



Research article

LX-BBSCA: Laplacian biogeography-based sine cosine algorithm for structural engineering design optimization

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Abstract: In this paper, an ensemble metaheuristic algorithm (denoted as LX-BBSCA) is introduced. It combines the strengths of Laplacian biogeography-based optimization (LX-BBO) and the sine cosine algorithm (SCA) to address structural engineering design optimization problems. Our primary objective is to mitigate the risk of getting stuck in local minima and accelerate the algorithm's convergence rate. We evaluate the proposed LX-BBSCA algorithm on a set of 23 benchmark functions, including both unimodal and multimodal problems of varying complexity and dimensions. Additionally, we apply LX-BBSCA to tackle five real-world structural engineering design problems, comparing the results with those obtained using other metaheuristics in terms of objective function values and convergence behavior. To ensure the statistical validity of our findings, we employ rigorous tests such as the t-test and the Wilcoxon rank test. The experimental outcomes consistently demonstrate that the ensemble LX-BBSCA algorithm outperforms not only the basic versions of BBO, SCA and LX-BBO but also other state-of-the-art metaheuristic algorithms.

Keywords: nature inspired algorithms; optimization; biogeography-based optimization (BBO); sine cosine algorithm (SCA); mutation operator; Laplacian migration operator

Mathematics Subject Classification: 90C59

1. Introduction

Optimization is considered a globally accepted tool for solving many real-world problems. Traditional optimization techniques have a limited scope in this regard as methods require continuity and differentiability knowledge of the objective function and constraints. Generally, real-world engineering problems are discontinuous and non-differentiable, and consequently, metaheuristic algorithms have been widely used in the applied sciences area because of their applicability and ease of use for a wide range of engineering problems [1–5].

Nature-inspired algorithms are optimization methods that are based on natural phenomena. Some popular metaheuristics in this class include the genetic algorithm (GA), ant colony optimization (ACO), particle swarm optimization (PSO) and grey wolf optimizer (GWO). These algorithms are rapidly evolving and growing. Based on the “no free lunch” theorem [6], this field has a lot of potential for researchers. It claims that no single algorithm is capable of solving all types of optimization problems, and each algorithm has its own advantages as well as some limitations.

Generally, exploration and exploitation are two important issues in stochastic algorithms [7]. Finding a new solution in the neighborhood of the existing solution is called exploration, whereas making a high change in the current position of the existing solutions to find a better fitness value is termed as exploitation. Some of the metaheuristic algorithms focus on exploration while many of them are better at exploitation. A balanced optimizer is one that provides a better exploration and extensive exploitation. To achieve this goal, the hybridization of two or more metaheuristic algorithms has proven to be a successful approach in the literature.

The sine cosine algorithm (SCA) [8] is a nature-inspired algorithm designed for solving complex optimization problems. It draws inspiration from the trigonometric functions of sine and cosine, which play a central role in the algorithm’s behavior. The SCA employs a population of candidate solutions represented as angles and uses the sine and cosine functions to update their positions iteratively. This approach facilitates both exploration and exploitation of the solution space. SCA is known for its simplicity, ease of implementation and efficiency in finding optimal or near-optimal solutions for a wide range of optimization problems, making it a valuable tool in optimization research and practical applications.

Biogeography-based optimization (BBO) [9] is a nature-inspired optimization algorithm rooted in the study of species distribution across geographic regions. It operates by modeling the migration and speciation of solutions within a solution space, akin to the movement of species between habitats. Each solution is associated with a habitat suitability index (HSI), which is based on suitability index variables (SIV) such as rainfall, vegetation and climate. The BBO employs migration and dispersal operators to facilitate the transfer of high-quality solutions to less explored regions while introducing randomness for exploration. The BBO is adaptable to various problem domains, accommodating both continuous and discrete optimization tasks. BBO has found applications in diverse fields, including engineering, machine learning and data mining, due to its ability to efficiently explore complex solution spaces, making it a valuable tool for real-world optimization challenges.

Laplacian BBO (LX-BBO) [10] is a variant of the BBO algorithm that incorporates Laplacian-based strategies to enhance its optimization capabilities. LX-BBO maintains the key principles of BBO, including the migration and speciation of solutions inspired by the species distribution in different habitats. However, LX-BBO introduces Laplacian operators to control the information flow among solutions in a population. These operators promote the sharing of useful information among the solutions while preventing overcrowding in the search space. By integrating Laplacian techniques, LX-BBO aims to improve the algorithm's convergence speed, exploration and exploitation abilities, making it a more effective tool for solving complex optimization problems in various domains.

Despite the remarkable advancements in metaheuristic algorithms, it remains a fundamental truth that no single algorithm can universally excel at solving all types of optimization problems. Recognizing this limitation, we hybridize the SCA and LX-BBO algorithms into the LX-BBSCA algorithm, seeking to harness the complementary attributes of both techniques. The proposed ensemble LX-BBSCA algorithm, combining the strengths of the SCA and LX-BBO, presents a powerful solution to optimization problems by leveraging the distinctive advantages of each constituent algorithm. Here are our motivations for presenting the LX-BBSCA algorithm:

- *Enhanced exploration and exploitation mechanisms:* On the one hand, SCA excels in exploration due to its inherent mechanisms inspired by trigonometric functions. It navigates the search space effectively, ensuring a wide exploration range. On the other hand, LX-BBO, with its migration and speciation strategies, is proficient at exploitation by refining solutions and guiding the algorithm toward optimal regions.
- *Balanced exploration-exploitation trade-off:* By combining SCA's exploration strength with LX-BBO's exploitation capabilities, LX-BBSCA achieves a refined balance between exploration and exploitation. It avoids premature convergence while ensuring efficient exploitation of discovered regions.
- *Fulfilling the limitation of single algorithms:* The recognition that no single algorithm universally excels at all problem types underscores the importance of hybridization to mitigate these limitations effectively.
- *Effective hybridization:* The integration of LX-BBO strategies into SCA enriches the algorithm's functionality by merging the strengths of two different paradigms, creating a more robust and versatile optimization tool. The inclusion of Laplacian operators from LX-BBO further refines the information sharing within the population, enhancing convergence speed and preventing stagnation in promising regions.
- *Applicability across domains in problem solving:* The adaptability of LX-BBSCA to various problem domains, ranging from engineering to data mining, highlights its applicability in addressing a wide array of real-world optimization challenges.
- *Potential for solution quality:* The hybridization opens doors to innovation in addressing complex problems, offering unique and potentially superior solutions.
- *Improved solution quality:* The incorporation of LX-BBO strategies into SCA enhances the quality of solutions, potentially providing more accurate and optimized results.
- *Practical implementation:* The hybrid LX-BBSCA algorithm maintains the simplicity and ease of implementation inherent in both methods, making it practical for practitioners. The combined LX-BBSCA algorithm not only inherits the merits of its constituent algorithms but also fills in their individual gaps, creating a unified and robust solution framework for tackling diverse optimization challenges in real-world applications.

The structure of this paper is as follows: In Section 2, we give a comprehensive review of the current literature, exploring the landscape of existing metaheuristic algorithms. Section 3 offers an in-depth description of LX-BBO and SCA and introduces our innovative ensemble LX-BBSCA algorithm. Then in Section 4, we meticulously project the outcomes of our experimentation, encompassing an extensive analysis of the results, comprehensive statistical evaluations, insightful convergence analyses and a thorough examination of the computational complexity. Section 5 takes a closer look at the application of the LX-BBSCA algorithm to tackle five real-world structural engineering challenges, providing a detailed account of the solutions obtained. Finally, in Section 6, we draw our conclusions, summarizing the key findings of this study and outlining avenues for future research directions.

2. Literature review

The existing literature provides a comprehensive overview of the widespread applicability and enhancements of the SCA in various optimization domains. The SCA, introduced by Mirjalili in 2016 [8], has garnered attention for its simplicity and effectiveness in solving global optimization problems, as validated by numerous researchers. Notable applications of SCA include its utilization by Wang et al. [11] and Tawahid et al. [12] to address multi-objective optimization problems. Furthermore, Reddy et al. [13] leveraged SCA to resolve profit-based unit commitment (PBUC) issues in competitive electricity markets, introducing a heuristic binary SCA method. Gupta and Deep [14] enhanced SCA by integrating PSO to improve its ability to locate global optima. Selim et al. [15] proposed a technique based on SCA and chaos map theory for allocating multiple distributed generators in distribution networks, demonstrating its practicality.

