AIMS Mathematics

http://www.aimspress.com/journal/Math

AIMS Mathematics, 8(10): 24484–24514.
DOI: 10.3934/math.20231249
Received: 19 April 2023
Revised: 07 July 2023
Accepted: 30 July 2023
Published: 18 August 2023

Research article

Cubic bipolar fuzzy VIKOR and ELECTRE-II algorithms for efficient freight transportation in Industry 4.0

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Abstract: The theory of cubic bipolar fuzzy sets (CBFSs) is a robust approach for dealing with vagueness and bipolarity in real-life circumstances. This theory provides a hybrid machine learning paradigm that can accurately describe two-sided contrasting features for medical diagnosis. The ELECTRE-II model, which is extensively used, is expanded in this article to include the cubic bipolar fuzzy (CBF) context. In order to produce a comprehensive preference ordering of actions, ELECTRE-II establishes two different forms of embedded outranking relations while taking into account the subjective human judgments. A huge number of applications have been created by its variations under various models, considering the CBF model's greater capacity to deal. For opinions in the adaptive CBF structure with unknown information, the CBF-ELECTRE-II group decision support method is described. With the use of proper CBF aggregation operations, the expert CBF views on each alternative and criterion are compiled in the first step. The approach then constructs weak and strong outranking relations and offers three distinct CBF outranking set kinds ("concordance", "indifferent" and "discordance" sets). Strong and weak outranking graphs serve as a visual depiction of the latter, which is finally studied by a rigorous iterative procedure that yields a preferred system. For these objectives, integrated CBF-VIKOR and CBF-ELECTRE-II techniques are developed for multi-criteria group decision making (MCDGM). Finally, suggested techniques are recommended to determine ranking index of efficient road freight transportation (FRT) in Industry 4.0. The ranking index and optimal decision are also computed with other techniques to demonstrate robustness of proposed MCDGM approach.

Keywords: cubic bipolar fuzzy information; CBF-VIKOR; CBF-ELECTRE-II; ranking index; MCGDM
Mathematics Subject Classification: 03E72, 90B50, 94D05

1. Introduction

Multi criteria group decision making (MCGDM) is an optimization approach in which a group of decision makers (DMs) unanimously choose a best choice from a collection of feasible choices under heterogeneous criterion. However, due to the inadequate data and inherent human judgments, this process entails ambiguous and vague information. The DMs often face difficulties to address two-sided contrasting features of bipolar information. Classical techniques are unable to determine the best option in the presence of vagueness and bipolarity. The abstraction of fuzzy set (FS) was first designed by Zadeh [1, 2] to address vagueness and since been FS theory has been constructively adopted to solve a extensive range of real-world issues. An intuitionistic fuzzy set (IFS) [3, 4] is a robust model to assign membership and non-membership grades in decision making problems. Table 1 lists numerous fuzzy set extensions that are helpful for grasping the concept of FS theory and its extension.

Table 1. Some extensions of fuzzy sets.

Fuzzy models	Researchers	Constraints
FS	Zadeh [1]	Membership values
Interval-valued fuzzy set (IVFS)	Zadeh [2]	Interval grading
IFS	Atanassov [3,4]	$\mu, \nu \in [0, 1], \mu + \nu \leq 1$
Pythagorean fuzzy set (PFS)	Yager [5,6]	$\mu, \nu \in [0, 1], \mu^2 + \nu^2 \leq 1$
q-Rung orthopair fuzzy set (q-ROFS)	Yager [7]	$\mu, \nu \in [0, 1], \mu^q + \nu^q \leq 1, q \geq 1$
Bipolar fuzzy set (BFS)	Zhang [8,9]	Positive grading $\mu^+ \in [0, 1]$ and negative grading $\mu^- \in [-1, 0]$
Cubic set (CS)	Jun et al. [10]	Hybrid model of FS and IVFS
Cubic bipolar fuzzy set (CBFS)	Riaz and Tehrim [11]	Hybrid model of CS and BFS

Many researchers have employed these models successfully in recent decades. All of these models were developed as a response to the imperative need to manage the instability intrinsic in the challenges that arise in the actual world. The discipline of multi-criteria decision making (MCDM) offers a robust approach to the DMs with assistance in the process of determining the appropriate action to take. In addition to this, it guarantees that the appropriate thought is given to two sided conflicting aspects of the problem in hand. The minimal components of MCDM are at least one or more decision makers, two distinct options and two essential criterion.

A significant amount of research effort has been put into the expansion and refinement of ELECTRE methods and these approaches have been deployed in a variety of real world applications. Various activities, also known as alternatives, are evaluated based on the suitable qualitative or quantitative scales associated with the criterion. On a more individual level, they frequently offer appraisals that are at odds with one another. The most significant benefit that it provides is the incorporation of the viewpoints of a number of DMs who share their professional competence in the fields that are pertinent to the discussion. As a direct consequence of this, this feature could end up being a more dependable and useful option. The following should be on a short list of traditional MCDM techniques: SIR [12], LAM [13], AHP [14], VIKOR [15], TOPSIS [16] and ELECTRE [17], which is the subject of this study. The ELECTRE strategy is a collection of outranking techniques that works best when there are many competing solutions and competing criteria [18]. In general, algorithms that are based on outranking perform out an inter-comparison of different options in a way that is systematic and is based on each criteria. The concordance and discordance are computed by contrasting the results obtained by evaluating every possible combination of possibilities in terms of a number of different factors. The concordance set illustrates the components (or criteria) that provide credence to the claim that one choice is superior to another. The discordance set, on the other hand, illustrates the feature (or standards) that run counter to the judgement that one option is more desirable than the other. Both the concordance and discordance sets are used to identify the comparison between the proposed relations, which are then included in the production of a more convincing proposition. In the years after the introduction of the ELECTRE strategy, other variations and additional outranking techniques have been developed [19]. Each one relates to details regarding the nature of the central problem. Almost every element of daily life, including the selection of environmentally friendly items, financial management and power projects, has seen extensive usage of the ELECTRE methods for MCDM. The ELECTRE-I strategy is appropriate when there are many options in the problem because it can establish a partial prioritising by choosing a promising alternatives set. The ELECTRE-I method fails to establish a preference ordering of activities, whereas the ELECTRE-II strategy succeeds by rating the alternatives [20].

Some extensions of ELECTRE technique are expressed in Table 2.

Table 2. Some important articles related to ELECTRE techniques.

Technique	Researchers	Decision making application
IVF-ELECTRE	Vahdani and Hadipour [21]	Maintenance and assessment
BF-ELECTRE-I	Akram et al. [22]	Healthcare diagnosis
CPF-ELECTRE-I	Akram et al. [23]	Interior design
Fuzzy ELECTRE I	Hatami et al. [24]	Group decision making
m-PFL-ELECTRE-I	Adeel et al. [25]	Analysis of salaries
Crisp-ELECTRE-I	Benayoun et al. [26]	Aircraft problem
Fuzzy-ELECTRE-I	Sevkli et al. [27]	Supplier selection
Fuzzy-ELECTRE-I	Rouyendegh and Erkan [28]	Academic staff selection
HF-ELECTRE-I	Chen et al. [29]	Project management
IF-ELECTRE-I	Rouyendegh [30]	Best plant selection
BF-ELECTRE-I	Akram et al. [31]	Medical diagnosis
ELECTRE	Wang and Triantaphyllou [32]	Ranking irregularities
PF-ELECTRE	Akram et al. [33]	Healthcare management

The two forms of integrated outranking links are established by ELECTRE-II by taking into account a set of concordance and discordance threshold values [34]. The graphs of strong and weak outranking show both in a visual way. Numerous ranking issues have been successfully solved using the ELECTRE-II method and its notable variations [35–38]. IF-ELECTRE II proposed by Victor and Rekha [39] to better understand gender inequality in society. Nimra and Riaz proposed CBF-topological structure, CBF-TOPSIS and CBF-ELECTRE-I to deal with CBF information and applied it decision analysis [40,41]. Riaz and Tehrim [42,43] proposed CBF-AOs for ranking index of alternative in MCGDM problems. Zhan et al. [44] developed PF-TOPSIS and Gwak et al. [45] proposed clustering analysis algorithm. Recently, Farid et al. [46] introduced the abstraction of information aggregation (IA) dynamic decision-making with T-spherical fuzzy sets for hierarchical medical diagnosis.

Table 3. Some extensions of ELECTRE and VIKOR technique.

Researchers	Techniques	Applications
Sooklall and Fonou-Dombeu [47]	ELECTRE-II & IV	Ontology ranking
Lin et al. [48]	Improved ELECTRE-II	Power generation technology
Chen and Pang [49]	ELECTRE-II	Electromagnet quality
Kirisci et al. [50]	Fermatean ELECTRE	Biomedical material selection
Sudipa [51]	ELECTRE-II	Analyze student constraint
Alinezhad and Khalili [52]	ELECTRE I–II–III Methods	MADM applications
Alshammari et al. [53]	TOPSIS and VIKOR	Rebotic agri-technique
Chen et al. [54]	VIKOR-GRA	Urban flood resilience
Topno et al. [55]	Integrated AHP-VIKOR	Municipal solid waste management
Pathak et al. [56]	VIKOR	Delivery performance
Liu et al. [57]	VIKOR	Intelligent distribution terminal
Samal and Dash [58]	TOPSIS and VIKOR	Ranking index model
Ismail and Felix [59]	VIKOR and TOPSIS	Sustainable development
Ic et al. [60]	AHP-modifined VIKOR	Financial performance
Zhou et al. [61]	extended VIKOR	Regional leading

Table 4. Bibliometric analysis of road freight transportation.

Researchers	Techniques	Applications
Yang et al. [62]	Bibliometric analysis	MCDM in shipping Industry 4.0
Krstic et al. [63]	Comprehensive distance based ranking (COBRA) method	reverse logistics technologies in Industry 4.0
Yavuz et al. [64]	HFS linguistic model	Evaluation of alternative-fuel vehicles
Farid and Riaz [65]	Prioritized interactive aggregation operators	Evaluation of efficient autonomous vehicles
Gružauskas et al. [66]	Optimization cost effective performance	Minimizing the trade-off with autonomous vehicles
Gerhátová et al. [67]	Bibliometric analysis	Implementation Industry 4.0 Railway transport
Qahtan et al. [68]	q-ROF rough sets model	Sustainable shipping transportation industry
Zhu et al. [69]	CO2 emissions future scenario simulation	Road freight transportation
Callefi et al. [70]	A multi-method study	Road freight transportation
Yilmaz [71]	IF-VIKOR method	Evaluating Industry 4.0 barriers
Bravo and Vidal [72]	Optimization models	Freight transportation function in supply chain

Some extensions of VIKOR and ELECTRE techniques towards different fuzzy models to solve MCDM problems under uncertainty are listed in the Table 3.

A brief bibliometric analysis of road freight transportation is given in Table 4.

Al-Quran [73] introduced T-spherical linear Diophantine fuzzy aggregation operators for multiple attribute decision-making. Al-Sharqi [74] proposed the notion of FP-interval complex neutrosophic soft sets and their applications under uncertainty. Al-Quran [75] developed a novel MADM approach with T-spherical hesitant fuzzy sets. Hanif et al. [76] introduced a new MCDM based on LDF graphs. Pamucar [77] suggested Dombi Bonferroni mean normalized weighted geometric operator. Many researchers extended fuzzy sets and soft sets towards MCDM such as bipolar fuzzy soft sets [78], almost convergence [79], soft union ideals and near-rings [80, 81], LDFS sine-trigonometric aggregation operators [82], LBWA and Z-MABAC methods [83].

1.1. Motivation

The following summarizes the major facts that guided this research work.

(1) In comparison with the ELECTRE-I methodology, the ELECTRE-II method generates a preferential ranking of the options available. The ability to provide a set of concordance and discordance barrier values is one of its features and it allows for two distinct kinds of engrained outranking relations to be implemented (weak and strong outranking relations). These embedded outranking interactions are shown in a clear and concise manner by the weak and strong outranking graphs, which are then used in order to derive decision outcomes. This demonstrates that we have a major edge over our competitors, which we intend to put to good use in this scenario.

(2) As problems get more complicated, MCGDM may be able to offer more dependable and persuasive answers, since it effectively makes use of expert opinions from those who are knowledgeable about the opposing sides of the underlying issue.

(3) CBFS increases the space that positive and negative grades are permitted to occupy along with IVFS and FS. For the purposes of MCDA, conflicting viewpoints can be accommodated more effectively.

(4) As a result, we draw the conclusion that the ELECTRE-II method combined with CBFS information will result in a MCGDM method that clearly outperforms previous approaches.

