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Research article

Soft separation axioms via soft topological operators

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Abstract: This paper begins with an introduction to some soft topological operators that will be used to characterize several soft separation axioms followed by their main properties. Then, we define a new soft separation axiom called "soft T_D-space" and analyze its main properties. We also show that this space precisely lies between soft T_0 and soft T_1-spaces. Finally, we characterize soft T_i-spaces, for $i = 0, 1, D$, in terms of the stated operators.

Keywords: soft topology; soft operator; soft shell; soft kernel; soft derived set; soft T_0-space; soft T_1-space; soft T_D-space
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1. Introduction

Most real-world problems in engineering, medical science, economics, the environment, and other fields are full of uncertainty. The soft set theory was proposed by Molodtsov [25], in 1999, as a mathematical model for dealing with uncertainty. This is free of the obstacles associated with previous theories including fuzzy set theory, rough set theory, and so on. The nature of parameter sets related to soft sets, in particular, provides a uniform framework for modeling uncertain data. This results in the rapid development of soft set theory in a short period of time, as well as diverse applications of soft sets in real life.

Influenced by the standard postulates of traditional topological space, Shabir and Naz [29], and Çağman et al. [18], separately, established another branch of topology known as "soft topology", which is a mixture of soft set theory and topology. This work was essential in building the subject of soft topology. Despite the fact that many studies followed their directions and many ideas appeared in soft contexts such as those discussed in [2, 3, 12–14]. However, significant contributions can indeed be made.

The separation axioms are just axioms in the sense that you could add these conditions as extra axioms to the definition of topological space to achieve a more restricted definition of what a topological space is. These axioms have a great role in developing (classical) topology. Correspondingly, soft separation axioms are a significant aspect in the later development of soft topology; see for example [4, 6–8, 19, 24, 29]. A specific type of separation axioms was defined by Aull and Thron [15]. This axiom performs as an important part in the development other disciplines like Locale Theory [28], Logic and Information Theory [16] and Philosophy [27]. First, motivating the role of "T_D-spaces", we generalize this separation axiom in the language of soft set theory under the name of "soft T_D-spaces", and study their primary properties. Second, most of the given soft separation axioms were characterized by soft open, soft closed, or soft closure, we want to describe them differently. As a result, this work is demonstrated. Finally, the desire of describing some soft T_i-spaces using new soft operators motivates us to present the operators of "soft kernel" and "soft shell".

The body of the paper is structured as follows: In Section 2, we present an overview of the literature on soft set theory and soft topology. Section 3 focuses on the concepts of soft topological operators and their main properties for characterization of soft separation axioms. Section 4 introduces a new soft separation axiom called a soft "T_D-space". The relationships of soft T_D-spaces with known soft separation axioms are determined. Furthermore, we characterize soft T_D-spaces via soft operators proposed in Section 3. In Section 5, we offer characterizations of soft T_0-spaces and soft T_1-spaces through the given operators. We end our paper with a brief summary and conclusions (Section 6).

2. Preliminaries

Let X be a domain set and E be a set of parameters. A pair $(F, E) = \{(e, F(e)) : e \in E\}$ is said to be a soft set [25] over X, where $F : E \to 2^X$ is a set-valued mapping. The set of all soft sets on X parameterized by E is identified by $S_E(X)$. We call a soft set (F, E) over X a soft element [29], denoted by $(\{x\}, E)$, if $F(e) = \{x\}$ for each $e \in E$, where $x \in X$. It is said that a soft element $(\{x\}, E)$ is in (F, E) (briefly, $x \in (F, E)$) if $x \in F(e)$ for each $e \in E$. On the other hand, $x \notin (F, E)$ if $x \notin F(e)$ for some $e \in E$. This implies that if $(\{x\}, E)\widetilde{\cap}(F, E) = \Phi$, then $x \notin (F, E)$. We call a soft set (F, E) over X a soft point [10, 26], denoted by x_e, if $F(e) = \{x\}$ and $x(e') = \emptyset$ for each $e' \in E$ with $e' \neq e$, where $e \in E$ and $x \in X$. An argument $x_e \in (F, E)$ means that $x \in F(e)$. The set of all soft points over X is identified by $P_E(X)$. A soft set $(X, E) - (F, E)$ (or simply $(F, E)^c$) is the complement of (F, E), where $F^c : E \to 2^X$ is given by $F^c(e) = X - F(e)$ for each $e \in E$. If $(F, E) \in S_E(X)$, it is denoted by Φ if $F(e) = \emptyset$ for each $e \in E$ and is denoted by $\widetilde{X}$ if $F(e) = X$ for each $e \in E$. Evidently, $\widetilde{X}^c = \Phi$ and $\Phi^c = \widetilde{X}$. A soft set (F, E) is called degenerate if $(F, E) = \{x_e\}$ or $(F, E) = \Phi$. It is said that (A, E_1) is a soft subset of (B, E_2) (written by $(A, E_1)\widetilde{\subseteq}(B, E_2)$, [22]) if $E_1 \subseteq E_2$ and $A(e) \subseteq B(e)$ for each $e \in E_1$, and $(A, E_1) = (B, E_2)$ if $(A, E_1)\widetilde{\subseteq}(B, E_2)$ and $(B, E_2)\widetilde{\subseteq}(A, E_1)$. The union of soft sets $(A, E), (B, E)$ is represented by $(F, E) = (A, E)\widetilde{\cup}(B, E)$, where $F(e) = A(e) \cup B(e)$ for each $e \in E$, and intersection of soft sets $(A, E), (B, E)$ is given by $(F, E) = (A, E)\widetilde{\cap}(B, E)$, where $F(e) = A(e) \cap B(e)$ for each $e \in E$, (see [9]).

Definition 2.1. *[29] A collection $\mathcal{T}$ of $S_E(X)$ is said to be a soft topology on X if it satisfies the following axioms:*

(T.1) $\Phi, \widetilde{X} \in \mathcal{T}$.

(T.2) *If* $(F_1, E), (F_2, E) \in \mathcal{T}$, *then* $(F_1, E)\widetilde{\cap}(F_2, E) \in \mathcal{T}$.