In hybridization efforts, SCA has been combined with various other algorithms to enhance its capabilities. Hota et al. [16] hybridized SCA with the extreme learning machine (ELM) for exchange rate prediction, whereas Sindhu et al. [17] proposed a strategy to improve SCA's accuracy. Zhu et al. [18] introduced the orthogonal learning strategy (OLSCA) to enhance SCA, and Gupta and Deep [19] introduced the HSCA variant to mitigate local optima entrapment. In engineering design applications, El-Shorbagy et al. [20] harnessed the power of hybridization in SCA-SSGA, and Chen et al. [21] improved SCA's performance through the Nelder-Mead simplex concept and opposition-based learning. Singh et al. [22] proposed the GWO-SCA algorithm, combining GWO and SCA effectively to solve benchmark functions and engineering fields. Fan et al. [23] introduced a hybrid method based on SCA and fruit fly optimization (SCA-FOA), Mahdad et al. (2017) used SCA to enhance power system security, and Kumar et al. [24] utilized hybrid Cauchy and Gaussian mutations. Moreover, Beururat et al. [25] developed an adaptive SCA, which is integrated with differential evaluation (ASCA-DE) to solve test problems for structural damage detection. Garg et al. [26] applied SCA to extract bioactive compounds, while Banerjee and Garg [27] proposed an algorithm incorporating five mutation operators into SCA for optimization tasks. Gupta and Deep [19] combined SCA and the artificial bee colony (ABC) algorithm, whereas Issa et al. [28] presented ASCA-PSO, a variant combining SCA with particle swarm optimization. Attia et al. [29] introduced Levy's flight into SCA to enhance its local search capabilities.

The extensive studies highlight the versatility and continuous improvements in the SCA and its various hybridizations, paving the way for enhanced optimization techniques across diverse domains. Das et al. [30] applied SCA to tackle fixed-head short-term hydrothermal scheduling problems, whereas Gupta and Deep [19] introduced the crossover approach (ISCA) to enhance SCA's exploration

and exploitation abilities. The combination of SSGA's exploitation merits with SCA's exploration capabilities was proposed by Turgut et al. [31] for shell and tube evaporator design. Finally, Elaziz et al. [32] adapted SCA to address complex nonlinear optimization problems.

One of the notable trends is the fusion of SCA with other metaheuristics to leverage the strengths of multiple algorithms, resulting in enhanced exploration and exploitation capabilities. It indicates a pragmatic approach to solving complex problems by capitalizing on the complementary attributes of different algorithms. Furthermore, the incorporation of various mutation strategies, including Cauchy, Gaussian and Levy mutations, has been demonstrated to augment SCA's ability to navigate intricate solution spaces efficiently. These mutation operators introduce diversity and adaptability into the optimization process, potentially mitigating convergence to local optima.

LX-BBO was extended by Garg and Deep [10], introducing constrained linear optimization capabilities, and Gupta and Deep [19] showcased LX-BBO in portfolio optimization. Furthermore, Garg et al. [33] demonstrated the effectiveness of incorporating LX-BBO into the salp swarm algorithm and the teaching-learning algorithm.

The literature underscores the importance of rigorous testing and validation through benchmark functions and practical engineering applications. By subjecting SCA and its variants to diverse testing scenarios, researchers have provided empirical evidence of their effectiveness in solving complex structural engineering design optimization problems. The diverse body of literature demonstrates the ever-evolving landscape of SCA and its hybridizations, reinforcing its status as a robust and versatile optimization tool. The reviewed literature highlights the potential directions of SCA, such as exploring hybridization with emerging metaheuristic algorithms and conducting more extensive real-world applications to further validate its practical utility. Furthermore, the LX-BBO algorithm has shown its effectiveness in terms of the constrained linear optimization capabilities. Therefore, due to the unique advantages of the SCA and LX-BBO algorithms, we are motivated to develop their hybridization into the proposed LX-BBSCA algorithm to simultaneously benefit from the advantages of both techniques while alleviating their drawbacks.

3. Proposed LX-BBSCA algorithm

3.1. Laplacian biogeography-based optimization (LX-BBO)

BBO [9] is based on the island biogeography theory, which is concerned with the migration, speciation and extinction of species within a habitat. It only considers the better living conditions for measuring the HSI. Personal interests, individual choices or societal norms, are not taken into consideration. A habitat or living place is considered a solution. The solution's fitness value is the number of species in the habitat. More species in a habitat is a notion of a good solution as this habitat has better living conditions and thus attracts species to move into the place. This phenomenon is termed a migration operator, as the better habitat is preferred over a worse solution, i.e., a habitat that has a smaller number of species count. Drastic change in weather (e.g., tsunami, earthquake or volcanic eruption) is termed the mutation operator in BBO. The details of these operators are given in the following subsections.

3.1.1. Migration operator

Based on emigration and immigration rates, previous solutions are replaced with better solutions. High emigration rates indicate a better solution and a high immigration rate gives a worse solution. This operator is an important part of the exploitation of the search space. A balanced and robust algorithm has a strong exploitation so as to find solutions in the neighborhood of the existing solution. In classical BBO, the migration operator lags behind to fully exploit the search space. It excels at exploitation but falls short at exploration. The BBO has undergone numerous modifications and hybridizations since its inception in order to improve its performance. There have been numerous efforts to raise the standard of BBO's exploitation. In an effort to better leverage the search space, Garg and Deep [34] collected various types of upgraded BBO utilizing different mutation and migration operators. A real-coded genetic algorithm's LX-crossover by Garg and Deep [10] produced a remarkable LX-BBO performance.

3.1.2. Laplacian migration operator

The restriction of the migration operator in the original BBO has been solved by the addition of the LX crossover from the real coded GA. The LX-BBO results have properly asserted that the program performed superbly. The LX-BBO migration operator selects two habitats (x_1 and x_2) using emigration and immigration rates. Then, these habitats will be combined via Laplace crossover to produce new habitats. The two new habitats, denoted as y_1 and y_2 , are produced as follows:

$$y_1^i = x_1^i + \beta(x_1^i - x_2^i) , \quad (1)$$

$$y_2^i = x_2^i + \beta(x_1^i - x_2^i) , \quad (2)$$

where β is a random number that follows the Laplace distribution and is generated as follows:

$$\beta = \begin{cases} a - b \log(u) & \text{if } u \leq 1/2 \\ a + b \log(u) & \text{if } u > 1/2 \end{cases} , \quad (3)$$

where u is a uniform random number within $[0, 1]$, $b > 0$ is called the scale parameter and $a \in R$ is a location parameter. The habitats y_1 and y_2 produce a new habitat z for the next generations as follows:

$$z = \gamma y_1^i + (1 - \gamma) y_2^i , \quad (4)$$

$$\gamma = \gamma_{min} + (\gamma_{max} - \gamma_{min})^k , \quad (5)$$

where γ_{min} and γ_{max} are in the range $[0, 1]$ and define the minimum and maximum bounds of β , i is the counter of the number of generations and k is a user-specific parameter. After each generation, an elitism mechanism is applied to select two habitats. It preserves the two best environments to replace the two worst solutions. The pseudo-code of LX-BBO is shown in Algorithm 1.

3.1.3. Mutation operator

Mutation in classical BBO is the tool for exploring new solutions. The newly generated solution by the mutation operator is based on the mutation rate and these rates are determined by a differential equation. A rigorous experimental study on the effects of different mutations on the performance of LX-BBO can be found in [35]. However, the results are similar for all mutation operators, and no outstanding breakthrough was established. This proves that the mutation operator needs to be efficient for the LX-BBO algorithm.

Algorithm 1. LX-BBO algorithm.

