - \mathfrak{B}_1(\zeta_\ell)) - 2\wp_1\mathbf{G}_1(\zeta_\ell,\mathcal{S}_\ell)(\mathfrak{B}_1(\zeta_\ell) - \mathfrak{B}_1(\zeta_{\ell-1}))}{\hbar} \\ -\frac{\wp_1\mathbf{G}_1(\zeta_{\ell-1},\mathcal{S}_{\ell-1})(\mathfrak{B}_1(\zeta_{\ell-1}) - \mathfrak{B}_1(\zeta_{\ell-2}))}{\hbar}\Big\}\mathfrak{I}^{\rho,\tau}_{3,\ell}\end{cases}\right] \\ &+ \frac{\rho\tau}{\mathbf{ABC}(\rho)\Gamma(\rho)}\left[\begin{cases}\mathcal{S}^*(\zeta_{\mathfrak{n}+1},\mathcal{S}^p_{\mathfrak{n}+1})\mathfrak{I}^{\rho,\tau}_{1,\mathfrak{n}} \\ +\mathcal{S}^*(\zeta_{\mathfrak{n}+1},\mathcal{S}^p_{\mathfrak{n}+1})\mathfrak{I}^{\rho,\tau}_{2,\mathfrak{n}} \\ -\mathcal{S}^*(\zeta_{\mathfrak{n}},\mathcal{S}_{\mathfrak{n}})\mathfrak{I}^{\rho,\tau}_{2,\mathfrak{n}} \\ +\Big\{\frac{\mathcal{S}^*(\zeta_{\mathfrak{n}+1},\mathcal{S}^p_{\mathfrak{n}+1}) - 2\mathcal{S}^*(\zeta_{\mathfrak{n}},\mathcal{S}_{\mathfrak{n}})}{2\hbar}\Big\}\mathfrak{I}^{\rho,\tau}_{3,\mathfrak{n}} \\ +\frac{\mathcal{S}^*(\zeta_{\mathfrak{n}-1},\mathcal{S}_{\mathfrak{n}-1})}{2\hbar^2}\mathfrak{I}^{\rho,\tau}_{3,\mathfrak{n}} \\ +\wp_1\mathbf{G}_1(\zeta_{\mathfrak{n}+1},\mathcal{S}^{p_1}_{\mathfrak{n}+1})(\mathfrak{B}_1(\zeta_{\mathfrak{n}+1}) - \mathfrak{B}_1(\zeta_{\mathfrak{n}}))\mathfrak{I}^{\rho,\tau}_{1,\mathfrak{n}} \\ +\wp_1\mathbf{G}_1(\zeta_{\mathfrak{n}+1},\mathcal{S}^{p_1}_{\mathfrak{n}+1})(\mathfrak{B}_1(\zeta_{\mathfrak{n}+1}) - \mathfrak{B}_1(\zeta_{\mathfrak{n}}))\mathfrak{I}^{\rho,\tau}_{2,\mathfrak{n}} \\ -\wp_1\mathbf{G}_1(\zeta_{\mathfrak{n}},\mathcal{S}^{p_1}_{\mathfrak{n}})(\mathfrak{B}_1(\zeta_{\mathfrak{n}}) - \mathfrak{B}_1(\zeta_{\mathfrak{n}-1}))\mathfrak{I}^{\rho,\tau}_{2,\mathfrak{n}} \\ +\frac{\wp_1\mathbf{G}_1(\zeta_{\mathfrak{n}+1},\mathcal{S}^{p_1}_{\mathfrak{n}+1})(\mathfrak{B}_1(\zeta_{\mathfrak{n}+1}) - \mathfrak{B}_1(\zeta_{\mathfrak{n}}))}{2\hbar}\mathfrak{I}^{\rho,\tau}_{3,\mathfrak{n}} \\ -2\frac{\wp_1\mathbf{G}_1(\zeta_{\mathfrak{n}},\mathcal{S}^{p_1}_{\mathfrak{n}})(\mathfrak{B}_1(\zeta_{\mathfrak{n}}) - \mathfrak{B}_1(\zeta_{\mathfrak{n}-1}))}{2\hbar}\mathfrak{I}^{\rho,\tau}_{3,\mathfrak{n}} \\ +\frac{\wp_1\mathbf{G}_1(\zeta_{\mathfrak{n}-1},\mathcal{S}^{p_1}_{\mathfrak{n}-1})(\mathfrak{B}_1(\zeta_{\mathfrak{n}-2}) - \mathfrak{B}_1(\zeta_{\mathfrak{n}-2}))}{2\hbar}\mathfrak{I}^{\rho,\tau}_{3,\mathfrak{n}}\end{cases}\right], \end{aligned} \tag{5.2}$$

$$
\begin{aligned}
\mathcal{I}_{\mathfrak{n}+1} = \mathcal{I}_0 + \frac{(1-\rho)}{\mathbf{ABC}(\rho)}\tau\zeta_{\mathfrak{n}+1}^{\tau-1}&\left[\begin{cases}\mathcal{I}^*(\zeta_{\mathfrak{n}+1},\mathcal{S}^p_{\mathfrak{n}+1},\mathcal{I}^p_{\mathfrak{n}+1})\\ +\wp_2\mathbf{G}_2(\zeta_{\mathfrak{n}+1},\mathcal{I}^p_{\mathfrak{n}+1})(\mathfrak{B}_2(\zeta_{\mathfrak{n}+2})-\mathfrak{B}_2(\zeta_{\mathfrak{n}+1}))\end{cases}\right]\\
+\frac{\rho\tau}{\mathbf{ABC}(\rho)\Gamma(\rho)}\sum_{\ell=0}^{\mathfrak{n}-1}&\left[\begin{cases}\mathcal{I}^*(\zeta_{\ell+1},\mathcal{S}_{\ell+1},\mathcal{I}_{\ell+1})\mathfrak{I}^{\rho,\tau}_{1,\ell}\\ +\frac{\mathcal{I}^*\left(\zeta_{\ell+1},\mathcal{S}_{\ell+1},\mathcal{I}_{\ell+1}\right)-\mathcal{I}^*\left(\zeta_{\ell},\mathcal{S}_{\ell},\mathcal{I}_{\ell}\right)}{\hbar}\mathfrak{I}^{\rho,\tau}_{2,\ell}\\ +\frac{\mathcal{I}^*\left(\zeta_{\ell+1},\mathcal{S}_{\ell+1},\mathcal{I}_{\ell+1}\right)-2\mathcal{I}^*\left(\zeta_{\ell},\mathcal{S}_{\ell},\mathcal{I}_{\ell}\right)+\mathcal{I}^*\left(\zeta_{\ell-1},\mathcal{S}_{\ell-1},\mathcal{I}_{\ell-1}\right)}{\hbar}\mathfrak{I}^{\rho,\tau}_{3,\ell}\end{cases}\right]\\
+\frac{\rho\tau}{\mathbf{ABC}(\rho)\Gamma(\rho)}\sum_{\ell=0}^{\mathfrak{n}-1}&\left[\begin{cases}\wp_2\mathbf{G}_2(\zeta_{\ell+1},\mathcal{I}_{\ell+1})(\mathfrak{B}_2(\zeta_{\ell+1})-\mathfrak{B}_2(\zeta_{\ell}))\mathfrak{I}^{\rho,\tau}_{1,\ell}\\ +\Big\{\wp_2\mathbf{G}_2(\zeta_{\ell+1},\mathcal{I}_{\ell+1})(\mathfrak{B}_2(\zeta_{\ell+1})-\mathfrak{B}_2(\zeta_{\ell}))\\ -\wp_2\mathbf{G}_2(\zeta_{\ell},\mathcal{I}_{\ell})(\mathfrak{B}_2(\zeta_{\ell})-\mathfrak{B}_2(\zeta_{\ell-1}))\Big\}\mathfrak{I}^{\rho,\tau}_{2,\ell}\\ +\Big\{\frac{\wp_2\mathbf{G}_2\left(\zeta_{\ell+1},\mathcal{I}_{\ell+1}\right)\left(\mathfrak{B}_2(\zeta_{\ell+1})-\mathfrak{B}_2(\zeta_{\ell})\right)-2\wp_2\mathbf{G}_2\left(\zeta_{\ell},\mathcal{I}_{\ell}\right)\left(\mathfrak{B}_2(\zeta_{\ell})-\mathfrak{B}_2(\zeta_{\ell-1})\right)}{\hbar}\\ -\frac{\wp_2\mathbf{G}_2\left(\zeta_{\ell-1},\mathcal{I}_{\ell-1}\right)\left(\mathfrak{B}_2(\zeta_{\ell-1})-\mathfrak{B}_2(\zeta_{\ell-2})\right)}{\hbar}\Big\}\mathfrak{I}^{\rho,\tau}_{3,\ell}\end{cases}\right]\\
+\frac{\rho\tau}{\mathbf{ABC}(\rho)\Gamma(\rho)}&\left[\begin{cases}\mathcal{I}^*(\zeta_{\mathfrak{n}+1},\mathcal{S}^p_{\mathfrak{n}+1},\mathcal{I}^p_{\mathfrak{n}+1})\mathfrak{I}^{\rho,\tau}_{1,\mathfrak{n}}\\ +\mathcal{I}^*(\zeta_{\mathfrak{n}+1},\mathcal{S}^p_{\mathfrak{n}+1},\mathcal{I}^p_{\mathfrak{n}+1})\mathfrak{I}^{\rho,\tau}_{2,\mathfrak{n}}\\ -\mathcal{I}^*(\zeta_{\mathfrak{n}},\mathcal{S}_{\mathfrak{n}},\mathcal{I}_{\mathfrak{n}})\mathfrak{I}^{\rho,\tau}_{2,\mathfrak{n}}\\ +\Big\{\frac{\mathcal{I}^*\left(\zeta_{\mathfrak{n}+1},\mathcal{S}^p_{\mathfrak{n}+1},\mathcal{I}^p_{\mathfrak{n}+1}\right)-2\mathcal{I}^*\left(\zeta_{\mathfrak{n}},\mathcal{S}_{\mathfrak{n}},\mathcal{I}_{\mathfrak{n}}\right)}{2\hbar}\Big\}\mathfrak{I}^{\rho,\tau}_{3,\mathfrak{n}}\\ +\frac{\mathcal{I}^*\left(\zeta_{\mathfrak{n}-1},\mathcal{S}_{\mathfrak{n}-1},\mathcal{I}_{\mathfrak{n}-1}\right)}{2\hbar^2}\mathfrak{I}^{\rho,\tau}_{3,\mathfrak{n}}\\ +\wp_2\mathbf{G}_2(\zeta_{\mathfrak{n}+1},\mathcal{I}^{p_1}_{\mathfrak{n}+1})(\mathfrak{B}_2(\zeta_{\mathfrak{n}+1})-\mathfrak{B}_2(\zeta_{\mathfrak{n}}))\mathfrak{I}^{\rho,\tau}_{1,\mathfrak{n}}\\ +\wp_2\mathbf{G}_2(\zeta_{\mathfrak{n}+1},\mathcal{I}^{p_1}_{\mathfrak{n}+1})(\mathfrak{B}_2(\zeta_{\mathfrak{n}+1})-\mathfrak{B}_2(\zeta_{\mathfrak{n}}))\mathfrak{I}^{\rho,\tau}_{2,\mathfrak{n}}\\ -\wp_2\mathbf{G}_2(\zeta_{\mathfrak{n}},\mathcal{I}^{p_1}_{\mathfrak{n}})(\mathfrak{B}_2(\zeta_{\mathfrak{n}})-\mathfrak{B}_2(\zeta_{\mathfrak{n}-1}))\mathfrak{I}^{\rho,\tau}_{2,\mathfrak{n}}\\ +\frac{\wp_2\mathbf{G}_2\left(\zeta_{\mathfrak{n}+1},\mathcal{I}^{p_1}_{\mathfrak{n}+1}\right)\left(\mathfrak{B}_2(\zeta_{\mathfrak{n}+1})-\mathfrak{B}_2(\zeta_{\mathfrak{n}})\right)}{2\hbar}\mathfrak{I}^{\rho,\tau}_{3,\mathfrak{n}}\\ -2\frac{\wp_2\mathbf{G}_2\left(\zeta_{\mathfrak{n}},\mathcal{I}^{p_1}_{\mathfrak{n}}\right)\left(\mathfrak{B}_2(\zeta_{\mathfrak{n}})-\mathfrak{B}_2(\zeta_{\mathfrak{n}-1})\right)}{2\hbar}\mathfrak{I}^{\rho,\tau}_{3,\mathfrak{n}}\\ +\frac{\wp_2\mathbf{G}_2\left(\zeta_{\mathfrak{n}-1},\mathcal{I}^{p_1}_{\mathfrak{n}-1}\right)\left(\mathfrak{B}_2(\zeta_{\mathfrak{n}-2})-\mathfrak{B}_2(\zeta_{\mathfrak{n}-2})\right)}{2\hbar}\mathfrak{I}^{\rho,\tau}_{3,\mathfrak{n}}\end{cases}\right],
\end{aligned}
\tag{5.3}
$$

