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*Research article*

# The new construction of knowledge measure on intuitionistic fuzzy sets and interval-valued intuitionistic fuzzy sets

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**Abstract:** As we all know, when describing knowledge measures in the context of intuitionistic fuzzy sets and interval-valued intuitionistic fuzzy sets, it is always considered as dual measures of entropy. However, information content and information clarity is closely related with the amount of knowledge. Motivated by this fact, in this study, we focus on a new axiomatic definition of knowledge measures for intuitionistic fuzzy sets and interval-valued intuitionistic fuzzy sets. First, we present the formulas of the knowledge measures using different abstract functions, and we proved these functions satisfy the axioms. On the basis of mathematical analysis and numerical examples, we further analyze the characteristics of the suggested knowledge measure. Finally, in order to demonstrate how rational and useful the system we developed is, we provide medical diagnoses and specific multi-attribute decision problems.

**Keywords:** intuitionistic fuzzy set; interval-valued intuitionistic fuzzy set; knowledge measure; multi-attribute decision making
**Mathematics Subject Classification:** 90B50, 91A35

## 1. Introduction

In the context of Zadeh fuzzy sets (FSs), intuitionistic fuzzy sets (IFSs) is introduced by Atanassov [1], where each element has a membership and non- membership degree. Intuitionistic fuzzy sets is extended to interval-valued intuitionistic fuzzy sets (IVIFSs) by Atanassov and Gargovin further [1], where the membership and non-membership degrees are not a real number, but they are represented by intervals. Since intuitionistic fuzzy sets and interval-valued fuzzy sets have greater flexibility with respect to uncertainty, they have been widely explored and applied in many different fields in the past decade. More recently, IFSs and IVIFSs have been widely used in many different frameworks such as uncertain decision-making and image threshold processing due totheir excellent agility and flexibility in dealing with uncertainty or fuzziness.

For entropy of the fuzzy set, which has been an active research direction in fuzzy set theory. Fuzzy entropy has attracted the attention of researchers since it was first mentioned by Zadeh. De Luca and Termini proposed the axiom of fuzzy entropy and defined it according to the Shannon function [2]. In entropy, there are three main structures, considering the uncertainty of the hesitation of the modeling of intuition and the use of Shannon entropy thought of probability and non-probability [3–6]. Therefore, in this paper, we no longer focus on the relationship between entropy and knowledge measure, but we construct a new axiomatic model in the context of knowledge measure to solve the problem that entropy cannot solve. Szmidt et al. conducted a pioneering exploration of the amount of knowledge transmitted by IFSs [7–10]. The concept of knowledge refers to the information that is considered useful in a particular environment, characterized by regularity, certainty and novelty. Guo et al. [11–14] believes that in the context of IFSs, it cannot be simply considered that entropy and knowledge measure have a certain logical relationship, and knowledge measure should be viewed from different perspectives. Some theories advocate information content, while others pay more attention to its inherent ambiguity [15–17]. From the above theories, we find that there is no axiomatic model of knowledge measure based on the combination of information content and information clarity.

In the latest research results, Guo and Xu [18] point out and demonstrate that in the context of IFSs and IVIFSs, at least information content and information clarity are related to them. Additionally, the axiom model is established using these two aspects.

In this paper, we study only the problems under IFSs and IVIFSs that are not related to entropy. A flexible knowledge axiomatic framework is established without going further to investigate the relationship between entropy and knowledge measures. And the application of multi-attribute decision making using the ranking method proposed by Xu et al. [19–21].

The rest of the paper is as follows: Section 2 focuses on reviewing the concepts and axioms of IFSs and IVIFSs. In Section 3, we develop the construction of a new knowledge measure framework using abstract functions in the context of IFSs and IVIFSs. Then, in Section 4, the superiority of our developed model is demonstrated by example experimental comparisons. Following that, applications of the novel model for multi-attribute decision making and medical diagnosis are described in Section 5 to demonstrate how well the suggested method of IFSs and IVIFSs works. Finally, in Section 6, a summary is presented.

## 2. Preliminaries

In this section, we briefly review the definitions of intuitionistic fuzzy sets and interval-valued intuitionistic fuzzy sets, which we will use in this paper. Let $X = \{x_1, x_2, \ldots, x_n\}$ be a universal of discourse.

**Definition 2.1.** [1] An intuitionistic fuzzy set (IFS) A in a universe $X$ is defined as the following form:

$$A = \{< x, \mu_A(x), \nu_A(x) > | x \in X\},$$

where $\mu_A : X \to [0, 1]$ and $\nu_A : X \to [0, 1]$ such that $0 \le \mu_A(x) + \nu_A(x) \le 1$ , for all $x \in X$, and they represent a membership degree and a non-membership degree for all $x \in A$.

An additional notion related to an IFSs is $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$ called a hesitation for all $x \in A$, and expressing a degree of lack of knowledge of whether $x \in A$, or not. It is obvious that $0 \le$

$\pi_A(x) \leq 1$ for all $x \in X$. For any IFSs A in $X = \{x_i | i = 1, 2, ..., n\}$, the complement set is expressed as $A^c = \{< x, \nu_A(x), \mu_A(x) > | \ x \in X\}$.

**Definition 2.2.** [1] An interval-valued intuitionistic fuzzy set (IVIFS) $\tilde{A}$ in a universe X is defined as the following form:

$$\tilde{A} = \{< x, \tilde{\mu}_A(x), \tilde{\nu}_A(x) > | \ x \in X\},$$

where $\tilde{\mu}_{\tilde{A}}(x) \subseteq [0, 1]$ and $\tilde{\nu}_{\tilde{A}}(x) \subseteq [0, 1]$ satisfying for all $x \in X$,

$$\sup \tilde{\mu}_{\tilde{A}}(x) + \sup \tilde{\nu}_{\tilde{A}}(x) \leq 1.$$

For the hesitation $\tilde{\pi}_{\tilde{A}}$,

$$\inf \tilde{\pi}_{\tilde{A}}(x) = 1 - \sup \tilde{\mu}_{\tilde{A}}(x) - \sup \tilde{\nu}_{\tilde{A}}(x),$$

and

$$\sup \tilde{\pi}_{\tilde{A}}(x) = 1 - \inf \tilde{\mu}_{\tilde{A}}(x) - \inf \tilde{\nu}_{\tilde{A}}(x),$$

if $\inf \tilde{\mu}_{\tilde{A}}(x) = \sup \tilde{\mu}_{\tilde{A}}(x)$ , $\inf \tilde{\nu}_{\tilde{A}}(x) = \sup \tilde{\nu}_{\tilde{A}}(x)$ for all $x \in A$. (sup is an abbreviation for supremum, meaning upper bound; inf is an abbreviation for infimum, meaning lower bound.) Then IVIFSs $\tilde{A}$ reduces to an IFSs.