Begin

Generation of a random population $H=\{H_1, H_2, H_3, \dots, H_n\}$

Determining the corresponding HSI values

Sort the population by the HSI values from best to worst

while ($t \leq Max_iteration$)

 for each habitat

 for each SIV

 Calculation of the emigration and immigration rates for each habitat

 Choosing H_i based on the immigration rate

 if H_i is chosen ($i = 1, 2, 3, \dots, n$)

$$H_i^1(SIV) = H_i(SIV) + \beta(H_i(SIV) - H_j(SIV))$$

$$H_i^2(SIV) = H_j(SIV) + \beta(H_i(SIV) - H_j(SIV))$$

$$\text{Updating } H_i(SIV) \leftarrow \gamma H_i^1(SIV) + (1 - \gamma) H_i^2(SIV)$$

$$\gamma = \gamma_{min} + (\gamma_{max} - \gamma_{min})^k$$

 if H_j is chosen ($j = 1, 2, 3, \dots, n$)

 Randomly selection of an SIV from H_i

 Replacing a randomly SIV in H_i with one from H_j

 end if

 end if

 Choosing a SIV in H_i based on the mutation rate μ

 if $H_j(SIV)$ is chosen

 Replacing a randomly selected SIV

 end if

 end for

end for

Calculation of the modified habitats HSI value again

Sort the population by the HSI values from best to worst

Replacing the worst solution using Elitism strategy

Sort the population by the HSI values from best to worst

$t=t+1$;

end while

End

3.2. Sine cosine algorithm (SCA)

The SCA was inspired by the sine and cosine functions from mathematics and is designed to solve optimization problems. It falls under the category of metaheuristic algorithms, which are used to find approximate solutions to optimization and search problems. This algorithm simulates the movement of sine and cosine waves to guide the search for optimal solutions in a given problem space. It is a population-based algorithm, where each potential solution is represented as an individual in the population. The algorithm iteratively updates the positions of these individuals in search of better solutions over time. Next, we present the basic outline of how the SCA works:

- *Initialization*: Initialize a random population of candidate solutions within the search space.
- *Evaluation*: Evaluate the fitness of the candidate solutions based on the objective function. The fitness value indicates how well each solution performs in terms of the objective.
- *Update positions*: Update the position of each candidate solution using the following equations, which are inspired by the sine and cosine functions:

$$X_i^{t+1} = \begin{cases} X_i^t + r_1 \times \sin(r_2) \times |r_3 P_i^t - X_i^t| \times r_4 \leq 0.5 \\ X_i^t + r_1 \times \cos(r_2) \times |r_3 P_i^t - X_i^t| \times r_4 > 0.5 \end{cases} \quad (6)$$

where X_i^t is the position of solution i in the current iteration, X_i^{t+1} is the position of the same solution in the next iteration, P_i^t is the destination position of the global optimal particle, r_1 linearly decreases from 2 to 0, and r_2 , r_3 and r_4 are uniformly distributed random numbers: r_2 and $r_3 \in [0, 2]$; $r_4 \in [0, 1]$.

- *Boundary handling*: It ensures that the updated positions remain within the feasible region of the problem by applying suitable boundary handling mechanisms.
- *Update the best solution*: It updates the best solution found so far based on the fitness values of the new candidate solutions.
- *Termination*: The algorithm terminates when a certain stop criterion is met, e.g., reaching a pre-defined number of iterations or achieving a satisfactory solution.

SCA is relatively simple to implement and has shown promising performance on different problems. However, its performance can vary depending on the problem's characteristics and parameter settings. As with any optimization algorithm, proper parameter tuning and problem-specific adaptations are crucial for achieving good results. This method considerably assists the search process in avoiding local optima, resulting in faster search times. Its exploration abilities are superior to its exploitation abilities. The pseudo-code of SCA is shown in Algorithm 2. In this pseudo-code, H_i^t denotes habitant i in iteration t .

Algorithm 2. Classical SCA algorithm.

```

Begin
Initialization of the solutions  $H_i(i = 1,2,3, \dots, n)$ 
Assessment of the fitness value of each solution
Selecting the best solution as  $G_{best}$ 
while ( $t \leq Max\_iteration$ )
  for  $i = 1:n$ 

```

```

Updating  $r_1$  to  $r_4$ :  $r_1$ : linearly decreasing from 2 to 0,  $r_2$  and  $r_3 \in [0, 2]$ ,  $r_4 \in [0, 1]$ 
Updating the position of solution  $i$  using Eq (6)
Assessment of the fitness value for solution  $i$ 
Checking the solution whether it goes beyond the search space; if yes, bring it back
if  $f(H_i^{t+1}) < f(H_i^t)$ 
    Updating the global best solution  $Gbest$ 
end if
end for
 $t=t+1$ ;
end while
return  $Gbest$ 
End

```

3.3. Proposed hybrid version of LX-BBO and SCA (LX-BBSCA)

The LX-BBO algorithm is good enough at altering the current population given to its migration operator, but it takes a long time to explore the entire search space. Therefore, a combined algorithm based on LX-BBO and SCA is designed to choose the most relevant feature set from the original feature. These algorithms are combined into a hybrid version, denoted as LX-BBSCA, where the mutation operator of LX-BBO is replaced with that of SCA. The pseudo-code of the LX-BBSCA is given in Algorithm 3.

Algorithm 3. Proposed LX-BBSCA algorithm.

```

Begin
Initialization of the solutions  $H_i(i = 1,2,3, \dots, n)$ 
Assessment of the fitness value of each solution
Selecting the best solution as  $Gbest$ 
while ( $t \leq Max\_iteration$ )
    SCA:
    for  $i = 1:n$ 
        Updating  $r_1$  to  $r_4$ :  $r_1$ : linearly decreasing from 2 to 0,  $r_2$  and  $r_3 \in [0,2]$ ,  $r_4 \in [0,1]$ 
        Updating the position of solution  $i$  using Eq (6)
        Assessment of the fitness value for solution  $i$ 
        Checking the solution whether it goes beyond the search space; if yes, bring it back
        if  $f(H_i^{t+1}) < f(H_i^t)$ 
            Updating the global best solution  $Gbest$ 
        end if
    end for
    LX-BBSCA:
    Calculate the immigration rate
    Select habitant  $H_i$ 
    for  $i = 1:n$ 
        for  $j = 1:D$ 

```

```

if rand <  $\lambda_k$ 
     $H_i^1(SIV) = H_i(SIV) + \beta(H_i(SIV) - H_j(SIV))$ 
     $H_i^2(SIV) = H_j(SIV) + \beta(H_i(SIV) - H_j(SIV))$ 
    Updating  $H_i(SIV) \leftarrow \gamma H_i^1(SIV) + (1 - \gamma)H_i^2(SIV)$ 
     $\gamma = \gamma_{min} + (\gamma_{max} - \gamma_{min})^k$ 
end if
if rand < 0.5 then
    Replacing the SIV of  $H_i$  with a randomly selected SIV
end if
end for
end for
for each habitat
    Recompute the HSI values
    Checking the solution whether it goes beyond the search space; if yes, bring it back
    if  $f(H_i^{t+1}) < f(H_i^t)$ 
        Updating the global best solution  $Gbest$ 
    end if
end for
 $t=t+1$ ;
end while
Return  $Gbest$ 
End

```

3.4. Computational complexity

The computational complexity of an algorithm is a criterion for determining its robustness with the least amount of effort. The computational complexity of the LX-BBO and LX-BBSCA algorithms can be described as follows: Initially, both algorithms consider the same population size N . Both algorithms have the same complexity when it comes to initializing input parameters such as population size, number of iterations, problem size and so on. After a while loop has been completed, the main mathematical measures for both algorithms begin. Because each solution is updated in the algorithms LX-BBO and LX-BBSCA, this computational complexity is the same for both methods. If the algorithm fails to achieve the desired accuracy, it will be continued for T iterations. Moreover, an objective function is used to determine each solution. As a result, the computational complexity in this step is $O(N)$. In the worst-case scenario, the overall complexity can be defined as $O(T \times N)$, although the complexity of the LX-BBSCA algorithm is not increased against the single LX-BBO algorithm.

4. Validation of LX-BBSCA for benchmark functions

The parameters set for the LX-BBSCA algorithm encompass a population size of 50 individuals and a total of 200 iterations. Importantly, LX-BBSCA maintains the integrity of the original parameters from both SCA and LX-BBO algorithms, deliberately excluding any new parameters. This choice aims to ensure a seamless transition and minimize the risk of conflicting settings between the integrated algorithms. Furthermore, to maintain a fair comparison with other metaheuristics, the LX-BBSCA

experimental setup mirrors the same population size and number of iterations as the other techniques: 50 populations and 200 iterations. This consistency in parameters across all methods establishes an equivalency in the number of fitness evaluations (NFV), allowing for an unbiased comparison between the different algorithms.

To validate the performance of the LX-BBSCA algorithm, it is used to solve 23 popular benchmark functions, as listed in Table 1. The problem set consists of 7 unimodal test functions (F1–F7) and 16 multimodal test functions (F8–F23).

Table 1. Benchmark functions.