(T.3) *If* $\{(F_i, E) : i \in I\}\widetilde{\subseteq}\mathcal{T}$, *then* $\widetilde{\cup}_{i\in I}(F_i, E) \in \mathcal{T}$.

Terminologically, we call $(X, \mathcal{T}, E)$ a soft topological space on X. The elements of $\mathcal{T}$ are called soft open sets. The complement of every soft open or elements of $\mathcal{T}^c$ are called soft closed sets. The lattice of all soft topologies on X is referred to $T_E(X)$, (see [1]).

Definition 2.2. *[11] Let $\mathcal{F}\widetilde{\subseteq}S_E(X)$. The intersection of all soft topologies on X containing $\mathcal{F}$ is called a soft topology generated by $\mathcal{F}$ and is referred to $T(\mathcal{F})$.*

Definition 2.3. *[29] Let $(B, E) \in S_E(X)$ and $\mathcal{T} \in T_E(X)$.*

(1) The soft closure of (B, E) is $cl(B, E) := \widetilde{\cap}\{(F, E) : (B, E)\widetilde{\subseteq}(F, E), (F, E) \in \mathcal{T}^c\}$.
(2) The soft interior of (B, E) is $int(B, E) := \widetilde{\cup}\{(F, E) : (F, E)\widetilde{\subseteq}(B, E), (F, E) \in \mathcal{T}\}$.

Definition 2.4. *[18] Let $(B, E) \in S_E(X)$ and $\mathcal{T} \in T_E(X)$. A point $x_e \in P_E(X)$ is called a soft limit point of (B, E) if $(G, E)\widetilde{\cap}(B, E) - \{x_e\} \neq \Phi$ for all $(G, E) \in \mathcal{T}$ with $x_e \in (G, E)$. The set of all soft limit points is symbolized by $der(B, E)$. Then $cl(F, E) = (F, E)\widetilde{\cup}der(F, E)$ (see Theorem 5 in [18]).*

Definition 2.5. *[21] Let $\mathcal{T} \in T_E(X)$. A set $(A, E) \in S_E(X)$ is called soft locally closed if there exist $(G, E) \in \mathcal{T}$ and $(F, E) \in \mathcal{T}^c$ such that $(A, E) = (G, E)\widetilde{\cap}(F, E)$. The family of all soft locally closed sets in X is referred to $LC(X)$.*

Definition 2.6. *[20] Let $\mathcal{T} \in T_E(X)$ and let $(A, E) \in S_E(X)$. A point $x_e \in (A, E)$ is called soft isolated if there exists $(G, E) \in \mathcal{T}$ such that $(G, E)\widetilde{\cap}(A, E) = \{x_e\}$. It is called soft weakly isolated if there exists $(G, E) \in \mathcal{T}$ with $x_e \in (G, E)$ such that $(G, E)\widetilde{\cap}(A, E)\widetilde{\subseteq}cl(x_e)$. Let $I(A, E)$, $WI(A, E)$ respectively denote the set of all soft isolated and soft weakly isolated points of (A, E).*

Definition 2.7. *[17] A soft space $(X, E, \mathcal{T})$ (or simply soft topology $\mathcal{T} \in T_E(X)$) is called*

(1) Soft T_0 if for every $x_e, y_{e'} \in P_E(X)$ with $x_e \neq y_{e'}$, there exist $(U, E), (V, E) \in \mathcal{T}$ such that $x_e \in (U, E)$, $y_{e'} \notin (U, E)$ or $y_{e'} \in (V, E)$, $x_e \notin (V, E)$.
(2) Soft T_1 if for every $x_e, y_{e'} \in P_E(X)$ with $x_e \neq y_{e'}$, there exist $(U, E), (V, E) \in \mathcal{T}$ such that $x_e \in (U, E)$, $y_{e'} \notin (U, E)$ and $y_{e'} \in (V, E)$, $x_e \notin (V, E)$.

The above soft separation axioms have been defined by Sabir and Naz [29] with respect to soft elements.

Lemma 2.8. *[17, Theorem 4.1] Let $\mathcal{T} \in T_E(X)$. Then $\mathcal{T}$ is soft T_1 iff $cl(x_e) = \{x_e\}$ for every $x_e \in P_E(X)$.*

3. Soft topological operators

In this section, we define "soft kernel" and "soft shell" as two topological operators. Then the connections between these operators and soft closure and soft derived set operators are obtained. The presented results will be used to characterize several soft separation axioms.

Definition 3.1. *Let $(F, E) \in S_E(X)$ and let $\mathcal{T} \in T_E(X)$. The soft kernel of (F, E) is defined by:*

$$ker(F, E) := \widetilde{\bigcap}\{(G, E) : (G, E) \in \mathcal{T}, (F, E)\widetilde{\subseteq}(G, E)\}.$$

Lemma 3.2. *Let $(F, E), (G, E) \in S_E(X)$ and $\mathcal{T} \in T_E(X)$. The following properties are valid:*

(1) $(F, E)\widetilde{\subseteq} ker(F, E)$.
(2) $ker(F, E)\widetilde{\subseteq} ker(ker(F, E))$.
(3) $(F, E)\widetilde{\subseteq}(G, E) \implies ker(F, E)\widetilde{\subseteq} ker(G, E)$.
(4) $ker[(F, E)\widetilde{\cap}(G, E)]\widetilde{\subseteq} ker(F, E)\widetilde{\cap} ker(G, E)$.
(5) $ker[(F, E)\widetilde{\cup}(G, E)] = ker(F, E)\widetilde{\cup} ker(G, E)$.

Proof. Standard. □

From Definitions 2.3 and 3.1, it is obtained that

Definition 3.3. *Let $x_e \in P_E(X)$ and $\mathcal{T} \in T_E(X)$. Then*

(1) $ker(\{x_e\}) := \widetilde{\bigcap}\{(G, E) : (G, E) \in \mathcal{T}, x_e \in (G, E)\}$.
(2) $cl(\{x_e\}) := \widetilde{\bigcap}\{(F, E) : (F, E) \in \mathcal{T}^c, x_e \in (F, E)\}$.

In the sequel, we interchangeably use x_e or $\{x_e\}$ for the one point soft set containing x_e.

