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Research article

Integrated decision-making methods based on 2-tuple linguistic m-polar fuzzy information

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Abstract: The 2-tuple linguistic m-polar fuzzy sets (2TLmFSs) are acknowledged to represent the multi-polar information owing to the practical structure of m-polar fuzzy sets with the help of linguistic terms. The TOPSIS and ELECTRE series are efficient and widely used methods for solving multi-attribute decision-making problems. This paper aim to augment the literature on multi-attribute group decision making focusing on the the strategic approaches of TOPSIS and ELECTRE-I methods for the 2TLmFSs. In the 2TLmF-TOPSIS method, the relative closeness index is used to rank the alternatives. For the construction of concordance and discordance sets, the superiority and inferiority of alternatives over each other are accessed by using the score and accuracy functions. In the 2TLmF ELECTRE-I, selection of the best alternative is made by the means of an outranking decision graph. At the final step of the 2TLmF ELECTRE-I method, a supplementary approach is developed for the linear ranking of alternatives based on the concordance and discordance outranking indices. The structure of the proposed techniques are illustrated by using a system flow diagram. Finally, two case studies are used to demonstrate the correctness, transparency, and effectiveness of the proposed methods for selecting highway construction project manager and the best textile industry.

Keywords: 2-tuple linguistic m-polar fuzzy set; 2-tuple linguistic m-polar fuzzy number; accuracy function; TOPSIS method; ELECTRE-I method
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1. Introduction

Decision-making is a technique of making choices by identifying decisions, gathering information, and solving difficulties to choose the best alternative. Multiple-attribute decision making (MADM) is a branch of operations research in which satisfactory solutions are selected based on various key aspects of competing criteria that may be selected as decision problems. This technique can often be used to solve many real-world problems related to the fields of social sciences, economics, medicine, and engineering. MADM is divided into single-expert decision-making vs group decision-making according to the number of experts. Multi-attribute group decision-making (MAGDM) is a technique in which a group of experts presents their preferences to achieve better outcomes than individuals. In general, MAGDM is preferable because the aggregation matrix is developed by using the crowd's decision on the final result. This study is inclined to explore the TOPSIS [1] and ELECTRE [2] methods thoroughly.

To address MADM and MAGDM problems the technique of order preferences by similarity to the ideal solution named (TOPSIS) is widely adopted. The TOPSIS approach works on the fundamental principle to select the best alternative that is closest to the positive ideal solution (PIS) and farthest from the negative ideal solution (NIS). In 1981, Hwang and Yoon [1] developed the TOPSIS technique to cope with decision-making issues. The classical TOPSIS approach only uses crisp data to tackle real-life issues. But, it is very rare in real-life decision-making to find crisp and precise data. The theory of fuzzy sets (FSs), pioneered by Zadeh [3], aimed to capture the ambiguous information to address the inherent fuzziness of real life situations. Firstly, Chen [4] merged the theories of TOPSIS method and FSs to develop new authentic technique to address the inconsistent data. In addition, Shen et al. [5] and Amiri [6] applied the fuzzy TOPSIS technique to pick suppliers and projects in oil field development, respectively. A significant contribution to FS theory was made by Atanassov [7] who introduced the concept of intuitionistic fuzzy set (IFS) that incorporated the non-membership ν and hesitation π degrees with the membership value μ, bounded by the condition $\mu + \nu \leq 1$. Boran et al. [8] proposed a decision-making approach based on TOPSIS method employing the adaptable structure of IFSs. Aloini et al. [9] also contributed to the IF-TOPSIS technique. Yager [10, 11] introduced the concept of Pythagorean fuzzy sets (PFSs) with membership μ, non-membership ν and hesitation $\pi = \sqrt{1 - \mu^2 - \nu^2}$, which relaxed the IFS condition $\mu + \nu \leq 1$ to $\mu^2 + \nu^2 \leq 1$. Further, Zhang and Xu [12] extended the TOPSIS framework for PFSs. Further, Akram et al. [13] provided a modified version of PF-TOPSIS method for group decision-making along with explanatory numerical examples. Yucesan and Gul [14] unfolded the application of PF-TOPSIS method in the field of medical.

The FS IFS and PFS were incompetent in dealing with imprecise and inconsistent multi-polar information. To overcome this limitation, Chen et al. [15] proposed the m-polar fuzzy set (mFS) that can deal with the multi-information for decision-making issues. Jana and Pal [16] presented some basic mF operations and utilized this idea for multi-attribute decision-making. Akram [17] is accredited to introduce the concept of mF graphs. Adeel et al. [18] extended the TOPSIS technique in mF environment for multi-attribute decision making. Akram and Adeel [19] introduced the hesitant m-polar fuzzy TOPSIS method for group decision making.