Function	Function name	Dim	Range	f_{min}
$f_1(x) = \sum_{i=1}^n x_i^2$	sphere	30	[-100, 100]	0
$f_2(x) = \sum_{i=1}^n x_i + \prod_{i=1}^n x_i $	Schwefel's 2.22	30	[-10, 10]	0
$f_3(x) = \sum_{i=1}^n x_i \left(\sum_{i=1}^n x_i \right)^2$	Schwefel's 1.20	30	[-100, 100]	0
$f_4(x) = \max \{ x_i , 1 \leq i \leq n\}$	Schwefel's 2.21	30	[-100, 100]	0
$f_5(x) = \sum_{i=1}^{n-1} [100(x_{i+1} - x_i^2)^2 + (x_i - 1)^2]$	Rosenbrock	30	[-30, 30]	0
$f_6(x) = \sum_{i=1}^n (x_i + 0.5)^2$	step	30	[-100, 100]	0
$f_7(x) = \sum_{i=1}^n ix_i^4 + random[0,1]$	Quartic Noise	30	[-1.28, 1.28]	0
$f_8(x) = \sum_{i=1}^n -x_i \sin(\sqrt{ x_i })$	Schwefel	10	[-500, 500]	-418*5
$f_9(x) = \sum_{i=1}^n [x_i^2 - 10 \cos(2\pi x_i) + 10]$	Rastrigin	30	[-5.12, 5.12]	0
$f_{10}(x) = -20 \exp\left(-0.2 \sqrt{\frac{1}{n} \sum_{i=1}^n x_i^2}\right) - \exp\left(\frac{1}{n} \sum_{i=1}^n \cos(2\pi x_i)\right) + 2 + e$	Ackley	30	[-32, 32]	0
$f_{11}(x) = \frac{1}{4000} \sum_{i=1}^n x_i^2 - \prod_{i=1}^n \cos\left(\frac{x_i}{\sqrt{i}}\right) + 1$	Griewank	30	[-600, 600]	0
$f_{12}(x) = \frac{\pi}{n} \left\{ 10 \sin(\pi y_i) + \sum_{i=1}^{n-1} (y_i - 1)^2 [1 + 10 \sin^2(\pi y_{i+1})] + (y_n - 1)^2 \right\} + \sum_{i=1}^n u(x_i, 10, 100, 4)$	Penalized 1	30	[-50, 50]	0
$f_{13}(x) = 0.1 \left\{ \sin^2(3\pi x_1) + \sum_{i=1}^n (x_i - 1)^2 [1 + \sin^2(3\pi x_i + 1)] + (x_n - 1)^2 [1 + \sin^2(2\pi x_n)] \right\} + \sum_{i=1}^n u(x_i, 5, 100, 4)$	Michalewicz	30	[-50, 50]	0

Continued on next page

Function	Function name	Dim	Range	f_{min}
$f_{14}(x) = \left(\frac{1}{500} + \sum_{j=1}^{25} \frac{1}{j + \sum_{i=1}^2 (x_i - a_{ij})^6} \right)^{-1}$	Shekel's	2	[-65, 65]	1
$f_{15}(x) = \sum_{i=1}^{11} \left[a_i - \frac{x_1(a_i^2 + b_i x_2)}{b_i^2 + b_i x_3 + x_4} \right]^2$	Foxholes	2	[-5, 5]	0.00030
$f_{16}(x) = -4x_1^2 - 2.1x_1^4 + \frac{1}{3}x_1^6 + x_1x_2 - 4x_2^4 + 4x_2^4$	Six-hump camel	3	[-5, 5]	0.398
$f_{17}(x) = (x_2 - \frac{5.1}{4\pi^2}x_1^2 + \frac{5}{\pi}x_1 - 6)^2 + 10(1 - \frac{1}{8\pi})\cos x_1 + 10$	Goldstein price	6	[-5, 5]	3
$f_{18}(x) = [1 + (x_1 + x_2 + 1)^2(19 - 14x_1 + 3x_1^2 - 14x_2 + 6x_1x_2 + 3x_2^2)] * [30 + (2x_1 - 3x_2)^2]$	Hartman's family-2	4	[0, 10]	-3.86
$f_{19}(x) = -c_i \exp(-\sum_{i=1}^6 a_{ij}(x_j - P_{ij})^2)$	sphere	3	[1, 3]	-3.32
$f_{20}(x) = -\sum_{i=1}^4 c_i \exp(-\sum_{i=1}^3 a_{ij}(x_j - P_{ij})^2)$	Hartmann 6-D	6	[0, 1]	-10.15
$f_{21}(x) = -\sum_{i=1}^5 [(X - a_i)(X - a_i)^T + C^i]^{-1}$	Shekel's	4	[0, 10]	-10.15
$f_{22}(x) = -\sum_{i=1}^7 [(X - a_i)(X - a_i)^T + C^i]^{-1}$	--	4	[0, 10]	-10.40
$f_{23}(x) = -\sum_{i=1}^{10} [(X - a_i)(X - a_i)^T + C^i]^{-1}$	--	4	[0, 10]	-10.53

In the following, the results of the proposed LX-BBSCA algorithm are compared to BBO and LX-BBO. The statistical parameters include the values for the objective function's average, the standard deviation (Std), the median, the minimum and maximum values, as well as the minimum and maximum for each run. The validity of the algorithms was evaluated using the Std and average. Furthermore, the minimum and maximum values of the objective function represent the best possible solutions to the given problem in terms of iterations. In Table 2, the calculations are computed for 30 independent trials and 1000 function evaluations.

Table 2. Objective function values obtained by LX-BBO, BBO and LX-BBSCA.

Function	Algorithm	Min	Max	Std	Average	Median
1	LXBBSCA	1.47E-08	0.003555	0.001019	0.0005	3.15E-05
	BBO	1777.031	5077.365	716.4427	3868.813	3941.62
	LX-BBO	1494.216	5044.481	865.3063	3672.611	3714.437
2	LXBBSCA	2.03E-06	0.004806	0.000867	0.000304	7.91E-05
	BBO	10.79175	21.72683	2.374523	17.00374	17.2588
	LX-BBO	11.32607	19.53608	2.057063	16.08974	16.2298
3	LXBBSCA	0.006513	13.14504	3.474535	1.89781	0.278841
	BBO	1934.879	5951.809	927.4112	3758.93	3920.839
	LX-BBO	2049.36	5171.657	816.2047	3564.492	3472.326
4	LXBBSCA	0.006297	0.446159	0.12764	0.113329	0.043289
	BBO	22.23739	42.85724	4.189265	34.69546	34.97368
	LX-BBO	19.27473	39.92089	4.00352	33.3647	33.24844
5	LXBBSCA	7.505546	706.7561	134.2925	40.99334	8.27666
	BBO	437754	3670960	763242.4	1595315	1487924
	LX-BBO	672237.7	2843717	561336.2	1672675	1735165
6	LXBBSCA	0.288575	0.979075	0.153504	0.703137	0.731203
	BBO	1932.084	6250.072	845.4571	3853.714	3971.152
	LX-BBO	987.377	5581.525	888.0294	3859.914	3877.024
7	LXBBSCA	0.001156	0.027382	0.006862	0.007034	0.003887
	BBO	0.26968	1.230022	0.230808	0.758662	0.709212
	LX-BBO	0.226661	1.362734	0.30781	0.809159	0.840667
8	LXBBSCA	-2827.44	-2036.08	166.5375	-2244.35	-2216.65
	BBO	-2829.83	-2121.53	181.1336	-2456.98	-2445.76
	LX-BBO	-2875.34	-2225.78	131.6692	-2432.53	-2413.6
9	LXBBSCA	2.33E-06	35.86702	10.85302	5.389507	0.14953
	BBO	46.81987	78.79715	6.388578	65.23132	65.17603
	LX-BBO	40.7377	74.05955	6.887215	62.28468	63.68432
10	LXBBSCA	6.82E-05	0.056023	0.010371	0.004476	0.001321
	BBO	13.59016	17.02045	0.828409	15.85072	16.04995
	LX-BBO	13.5316	17.03005	0.923063	15.67928	15.87936
11	LXBBSCA	0.00015	0.863571	0.231311	0.330496	0.313525
	BBO	22.21162	48.54429	7.655131	35.48117	35.48288
	LX-BBO	11.99046	48.13176	7.368682	36.91168	37.60413
12	LXBBSCA	0.066566	0.350893	0.070983	0.182471	0.179451
	BBO	42333.56	2511312	685507	686604.5	373467.9
	LX-BBO	266.1032	2036870	468695.3	488556.4	353698.8
13	LXBBSCA	0.199118	0.547923	0.085934	0.408174	0.401248
	BBO	697435.3	12295841	2913431	4140497	3615374
	LX-BBO	475863.4	9962896	2547311	4441777	4136487
14	LXBBSCA	0.998004	1.009151	0.002023	0.998519	0.998021
	BBO	0.998006	1.027821	0.006486	1.000994	0.998702
	LX-BBO	0.998004	1.176247	0.041217	1.019683	1.001262
15	LXBBSCA	0.000522	0.001586	0.000381	0.001096	0.001103
	BBO	0.000736	0.004441	0.000861	0.002095	0.00193
	LX-BBO	0.000443	0.005199	0.001079	0.002494	0.002101