1.2. Research objectives and highlights

The primary intention of this research investigation is to develop a suggestion of MCGDM for CBF information. It requires two crucial actions to complete. In the first stage of the CBF assessment process, DMs evaluate how well the various options perform in relation to each criteria in the form of CBF decision matrices. This phase of the CBF evaluation process takes place in the first stage. During the second stage, which is a CBF ranking phase, we make adjustments to the ELECTRE-II model in accordance with the CBF.

The highlights of this research work are given as follows.

(1) Modeling uncertainties with cubic bipolar fuzzy information.

(2) New algorithms are developed for CBF-VIKOR and CBF-ELECTRE-II techniques.

(3) Robust MCGDM approach is proposed for efficient RFT in Industry 4.0.

(4) Ranking index for feasible alternatives is determined with score function to seek optimal alternative.

Major objectives may be described as follows.

(1) We introduced a robust extension of ELECTRE-II model that functions as a MCGDM framework for CBF information. By splitting the decision-making process into two basic steps, interpretation and ranking of objects, the innovative CBF-ELECTRE-II technique streamlines it.

(2) There are three different kinds of outranking sets, two different kinds of outranking matrices, two different kinds of outranking relations and two different kinds of outranking graphs in the CBF-ELECTRE-II method's basic structure. Each one is demonstrated in a CBF environment.

(3) In CBF-ELECTRE-II, the iterative process of determining the outranking graphs is taught in a clear and accessible manner.

(4) The two-phase approach for facilitating MCGDM is proposed. This aids in creating a step-by-step knowledge of how we intend to solve the issue.

(5) A CBF-VIKOR technique is developed for robust MCDGM process. The optimal decision is verified by some existing techniques.

(6) To prove the superiority of our methods, a thorough comparison of the new methodology with the previous procedures is offered.

The rest of the paper is ordered as follows. Section 2 contains literature review of some rudiments of CBFS and their operational laws under P(R)-order. Section 3 provides algorithms of VIKOR and ELECTRE II techniques to address CBF information. Section 4 presents case study and discussion of a FRT. In Section 5, a robust MCGDM application to FRT selection in Industry 4.0 is presented. Section 6 presents a comparative analysis to discuss the robustness of suggested methodologies. Section 7 gives the conclusions of the work and indicates possible extension areas.

2. Some fundamental notions

In this section, we review some rudiments of CBFSs and their operational laws, such as inclusion, intersection, union, sum, product, scalar multiplication and exponents, under P(R)-order.

Definition 2.1. [42, 43]
Let V be a non-empty set. A CBFS $C^{\urcorner}$ in V is defined as follows,

$$C^{\urcorner} = \{\langle a,\ \mathcal{P}^{\urcorner} = [\mathcal{P}_l(a),\ \mathcal{P}_u(a)],\ \mathcal{N}^{\urcorner} = [\mathcal{N}_l(a),\ \mathcal{N}_u(a)],\ \lambda^{\urcorner}(a),\ \mu^{\urcorner}(a)\rangle |\ a \in V\},$$

where $[\mathcal{P}_l(a),\ \mathcal{P}_u(a)] \subseteq [0,\ 1]$ and $[\mathcal{N}_l(a),\ \mathcal{N}_u(a)] \subseteq [-1,\ 0]$, $\lambda^{\urcorner} : V \to [0,\ 1]$ and $\mu^{\urcorner} : V \to [-1,\ 0]$.

Definition 2.2. Equality: [42, 43]
Two $CBFS$ s $C_1^{\neg} = \langle a,\ \mathcal{P}_1,\ \mathcal{N}_1,\ \lambda_1^{\neg},\ \mu_1^{\neg}\rangle$ and $C_2^{\neg} = \langle a,\ \mathcal{P}_2,\ \mathcal{N}_2,\ \lambda_2^{\neg},\ \mu_2^{\neg}\rangle$ are said to be equal iff

$$\mathcal{P}_1 = \mathcal{P}_2,\ \mathcal{N}_1 = \mathcal{N}_2,\ \lambda_1^{\neg} = \lambda_2^{\neg},\ \mu_1^{\neg} = \mu_2^{\neg}.$$

Definition 2.3. P-Order: [42, 43]
Let $C_1^{\neg} = \langle a,\ \mathcal{P}_1,\ \mathcal{N}_1,\ \lambda_1^{\neg},\ \mu_1^{\neg}\rangle$ and $C_2^{\neg} = \langle a,\ \mathcal{P}_2,\ \mathcal{N}_2,\ \lambda_2^{\neg},\ \mu_2^{\neg}\rangle$ be two $CBFS$s. Then $C_1^{\neg}$ is a subset of $C_2^{\neg}$ with P-order written as $C_1^{\neg} \subseteq_P C_2^{\neg}$ iff

$$\mathcal{P}_1 \subseteq \mathcal{P}_2,\ \ \mathcal{N}_1 \supseteq \mathcal{N}_2,\ \ \lambda_1^{\neg} \le \lambda_2^{\neg},\ \ \mu_1^{\neg} \ge \mu_2^{\neg}.$$

Definition 2.4. P-Union: [42, 43]
Let $C_1^{\neg} = \langle a,\ \mathcal{P}_1,\ \mathcal{N}_1,\ \lambda_1^{\neg},\ \mu_1^{\neg}\rangle$ and $C_2^{\neg} = \langle a,\ \mathcal{P}_2,\ \mathcal{N}_2,\ \lambda_2^{\neg},\ \mu_2^{\neg}\rangle$ be two $CBFS$s. The P-union of two $CBFS$s is defined as

$$C_1^{\neg} \bigcup_P C_2^{\neg} = \Big\{\langle a,\ \mathcal{P}_1 \cup \mathcal{P}_2,\ \mathcal{N}_1 \cap \mathcal{N}_2,\ \max(\lambda_1^{\neg}, \lambda_2^{\neg}),\ \min(\mu_1^{\neg}, \mu_2^{\neg}) > \mid a \in V\Big\}.$$

Definition 2.5. P-Intersection: [42, 43]
Let $C_1^{\neg} = \langle a,\ \mathcal{P}_1,\ \mathcal{N}_1,\ \lambda_1^{\neg},\ \mu_1^{\neg}\rangle$ and $C_2^{\neg} = \langle a,\ \mathcal{P}_2,\ \mathcal{N}_2,\ \lambda_2^{\neg},\ \mu_2^{\neg}\rangle$ be two $CBFS$s. The P-intersection of two $CBFS$s is written as:

$$C_1^{\neg} \bigcap_P C_2^{\neg} = \Big\{\langle a,\ \mathcal{P}_1 \cap \mathcal{P}_2,\ \mathcal{N}_1 \cup \mathcal{N}_2,\ \min(\lambda_1^{\neg}, \lambda_2^{\neg}),\ \max(\mu_1^{\neg}, \mu_2^{\neg}) > \mid a \in V\Big\}.$$

Definition 2.6. P-Ring Sum: [42, 43]
Let $C_1^{\neg} = \langle a,\ \mathcal{P}_1,\ \mathcal{N}_1,\ \lambda_1^{\neg},\ \mu_1^{\neg}(a)\rangle$ and $C_2^{\neg} = \langle a,\ \mathcal{P}_2,\ \mathcal{N}_2,\ \lambda_2^{\neg},\ \mu_2^{\neg}(a)\rangle$ be two $CBFS$s. Then,

$$\begin{aligned} C_1^{\neg} \bigoplus_P C_2^{\neg} \ = \ & \Big\{\langle a,\ [\mathcal{P}_{1l}(a) + \mathcal{P}_{2l}(a) - \mathcal{P}_{1l}(a) * \mathcal{P}_{2l}(a),\ \mathcal{P}_{1u}(a) + \mathcal{P}_{2u}(a) - \mathcal{P}_{1u}(a) * \mathcal{P}_{2u}(a)],\ [-\mathcal{N}_{1l}(a) * \mathcal{N}_{2l}(a), \\ & - \mathcal{N}_{1u}(a) * \mathcal{N}_{2u}(a)],\ \lambda_1^{\neg}(a) + \lambda_2^{\neg}(a) - \lambda_1^{\neg}(a) * \lambda_2^{\neg}(a),\ \ -\mu_1^{\neg}(a) * \mu_2^{\neg}(a) > \mid a \in V\Big\}. \end{aligned}$$

Definition 2.7. P-Ring Product: [42, 43]
Let $C_1^{\neg} = \langle a,\ \mathcal{P}_1,\ \mathcal{N}_1,\ \lambda_1^{\neg}(a),\ \mu_1^{\neg}(a)\rangle$ and $C_2^{\neg} = \langle a,\ \mathcal{P}_2,\ \mathcal{N}_2,\ \lambda_2^{\neg}(a),\ \mu_2^{\neg}(a)\rangle$ be two $CBFS$s. Then,

$$\begin{aligned} C_1^{\neg} \bigotimes_P C_2^{\neg} \ = \ & \Big\{\langle a,\ [\mathcal{P}_{1l}(a) * \mathcal{P}_{2l}(a),\ \mathcal{P}_{1u}(a) * \mathcal{P}_{2u}(a)],\ [-(-\mathcal{N}_{1l}(a) - \mathcal{N}_{2l}(a) + \mathcal{N}_{1l}(a) * \mathcal{N}_{2l}(a)), -(-\mathcal{N}_{1u}(a) \\ & - \mathcal{N}_{2u}(a) + \mathcal{N}_{1u}(a) * \mathcal{N}_{2u}(a))],\ \lambda_1^{\neg}(a) * \lambda_2^{\neg}(a),\ \ -(-\mu_1^{\neg}(a) - \mu_2^{\neg}(a) - \mu_1^{\neg}(a) * \mu_2^{\neg}(a)) > \mid a \in V\Big\}. \end{aligned}$$

Definition 2.8. P-Constant Power: [42, 43]
Let $C^{\neg} = \langle a,\ \mathcal{P},\ \mathcal{N},\ \lambda^{\neg},\ \mu^{\neg}\rangle$ be $CBFS$. Then,

$$C^{\neg^k} = \Big\{\Big\langle a,\ [(\mathcal{P}_l(a))^k,\ (\mathcal{P}_u(a))^k],\ [-(1-(1-\mathcal{N}_l(a))^k),\ -(1-(1-\mathcal{N}_u(a))^k)],\ (\lambda^{\neg}(a))^k,\ -(1-(1-\mu^{\neg}(a))^k)\Big\rangle \mid a \in V\Big\}.$$

Definition 2.9. P-Scalar Product: [42, 43]
Let $C^{\neg} = \langle a,\ \mathcal{P},\ \mathcal{N},\ \lambda^{\neg},\ \mu^{\neg}\rangle$ be a $CBFS$. Then,

$$k * C^{\neg} = \Big\{\langle a,\ [1-(1-\mathcal{P}_l(a))^k,\ 1-(1-\mathcal{P}_u(a))^k],\ [-(\mathcal{N}_l(a))^k,\ -(\mathcal{N}_u(a))^k],\ 1-(1-\lambda^{\neg}(a))^k,\ -(\mu^{\neg}(a))^k\rangle\Big\}.$$

Definition 2.10. R-Order: [42, 43]
Let $C_1^{\neg} = \langle a,\ \mathcal{P}_1,\ \mathcal{N}_1,\ \lambda_1^{\neg}(a),\ \mu_1^{\neg}(a)\rangle$ and $C_2^{\neg} = \langle a,\ \mathcal{P}_2,\ \mathcal{N}_2,\ \lambda_2^{\neg}(a),\ \mu_2^{\neg}(a)\rangle$ be two $CBFS$s. Then $C_1^{\neg}$ is said to be subset of $C_2^{\neg}$ with R-order written as $C_1^{\neg} \subseteq_R C_2^{\neg}$ iff

$$\mathcal{P}_1 \subseteq \mathcal{P}_2,\ \mathcal{N}_1 \supseteq \mathcal{N}_2,\ \lambda_1^{\neg}(a) \geq \lambda_2^{\neg}(a),\ \mu_1^{\neg}(a) \leq \mu_2^{\neg}(a).$$

Definition 2.11. R-Union: [42, 43]
Let $C_1^{\neg} = \langle a,\ \mathcal{P}_1,\ \mathcal{N}_1,\ \lambda_1^{\neg}(a),\ \mu_1^{\neg}(a)\rangle$ and $C_2^{\neg} = \langle a,\ \mathcal{P}_2,\ \mathcal{N}_2,\ \lambda_2^{\neg}(a),\ \mu_2^{\neg}(a)\rangle$ be two $CBFS$. Then, the R-union of two $CBFS$-sets are defined as:

$$C_1^{\neg}\bigcup_R C_2^{\neg} = \Big\{\langle a,\ \mathcal{P}_1 \cup \mathcal{P}_2,\ \mathcal{N}_1 \cap \mathcal{N}_2,\ \ \min(\lambda_1^{\neg}(a), \lambda_2^{\neg}(a)),\ \ \max(\mu_1^{\neg}(a), \mu_2^{\neg}(a))\rangle |\ a \in V\Big\}.$$