$$
\begin{aligned}
\mathcal{H}_{\mathfrak{n}+1} &= \mathcal{H}_0 + \frac{(1-\rho)}{\mathbf{ABC}(\rho)}\tau\zeta_{\mathfrak{n}+1}^{\tau-1}\left[\begin{cases}\mathcal{H}^*(\zeta_{\mathfrak{n}+1}, \mathcal{I}_{\mathfrak{n}+1}^{p}, \mathcal{H}_{\mathfrak{n}+1}^{p}) \\ +\wp_3\mathbf{G}_4(\zeta_{\mathfrak{n}+1}, \mathcal{H}_{\mathfrak{n}+1}^{p})(\mathfrak{B}_3(\zeta_{\mathfrak{n}+2}) - \mathfrak{B}_3(\zeta_{\mathfrak{n}+1}))\end{cases}\right] \\
&+ \frac{\rho\tau}{\mathbf{ABC}(\rho)\Gamma(\rho)}\sum_{\ell=0}^{\mathfrak{n}-1}\left[\begin{cases}\mathcal{H}^*(\zeta_{\ell+1}, \mathcal{I}_{\ell+1}, \mathcal{H}_{\ell+1})\mathfrak{I}_{1,\ell}^{\rho,\tau} \\ +\frac{\mathcal{H}^*\left(\zeta_{\ell+1},\mathcal{I}_{\ell+1},\mathcal{H}_{\ell+1}\right)-\mathcal{H}^*\left(\zeta_{\ell},\mathcal{I}_{\ell},\mathcal{H}_{\ell}\right)}{\hbar}\mathfrak{I}_{2,\ell}^{\rho,\tau} \\ +\frac{\mathcal{H}^*\left(\zeta_{\ell+1},\mathcal{I}_{\ell+1},\mathcal{H}_{\ell+1}\right)-2\mathcal{H}^*\left(\zeta_{\ell},\mathcal{I}_{\ell},\mathcal{H}_{\ell}\right)+\mathcal{H}^*\left(\zeta_{\ell-1},\mathcal{I}_{\ell-1},\mathcal{H}_{\ell-1}\right)}{\hbar}\mathfrak{I}_{3,\ell}^{\rho,\tau}\end{cases}\right] \\
&+ \frac{\rho\tau}{\mathbf{ABC}(\rho)\Gamma(\rho)}\sum_{\ell=0}^{\mathfrak{n}-1}\left[\begin{cases}\wp_3\mathbf{G}_3(\zeta_{\ell+1}, \mathcal{H}_{\ell+1})(\mathfrak{B}_3(\zeta_{\ell+1}) - \mathfrak{B}_3(\zeta_{\ell}))\mathfrak{I}_{1,\ell}^{\rho,\tau} \\ +\begin{Bmatrix}\wp_3\mathbf{G}_3(\zeta_{\ell+1}, \mathcal{H}_{\ell+1})(\mathfrak{B}_3(\zeta_{\ell+1}) - \mathfrak{B}_3(\zeta_{\ell})) \\ -\wp_3\mathbf{G}_3(\zeta_{\ell}, \mathcal{H}_{\ell})(\mathfrak{B}_3(\zeta_{\ell}) - \mathfrak{B}_3(\zeta_{\ell-1}))\end{Bmatrix}\mathfrak{I}_{2,\ell}^{\rho,\tau} \\ +\begin{Bmatrix}\frac{\wp_3\mathbf{G}_3\left(\zeta_{\ell+1},\mathcal{H}_{\ell+1}\right)\left(\mathfrak{B}_3(\zeta_{\ell+1})-\mathfrak{B}_3(\zeta_{\ell})\right)-2\wp_3\mathbf{G}_3\left(\zeta_{\ell},\mathcal{H}_{\ell}\right)\left(\mathfrak{B}_3(\zeta_{\ell})-\mathfrak{B}_3(\zeta_{\ell-1})\right)}{\hbar} \\ -\frac{\wp_3\mathbf{G}_3\left(\zeta_{\ell-1},\mathcal{H}_{\ell-1}\right)\left(\mathfrak{B}_3(\zeta_{\ell-1})-\mathfrak{B}_3(\zeta_{\ell-2})\right)}{\hbar}\end{Bmatrix}\mathfrak{I}_{3,\ell}^{\rho,\tau}\end{cases}\right] \\
&+ \frac{\rho\tau}{\mathbf{ABC}(\rho)\Gamma(\rho)}\left[\begin{cases}\mathcal{H}^*(\zeta_{\mathfrak{n}+1}, \mathcal{I}_{\mathfrak{n}+1}^{p}, \mathcal{H}_{\mathfrak{n}+1}^{p})\mathfrak{I}_{1,\mathfrak{n}}^{\rho,\tau} \\ +\mathcal{H}^*(\zeta_{\mathfrak{n}+1}, \mathcal{I}_{\mathfrak{n}+1}^{p}, \mathcal{H}_{\mathfrak{n}+1}^{p})\mathfrak{I}_{2,\mathfrak{n}}^{\rho,\tau} \\ -\mathcal{H}^*(\zeta_{\mathfrak{n}}, \mathcal{I}_{\mathfrak{n}}, \mathcal{H}_{\mathfrak{n}})\mathfrak{I}_{2,\mathfrak{n}}^{\rho,\tau} \\ +\begin{Bmatrix}\frac{\mathcal{H}^*\left(\zeta_{\mathfrak{n}+1},\mathcal{I}_{\mathfrak{n}+1}^{p},\mathcal{H}_{\mathfrak{n}+1}^{p}\right)-2\mathcal{H}^*\left(\zeta_{\mathfrak{n}},\mathcal{I}_{\mathfrak{n}},\mathcal{H}_{\mathfrak{n}}\right)}{2\hbar}\end{Bmatrix}\mathfrak{I}_{3,\mathfrak{n}}^{\rho,\tau} \\ +\frac{\mathcal{H}^*\left(\zeta_{\mathfrak{n}-1},\mathcal{I}_{\mathfrak{n}-1},\mathcal{H}_{\mathfrak{n}-1}\right)}{2\hbar^2}\mathfrak{I}_{3,\mathfrak{n}}^{\rho,\tau} \\ +\wp_3\mathbf{G}_3(\zeta_{\mathfrak{n}+1}, \mathcal{H}_{\mathfrak{n}+1}^{p_1})(\mathfrak{B}_3(\zeta_{\mathfrak{n}+1}) - \mathfrak{B}_3(\zeta_{\mathfrak{n}}))\mathfrak{I}_{1,\mathfrak{n}}^{\rho,\tau} \\ +\wp_3\mathbf{G}_3(\zeta_{\mathfrak{n}+1}, \mathcal{H}_{\mathfrak{n}+1}^{p_1})(\mathfrak{B}_3(\zeta_{\mathfrak{n}+1}) - \mathfrak{B}_3(\zeta_{\mathfrak{n}}))\mathfrak{I}_{2,\mathfrak{n}}^{\rho,\tau} \\ -\wp_3\mathbf{G}_3(\zeta_{\mathfrak{n}}, \mathcal{H}_{\mathfrak{n}}^{p_1})(\mathfrak{B}_3(\zeta_{\mathfrak{n}}) - \mathfrak{B}_3(\zeta_{\mathfrak{n}-1}))\mathfrak{I}_{2,\mathfrak{n}}^{\rho,\tau} \\ +\frac{\wp_3\mathbf{G}_3\left(\zeta_{\mathfrak{n}+1},\mathcal{H}_{\mathfrak{n}+1}^{p_1}\right)\left(\mathfrak{B}_3(\zeta_{\mathfrak{n}+1})-\mathfrak{B}_3(\zeta_{\mathfrak{n}})\right)}{2\hbar}\mathfrak{I}_{3,\mathfrak{n}}^{\rho,\tau} \\ -2\frac{\wp_3\mathbf{G}_3\left(\zeta_{\mathfrak{n}},\mathcal{H}_{\mathfrak{n}}^{p_1}\right)\left(\mathfrak{B}_3(\zeta_{\mathfrak{n}})-\mathfrak{B}_3(\zeta_{\mathfrak{n}-1})\right)}{2\hbar}\mathfrak{I}_{3,\mathfrak{n}}^{\rho,\tau} \\ +\frac{\wp_3\mathbf{G}_3\left(\zeta_{\mathfrak{n}-1},\mathcal{H}_{\mathfrak{n}-1}^{p_1}\right)\left(\mathfrak{B}_3(\zeta_{\mathfrak{n}-2})-\mathfrak{B}_3(\zeta_{\mathfrak{n}-2})\right)}{2\hbar}\mathfrak{I}_{3,\mathfrak{n}}^{\rho,\tau}\end{cases}\right],
\end{aligned} \tag{5.4}
$$