Continued on next page

Function	Algorithm	Min	Max	Std	Average	Median
16	LXBBSCA	-1.03162	-1.03129	8.75E-05	-1.03155	-1.03158
	BBO	-1.03152	-1.01417	0.004388	-1.02788	-1.02916
	LX-BBO	-1.03136	-1.02386	0.002109	-1.02922	-1.02999
17	LXBBSCA	0.397887	0.407469	0.002234	0.399293	0.39849
	BBO	0.397969	0.402253	0.001318	0.399398	0.398989
	LX-BBO	0.397943	0.405405	0.002047	0.399853	0.39913
18	LXBBSCA	3.000001	3.001711	0.000314	3.000123	3.000031
	BBO	3.000651	3.301352	0.070737	3.062178	3.039374
	LX-BBO	3.001377	3.208721	0.060075	3.055511	3.026186
19	LXBBSCA	-3.86153	-3.85282	0.001996	-3.85482	-3.85442
	BBO	-3.8626	-3.85139	0.002833	-3.8588	-3.85956
	LX-BBO	-3.86254	-3.84367	0.004217	-3.85762	-3.8583
20	LXBBSCA	-3.12678	-1.68596	0.347211	-2.88178	-3.00666
	BBO	-3.22558	-2.95942	0.061642	-3.0718	-3.05945
	LX-BBO	-3.19627	-2.91769	0.056134	-3.08601	-3.08811
21	LXBBSCA	-5.85902	-0.87934	1.738807	-3.60057	-4.69579
	BBO	-8.40559	-2.53527	1.188802	-4.05523	-3.75194
	LX-BBO	-7.07924	-2.17446	1.127562	-3.73127	-3.67004
22	LXBBSCA	-8.8799	-0.90652	1.968125	-4.25467	-4.60085
	BBO	-5.87171	-2.48824	0.819649	-3.90592	-3.90487
	LX-BBO	-8.17891	-2.59073	1.288829	-4.16432	-3.82846
23	LXBBSCA	-6.73001	-0.94429	1.530706	-4.24441	-4.38713
	BBO	-7.76979	-2.86094	1.138833	-4.29905	-4.01435
	LX-BBO	-6.95564	-2.57408	1.23627	-4.39294	-4.20171

4.1. Unimodal test functions

If a unimodal function is monotonically increasing and monotonically decreasing, respectively, then the function has one global minimum or maximum value. The unimodal functions (F1–F7) have one global solution. So, they are taken to investigate the exploitation capacity. As a result, they are used to assess the convergence rate of the heuristic optimization algorithms. In terms of mean, Std, median, minimum and maximum values of the result. To solve a unimodal problem, the hybrid method LX-BBSCA can find the most feasible and comprehensive solutions. In all problems, the LX-BBSCA algorithm outperforms the other algorithms. As a result, it has a strong exploitation ability and great performance when it locates the overall solution of the unimodal benchmark function. In Table 2, the objective function values are analyzed over 30 different runs, and their mean, mode, Std, best values and worst values are reported. The results in the first seven rows give the results for unimodal functions. Clearly, LXBBSCA has obtained better results than its counterparts. The results indicate that LXBBSCA has better exploitation capability than LXBBBO and BBO.

4.2. Multimodal test functions

The multimodal functions (F8–F13) have a lot of local optimal points. These functions are typically tested to assess the exploration ability of the different methods, as having a lot of local optima values raises the possibility of stagnation of an algorithm. Multimodal functions with numerous local minima are notoriously difficult to optimize. The proposed variant's superiority and ability on multiple

dimensions have been demonstrated using Std, average, and minimum and maximum objective function values. The final results are significantly more relevant for these functions since they represent the algorithm's ability to escape undesirable local optima and obtain a good solution near the global optimum.

The results in Table 2 show that the LX-BBSCA algorithm outperforms BBO, LX-BBO, and SCA, for all performance measures in the multimodal functions (F8–F13). Furthermore, the test results reveal that the novel hybrid method's strong exploration capability allows us to extensively search the space and find viable search areas. In Table 2, the results are demonstrated for multimodal functions in rows 8–13. The analysis of these problems clearly indicates the better performance of LXBBCA as the objective function values of LX-BBSCA are smaller than those obtained by LX-BBO and BBO. Since the problems are minimization ones, the results are encouraging in order to claim that LX BBCA has a better exploration strategy than its counterparts. Multimodal problems need a better extensive search in the search space.

The test problems (F14–F23) are multimodal functions that are typically used to evaluate the search algorithm's ability to balance exploration and exploitation. The main difference between functions F8–F13 and functions with a few local minima, i.e., F14–F23, is that functions F14–F23 are simpler than functions F8–F13 due to their low dimensionality and fewer local minima. BBO and LX-BBO algorithms were significantly outperformed by LX-BBSCA.

These findings illustrate that the proposed LX-BBSCA algorithm outperforms the traditional method in terms of solution quality. The new hybrid approach LX-BSCA prevents the search process from converging prematurely to a local optimal point and allows exploration of the search path. In Table 2, rows 14–23, the results are given for multimodal functions with varying dimensions. Testing the algorithm on varying dimensionality problems is the key to finding the usability and applicability of the algorithm for complex real-world problems. The results in Table 2 clearly show the better performance of the proposed method over its basic versions.

4.3. Statistical analysis

To check the performance of the algorithms, the t-test and the Wilcoxon rank test are performed to compare the significance of the LX-BBSCA and LX-BBO algorithms.

4.3.1. t-test analysis

A paired t-test, also referred to as a paired-samples t-test or a dependent-samples t-test, is a statistical test employed to assess whether there exists a significant disparity between the means of two interconnected groups. This test is particularly useful when one has two sets of observations that are paired in some way, such as before-and-after measurements on the same subjects or matched data points. It is appropriate when the data are assumed to be approximately normally distributed and the differences between the paired observations are approximately normally distributed as well.

Table 3 shows the results obtained after using the t-test for the pair of LX-BBSCA and LX-BBO algorithms. The p-values are significant because they provide a common language for the test results. A p-value is a number ranging from 0 to 1 that can be used to determine the statistical significance of a set of data.

- If the p-value is larger than 0.05, the difference described is not significant.

- If the p-value is less than 0.05 but greater than 0.001, the difference is significant.
- If the p-value is less than 0.001, the difference is very significant.

The t-test results of the LX-BBSCA and LX-BBO are shown in Table 3, in which, “a+”, “a” and “b” mean the difference is “very significant”, “significant” and “no significance”, respectively.

Table 3. t-test results for LX-BBSCA and LX-BBO.

Function	Mean	Std deviation	Std error mean	95% Confidence interval		Significance	
				Lower	Upper	p	Conclusion
F1	3672.61	865.306	157.56	3995.72	3349.5	0	a+
F2	16.0892	2.057	0.37558	16.8574	15.3211	0	a+
F3	3562.59	816.499	149.071	3867.48	3257.71	0	a+
F4	33.2513	4.02652	0.73514	34.7549	31.7478	0	a+
F5	1672634	561349	102488	1882245	1463023	0	a+
F6	3859.21	888.051	162.135	4190.81	3527.61	0	a+
F7	8E+07	0.30838	0.0563	9172777	6.9E+07	0	a+
F8	188.181	206.407	37.6847	11.1075	265.255	0	a+
F9	56.895	11.0961	2.02585	61.03	52.7518	0	a+
F10	15.6748	0.92076	0.16811	16.0186	15.3309	0	a+
F11	-36.581	7.38748	1.34876	39.3397	33.8227	0	a+
F12	-488556	468695	85571.7	663570	313542	0	a+
F13	4441776	2547311	465073	5392958	3490595	0	a+
F14	0.0211647	0.04141	0.00756	-4E+06	0.005701113	0.009	a
F15	13988	0.00108	0.0002	-0.0018	-0.001	0	a+
F16	0.00233	0.00213	0.00039	0.00313	0.00154	0	a+
F17	0.00055987	0.00317	0.00058	17442	0.00062	0.034	a
F18	0.05539	0.05998	0.01095	0.07779	0.03299001	0	a+
F19	0.00279	0.00447	0.00082	0.00112	0.00446	0.002	a
F20	200620	0.35852	0.06546	0.06675	0.3345	0.005	a
F21	0.15514	2.13066	0.38901	6.4E+09	0.95075	0.693	b
F22	0.035	2.2221	0.40571	-0.8648	0.79477	0.932	b
F23	0.14853	2.1494	0.39243	0.65407	0.95114	0.708	b

4.3.2. Wilcoxon rank-sum test

Wilcoxon’s rank-sum test (WRST) is utilized to provide a more statistical evaluation of the suggested approach. This is a non-parametric test that determines whether LX-BBSCA and LX-BBO have a significant difference. For a statistically significant correlation, the Wilcoxon test was used since the median value is more essential in describing the algorithm’s efficiency, and this test may be applied without knowing the distribution of the data set. The statistical comparison between LX-BBO and the proposed LX-BBSCA algorithm is done in Table 4 at a significance level of 5%. The acquired p-values corresponding to all test problems are also presented in this table. The “+” sign is used to show that the LX-BBSCA is statistically significantly better than LX-BBO, and the “-” sign is considered to mention that LX-BBO is statistically significantly better than LX-BBSCA. Table 4

shows that out of 23 benchmark test problems, the proposed LX-BBSCA algorithm significantly outperforms LX-BBO in 17 test functions, while LX-BBO significantly outperforms LX-BBSCA in the remaining seven problems.