Definition 2.12. R-Intersection: [42, 43]
Let $C_1^{\neg} = \langle a,\ \mathcal{P}_1,\ \mathcal{N}_1,\ \lambda_1^{\neg}(a),\ \mu_1^{\neg}(a)\rangle$ and $C_2^{\neg} = \langle a,\ \mathcal{P}_2,\ \mathcal{N}_2,\ \lambda_2^{\neg}(a),\ \mu_2^{\neg}(a)\rangle$ be two $CBFS$s. The R-intersection of two $CBFS$-sets are:

$$C_1^{\neg}\bigcap_R C_2^{\neg} = \Big\{\langle a,\ \mathcal{P}_1 \cap \mathcal{P}_2,\ \mathcal{N}_1 \cup \mathcal{N}_2,\ \ \max(\lambda_1^{\neg}(a), \lambda_2^{\neg}(a)),\ \ \min(\mu_1^{\neg}(a), \mu_2^{\neg}(a))\rangle |\ a \in V\Big\}.$$

Definition 2.13. R-Ring Sum: [42, 43]
Let $C_1^{\neg} = \langle a,\ \mathcal{P}_1,\ \mathcal{N}_1,\ \lambda_1^{\neg}(a),\ \mu_1^{\neg}(a)\rangle$ and $C_2^{\neg} = \langle a,\ \mathcal{P}_2,\ \mathcal{N}_2,\ \lambda_2^{\neg}(a),\ \mu_2^{\neg}(a)\rangle$ be two $CBFS$s. Then,

$$\begin{aligned} C_1^{\neg}\bigoplus_R C_2^{\neg} \ =\ & \Big\{\Big\langle a,\ [\mathcal{P}_{1l}(a) + \mathcal{P}_{2l}(a) - \mathcal{P}_{1l}(a) * \mathcal{P}_{2l}(a),\ \mathcal{P}_{1u}(a) + \mathcal{P}_{2u}(a) - \mathcal{P}_{1u}(a) * \mathcal{P}_{2u}(a)],\ [-\mathcal{N}_{1l}(a) * \mathcal{N}_{2l}(a), \\ & - \mathcal{N}_{1u}(a) * \mathcal{N}_{2u}(a)],\ \lambda_1^{\neg}(a) * \lambda_2^{\neg}(a),\ \ -(-\mu_1^{\neg}(a)(a) - \mu_2^{\neg}(a)(a) + \mu_1^{\neg}(a)(a) * \mu_2^{\neg}(a)(a))\rangle |\ a \in V\Big\}. \end{aligned}$$

Definition 2.14. R-Ring Product: [42, 43]
Let $C_1^{\neg} = \langle a,\ \mathcal{P}_1,\ \mathcal{N}_1,\ \lambda_1^{\neg}(a),\ \mu_1^{\neg}(a)(a)\rangle$ and $C_2^{\neg} = \langle a,\ \mathcal{P}_2,\ \mathcal{N}_2,\ \lambda_2^{\neg}(a),\ \mu_2^{\neg}(a)(a)\rangle$ be two $CBFS$s. Then,

$$\begin{aligned} C_1^{\neg}\bigotimes_R C_2^{\neg} \ =\ & \Big\{\langle a,\ [\mathcal{P}_{1l}(a)\mathcal{P}_{2l}(a),\ \mathcal{P}_{1u}(a)\mathcal{P}_{2u}(a)],\ [-(-\mathcal{N}_{1l}(a) - \mathcal{N}_{2l}(a) - \mathcal{N}_{1l}(a) * \mathcal{N}_{2l}(a)),\ \ -(-\mathcal{N}_{1u}(a) - \mathcal{N}_{2u}(a) \\ & -\mathcal{N}_{1u}(a) * \mathcal{N}_{2u}(a))],\ \lambda_1^{\neg}(a) + \lambda_2^{\neg}(a) - \lambda_1^{\neg}(a) * \lambda_2^{\neg}(a),\ \ -\mu_1^{\neg}(a)(a) * \mu_2^{\neg}(a)(a)\rangle |\ a \in V\Big\}. \end{aligned}$$

Definition 2.15. k-Scalar Power: [42, 43]
Let $C^{\neg} = \langle a,\ \mathcal{P},\ \mathcal{N},\ \lambda^{\neg},\ \mu^{\neg}\rangle$ be a $CBFS$ then

$$C^{\neg^k} = \Big\{\langle a,\ [(\mathcal{P}_l(a))^k,\ (\mathcal{P}_u(a))^k],\ [-(1-(1-\mathcal{N}_l(a))^k),\ -(1-(1-\mathcal{N}_u(a))^k)],\ 1-(1-\lambda^{\neg}(a))^k,\ -(-\mu^{\neg}(a))^k |\ a \in V\Big\}.$$

Definition 2.16. k-scalar Product: [42, 43]
Let $C^{\neg} = \langle a,\ \mathcal{P},\ \mathcal{N},\ \lambda^{\neg},\ \mu^{\neg}\rangle$ be a $CBFS$ then

$$k*C^{\neg} = \Big\{\langle a,\ [1-(1-\mathcal{P}_l(a))^k,\ 1-(1-\mathcal{P}_u(a))^k],\ [-(-\mathcal{N}_l(a))^k,\ -(-\mathcal{N}_u(a))^k],\ (\lambda^{\neg}(a))^k,\ -(1-(1-\mu^{\neg}(a))^k)\rangle |\ a \in V\Big\}.$$

Example 2.17. Consider any three $CBFNs$, $C_1^{\urcorner}, C_2^{\urcorner}$ and $C_3^{\urcorner}$ as follows:

$$C_1^{\urcorner} = \langle [0.75\ ,\ 0.85],\ [-0.85\ ,\ -0.75],\ 0.80,\ -0.70\rangle,$$
$$C_2^{\urcorner} = \langle [0.35\ ,\ 0.55],\ [-0.60\ ,\ -0.45],\ 0.45,\ -0.57\rangle,$$
$$C_3^{\urcorner} = \langle [0.65\ ,\ 0.75],\ [-0.30\ ,\ -0.10],\ 0.50,\ -0.35\rangle,$$

and $K = 3$. We compute the results as follow.

(1) $C_2^{\urcorner} \subset_P C_1^{\urcorner};\ \ C_3^{\urcorner} \subset_P C_1^{\urcorner}$.

(2) $C_2^{\urcorner} \bigcup_P C_1^{\urcorner} = \langle [0.75,\ 0.85],\ [-0.85,\ -0.75],\ 0.85,\ -0.70\rangle = C_1^{\urcorner}$.

(3) $C_3^{\urcorner} \bigcup_P C_1^{\urcorner} = C_1^{\urcorner}$.

(4) $C_2^{\urcorner} \bigcup_P C_3^{\urcorner} = \langle [0.65,\ 0.75],\ [-0.60,\ -0.45],\ 0.50,\ -0.57\rangle$.

(5) $C_2^{\urcorner} \bigcap_P C_1^{\urcorner} = \langle [0.35,\ 0.55],\ [-0.60,\ -0.45],\ 0.45,\ -0.57\rangle = C_2^{\urcorner}$.

(6) $C_3^{\urcorner} \bigcup_P C_1^{\urcorner} = C_3^{\urcorner}$.

(7) $C_2^{\urcorner} \bigcup_P C_3^{\urcorner} = \langle [0.35,\ 0.55],\ [-0.30,\ -0.10],\ 0.45,\ -0.35\rangle$.

(8) $C_1^{\urcorner} \bigoplus_P C_2^{\urcorner} = \langle [0.8375,\ 0.9325],\ [-0.5100,\ -0.3375],\ 0.8900,\ -0.3990\rangle$.

(9) $C_1^{\urcorner} \bigotimes_P C_2^{\urcorner} = \langle [0.2625,\ 0.4675],\ [-0.9400,\ -0.8625],\ 0.3600, -0.8710\rangle$.

(10) $C_3^{\urcorner K} = \langle [0.2745,\ 0.4219],\ [-0.6570,\ -0.2710],\ 0.1250,\ -0.7254\rangle$.

(11) $K * C_3^{\urcorner} = \langle [0.9571,\ 0.9844],\ [-0.0270,\ -0.0010],\ 0.8750,\ -0.0429\rangle$.

Similarly, we obtain the following results.

(1) $C_i^{\urcorner} \subset_R C_j^{\urcorner}$ for all i & j.

(2) $C_1^{\urcorner} \bigcup_R C_2^{\urcorner} = \langle [0.75,\ 0.85],\ [-0.85,\ -0.75],\ 0.45,\ -0.57\rangle$.

(3) $C_1^{\urcorner} \bigcup_R C_3^{\urcorner} = \langle [0.75,\ 0.85],\ [-0.85,\ -0.75],\ 0.50,\ -0.35\rangle$.

(4) $C_3^{\urcorner} \bigcup_R C_2^{\urcorner} = \langle [0.65,\ 0.75],\ [-0.80,\ -0.45],\ 0.45,\ -0.35\rangle$.

(5) $C_1^{\urcorner} \bigcap_R C_2^{\urcorner} = \langle [0.35,\ 0.55],\ [-0.60,\ -0.45],\ 0.80,\ -0.70\rangle$.

(6) $C_1^{\urcorner} \bigcup_R C_3^{\urcorner} = \langle [0.65,\ 0.75],\ [-0.30,\ -0.10],\ 0.80,\ -0.70\rangle$.

(7) $C_3^{\urcorner} \bigcup_R C_2^{\urcorner} = \langle [0.35,\ 0.55],\ [-0.30,\ -0.10],\ 0.50,\ -0.57\rangle$.

(8) $C_1^{\urcorner} \bigoplus_R C_2^{\urcorner} = \langle [0.8375,\ 0.9325],\ [-0.5100,\ 0.3377],\ 0.3600,\ -0.8710\rangle$.

(9) $C_1^{\urcorner} \bigotimes_R C_2^{\urcorner} = \langle [0.2625,\ 0.4675],\ [-.9400,\ -0.5625],\ 0.8900,\ -0.3990\rangle$.

(10) $C_3^{\urcorner K} = \langle [0.2745,\ 0.4219],\ [-0.6570,\ -0.2710],\ 0.8750,\ -0.0429\rangle$.

(11) $K * C_3^{\urcorner} = \langle [0.9571,\ 0.9844],\ [-0.0270,\ -0.0010],\ 0.1250,\ -0.7254\rangle$.

Now, we will calculate the score function of $C_1^{\urcorner},\ C_2^{\urcorner},\ C_3^{\urcorner}$:

$$S_P(C_1^{\urcorner}) = -0.0167; S_P(C_2^{\urcorner}) = -0.0050; S_P(C_3^{\urcorner}) = 0.1417.$$
$$S_R(C_1^{\urcorner}) = 0.0167; S_R(C_2^{\urcorner}) = -0.0450; S_R(C_3^{\urcorner}) = 0.1917.$$
$$A(C_1^{\urcorner}) = 0.25; A(C_2^{\urcorner}) = 0.1450; A(C_3^{\urcorner}) = 0.3083.$$

3. Propounded techniques

In this section, we will discuss some propounded MCDM techniques to deal CBF data.