$$
\begin{aligned}
\mathcal{D}_{\mathfrak{n}+1} = {} & \mathcal{D}_0 + \frac{(1-\rho)}{\mathbf{ABC}(\rho)}\tau\zeta_{\mathfrak{n}+1}^{\tau-1}\left[\left\{\begin{array}{l}
\mathcal{D}^*(\zeta_{\mathfrak{n}+1}, \mathcal{I}^p_{\mathfrak{n}+1}, \mathcal{H}^p_{\mathfrak{n}+1}, \mathcal{D}^p_{\mathfrak{n}+1}) \\
+\wp_4\mathbf{G}_4(\zeta_{\mathfrak{n}+1}, \mathcal{D}^p_{\mathfrak{n}+1})(\mathfrak{B}_4(\zeta_{\mathfrak{n}+2}) - \mathfrak{B}_4(\zeta_{\mathfrak{n}+1}))
\end{array}\right.\right] \\
& + \frac{\rho\tau}{\mathbf{ABC}(\rho)\Gamma(\rho)}\sum_{\ell=0}^{\mathfrak{n}-1}\left[\left\{\begin{array}{l}
\mathcal{D}^*(\zeta_{\ell+1}, \mathcal{I}_{\ell+1}, \mathcal{H}_{\ell+1}, \mathcal{D}_{\ell+1})\mathfrak{I}^{\rho,\tau}_{1,\ell} \\
+\frac{\mathcal{D}^*\left(\zeta_{\ell+1}, \mathcal{I}_{\ell+1}, \mathcal{H}_{\ell+1}, \mathcal{D}_{\ell+1}\right) - \mathcal{D}^*\left(\zeta_{\ell}, \mathcal{I}_{\ell}, \mathcal{H}_{\ell}, \mathcal{D}_{\ell}\right)}{\hbar}\mathfrak{I}^{\rho,\tau}_{2,\ell} \\
+\frac{\mathcal{D}^*\left(\zeta_{\ell+1}, \mathcal{I}_{\ell+1}, \mathcal{H}_{\ell+1}, \mathcal{D}_{\ell+1}\right) - 2\mathcal{D}^*\left(\zeta_{\ell}, \mathcal{I}_{\ell}, \mathcal{H}_{\ell}, \mathcal{D}_{\ell}\right) + \mathcal{D}^*\left(\zeta_{\ell-1}, \mathcal{I}_{\ell-1}, \mathcal{H}_{\ell-1}, \mathcal{D}_{\ell-1}\right)}{\hbar}\mathfrak{I}^{\rho,\tau}_{3,\ell}
\end{array}\right.\right] \\
& + \frac{\rho\tau}{\mathbf{ABC}(\rho)\Gamma(\rho)}\sum_{\ell=0}^{\mathfrak{n}-1}\left[\left\{\begin{array}{l}
\wp_4\mathbf{G}_4(\zeta_{\ell+1}, \mathcal{D}_{\ell+1})(\mathfrak{B}_4(\zeta_{\ell+1}) - \mathfrak{B}_4(\zeta_{\ell}))\mathfrak{I}^{\rho,\tau}_{1,\ell} \\
+\left\{\begin{array}{l}\wp_4\mathbf{G}_4(\zeta_{\ell+1}, \mathcal{D}_{\ell+1})(\mathfrak{B}_4(\zeta_{\ell+1}) - \mathfrak{B}_4(\zeta_{\ell})) \\
-\wp_4\mathbf{G}_4(\zeta_{\ell}, \mathcal{D}_{\ell})(\mathfrak{B}_4(\zeta_{\ell}) - \mathfrak{B}_4(\zeta_{\ell-1}))\end{array}\right\}\mathfrak{I}^{\rho,\tau}_{2,\ell} \\
+\left\{\begin{array}{l}\frac{\wp_4\mathbf{G}_4\left(\zeta_{\ell+1}, \mathcal{D}_{\ell+1}\right)\left(\mathfrak{B}_4(\zeta_{\ell+1}) - \mathfrak{B}_4(\zeta_{\ell})\right) - 2\wp_4\mathbf{G}_4\left(\zeta_{\ell}, \mathcal{D}_{\ell}\right)\left(\mathfrak{B}_4(\zeta_{\ell}) - \mathfrak{B}_4(\zeta_{\ell-1})\right)}{\hbar} \\
-\frac{\wp_4\mathbf{G}_4\left(\zeta_{\ell-1}, \mathcal{D}_{\ell-1}\right)\left(\mathfrak{B}_4(\zeta_{\ell-1}) - \mathfrak{B}_4(\zeta_{\ell-2})\right)}{\hbar}\end{array}\right\}\mathfrak{I}^{\rho,\tau}_{3,\ell}
\end{array}\right.\right] \\
& + \frac{\rho\tau}{\mathbf{ABC}(\rho)\Gamma(\rho)}\left[\left\{\begin{array}{l}
\mathcal{D}^*(\zeta_{\mathfrak{n}+1}, \mathcal{I}^p_{\mathfrak{n}+1}, \mathcal{H}^p_{\mathfrak{n}+1}, \mathcal{D}^p_{\mathfrak{n}+1})\mathfrak{I}^{\rho,\tau}_{1,\mathfrak{n}} \\
+\mathcal{D}^*(\zeta_{\mathfrak{n}+1}, \mathcal{I}^p_{\mathfrak{n}+1}, \mathcal{H}^p_{\mathfrak{n}+1}, \mathcal{D}^p_{\mathfrak{n}+1})\mathfrak{I}^{\rho,\tau}_{2,\mathfrak{n}} \\
-\mathcal{D}^*(\zeta_{\mathfrak{n}}, \mathcal{I}_{\mathfrak{n}}, \mathcal{H}_{\mathfrak{n}}, \mathcal{D}_{\mathfrak{n}})\mathfrak{I}^{\rho,\tau}_{2,\mathfrak{n}} \\
+\left\{\frac{\mathcal{D}^*\left(\zeta_{\mathfrak{n}+1}, \mathcal{I}^p_{\mathfrak{n}+1}, \mathcal{H}^p_{\mathfrak{n}+1}, \mathcal{D}^p_{\mathfrak{n}+1}\right) - 2\mathcal{D}^*\left(\zeta_{\mathfrak{n}}, \mathcal{I}_{\mathfrak{n}}, \mathcal{H}_{\mathfrak{n}}, \mathcal{D}_{\mathfrak{n}}\right)}{2\hbar}\right\}\mathfrak{I}^{\rho,\tau}_{3,\mathfrak{n}} \\
+\frac{\mathcal{D}^*\left(\zeta_{\mathfrak{n}-1}, \mathcal{I}_{\mathfrak{n}-1}, \mathcal{H}_{\mathfrak{n}-1}, \mathcal{D}_{\mathfrak{n}-1}\right)}{2\hbar^2}\mathfrak{I}^{\rho,\tau}_{3,\mathfrak{n}} \\
+\wp_4\mathbf{G}_4(\zeta_{\mathfrak{n}+1}, \mathcal{D}^{p_1}_{\mathfrak{n}+1})(\mathfrak{B}_4(\zeta_{\mathfrak{n}+1}) - \mathfrak{B}_4(\zeta_{\mathfrak{n}}))\mathfrak{I}^{\rho,\tau}_{1,\mathfrak{n}} \\
+\wp_4\mathbf{G}_4(\zeta_{\mathfrak{n}+1}, \mathcal{D}^{p_1}_{\mathfrak{n}+1})(\mathfrak{B}_4(\zeta_{\mathfrak{n}+1}) - \mathfrak{B}_4(\zeta_{\mathfrak{n}}))\mathfrak{I}^{\rho,\tau}_{2,\mathfrak{n}} \\
-\wp_4\mathbf{G}_4(\zeta_{\mathfrak{n}}, \mathcal{D}^{p_1}_{\mathfrak{n}})(\mathfrak{B}_4(\zeta_{\mathfrak{n}}) - \mathfrak{B}_4(\zeta_{\mathfrak{n}-1}))\mathfrak{I}^{\rho,\tau}_{2,\mathfrak{n}} \\
+\frac{\wp_4\mathbf{G}_4\left(\zeta_{\mathfrak{n}+1}, \mathcal{D}^{p_1}_{\mathfrak{n}+1}\right)\left(\mathfrak{B}_4(\zeta_{\mathfrak{n}+1}) - \mathfrak{B}_4(\zeta_{\mathfrak{n}})\right)}{2\hbar}\mathfrak{I}^{\rho,\tau}_{3,\mathfrak{n}} \\
-2\frac{\wp_4\mathbf{G}_4\left(\zeta_{\mathfrak{n}}, \mathcal{D}^{p_1}_{\mathfrak{n}}\right)\left(\mathfrak{B}_4(\zeta_{\mathfrak{n}}) - \mathfrak{B}_4(\zeta_{\mathfrak{n}-1})\right)}{2\hbar}\mathfrak{I}^{\rho,\tau}_{3,\mathfrak{n}} \\
+\frac{\wp_4\mathbf{G}_4\left(\zeta_{\mathfrak{n}-1}, \mathcal{D}^{p_1}_{\mathfrak{n}-1}\right)\left(\mathfrak{B}_4(\zeta_{\mathfrak{n}-2}) - \mathfrak{B}_4(\zeta_{\mathfrak{n}-2})\right)}{2\hbar}\mathfrak{I}^{\rho,\tau}_{3,\mathfrak{n}}
\end{array}\right.\right], \qquad (5.5)
\end{aligned}
$$