Table 4. Wilcoxon test for LX-BBSCA and LX-BBO with 5% level of significance.

Function	Wilcoxon's Z value	p	Conclusion
F1	-4.782	0.000	+
F2	-4.782	0.000	+
F3	-4.782	0.000	+
F4	-4.782	0.000	+
F5	-4.782	0.000	+
F6	-4.782	0.000	+
F7	-3.815	0.000	+
F8	-4.78	0.000	+
F8	-4.782	0.000	+
F9	-4.782	0.000	+
F10	-4.782	0.000	+
F11	-4.782	0.000	+
F12	-4.782	0.000	+
F13	-3.815	0.000	+
F14	-3.78	0.000	+
F15	-3.887	0.000	+
F16	-0.234	0.000	+
F17	0.256	0.245	-
F18	0.345	0.000	+
F19	0.332	0.003	-
F20	0.3075	0.002	-
F21	-0.237	0.813	-
F22	0.319	0.943	-
F23	0.319	0.750	-

5. Applying LX-BBSCA to structural engineering design problems

This section evaluates the performance of the proposed LX-BBSCA algorithm against the existing metaheuristic algorithms for five problems, including three structural engineering problems and two reliability problems.

5.1. Problem 1: Gear train problem

Analyzing the connections between several gears in a mechanical system is the gear train problem. Between spinning shafts, motion and power are transferred using gears. They come in a variety of shapes and sizes, and depending on how they are arranged in a gear train, they can offer varying mechanical benefits, speed ratios, and torque variations. A gear train is defined as a set of gears that consists of two pairs of gears and has an overall reduction ratio that as closely as possible approaches

a certain value while keeping in mind that the total number of teeth in all four gears must not exceed a predetermined limit. The problem can be mathematically stated as follows:

$$\text{minimize } f_1(x) = \left(\frac{1}{6.931} - \frac{x_1 x_3}{x_2 x_4} \right)^2, \quad x = (x_1, x_2, x_3, x_4), \quad (7)$$

$$\text{subject to } 12 \leq (x_1, x_2, x_3, x_4) \leq 60, \quad (8)$$

where x_1 , x_2 , x_3 and x_4 , are the number of teeth for four gears on a gearwheel.

The results obtained with LX-BBSCA, LX-BBO, BBO and SCA, as well as other metaheuristic algorithms, are reported in Table 5. The results show the superiority of the LX-BBSCA algorithm in solving the gear train problem by obtaining a better fitness value. Figure 1 shows the convergence graph for the problem.

Table 5. Comparison of different algorithms for the gear train problem.

Technique	x_1	x_2	x_3	x_4	Objective function value	The largest value of the objective function
LX-BBSCA	12	60	17.611	24.41	1.51×10^{-11}	1000
LX-BBO	23	39	12	50.08	2.58×10^{-11}	1000
BBO	14.66	60	16.67	28.25	2.68×10^{-11}	1000
SCA	18	26	12	60	7.5×10^{-10}	300
CS	16	43	19	49	2.7×10^{-11}	5,000
GWO	17	54	22	48	1.1×10^{-10}	700

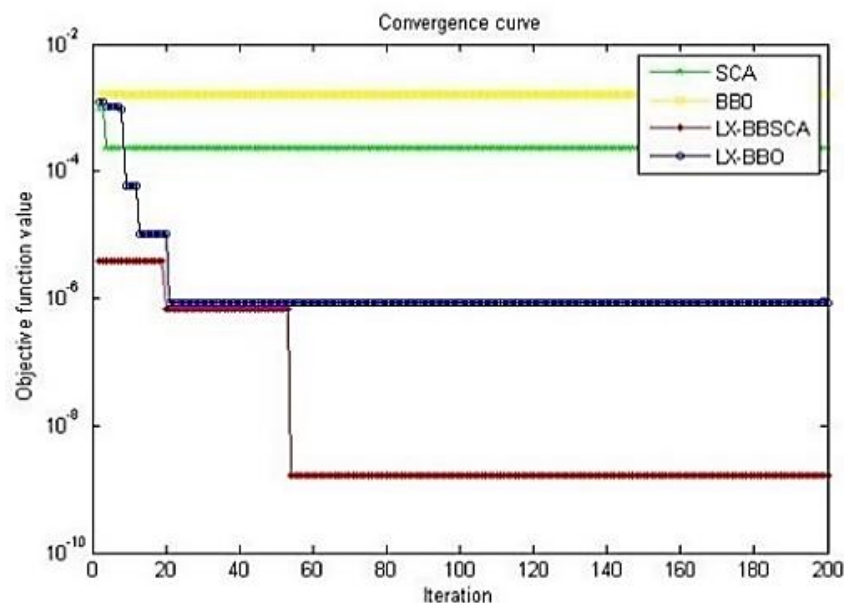


Figure 1. Convergence graph of different algorithms for the gear train problem.

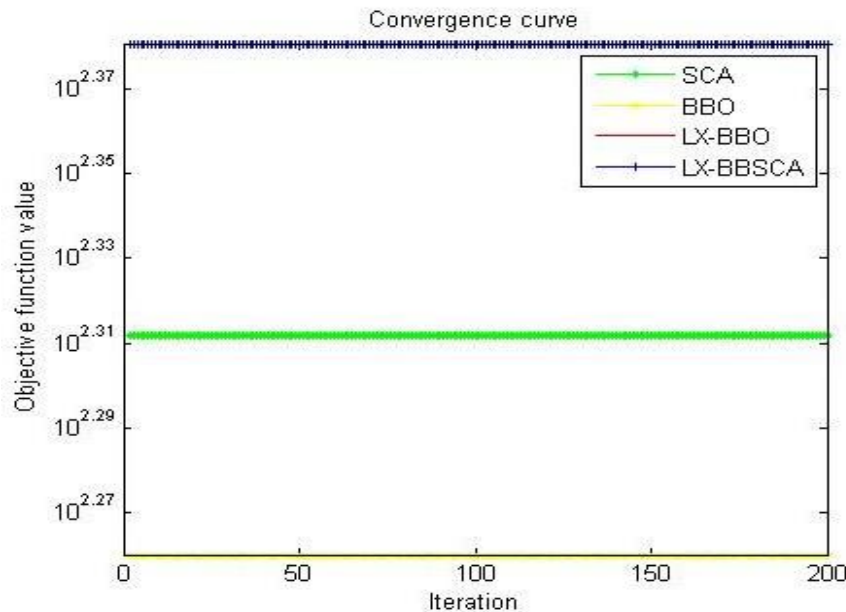


Figure 2. Convergence graph of different algorithms for the gas production problem.

5.2. Problem 2: Gas production problem

The level of gas production capacity that maximizes the effectiveness and profitability of a gas production facility or operation is referred to as the optimal capacity of gas production. Finding the right balance between production costs, market demand, infrastructural capabilities and other elements that have an impact on the efficiency of the gas production process is necessary to determine the appropriate capacity. This problem consists of choosing the best production capacities for a system that generates and contains gas. The problem can be expressed as follows:

$$\begin{aligned} \text{minimize } f_2(x) = & 61.8 + 5.72x_1 + 0.2623\left[(40 - x_1)\ln\left(\frac{x_2}{200}\right)\right]^{-0.85} \\ & + 0.087(40 - x_1)\ln\left(\frac{x_2}{200}\right) + 700.23x_2^{-0.75}, \end{aligned} \quad (9)$$

$$\begin{aligned} \text{subject to } & 17.5 \leq x_1 \leq 40, \\ & 300 \leq x_2 \leq 600, \end{aligned} \quad (10)$$

where x_1 and x_2 are the capacity of production facilities.