Various aspects of daily life activities can not be evaluated accurately in a quantitative form but possibly in a qualitative one. Most people would like to express their opinions in dialogues, as good or bad. In this way, the quantification burden can be replaced by the qualitative concept. By utilizing the linguistic preferences, Liu et al. [20] and Akram et al. [21] contributed for group decision making. Xu et al. [22] and Zhu et al. [23] have made a significant contribution for linguistic preferences of data in decision analysis. Further, we can obtain more accurate results by using the 2-tuple linguistic

(2TL) concept in decision-making. Firstly, Herrera and Martínez [24] introduced words processing by using a 2-tuple fuzzy linguistic representation model. For further contribution in 2TL environment, the reader can consult [25–27]. Akram et al. [28] introduced the combined concept of a 2-tuple linguistic and m-polar fuzzy set for decision analysis. In addition, Wei [29] provided the TOPSIS approach for multiple attribute group decision-making based on 2-tuple linguistic fuzzy information.

Benayoun et al. [2] initiated the family of ELimination and Choice translating reality (ELECTRE) methods by presenting ELECTRE-I method. Figueira et al. [30] addressed the variants of ELECTRE strategies. The ELECTRE-I technique in fuzzy environment was first extended by Hatami-Marbini and Tavana [31]. Further, the fuzzy ELECTRE technique was applied for academic staff selection by Rouyendegh and Erkan [32]. For analyzing mobile payment business models, Asghari et al. [33] used the fuzzy ELECTRE technique. Kheirkhah and Dehghani [34] employed the fuzzy ELECTRE technique to assess the quality of public transportation. Chen and Wu [35] put forward the ELECTRE-I approach for the practical and competent framework of IFSs. Akram et al. [36] presented the ELECTRE-I method for multi-criteria group decision making in Pythagorean fuzzy environment. Further, Akram et al. [37] presented the extended version of the ELECTRE-I method in m-polar fuzzy environment. Adeel et al. [38] introduced mHF ELECTRE-I and HmF ELECTRE-I method for multi-criteria decision-making. Adeel et al. [39] contributed to group decision-making by presenting the m-polar fuzzy linguistic ELECTRE-I method. For the related work, the readers are suggested to [40–45].

We now turn to the motivation and key significance of the planned research effort. All existing decision-making processes are successful and appropriate when the decision data are in precise or vaguely imprecise form, but they cannot be exploited when the decision problem contains multipolar imprecise data and 2-tuple linguistic information. We are inspired to extend the proposed MAGDM method based on 2TLmF information for the following reasons.

- The 2TLmF set has a wide range of applications because it combines the advantages of 2TL and m-polar fuzzy sets. However, handling 2-tuple linguistic methods, especially the 2TLmF MAGDM method, in the multi-pole fuzzy case remains a hurdle for us, which we address in this work.
- Existing strategies to solve the MAGDM problem are limited to dealing with m-polar ambiguous information. These methods cannot account for 2-tuple linguistic data. Therefore, information may be lost, leading to undesirable consequences. However, existing technical limitations can be addressed using the newly proposed work.
- The limited literature on 2TLmFS is a major incentive of our research as there is no known decision method, based on TOPSIS or ELECTRE approach, for processing 2TLmF data. Therefore, 2TLmF-TOPSIS and 2TLmF ELECTRE-I models are developed to address this research gap, which combine multi-polarity and 2-tuple linguistic terminology without adding any complexity.

In this article, we offer two strategies, 2TLmF-TOPSIS and 2TLmF ELECTRE-I, based on the 2TLmF information as well as the combined flowchart for proposed work, to extract motivation from the 2TLmFS. These two methods collect the information both linguistically and multi-polar numeric values, which contribute to produce more reliable results. In 2TLmF-TOPSIS, we evaluate the normalized Euclidean distance to access the distance among alternatives by using 2TLmF aggregated weighted decision matrix, 2TLmFPIS, and 2TLmFNIS respectively. Finally, we rank the alternatives by using the relative closeness index. Nevertheless, in the 2TLmF ELECTRE-I method, we can compare the 2TLmF numbers by using their 2TLmF score and 2TLmF accuracy function. Firstly, we obtain the 2TLmF-concordance and 2TLmF-discordance sets by comparing the superiority and inferiority of every alternative over the other. Then, by utilizing these concordance and discordance sets we can obtain the concordance and discordance matrices. Further, we can construct the 2TLmF concordance dominance and 2TLmF discordance dominance matrix by comparing concordance and discordance levels with 2TLmF-concordance and 2TLmF-discordance indices. Ultimately, we use the outranking graph to select the most suitable alternative.