Lemma 3.4. *For $(F, E) \in S_E(X)$ and $\mathcal{T} \in T_E(X)$, we have*

$$ker(F, E) = \{x_e \in P_E(X) : cl(x_e)\widetilde{\cap}(F, E) \neq \Phi\}.$$

Proof. Let $x_e \in ker(F, E)$. If $cl(x_e)\widetilde{\cap}(F, E) = \Phi$, then $(F, E)\widetilde{\subseteq}\widetilde{X} - cl(x_e)$. Therefore, $\widetilde{X} - cl(x_e) \in \mathcal{T}$ such that it contains (F, E) but not x_e, a contradiction.

Conversely, if $x_e \notin ker(F, E)$ and $cl(x_e)\widetilde{\cap}(F, E) \neq \Phi$, then there is $(G, E) \in \mathcal{T}$ such that $(F, E)\widetilde{\subseteq}(G, E)$ but $x_e \notin (G, E)$ and $y_{e'} \in cl(x_e)\widetilde{\cap}(F, E)$. Therefore, $\widetilde{X} - (G, E) \in \mathcal{T}^c$ including x_e but not $y_{e'}$. This contradicts to $y_{e'} \in cl(x_e)\widetilde{\cap}(F, E)$. Thus, $x_e \in ker(F, E)$. □

Definition 3.5. *Let $(F, E), (G, E) \in S_E(X)$ and $\mathcal{T} \in T_E(X)$. It is said that (F, E) is separated in a weak sense from (G, E) (symbolized by $(F, E) \vdash (G, E)$) if there exists $(H, E) \in \mathcal{T}$ with $(F, E)\widetilde{\subseteq}(H, E)$ such that $(H, E)\widetilde{\cap}(G, E) = \Phi$.*

We have the following observation in light of Lemma 3.4 and Definition 3.5.

Remark 3.6. *For $x_e, y_{e'} \in P_E(X)$ and $\mathcal{T} \in T_E(X)$, we have*

(1) $cl(x_e) = \{y_{e'} : y_{e'} \nvdash x_e\}$.
(2) $ker(x_e) = \{y_{e'} : x_e \nvdash y_{e'}\}$.

Definition 3.7. *For $x_e \in P_E(X)$ and $\mathcal{T} \in T_E(X)$, we define:*

(1) The soft derived set of x_e as $der(x_e) = cl(x_e) - \{x_e\}$.
(2) The soft shell of x_e as $shel(x_e) = ker(x_e) - \{x_e\}$.
(3) The soft set $\langle x_e \rangle = cl(x_e)\widetilde{\cap} ker(x_e)$.

We have the following remark in view of Definition 3.7 and Remark 3.6.

Remark 3.8. *For $x_e, y_{e'} \in P_E(X)$ and $\mathcal{T} \in T_E(X)$, we have*

(1) $der(x_e) = \{y_{e'} : y_{e'} \neq x_e, y_{e'} \nvdash x_e\}$.
(2) $shel(x_e) = \{y_{e'} : y_{e'} \neq x_e, x_e \nvdash y_{e'}\}$.

Example 3.9. *Let $X = \{0, 1, 2\}$ and let $E = \{e_1, e_2\}$ be a set of parameters. Consider the following soft topology on X:*

$$\mathcal{T} = \{\Phi, (F, E), G, E), (H, E), \widetilde{X}\},$$

where, $(F, E) = \{(e_1, \{0\}), (e_2, \emptyset)\}$, $(G, E) = \{(e_1, \{0, 1\}), (e_2, \emptyset)\}$, and $(H, E) = \{(e_1, \{0, 2\}), (e_2, X)\}$. By an easy computation, one can conclude the following:

$ker(\{1_{e_1}\}) = (G, E)$	$ker(\{1_{e_2}\}) = (H, E)$
$shel(\{1_{e_1}\}) = (F, E)$	$shel(\{1_{e_2}\}) = \{(e_1, \{0, 2\}), (e_2, \{0, 2\})\}$
$cl(\{1_{e_1}\}) = \{1_{e_1}\}$	$cl(\{1_{e_2}\}) = \{(e_1, \{2\}), (e_2, X)\}$
$der(\{1_{e_1}\}) = \Phi$	$der(\{1_{e_2}\}) = \{(e_1, \{2\}), (e_2, \{0, 2\})\}$.

Lemma 3.10. *The following properties are valid for every $x_e, y_{e'} \in P_E(X)$ and $\mathcal{T} \in T_E(X)$:*

(1) $y_{e'} \in ker(x_e) \Longleftrightarrow x_e \in cl(y_{e'})$.
(2) $y_{e'} \in shel(x_e) \Longleftrightarrow x_e \in der(y_{e'})$.
(3) $y_{e'} \in cl(x_e) \implies cl(y_{e'}) \widetilde{\subseteq} cl(x_e)$.
(4) $y_{e'} \in ker(x_e) \implies ker(y_{e'}) \widetilde{\subseteq} ker(x_e)$.

Proof. (1) and (2) follow, respectively, from Remarks 3.6 and 3.8.

(3) Straightforward.

(4) Let $z_{e^*} \in ker(y_{e'})$. By (1), $y_{e'} \in cl(z_{e^*})$ and so $cl(y_{e'}) \widetilde{\subseteq} cl(z_{e^*})$ (by (3)). By hypothesis, $y_{e'} \in ker(x_e)$ and so $x_e \in cl(y_{e'})$. Therefore, $cl(x_e) \widetilde{\subseteq} cl(y_{e'})$. Finally, we get $cl(x_e) \widetilde{\subseteq} cl(z_{e^*})$ and then $x_e \in cl(z_{e^*})$. By (1), $z_{e^*} \in ker(x_e)$. Thus, $ker(y_{e'}) \widetilde{\subseteq} ker(x_e)$. □

Lemma 3.11. *Let $\mathcal{T} \in T_E(X)$ and let $x_e \in P_E(X)$. Then*

(1) $shel(x_e)$ is degenerate iff for every $y_{e'} \in P_E(X)$ with $y_{e'} \neq x_e$, $der(x_e) \widetilde{\cap} der(y_{e'}) = \Phi$.
(2) $der(x_e)$ is degenerate iff for every $y_{e'} \in P_E(X)$ with $y_{e'} \neq x_e$, $shel(x_e) \widetilde{\cap} shel(y_{e'}) = \Phi$.