For convenience, let $\tilde{\mu}_{\tilde{A}}(x) = [\mu^L_{\tilde{A}}(x), \mu^U_{\tilde{A}}(x)]$, $\tilde{\nu}_{\tilde{A}}(x) = [\nu^L_{\tilde{A}}(x), \nu^U_{\tilde{A}}(x)]$, and $\tilde{\pi}_{\tilde{A}}(x) = [\pi^L_{\tilde{A}}(x), \pi^U_{\tilde{A}}(x)]$ such that $\mu^U_{\tilde{A}}(x) + \nu^U_{\tilde{A}}(x) \leq 1$ for all $x \in A$. The complement set $\tilde{A}^c = \{< x, \tilde{\nu}_A(x), \tilde{\mu}_A(x) > | \ x \in X\}$.

In order to express more simply in the following definitions, let $IFS\ (X)$ represent a set comprising all IFSs on $X$ and $IVIFS\ (X)$ express the set of IVIFSs on $X$.

**Definition 2.3.** [18] Let $A, B \in IFS(X)$. A mapping $KM_{IFS}\colon IFS\ (X) \to [0, 1]$ is called a knowledge measure on $IFS\ (X)$, if $KM_{IFS}$ has the following properties:

($KM_{IFS}1$) $K(A) = 1$ iff $A$ is a crisp set.

($KM_{IFS}2$) $K(A) = 0$ iff $\pi_A(x_i) = 1$ for all $x_i \in X$.

($KM_{IFS}3$) $K(A) \geq K(B)$ if $A$ has more information content and greater information clarity than $B$, i.e., $\mu_A(x_i) + \nu_A(x_i) \geq \mu_B(x_i) + \nu_B(x_i)$ and $|\mu_A(x_i) - \nu_A(x_i)| \geq |\mu_B(x_i) - \nu_B(x_i)|$ for all $x_i \in X (i = 1, 2, ..., n)$.

($KM_{IFS}4$) $K(A^c) = K(A)$.

The concept we are dedicated to measure is the partial order relationship between knowledge measure and IFSs, i.e., the information content and information clarity, denoted by $\mu_A(x_i) + \nu_A(x_i)$ and $|\mu_A(x_i) - \nu_A(x_i)|$, respectively.

IFSs describe fuzzy information through membership, non membership and hesitation, which can effectively avoid the loss of information. Therefore, the research on the information measure of IFSs is relatively extensive.

In the early stage, researchers were devoted to studying the entropy of IFSs with membership and non-membership. Zang and Li [13] proposed:

$$E_{ZL}(A) = 1 - \frac{1}{n} \sum_{i=1}^{n} |\mu_A(x_i) - \nu_A(x_i)|. \tag{2.1}$$

It is clear that reluctance occurs despite the entropy indicated earlier. This behavior is prone to information loss, hence Szmidt and Kacprzyk [13] et al. presented alternative entropy models, which are as follows:

$$E_{SK}(A) = \frac{1}{n} \sum_{i=1}^{n} \frac{\min(\mu_A(x_i), \nu_A(x_i)) + \pi_A(x_i)}{\max(\mu_A(x_i), \nu_A(x_i)) + \pi_A(x_i)}. \tag{2.2}$$

$$E_{HC}^{2}(A)=\frac{1}{n}\sum_{i=1}^{n}(1-(\mu_A(x_i))^2-(\nu_A(x_i))^2-(\pi_A(x_i))^2). \tag{2.3}$$

The degree of ambiguity of an object can also be described using knowledge metrics. Based on the entropies indicated above, Guo et al. [7–9] suggested the axiomatic model below.

$$K_G(A)=1-\frac{1}{2n}\sum_{i=1}^{n}(1-|\mu_A(x_i)-\nu_A(x_i)|)(1+\pi_A(x_i)). \tag{2.4}$$

$$K_{skb}(A)=1-\frac{1}{2\pi}\left(\sum_{i=1}^{n}\frac{\min(\mu_A(x_i),\nu_A(x_i))+\pi_A(x_i)}{\max(\mu_A(x_i),\nu_A(x_i))+\pi_A(x_i)}+\pi_A(x_i)\right). \tag{2.5}$$

$$K_N(A)=\frac{1}{n\sqrt{2}}\sum_{i=1}^{n}\sqrt{(\mu_A(x_i))^2+(\nu_A(x_i))^2+(\mu_A(x_i)+\nu_A(x_i))^2}. \tag{2.6}$$

## 3. Construct knowledge measures for IFSs and IVIFSs

In this section, we obtain two new overall expression forms based on the axiomatic definition and properties of IFSs and IVIFSs knowledge measures using abstract functions.

### *3.1. A new knowledge measure for IFSs*

Knowledge measures are usually used to describe the knowledge content of IFSs. In this part, a new formula of knowledge measures is constructed mainly based on the binary aggregation function.

**Definition 3.1.** $f:[0,1]^2\to[0,1]$ be a binary aggregation function and $x,y\in[0,1]$, if it satisfies the following conditions:

(1) $f(x,y)$ is a strictly monotone increasing function,

(2) $f(1,1)=1$,

(3) $f(0,0)=0$.

We construct a new IFSs knowledge measure with a binary abstraction function on $IFS(X)$.

IFSs describe fuzzy information through membership, non-membership and hesitation, which can effectively avoid the loss of information. Therefore, the research on the information measure of IFSs is relatively extensive. Currently, many scholars have achieved extensive results in information measures and distance measures.

**Theorem 3.1.** Let$f=[0,1]^2\to[0,1]$ be a binary aggregation real function, then, for all $A\in IFS(X)$, the function $KM_{IFS}$: $IFS(X)^2\to[0,1]$ is defined by:

$$KM_{IFS}(A_i)=f(a_i,b_i), \tag{3.1}$$

is a knowledge measure for IFSs, where $a_i=\mu_A(x_i)+\nu_A(x_i)$; $b_i=|\mu_A(x_i)-\nu_A(x_i)|$ $(i=1,2,...,n)$.

*Proof.* As a meaningful knowledge measure of IFSs , $KM_{IFS}(A_i)$ should satisfy the axioms of $KM_{IFS}1\sim4$ for Definition 2.3.

($\mathbf{KM_{IFS}1}$): Let $A_i$ be a crisp set, for any $x_i \in X$, $\mu_A(x_i) = 1$, $\nu_A(x_i) = 0$ or $\nu_A(x_i) = 1$, $\mu_A(x_i) = 0$ it implies,

$$\mu_A(x_i) + \nu_A(x_i) = 1, |\mu_A(x_i) - \nu_A(x_i)| = 1,$$

thus $KM_{IFS}(A_i) = 1$.