3.1. VIKOR method

This subsection details the fundamental framework of CBF-VIKOR, including its essential procedures, formulations and terminology at the most fundamental level. The acronym VIKOR, which stands for Vlse Kriterijumska Optimizacija Kompromisno Resenje, is a Serbian term that refers to multiple optimization, conflict and compromise factors. It is assumed that compromise is a suitable method for conflict resolution, the person making the choice appears to seek a solution that is as near to the ideal as is feasible and the options are evaluated according to all of the signs. When determining the order of the available choices, VIKOR considers whatever compromise gets the closest to achieving the desired end result.
Step 1: Analyze the issue and set the group of DMs, alternatives and criterions.
Step 2: Get the decision matrix for each decision maker opinion for each alternative verse criteria.
Step 3: Generate(Calculate) the weights and normalized it.
Step 4: Aggregate the decision matrices by using formula.

$$CBFG(C_1^{\urcorner},...,C_k^{\urcorner}) = \left\langle [\gimel_{k=1}^{n}(\mathcal{P}_{l_k}), \gimel_{k=1}^{n}(\mathcal{P}_{u_k})], [-(1-\gimel_{k=1}^{n}(1-\mathcal{N}_{l_k})), -(1-\gimel_{k=1}^{n}(1-\mathcal{N}_{u_k}))], \gimel_{k=1}^{n}(\lambda_k^{\urcorner}), -(1-\gimel_{k=1}^{n}(1-\mu_k^{\urcorner}))\right\rangle. \tag{3.1}$$

Step 5: Evaluate the PIS and NIS by using formula:

$$d(C_1^{\urcorner}, C_2^{\urcorner}) = \frac{1}{6}\sqrt{(\mathcal{P}_{l_1} - \mathcal{P}_{l_2})^2 + (\mathcal{P}_{u_1} - \mathcal{P}_{u_2})^2 + (\mathcal{N}_{l_1} - \mathcal{N}_{l_2})^2 + (\mathcal{N}_{u_1} - \mathcal{N}_{u_2})^2 + (\lambda_1^{\urcorner} - \lambda_2^{\urcorner})^2 + (\mu_1^{\urcorner} - \mu_2^{\urcorner})^2}.$$

Step 6: Evaluate the "group utility" value $\mathcal{S}_i$, the "individual regret value" $\mathcal{R}_i$ and "compromise value" $\mathcal{Q}_i$ by making use of listed formulas (3.2)–(3.4).

$$\mathcal{S}_i = \sum_{k=1}^{n} w_l \left(\frac{d(\rho_k^+ - \rho_{ik})}{d(\rho_k^+, \rho_k^-)} \right), \tag{3.2}$$

$$\mathcal{R}_i = \max_{k=1}^{n}\{w_l \left(\frac{d(\rho_k^+ - \rho_{ik})}{d(\rho_k^+, \rho_k^-)} \right)\}, \tag{3.3}$$

$$\mathcal{Q}_i = K\left(\frac{\mathcal{S}_i - S^-}{S^+ - S^-} \right) + (1-K)\left(\frac{\mathcal{R}_i - R^-}{R^+ - R^-} \right), \tag{3.4}$$

where $S^+ = \max_i \mathcal{S}_i, S^- = \min_i \mathcal{S}_i, R^+ = \max_i \mathcal{R}_i$, and $R^- = \min_i \mathcal{R}_i$. In order to select a compromise solution by majority vote, the value of the decision mechanism's coefficient $K \in [0,\ 1]$. The weight of the j^{th} criterion, expressed as w_j, indicates its relative importance.
Step 7: Consider your options carefully and come up with a compromise. Make three ranking lists S, R and Q. The alternative f will be deemed the compromise option if it scores highest in $Q[.]$ and simultaneously meets the criteria:

The flow chart of CBF-VIKOR algorithm is presented in Figure 1.

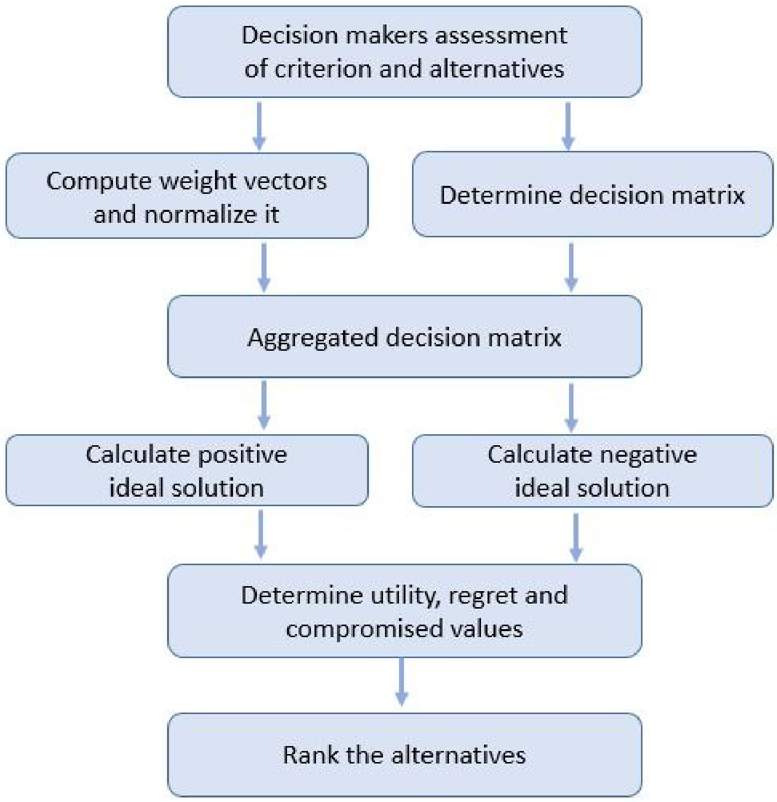

Figure 1. Pictorial algorithm of CBF-VIKOR.

3.2. ELECTRE-II method

This section entails the formulation and fundamental terms as well as the main structure of CBF-ELECTRE-II.

3.2.1. Basic data

Analyze the issue and set the group of DMs, alternatives and criterions.

3.2.2. CBF-concordance, CBF-disconcordance and CBF-indifference matrices

The argument that ρm is at least as excellent as ρn is used to establish the outranking relation between any two alternatives. Two indices, the concordance index and the disconcordance index, are used in outranking-based approaches. These indices outline the factors that are both in favor of and against an outranking problem.

3.2.3. CBF-concordance sets

The list of criteria for concordance includes subscripts that highlight the alternatives (ρ_m, ρ_n) where $m, n = 1, 2, ..., 6$. The CBF-concordance set is categorized into eight sets $(\mathfrak{B}_{\psi_k})$ expressed in Eqs (3.5)–(3.12) if $\mathcal{P}_{l_1} \geq \mathcal{P}_{l_2}$ and $\mathcal{P}_{u_1} \geq \mathcal{P}_{u_2}$.

$$\mathfrak{B}_{\psi_1} \quad = \quad \{b|\mathcal{N}_{l_1} \geq \mathcal{N}_{l_2}, \mathcal{N}_{u_1} \geq \mathcal{N}_{u_2}, \lambda_1^{\urcorner} \geq \lambda_2^{\urcorner}, \mu_1^{\urcorner} < \mu_2^{\urcorner}\}, \tag{3.5}$$

$$\begin{aligned}
\mathfrak{B}_{\psi_2} &= \{b|\mathcal{N}_{l_1} < \mathcal{N}_{l_2}, \mathcal{N}_{u_1} < \mathcal{N}_{u_2}, \lambda_1^\urcorner < \lambda_2^\urcorner, \mu_1^\urcorner < \mu_2^\urcorner\}, && (3.6)\\
\mathfrak{B}_{\psi_3} &= \{b|\mathcal{N}_{l_1} < \mathcal{N}_{l_2}, \mathcal{N}_{u_1} < \mathcal{N}_{u_2}, \lambda_1^\urcorner \geq \lambda_2^\urcorner, \mu_1^\urcorner \geq \mu_2^\urcorner\}, && (3.7)\\
\mathfrak{B}_{\psi_4} &= \{b|\mathcal{N}_{l_1} < \mathcal{N}_{l_2}, \mathcal{N}_{u_1} < \mathcal{N}_{u_2}, \lambda_1^\urcorner < \lambda_2^\urcorner, \mu_1^\urcorner \geq \mu_2^\urcorner\}, && (3.8)\\
\mathfrak{B}_{\psi_5} &= \{b|\mathcal{N}_{l_1} \geq \mathcal{N}_{l_2}, \mathcal{N}_{u_1} \geq \mathcal{N}_{u_2}, \lambda_1^\urcorner \geq \lambda_2^\urcorner, \mu_1^\urcorner < \mu_2^\urcorner\}, && (3.9)\\
\mathfrak{B}_{\psi_6} &= \{b|\mathcal{N}_{l_1} \geq \mathcal{N}_{l_2}, \mathcal{N}_{u_1} \geq \mathcal{N}_{u_2}, \lambda_1^\urcorner \geq \lambda_2^\urcorner, \mu_1^\urcorner \geq \mu_2^\urcorner\}, && (3.10)\\
\mathfrak{B}_{\psi_7} &= \{b|\mathcal{N}_{l_1} \geq \mathcal{N}_{l_2}, \mathcal{N}_{u_1} \geq \mathcal{N}_{u_2}, \lambda_1^\urcorner < \lambda_2^\urcorner, \mu_1^\urcorner \geq \mu_2^\urcorner\}, && (3.11)\\
\mathfrak{B}_{\psi_8} &= \{b|\mathcal{N}_{l_1} \geq \mathcal{N}_{l_2}, \mathcal{N}_{u_1} \geq \mathcal{N}_{u_2}, \lambda_1^\urcorner < \lambda_2^\urcorner, \mu_1^\urcorner \geq \mu_2^\urcorner\}. && (3.12)
\end{aligned}$$

3.2.4. CBF-indifference set

It is possible that both the alternatives ρ_m and ρ_n will have the same accuracy degree and score degree, i.e. they will be equally indifferent to one another. The CBF-indifference set $\mathfrak{B}^{=}_{mn}$ is defined as follows in order to represent this difference relation:

$$\mathfrak{B}^{=}_{mn} = \{b|\mathcal{P}_{l_1} = \mathcal{P}_{l_2}, \mathcal{P}_{u_1} = \mathcal{P}_{u_2}, \mathcal{N}_{l_1} = \mathcal{N}_{l_2}, \mathcal{N}_{u_1} = \mathcal{N}_{u_2}, \lambda_1^\urcorner = \lambda_2^\urcorner, \mu_1^\urcorner = \mu_2^\urcorner\}. \tag{3.13}$$

3.2.5. CBF-disconcordance sets

For the two alternatives $(\rho_m, \rho_n)(m, n = 1, 2, ..., 6; m \neq n)$, the CBF-disconcordance set comprises of the indicators which oppose the assertion that ρ_m is outperforming ρ_n. The sets $\mathfrak{B}_{\Delta_k}$ are defined in the Eqs (3.14)–(3.21) if $\mathcal{P}_{l_1} < \mathcal{P}_{l_2}, \mathcal{P}_{u_1} < \mathcal{P}_{u_2}$.

$$\begin{aligned}
\mathfrak{B}_{\Delta_1} &= \{b|\mathcal{N}_{l_1} < \mathcal{N}_{l_2}, \mathcal{N}_{u_1} < \mathcal{N}_{u_2}, \lambda_1^\urcorner < \lambda_2^\urcorner, \mu_1^\urcorner < \mu_2^\urcorner\}, && (3.14)\\
\mathfrak{B}_{\Delta_2} &= \{b|\mathcal{N}_{l_1} < \mathcal{N}_{l_2}, \mathcal{N}_{u_1} < \mathcal{N}_{u_2}, \lambda_1^\urcorner < \lambda_2^\urcorner, \mu_1^\urcorner \geq \mu_2^\urcorner\}, && (3.15)\\
\mathfrak{B}_{\Delta_3} &= \{b|\mathcal{N}_{l_1} < \mathcal{N}_{l_2}, \mathcal{N}_{u_1} < \mathcal{N}_{u_2}, \lambda_1^\urcorner \geq \lambda_2^\urcorner, \mu_1^\urcorner < \mu_2^\urcorner\}, && (3.16)\\
\mathfrak{B}_{\Delta_4} &= \{b|\mathcal{N}_{l_1} < \mathcal{N}_{l_2}, \mathcal{N}_{u_1} < \mathcal{N}_{u_2}, \lambda_1^\urcorner \geq \lambda_2^\urcorner, \mu_1^\urcorner \geq \mu_2^\urcorner\}, && (3.17)\\
\mathfrak{B}_{\Delta_5} &= \{b|\mathcal{N}_{l_1} \geq \mathcal{N}_{l_2}, \mathcal{N}_{u_1} \geq \mathcal{N}_{u_2}, \lambda_1^\urcorner < \lambda_2^\urcorner, \mu_1^\urcorner < \mu_2^\urcorner\}, && (3.18)\\
\mathfrak{B}_{\Delta_6} &= \{b|\mathcal{N}_{l_1} \geq \mathcal{N}_{l_2}, \mathcal{N}_{u_1} \geq \mathcal{N}_{u_2}, \lambda_1^\urcorner < \lambda_2^\urcorner, \mu_1^\urcorner \geq \mu_2^\urcorner\}, && (3.19)\\
\mathfrak{B}_{\Delta_7} &= \{b|\mathcal{N}_{l_1} \geq \mathcal{N}_{l_2}, \mathcal{N}_{u_1} \geq \mathcal{N}_{u_2}, \lambda_1^\urcorner \geq \lambda_2^\urcorner, \mu_1^\urcorner < \mu_2^\urcorner\}, && (3.20)\\
\mathfrak{B}_{\Delta_8} &= \{b|\mathcal{N}_{l_1} \geq \mathcal{N}_{l_2}, \mathcal{N}_{u_1} \geq \mathcal{N}_{u_2}, \lambda_1^\urcorner \geq \lambda_2^\urcorner, \mu_1^\urcorner \geq \mu_2^\urcorner\}. && (3.21)
\end{aligned}$$

3.2.6. CBF-concordance matrix

The concordance indices, denoted by ψ_{mn} in the range of [0,1], are used to form the CBF-concordance matrix. Equation (3.22) is used to calculate the index ψ_{mn}, where $\eta'_b \in [0, 1]$ are the normalized weights related to the b^{th} criteria.

$$\psi_{mn} = \Sigma_k\Big(\omega_{\mathfrak{B}_{\psi_k}} \times \Sigma_{b \in \mathfrak{B}_{\psi_k}}\Big), \tag{3.22}$$

where ω_i are the respective weights assigned to the CBF-concordance sets specified by the experts.