The rest of the proposed research article is organized as: Section 2 reviews some basic concepts about 2-tuple linguistic (2TL) representation models, m-polar fuzzy sets (mF) and 2TLmFS. In Section 3, we introduce the mathematical model of the 2TLmF-TOPSIS approach for MAGDM. In Section 4, two numerical examples related to selection of highway construction project managers and the textile industry are taken into account by using the 2TLmF-TOPSIS approach. In Section 5, we present the 2TLm ELECTRE-I approach with a combined flowchart for 2TLmF-TOPSIS and 2TLm ELECTRE-I approaches. Section 6 addresses the same practical difficulties already discussed in Section 3, solved by using the 2TLmF ELECTRE-I method. Furthermore, in Section 7, we conduct a comparative study to examine the effectiveness of the proposed method. The Section 8 contains the contributions and limitations of the proposed work. The Section 9 gives conclusions and future directions.

2. 2-tuple linguistic m-polar fuzzy sets

This section contains basic definitions that are necessary for this paper.

Definition 2.1. [24] Let $\wp = \{c_j \mid j = 0, \ldots, \sigma\}$ be a set of linguistic terms and $\psi \in [0, \sigma]$ be a number value illustrating the aggregation result of linguistic symbolic. Then the 2-tuple linguistic information equivalent to ψ is obtained by using function Λ defined as

$$\Lambda : [0, \sigma] \to \wp \times [-0.5, 0.5),$$

$$\Lambda(\psi) = \begin{cases} c_j, \; j = \text{round}(\psi) \\ \varrho = \psi - j, \; \varrho \in [-0.5, 0.5). \end{cases} \tag{2.1}$$

Definition 2.2. [24] Let $\wp = \{c_j | j = 0, \ldots, \sigma\}$ be a set of linguistic terms and (c_j, ϱ_j) be a 2-tuple, then the function Λ^{-1} which restore the 2-tuple value to its equivalent numerical value $\psi \in [0, \sigma] \subset \mathbb{R}$ is defined as

$$\begin{aligned} \Lambda^{-1} &: \wp \times [-0.5, 0.5) \to [0, \sigma], \\ &\Lambda^{-1}(c_j, \varrho) = j + \varrho = \psi. \end{aligned} \tag{2.2}$$

Definition 2.3. [24,25] Consider (c_t, ϱ_1) and (c_u, ϱ_2) be two 2TL values. Then,

(1) For $t < u$, we have, $(c_t, \varrho_1) < (c_u, \varrho_2)$.

(2) If t=u, then
a) For $\varrho_1 = \varrho_2 \rightarrow (c_t, \varrho_1)$ and (c_u, ϱ_2) both are same.
b) For $\varrho_1 < \varrho_2 \rightarrow (c_t, \varrho_1)$ is less than (c_u, ϱ_2).
c) For $\varrho_1 > \varrho_2 \rightarrow (c_t, \varrho_1)$ is greater than (c_u, ϱ_2).

Definition 2.4. [15] An mF set $\hat{\c{C}}$ on non-empty set Y is a mapping $\hat{\c{C}} : Y \rightarrow [0,1]^m$. The membership value for every element $y \in Y$ is represented as

$$\hat{\c{C}} = (p_1 \circ \c{C}(y), p_2 \circ \c{C}(y), \ldots, p_m \circ \c{C}(y)).$$

Here $p_i \circ \c{C} : [0,1]^m \rightarrow [0,1]$ is the $i-$th projection mapping.

Where,

- The m-th power of [0, 1] is a poset with the point-wise order $\leq$, where m is any arbitrary ordinal number (For convince, $m = \{n|n < m\}$ when $m > 0$), $\leq$ is defined by $x \leq y \Leftrightarrow p_i(x) \leq p_i(y)$ for each $i \in m$ ($x, y \in [0,1]^m$).
- In the $[0,1]^m$, the greatest value is $\mathbf{1} = (1,1,\ldots,1)$ and the smallest value is $\mathbf{0} = (0,0,\ldots,0)$.
- For convince the mF number is represented as, $\hat{\c{C}} = (p_1 \circ \c{C}, \ldots, p_m \circ \c{C})$.