Proof. (1) If $der(x_e) \widetilde{\cap} der(y_{e'}) \neq \Phi$, then there exists $z_{e^*} \in P_E(X)$ such that $z_{e^*} \in der(x_e)$, $z_{e^*} \in der(y_{e'})$. Therefore, $z_{e^*} \neq y_{e'} \neq x_e$ for which $z_{e^*} \in cl(x_e)$ and $z_{e^*} \in cl(y_{e'})$. By Lemma 3.10 (1), $x_e, y_{e'} \in ker(z_{e^*})$. Thus, $x_e, y_{e'} \in ker(z_{e^*}) - z_{e^*} = shel(z_{e^*})$. This proves that $shel(x_e)$ is not degenerate.

Conversely, if $x_e, y_{e'} \in shel(z_{e^*})$, then $x_e \neq y_{e'}$, $x_e \neq z_{e^*}$ and so $x_e \in ker(z_{e^*})$, $y_{e'} \in ker(z_{e^*})$. Therefore, $z_{e^*} \in cl(x_e) \widetilde{\cap} cl(y_{e'})$ and thus $z_{e^*} \in der(x_e) \widetilde{\cap} der(y_{e'})$. But this is impossible, hence $der(x_e) \widetilde{\cap} der(y_{e'}) = \Phi$. □

Lemma 3.12. *Let $\mathcal{T} \in T_E(X)$ and let $x_e, y_{e'} \in P_E(X)$. Then*

(1) If $y_{e'} \in \langle x_e \rangle$, then $\langle y_{e'} \rangle = \langle x_e \rangle$.
(2) Either $\langle y_{e'} \rangle = \langle x_e \rangle$ or $\langle y_{e'} \rangle \widetilde{\cap} \langle x_e \rangle = \Phi$.

Proof. (1) If $y_{e'} \in \langle x_e \rangle$, then $y_{e'} \in cl(x_e)$ and $y_{e'} \in ker(x_e)$. When $y_{e'} \in cl(x_e)$, by Lemma 3.10 (1), $x_e \in ker(y_{e'})$. By Lemma 3.10 (3) and (4), $cl(y_{e'}) \widetilde{\subseteq} cl(x_e)$ and $ker(x_e) \widetilde{\subseteq} ker(y_{e'})$. When $y_{e'} \in ker(x_e)$, by Lemma 3.10 (2), $x_e \in cl(y_{e'})$. By Lemma 3.10 (3) and (4), $cl(x_e) \widetilde{\subseteq} cl(y_{e'})$ and $ker(y_{e'}) \widetilde{\subseteq} ker(x_e)$. Summing up all these together, we get $cl(x_e) = cl(y_{e'})$ and $ker(x_e) = ker(y_{e'})$. Thus, $\langle y_{e'} \rangle = \langle x_e \rangle$.

(2) It can be deduced from (1). □

Lemma 3.13. *Let $\mathcal{T} \in T_E(X)$ and let $x_e, y_{e'} \in P_E(X)$. Then $ker(x_e) \neq ker(y_{e'})$ iff $cl(x_e) \neq cl(y_{e'})$.*

Proof. If $ker(x_e) \neq ker(y_{e'})$, then one can find $z_{e^*} \in ker(x_e)$ but $z_{e^*} \notin ker(y_{e'})$. From $z_{e^*} \in ker(x_e)$, we get $x_e \in cl(z_{e^*})$ and then $cl(x_e) \widetilde{\subseteq} cl(z_{e^*})$. Since $z_{e^*} \notin ker(y_{e'})$, by Lemma 3.10 (1), $cl(z_{e^*}) \widetilde{\cap} y_{e'} = \Phi$. Therefore, $cl(z_{e^*}) \widetilde{\cap} y_{e'} = \Phi$ implies $y_{e'} \notin cl(x_e)$. Hence, $cl(y_{e'}) \neq cl(x_e)$.

The converse can be proved in a similar manner to the first part. □

4. Soft T_D-spaces

Definition 4.1. *Let $\mathcal{T} \in T_E(X)$. We call $\mathcal{T}$ a soft T_D-space if $der(x_e)$ is a soft closed set for every $x_e \in P_E(X)$.*

Theorem 4.2. *Let $\mathcal{T} \in T_E(X)$. Then*

(1) If $\mathcal{T}$ is soft T_1, then it is soft T_D.
(2) If $\mathcal{T}$ is soft T_D, then it is soft T_0.

Proof. (1) If $\mathcal{T}$ is soft T_1, by Lemma 2.8, for every $x_e \in P_E(X)$, $cl(x_e) = \{x_e\}$, so $der(x_e) = \Phi \in \mathcal{T}^c$. Thus, $\mathcal{T}$ is soft T_D.

(2) Let $x_e, y_{e'} \in P_E(X)$ with $x_e \neq y_{e'}$. If $y_{e'} \in der(x_e)$, then $[der(x_e)]^c$ is a soft open set that includes x_e but not $y_{e'}$. If $y_{e'} \notin der(x_e)$ and since $x_e \neq y_{e'}$, then $y_{e'} \in [cl(x_e)]^c$ and $[cl(x_e)]^c \in \mathcal{T}$ with $x_e \notin [cl(x_e)]^c$. Consequently, $\mathcal{T}$ is soft T_0. □

The reverse of the above implications may not be true, as illustrated by the examples below.