The Concondance matrix is given in Table 5.

Table 5. Concondance matrix.

ψ	ρ_1	ρ_2	$\cdots$	ρ_{s-1}	ρ_s
ρ_1	-	ψ_{12}	$\cdots$	$\psi_{1(s-1)}$	ψ_{1s}
ρ_2	ψ_{21}	-	$\cdots$	$\psi_{2(s-1)}$	ψ_{2s}
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
ρ_{s-1}	$\psi_{(s-1)1}$	$\psi_{(s-1)2}$	$\cdots$	-	$\psi_{(s-1)s}$
ρ_s	ψ_{s1}	ψ_{s2}	$\cdots$	$\psi_{s(s-1)}$	-

3.2.7. CBF-disconcordance matrix

The disconcordance indices $\delta_{mn} \in [0, 1]$ make up the CBF-disconcordance matrix $\Delta = (\delta_{mn})_{stimess}$. The indices δ_{mn} express how strongly one choice outranks another (ρ_n over ρ_m). In other words, the evaluation of ρ_m is poorer than ρ_n the greater the value of δ_{mn}. Equation (3.23) is used to get the index δ_{mn}.

$$d(\rho_m, \rho_n) = \sqrt{(\mathcal{P}_{l_1} - \mathcal{P}_{l_2})^2 + (\mathcal{P}_{u_1} - \mathcal{P}_{u_2})^2 + (\mathcal{N}_{l_1} - \mathcal{N}_{l_2})^2 + (\mathcal{N}_{u_1} - \mathcal{N}_{u_2})^2 + (\lambda_1^{\urcorner} - \lambda_2^{\urcorner})^2 + (\mu_1^{\urcorner} - \mu_2^{\urcorner})^2}. \tag{3.23}$$

$$\delta_{mn} = \frac{\max\{\omega_{\mathfrak{B}_{\Delta_k}} \times d(\rho_m, \rho_n)\}}{\max\{d(\rho_m, \rho_n)\}}. \tag{3.24}$$

3.2.8. Ranking of alternatives

By creating two embedding relations—strong and weak outranking, denoted as $\mathcal{O}^S$ and $\mathcal{O}^W$—the ELECTRE-II approach allows for the preference ordering of alternatives. These outranking were created by combining elemental memberships that were concordant and discordant. The threshold values ψ^-, ψ^0 & ψ^+ represent three strictly rising degrees of concordance, or low, average and high levels are integers. Additionally, δ^0 & δ^+ must represent strictly decreasing levels of disconcordance, such as average and low levels. Outranking of alternatives is shown in Table 6.

Table 6. Outranking of alternatives.

Strong $\rho_m \mathcal{O}^S \rho_n$	Weak $\rho_m \mathcal{O}^W \rho_n$
$\psi_{mn} \geq \psi^+$	$\psi_{mn} \geq \psi^-$
$\delta_{mn} \leq \delta^+$	$\delta_{mn} \leq \delta^0$
$\psi_{mn} \geq \psi_{nm}$	$\psi_{mn} \geq \psi_{nm}$

3.2.9. Exploration of outranking graphs

The two embedded outranking relationships are taken into consideration throughout the ranking process via the ELECTRE-II approach. For the strong outranking connection $\mathcal{O}^S$, draw the strong outranking graph $\mathcal{G}^S = (V_S, E_S)$ and for the weak outranking relationship $\mathcal{OW}$, draw the weak outranking graph $\mathcal{G}^W = (V_W, E_W)$. The collection of directed arcs between the two alternatives, E_S

and E_W, respectively, indicate the outranking in accordance with the principles listed in Table 6. The first step is to construct a forward ordering (φ') and a reverse ordering (φ"), with an average ordering (φ) serving as the final ranking.

Forward ordering φ'

Consider the set of vertices $V_S = \rho_1, \rho_2, ..., \rho_6$ and $\mathcal{T}(V)$ be the subset of V_S, the following is a breakdown of the phases involved in the forward ordering process:

(1) First, identify the no-precedent and incoming arrow vertices of the strong outranking graph $\mathcal{G}^S$. Put these vertices together into a set represented by $\mathcal{H}(V)$.

(2) Find the arcs from E_W with both endpoints from $\mathcal{H}(V)$ in the weak outranking graph $\mathcal{G}^W$. Assign this set the value $\overline{V^f}$ and create the graph $(\mathcal{H}(V), \overline{V^f})$.

(3) Create the set $\mathcal{F}V$, which is the set of non-dominated solutions that may be referred to as the v^{th} iteration, consisting of vertices that have no predecessor in the graph $(\mathcal{H}(V), \overline{V^f})$.

(4) Use the below-described iterative approach to build the forward ordering φ'.

a Initiate with $v = 1$ and $V(1) = V_S$.

b Follow the above (1), (2), (3), (4), we determine the sets V_S & $\mathcal{F}(V)$.

c Give an alternative ρ_k the order v as follows: $\varphi'(rho_k) = v \forall \mathcal{F}(V)$.

d By determining $V(v + 1) = V(v) - \mathcal{F}(v)$ and deleting all arcs from graphs $\mathcal{G}^S$ & $\mathcal{G}^W$, coming from the alternatives in sets $\mathcal{F}(v)$, the forward-ranked alternatives are eliminated from the system. All the options are rated if $V(v + 1) = \emptyset$. Set $v = v + 1$ and proceed to step 2 if $V(v + 1) \neq \emptyset$.

Reverse ordering φ"

The processes involved in reversing the ordering of φ" may be illustrated as:

a By flipping the arc directions E_S in $\mathcal{G}^S$ and E_W in $\mathcal{G}^W$, you may get the mirror image of the outranking relations.

b Utilize the mean of the above-mentioned inverted graphs to derive an ordering, $\varpi(\rho k)$ and proceed as described in the preceding φ' forward ordering demonstration.

c Summarise the sequence in which things should be done by setting: $\varphi" = 1 + max(\varpi(\rho_k)) - \varpi(\rho_k)$.

Average ordering φ

Establish the average ordering φ as follows:

$$\varphi(\rho_k) = \frac{\varphi'(\rho_k) + \varphi"(\rho_k)}{2}. \tag{3.25}$$

The flow chart of CBF ELECTRE-II algorithm is shown in Figure 2.

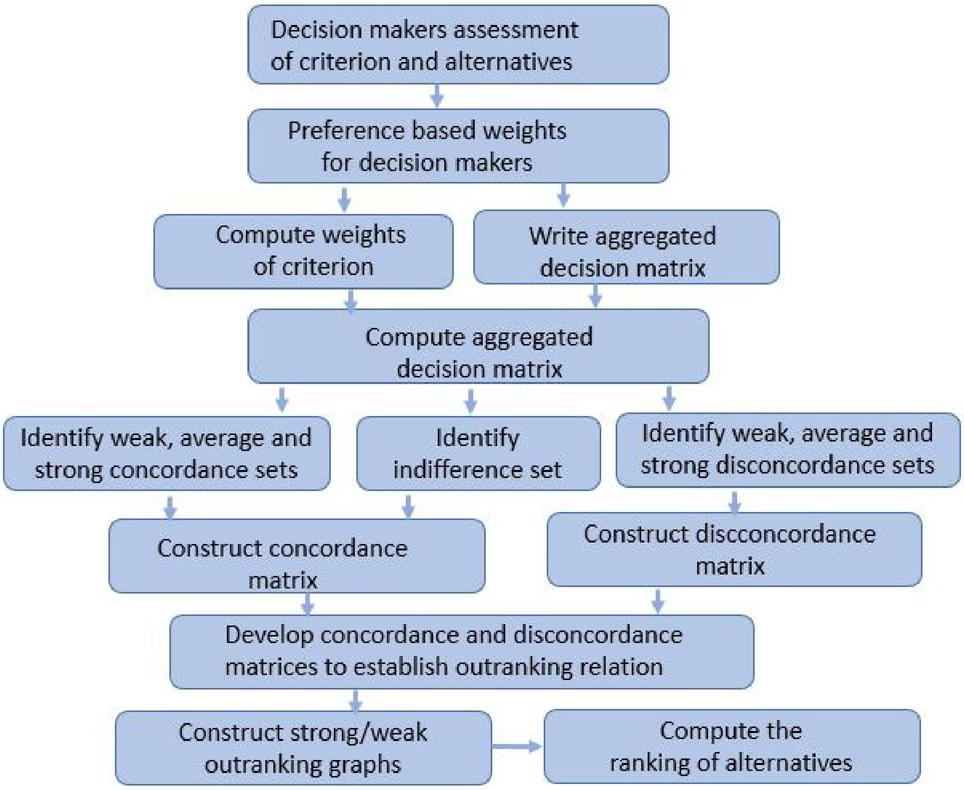

Figure 2. Flow chart of CBF-ELECTRE-II.

4. MCGDM application

Road freight transportation (RFT) in Industry 4.0 enables various smart features of business promotion with end-to-end (E2E) visibility, digitization and undoubtedly supply chain operations as well as tracking, control and trustworthy logistics recognition. Nowadays, the growing challenges of sustainable planning and decision management in RFT have put pressure on steakholders of Industry 4.0. Managing the cybersecurity and risk factors of autonomous vehicles is more complex in the logistics industry because supply chain requires sustainability, accuracy and cost efficiency. The selection of autonomous FRT companies depend on various types of preferences and constraints, including distance, nature of goods, size and volume of goods, flexibility of various modes of transportation, priorities and cost.

The manufacturing industry and everyday living both rely substantially on the ability to transport things. It produces materials for both industrial and domestic use, making it an indispensable part of modern society. However, autonomous vehicles have a major effect on the environment as a whole. The promotion of ecologically friendly technology, autonomous vehicles facilitated close monitoring of transportation patterns.

Artificial Intelligence (AI) and Machine Learning (ML) have promoted and assisted the Industry 4.0. Supply chain and logistics are greatly regulated by advances in AI approaches. By optimizing and streamlining an extensive range of business operations, this modern technology helps organizations save both time and money. Building efficient and trustworthy modes AI/ML tools has increased the interest of investors in investing more on FRT.

Overall, defining and optimizing any end-to-end supply chain method from an environmental viewpoint requires attention to major existing concerns.

(1) Balance supply and demand according to market need. Supply of needed products as well as not keeping superfluous products for maximum profit of all business partners.

(2) Improve quality and reduce pollution. Significantly reduce carbon emissions.

(3) Autonomous vehicle network for freight transportation. Smart storage of liquid, gas, solid and dry products.

(4) Sustainable environmental resources. Efficient performance with reliability.

(5) Recycling and reuse of waste material.

(6) Cost effective performance with stimulate clean energy resources.

(7) Adoption of ML/AI tools. E2E visibility with tracking systems. Use of robotics to minimize labor cost.

4.1. Numerical example

Consider the problem of capital investment in the ranking of efficient FRT companies.

Assume there are six options $\wp_1,\ \wp_2,\ \wp_3, \wp_4, \wp_5,\ \wp_6$ of FRT companies that business experts must evaluate based on four criterion $\zeta_1,\ \zeta_2,\ \zeta_3,\ \zeta_4$, where

ζ_1 = Smart technologies, cloud computing, robotics, networking,

ζ_2 = E2E visibility, tracking systems,

ζ_3 = Automation, cost efficiency, save time and money,

ζ_4 = Reliability, sustainability, clean environment.

Three DMs are called to put their expert opinions for MCGDM framework. Freight transportation selection criteria is given in Figure 3.

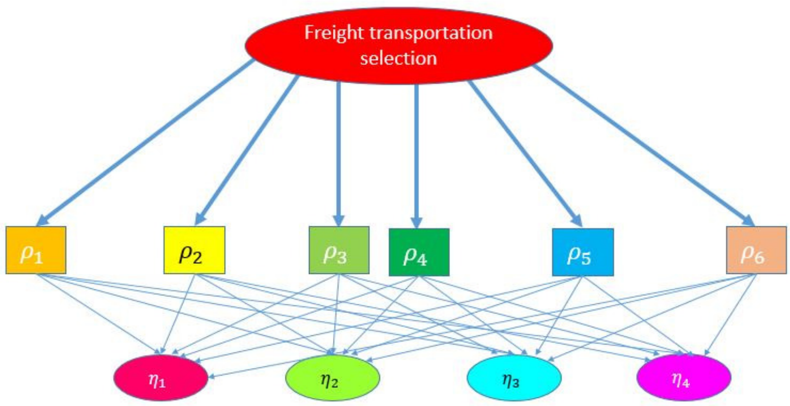

Figure 3. Freight transportation selection criteria.