The results obtained with different algorithms are reported in Table 6. The obtained results demonstrate that the LX-BBSCA outperforms other techniques by obtaining a higher objective function value. Figure 2 shows the convergence graph for the problem.

Table 6. Comparison of different algorithms for the gas production problem.

Technique	x_1	x_2	Objective function value	The largest value of the objective function
LX-BBSCA	26	600	170.1	1000
LX-BBO	17.5	400	166.3	1000
BBO	40	529	168.8	1000
SCA	17.5	600	169.843	300
CS	17.5	593	169.996	324
GWO	17.51	600	169.818	342

5.3. Problem 3: Beam design problem

When designing a beam, the right dimensions, materials and configurations must be chosen to guarantee that the beam can safely carry the applied loads and satisfy certain design requirements. Beams are structural components used to distribute and carry loads to supports. The problem of beam design is multi-objective and can be stated as follows:

- The area of the beam's cross-section that for a given length reflects its volume.
- The beam's static deflection for the displacement under force P.

Both criteria should be kept to a minimum. These criteria are obviously incompatible because the smallest cross-section area yields the greatest deflection and vice-versa. The mathematical model of this problem is as follows:

$$\text{minimize } f_3(x) = \frac{5000}{\frac{x_2(x_2-2x_4)}{12} + \frac{x_1x_4^3}{6} + 2x_1x_1\left(\frac{x_2-x_4}{2}\right)^2}, \quad x = (x_1, x_2, x_3, x_4), \quad (11)$$

$$\text{subject to } 2x_1x_3 + x_3(x_2 - 2x_4) \leq 300,$$

$$\frac{18x_2 \times 10^4}{x_3(x_2 - 2x_4)^3 + 2x_1x_3(4x_4^2 + 3x_2(x_2 - 2x_4))} + \frac{15x_210^3}{(x_2 - 2x_4)x_3^3 + 2x_1x_1^3} \leq 6, \quad (12)$$

$$10 \leq x_1 \leq 80,$$

$$10 \leq x_2 \leq 50,$$

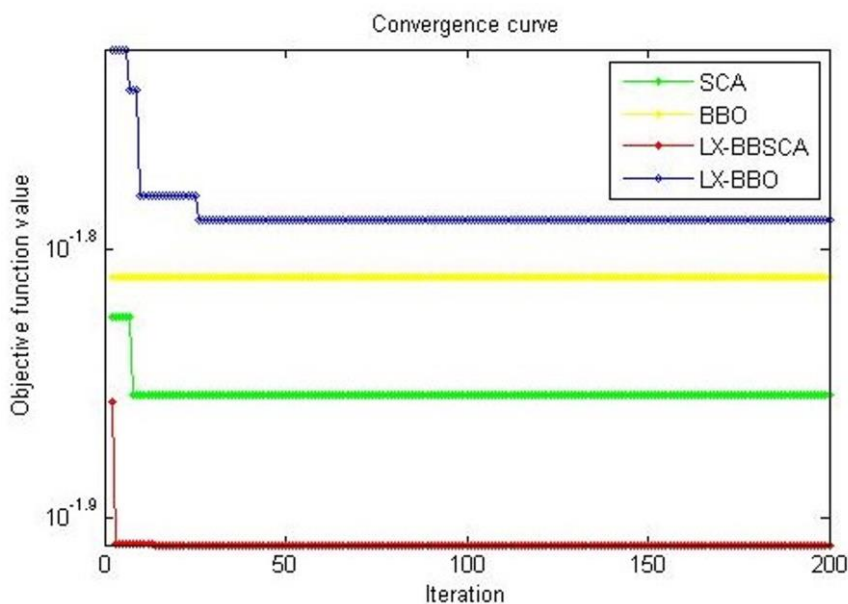
$$0.9 \leq x_3, x_4 \leq 5,$$

where x_1 , x_2 , x_3 and x_4 , are the height or width of different beams.

The LX-BBSCA, LX-BBO, SCA, and BBO, as well as other metaheuristics, are used to solve the beam design problem. The obtained results by different algorithms are summarized in Table 7. The results demonstrate that the proposed LX-BBSCA method as well as its basic algorithms (i.e., SCA, BBO, and LX-BBO) are superior to the other techniques. Furthermore, the convergence graphs of the SCA, BBO, LX-BBO, and LX-BBSCA methods are provided in Figure 3, which show that the LX-BBSCA algorithm outperforms the other techniques by obtaining a lower objective function value.

Table 7. Comparison of different algorithms for the beam design problem.

Technique	x_1	x_2	x_3	x_4	Objective function value
LX-BBSCA	80	50	0.9	0.5	0.0123
LX-BBO	80	50	0.9	0.5	0.0123
BBO	80	50	0.9	0.5	0.0123
SCA	80	50	0.9	0.5	0.0123
CS	80	50	0.9	2.32	0.013
GWO	80	50	0.9	2.32	0.013

**Figure 3.** Convergence graph of different algorithms for the beam design problem.

5.4. Problem 4: Space capsule problem

The enormous engineering task of designing a life support system for a space capsule entails assuring the survival and well-being of astronauts throughout their mission in the hostile environment of space. Everything an astronaut needs to breathe, eat, drink, maintain body temperature, eliminate waste and stay safe must be provided by the life support system of the space capsule. Due to the system's redundant component's entirely recidivist structure, this dependability issue is challenging to eradicate. This system is made up of four modules, each with a reliability of r_j , $j = 1,2,3,4$. It necessitates the success of a single path, and as such, it is split into two subsystems. Each subsystem comprises components arranged in series with component two, creating two identical paths. In this configuration, component three acts as a backup for the third path within this pair, while component one is complemented by parallel component four. The phase formed by elements one and four is in series with element two. Finally, these two identical paths run in parallel, ensuring that one of them guarantees the output. As a final expression, the reliability of the overall system can be defined as:

$$R_5 = 1 - r_3 \times [(1 - r_1)(1 - r_4)]^2 - (1 - r_3)[1 - r_2 \{1 - (1 - r_1)(1 - r_4)\}]^2, \quad (13)$$

while the system cost can be calculated as:

$$C_5 = 2k_1r_1^{p_1} + 2k_2r_2^{p_2} + 2k_3r_3^{p_3} + 2k_4r_4^{p_4}, \quad (14)$$

where $k_1 = k_2 = 100$, $k_3 = 200$, $k_4 = 150$ and $p_1 = p_2 = p_3 = p_4 = 0.6$.

The problem can be mathematically expressed as follows:

$$\text{minimize } f_4 = C_5 = 2k_1r_1^{p_1} + 2k_2r_2^{p_2} + 2k_3r_3^{p_3} + 2k_4r_4^{p_4}, \quad (15)$$

$$\text{subject to } 0.5 \leq r_j \leq 1 (\forall j = 1,2,3,4), \quad (16)$$

$$0.9 \leq R_5 \leq 1.$$

The results of the different algorithms are given in Table 8 and Figure 4. The results show that the LX-BBSCA algorithm slightly reduced the objective function value (i.e., Cost) by 0.01 against the SCA, BBO, and LX-BBO techniques. Furthermore, the LX-BBSCA algorithm outperforms CS and GWO by 16.81 and 16.74, respectively.

Table 8. Comparison of different algorithms for the space capsule problem.

