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Research article

New results on finite-/fixed-time synchronization of delayed memristive neural networks with diffusion effects

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Abstract: In this paper, we further investigate the finite-/fixed-time synchronization (FFTS) problem for a class of delayed memristive reaction-diffusion neural networks (MRDNNs). By utilizing the state-feedback control techniques, and constructing a general Lyapunov functional, with the help of inequality techniques and the finite-time stability theory, novel criteria are established to realize the FFTS of the considered delayed MRDNNs, which generalize and complement previously known results. Finally, a numerical example is provided to support the obtained theoretical results.

Keywords: memristive neural network; finite-/fixed-time synchronization; diffusion effect
Mathematics Subject Classification: 34K39, 93D05

1. Introduction

In 1971, Chua [1] firstly proposed the definition of memristor, and he was sure that it is the fourth fundamental circuit element except for other three basic circuit elements: Capacitor, inductor and resistor. Until 2008, Hewlett-Packard Lab declared that they realized the physical prototype of the memristor and published their experimental findings on Nature [2]. In consideration of its brilliant superiorities of nanoscale dimension, compatible with logical operation and information storage, and low power consumption, scientific researchers usually replace conventional resistor with memristor in biological or artificial neural networks, this is the so-called memristive neural networks (MNNs). Since MNNs can be seen as a class of state dependent discontinuous dynamical system, and they can show several distinctive dynamics characteristics. Therefore, the rich and complex dynamic behaviors of MNNs have received phenomenal worldwide attention in the past years, see [3–9] and the references therein.

Diffusion effects may have influence on dynamical behaviors of MNNs. As shown in [10–12], the diffusion terms directly have influence on the existence and stability of equilibrium or (anti-)periodic solution of (high-order) Hopfield neural network models. In [13, 14], the dissipativity and passivity analysis of coupled reaction-diffusion neural networks are rigorously analyzed. Recently, many excellent synchronization results of reaction-diffusion neural networks are extensively studied since their wide applications are found in many fields including such as artificial intelligence, face recognition, emotion analysis and image identification, see for example, adaptive synchronization [15], exponential synchronization [16], pinning synchronization [17], general decay lag anti-synchronization [18], and so on.

It is well known that the aforementioned synchronization modes are usually realized in an infinite time interval. However, in some practical applications such as robotics, a standard problem in system theory is conceiving controllers to drive a system to a given positions as quickly as possible. In view of this situation, the finite-time control technique is a powerful and useful tool to solve various engineering problems, and it has demonstrated better robustness and disturbance rejection properties [19, 20]. Although the results of many studies indicated that the finite-time synchronization have the optimal convergence time, the halting (or settling) time of finite-time synchronization is closely related to the initial value. Actually, the initial information of many practical systems are unknown and difficult to measure, which makes us hard to estimate the halting time. In order to overcome this shortcoming, Polyakov [21] initially proposed the concept of fixed-time stability, which requires systems are global finite-time stable, and the halting time is independent of initial values and can be bounded by an upper bound. Recently, the FFTS problem of neural networks are extensively investigated under a unified framework, see [22–27] and the references therein. In particular, the FFTS problem for a class of delayed MRDNNs were established based on the constructed Lyapunov functional in terms of L^2-norm [28], however, such a type of Lyapunov functional is relatively conservative.

Inspired by the aforementioned discussions, the main purpose of this paper is to extend the obtained results in [28] to a more general case. Specifically, we further study the FFTS for a class of delayed MRDNNs by constructing a Lyapunov functional in terms of L^p-norm ($p \geq 2$), with the help of theories of finite-time stability, Green's formula, and inequality techniques, novel criteria are established to ensure the finite-/fixed-time stability of error neural networks, and then FFTS of delayed MRDNNs can be realized under the drive-response framework.

The paper is outlined as follows. In Section 2, model description and necessary preliminaries are presented. In Section 3, the FFTS of the addressed network system is studied. In Section 4, numerical simulations are performed to validate the applicability of the theoretical results. Finally, the conclusions are drawn in Section 5.

2. Model description and preliminaries

As shown in [28], we consider the following delayed MRDNNs:

$$\frac{\partial u_j(t,z)}{\partial t} = \sum_{l=1}^{m} D_{jl} \frac{\partial^2 u_j(t,z)}{\partial z_l^2} - c_j u_j(t,z) + \sum_{k=1}^{n} a_{jk}(u) f_k(u_k(t,z)) + \sum_{k=1}^{n} b_{jk}(u) g_k(u_k(t-\tau,z)) + I_j, \ j = 1, 2, ..., n, \tag{2.1}$$