Linguistic terms are given in Table 7, decision matrices are expressed in Table 8 and fuzzy values of alternative v/s criterions id listed in Table 9.

Fuzzy maximum and minimum values of alternatives are given in Table 10.

Table 7. Linguistic terms.

Linguistics term	Tally marks	Associated values		
Best	$\clubsuit$	$\langle[0.90, 1.00], [-1, -0.90], 0.85, -0.75\rangle$	to	$\langle[1.00, 1.00], [-1.00, -1.00], 1, -1\rangle$
Good	$\diamondsuit$	$\langle[0.75, 0.90], [-0.90, -0.70], 0.65, -0.65\rangle$	to	$\langle[0.90, 1.00], [-1, -0.90], 0.85, -0.75\rangle$
Average	$\heartsuit$	$\langle[0.50, 0.75], [-0.70, -0.60], 0.55, -0.45\rangle$	to	$\langle[0.75, 0.90], [-0.90, -0.70], 0.65, -0.65\rangle$
Bad	$\spadesuit$	$\langle[0.35, 0.50], [-0.60, -0.35], 0.45, -0.40\rangle$	to	$\langle[0.50, 0.75], [-0.70, -0.60], 0.55, -0.45\rangle$
Worst	$\P$	$\langle[0.00, 0.35], [-0.35, -0.01], 0.25, -0.20\rangle$	to	$\langle[0.35, 0.50], [-0.60, -0.35], 0.45, -0.40\rangle$
Prohibited	$\star$	$\langle[0.00, 0.00], [0.00, 0.00], 0, 0\rangle$	to	$\langle[0.00, 0.35], [-0.35, -0.01], 0.25, -0.20\rangle$

Table 8. Decision matrices.

D_1	$\wp_1$	$\wp_2$	$\wp_3$	$\wp_4$	$\wp_5$	$\wp_6$	D_2	$\wp_1$	$\wp_2$	$\wp_3$	$\wp_4$	$\wp_5$	$\wp_6$	D_3	$\wp_1$	$\wp_2$	$\wp_3$	$\wp_4$	$\wp_5$	$\wp_6$
ζ_1	$\P$	$\spadesuit$	$\P$	$\heartsuit$	$\diamondsuit$	$\clubsuit$	ζ_1	$\star$	$\P$	$\P$	$\star$	$\heartsuit$	$\P$	ζ_1	$\P$	$\spadesuit$	$\star$	$\spadesuit$	$\star$	$\heartsuit$
ζ_2	$\clubsuit$	$\star$	$\P$	$\heartsuit$	$\diamondsuit$	$\spadesuit$	ζ_2	$\star$	$\diamondsuit$	$\heartsuit$	$\P$	$\diamondsuit$	$\star$	ζ_2	$\heartsuit$	$\spadesuit$	$\P$	$\spadesuit$	$\star$	$\diamondsuit$
ζ_3	$\star$	$\P$	$\clubsuit$	$\diamondsuit$	$\spadesuit$	$\heartsuit$	ζ_3	$\diamondsuit$	$\heartsuit$	$\star$	$\spadesuit$	$\heartsuit$	$\heartsuit$	ζ_3	$\star$	$\P$	$\P$	$\P$	$\P$	$\star$
ζ_4	$\P$	$\diamondsuit$	$\spadesuit$	$\star$	$\heartsuit$	$\spadesuit$	ζ_4	$\spadesuit$	$\diamondsuit$	$\clubsuit$	$\P$	$\star$	$\diamondsuit$	ζ_4	$\P$	$\star$	$\heartsuit$	$\diamondsuit$	$\spadesuit$	$\star$

Table 9. Fuzzy values of alternative v/s criterions.

	$\wp_1$	$\wp_2$	$\wp_3$
ζ_1	⟨[0.02 , 0.13] , [-0.96 , -0.83] , 0.83 , -0.07 ⟩	⟨[0.15 , 0.28] , [-0.98 , -0.79] , 0.85 , -0.21 ⟩	⟨[0.00 , 0.04] , [-0.98 , -0.96] , 0.63 , -0.07 ⟩
ζ_2	⟨[0.13 , 0.21] , [-0.87 , -0.79] , 0.90 , -0.17 ⟩	⟨[0.09 , 0.15] , [-0.89 , 0.79] , 0.96 , -0.01 ⟩	⟨[0.02 , 0.07] , [-0.93 , -0.87] , 0.72 , -0.37 ⟩
ζ_3	⟨[0.07 , 0.13] , [-0.87 , -0.72] , 0.83 , -0.10 ⟩	⟨[0.01 , 0.04] , [-0.98 ,-0.93] , 0.99 , -0.09 ⟩	⟨[0.04 , 0.13] , [-0.96 , -0.87] , 0.93 , -0.17 ⟩
ζ_4	⟨[0.21 , 0.37] , [-0.87 , -0.72] , 0.96 , -0.10 ⟩	⟨[0.05 , 0.09] , [-0.70 , 0.00] , 0.93 , -0.09 ⟩	⟨[0.17 , 0.28] , [-0.87 , -0.72] , 0.83 , -0.17 ⟩
ρ^+	⟨[0.21 , 0.37] , [-0.87 , -0.72] , 0.96 , -0.07 ⟩	⟨[0.15 , 1.00] , [-0.79 , 0,00] , 0.99 , -0.01 ⟩	⟨[0.17 , 0.28] , [-0.87 , -0.72] , 0.93 , -0.07 ⟩
ρ^-	⟨[0.02 , 0.13] , [-0.96 , -0.83] , 0.83 , -0.17 ⟩	⟨[0.01 , 0.04] , [-0.98 , -0.98] , 0.85 , -0.21 ⟩	⟨[0.00 , 0.04] , [-0.98 , -0.96] , 0.63 -0.37 ⟩

	$\wp_4$	$\wp_5$	$\wp_6$
ζ_1	⟨[0.07 , 0.11] , [-0.98 , -0.85] , 0.95 , -0.04 ⟩	⟨[0.13 , 0.28] , [-0.90 , -0.72] , 0.93 , -0.10 ⟩	⟨[0.04 , 0.13] , [-0.96 , -0.87] , 0.79 , -0.17 ⟩
ζ_2	⟨[0.05 , 0.15] , [-0.96 , -0.89] , 0.94 , -0.07 ⟩	⟨[0.02 , 0.07] , [-0.98 , -0.90] , 0.72 , -0.21 ⟩	⟨[0.04 , 0.21] , [-0.96 , -0.83] , 0.87 , -0.17 ⟩
ζ_3	⟨[0.00 , 0.02] , [-0.99 , -0.93] , 0.79 , -0.11 ⟩	⟨[0.02 , 0.07] , [-0.98 , -0.96] , 0.72 , -0.21 ⟩	⟨[0.07 , 0.13] , [-0.93 , -0.83] , 0.83 , -0.13 ⟩
ζ_4	⟨[0.00 , 0.04] , [-0.85 , -0.79] , 0.79 , -0.21 ⟩	⟨[0.02 , 0.07] , [-0.98 , -0.90] , 0.79 , -0.28 ⟩	⟨[0.17 , 0.37] , [-0.79 , -0.00] , 0.96 , -0.02 ⟩
ρ^+	⟨[0.07 , 0.15] , [-0.85 , -0.79] , 0.95 , -0.04 ⟩	⟨[0.13 , 0.28] , [-0.83 , -0.72] , 0.93 , -0.10 ⟩	⟨[0.17 , 0.37] , [-0.79 , 0.00] , 0.96 , -0.02 ⟩
ρ^-	⟨[0.00 , 0.02] , [-0.99 , -0.98] , 0.79 , -0.21 ⟩	⟨[0.02 , 0.07] , [-0.98 , -0.96] , 0.72 , -0.28 ⟩	⟨[0.04 , 0.13] , [-0.96 , -0.87] , 0.79 , -0.17 ⟩

Table 10. Fuzzy maximum and minimum values of alternatives.

ζ_i	Maximum	Minimum
ζ_1	$\langle[0.15,\ 0.28],\ [-0.98,\ -0.96],\ 0.95,\ -0.21\rangle$	$\langle[0.00,\ 0.04],\ [-0.90,\ -0.72],\ 0.63,\ -0.04\rangle$
ζ_2	$[0.13,\ 0.21],\ [-0.98,\ -0.90],\ 0.96,\ -0.37$	$[0.02,\ 0.07],\ [-0.87,\ -0.79],\ 0.72,\ -0.01$
ζ_3	$[0.07,\ 0.13],\ [-0.99,\ -0.96],\ 0.99,\ -0.21$	$[0.00,\ 0.02],\ [-0.87,\ -0.72],\ 0.79,\ -0.10$
ζ_4	$[0.21,\ 0.37],\ [-0.98,\ -0.90],\ 0.96,\ -0.28$	$[0.00,\ 0.04],\ [-0.70,\ -0.00],\ 0.79,\ -0.02$

4.2. CBF-VIKOR technique

Using formula

$$d(\eta^+\ ,\ \eta^-) = \sqrt{\frac{(\mathcal{P}_l^+ - \mathcal{P}_l^-)^2 + (\mathcal{P}_u^+ - \mathcal{P}_u^-)^2 + (\mathcal{N}_l^+ - \mathcal{N}_l^-)^2 + (\mathcal{N}_u^+ - \mathcal{N}_u^-)^2 + (\lambda^+ - \lambda^-)^2 + (\mu^+ - \mu^-)^2}{6}}. \tag{4.1}$$

We have PIS and NIS as given in Table 11 and distance between alternatives and positive ideal solution is expressed in Table 12.

Table 11. Difference between PIS and NIS.

Distances	Values
$d(\zeta_1^+\ ,\ \zeta_1^-)$	0.2142
$d(\zeta_2^+\ ,\ \zeta_2^-)$	0.2013
$d(\zeta_3^+\ ,\ \zeta_3^-)$	0.1714
$d(\zeta_4^+\ ,\ \zeta_4^-)$	0.4355

Table 12. Distance between alternatives and positive ideal solution.

Distances	$d(\eta_1^+\ ,\ \zeta_{11})$	$d(\eta_2^+\ ,\ \zeta_{12})$	$d(\eta_3^+\ ,\ \zeta_{13})$	$d(\eta_4^+\ ,\ \zeta_{14})$	$d(\eta_5^+\ ,\ \zeta_{15})$	$d(\eta_6^+\ ,\ \zeta_{16})$
Values	0.12295	0.08052	0.18353	0.11277	0.11321	0.10824
Distances	$d(\eta_1^+\ ,\ \zeta_{21})$	$d(\eta_2^+\ ,\ \zeta_{22})$	$d(\eta_3^+\ ,\ \zeta_{23})$	$d(\eta_4^+\ ,\ \zeta_{24})$	$d(\eta_5^+\ ,\ \zeta_{25})$	$d(\eta_6^+\ ,\ \zeta_{26})$
Values	0.10630	0.16073	0.12430	0.12968	0.13839	0.10124
Distances	$d(\eta_1^+\ ,\ \zeta_{31})$	$d(\eta_2^+\ ,\ \zeta_{32})$	$d(\eta_3^+\ ,\ \zeta_{33})$	$d(\eta_4^+\ ,\ \zeta_{34})$	$d(\eta_5^+\ ,\ \zeta_{35})$	$d(\eta_6^+\ ,\ \zeta_{36})$
Values	0.13522	0.06721	0.05017	0.10638	0.11482	0.09354
Distances	$d(\eta_1^+\ ,\ \zeta_{41})$	$d(\eta_2^+\ ,\ \zeta_{42})$	$d(\eta_3^+\ ,\ \zeta_{43})$	$d(\eta_4^+\ ,\ \zeta_{44})$	$d(\eta_5^+\ ,\ \zeta_{45})$	$d(\eta_6^+\ ,\ \zeta_{46})$
Values	0.11321	0.41421	0.11776	0.18965	0.16073	0.39056

$S_3 = \sum_{j=1}^{m} w_j(\frac{d(\eta_j^+ - \zeta_{1j})}{d(\eta_j^+ - \eta^-)})$,
$S_3 = 0.2 \times \frac{0.18353}{0.2142} + 0.3 \times \frac{0.12430}{0.2013} + 0.4 \times \frac{0.05017}{0.1714} + 0.1 \times \frac{0.11776}{0.4355}$,
$S_3 = 0.5007$.

$R_3 = \max_{j=1}^{m} w_j(\frac{d(\eta_j^+ - \zeta_{1j})}{d(\eta_j^+ - \eta^-)})$,
$R_3 = \max\{0.2 \times \frac{0.18353}{0.2142},\ 0.3 \times \frac{0.12430}{0.2013},\ 0.4 \times \frac{0.05017}{0.1714},\ 0.1 \times \frac{0.11776}{0.4355}$,
$R_3 = 0.1852$.

$$Q_i = \kappa\left(\frac{S_i - S^-}{S^+ - S^-}\right) + (1-\kappa)\left(\frac{R_i - R^-}{R^+ - R^-}\right). \tag{4.2}$$

Fix $\kappa = 0.5$. The utility, regret and compromise values are listed in Table 13.