$$\begin{aligned}
\mathcal{P}_{\mathfrak{n}+1} &= \mathcal{P}_0 + \frac{(1-\rho)}{\mathbf{ABC}(\rho)}\tau\zeta_{\mathfrak{n}+1}^{\tau-1}\left[\begin{cases}\mathcal{P}^*(\zeta_{\mathfrak{n}+1}, \mathcal{I}_{\mathfrak{n}+1}^{p}, \mathcal{D}_{\mathfrak{n}+1}^{p}, \mathcal{P}_{\mathfrak{n}+1}^{p}) \\ +\wp_5\mathbf{G}_5(\zeta_{\mathfrak{n}+1}, \mathcal{P}_{\mathfrak{n}+1}^{p})(\mathfrak{B}_5(\zeta_{\mathfrak{n}+2}) - \mathfrak{B}_5(\zeta_{\mathfrak{n}+1}))\end{cases}\right] \\
&+ \frac{\rho\tau}{\mathbf{ABC}(\rho)\Gamma(\rho)}\sum_{\ell=0}^{\mathfrak{n}-1}\left[\begin{cases}\mathcal{P}^*(\zeta_{\ell+1}, \mathcal{I}_{\ell+1}, \mathcal{D}_{\ell+1}, \mathcal{P}_{\ell+1})\mathfrak{I}_{1,\ell}^{\rho,\tau} \\ +\frac{\mathcal{P}^*\left(\zeta_{\ell+1},\mathcal{I}_{\ell+1},\mathcal{D}_{\ell+1},\mathcal{P}_{\ell+1}\right)-\mathcal{P}^*\left(\zeta_{\ell},\mathcal{I}_{\ell},\mathcal{D}_{\ell},\mathcal{P}_{\ell}\right)}{\hbar}\mathfrak{I}_{2,\ell}^{\rho,\tau} \\ +\frac{\mathcal{P}^*\left(\zeta_{\ell+1},\mathcal{I}_{\ell+1},\mathcal{D}_{\ell+1},\mathcal{P}_{\ell+1}\right)-2\mathcal{P}^*\left(\zeta_{\ell},\mathcal{I}_{\ell},\mathcal{D}_{\ell},\mathcal{P}_{\ell}\right)+\mathcal{P}^*\left(\zeta_{\ell-1},\mathcal{I}_{\ell-1},\mathcal{D}_{\ell-1},\mathcal{P}_{\ell-1}\right)}{\hbar}\mathfrak{I}_{3,\ell}^{\rho,\tau}\end{cases}\right] \\
&+ \frac{\rho\tau}{\mathbf{ABC}(\rho)\Gamma(\rho)}\sum_{\ell=0}^{\mathfrak{n}-1}\left[\begin{cases}\wp_5\mathbf{G}_5(\zeta_{\ell+1}, \mathcal{P}_{\ell+1})(\mathfrak{B}_5(\zeta_{\ell+1}) - \mathfrak{B}_5(\zeta_{\ell}))\mathfrak{I}_{1,\ell}^{\rho,\tau} \\ +\Big\{\wp_5\mathbf{G}_5(\zeta_{\ell+1}, \mathcal{P}_{\ell+1})(\mathfrak{B}_5(\zeta_{\ell+1}) - \mathfrak{B}_5(\zeta_{\ell})) \\ -\wp_5\mathbf{G}_5(\zeta_{\ell}, \mathcal{P}_{\ell})(\mathfrak{B}_5(\zeta_{\ell}) - \mathfrak{B}_5(\zeta_{\ell-1}))\Big\}\mathfrak{I}_{2,\ell}^{\rho,\tau} \\ +\Big\{\frac{\wp_5\mathbf{G}_5\left(\zeta_{\ell+1},\mathcal{P}_{\ell+1}\right)\left(\mathfrak{B}_5(\zeta_{\ell+1})-\mathfrak{B}_5(\zeta_{\ell})\right)-2\wp_5\mathbf{G}_5\left(\zeta_{\ell},\mathcal{P}_{\ell}\right)\left(\mathfrak{B}_5(\zeta_{\ell})-\mathfrak{B}_5(\zeta_{\ell-1})\right)}{\hbar} \\ -\frac{\wp_5\mathbf{G}_5\left(\zeta_{\ell-1},\mathcal{P}_{\ell-1}\right)\left(\mathfrak{B}_5(\zeta_{\ell-1})-\mathfrak{B}_5(\zeta_{\ell-2})\right)}{\hbar}\Big\}\mathfrak{I}_{3,\ell}^{\rho,\tau}\end{cases}\right] \\
&+ \frac{\rho\tau}{\mathbf{ABC}(\rho)\Gamma(\rho)}\left[\begin{cases}\mathcal{D}^*(\zeta_{\mathfrak{n}+1}, \mathcal{I}_{\mathfrak{n}+1}^{p}, \mathcal{D}_{\mathfrak{n}+1}^{p}, \mathcal{P}_{\mathfrak{n}+1}^{p})\mathfrak{I}_{1,\mathfrak{n}}^{\rho,\tau} \\ +\mathcal{P}^*(\zeta_{\mathfrak{n}+1}, \mathcal{I}_{\mathfrak{n}+1}^{p}, \mathcal{D}_{\mathfrak{n}+1}^{p}, \mathcal{P}_{\mathfrak{n}+1}^{p})\mathfrak{I}_{2,\mathfrak{n}}^{\rho,\tau} \\ -\mathcal{P}^*(\zeta_{\mathfrak{n}}, \mathcal{I}_{\mathfrak{n}}, \mathcal{D}_{\mathfrak{n}}, \mathcal{P}_{\mathfrak{n}})\mathfrak{I}_{2,\mathfrak{n}}^{\rho,\tau} \\ +\Big\{\frac{\mathcal{P}^*\left(\zeta_{\mathfrak{n}+1},\mathcal{I}_{\mathfrak{n}+1}^{p},\mathcal{D}_{\mathfrak{n}+1}^{p},\mathcal{P}_{\mathfrak{n}+1}^{p}\right)-2\mathcal{P}^*\left(\zeta_{\mathfrak{n}},\mathcal{I}_{\mathfrak{n}},\mathcal{D}_{\mathfrak{n}},\mathcal{P}_{\mathfrak{n}}\right)}{2\hbar}\Big\}\mathfrak{I}_{3,\mathfrak{n}}^{\rho,\tau} \\ +\frac{\mathcal{P}^*\left(\zeta_{\mathfrak{n}-1},\mathcal{I}_{\mathfrak{n}-1},\mathcal{D}_{\mathfrak{n}-1},\mathcal{P}_{\mathfrak{n}-1}\right)}{2\hbar^2}\mathfrak{I}_{3,\mathfrak{n}}^{\rho,\tau} \\ +\wp_5\mathbf{G}_5(\zeta_{\mathfrak{n}+1}, \mathcal{P}_{\mathfrak{n}+1}^{p_1})(\mathfrak{B}_5(\zeta_{\mathfrak{n}+1}) - \mathfrak{B}_5(\zeta_{\mathfrak{n}}))\mathfrak{I}_{1,\mathfrak{n}}^{\rho,\tau} \\ +\wp_5\mathbf{G}_5(\zeta_{\mathfrak{n}+1}, \mathcal{P}_{\mathfrak{n}+1}^{p_1})(\mathfrak{B}_5(\zeta_{\mathfrak{n}+1}) - \mathfrak{B}_5(\zeta_{\mathfrak{n}}))\mathfrak{I}_{2,\mathfrak{n}}^{\rho,\tau} \\ -\wp_5\mathbf{G}_5(\zeta_{\mathfrak{n}}, \mathcal{P}_{\mathfrak{n}}^{p_1})(\mathfrak{B}_5(\zeta_{\mathfrak{n}}) - \mathfrak{B}_5(\zeta_{\mathfrak{n}-1}))\mathfrak{I}_{2,\mathfrak{n}}^{\rho,\tau} \\ +\frac{\wp_5\mathbf{G}_5\left(\zeta_{\mathfrak{n}+1},\mathcal{P}_{\mathfrak{n}+1}^{p_1}\right)\left(\mathfrak{B}_5(\zeta_{\mathfrak{n}+1})-\mathfrak{B}_5(\zeta_{\mathfrak{n}})\right)}{2\hbar}\mathfrak{I}_{3,\mathfrak{n}}^{\rho,\tau} \\ -2\frac{\wp_5\mathbf{G}_5\left(\zeta_{\mathfrak{n}},\mathcal{P}_{\mathfrak{n}}^{p_1}\right)\left(\mathfrak{B}_5(\zeta_{\mathfrak{n}})-\mathfrak{B}_5(\zeta_{\mathfrak{n}-1})\right)}{2\hbar}\mathfrak{I}_{3,\mathfrak{n}}^{\rho,\tau} \\ +\frac{\wp_5\mathbf{G}_5\left(\zeta_{\mathfrak{n}-1},\mathcal{P}_{\mathfrak{n}-1}^{p_1}\right)\left(\mathfrak{B}_5(\zeta_{\mathfrak{n}-2})-\mathfrak{B}_5(\zeta_{\mathfrak{n}-2})\right)}{2\hbar}\mathfrak{I}_{3,\mathfrak{n}}^{\rho,\tau}\end{cases}\right],
\end{aligned} \tag{5.6}$$