where n is the number of neurons, Ω is a bounded domain with smooth boundary $\partial\Omega$ in $\mathbb{R}^m$ and the space vector $z = (z_1, z_2, \ldots, z_m)^{\mathrm{T}} \in \Omega$, $u_j(t,z)$ corresponds to the state of the jth neuron at time t and in space $z \in \Omega$, $D_{jl} \geq 0$ is the transmission diffusion coefficient along the jth neuron, $c_j > 0$ represents the neural self-inhibition, I_j is the external input or bias, $\tau > 0$ is the transmission delay among neurons, $f_k(u_k(t,z))$ and $g_k(u_k(t-\tau,z))$ correspond to neuronal activation functions of k-th neuron without and with time delay, respectively; The feedback connection weight $a_{jk}(u)$ is the function of $f_{jk}(t,z) \triangleq f_k(u_k(t,z)) - u_j(t,z)$ and the delayed feedback connection weight $b_{jk}(u)$ is the function of $g_{jk}(t,z) \triangleq g_k(u_k(t-\tau,z)) - u_j(t,z)$. They are defined as

$$a_{jk}(u) = \begin{cases} \hat{a}_{jk}, & D^- f_{jk}(t,z) > 0, \\ \check{a}_{jk}, & D^- f_{jk}(t,z) < 0, \\ a_{jk}(t^-, z), & D^- f_{jk}(t,z) = 0, \end{cases}$$

and

$$b_{jk}(u) = \begin{cases} \hat{b}_{jk}, & D^- g_{jk}(t,z) > 0, \\ \check{b}_{jk}, & D^- g_{jk}(t,z) < 0, \\ b_{jk}(t^-, z), & D^- g_{jk}(t,z) = 0, \end{cases}$$

where $D^- f_{jk}(t,z)$ and $D^- g_{jk}(t,z)$ respectively denote the upper left Dini-derivation of $f_{jk}(t,z)$ and $g_{jk}(t,z)$ with regard to t, $\hat{a}_{jk}, \check{a}_{jk}, \hat{b}_{jk}, \check{b}_{jk}$ are known constants, $a_{jk}(t^-,z), b_{jk}(t^-,z)$ are the left limit of a_{jk} and b_{jk} with regard to t. We refer to [7, 16] for more information about the characterization of memristive neural networks.

The Dirichlet boundary condition and initial value of system (2.1) are given by

$$\begin{aligned} u_j(t,z) &= 0, \quad (t,z) \in [-\tau, \infty) \times \partial\Omega, \\ u_j(s,z) &= \varphi_j(s,z), \quad (s,z) \in [-\tau, 0] \times \Omega, \end{aligned} \tag{2.2}$$

where $\varphi_j(s,z) \in C([-\tau,0] \times \Omega, \mathbb{R})$.

According to the drive-response framework proposed by Pecora and Carroll in [29], we regard MRDNNs (2.1) as the drive system, and the corresponding response system for (2.1) can be described by the following form:

$$\begin{aligned} \frac{\partial \widetilde{u}_j(t,z)}{\partial t} = &\sum_{l=1}^{m} D_{jl} \frac{\partial^2 \widetilde{u}_j(t,z)}{\partial z_l^2} - c_j \widetilde{u}_j(t,z) + \sum_{k=1}^{n} a_{jk}(\widetilde{u}) f_k(\widetilde{u}_k(t,z)) \\ &+ \sum_{k=1}^{n} b_{jk}(\widetilde{u}) g_k(\widetilde{u}_k(t-\tau,z)) + I_j + W_j(t,z), \ j = 1,2,...,n, \end{aligned} \tag{2.3}$$

where $W_j(t,z)$ is the appropriate control input that will be designed in order to obtain a certain control objective.

As to system (2.3), the Dirichlet boundary and initial value conditions are shown as follows:

$$\begin{aligned} \widetilde{u}_j(t,z) &= 0, \quad (t,z) \in [-\tau, \infty) \times \partial\Omega, \\ \widetilde{u}_j(s,z) &= \psi_j(s,z), \quad (s,z) \in [-\tau, 0] \times \Omega, \end{aligned} \tag{2.4}$$

in which $\psi_j(s,z) \in C([-\tau,0] \times \Omega, \mathbb{R})$, $j = 1, 2, \ldots, n$.

Let us define the synchronization error function as $e_j(t,z) = \widetilde{u}_j(t,z) - u_j(t,z)$, and then subtracting (2.3) from (2.1) yields the error system:

$$\begin{aligned}\frac{\partial e_j(t,z)}{\partial t} &= \sum_{l=1}^{m} D_{jl}\frac{\partial^2 e_j(t,z)}{\partial z_l^2} - c_j e_j(t,z) + \sum_{k=1}^{n}\Big[a_{jk}(\widetilde{u})f_k(\widetilde{u}_k(t,z)) - a_{jk}(u)f_k(u_k(t,z))\Big]\\ &\quad + \sum_{k=1}^{n}\Big[b_{jk}(\widetilde{u})g_k(\widetilde{u}_k(t-\tau,z)) - b_{jk}(u)g_k(u_k(t-\tau,z))\Big] + W_j(t,z)\\ &= \sum_{l=1}^{m} D_{jl}\frac{\partial^2 e_j(t,z)}{\partial z_l^2} - c_j e_j(t,z) + \sum_{k=1}^{n} a_{jk}(\widetilde{u})\Big[f_k(\widetilde{u}_k(t,z)) - f_k(u_k(t,z))\Big]\\ &\quad + \sum_{k=1}^{n}[a_{jk}(\widetilde{u}) - a_{jk}(u)]f_k(u_k(t,z)) + \sum_{k=1}^{n}[b_{jk}(\widetilde{u}) - b_{jk}(u)]g_k(u_k(t-\tau,z))\\ &\quad + \sum_{k=1}^{n} b_{jk}(\widetilde{u})\Big[g_k(\widetilde{u}_k(t-\tau,z)) - g_k(u_k(t-\tau,z))\Big] + W_j(t,z),\ \ j = 1,2,...,n. \end{aligned} \tag{2.5}$$

The Dirichlet boundary condition and initial value associated with error system (2.5) is

$$\begin{aligned} e_j(t,z) &= 0, \qquad (t,z) \in [-\tau,\infty)\times\partial\Omega,\\ e_j(s,z) &= \psi_j(s,z) - \varphi_j(s,z), \quad (s,z)\in[-\tau,0]\times\Omega. \end{aligned} \tag{2.6}$$

It is evident to see that the finite-/fixed-time synchronization problem of drive-response systems (2.1) and (2.3) is equivalent to the finite-/fixed-time stability problem of error system (2.5). In order to achieve such a synchronization goal, the following preliminaries including assumptions, definitions and lemmas are needed.

Assumption 2.1. *The activation functions f_k and g_k are globally Lipschitz continuous and bounded, that is, there exist positive constants L_k^f, L_k^g, F_k and G_k such that*

$$|f_k(x) - f_k(y)| \le L_k^f|x-y|,$$

$$|g_k(x) - g_k(y)| \le L_k^g|x-y|,$$

and

$$|f_k(x)| \le F_k,\ |g_k(x)| \le G_k$$

hold for any $x, y \in \mathbb{R}$.

Definition 2.1 (see [25])**.** *Drive system* (2.1) *and response system* (2.3) *are said to achieve finite-time (or fixed-time) synchronization, if there exists a halting time $\mathbb{T}$ depending (or independent) on the initial values,*

$$\lim_{t\to\mathbb{T}} \| e(t,\cdot) \| = 0$$

and

$$\| e(t,\cdot) \| = 0,\ for \quad t \ge \mathbb{T},$$

where $\|\cdot\|$ denotes some norm.

Lemma 2.2 (see [24, 26]). *Suppose that* $V(x(t)) : \mathbb{R}^n \to \mathbb{R}$ *is C-regular, and that* $x(t) : [0, \infty) \to \mathbb{R}^n$ *is absolutely continuous on any compact interval of* $[0, \infty)$. *Let* $\mathscr{K}(V) = k_1V^\alpha + k_2V^\beta$, *where* $k_1, k_2 > 0$, *If* $D^+V(x) \leq -\mathscr{K}(V)$ *and* $\mathbb{T} = \int_0^{V(x(0))} \frac{1}{\mathscr{K}(\sigma)} d\sigma < \infty$, *then,*

(1) if $0 \leq \alpha, \beta < 1$, $V(x(t))$ *will reach zero in a finite time and the halting time is bounded by*

$$\mathbb{T} \leq \min\Big\{\frac{V^{1-\alpha}(x(0))}{k_1(1-\alpha)}, \frac{V^{1-\beta}(x(0))}{k_2(1-\beta)}\Big\};$$

(2) if $\alpha > 1, 0 \leq \beta < 1$, $V(x(t))$ *will reach zero in a fixed time and the halting time is bounded by*

$$\mathbb{T} \leq \frac{1}{k_1}\frac{1}{\alpha-1} + \frac{1}{k_2}\frac{1}{1-\beta}.$$

Lemma 2.3 (see [26, 30]). *Suppose*

$$\Omega = \{z = (z_1, z_2, \cdots, z_m)^{\mathrm{T}} | h_l^m \leq z_l \leq h_l^M, l = 1, 2, \cdots, m\}$$

is a bounded compact set with smooth boundary $\partial\Omega$ *and mes* $\Omega > 0, \varphi(z) \in C^1(\Omega)$ *is a real-valued function satisfying* $\varphi(z)|_{\partial\Omega} = 0$, *then*

$$\int_\Omega \varphi^2(z)dz \leq \Big(\frac{h_l^M - h_l^m}{\pi}\Big)^2 \int_\Omega \Big(\frac{\partial\varphi}{\partial z_l}\Big)^2 dz.$$