The final ranking is drawn as $\wp_3 \geq \wp_6 \geq \wp_2 \geq \wp_5 \geq \wp_4 \geq \wp_1$. The optimum choice is $\wp_3$.

Table 13. Utility, regret and compromise values.

	S_i	R_i	Q_i(4.2)	Ranking
$\wp_1$	0.6148	0.3156	1.0000	6^{th}
$\wp_2$	0.5667	0.2395	0.4974	3^{rd}
$\wp_3$	0.5007	0.1852	0.0000	1^{st}
$\wp_4$	0.5904	0.2683	0.7117	5^{th}
$\wp_5$	0.5902	0.2180	0.5180	4^{th}
$\wp_6$	0.5599	0.2183	0.3863	2^{nd}
minimum	0.6148	0.3156		
maximum	0.5007	0.1852		

4.3. CBF-ELECTRE-II technique

4.3.1. Concordance matrix

The Concordance matrices is presented in Table 14. Indifference matrix is given in Table 15.

$$W_\psi = \{\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \frac{1}{7}, \frac{1}{8}, \frac{1}{9}\},$$

$$\zeta_b = \{0.2, 0.3, 0.4, 0.1\}.$$

The concordance membership grades matrix is given in Table 16.

4.3.2. Disconcordance matrix

The disconcordance matrices is expressed in Tables 17 and 18.

4.4. Ranking the alternatives

To rank the alternatives, first, we will fix the threshold values $\psi^- = 0.3$, $\psi^0 = 0.5$, $\psi^* = 0.9$, $\Delta^0 = 0.7$, and $\Delta^* = 0.5$ which satisfies $0 < \psi^- = 0.3 < \psi^0 = 0.5 < \psi^* = 0.9, < 1$ and $1 > \Delta^0 = 0.7 > \Delta^* = 0.5 > 0$. By using relations mentioned in Table 6 we have the weak relation between alternatives listed in Table 19 and strong relation between alternatives in Table 20.

By applying algorithm for ordering listed in Subsubsection (3.2.7) we have the graphs and its mirror graphs for strong and weak relation along with their rankings given in Table 21.

Strong relation based graph is expressed in Figure 4, strong relation based mirror graph is given in Figure 5, weak relation based graph is expressed in Figure 6 and Weak relation based mirror graph is given in Figure 7.

The final ranking is $\wp_3 \geq \wp_1 \geq \wp_2 \geq \wp_5 \geq \wp_6 \geq \wp_4$. The optimum choice is $\wp_3$.

Table 14. Concordance matrices.

$\mathfrak{B}_{\psi_1}$	$\wp_1$	$\wp_2$	$\wp_3$	$\wp_4$	$\wp_5$	$\wp_6$	$\mathfrak{B}_{\psi_2}$	$\wp_1$	$\wp_2$	$\wp_3$	$\wp_4$	$\wp_5$	$\wp_6$
$\wp_1$	-	Λ	{4}	Λ	Λ	{4}	$\wp_1$	-	Λ	Λ	Λ	Λ	{4}
$\wp_2$	Λ	-	Λ	Λ	Λ	Λ	$\wp_2$	Λ	-	Λ	Λ	Λ	Λ
$\wp_3$	Λ	Λ	-	{3}	Λ	Λ	$\wp_3$	Λ	{4}	-	Λ	Λ	Λ
$\wp_4$	{1}	Λ	Λ	-	Λ	Λ	$\wp_4$	Λ	Λ	Λ	-	Λ	Λ
$\wp_5$	{1}	Λ	{1}	{4}	-	Λ	$\wp_5$	{1}	{3}	{2}	{4}	-	Λ
$\wp_6$	Λ	Λ	Λ	Λ	Λ	-	$\wp_6$	Λ	Λ	Λ	Λ	Λ	-
$\mathfrak{B}_{\psi_3}$	$\wp_1$	$\wp_2$	$\wp_3$	$\wp_4$	$\wp_5$	$\wp_6$	$\mathfrak{B}_{\psi_4}$	$\wp_1$	$\wp_2$	$\wp_3$	$\wp_4$	$\wp_5$	$\wp_6$
$\wp_1$	-	Λ	{4}	Λ	Λ	Λ	$\wp_1$	-	Λ	Λ	Λ	Λ	Λ
$\wp_2$	Λ	-	Λ	Λ	Λ	Λ	$\wp_2$	Λ	-	Λ	Λ	Λ	Λ
$\wp_3$	Λ	Λ	-	Λ	Λ	Λ	$\wp_3$	Λ	Λ	-	Λ	Λ	{1}
$\wp_4$	Λ	Λ	Λ	-	Λ	Λ	$\wp_4$	Λ	Λ	Λ	-	Λ	Λ
$\wp_5$	Λ	Λ	{2}	Λ	-	Λ	$\wp_5$	Λ	Λ	{2}	Λ	-	Λ
$\wp_6$	Λ	{4}	Λ	Λ	Λ	-	$\wp_6$	Λ	Λ	Λ	Λ	Λ	-
$\mathfrak{B}_{\psi_5}$	$\wp_1$	$\wp_2$	$\wp_3$	$\wp_4$	$\wp_5$	$\wp_6$	$\mathfrak{B}_{\psi_6}$	$\wp_1$	$\wp_2$	$\wp_3$	$\wp_4$	$\wp_5$	$\wp_6$
$\wp_1$	-	Λ	{1}	Λ	Λ	{2}	$\wp_1$	-	Λ	{1, 2, 4}	{3}	{2, 3, 4}	{2, 3}
$\wp_2$	Λ	-	{1}	Λ	Λ	Λ	$\wp_2$	Λ	-	{2}	{2, 3, 5}	{2, 4}	{2}
$\wp_3$	Λ	Λ	-	{3}	{2}	Λ	$\wp_3$	Λ	Λ	-	Λ	{2, 3, 4}	Λ
$\wp_4$	Λ	Λ	Λ	-	Λ	Λ	$\wp_4$	Λ	Λ	{1}	-	Λ	Λ
$\wp_5$	{1}	Λ	{1}	Λ	-	Λ	$\wp_5$	{1}	Λ	Λ	Λ	-	{1}
$\wp_6$	Λ	Λ	{1}	{3}	Λ	-	$\wp_6$	Λ	Λ	{4}	{4}	{2, 3, 4}	-
$\mathfrak{B}_{\psi_7}$	$\wp_1$	$\wp_2$	$\wp_3$	$\wp_4$	$\wp_5$	$\wp_6$	$\mathfrak{B}_{\psi_8}$	$\wp_1$	$\wp_2$	$\wp_3$	$\wp_4$	$\wp_5$	$\wp_6$
$\wp_1$	-	{2, 3}	Λ	{2}	Λ	Λ	$\wp_1$	-	Λ	Λ	Λ	Λ	{3}
$\wp_2$	Λ	-	Λ	{1}	Λ	Λ	$\wp_2$	Λ	-	Λ	Λ	Λ	Λ
$\wp_3$	Λ	Λ	-	Λ	{2}	Λ	$\wp_3$	Λ	Λ	-	Λ	Λ	Λ
$\wp_4$	Λ	Λ	Λ	-	Λ	Λ	$\wp_4$	Λ	Λ	Λ	-	Λ	Λ
$\wp_5$	Λ	Λ	Λ	{1}	-	Λ	$\wp_5$	Λ	Λ	Λ	Λ	-	Λ
$\wp_6$	{1}	Λ	Λ	Λ	Λ	-	$\wp_6$	Λ	Λ	{3}	Λ	Λ	-

Table 15. Indifference matrix.

$\mathfrak{B}^{=}_{\psi}$	$\wp_1$	$\wp_2$	$\wp_3$	$\wp_4$	$\wp_5$	$\wp_6$
$\wp_1$	-	Λ	Λ	Λ	Λ	Λ
$\wp_2$	Λ	-	Λ	Λ	Λ	Λ
$\wp_3$	Λ	Λ	-	Λ	Λ	Λ
$\wp_4$	Λ	Λ	Λ	-	Λ	Λ
$\wp_5$	Λ	Λ	Λ	Λ	-	Λ
$\wp_6$	Λ	Λ	Λ	Λ	Λ	-

Table 16. Concordance membership grades matrix.

ψ	$\wp_1$	$\wp_2$	$\wp_3$	$\wp_4$	$\wp_5$	$\wp_6$
$\wp_1$	-	0	0.1607	0.0946	0.1143	0.2778
$\wp_2$	0	-	0.0762	0.1393	0.0571	0.0429
$\wp_3$	0	0.3333	-	0.2667	0.1589	0
$\wp_4$	0.1000	0	0.0286	-	0	0
$\wp_5$	0.1536	0.2000	0.3683	0.1083	-	0.0286
$\wp_6$	0.2250	0.0250	0.0921	0.0810	0.1143	-

Table 17. Disconcordance matrices.

$\mathfrak{B}_{\Delta_1}$	$\wp_1$	$\wp_2$	$\wp_3$	$\wp_4$	$\wp_5$	$\wp_6$	$\mathfrak{B}_{\Delta_2}$	$\wp_1$	$\wp_2$	$\wp_3$	$\wp_4$	$\wp_5$	$\wp_6$
$\wp_1$	-	Λ	Λ	Λ	Λ	Λ	$\wp_1$	-	Λ	Λ	Λ	$\{1\}$	Λ
$\wp_2$	$\{4\}$	-	Λ	Λ	Λ	Λ	$\wp_2$	Λ	-	Λ	Λ	Λ	Λ
$\wp_3$	$\{1,2,4\}$	$\{2\}$	-	$\{1\}$	Λ	$\{4\}$	$\wp_3$	$\{1\}$	$\{1\}$	-	Λ	$\{1\}$	$\{1\}$
$\wp_4$	$\{3\}$	$\{2,3,4\}$	Λ	-	Λ	$\{4\}$	$\wp_4$	Λ	Λ	$\{3\}$	-	Λ	$\{3\}$
$\wp_5$	$\{2,3,4\}$	$\{2,4\}$	$\{3,4\}$	$\{2\}$	-	$\{2,3,4\}$	$\wp_5$	Λ	Λ	$\{2\}$	Λ	-	Λ
$\wp_6$	$\{1,3\}$	$\{2\}$	Λ	Λ	$\{1\}$	-	$\wp_6$	$\{1\}$	Λ	Λ	Λ	Λ	-
$\mathfrak{B}_{\Delta_3}$	$\wp_1$	$\wp_2$	$\wp_3$	$\wp_4$	$\wp_5$	$\wp_6$	$\mathfrak{B}_{\Delta_4}$	$\wp_1$	$\wp_2$	$\wp_3$	$\wp_4$	$\wp_5$	$\wp_6$
$\wp_1$	-	Λ	Λ	Λ	Λ	Λ	$\wp_1$	-	Λ	Λ	Λ	Λ	Λ
$\wp_2$	Λ	-	Λ	Λ	Λ	Λ	$\wp_2$	$\{2,3\}$	-	$\{2\}$	Λ	Λ	$\{3\}$
$\wp_3$	$\{3\}$	Λ	-	Λ	Λ	$\{3\}$	$\wp_3$	Λ	Λ	-	Λ	Λ	Λ
$\wp_4$	Λ	Λ	Λ	-	Λ	Λ	$\wp_4$	Λ	Λ	Λ	-	$\{1\}$	Λ
$\wp_5$	Λ	Λ	Λ	Λ	-	Λ	$\wp_5$	Λ	Λ	$\{2\}$	Λ	-	Λ
$\wp_6$	$\{3\}$	Λ	Λ	Λ	Λ	-	$\wp_6$	Λ	Λ	Λ	Λ	Λ	-
$\mathfrak{B}_{\Delta_5}$	$\wp_1$	$\wp_2$	$\wp_3$	$\wp_4$	$\wp_5$	$\wp_6$	$\mathfrak{B}_{\Delta_6}$	$\wp_1$	$\wp_2$	$\wp_3$	$\wp_4$	$\wp_5$	$\wp_6$
$\wp_1$	-	Λ	Λ	Λ	Λ	Λ	$\wp_1$	-	$\{1\}$	Λ	Λ	Λ	$\{3\}$
$\wp_2$	Λ	-	Λ	Λ	Λ	$\{4\}$	$\wp_2$	$\{4\}$	-	Λ	Λ	Λ	Λ
$\wp_3$	$\{4\}$	Λ	-	Λ	Λ	$\{4\}$	$\wp_3$	Λ	Λ	-	Λ	Λ	Λ
$\wp_4$	Λ	Λ	Λ	-	Λ	Λ	$\wp_4$	Λ	Λ	Λ	-	$\{4\}$	Λ
$\wp_5$	Λ	Λ	Λ	Λ	-	Λ	$\wp_5$	Λ	Λ	Λ	Λ	-	Λ
$\wp_6$	Λ	Λ	Λ	Λ	Λ	-	$\wp_6$	$\{4\}$	Λ	Λ	Λ	Λ	-
$\mathfrak{B}_{\Delta_7}$	$\wp_1$	$\wp_2$	$\wp_3$	$\wp_4$	$\wp_5$	$\wp_6$	$\mathfrak{B}_{\Delta_8}$	$\wp_1$	$\wp_2$	$\wp_3$	$\wp_4$	$\wp_5$	$\wp_6$
$\wp_1$	-	Λ	Λ	Λ	Λ	Λ	$\wp_1$	-	Λ	Λ	Λ	Λ	$\{1,3\}$
$\wp_2$	Λ	-	Λ	Λ	Λ	Λ	$\wp_2$	Λ	-	$\{4\}$	Λ	$\{3\}$	Λ
$\wp_3$	Λ	Λ	-	Λ	$\{2\}$	Λ	$\wp_3$	Λ	Λ	-	Λ	Λ	Λ
$\wp_4$	Λ	Λ	Λ	-	Λ	Λ	$\wp_4$	Λ	Λ	Λ	-	$\{4\}$	Λ
$\wp_5$	Λ	Λ	Λ	Λ	-	Λ	$\wp_5$	Λ	$\{1\}$	Λ	Λ	-	Λ
$\wp_6$	Λ	Λ	Λ	Λ	Λ	-	$\wp_6$	Λ	Λ	Λ	Λ	Λ	-

Table 18. Disconcordance matrix.