On the other hand, let $KM_{IFS}(A_i) = 1$, for any $x_i \in X$,

$$\mu_A(x_i) + \nu_A(x_i) = 1, |\mu_A(x_i) - \nu_A(x_i)| = 1,$$

which means

$$\mu_A(x_i) = 1, \nu_A(x_i) = 0,$$

and this implies $A_i$ is a crisp set.

($\mathbf{KM_{IFS}2}$): Let $\pi_A(x_i) = 1$, it implies $\mu_A(x_i) = \nu_A(x_i) = 0$ for any $x_i \in X$. That

$$\mu_A(x_i) + \nu_A(x_i) = 0, |\mu_A(x_i) - \nu_A(x_i)| = 0,$$

thus $KM_{IFS}(A_i) = 0$.

On the other hand, let $KM_{IFS}(A_i) = 0$, this implies that

$$\mu_A(x_i) + \nu_A(x_i) = 0, |\mu_A(x_i) - \nu_A(x_i)| = 0,$$

for any $x_i \in X$,

$$\mu_A(x_i) + \nu_A(x_i) = 1, \pi_A(x_i) = 1.$$

($\mathbf{KM_{IFS}3}$): For any $A_i, B_i \in IFS(X)$ let

$$\mu_A(x_i) + \nu_A(x_i) \geq \mu_B(x_i) + \nu_B(x_i),$$

$$|\mu_A(x_i) - \nu_A(x_i)| \geq |\mu_B(x_i) - \nu_B(x_i)|.$$

$f(a_i, b_i)$ is monotonically increasing with respect to $a_i$ and $b_i$. Thus, $KM_{IFS}(A_i) \geq KM_{IFS}(B_i)$.

($\mathbf{KM_{IFS}4}$): By the definition of $A^c$, thus $KM_{IFS}(A) = KM_{IFS}(A^c)$.

For Theorem 3.1, we can develop different formulas to calculate knowledge measures on IFSs, using different functions $f : [0, 1]^2 \to [0, 1]$, is a strictly monotone increasing function as follows: (1) $f(x, y) = \frac{x+y}{1+xy}$; (2) $f(x, y) = \frac{1}{2}(x + xy)$; (3) $f(x, y) = \frac{1}{2}x + \alpha y + (\frac{1}{2} - \alpha)xy (0 \leq \alpha \leq 1)$; (4) $f(x, y) = \frac{\frac{1}{2}(x+y)}{2-xy}$.

Equation (3.1) describes a parametric model for a single element belonging to an IFS. For any $A \in IFS(X)$:

$$KM_{IFS}(A) = \frac{1}{n}\sum_{i=1}^{n} KM_{IFS}(A_i) = \frac{1}{n}\sum_{i=1}^{n} f(a_i, b_i). \tag{3.2}$$

Then four knowledge measures are obtained based on Eq (3.1):

$$KM_{IFS}(A) = \frac{1}{n}\sum_{i=1}^{n} \frac{(\mu_A(x_i) + \nu_A(x_i)) + |\mu_A(x_i) - \nu_A(x_i)|}{1 + (\mu_A(x_i) + \nu_A(x_i))|\mu_A(x_i) - \nu_A(x_i)|}. \tag{3.3}$$

$$KM_{IFS}(A) = \frac{1}{n}\sum_{i=1}^{n}\frac{1}{2}[(\mu_A(x_i) + \nu_A(x_i)) + (\mu_A(x_i) + \nu_A(x_i))|\mu_A(x_i) - \nu_A(x_i)|]. \tag{3.4}$$

$$KM_{IFS}(A) = \frac{1}{n}\sum_{i=1}^{n}[\frac{1}{2}(\mu_A(x_i) + \nu_A(x_i) + \alpha|\mu_A(x_i) - \nu_A(x_i)| + (\frac{1}{2} - \alpha)(\mu_A(x_i) + \nu_A(x_i))|\mu_A(x_i) - \nu_A(x_i)|]. \tag{3.5}$$

Therefore, it can be concluded that Eq (3.5) can cover the knowledge measure proposed in [18].

$$KM_{IFS}(A) = \frac{1}{n}\sum_{i=1}^{n}\frac{\frac{1}{2}[(\mu_A(x_i) + \nu_A(x_i)) + |\mu_A(x_i) - \nu_A(x_i)|]}{2 - (\mu_A(x_i) + \nu_A(x_i))|\mu_A(x_i) - \nu_A(x_i)|}. \tag{3.6}$$

This section presents an abstract function-based knowledge measure for IFSs to calculate the knowledge that is conveyed by IFSs. Knowledge is a key statistic for analyzing intuitionistic fuzzy information.

The new knowledge measure is created from a mathematical perspective, which considerably broadens the study's focus to include the actual demands of people in many situations including unknowns.

### *3.2. A new knowledge measure for IVIFSs*

The knowledge measure of IVIFSs using a quadratic abstraction function is constructed based on the knowledge measure of IFSs.

**Definition 3.2.** Let $\tilde{A}, \tilde{B} \in IVIFS(X)$, a mapping $KM_{IVIFS}\colon IVIFS(X) \to [0,1]$ is called a knowledge measure on $IVIFS(X)$, if $KM_{IVIFS}$ has the following properties:

($KM_{IVIFS}1$) $K(\tilde{A}) = 1$ iff $\tilde{A}$ is a crisp set, i.e., $\tilde{A}(x_i) = \langle[1,1],[0,0]\rangle$ or $\tilde{A}(x_i) = \langle[0,0],[1,1]\rangle$ for any $x_i \in X$.

($KM_{IVIFS}2$) $K(\tilde{A}) = 0$ iff $\tilde{\pi}_{\tilde{A}}(x_i) = [1,1]$ for any $x_i \in \tilde{A}$.

($KM_{IVIFS}3$) $K(\tilde{A}) \geq K(\tilde{B})$ if $\tilde{A}$ has more information content and greater information clarity than $\tilde{B}$, i.e., $\mu^L_{\tilde{A}}(x_i)+\nu^L_{\tilde{A}}(x_i)+\mu^U_{\tilde{A}}(x_i)+\nu^U_{\tilde{A}}(x_i) \geq \mu^L_{\tilde{B}}(x_i)+\nu^L_{\tilde{B}}(x_i)+\mu^U_{\tilde{B}}(x_i)+\nu^U_{\tilde{B}}(x_i)$ and $|\mu^L_{\tilde{A}}(x_i)-\nu^L_{\tilde{A}}(x_i)|+|\mu^U_{\tilde{A}}(x_i)-\nu^U_{\tilde{A}}(x_i)| \geq |\mu^L_{\tilde{B}}(x_i)-\nu^L_{\tilde{B}}(x_i)|+|\mu^U_{\tilde{B}}(x_i)-\nu^U_{\tilde{B}}(x_i)|$.