where

$$\begin{aligned}
\mathcal{S}_{\mathfrak{n}+1}^{p_1} &= \mathcal{S}_0 + \frac{1-\rho}{\mathbf{ABC}(\rho)}\tau\zeta_{\mathfrak{n}+1}^{\tau-1}\left[\begin{cases}\mathcal{S}^*(\zeta_{\mathfrak{n}}, \mathcal{S}_{\mathfrak{n}}) \\ +\tau\wp_1\mathbf{G}_1(\zeta_{\mathfrak{n}}, \mathcal{S}_{\mathfrak{n}})(\mathfrak{B}_1(\zeta_{\mathfrak{n}}) - \mathfrak{B}_1(\zeta_{\mathfrak{n}-1}))\end{cases}\right] \\
&+ \frac{\rho\tau}{\mathbf{ABC}(\rho)\Gamma(\rho)}\sum_{\ell=0}^{\mathfrak{n}}\left[\begin{cases}\mathcal{S}^*(\zeta_{\ell}, \mathcal{S}_{\ell})\mathfrak{I}_{1,\ell}^{\rho,\tau} \\ +\wp_1\mathbf{G}_1(\zeta_{\ell}, \mathcal{S}_{\ell})(\mathfrak{B}_1(\zeta_{\ell}) - \mathfrak{B}_1(\zeta_{\ell-1}))\mathfrak{I}_{1,\ell}^{\rho,\tau}\end{cases}\right], \\
\mathcal{I}_{\mathfrak{n}+1}^{p_1} &= \mathcal{I}_0 + \frac{1-\rho}{\mathbf{ABC}(\rho)}\tau\zeta_{\mathfrak{n}+1}^{\tau-1}\left[\begin{cases}\mathcal{I}^*(\zeta_{\mathfrak{n}}, \mathcal{S}_{\mathfrak{n}}, \mathcal{I}_{\mathfrak{n}}) \\ +\tau\wp_2\mathbf{G}_2(\zeta_{\mathfrak{n}}, \mathcal{I}_{\mathfrak{n}})(\mathfrak{B}_2(\zeta_{\mathfrak{n}}) - \mathfrak{B}_2(\zeta_{\mathfrak{n}-1}))\end{cases}\right] \\
&+ \frac{\rho\tau}{\mathbf{ABC}(\rho)\Gamma(\rho)}\sum_{\ell=0}^{\mathfrak{n}}\left[\begin{cases}\mathcal{I}^*(\zeta_{\ell}, \mathcal{S}_{\ell}, \mathcal{I}_{\ell})\mathfrak{I}_{1,\ell}^{\rho,\tau} \\ +\wp_2\mathbf{G}_2(\zeta_{\ell}, \mathcal{I}_{\ell})(\mathfrak{B}_2(\zeta_{\ell}) - \mathfrak{B}_2(\zeta_{\ell-1}))\mathfrak{I}_{1,\ell}^{\rho,\tau}\end{cases}\right],
\end{aligned}$$

$$\mathcal{H}^{p_1}_{\mathfrak{n}+1} = \mathcal{H}_0 + \frac{1-\rho}{\mathbf{ABC}(\rho)}\tau\zeta^{\tau-1}_{\mathfrak{n}+1}\left[\begin{cases}\mathcal{H}^*(\zeta_{\mathfrak{n}}, \mathcal{I}_{\mathfrak{n}}, \mathcal{H}_{\mathfrak{n}}) \\ +\tau\wp_3\mathbf{G}_3(\zeta_{\mathfrak{n}}, \mathcal{H}_{\mathfrak{n}})(\mathfrak{B}_3(\zeta_{\mathfrak{n}}) - \mathfrak{B}_3(\zeta_{\mathfrak{n}-1}))\end{cases}\right]$$
$$+ \frac{\rho\tau}{\mathbf{ABC}(\rho)\Gamma(\rho)}\sum_{\ell=0}^{\mathfrak{n}}\left[\begin{cases}\mathcal{H}^*(\zeta_\ell, \mathcal{I}_\ell, \mathcal{H}_\ell)\mathfrak{I}^{\rho,\tau}_{1,\ell} \\ +\wp_3\mathbf{G}_3(\zeta_\ell, \mathcal{H}_\ell)(\mathfrak{B}_3(\zeta_\ell) - \mathfrak{B}_3(\zeta_{\ell-1}))\mathfrak{I}^{\rho,\tau}_{1,\ell}\end{cases}\right],$$
$$\mathcal{D}^{p_1}_{\mathfrak{n}+1} = \mathcal{D}_0 + \frac{1-\rho}{\mathbf{ABC}(\rho)}\tau\zeta^{\tau-1}_{\mathfrak{n}+1}\left[\begin{cases}\mathcal{D}^*(\zeta_{\mathfrak{n}}, \mathcal{I}_{\mathfrak{n}}, \mathcal{H}_{\mathfrak{n}}, \mathcal{D}_{\mathfrak{n}}) \\ +\tau\wp_4\mathbf{G}_4(\zeta_{\mathfrak{n}}, \mathcal{D}_{\mathfrak{n}})(\mathfrak{B}_4(\zeta_{\mathfrak{n}}) - \mathfrak{B}_4(\zeta_{\mathfrak{n}-1}))\end{cases}\right]$$
$$+ \frac{\rho\tau}{\mathbf{ABC}(\rho)\Gamma(\rho)}\sum_{\ell=0}^{\mathfrak{n}}\left[\begin{cases}\mathcal{D}^*(\zeta_\ell, \mathcal{I}_\ell, \mathcal{H}_\ell, \mathcal{D}_\ell)\mathfrak{I}^{\rho,\tau}_{1,\ell} \\ +\wp_4\mathbf{G}_4(\zeta_\ell, \mathcal{D}_\ell)(\mathfrak{B}_4(\zeta_\ell) - \mathfrak{B}_4(\zeta_{\ell-1}))\mathfrak{I}^{\rho,\tau}_{1,\ell}\end{cases}\right],$$
$$\mathcal{P}^{p_1}_{\mathfrak{n}+1} = \mathcal{P}_0 + \frac{1-\rho}{\mathbf{ABC}(\rho)}\tau\zeta^{\tau-1}_{\mathfrak{n}+1}\left[\begin{cases}\mathcal{P}^*(\zeta_{\mathfrak{n}}, \mathcal{I}_{\mathfrak{n}}, \mathcal{D}_{\mathfrak{n}}, \mathcal{P}_{\mathfrak{n}}) \\ +\tau\wp_5\mathbf{G}_5(\zeta_{\mathfrak{n}}, \mathcal{P}_{\mathfrak{n}})(\mathfrak{B}_5(\zeta_{\mathfrak{n}}) - \mathfrak{B}_5(\zeta_{\mathfrak{n}-1}))\end{cases}\right]$$
$$+ \frac{\rho\tau}{\mathbf{ABC}(\rho)\Gamma(\rho)}\sum_{\ell=0}^{\mathfrak{n}}\left[\begin{cases}\mathcal{P}^*(\zeta_\ell, \mathcal{I}_\ell, \mathcal{D}_\ell, \mathcal{P}_\ell)\mathfrak{I}^{\rho,\tau}_{1,\ell} \\ +\wp_5\mathbf{G}_5(\zeta_\ell, \mathcal{P}_\ell)(\mathfrak{B}_5(\zeta_\ell) - \mathfrak{B}_5(\zeta_{\ell-1}))\mathfrak{I}^{\rho,\tau}_{1,\ell}\end{cases}\right], \tag{5.7}$$