Lemma 2.4 (see [26]). *If* $a_1, a_2, \cdots, a_n$ *are positive numbers, then*

$$na_1a_2\cdots a_n \leq a_1^n + a_2^n + \cdots + a_n^n.$$

3. Main results

In this section, new criteria will be established to realize the finite-/fixed-time synchronization between the drive MRDNNs (2.1) and response MRDNNs (2.3) by constructing a novel Lyapunov functional. To present the main results of this section, we first design the following controller:

$$\begin{aligned} W_j(t,z) = &- P_{j1}e_j(t,z) - P_{j2}sign(e_j(t,z)) - P_{j3}(\parallel e(t,\cdot)\parallel^{1-p} + \parallel e(t,\cdot)\parallel^{p(\alpha-1)})e_j(t,z) \\ &- \sum_{k=1}^n P_{j4}sign(e_j(t,z))\big|e_k(t-\tau,z)\big|, \ j = 1, 2, ..., n, \end{aligned} \tag{3.1}$$

where

$$\parallel e(t,\cdot) \parallel = \Big(\int_\Omega \sum_{j=1}^n |e_j(t,z)|^p dz\Big)^{\frac{1}{p}} \text{ and } p \geq 2,$$

the feedback gain parameters $P_{j1}, P_{j2}, P_{j3}, P_{j4}$ are positive constants determined later, and $\alpha \geq 0$. For convenience, we denote

$$\bar{a}_{jk} = \max\{|\hat{a}_{jk}|, |\check{a}_{jk}|\}, \bar{b}_{jk} = \max\{|\hat{b}_{jk}|, |\check{b}_{jk}|\},$$

$$\acute{a}_{jk} = \max\{\hat{a}_{jk}, \check{a}_{jk}\}, \grave{a}_{jk} = \min\{\hat{a}_{jk}, \check{a}_{jk}\},$$

$$\acute{b}_{jk} = \max\{\hat{b}_{jk}, \check{b}_{jk}\}, \grave{b}_{jk} = \min\{\hat{b}_{jk}, \check{b}_{jk}\}.$$

Remark 3.1. *It is noteworthy that the designed negative exponential controller* (3.1) *is reasonable (see [28]), which is essentially different from those without diffusive effects. On the other hand, the controller* (3.1) *with one exponential parameter α is easier to adjust than the controller with multiple exponential parameters in practical applications.*

We are now in a position to state the main theoretical result as follows.

Theorem 3.1. *If Assumption 2.1 hold, and the control parameters satisfy*

$$\begin{aligned} P_{j1} &\geq -\sum_{l=1}^{m} \frac{D_{jl}\pi^2}{(h_l^M - h_l^m)^2} - c_j + \overline{a}_{jj} L_j^f + \mathscr{A}_j, \\ P_{j2} &\geq \sum_{k=1}^{n} (|\acute{a}_{jk} - \grave{a}_{jk}| F_k + |\acute{b}_{jk} - \grave{b}_{jk}| G_k), \\ P_{j4} &\geq \sum_{k=1}^{n} \overline{b}_{jk} L_k^g, \end{aligned} \tag{3.2}$$

where

$$\mathscr{A}_j = \frac{1}{p}\Big[\sum_{\substack{k=1\\k\neq j}}^{n} \sum_{m=1}^{p-1} (\overline{a}_{jk})^{p\xi_m} (L_k^f)^{p\sigma_m} + \sum_{\substack{k=1\\k\neq j}}^{n} (\overline{a}_{kj})^{p\xi_p} (L_j^f)^{p\sigma_p}\Big],$$

$$\sum_{m=1}^{p} \xi_m = \sum_{m=1}^{p} \sigma_m = 1.$$

Then,

(I) the drive MRDNNs (2.1) *and response MRDNNs* (2.3) *are finite-timely synchronous when* $0 \leq \alpha < 1$*;*

(II) the drive MRDNNs (2.1) *and response MRDNNs* (2.3) *are fixed-timely synchronous when* $\alpha > 1$.

Proof. Consider the following Lyapunov functional:

$$V(t) = \int_{\Omega} \sum_{j=1}^{n} |e_j(t,z)|^p dz, \quad p \geq 2.$$

Calculating the upper right Dini-derivative of $V(t)$ along the trajectories of system (2.5), we obtain