($KM_{IVIFS}4$) $K(\tilde{A}^C) = K(\tilde{A})$.

In particular, according to $\tilde{\mu}_{\tilde{A}}(x_i) = \tilde{\nu}_{\tilde{A}}(x_i) = [\frac{1}{3}, \frac{1}{3}]$ proposed in [13], which means IFSs have both the greatest information content and least information clarity.

**Definition 3.3.** That $g : [0,1]^4 \to [0,1]$ be a quaternary aggregation function and $x, y, m, n \in [0,1]$, if it satisfies the following conditions:

(1) $g(x, y, m, n)$ is a strictly monotone increasing function,

(2) $g(1,1,1,1) = 1$,

(3) $g(0,0,0,0) = 0$.

We construct a new IVIFSs knowledge measure formula based on a quaternary abstraction function on $IVIFS(X)$.

**Theorem 3.2.** Let $g : [0,1]^4 \rightarrow [0,1]$ be a quaternary aggregation real function, then, for all $\tilde{A} \in IVIFS(X)$, the function $KM_{IVIFS}$: $IVIFS(X)^4 \rightarrow [0,1]$ defined by:

$$KM_{IVIFS}(\tilde{A}_i) = g(h_i, p_i, q_i, s_i), \tag{3.7}$$

is a knowledge measure for IVIFSs, where $h_i = \mu^L_{\tilde{A}}(x_i)+\nu^L_{\tilde{A}}(x_i)$; $p_i = \mu^U_{\tilde{A}}(x_i)+\nu^U_{\tilde{A}}(x_i)$; $q_i = |\mu^L_{\tilde{A}}(x_i)-\nu^L_{\tilde{A}}(x_i)|$; $s_i = |\mu^U_{\tilde{A}}(x_i) - \nu^U_{\tilde{A}}(x_i)|$ $(i = 1, 2, ..., n)$.

*Proof.* As a meaningful knowledge measure of IVIFSs , $KM_{IVIFS}(\tilde{A}_i)$ should be strictly comply with the axioms of $KM_{IVIFS}1 \sim 4$ for Definition 3.2.

($\mathbf{KM_{IVIFS}1}$): Let $\tilde{A}_i$ be a crisp set, it implies $\tilde{\mu}_{\tilde{A}}(x_i) = [1,1]$, or $\tilde{\nu}_{\tilde{A}}(x_i) = [1,1]$ for any $x_i \in X$, that

$$\mu^L_{\tilde{A}}(x_i) + \nu^L_{\tilde{A}}(x_i) = 1, \mu^U_{\tilde{A}}(x_i) + \nu^U_{\tilde{A}}(x_i) = 1,$$

and

$$|\mu^L_{\tilde{A}}(x_i) - \nu^L_{\tilde{A}}(x_i)| = 1, |\mu^U_{\tilde{A}}(x_i) - \nu^U_{\tilde{A}}(x_i)| = 1,$$

thus $KM_{IVIFS}(\tilde{A}_i) = 1$.

On the other hand, let $KM_{IVIFS}(\tilde{A}_i) = 1$ for any $x_i \in X$ that

$$\mu^L_{\tilde{A}}(x_i) + \nu^L_{\tilde{A}}(x_i) = 1, \mu^U_{\tilde{A}}(x_i) + \nu^U_{\tilde{A}}(x_i) = 1,$$

and

$$|\mu^L_{\tilde{A}}(x_i) - \nu^L_{\tilde{A}}(x_i)| = 1, |\mu^U_{\tilde{A}}(x_i) - \nu^U_{\tilde{A}}(x_i)| = 1.$$

For any $x_i \in X$, $\mu^L_{\tilde{A}}(x_i) = \mu^U_{\tilde{A}}(x_i) = 1$, or $\nu^L_{\tilde{A}}(x_i) = \nu^U_{\tilde{A}}(x_i) = 1$, which means

$$\mu_{\tilde{A}}(x_i) = \langle [1,1], [0,0] \rangle, \nu_{\tilde{A}}(x_i) = \langle [0,0], [1,1] \rangle,$$

and this implies $\tilde{A}_i$ is a crisp set.

($\mathbf{KM_{IIVFS}2}$): Let $\tilde{\pi}_{\tilde{A}}(x_i) = [1,1]$ for any $x_i \in X$, it implies $\tilde{\mu}_{\tilde{A}}(x_i) = \tilde{\nu}_{\tilde{A}}(x_i) = [0,0]$. That $\mu^L_{\tilde{A}}(x_i) + \nu^L_{\tilde{A}}(x_i) = 0$, $\mu^U_{\tilde{A}}(x_i) + \nu^U_{\tilde{A}}(x_i) = 0$ and $|\mu^L_{\tilde{A}}(x_i) - \nu^L_{\tilde{A}}(x_i)| = 0$, $|\mu^U_{\tilde{A}}(x_i) - \nu^U_{\tilde{A}}(x_i)| = 0$, thus $KM_{IIVFS}(\tilde{A}_i) = 0$. On the other hand, let $KM_{IVIFS}(\tilde{A}_i) = 0$, this implies that $\tilde{\mu}_{\tilde{A}}(x_i) = \tilde{\nu}_{\tilde{A}}(x_i) = [0,0]$ $\tilde{\pi}_{\tilde{A}}(x_i) = [1,1]$ for any $x_i \in X$.

($\mathbf{KM_{IVIFS}3}$): For any $\tilde{A}_i, \tilde{B}_i \in IVIFS(X)$, for all $x_i \in X$, let

$$\mu^L_{\tilde{A}}(x_i) + \nu^L_{\tilde{A}}(x_i) + \mu^U_{\tilde{A}}(x_i) + \nu^U_{\tilde{A}}(x_i) \geq \mu^L_{\tilde{B}}(x_i) + \nu^L_{\tilde{B}}(x_i) + \mu^U_{\tilde{B}}(x_i) + \nu^U_{\tilde{B}}(x_i),$$

and

$$|\mu^L_{\tilde{A}}(x_i) - \nu^L_{\tilde{A}}(x_i)| + |\mu^U_{\tilde{A}}(x_i) - \nu^U_{\tilde{A}}(x_i)| \geq |\mu^L_{\tilde{B}}(x_i) - \nu^L_{\tilde{B}}(x_i)| + |\mu^U_{\tilde{B}}(x_i) - \nu^U_{\tilde{B}}(x_i)|,$$

is monotonically increasing with respect to $h_i, p_i, q_i$ and $s_i$. Thus $KM_{IVIFS}(\tilde{A}_i) \geq KM_{IVIFS}(\tilde{B}_i)$.