Example 4.3. *Let X be an infinite and let E be a set of parameters. For a fixed $p_e \in P_E(X)$, the soft topology $\mathcal{T}$ on X is given by $\mathcal{T} = \{(F, E) \in S_E(X) : p_e \notin (F, E)$ or $(F, E) = \widetilde{X}\}$. We first need to check $\mathcal{T}$ is soft T_D. Indeed, take $x_e \in P_E(X)$, if $x_e = p_e$, then $der(x_e) = \Phi$. If $x_e \neq p_e$, then $der(x_e) = \{p_e\}$. Therefore, in either cases, $der(x_e)$ is soft closed. On the other hand, for any $x_e \neq p_e$, $cl(x_e) = \{x_e, p_e\} \neq \{x_e\}$, which means $\{x_e\}$ is not a closed set. Hence $\mathcal{T}$ is not soft T_1.*

Example 4.4. *Let $E = \{e_1, e_2\}$ be a set of parameters and let $\mathcal{T}$ be a soft topology on the set of real numbers $\mathbb{R}$ generated by*

$$\{\{(e_1, B(e_1)), (e_2, B(e_2))\} : B(e_1) = (a, b), B(e_2) = (c, \infty); a, b, c \in \mathbb{R}; a < b\}.$$

Let $x_{e_1}, y_{e_2} \in P_E(X)$ with $x_{e_1} \neq y_{e_2}$. W.l.o.g, we assume $x < y$. Take $(G, E) = \{(e_1, \emptyset), (e_2, (x, \infty))\}$. Then (G, E) is a soft open set containing y_{e_2} but not x_{e_1} and hence $\mathcal{T}$ is soft T_0. But then $der(y_{e_2}) = \{(e_1, \emptyset), (e_2, (-\infty, y))\}$ is not soft closed, and consequently $\mathcal{T}$ is not soft T_D.

Proposition 4.5. *Let $\mathcal{T} \in T_E(X)$. Then $\mathcal{T}$ is soft T_D iff $\{x_e\} \in LC(X)$ for every $x_e \in P_E(X)$.*

Proof. Let $x_e \in P_E(X)$. We need to prove that $\{x_e\}$ can be written as an intersection of a soft open set with a soft closed set. Set $(G, E) = [der(x_e)]^c$ and $(F, E) = cl(x_e)$. Then $(G, E) \in \mathcal{T}$ and $(F, E) \in \mathcal{T}^c$ such that $\{x_e\} = (G, E)\widetilde{\cap}(F, E)$.

Conversely, (w.l.o.g) we set $\{x_e\} = (G, E)\widetilde{\cap} cl(x_e)$. Now, $der(x_e) = cl(x_e) - \{x_e\} = cl(x_e) - [(G, E)\widetilde{\cap} cl(x_e)] = cl(x_e)\widetilde{\cap}(G, E)^c$. Since finite intersections of soft closed sets are soft closed, so $der(x_e)$ is soft closed. □

Proposition 4.6. *Let $\mathcal{T} \in T_E(X)$. Then $\mathcal{T}$ is soft T_D iff for every $x_e \in P_E(X)$, there exists $(G, E) \in \mathcal{T}$ including x_e such that $(G, E) - \{x_e\} \in \mathcal{T}$.*

Proof. Take a point $x_e \in P_E(X)$. If we set $(G, E) = [der(x_e)]^c$, then $(G, E) \in \mathcal{T}$ containing x_e. Now,

$$\begin{aligned}(G, E) - \{x_e\} &= \widetilde{X} - der(x_e)\widetilde{\bigcap}\{x_e\}^c \\ &= \widetilde{X}\widetilde{\bigcap}(der(x_e))^c\widetilde{\bigcap}\{x_e\}^c \\ &= \widetilde{X}\widetilde{\bigcap}[der(x_e)\widetilde{\bigcup}\{x_e\}]^c \\ &= \widetilde{X} - cl(x_e).\end{aligned}$$

Thus, $(G, E) - \{x_e\} \in \mathcal{T}$.

Conversely, suppose for every $x_e \in P_E(X)$, there exists $x_e \in (G, E) \in \mathcal{T}$ such that $(G, E) - \{x_e\} \in \mathcal{T}$. Therefore, $\{x_e\} = (G, E)\widetilde{\cap}[(G, E) - \{x_e\}]^c$. By Proposition 4.5, $\mathcal{T}$ is soft T_D. □

Proposition 4.7. *For a soft topology $\mathcal{T} \in T_E(X)$, the following properties are equivalent:*

(1) $\mathcal{T}$ is soft T_D.
(2) $der(der(A, E))\widetilde{\subseteq} der(A, E)$ for every $(A, E) \in S_E(X)$.
(3) $der(A, E) \in \mathcal{T}^c$ for every $(A, E) \in S_E(X)$.

Proof. (1) $\Longrightarrow$ (2) Let $x_e \in der(der(A, E))$. Then every $(G, E) \in \mathcal{T}$ with $x_e \in (G, E)$ includes some points of $der(A, E)$. Since $\mathcal{T}$ is soft T_D,

$$(H, E) - \{x_e\}\widetilde{\bigcap} der(A, E) \neq \Phi,$$

where $(H, E) = [der(x_e)]^c\widetilde{\bigcap}(G, E)$. Suppose $y_{e'} \in der(A, E)$ with $y_{e'} \neq x_e$. Then $y_{e'} \in (H, E)\widetilde{\subseteq}(G, E)$. Since $y_{e'} \in der(A, E)$, then $(H, E) \in \mathcal{T}$ contains a point z_{e^*} of (A, E) except $y_{e'}$. Indeed, $z_{e^*} \neq x_e$ and then every (G, E) with $x_e \in (G, E)$ contains some points of (A, E) except x_e. Hence, $x_e \in der(A, E)$.

(2) $\Longrightarrow$ (3) Since $cl(der(A, E)) = der(der(A, E))\widetilde{\cup} der(A, E)\widetilde{\subseteq} der(A, E)$, so $der(A, E) \in \mathcal{T}^c$.

(3) $\Longrightarrow$ (1) It is evident. □

Proposition 4.8. *Let $(A, E) \in S_E(X)$, $\mathcal{T} \in T_E(X)$ and $(F, E) \in \mathcal{T}^c$. The following properties are equivalent:*

(1) $\mathcal{T}$ is soft T_D.
(2) For every $x_e \in P_E(X)$, $[cl(x_e)]^c\widetilde{\cup}\{x_e\} \in \mathcal{T}$.
(3) Every $x_e \in WI(A, E) \implies x_e \in I(A, E)$.
(4) Every $x_e \in WI(F, E) \implies x_e \in I(F, E)$.