$$\begin{aligned} D^+V(t) = \int_{\Omega} \sum_{j=1}^{n} p|e_j(t,z)|^{p-2} e_j(t,z) \Big\{ &\sum_{l=1}^{m} D_{jl} \frac{\partial^2 e_j(t,z)}{\partial z_l^2} - c_j e_j(t,z) \\ &+ \sum_{k=1}^{n} a_{jk}(\overline{u}) \Big[f_k(\overline{u}_k(t,z)) - f_k(u_k(t,z)) \Big] + \sum_{k=1}^{n} [a_{jk}(\overline{u}) - a_{jk}(u)] f_k(u_k(t,z)) \\ &+ \sum_{k=1}^{n} [b_{jk}(\overline{u}) - b_{jk}(u)] g_k(u_k(t-\tau,z)) + \sum_{k=1}^{n} b_{jk}(\overline{u}) \Big[g_k(\overline{u}_k(t-\tau,z)) - g_k(u_k(t-\tau,z)) \Big] \\ &- P_{j1} e_j(t,z) - P_{j2} sign(e_j(t,z)) - P_{j3} (\|e(t,\cdot)\|^{1-p} + \|e(t,\cdot)\|^{p(\alpha-1)}) e_j(t,z) \end{aligned}$$

$$
\begin{aligned}
&- \sum_{k=1}^{n} P_{j4} sign(e_j(t,z))|e_k(t-\tau,z)|\Big\}dz \\
\leq& \int_\Omega \sum_{j=1}^{n} p|e_j(t,z)|^{p-2} e_j(t,z) \sum_{l=1}^{m} D_{jl} \frac{\partial^2 e_j(t,z)}{\partial z_l^2} dz - \int_\Omega \sum_{j=1}^{n} pc_j |e_j(t,z)|^p dz \\
&+ \int_\Omega \sum_{j=1}^{n}\sum_{k=1}^{n} p\overline{a}_{jk}|e_j(t,z)|^{p-1}\Big|f_k(\widetilde{u}_k(t,z)) - f_k(u_k(t,z))\Big|dz \\
&+ \int_\Omega \sum_{j=1}^{n}\sum_{k=1}^{n} p|\acute{a}_{jk} - \grave{a}_{jk}||e_j(t,z)|^{p-1}|\Big|f_k(u_k(t,z))\Big|dz \\
&+ \int_\Omega \sum_{j=1}^{n}\sum_{k=1}^{n} p|\acute{b}_{jk} - \grave{b}_{jk}||e_j(t,z)|^{p-1}|\Big|g_k(u_k(t-\tau,z))\Big|dz \\
&+ \int_\Omega \sum_{j=1}^{n}\sum_{k=1}^{n} p\overline{b}_{jk}|e_j(t,z)|^{p-1}\Big|g_k(\widetilde{u}_k(t-\tau,z)) - g_k(u_k(t-\tau,z))\Big|dz \\
&- \int_\Omega \sum_{j=1}^{n} pP_{j1}|e_j(t,z)|^p dz - \int_\Omega \sum_{j=1}^{n} pP_{j2}|e_j(t,z)|^{p-1}dz \\
&- \int_\Omega \sum_{j=1}^{n}\sum_{k=1}^{n} pP_{j4}|e_j(t,z)|^{p-1}|e_k(t-\tau,z)|dz \\
&- \min_{1\leq j\leq n}\{pP_{j3}\}(\|e(t,\cdot)\| + \|e(t,\cdot)\|^{p\alpha}).
\end{aligned}
\tag{3.3}
$$

First, integrating by parts, and in view of Lemma 2.3, we have

$$
\sum_{j=1}^{n} \int_\Omega e_j(t,z)\Big(\sum_{l=1}^{m} D_{jl} \frac{\partial^2 e_j(t,z)}{\partial z_l^2}\Big)dz \leq -\sum_{j=1}^{n}\sum_{l=1}^{m} \frac{D_{jl}\pi^2}{(h_l^M - h_l^m)^2} \int_\Omega e_j^2(t,z)dz. \tag{3.4}
$$

Moreover, it follows from Assumption 2.1 and Lemma 2.4 that

$$
\begin{aligned}
&\sum_{j=1}^{n}\sum_{k=1}^{n} p\overline{a}_{jk}|e_j(t,z)|^{p-1}\Big|f_k(\widetilde{u}_k(t,z)) - f_k(u_k(t,z))\Big| \\
\leq& \sum_{j=1}^{n}\sum_{k=1}^{n} p\overline{a}_{jk}|e_j(t,z)|^{p-1} L_k^f |e_k(t,z)| \\
=& \sum_{j=1}^{n} p\overline{a}_{jj} L_j^f |e_j(t,z)|^p + \sum_{j=1}^{n}\sum_{\substack{k=1\\k\neq j}}^{n} p\overline{a}_{jk} L_k^f |e_j(t,z)|^{p-1}|e_k(t,z)| \\
=& \sum_{j=1}^{n} p\overline{a}_{jj} L_j^f |e_j(t,z)|^p + \sum_{j=1}^{n}\sum_{\substack{k=1\\k\neq j}}^{n} p\Big(\prod_{m=1}^{p-1} (\overline{a}_{jk})^{\xi_m}(L_k^f)^{\sigma_m}|e_j(t,z)|\Big)(\overline{a}_{jk})^{\xi_p}(L_k^f)^{\sigma_p}|e_k(t,z)| \\
\leq& \sum_{j=1}^{n} p\overline{a}_{jj} L_j^f |e_j(t,z)|^p + \sum_{j=1}^{n}\sum_{\substack{k=1\\k\neq j}}^{n} \Big(\sum_{m=1}^{p-1} (\overline{a}_{jk})^{p\xi_m}(L_k^f)^{p\sigma_m}|e_j(t,z)|^p + (\overline{a}_{jk})^{p\xi_p}(L_k^f)^{p\sigma_p}|e_k(t,z)|^p\Big)
\end{aligned}
$$