($\mathbf{KM_{IVIFS}4}$): By the definition of $\tilde{A}^c$, thus $KM_{IVIFS}(\tilde{A}) = KM_{IVIFS}(\tilde{A}^c)$.

For Theorem 3.2 a series of knowledge measures can be obtained according to different expressions of function. Specially, we can consider $g : [0,1]^4 \rightarrow [0,1]$ as: (1) $g(x, y, m, n) = \frac{1}{4}(x + y +$

$m+n$); (2) $g(x,y,m,n) = \frac{1}{2}(\sqrt{x^2+y^2+m^2+n^2})$; (3) $g(x,y,m,n) = \frac{1}{4}(x+y)+\frac{1}{2}(m+n)-\frac{1}{8}(x+y)(m+n)$; (4) $g(x,y,m,n) = xymn$.

Equation (3.7) describes a parametric model for a single element belonging to an IVIFS. For any $\tilde{A} \in IVIFS(X)$, as follow:

$$KM_{IVIFS}(\tilde{A}) = \frac{1}{n}\sum_{i=1}^{n} KM_{IVIFS}(\tilde{A}_i) = \frac{1}{n}\sum_{i=1}^{n} g(h_i, p_i, q_i, s_i). \tag{3.8}$$

Then, four knowledge measures are obtained based on Eq (3.8)

$$\begin{aligned} KM_{IVIFS}(\tilde{A}) =& \frac{1}{n}\sum_{i=1}^{n}\frac{1}{4}[\mu^L_{\tilde{A}}(x_i) + \nu^L_{\tilde{A}}(x_i) + \mu^U_{\tilde{A}}(x_i) + \nu^U_{\tilde{A}}(x_i) \\ &+ |\mu^L_{\tilde{A}}(x_i) - \nu^L_{\tilde{A}}(x_i)| + |\mu^U_{\tilde{A}}(x_i) - \nu^U_{\tilde{A}}(x_i)|]. \end{aligned} \tag{3.9}$$

$$\begin{aligned} KM_{IVIFS}(\tilde{A}) =& \frac{1}{n}\sum_{i=1}^{n}\frac{1}{2}((\mu^L_{\tilde{A}}(x_i) + \nu^L_{\tilde{A}}(x_i))^2 + (\mu^U_{\tilde{A}}(x_i) + \nu^U_{\tilde{A}}(x_i))^2 \\ &+ |\mu^L_{\tilde{A}}(x_i) - \nu^L_{\tilde{A}}(x_i)|^2 + |\mu^L_{\tilde{A}}(x_i) - \nu^L_{\tilde{A}}(x_i)|^2)^{\frac{1}{2}}. \end{aligned} \tag{3.10}$$

$$\begin{aligned} KM_{IVIFS}(\tilde{A}) =& \frac{1}{n}\sum_{i=1}^{n}(\mu^L_{\tilde{A}}(x_i) + \nu^L_{\tilde{A}}(x_i))(\mu^U_{\tilde{A}}(x_i) + \nu^U_{\tilde{A}}(x_i)) \\ &|\mu^L_{\tilde{A}}(x_i) - \nu^L_{\tilde{A}}(x_i)||\mu^U_{\tilde{A}}(x_i) - \nu^U_{\tilde{A}}(x_i)|. \end{aligned} \tag{3.11}$$

$$\begin{aligned} KM_{IVIFS}(\tilde{A}) =& \frac{1}{n}\sum_{i=1}^{n}\frac{1}{4}(\mu^L_{\tilde{A}}(x_i) + \nu^L_{\tilde{A}}(x_i) + \mu^U_{\tilde{A}}(x_i) + \nu^U_{\tilde{A}}(x_i)) \\ &+ \frac{1}{2}(|\mu^L_{\tilde{A}}(x_i) - \nu^L_{\tilde{A}}(x_i)| + |\mu^U_{\tilde{A}}(x_i) - \nu^U_{\tilde{A}}(x_i)|) \\ &- \frac{1}{8}(\mu^L_{\tilde{A}}(x_i) + \nu^L_{\tilde{A}}(x_i) + \mu^U_{\tilde{A}}(x_i) + \nu^U_{\tilde{A}}(x_i)) \\ &(|\mu^L_{\tilde{A}}(x_i) - \nu^L_{\tilde{A}}(x_i)| + |\mu^U_{\tilde{A}}(x_i) - \nu^U_{\tilde{A}}(x_i)|). \end{aligned} \tag{3.12}$$

In this paper, only two aspects of information content and information clarity of knowledge measure are defined. Because $\tilde{\pi}_{\tilde{A}}(x) = 1 - (\tilde{\mu}_{\tilde{A}}(x) + \tilde{\nu}_{\tilde{A}}(x))$, we can regard it as formula hidden information content.

It is easy to see that the literature [8] is covered by the Eq (3.12) we developed.

The knowledge measure for IFSs is expanded to the context of IVIFSs in this section. Using a quaternionic aggregation function, which can be created when various conforming functions are chosen, a new knowledge measure model is built. This considerably increases its validity and breadth.

## 4. Numerical examples

In this section, we test the effectiveness and performance of the developed knowledge measure with a series of numerical experiments through a comparative analysis of other knowledge measures [7–9] and entropy [13] models. For convenience, the following cases all use $KM_{IFS}(A) =$

$\frac{1}{n}\sum_{i=1}^{n}\frac{1}{2}[(\mu_A(x_i)+\nu_A(x_i))+(\mu_A(x_i)+\nu_A(x_i))|\mu_A(x_i)-\nu_A(x_i)|]$ , to calculate the knowledge measure of IFSs, so as to verify the effectiveness of our developed Eq (3.4).

**Example 4.1.** Four IFSs defined in $X$ are given as: $A_1=\langle x,0.5,0.5\rangle$, $A_2=\langle x,0.35,0.35\rangle$, $A_3=\langle x,0.25,0.25\rangle$, $A_4=\langle x,0,0\rangle$.

According to the six existing measures and our proposed knowledge measure, we can calculate the amount of knowledge. The comparative results are shown in Table 1.

**Table 1.** Analysis results by different measuring models for IFSs.

| $A_i$ | $E_{SK}$ | $E_{ZL}$ | $E_{HC}^2$ | $K_N$ | $K_{SKB}$ | $K_G$ | $KM_{IFS}$ |
|---|---|---|---|---|---|---|---|
| $A_1$ | 1 | 1 | 0.500 | 0.866 | 0.50 | 0.50 | 0.50 |
| $A_2$ | 1 | 1 | 0.665 | 0.606 | 0.35 | 0.35 | 0.35 |
| $A_3$ | 1 | 1 | 0.625 | 0.433 | 0.25 | 0.25 | 0.25 |
| $A_4$ | 1 | 1 | 0 | 0 | 0 | 0 | 0 |

From Table 1, $E_{ZL}(A_i)=E_{SK}(A_i)=1$ cannot discriminate the amount of knowledge of IFSs. Therefore, the model we developed $KM_{IFS}$ and $K_N$, $K_{SKB}$, $K_G$ are obviously more clear and intuitive, and can surely be useful to distinguish between these IFSs in terms of the amount of knowledge associated with them, just as we would expect.