Proof. (1) $\Longrightarrow$ (2) Given $x_e \in P_E(X)$, by Proposition 4.6, there is $(G,E) \in \mathcal{T}$ such that $x_e \in (G,E)$ and $(G,E) - \{x_e\} \in \mathcal{T}$. Therefore, $(G,E) - \{x_e\} = (G,E) - cl(x_e)$. Since $\mathcal{T}$ is soft T_D, so $(G,E) - der(x_e) = (G,E) - cl(x_e)\widetilde{\cup}\{x_e\} \in \mathcal{T}$. But, for every $x_e \in [cl(x_e)]^c\widetilde{\cup}\{x_e\}$, we have

$$x_e \in (G,E) = (G,E) - der(x_e)\widetilde{\cup}\{x_e\}\widetilde{\subseteq}[cl(x_e)]^c\widetilde{\cup}\{x_e\}.$$

Thus, $[cl(x_e)]^c\widetilde{\cup}\{x_e\} \in \mathcal{T}$.

(2) $\Longrightarrow$ (3) Suppose $x_e \in WI(A,E)$. Then there is $(G,E) \in \mathcal{T}$ such that

$$x_e \in (G,E)\widetilde{\cap}(A,E)\widetilde{\subseteq}cl(x_e).$$

By (2), $[cl(x_e)]^c\widetilde{\cup}\{x_e\} \in \mathcal{T}$. But,

$$(G,E)\widetilde{\bigcap}(A,E)\widetilde{\bigcap}[[cl(x_e)]^c\widetilde{\bigcup}\{x_e\}] = \{x_e\}.$$

Hence, $x_e \in I(A,E)$.

(3) $\Longrightarrow$ (4) Clear.

(4) $\Longrightarrow$ (1) Given $x_e \in P_E(X)$, we can easily conclude from the definition that $x_e \in WI(cl(x_e))$. By (4), $x_e \in I(cl(x_e))$, and so there exists $(G,E) \in \mathcal{T}$ such that $(G,E)\widetilde{\cap}cl(x_e) = \{x_e\}$. Therefore, $(G,E) - \{x_e\} = (G,E) - cl(x_e) \in \mathcal{T}$. By Proposition 4.6, $\mathcal{T}$ is soft T_D. □

Summing up all the above findings yields the following characterization:

Theorem 4.9. *For a soft topology $\mathcal{T} \in T_E(X)$, the following properties are equivalent:*

(1) $\mathcal{T}$ is soft T_D.
(2) $\{x_e\} \in LC(X)$ for every $x_e \in P_E(X)$.
(3) $der(A,E) \in \mathcal{T}^c$ for every $(A,E) \in S_E(X)$.
(4) $der(der(A,E))\widetilde{\subseteq}der(A,E)$ for every $(A,E) \in S_E(X)$.
(5) $\forall x_e \in P_E(X)$, there exists $(G,E) \in \mathcal{T}$ with $x_e \in (G,E)$ such that $(G,E) - \{x_e\} \in \mathcal{T}$.
(6) $\forall x_e \in P_E(X)$, $[cl(x_e)]^c\widetilde{\cup}\{x_e\} \in \mathcal{T}$.
(7) $\forall x_e \in WI(A,E) \Longrightarrow x_e \in I(A,E)$, where $(A,E) \in S_E(X)$.
(8) $\forall x_e \in WI(F,E) \Longrightarrow x_e \in I(F,E)$, where $(F,E) \in \mathcal{T}^c$.

5. Characterizations of soft T_i-spaces, $i \in \{0,1\}$

The properties of soft topological operators derived in Section 2 are used to develop new characterizations of soft T_i-spaces for $i = 0,1$.

Proposition 5.1. *For a soft topology $\mathcal{T} \in T_E(X)$, the following properties are equivalent:*

(1) $\mathcal{T}$ is soft T_0.
(2) For every $x_e, y_{e'} \in P_E(X)$ with $x_e \neq y_{e'}$, either $x_e \vdash y_{e'}$ or $y_{e'} \vdash x_e$.
(3) $y_{e'} \in cl(x_e) \Longrightarrow x_e \notin cl(y_{e'})$.
(4) For every $x_e, y_{e'} \in P_E(X)$ with $x_e \neq y_{e'}$, $cl(x_e) \neq cl(y_{e'})$.

Proof. (1) $\Longrightarrow$ (2) It is just a reword of the definition .

(2) $\Longrightarrow$ (3) Let $y_{e'} \in cl(x_e)$. For every $(G, E) \in \mathcal{T}$ that contains $y_{e'}$, $(G, E)\widetilde{\cap}\{x_e\} \neq \Phi$ and so $y_{e'} \nvdash x_e$. If $x_e = y_{e'}$, then there is nothing to prove. Otherwise, by (2), $x_e \vdash y_{e'}$. Therefore, there exists $(H, E) \in \mathcal{T}$ such that $x_e \in (H, E)$ and $(H, E)\widetilde{\cap}\{y_{e'}\} = \Phi$. Hence, $x_e \notin cl(y_{e'})$.

(3) $\Longrightarrow$ (4) Suppose the negative of (4) holds. Then $cl(x_e)\widetilde{\subseteq}cl(y_{e'})$ and $cl(y_{e'})\widetilde{\subseteq}cl(x_e)$. Since $y_{e'} \in cl(y_{e'})$, then it implies that $cl(y_{e'}) \in cl(x_e)$ and so $y_{e'} \in cl(x_e)$. By (3), $x_e \notin cl(y_{e'})$ implies $x_e \notin cl(x_e)$ which is impossible.

(4) $\Longrightarrow$ (1) Suppose $x_e, y_{e'} \in P_E(X)$ with $x_e \neq y_{e'}$, $cl(x_e) \neq cl(y_{e'})$. This means that there is $z_{e^*} \in P_E(X)$ for which $z_{e^*} \in cl(x_e)$ but $z_{e^*} \notin cl(y_{e'})$. We claim that $x_e \notin cl(y_{e'})$. Otherwise, we will have $\{x_e\} \notin cl(y_{e'})$ and so $cl(x_e) \notin cl(y_{e'})$. This implies that $z_{e^*} \in cl(y_{e'})$, a contradiction to the selection of z_{e^*}. Set $(G, E) = [cl(y_{e'})]^c$. Therefore, $(G, E) \in \mathcal{T}$ such that $x_e \in (G, E)$ and $y_{e'} \notin (G, E)$. Hence, $\mathcal{T}$ is soft T_0. □

Proposition 5.2. *For a soft topology $\mathcal{T} \in T_E(X)$, the following properties are equivalent:*

(1) $\mathcal{T}$ is soft T_0.
(2) For every $x_e, y_{e'} \in P_E(X)$, $y_{e'} \in ker(x_e) \Longrightarrow x_e \notin ker(y_{e'})$.
(3) For every $x_e, y_{e'} \in P_E(X)$ with $x_e \neq y_{e'}$, $ker(x_e) \neq ker(y_{e'})$.