$$= \sum_{j=1}^{n} p\overline{a}_{jj}L_j^f|e_j(t,z)|^p + \sum_{j=1}^{n}\Big(\sum_{\substack{k=1\\k\neq j}}^{n}\sum_{m=1}^{p-1}(\overline{a}_{jk})^{p\xi_m}(L_k^f)^{p\sigma_m} + \sum_{\substack{k=1\\k\neq j}}^{n}(\overline{a}_{kj})^{p\xi_p}(L_j^f)^{p\sigma_p}\Big)|e_j(t,z)|^p$$

$$= \sum_{j=1}^{n} p(\overline{a}_{jj}L_j^f + \mathscr{A}_j)|e_j(t,z)|^p, \tag{3.5}$$

where

$$\sum_{m=1}^{p}\xi_m = \sum_{m=1}^{p}\sigma_m = 1,$$

$$\sum_{\substack{k=1\\k\neq j}}^{n}\sum_{m=1}^{p-1}(\overline{a}_{jk})^{p\xi_m}(L_k^f)^{p\sigma_m} + \sum_{\substack{k=1\\k\neq j}}^{n}(\overline{a}_{kj})^{p\xi_p}(L_j^f)^{p\sigma_p} = p\mathscr{A}_j.$$

On the other hand, by Assumption 2.1, we have

$$\sum_{j=1}^{n}\sum_{k=1}^{n} p|\acute{a}_{jk} - \grave{a}_{jk}||e_j(t,z)|^{p-1}\Big|f_k(u_k(t,z))\Big| \le \sum_{j=1}^{n}\sum_{k=1}^{n} p|\acute{a}_{jk} - \grave{a}_{jk}|F_k|e_j(t,z)|^{p-1}, \tag{3.6}$$

$$\sum_{j=1}^{n}\sum_{k=1}^{n} p|\acute{b}_{jk} - \grave{b}_{jk}||e_j(t,z)|^{p-1}\Big|g_k(u_k(t-\tau,z))\Big| \le \sum_{j=1}^{n}\sum_{k=1}^{n} p|\acute{b}_{jk} - \grave{b}_{jk}|G_k|e_j(t,z)|^{p-1}, \tag{3.7}$$

and

$$\sum_{j=1}^{n}\sum_{k=1}^{n} p\overline{b}_{jk}|e_j(t,z)|^{p-1}\Big|g_k(\widetilde{u}_k(t-\tau,z)) - g_k(u_k(t-\tau,z))\Big|$$
$$\le \sum_{j=1}^{n}\sum_{k=1}^{n} p\overline{b}_{jk}|e_j(t,z)|^{p-1}L_k^g|e_k(t-\tau,z)|. \tag{3.8}$$

Substituting (3.4)–(3.8) into (3.3) and combining with (3.2) produces

$$\begin{aligned} D^+V(t) \le &\int_\Omega \sum_{j=1}^{n} p\Big[-\sum_{l=1}^{m}\frac{D_{jl}\pi^2}{(h_l^M - h_l^m)^2} - c_j + \overline{a}_{jj}L_j^f + \mathscr{A}_j - P_{j1}\Big]|e_j(t,z)|^p dz \\ &+ \int_\Omega \sum_{j=1}^{n} p\Big[\sum_{k=1}^{n}\big(|\acute{a}_{jk} - \grave{a}_{jk}|F_k + |\acute{b}_{jk} - \grave{b}_{jk}|G_k\big) - P_{j2}\Big]|e_j(t,z)|^{p-1}dz \\ &+ \int_\Omega \sum_{j=1}^{n} p\Big(\sum_{k=1}^{n}\overline{b}_{jk}L_k^g - P_{j4}\Big)|e_j(t,z)|^{p-1}|e_k(t-\tau,z)|dz \\ &- \min_{1\le j\le n}\{pP_{j3}\}\big(\|e(t,\cdot)\| + \|e(t,\cdot)\|^{p\alpha}\big) \\ \le &- \min_{1\le j\le n}\{pP_{j3}\}\big(V^{\frac{1}{p}}(t) + V^{\alpha}(t)\big). \end{aligned} \tag{3.9}$$