where

$$\mathfrak{I}^{\rho,\tau}_{1,\ell} = \frac{((\mathfrak{n}+1)\hbar)^{\rho-1}}{\tau}\left[\begin{cases}((\ell+1)\hbar)^\tau\,{}_2\mathbf{F}_1\Big([\tau, 1-\rho], [1+\tau], \frac{\ell+1}{\mathfrak{n}+1}\Big) \\ -(\ell\hbar)^\tau\,{}_2\mathbf{F}_1\Big([\tau, 1-\rho], [1+\tau], \frac{\ell}{\mathfrak{n}}\Big)\end{cases}\right],$$
$$\mathfrak{I}^{\rho,\tau}_{2,\ell} = \frac{((\mathfrak{n}+1)\hbar)^{\rho-1}}{\tau(\tau+1)}\left[\begin{cases}\tau((\ell+1)\hbar)^{\tau+1}\,{}_2\mathbf{F}_1\Big([1+\tau, 1-\rho], [2+\tau], \frac{\ell+1}{\mathfrak{n}+1}\Big) \\ -(1+\tau)((\ell+1)\hbar)^{\tau+1}\,{}_2\mathbf{F}_1\Big([\tau, 1-\rho], [1+\tau], \frac{\ell+1}{\mathfrak{n}}\Big) \\ -\tau((\ell)\hbar)^{\tau+1}\,{}_2\mathbf{F}_1\Big([1+\tau, 1-\rho], [2+\tau], \frac{\ell}{\mathfrak{n}+1}\Big) \\ +\hbar((\ell)\hbar)^\tau(1+\tau)(1+\ell)\,{}_2\mathbf{F}_1\Big([\tau, 1-\rho], [1+\tau], \frac{\ell}{\mathfrak{n}}\Big)\end{cases}\right],$$
$$\mathfrak{I}^{\rho,\tau}_{3,\ell} = \frac{((\ell+1)\hbar)^{\rho-1}}{\tau(\tau+1)(\tau+2)}\left[\begin{cases}\tau(\tau+1)((\ell+1)\hbar)^{\tau+2}\,{}_2\mathbf{F}_1\Big([2+\tau, 1-\rho], [3+\tau], \frac{\ell+1}{\mathfrak{n}+1}\Big) \\ -2\tau(\tau+2)\Big(\ell+\frac{1}{2}\Big)\hbar((\ell+1)\hbar)^{\tau+1}\,{}_2\mathbf{F}_1\Big([1+\tau, 1-\rho], [2+\tau], \frac{\ell+1}{\mathfrak{n}+1}\Big) \\ +\ell\hbar(\tau+1)(\tau+2)((\ell+1)\hbar)^{\tau+1}\,{}_2\mathbf{F}_1\Big([\tau, 1-\rho], [1+\tau], \frac{\ell+1}{\mathfrak{n}}\Big) \\ +2\tau(\tau+2)\Big(\ell+\frac{1}{2}\Big)\hbar((\hbar\ell))^{\tau+1}\,{}_2\mathbf{F}_1\Big([1+\tau, 1-\rho], [2+\tau], \frac{\ell}{\mathfrak{n}+1}\Big) \\ -\tau(\tau+1)(\ell\hbar)^{\tau+2}\,{}_2\mathbf{F}_1\Big([2+\tau, 1-\rho], [3+\tau], \frac{\ell}{\mathfrak{n}}\Big) \\ -\hbar(\tau+1)(\tau+2)(\ell+1)(\ell\hbar)^{\tau+1}\,{}_2\mathbf{F}_1\Big([\tau, 1-\rho], [1+\tau], \frac{\ell}{\mathfrak{n}+1}\Big)\end{cases}\right] \tag{5.8}$$

5.1. Results and discussion

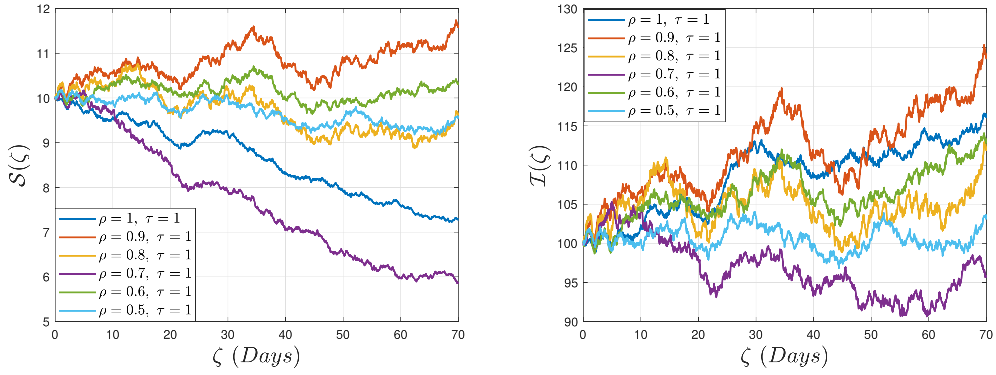
In this section, a series of experiments for the EV scheme (5.1) are carried out by utilizing the F-F derivative operator. Taking into account the F-F Atangana-Baleanu-Caputo operator and white noise, we employed the flawless revolutionary numerical scheme proposed by Atangana and Araz [32]. When doing numerical simulations of a system, one cannot be assured of the attribute values to be selected. Although considerable efforts are taken to reduce inaccuracies, the majority of evidence collection techniques employed to select system parameter values do contain some inconsistencies. Table 1 lists the specifications of parameters used in analytical simulation, with the distinguishing characteristics of random densities $\wp_\ell, \ell = 1, ..., 5$ and ICs $\mathcal{S} = 98000$, $\mathcal{I} = 1500$, $\mathcal{H} = 1000$, $\mathcal{D} = 300$, $\mathcal{P} = 600$.

Table 1. List of parameters.

$Symbols$	Explanation	Values
π	Recruitment level	200
ϵ_1	Rate of dread	0.35
ς	Natural death rate	0.000296
η_1	Rate of infection-related restoration	0.33
η_2	Hospitalization rates for contagious diseases	0.019
η_3	Mortality of the afflicted due to infection	0.5
ω_1	Proportion of hospitalized patient's restoration	0.85
ω_2	Hospitalized patient's deaths due to illness	0.25
ϱ	Rate at which bodies are disposed	0.0095
δ_1	Disease excretion rate from affected patients	0.056
δ_2	Rate of infection transmission by the deceased	0.045
ν	Rate of virus elimination from the surroundings	0.016
κ	Half saturation level	27
γ_1	Efficacious rate of personal interaction	0.57
γ_2	Efficient proportion of human-pathogen interaction	0.65
ϑ_1	Infected people's rate of infection	0.07
ϑ_2	Continuity of the dead people's infections	1.3

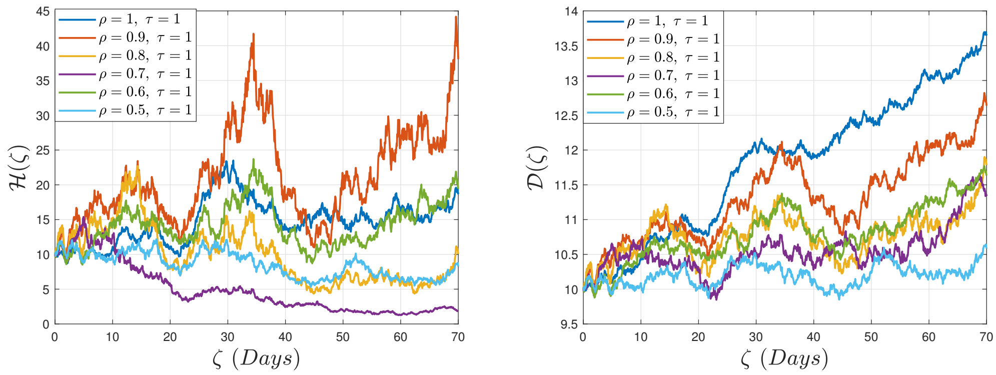
Figures 3–5 displays the associations connecting the main relevant characteristics and the contaminated community. As the incidence of person-to-person interaction rises, we see an elevation in the estimated prevalence in Figure 3(a). This is due to the ease at which EV disease can spread from susceptible people to the infected given the random densities $\wp_1 = 0.06$, $\wp_2 = 0.07$, $\wp_3 = 0.08$, $\wp_4 = 0.09$ and $\wp_5 = 0.06$, respectively. As shown in Figure 3(b), anxiety is adversely connected to the community that is affected, but some preventive strategies, including incarceration, interaction monitoring and media advertising, are sometimes necessary to eradicate the EV. Figure 4(a) depicts the hospitalized class $\mathcal{H}(\zeta)$ as increasing initially before becoming stationary at various F-F orders. The pathogenic class $\mathcal{P}(\zeta)$ manages the infection and restores it to equilibrium.

Figures 6–8 show the dynamic behavior of the infections in the surroundings $\mathcal{P}(\zeta)$ as a measure of the characteristics ϱ and ϵ_1, which were significantly weakly connected to $\mathcal{P}(\zeta)$, in Figures 3–5 when random densities were $\wp_1 = 0.06$, $\wp_2 = 0.07$, $\wp_3 = 0.08$, $\wp_4 = 0.09$ and $\wp_5 = 0.06$, respectively. The graphs demonstrate that an improvement in the handling and removal of EV victims' remains as well as a high standard of death phobia will aid in reducing the number of viruses in the surroundings. As a result, the risk of transmission may drop. Figure 7 depicts the asymptotic or hospitalized class $\mathcal{H}(\zeta)$, which exhibits a decline in behavior patterns similar to the susceptible class at different fractal dimensions.



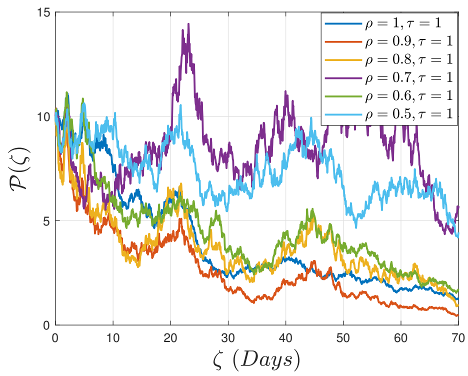
(a) **(b)**

Figure 3. Dynamical behavior of the stochastic F-F EV model (5.1) for susceptible individuals $\mathcal{S}(\zeta)$ and infectious people $\mathcal{I}(\zeta)$ when ρ falls significantly and $\tau = 1$.