**Example 4.2.** Nine IFSs defined in $X$ are considered. Let $B_1=\langle x,0.95,0.00\rangle$, $B_2=\langle x,0.90,0.05\rangle$, $B_3=\langle x,0.85,0.10\rangle$, $B_4=\langle x,80,0.10\rangle$, $B_5=\langle x,0.75,0.20\rangle$, $B_6=\langle x,0.70,0.25\rangle$, $B_7=\langle x,0.65,0.30\rangle$, $B_8=\langle x,0.60,0.35\rangle$, $B_9=\langle x,0.55,0.40\rangle$. In this data set, it is characterised by information content equal to 0.95 and a gradual decrease in information clarity. The comparative results are given in Table 2.

**Table 2.** Analysis results by different measuring models for IFSs.

| $B_i$ | $E_{SK}$ | $E_{ZL}$ | $S_{CH}^2$ | $K_N$ | $K_{SKB}$ | $K_G$ | $KM_{IFS}$ |
|---|---|---|---|---|---|---|---|
| $B_1$ | 0.0500 | 0.0500 | 0.0950 | 0.9500 | 0.9500 | 0.9738 | 0.9263 |
| $B_2$ | 0.1052 | 0.1500 | 0.1850 | 0.9260 | 0.9224 | 0.9212 | 0.8788 |
| $B_3$ | 0.1667 | 0.2500 | 0.2650 | 0.9042 | 0.8917 | 0.8688 | 0.8313 |
| $B_4$ | 0.2353 | 0.3500 | 0.3350 | 0.8846 | 0.8573 | 0.8163 | 0.7838 |
| $B_5$ | 0.3125 | 0.4500 | 0.3950 | 0.8675 | 0.8188 | 0.7638 | 0.7363 |
| $B_6$ | 0.4000 | 0.5500 | 0.4450 | 0.8529 | 0.7750 | 0.7113 | 0.6888 |
| $B_7$ | 0.5000 | 0.6500 | 0.4850 | 0.8411 | 0.7250 | 0.6588 | 0.6412 |
| $B_8$ | 0.6154 | 0.7500 | 0.5150 | 0.8322 | 0.6673 | 0.6062 | 0.5938 |
| $B_9$ | 0.7500 | 0.8500 | 0.5350 | 0.8261 | 0.6000 | 0.5538 | 0.4988 |

From Table 2, when the information content is equal to 0.9, the entropy of IFSs gradually increases as the information clarity decreases from $B_1$ to $B_9$, as can be seen from $E_{ZL}$, $E_{SK}$ and $E_{CH}^2$. However, the knowledge measure of IFSs shows a decreasing trend, as shown in $K_N$, $K_{SKB}$, $K_G$ and $KM_{IFS}$, as shown in Figure 1. In contrast to $K_{SKB}$, $K_G$ and $KM_{IFS}$, the value of $K_N$ is obviously too large, which indicates that $K_N$ only considers the information content but ignores the information clarity. It is important to note that knowledge measures are related to both information content and information clarity, and can not be considered in a single way. Clearly, the model we have developed performs well in this case.

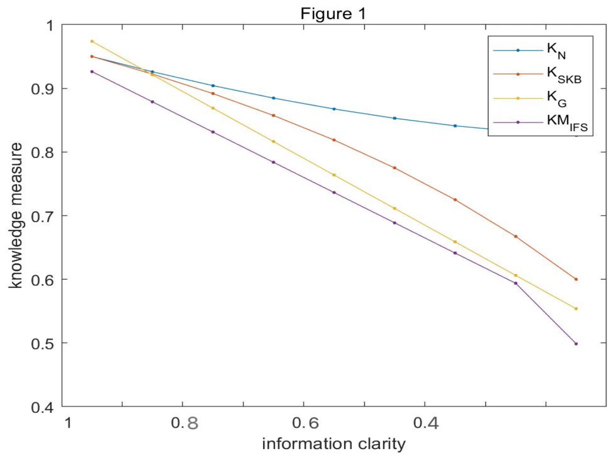


**Figure 1.** Knowledge measure and information clarity.

**Example 4.3.** Characterized by information clarity is equal to 0.2 and the increasing values of the information content. Let $C_1 = \langle x, 0.95, 0.00\rangle$, $C_2 = \langle x, 0.90, 0.05\rangle$, $C_3 = \langle x, 0.85, 0.10\rangle$, $C_4 = \langle x, 80, 0.10\rangle$, $C_5 = \langle x, 0.75, 0.20\rangle$, $C_6 = \langle x, 0.70, 0.25\rangle$, $C_7 = \langle x, 0.65, 0.30\rangle$, $C_8 = \langle x, 0.60, 0.35\rangle$, $C_9 = \langle x, 0.55, 0.40\rangle$. The analysis results are shown in Table 3, as follow:

**Table 3.** Analysis results by different measuring models for IFSs.

| $C_i$ | $E_{SK}$ | $E_{ZL}$ | $S^2_{CH}$ | $K_N$ | $K_{SKB}$ | $K_G$ | $KM_{IFS}$ |
|---|---|---|---|---|---|---|---|
| $C_1$ | 0.8000 | 0.8000 | 0.3200 | 0.2000 | 0.2000 | 0.2800 | 0.1600 |
| $C_2$ | 0.7895 | 0.8000 | 0.4450 | 0.2784 | 0.2553 | 0.3200 | 0.1800 |
| $C_3$ | 0.7778 | 0.8000 | 0.5400 | 0.3606 | 0.3111 | 0.3600 | 0.2400 |
| $C_4$ | 0.7647 | 0.8000 | 0.6050 | 0.4444 | 0.3676 | 0.4000 | 0.3000 |
| $C_5$ | 0.7500 | 0.8000 | 0.6400 | 0.5292 | 0.4250 | 0.4400 | 0.3600 |
| $C_6$ | 0.7333 | 0.8000 | 0.6450 | 0.6144 | 0.4833 | 0.4800 | 0.4200 |
| $C_7$ | 0.7143 | 0.8000 | 0.6200 | 0.7000 | 0.5429 | 0.5200 | 0.4800 |
| $C_8$ | 0.6923 | 0.8000 | 0.5650 | 0.7858 | 0.6038 | 0.5600 | 0.5400 |
| $C_9$ | 0.6667 | 0.8000 | 0.4800 | 0.8718 | 0.6667 | 0.6000 | 0.6000 |

From Table 3, when the information clarity is equal to 0.2, the entropy of these IFSs decreases as the information content decreases from $C_1$ to $C_9$, as shown by $E_{SK}$. However, during this process, $E_{ZL} = 0.8$, which indicates that in this case it is not possible to use entropy to judge the amount of knowledge contained in the ifs. The reason for the decrease in the amount of entropy is due to the less information clarity of these IFSs throughout the process. Therefore, we find that the increase in the knowledge measure is also limited, as in the case of $K_{SKB}, K_G$ and $KM_{IFS}$. The $K_N$ model produces larger values compared to the other knowledge measures. The model we developed produced more intuitive and convincing results compared to $K_N$.