Case I. If $0 \leq \alpha < 1$, one notices from the Lemma 2.2 that the finite-time synchronization of delayed MRDNNs (2.1) and (2.3) is achieved under the designed controller (3.1), and the halting time can be bounded by

$$\begin{aligned}
\mathbb{T}_1 &\leq \int_0^{V(0)} \frac{1}{\min\limits_{1\leq j\leq n}\{pP_{j3}\}V^{\frac{1}{p}} + \min\limits_{1\leq j\leq n}\{pP_{j3}\}V^{\alpha}} dV \\
&\leq \min\left\{ \int_0^{V(0)} \frac{1}{\min\limits_{1\leq j\leq n}\{pP_{j3}\}V^{\frac{1}{p}}} dV, \int_0^{V(0)} \frac{1}{\min\limits_{1\leq j\leq n}\{pP_{j3}\}V^{\alpha}} dV \right\} \\
&= \min\left\{ \frac{pV^{1-\frac{1}{p}}(0)}{p-1}, \frac{V^{1-\alpha}(0)}{1-\alpha} \right\} \frac{1}{\min\limits_{1\leq j\leq n}\{pP_{j3}\}}.
\end{aligned} \tag{3.10}$$

Case II. If $\alpha > 1$, due to Lemma 2.2, the fixed-time synchronization of delayed MRDNNs (2.1) and (2.3) is guaranteed under the controller (3.1). Moreover, the halting time is estimated by

$$\mathbb{T}_2 \leq \frac{p\alpha - 1}{\min\limits_{1\leq j\leq n}\{pP_{j3}\}(\alpha-1)(p-1)}.$$

The proof is complete. □

Remark 3.2. *In [28], the authors only studied the MRDNNs* (2.1)*, and considered the Lyapunov functional in terms of L^2-norm. However, in this paper, by virtue of inequality techniques, we construct a more general Lyapunov functional. So the results given in this paper are less conservative and extend the previous works.*

4. Numerical simulation

In this section, we shall use numerical simulations to verify the practicability of the theoretical results.

Example 4.1. Let $\Omega = [-3, 3], n = 2, m = 1$, and consider the delayed drive-response memristive NNs (2.1)–(2.3) with two neurons, choose the system parameters: $D_1 = 1.2, D_2 = 1.4, c_1 = 0.7, c_2 = 0.8, I_1 = 0.8, I_2 = 1.2, \tau = 0.6, f_k(\cdot) = \sin(\cdot)$ and $g_k(\cdot) = \tanh(\cdot), k = 1, 2$. It is readily seen that the activation functions f_k, g_k satisfy Assumption (2.1) with $L_k^f = L_k^g = 1$, and $F_k = G_k = 1, k = 1, 2$, the memristive connection weights are taken as follows:

$$a_{jk}(u) = \begin{cases} \hat{a}_{jk}, & D^- f_{jk}(t,z) > 0, \\ \check{a}_{jk}, & D^- f_{jk}(t,z) < 0, \\ a_{jk}(t^-, z), & D^- f_{jk}(t,z) = 0, \end{cases}$$

and

$$b_{jk}(u) = \begin{cases} \hat{b}_{jk}, & D^- g_{jk}(t,z) > 0, \\ \check{b}_{jk}, & D^- g_{jk}(t,z) < 0, \\ b_{jk}(t^-, z), & D^- g_{jk}(t,z) = 0, \end{cases}$$

where

$$\overline{A} = (\hat{a}_{jk})_{2\times 2} = \begin{pmatrix} 3.6 & 2.8 \\ 4.5 & 3.9 \end{pmatrix}, \quad \underline{A} = (\check{a}_{jk})_{2\times 2} = \begin{pmatrix} 2.2 & -1.8 \\ -1.9 & 3.7 \end{pmatrix},$$

$$\overline{B} = (\hat{b}_{jk})_{2\times 2} = \begin{pmatrix} 1.7 & 2.9 \\ 1.8 & 2.8 \end{pmatrix}, \quad \underline{B} = (\check{b}_{jk})_{2\times 2} = \begin{pmatrix} -1.2 & 2.7 \\ -0.2 & -0.6 \end{pmatrix},$$

the boundary-initial value conditions are given as

$$u_j(t,-3) = u_j(t,3) = 0,\ \widetilde{u}_j(t,-3) = \widetilde{u}_j(t,3) = 0,$$

$$u_1 = 2.4\sin(\pi z),\ u_2 = -4.2\cos(\frac{\pi z}{2}),\ \widetilde{u}_1 = 3.2\sin(\pi z),\ \widetilde{u}_2 = 1.5\sin(\pi z),$$

for $(t,z) \in [-0.6, 0] \times [-3, 3]$. The controller parameters in (3.1) are chosen as