Δ	$\wp_1$	$\wp_2$	$\wp_3$	$\wp_4$	$\wp_5$	$\wp_6$
$\wp_1$	-	0.1429	0.0000	0.0000	0.33333	0.3545
$\wp_2$	0.7645	-	0.2267	0.0000	0.1111	0.3005
$\wp_3$	1.0000	0.8087	-	0.5000	0.3778	0.8107
$\wp_4$	0.5000	0.7089	0.3333	-	0.3627	0.5662
$\wp_5$	1.0000	0.7047	1.0000	0.5000	-	0.7102
$\wp_6$	0.5195	0.5000	0.0000	0.0000	0.5000	-

Table 19. Weak relation.

ϑ^W	$\wp_1$	$\wp_2$	$\wp_3$	$\wp_4$	$\wp_5$	$\wp_6$
$\wp_1$	-	√	√	√	√	√
$\wp_2$	-	-	√	√	√	√
$\wp_3$	-	-	-	√	√	-
$\wp_4$	√	-	√	-	√	√
$\wp_5$	-	-	-	-	-	-
$\wp_6$	√	√	√	√	√	-

Table 20. Strong relation.

ϑ^W	$\wp_1$	$\wp_2$	$\wp_3$	$\wp_4$	$\wp_5$	$\wp_6$
$\wp_1$	-	√	√	√	√	√
$\wp_2$	-	-	√	√	√	√
$\wp_3$	-	-	-	√	√	-
$\wp_4$	√	-	√	-	√	-
$\wp_5$	-	-	-	√	-	-
$\wp_6$	√	√	√	√	√	-

Table 21. Ranking.

	$\wp_1$	$\wp_2$	$\wp_3$	$\wp_4$	$\wp_5$	$\wp_6$
$\varphi^{'}$	3	4	1	6	2	5
$\varphi^{''}$	2	1	3	6	4	5
φ	2	3	1	6	4	5

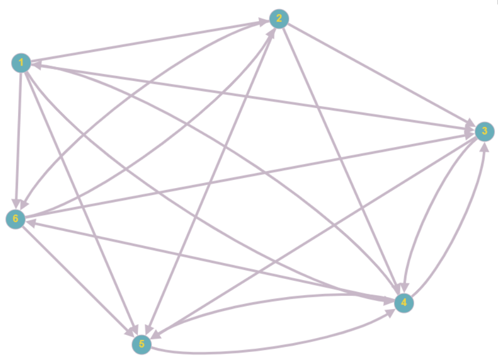

Figure 4. Strong relation based graph.

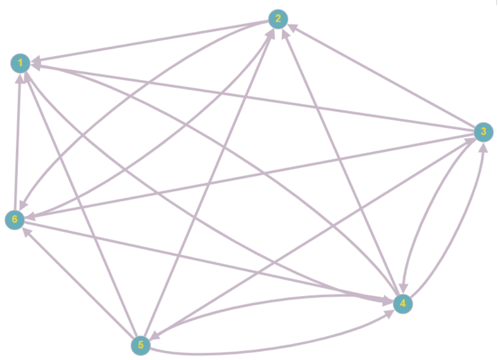

Figure 5. Strong relation based mirror graph.

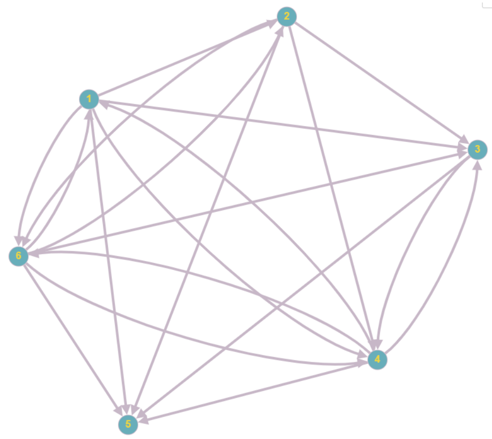

Figure 6. Weak relation based graph.

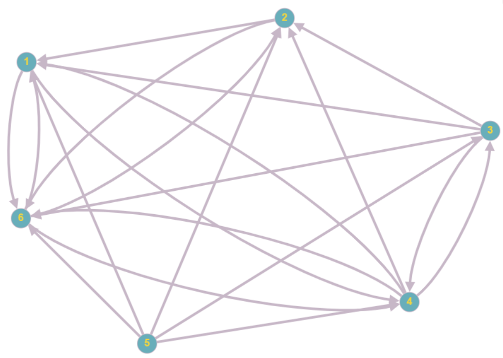

Figure 7. Weak relation based mirror graph.

5. Comparison analysis

In this study, we discovered a method of decision-making that combines the CBF model and the outranking ELECTRE-II methodology. A brief comparison of the CBF-ELECTRE-II with CBF-TOPSIS method and CBF-ELECTRE-I method is given in Table 22 and Figure 8 to demonstrate the advantages of the proposed PF-ELECTRE-II.

Table 22. Comparative analysis of CBF-VIKOR method and CBF-ELECTRE-II with other techniques.

Sr. No	Technique	Ranking
1	CBF-VIKOR (Proposed)	$\wp_3 \geq \wp_6 \geq \wp_2 \geq \wp_5 \geq \wp_4 \geq \wp_1$
2	CBF-ELECTRE-II (Proposed)	$\wp_3 \geq \wp_1 \geq \wp_2 \geq \wp_5 \geq \wp_6 \geq \wp_4$
3	CBF-ELECTRE [40]	$\wp_3 \geq \wp_2 \geq \wp_6 \geq \wp_5 \geq \wp_4 \geq \wp_1$
4	CBF-TOPSIS [40]	$\wp_3 \geq \wp_6 \geq \wp_5 \geq \wp_4 \geq \wp_1 \geq \wp_2$
5	CBF-SIR [41]	$\wp_3 \geq \wp_1 \geq \wp_2 \geq \wp_5 \geq \wp_6 \geq \wp_4$
6	CBF-LAM [41]	$\wp_3 \geq \wp_2 \geq \wp_5 \geq \wp_1 \geq \wp_4 \geq \wp_6$

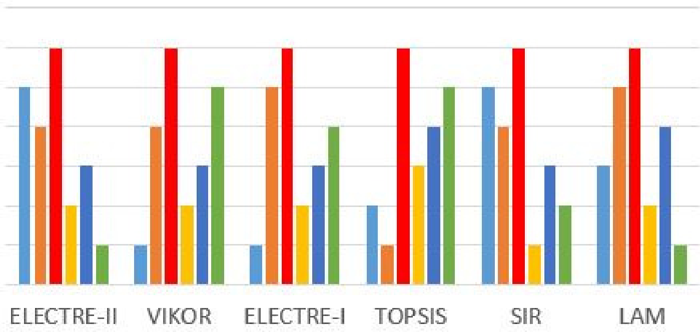

Figure 8. Comparative analysis of ranking by some MCGDM methods.

5.1. Insights and limitations of CBF-ELECTRE-II method

The following are the main conclusions and restrictions of the suggested CBF-ELECTRE-II model:

(1) A $CBFS$ is a robust model for handling bipolarity and fuzziness (NMGs). To specify the extent to which a given property does not belong, we use an NMG, which is defined by a negative interval and a negative number and a PMG, which is defined by a positive interval and a positive number, to indicate the degree to which a given property belongs (or satisfaction level of its counter property). In order to expand the decision space available to DM while comparing candidates against predetermined criteria, CBF-ELECTRE-II makes use of the CBF model.

(2) A method for supporting group decisions is CBF-ELECTRE-II, which adjusts the collective choice opinions to standard form using criteria related to cost and benefit-type standards, making it appropriate for all sorts of benefits of choice issues.

(3) The criteria weights and DMs are derived by the simplest method for collecting opinions.

(4) The ranking module of ELECTRE-II iteration processed of exploring outranking graphs is described in the finest and most understandable conceivable way, which enriches the viewing experience.

(5) To create a step-by-step knowledge for problem solving, a diagrammatic model of the group decision supporting system with two stages is offered.

(6) When there are a variety of choices to choose from in the challenge, it is not always straightforward to provide the proper three kinds of concordance threshold values and two kinds of discordance threshold values.

5.2. Advantages and dominance of the proposed Method

In the following section, we will examine the benefits and drawbacks of the proposed MCGDM methods. The proposed CBFS MCDGM are more accurate and reliable and cover drawbacks of existing methods.

(1) **Accuracy and supremacy:** For a variety of input data types, the provided MCGDM frameworks are appropriate and applicable. The techniques are capable of managing uncertainties and ambiguities as well as resolving defects in the input data and shows high accuracy comparatively

with other methods. As a hybrid structured set, the CBFSs can be used to collect information on a large scale under different criteria against each alternative.

(2) **Managing several criterion with efficiency:** The decision support system problems involve several criteria and input data dependent on specific situations. The CBFSs that have been suggested are straightforward and uncomplicated, allowing for their seamless application in any situation involving different alternatives and criteria.

(3) **Superiority and flexibility:** Our algorithms are characterized by their simplicity, flexibility and superiority over other hybrid fuzzy sets and operators. Their high flexibility allows administrators to conduct comparative analysis at multiple levels, resulting in more optimal solutions. As a result of this study, a systematic approach to selecting the best algorithm from a list of algorithms. Our proposed method is less sensitive to input and output data variations, making it a valuable tool for managers who must deal with high levels of uncertainty and vagueness when evaluating options.

6. Conclusions

The absence of ranking judgments for real-world problems is effectively solved by the ELECTRE-II approach, an expanded version of the original ELECTRE method. When it is difficult to identify a single decision maker with sufficient training to accurately understand the entire problem and its constraints, decisions made using information from a group of DMs generate more dependable results. We show how ELECTRE-II and CBFS can be used in tandem when making group decisions. The CBF-ELECTRE-II model is presented for CBF information to tackle challenges in daily life. In a CBF context, the fundamental architecture of the ELECTRE-II model is defined, along with a detailed step-by-step process that has two fundamental steps: (i) group opinion aggregation and (ii) ranking mechanism. To verify the efficacy of this method, it is utilized to the issue of RFT problem in Industry 4.0. An existing method is then briefly juxtaposed with the choice outcomes. Our method is intended for application in any group decision-making environment, including but not limited to industrial engineering, the health sciences, corporate management and similar fields. This key contributions of this study are listed below.

(1) Modeling uncertain and sensitive information with cubic bipolar fuzzy hybrid model.

(2) It describes benefits of utilizing CBFSs.

(3) This investigated several significant concepts related to CBFSs.

(4) An application of RFT in Industry 4.0 is presented that leads to new MCGDM methods to seek reasonable decisions in a timely manner.

(5) Such application enables organizations to quickly access required information and respond to inquiries or concerns promptly.

(6) It also helps organizations to efficiently manage large amounts of data and information.

Conflict of interest

The authors declare that they have no conflicts of interest.

Acknowledgments

This work was supported by the Deanship of Scientific Research, Vice Presidency for Graduate Studies and Scientific Research, King Faisal University, Saudi Arabia [Grant No.GRANT3873].

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