In conclusion, the model we constructed, $KM_{IFS}$, also performs better throughout, with excellent stability and dependability. However, in the example we have given, $E_{SK}$ performs well in the aforementioned entropy and is clearly superior altogether.

## 5. Application of IFSs and IVIFSs

The knowledge measures of IFSs and IVIFSs can be employed as an assessment tool for dealing with numerous practical difficulties, according to prior studies. The following section will use medical diagnosis issues and the use of IVIFS' multi-attribute decision making to demonstrate the usefulness and viability of our built model.

### *5.1. Application of IVIFSs medical diagnosis*

In this section, tests are carried out on the medical diagnosis problem to show the efficacy of our proposed approach. Researchers have focused on IFSs for the purpose of medical diagnosis, and a variety of metrics between IFSs have been applied to the issue of classifying patients' symptoms, providing an alternative diagnosis for the doctor's decision-making. These metrics include similarity metrics and standardised Hamming and Euclidean distances. We apply the newly defined knowledge measures to medical diagnostics in this part. Let's think about a well-known issue [21].

Describe this medical diagnostic problem [21] as follows: A set of diagnoses D={Viral Fever, Malaria, Typhoid, Stomach Disease,Chest Disease}, denoting five possible diseases, and a set of patients P ={Amy, Bob, Cindy, Davie}, considering five medical symptoms S={Fever, Headache, Stomachache, Cough, Chest Pain}. Table 4 and Table 5 show the characteristic symptoms of the diagnosis and the symptoms of each patient, respectively. Each element is described by the form of a pair of numbers corresponding to the membership $\mu$ and nonmembership $\nu$.

**Table 4.** Diagnostic symptoms.

| | Viral fever | Malaria | Typhoid | Stomach | Chest |
|---|---|---|---|---|---|
| Fever | $< 0.4, 0.0 >$ | $< 0.7, 0.0 >$ | $< 0.3, 0.3 >$ | $< 0.1, 0.7 >$ | $< 0.1, 0.8 >$ |
| Headache | $< 0.3, 0.5 >$ | $< 0.2, 0.6 >$ | $< 0.6, 0.1 >$ | $< 0.2, 0.4 >$ | $< 0.0, 0.8 >$ |
| Stomach | $< 0.1, 0.7 >$ | $< 0.0, 0.9 >$ | $< 0.2, 0.7 >$ | $< 0.8, 0.0 >$ | $< 0.2, 0.8 >$ |
| Cough | $< 0.4, 0.3 >$ | $< 0.7, 0.0 >$ | $< 0.2, 0.6 >$ | $< 0.2, 0.7 >$ | $< 0.2, 0.8 >$ |
| Chest | $< 0.1, 0.7 >$ | $< 0.1, 0.8 >$ | $< 0.1, 0.9 >$ | $< 0.2, 0.7 >$ | $< 0.8, 0.1 >$ |

**Table 5.** Patients symptoms.

| Name | Temperature | Headache | Stomach | Cough | Chest |
|---|---|---|---|---|---|
| Amy | $< 0.8, 0.1 >$ | $< 0.6, 0.1 >$ | $< 0.2, 0.8 >$ | $< 0.6, 0.1 >$ | $< 0.1, 0.6 >$ |
| Bob | $< 0.0, 0.8 >$ | $< 0.4, 0.4 >$ | $< 0.6, 0.1 >$ | $< 0.1, 0.7 >$ | $< 0.1, 0.8 >$ |
| Cindy | $< 0.8, 0.1 >$ | $< 0.8, 0.1 >$ | $< 0.0, 0.6 >$ | $< 0.2, 0.7 >$ | $< 0.0, 0.5 >$ |
| Davie | $< 0.6, 0.1 >$ | $< 0.5, 0.4 >$ | $< 0.3, 0.4 >$ | $< 0.7, 0.2 >$ | $< 0.3, 0.4 >$ |

Our proposed knowledge measures Eq (3.4) for each patient from the set of possible diagnoses are presented in Table 6, respectively. The proper diagnosis has been noted in bold.

**Table 6.** Diagnostic results.

| Name | Viral fever | Malaria | Typhoid | Stomach | Chest |
|---|---|---|---|---|---|
| Amy | 0.472 | 0.525 | 0.800 | 0.730 | 0.525 |
| Bob | 0.720 | 0.400 | 0.525 | 0.375 | 0.765 |
| Cindy | 0.765 | 0.765 | 0.480 | 0.675 | 0.575 |
| Davie | 0.385 | 0.495 | 0.395 | 0.675 | 0.585 |

From the Table 6, the proper diagnoses are Amy-Viral fever, Bob-Stomach, Cindy-Typhoid and Davie-Viral fever, which coincide with the results of [21]. It further illustrate, the practicality and validity of the knowledge measures of our proposed IFSs.

### *5.2. Application of IVIFSs multi-attribute decision making*

In this section, we apply knowledge measures to solve multiple-attribute decision making-problems. In practical applications, the attribute weights are known or completely unknown in most cases. Therefore, it is necessary to determine the attribute weights when we solve decision-making problems. Attribute weights are usually assigned by experts based on experience and intellectual background. However, there is a strong subjectivity in this approach, which leads to a lack of applicability of weight information. Therefore, we propose a new model to further determine the attribute weights using knowledge measures. We used a real- life example in [8], which is basically, described as follows.

The urban development department plans to choose an air conditioning system to be installed in library. The contractor proposed four solutions $A_i (i = 1, 2, 3, 4)$ for this problem, and five attributes $C_j (j = 1, 2, 3, 4, 5)$ ($C_1$-performance, $C_2$-maintainbility, $C_3$-flexibility, $C_4$-cost, $C_5$-safety).