$$P_{11} = 8.2,\ P_{12} = 10.1,\ P_{21} = 7.2,\ P_{22} = 16.5,\ P_{13} = 0.5,\ P_{23} = 0.25,\ P_{14} = 11.3,\ P_{24} = 10.5$$

and

$$p = 2,\ \xi_1 = 0.4,\ \xi_2 = 0.6,\ \sigma_1 = \sigma_2 = 0.5,\ \alpha = 2.$$

Notice that

$$\mathscr{A}_1 = \frac{1}{p}\Big[(\overline{a}_{12})^{p\xi_1}(L_2^f)^{p\sigma_1} + (\overline{a}_{21})^{p\xi_2}(L_1^f)^{p\sigma_2}\Big] \approx 4.1791,$$

$$\mathscr{A}_2 = \frac{1}{p}\Big[(\overline{a}_{21})^{p\xi_1}(L_1^f)^{p\sigma_1} + (\overline{a}_{12})^{p\xi_2}(L_2^f)^{p\sigma_2}\Big] \approx 3.3856,$$

$$P_{11} = 8.2 > 6.7504 \approx -\frac{d_1\pi^2}{6^2} - c_1 + \overline{a}_{11}L_1^f + \mathscr{A}_1,$$

$$P_{21} = 7.2 > 6.1022 \approx -\frac{d_2\pi^2}{6^2} - c_2 + \overline{a}_{22}L_2^f + \mathscr{A}_2,$$

$$P_{12} = 10.1 > 9.1 = \sum_{k=1}^{2}\Big[|\acute{a}_{1k} - \grave{a}_{1k}|F_k + |\acute{b}_{1k} - \grave{b}_{1k}|G_k\Big],$$

$$P_{22} = 16.5 > 12 = \sum_{k=1}^{2}\Big[|\acute{a}_{2k} - \grave{a}_{2k}|F_k + |\acute{b}_{2k} - \grave{b}_{2k}|G_k\Big],$$

$$P_{14} = 11.3 > 4.6 = \overline{b}_{11}L_1^g + \overline{b}_{12}L_2^g,$$

$$P_{24} = 10.5 > 4.6 = \overline{b}_{21}L_1^g + \overline{b}_{22}L_2^g,$$

which means that all conditions in Theorem 3.1 hold. Therefore it can be concluded that the drive-response MRDNNs (2.1)–(2.3) can realize fixed-time synchronization, and the simulated synchronization error trajectories are presented in Figure 1, which are consistent with the theoretical results.

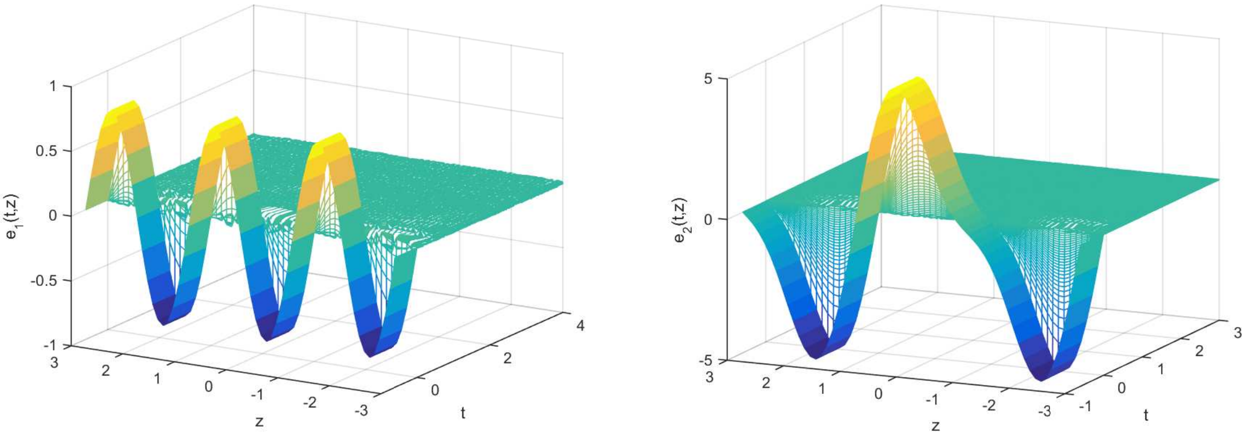

Figure 1. Spatiotemporal evolution of $e_i(t, z), i = 1, 2$.

5. Conclusions

In this paper, the FFTS criteria for the considered MRDNNs with discrete delay have been proposed. To do this, a negative exponent feedback controller was designed and a general Lyapunov functional ($p \geq 2$) was constructed to realize the FFTS behaviors. Finally, a numerical example has been given to show the effectiveness and usefulness of the presented criteria. The next work will consider the FFTS issue of the consider model with leakage delays and distributed delays.

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Conflict of interest

The authors declare that they have no conflicts of interest to this work.

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