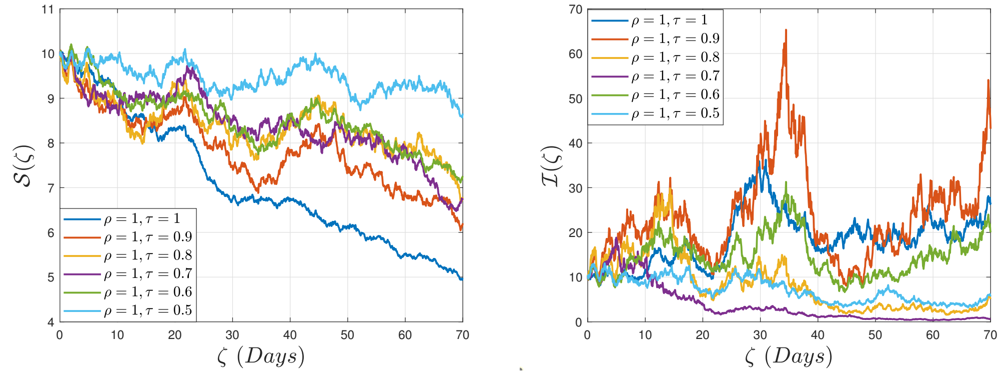
(a) **(b)**

Figure 4. Dynamical behavior of the stochastic F-F EV model (5.1) for hospitalized individuals $\mathcal{H}(\zeta)$ and the deceased $\mathcal{D}(\zeta)$ when ρ falls significantly and $\tau = 1$.



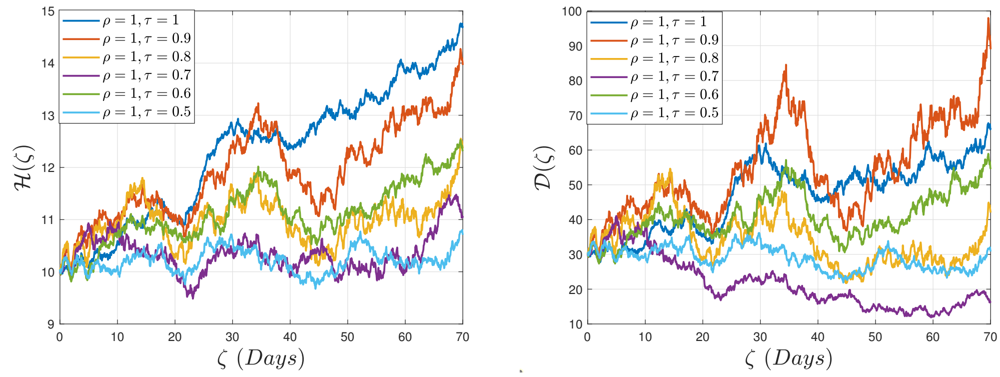
(a)

Figure 5. Dynamical behavior of the stochastic F-F EV model (5.1) for pathogens in the surroundings $\mathcal{P}(\zeta)$ when ρ falls significantly and $\tau = 1$.



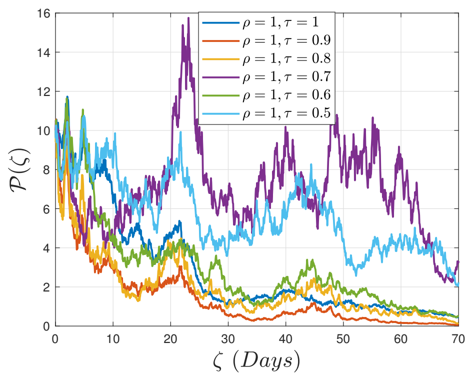
(a) **(b)**

Figure 6. Dynamical behavior of the stochastic F-F EV model (5.1) for susceptible individuals $\mathcal{S}(\zeta)$ and infectious people $\mathcal{I}(\zeta)$ when τ falls significantly and $\rho = 1$.



(a) **(b)**

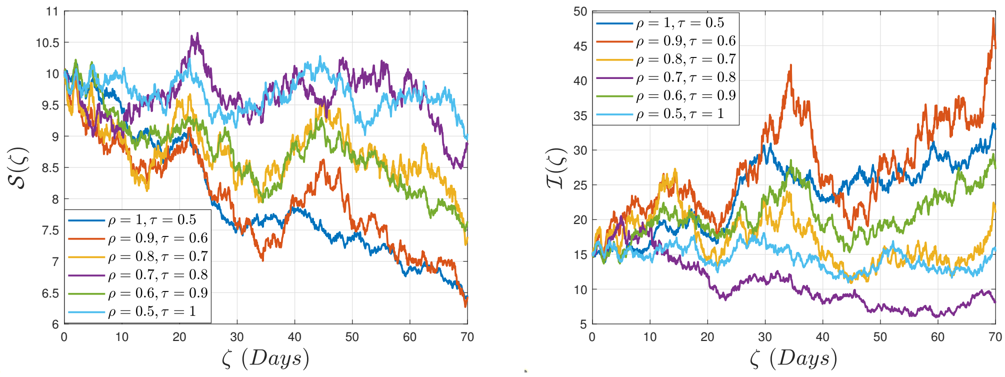
Figure 7. Dynamical behavior of the stochastic F-F EV model (5.1) for hospitalized individuals $\mathcal{H}(\zeta)$ and the deceased $\mathcal{D}(\zeta)$ when τ falls significantly and $\rho = 1$.



(a)

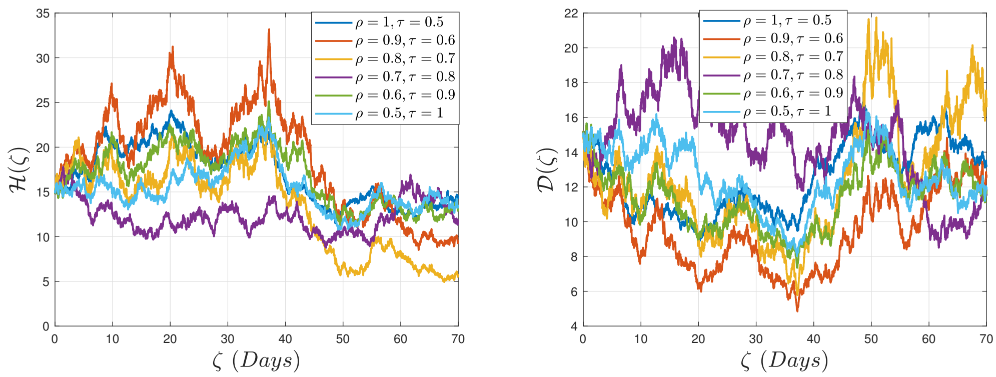
Figure 8. Dynamical behavior of the stochastic F-F EV model (5.1) for the pathogen in the surroundings $\mathcal{P}(\zeta)$ when τ falls significantly and $\rho = 1$.

By changing the intensity of dread while keeping the remaining factors fixed, the graphs Figures 9–11 were created with the random densities $\wp_1 = 1.2$, $\wp_2 = 1.4$, $\wp_3 = 1.6$, $\wp_4 = 1.8$ and $\wp_5 = 1.9$, respectively. The reductions in infectious cases, hospital admissions and fatalities that are shown when the amount of anxiety rises demonstrates how the mechanisms of bacterial contamination are impacted by the dread of EV disease mortality. As well, Figure 10(a) illustrates the effect of dread on environmental infections. Figure 10(b) depicts the classification of deceased people from the aforementioned pandemic at different fractional orders and fractal dimensions. As the sense of anxiety rises, it demonstrates a diminution in the number of germs in the surroundings. This is exacerbated by the fact that increased fear alters individual interactions, reducing the amount of viruses, diseased people and dead people pollutants in the atmosphere.



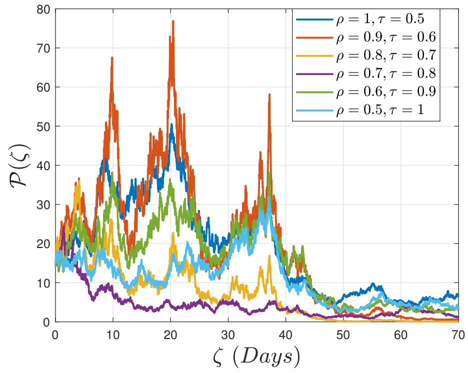
(a) **(b)**

Figure 9. Dynamical behavior of the stochastic F-F EV model (5.1) for susceptible individuals $\mathcal{S}(\zeta)$ and infectious people $\mathcal{I}(\zeta)$ when ρ increases and τ decreases significantly.



(a) **(b)**

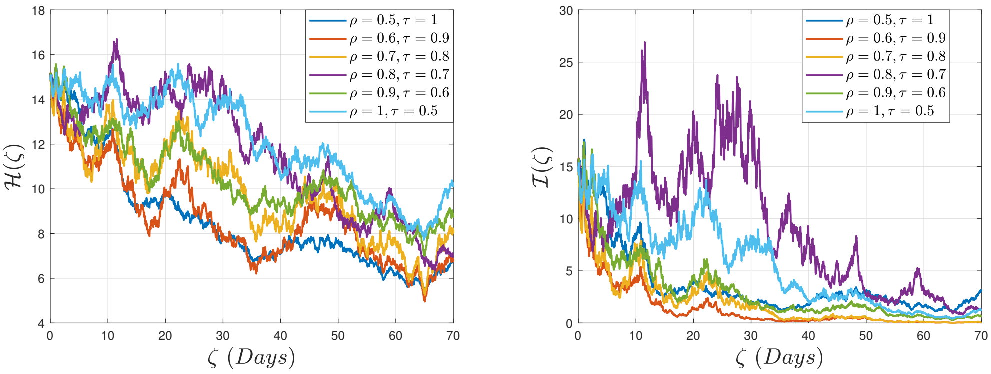
Figure 10. Dynamical behavior of the stochastic F-F EV model (5.1) for hospitalized individuals $\mathcal{S}(\zeta)$ and the deceased $\mathcal{D}(\zeta)$ when ρ increases and τ decreases significantly.