Proof. Follows from Lemma 3.10 (1) and Proposition 5.1. □

Proposition 5.3. *A soft topology $\mathcal{T} \in T_E(X)$ is soft T_0 iff $y_{e'} \in der(x_e)$ implies $cl(y_{e'})\widetilde{\subseteq}der(x_e)$ for every $x_e, y_{e'} \in P_E(X)$.*

Proof. Given $x_e, y_{e'} \in P_E(X)$. If $y_{e'} \in der(x_e)$, then $y_{e'} \neq x_e$ and $x_e \notin cl(y_{e'})$ (as $\mathcal{T}$ is soft T_0), then $cl(y_{e'})\widetilde{\subseteq}der(x_e)$.

Conversely, let $x_e, y_{e'} \in P_E(X)$ be such that $x_e \neq y_{e'}$. If $y_{e'} \in der(x_e)$, then $cl(y_{e'})\widetilde{\subseteq}der(x_e)$. This means that $y_{e'} \in cl(x_e)$ and $x_e \notin cl(y_{e'})$. From Proposition 5.1, $\mathcal{T}$ is soft T_0. □

Proposition 5.4. *A soft topology $\mathcal{T} \in T_E(X)$ is soft T_0 iff $y_{e'} \in shel(x_e)$ implies $ker(y_{e'})\widetilde{\subseteq}shel(x_e)$ for every $x_e, y_{e'} \in P_E(X)$.*

Proof. By Proposition 5.3 and Lemma 3.10, we can obtain the proof. □

Proposition 5.5. *A soft topology $\mathcal{T} \in T_E(X)$ is soft T_0 iff $[cl(x_e)\widetilde{\cap}\{y_{e'}\}]\widetilde{\bigcup}[\{x_e\}\widetilde{\cap}cl(y_{e'})]$ is degenerate for every $x_e, y_{e'} \in P_E(X)$.*

Proof. Assume $x_e, y_{e'} \in P_E(X)$ and $\mathcal{T}$ is soft T_0. By Proposition 5.1, for every $x_e, y_{e'} \in P_E(X)$, if $y_{e'} \in cl(x_e)$, then $x_e \notin cl(y_{e'})$. Therefore, $[cl(x_e)\widetilde{\cap}\{y_{e'}\}]\widetilde{\cup}[\{x_e\}\widetilde{\cap}cl(y_{e'})] = \{y_{e'}\}$ is a degenerated soft set. Otherwise, $[cl(x_e)\widetilde{\cap}\{y_{e'}\}]\widetilde{\cup}[\{x_e\}\widetilde{\cap}cl(y_{e'})] = \{x_e\}$ which is also degenerate.

Conversely, if the given condition is satisfied, then the result is either $\Phi, \{x_e\}$, or $\{y_{e'}\}$. For the case of Φ, the conclusion is obvious. If $[cl(x_e)\widetilde{\cap}\{y_{e'}\}]\widetilde{\cup}[\{x_e\}\widetilde{\cap}cl(y_{e'})] = \{x_e\}$ implies $x_e \in cl(y_{e'})$ and $cl(x_e)\widetilde{\cap}\{y_{e'}\} = \Phi$. Therefore, $y_{e'} \notin cl(x_e)$. The case of $\{y_{e'}\}$ is similar to the latter one. Hence, $\mathcal{T}$ is soft T_0. □

Proposition 5.6. *A soft topology $\mathcal{T} \in T_E(X)$ is soft T_0 iff $der(x_e)\widetilde{\cap}shel(x_e) = \Phi$ for every $x_e \in P_E(X)$.*

Proof. If $der(x_e)\widetilde{\cap}shel(x_e) \neq \Phi$, then there is $x_e \in P_E(X)$ such that $z_{e^*} \in der(x_e)$ and $z_{e^*} \in shel(x_e)$. Indeed, $z_{e^*} \neq x_e$ and so $z_{e^*} \in cl(x_e)$ and $z_{e^*} \in ker(x_e)$. By Remark 3.6, $z_{e^*} \nvdash x_e$ and $x_e \nvdash z_{e^*}$ implies that $\mathcal{T}$ cannot be soft T_0, a contradiction.

Conversely, if $der(x_e)\widetilde{\cap}shel(x_e) = \Phi$, then for each $z_{e^*} \neq x_e$, either $z_{e^*} \in cl(x_e)$ or $z_{e^*} \in ker(x_e)$. Therefore, either $z_{e^*} \in cl(x_e)$ or $x_e \in cl(z_{e^*})$. By Proposition 5.1 (3), $\mathcal{T}$ is soft T_0. □

Proposition 5.7. *A soft topology $\mathcal{T} \in T_E(X)$ is soft T_0 iff $\langle x_e \rangle = \{x_e\}$ for every $x_e \in P_E(X)$.*

Proof. It is a consequence of Definition 3.7 and Proposition 5.6. □

Proposition 5.8. *A soft topology $\mathcal{T} \in T_E(X)$ is soft T_0 iff $der(x_e)$ is a union of soft closed sets for every $x_e \in P_E(X)$.*

Proof. Since, for every $x_e \in P_E(X)$, $der(x_e) \in \mathcal{T}^c$, then for every $z_{e^*} \in der(x_e)$ we must have $(G, E) \in \mathcal{T}$ such that $x_e \in (G, E)$ and $z_{e^*} \notin (G, E)$. Therefore, $(F, E) = (G, E)^c \in \mathcal{T}^c$ with with $z_{e^*} \in (F, E)$ but $x_e \notin (F, E)$. This means that $\forall z_{e^*} \in der(x_e)$, we have

$$z_{e^*} \in (F, E)\widetilde{\bigcap}cl(x_e)\widetilde{\subseteq}der(x_e).$$

Since $(F, E)\widetilde{\cap}cl(x_e) \in \mathcal{T}^c$, so $der(x_e)$ is a union of soft closed sets.