Technique	r_1	r_2	r_3	r_4	Cost	Reliability
LX-BBSCA	0.5	0.5	0.5	0.5	725.08	0.9991
LX-BBO	0.5	0.5	0.5	0.5	725.09	0.9871
BBO	0.5	0.5	0.5	0.5	725.09	0.9645
SCA	0.5	0.5	0.5	0.5	725.09	0.9528
CS	0.5009	0.8377	0.5005	0.5001	741.9	0.9001
GWO	0.5001	0.8389	0.5001	0.5001	741.83	0.9

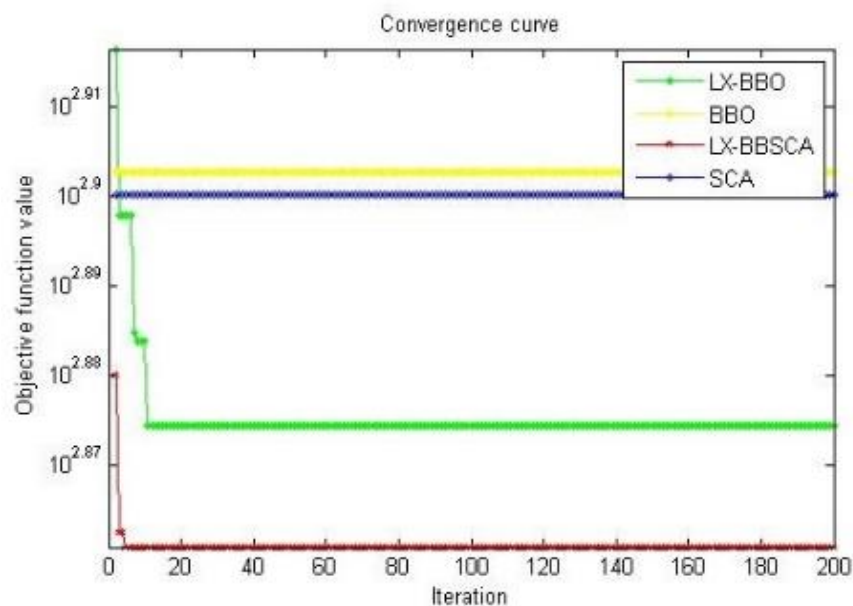


Figure 4. Convergence graph of different algorithms for the space capsule problem.

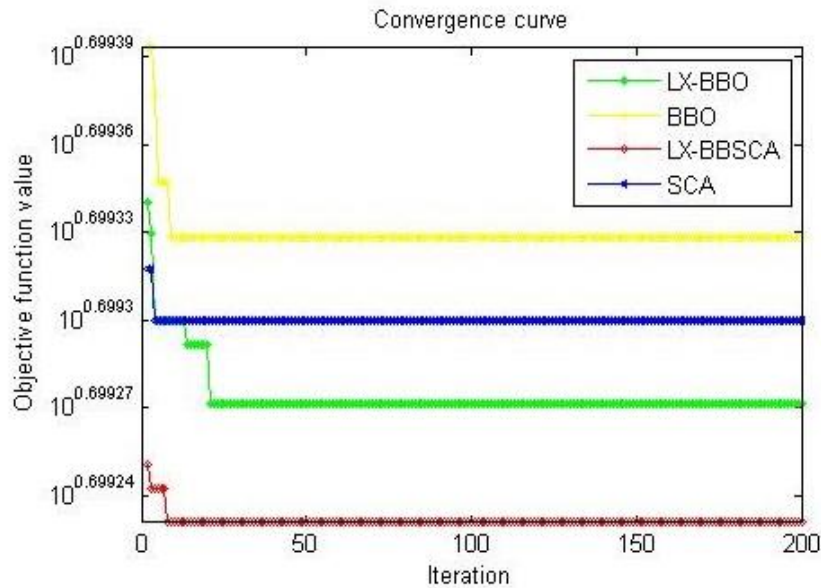


Figure 5. Convergence graph of different algorithms for the complex bridge network problem.

5.5. Problem 5: Complex bridge network problem

An interconnected network of bridges that can efficiently handle traffic flow, address geographic constraints and satisfy numerous engineering requirements is required when designing a complex bridge network. The current issue is a nonlinear multi-objective optimization problem, and the ideal solution is one that maximizes reliability while minimizing expenses. The system consists of five dimensions, each with a reliability of r_j , $j = 1, 2, \dots, 5$.

$$\text{minimize } f_5 = C_5 = \sum_{j=1}^5 a_j \exp\left(\frac{\beta_j}{1-r_j}\right), \quad (17)$$

$$\text{subject to } 0.5 \leq r_j \leq 1 \quad \forall j = 1, 2, 3, 4,$$

$$0.9 \leq r_5 \leq 1,$$

$$a_j = 1 \text{ and } \beta_j = 0.0003 \quad \forall j = 1, 2, 3, 4, 5,$$

(18)

$$R_S(r_1, r_2, r_3, r_4, r_5)$$

$$= r_1 r_4 + r_2 r_5 + r_1 r_3 r_5 + r_2 r_3 r_4 - r_2 r_3 r_4 r_5 + 2r_1 r_2 r_3 r_4 r_5 - r_1 r_3 r_4 r_5 \\ - r_1 r_2 r_3 r_5 - r_1 r_2 r_4 r_5 - r_1 r_2 r_3 r_4.$$

Table 9 presents the results of the different techniques for the complex bridge network problem. Also, Figure 5 depicts the convergence graph for the LX-BBSCA, LX-BBO, BBO, and SCA algorithms. The results demonstrate the superiority of the proposed LX-BBSCA algorithm against all compared techniques in reducing the objective function value.

Table 9. Comparison of different algorithms for the complex bridge network problem.

Technique	r_1	r_2	r_3	r_4	r_5	Cost	Reliability
LX-BBSCA	0.5	0.5	0.5	0.5	0.50	5.003	0.9901
LX-BBO	0.5	0.5	0.5	0.5	0.50	5.003	0.9540
BBO	0.5	0.5	0.5	0.5	0.50	5.003	0.9566
SCA	0.5	0.5	0.5	0.5	0.50	5.003	0.9508
CS	0.9349	0.9355	0.7853	0.941	0.92	5.019	0.9000
GWO	0.9595	0.8798	0.7908	0.973	0.89	5.025	0.9911

5.6. Discussion

In this section, we demonstrate the effectiveness of the LX-BBSCA algorithm by comparing its results (objective function value) against the LX-BBO, BBO, SCA, CS, and GWO algorithms. The data within Table 10 presents a compelling narrative of the superiority of the LX-BBSCA algorithm over well-established optimization techniques. Across all Problems 1–5, LX-BBSCA consistently showcases a significant positive improvement rate in comparison to its counterparts. Notably, LX-BBSCA exhibits remarkable performance enhancements, particularly evident in outperforming LX-BBO, BBO and SCA, establishing its dominance in a majority of the problem types examined. Even when compared against competitive methodologies like CS and GWO, LX-BBSCA maintains a consistent edge, highlighting its robustness in solving structural engineering design optimization problems. These findings underscore the LX-BBSCA's capability to significantly enhance optimization outcomes, portraying LX-BBSCA as a promising and reliable contender in the field of metaheuristic algorithms.

Table 10. Improvement rate (%) of the proposed LX-BBSCA algorithm against other algorithms.

Technique	Problem 1	Problem 2	Problem 3	Problem 4	Problem 5
LX-BBO	+ 41.5	+ 2.3	0	+ 0.01	0
BBO	+ 43.7	+ 0.8	0	+ 0.01	0
SCA	+ 98	+ 0.2	0	+ 0.01	0
CS	+ 44.1	+ 0.1	+ 5.4	+ 2.27	+ 0.32
GWO	+ 86.3	+ 0.2	+ 5.4	+ 2.25	+ 0.44

6. Conclusions

In this study, a hybrid metaheuristic-driven optimization algorithm LX-BBSCA has been created by combining SCA and LX-BBO, which coordinates both local and global search and incorporates the advantages of both algorithms. As a result, the novel optimization algorithm is an effective algorithm for searching the solution space. We can draw the conclusion that the new hybrid technique is capable of maintaining a proper balance between exploration and exploitation. Five engineering design issues were resolved using the suggested algorithm. The new hybrid approach surpasses the existing techniques in terms of convergence speed and solution quality. According to the quantitative statistical data, it can be demonstrated that the proposed algorithm may be employed as a successful and proficient ensemble metaheuristic algorithm for real-world tasks involving a complicated search area.

In this paper, LX-BBSCA has been presented as a single-objective ensemble metaheuristic algorithm. As a future work, this combination can be extended to multi-objective problems by means of the multi-objective mechanisms such as Pareto fronts [36–38]. Moreover, we have considered all parameters of the structural optimization problems to be deterministic. In future works, robust optimization [39], stochastic programming [2], fuzzy logic [40,41], machine learning [42–45], stochastic games and regime-switching models [46], can be applied to handle the uncertainties of the parameters. Moreover, ensemble heuristic-metaheuristic techniques [3,7,47] can be applied to guide the search process of the metaheuristic algorithm with the knowledge available in the problem model. Another future research direction is to apply the proposed algorithm for convex optimization [48], integer programming problems and constrained optimization problems [49,50].

Use of AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

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Conflict of interest

Prof. Dr. Werner is the Guest Editor of special issue “Discrete Optimization: Theory, Algorithms and New Applications” for AIMS Mathematics. Prof. Dr. Werner was not involved in the editorial review and the decision to publish this article.

The authors declare no conflict of interest.

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