(a)

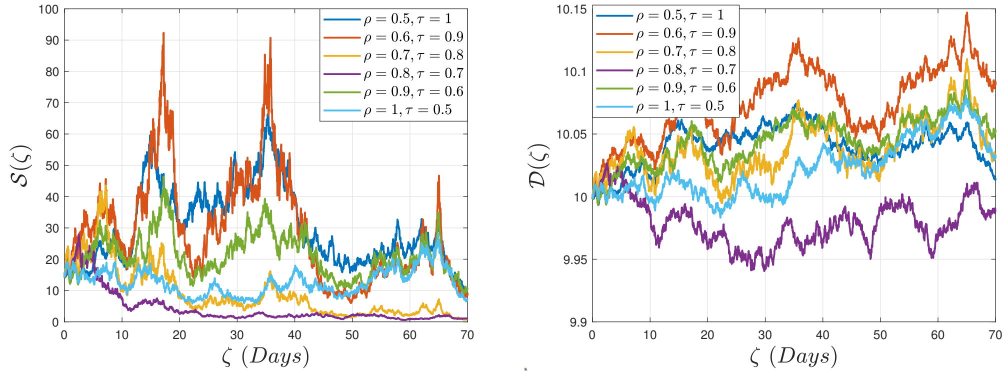
Figure 11. Dynamical behavior of the stochastic F-F EV model (5.1) for pathogens in the surroundings $\mathcal{P}(\zeta)$ when ρ increases and τ decreases significantly.

The reproduction number $\mathbb{R}_0^s$ rises as the successful interaction levels do, as seen in Figures 12–14 when the random densities were $\wp_1 = 1.2$, $\wp_2 = 1.4$, $\wp_3 = 1.6$, $\wp_4 = 1.8$ and $\wp_5 = 1.9$, respectively. As a result, when acquaintance patterns have taken hold, transmissibility levels are also significant. The infections in the atmosphere are shown to contribute to the spread of infection in Figure 12(b). The identification of perished victims of the aforesaid outbreak at various fractional orders and fractal dimensions is shown in Figure 13(b). Numerous victims get the infection, as evidenced by the fact that the EV diseased and mortalities continue to release viruses into the environment.



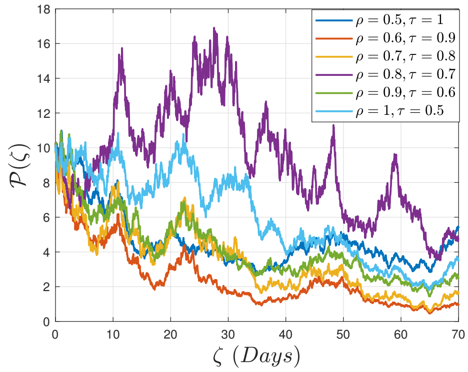
(a) **(b)**

Figure 12. Dynamical behavior of the stochastic F-F EV model (5.1) for susceptible individuals $\mathcal{S}(\zeta)$ and infectious people $\mathcal{I}(\zeta)$ when ρ decreases and τ increases significantly.



(a) **(b)**

Figure 13. Dynamical behavior of the stochastic F-F EV model (5.1) for hospitalized individuals $\mathcal{H}(\zeta)$ and the deceased $\mathcal{D}(\zeta)$ when ρ decreases and τ increases significantly.

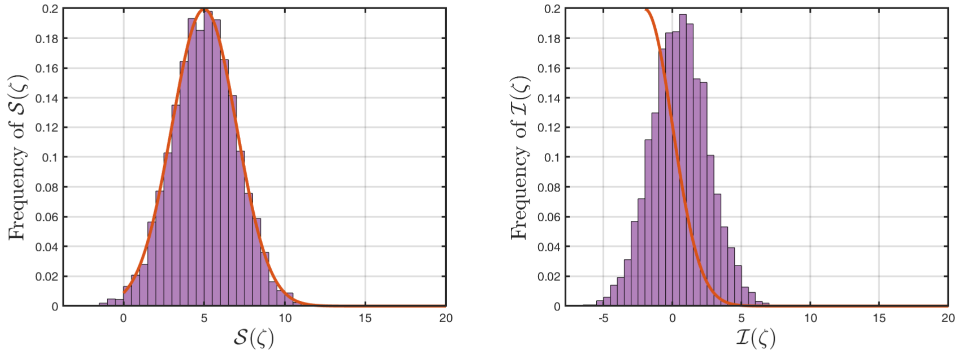


(a)

Figure 14. Dynamical behavior of the stochastic F-F EV model (5.1) for pathogens in the surroundings $\mathcal{P}(\zeta)$ when ρ decreases and τ increases significantly.

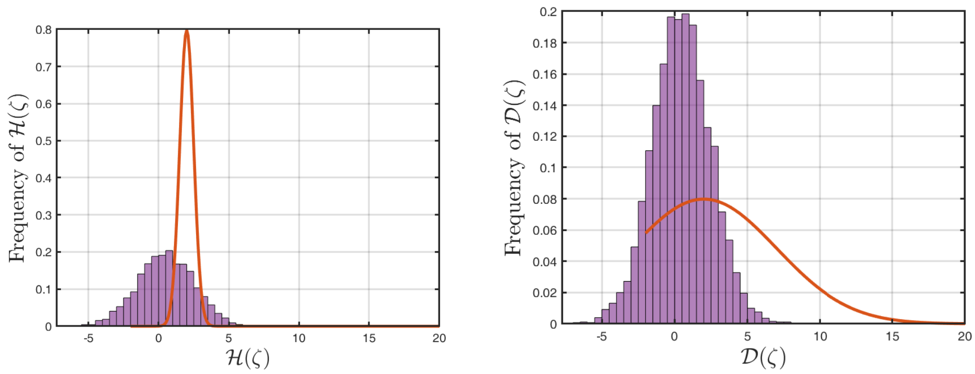
In Figures 15–16, we estimated $\mathbb{R}_0^s > 1$ by using the stochastic system (3.3); the characteristic variables are presented in Table 1, which guarantees that the EV sustained requirements have been fulfilled. The contamination of the mechanism described (3.3) will stay in the population, as shown in Figure 17, supporting the findings of Theorem 4.3. We tracked 5000 attempts at $\zeta = 400$, and then determined the mean and variance for normal distribution $\mathcal{N}(mean, variance)$. According to Theorem 4.3, Figures 15–16 illustrates that the model (3.3) exhibits an ergodic stationary distribution.

As a consequence of these illustrated outcomes, we arrive at the conclusion that by using this novel F-F formulation concept, it is important to investigate highly rigorous effects and convey a broader awareness of the challenges that arise in scientific disciplines as well as in real life.



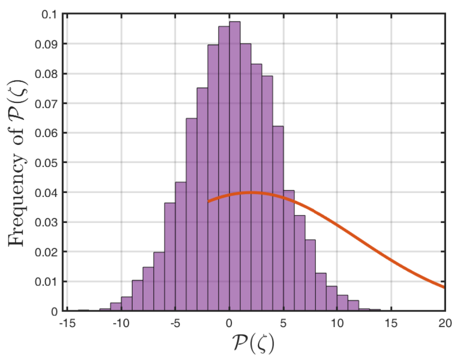
(a) **(b)**

Figure 15. Dynamical behavior of the stochastic F-F EV model (5.1) for susceptible individuals $\mathcal{S}(\zeta)$ and infectious people $\mathcal{I}(\zeta)$ when $\rho = 1 = \tau$ and there is the standard normal distribution $\mathcal{N}(0, 1)$.



(a) **(b)**

Figure 16. Dynamical behavior of the stochastic F-F EV model (5.1) for hospitalized individuals $\mathcal{H}(\zeta)$ and the deceased $\mathcal{D}(\zeta)$ when $\rho = 1 = \tau$ and there is the standard normal distribution $\mathcal{N}(0, 1)$.



(a)

Figure 17. Dynamical behavior of the stochastic F-F EV model (5.1) for pathogens in the surroundings $\mathcal{P}(\zeta)$ when $\rho = 1 = \tau$ and there is the standard normal distribution $\mathcal{N}(0, 1)$.

6. Conclusions

In this study, we proposed and investigated a six-cohort stochastic mathematical method that incorporates stochastic modeling techniques to depict how the evolution of the EV virus affects the mechanisms of the spread of infection while also accounting for environmental occurrences. Taking into consideration the F-F derivative, having the generalized Mittag-Leffler kernel and environmental noise were the key components of the system's examination. According to the interpretation of the stochastic Lyapunov candidate, a positive solution to the system was addressed. Furthermore, we demonstrated that the model's solution has an ergodic stationary distribution whenever $\mathbb{R}_0^s > 1$. Additionally, our research indicates that the EV virus epidemic may become obsolete whenever $\tilde{\mathbf{q}} > \frac{\wp_1^2 \vee \wp_2^2 \vee \wp_3^2 \vee \wp_4^2 \vee \wp_5^2}{2}$ and $\mathbb{R}_0^D < 1$. After conducting several experiments utilizing the stochastic F-F derivative approach with a novel methodology to sustain the theoretical aspects, it was found that both the eradication and consistency of pathogens are influenced by the level of white noise, with the eradication of pathogens increasing as the level of white noise intensifies and the perseverance of illnesses increasing as the level of white noise reduces. The system can be enhanced by taking into account infection proliferation in the surroundings apart from the transmission patterns discussed in this article. Even though the approach has several limitations, taking dread and environmental propagation into account is crucial for the treatment and prevention of EV virus disease. In future research, we can extend the model by incorporating Lévy noise and fractional calculus theory.

Conflict of interest

The authors declare no conflicts of interest.

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