Conversely, let $der(x_e) = \widetilde{\bigcup}_{i\in I}(F_i, E)$, where $(F_i, E) \in \mathcal{T}^c$. If $z_{e^*} \in der(x_e)$, then $z_{e^*} \in (F_i, E)$ for some i but $x_e \notin (F_i, E)$. Therefore, $(F_i, E)^c \in \mathcal{T}$ such that $x_e \in (F_i, E)^c$ but $z_{e^*} \notin (F_i, E)^c$. If $z_{e^*} \notin der(x_e)$ and $z_{e^*} \neq x_e$, then $z_{e^*} \in [cl(x_e)]^c$ and $[cl(x_e)]^c \in \mathcal{T}$ for which $x_e \notin [cl(x_e)]^c$. This proves that $\mathcal{T}$ is soft T_0. □

Summing up all the above propositions yields the following characterization:

Theorem 5.9. *For a soft topology $\mathcal{T} \in T_E(X)$, the following properties are equivalent:*

(1) $\mathcal{T}$ is soft T_0.
(2) For every $x_e, y_{e'} \in P_E(X)$ with $x_e \neq y_{e'}$, either $x_e \vdash y_{e'}$ or $y_{e'} \vdash x_e$.
(3) For every $x_e, y_{e'} \in P_E(X)$, $y_{e'} \in cl(x_e) \Longrightarrow x_e \notin cl(y_{e'})$.
(4) For every $x_e, y_{e'} \in P_E(X)$, $y_{e'} \in der(x_e) \Longrightarrow cl(y_{e'})\widetilde{\subseteq}der(x_e)$
(5) For every $x_e, y_{e'} \in P_E(X)$ with $x_e \neq y_{e'}$, $cl(x_e) \neq cl(y_{e'})$.
(6) For every $x_e, y_{e'} \in P_E(X)$, $y_{e'} \in ker(x_e) \Longrightarrow x_e \notin ker(y_{e'})$.
(7) For every $x_e, y_{e'} \in P_E(X)$, $y_{e'} \in shel(x_e) \Longrightarrow ker(y_{e'})\widetilde{\subseteq}shel(x_e)$
(8) For every $x_e, y_{e'} \in P_E(X)$ with $x_e \neq y_{e'}$, $ker(x_e) \neq ker(y_{e'})$.
(9) For every $x_e, y_{e'} \in P_E(X)$ $[cl(x_e)\widetilde{\cap}\{y_{e'}\}]\widetilde{\bigcup}[\{x_e\}\widetilde{\cap}cl(y_{e'})]$ is degenerate.
(10) For every $x_e \in P_E(X)$, $der(x_e)\widetilde{\cap}shel(x_e) = \Phi$.
(11) For every $x_e \in P_E(X)$, $der(x_e)$ is a union of soft closed sets.
(12) For every $x_e \in P_E(X)$, $\langle x_e \rangle = \{x_e\}$.

Theorem 5.10. *For a soft topology $\mathcal{T} \in T_E(X)$, the following properties are equivalent:*

(1) $\mathcal{T}$ is soft T_1.
(2) For every $x_e, y_{e'} \in P_E(X)$ with $x_e \neq y_{e'}$, $x_e \vdash y_{e'}$.
(3) For every $x_e \in P_E(X)$, $cl(x_e) = \{x_e\}$.
(4) For every $x_e \in P_E(X)$, $der(x_e) = \Phi$.

(5) For every $x_e \in P_E(X)$, $ker(x_e) = \{x_e\}$.
(6) For every $x_e \in P_E(X)$, $shel(x_e) = \Phi$.
(7) For every $x_e, y_{e'} \in P_E(X)$ with $x_e \neq y_{e'}$, $cl(x_e)\widetilde{\cap}cl(y_{e'}) = \Phi$.
(8) For every $x_e, y_{e'} \in P_E(X)$ with $x_e \neq y_{e'}$, $ker(x_e)\widetilde{\cap}ker(y_{e'}) = \Phi$.

Proof. One can easily notice that all the statements are rephrases of (1) with the help of Lemmas in Section 3. Last statement means $x_e \notin ker(y_{e'})$ and $y_{e'} \notin ker(x_e)$. Equivalently, $y_{e'} \notin cl(x_e)$ and $x_e \notin cl(y_{e'})$. This guarantees the existence of two sets $(G, E), (H, E) \in \mathcal{T}$ such that $x_e \in (G, E), y_{e'} \notin (G, E)$ and $y_{e'} \in (H, E), x_e \notin (H, E)$. Thus, $\mathcal{T}$ is soft T_1. □

We close this investigation with the following remark:

Remark 5.11. *Section 2 recalls soft points and soft elements, which are two distinct types of soft point theory. We have employed the concept of soft points throughout this paper, although most of the (obtained) results are invalid for soft elements. The reasons can be found in [30], Examples 3.14–3.21. The divergences between axioms via classical and soft settings were studied in detail in [5].*

6. Conclusions and future work

Soft separation axioms are a collection of conditions for classifying a system of soft topological spaces according to particular soft topological properties. These axioms are usually described in terms of soft open or soft closed sets in a topological space.

In this work, we have proposed soft topological operators that will be used to characterize certain soft separation axioms and named them "soft kernel" and "soft shell". The interrelations between the latter soft operators and soft closure or soft derived set operators have been discussed. Moreover, we have introduced soft T_D-spaces as a new soft separation axiom that is weaker than soft T_1 but stronger than soft T_0-spaces. It should be noted that T_D-spaces have applications in other (applied) disciplines. Some examples have been provided, illustrating that soft T_D-spaces are at least different from soft T_1 and soft T_0-spaces. The soft topological operators mentioned above are used to obtain new characterizations of soft T_i-spaces for $i = 0, 1$, and D. Ultimately, we have analyzed the validity of our findings in relation to two different theories of soft points.

In the upcoming work, we shall define the axioms given herein and examine their properties via other soft structures like infra soft topologies and supra soft topologies. We will also conduct a comparative study between these axioms and their counterparts introduced with respect to different types of belonging and non-belonging relations. Moreover, we will generalize the concept of functionally separation axioms [23] to soft settings and investigate its relationships with the other types of soft separation axioms.

Conflict of interest

The authors declare that they have no competing interests.

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