The committee is formed of four experts $D_k (k = 1, 2, 3, 4)$ to provide review comments. Let the weight vector of the experts be $\lambda = (0.3, 0.2, 0.3, 0.2)^T$. The weights $\omega_{ij}$ of the attributes are unknown. The individual opinions of $D_k$ on $A_i$ with respect to $C_j$ are provided in terms of IVIFSs decision matrix, denoted by $\tilde{R}^k = (\tilde{r}^k_{ij})_{4\times5} (k = 1, 2, 3, 4)$, where $\tilde{r}^k_{ij} = (\tilde{\mu}^k_{ij}, \tilde{\nu}^k_{ij})$ are expressed as IVIFSs.

Especially, we use the proposed model Eq (3.9) to calculate the weight of attributes.

**Step 1.** Using the weighted average operator (IIFWA) [19,20] for IVIFSs, all the attributes are aggregated to form the following IVIFSs decision matrix $\tilde{R}^k = (\tilde{r}^k_{ij})_{4\times5} (k = 1, 2, 3, 4)$, where

$$\begin{aligned} \tilde{r}_{ij} = IIFWA_\lambda(r^1_{ij}, r^2_{ij}, ..., r^4_{ij}) = ([1 - \Pi^4_{k=1}(1 - \mu^{Lk}_{ij})^{\lambda_k}, \\ 1 - \Pi^4_{k=1}(1 - \mu^{Uk}_{ij})^{\lambda_k}], [\prod_{k=1}^{4}(\nu^{Lk}_{ij})^{\lambda_k}, \prod_{k=1}^{4}(\nu^{Uk}_{ij})^{\lambda_k}]. \end{aligned} \tag{5.1}$$

Thus, the aggregation matrix is shown as

$$\tilde{R}=\begin{bmatrix}([0.476,0.638],[0.162,0.281])([0.300,0.462],[0.346,0.464])([0.622,0.723],\\ [0.123,0.226])([0.500,0.633],[0.100,0.266])([0.165,0.379],[0.362,0.535])\\ ([0.352,0.472],[0.325,0.494])([0.131,0.314],[0.267,0.452])([0.332,0.432],\\ [0.384,0.485])([0.171,0.304],[0.549,0.675])([0.633,0.755],[0.123,0.226])\\ ([0.346,0.495],[0.300,0.428])([0.633,0.783],[0.100,0.217])([0.549,0.774],\\ [0.100,0.226])([0.432,0.616],[0.217,0.318])([0.532,0.675],[0.176,0.298])\\ ([0.300,0.432],[0.325,0.485])([0.100,0.221],[0.628,0.779])([0.100,0.231],\\ [0.535,0.769])([0.221,0.321],[0.428,0.606])([0.193,0.315],[0.528,0.654])\end{bmatrix}$$

**Step 2.** The knowledge measure of $C_j$ $(j = 1,2,3,4,5)$ is calculated by Eq (3.9). The results of the computation are normalised ($\omega_i = \frac{\omega_{ij}}{\sum_{i=1}^{5}\omega_{ij}}$) to obtain the weight vector $\omega = (0.166, 0.198, 0.220, 0.202, 0.212)^T$ for all attributes of $\tilde{R}$.

**Step 3.** With $\omega$ and $\tilde{R}$, make a group assessment $\tilde{r}_i$ on each $A_i$ by using the IIFWA operator again, as follow:

$$\begin{aligned}\tilde{r}_i = IIFWA_\lambda(r_{i1}, r_{i2}, r_{i3}, r_{i4}, r_{i5}) = ([1-\Pi_{j=1}^{5}(1-\mu_{ij}^{L})^{\omega_j},\\ 1-\Pi_{j=1}^{5}(1-\mu_{ij}^{U})^{\omega_j}],[\prod_{j=1}^{5}(\nu_{ij}^{L})^{\omega_j},\prod_{j=1}^{5}(\nu_{ij}^{U})^{\omega_j}]),\end{aligned} \tag{5.2}$$

where $i = 1,2,3,4$.

Group assessments of the alternatives can be shown as

$$\begin{aligned}\tilde{r}_1 &= ([0.434, 0.584], [0.192, 0.336]),\\ \tilde{r}_2 &= ([0.357, 0.493], [0.292, 0.437]),\\ \tilde{r}_3 &= ([0.515, 0.692], [0.157, 0.282]),\\ \tilde{r}_4 &= ([0.181, 0.302], [0.486, 0.657]).\end{aligned}$$

**Step 4.** Evaluate the values of $\tilde{r}_i (i = 1,2,3,4)$ by using the method in [19,20],the specific form is as follows:

$$Z_{IVIFV}(\tilde{r}_i) = \frac{1}{2}(1-\frac{1}{4}(\pi_{\tilde{r}_i}^{L}+\pi_{\tilde{r}_i}^{U}))(\mu_{\tilde{r}_i}^{L}+\mu_{\tilde{r}_i}^{U}+\frac{1}{2}(\pi_{\tilde{r}_i}^{L}+\pi_{\tilde{r}_i}^{U})), \tag{5.3}$$

where the larger the value of $Z_{IVIFV}(\tilde{r}_i) \in [0,1]$, the better $\tilde{r}_i$. Then we have

$$\begin{aligned}Z_{IVIFV}(\tilde{r}_1) = 0.552, Z_{IVIFV}(\tilde{r}_2) = 0.482,\\ Z_{IVIFV}(\tilde{r}_3) = 0.635, Z_{IVIFV}(\tilde{r}_4) = 0.313.\end{aligned}$$

The results indicate that $Z_{IVIFV}(\tilde{r}_3) > Z_{IVIFV}(\tilde{r}_1) > Z_{IVIFV}(\tilde{r}_2) > Z_{IVIFV}(\tilde{r}_4)$. Obviously, the following ranking is obtained that $A_3 \succ A_1 \succ A_2 \succ A_4$.

Obviously, from Table 7, it can be seen that our scheme ranking results are consistent with [19]. It is further demonstrated that the knowledge measure proposed by our method is a feasible measure.

**Table 7.** Comparison results of methods.

| | weighting vector | program ranking |
|---|---|---|
| Guo[8] | $\omega = (0.161, 0.199, 0.223, 0.204, 0.213)^T$ | $A_3 \succ A_1 \succ A_2 \succ A_4$ |
| Our Model | $\omega = (0.166, 0.198, 0.220, 0.202, 0.212)^T$ | $A_3 \succ A_1 \succ A_2 \succ A_4$ |

## 6. Conclusions

Based on axiomatic definitions of information content and information clarity, we design knowledge measure models for IFSs and IVIFSs. A variety of knowledge measuring issues can be addressed by extending the two formulations we present. Through numerical examples, the validity and viability of the knowledge measurement models are shown. Finally, applications to medical diagnosis and multi-attributed decision-making issues show how effective the method is. Future research can focus on the connection between knowledge metrics and the entropy of IFSs and IVIFSs.

## Use of AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

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## Conflict of interest

This research does not involve any conflicts of interest.

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