Definition 2.5. [15] Let $\hat{\c{C}}_1 = (p_1 \circ \c{C}_1, \ldots, p_m \circ \c{C}_1)$, and $\hat{\c{C}}_2 = (p_1 \circ \c{C}_2, \ldots, p_m \circ \c{C}_2)$ be two m-polar fuzzy numbers. Then by using score $\mathbb{S}$ and accuracy function $\mathbb{H}$ we have:

(1) If $\mathbb{S}(\hat{\c{C}}_1) < \mathbb{S}(\hat{\c{C}}_2)$, than $\hat{\c{C}}_1 < \hat{\c{C}}_2$.
(2) If $\mathbb{S}(\hat{\c{C}}_1) > \mathbb{S}(\hat{\c{C}}_2)$, than $\hat{\c{C}}_1 > \hat{\c{C}}_2$.
(3) If $\mathbb{S}(\hat{\c{C}}_1) = \mathbb{S}(\hat{\c{C}}_2)$ and $\mathbb{H}(\hat{\c{C}}_1) = \mathbb{H}(\hat{\c{C}}_2)$, than $\hat{\c{C}}_1 = \hat{\c{C}}_2$.
(4) If $\mathbb{S}(\hat{\c{C}}_1) = \mathbb{S}(\hat{\c{C}}_2)$ but $\mathbb{H}(\hat{\c{C}}_1) < \mathbb{H}(\hat{\c{C}}_2)$, than $\hat{\c{C}}_1 < \hat{\c{C}}_2$.
(5) If $\mathbb{S}(\hat{\c{C}}_1) = \mathbb{S}(\hat{\c{C}}_2)$ but $\mathbb{H}(\hat{\c{C}}_1) > \mathbb{H}(\hat{\c{C}}_2)$, than $\hat{\c{C}}_1 > \hat{\c{C}}_2$.

Definition 2.6. [28] A 2TLmF set $\hat{\Phi}$ on a nonempty set $\mathbb{X}$ is defined as

$$\hat{\Phi} = \{< x, ((c_{\phi_1}(x), \varrho_1(x)), (c_{\phi_2}(x), \varrho_2(x)), \ldots, (c_{\phi_m}(x), \varrho_m(x))) >: x \in \mathbb{X}\},$$

where, the membership degree is denoted as, $(c_{\phi_i}(x), \varrho_i(x))$ with the conditions $c_{\phi_i}(x) \in \hat{\Phi}$, $\varrho_i(x) \in [-0.5, 0.5)$, $0 \leq \Lambda^{-1}(c_{\phi_i}(x), \varrho_i(x)) \leq \sigma, i = 1,2,\ldots,m$.

Conveniently, we say $\chi = ((c_{\phi_1}, \varrho_1), (c_{\phi_2}, \varrho_2), \ldots, (c_{\phi_m}, \varrho_m))$, a 2-tuple linguistic m-polar fuzzy number.

Definition 2.7. [28] The *score function* $\mathbb{S}$ of a 2TL m-polar fuzzy number, $\hat{\chi} = ((c_{\phi_1}, \varrho_1), (c_{\phi_2}, \varrho_2), \ldots, (c_{\phi_m}, \varrho_m))$, is defined as

$$\mathbb{S}(\hat{\chi}) = \Lambda\Big(\frac{\sigma}{m}\sum_{r=1}^{m}(\frac{\Lambda^{-1}(c_{\phi_r}, \varrho_r)}{\sigma})\Big), \quad \Lambda^{-1}(\mathbb{S}(\hat{\chi})) \in [0, \sigma].$$

Definition 2.8. [28] The *accuracy function* $\mathbb{H}$ of a 2TL m-polar fuzzy number, $\hat{\chi} = ((c_{\phi_1}, \varrho_1), (c_{\phi_2}, \varrho_2), \ldots, (c_{\phi_m}, \varrho_m))$, is defined as

$$\mathbb{H}(\hat{\chi}) = \Lambda\Big(\frac{\sigma}{m}\sum_{r=1}^{m}(-1)^r((\frac{\Lambda^{-1}(c_{\phi_r}, \varrho_r)}{\sigma}) - 1)\Big), \quad \Lambda^{-1}(\mathbb{H}(\hat{\chi})) \in [0, \sigma].$$

Definition 2.9. [28] Let $\chi_1 = ((c_{\phi_1^1}, \varrho_1^1), (c_{\phi_2^1}, \varrho_2^1), \ldots, (c_{\phi_m^1}, \varrho_m^1))$, and $\chi_2 = ((c_{\phi_1^2}, \varrho_1^2), (c_{\phi_2^2}, \varrho_2^2), \ldots, (c_{\phi_m^2}, \varrho_m^2))$, be two 2TL$m$F numbers. we define the following operations on 2TLmF numbers as:

(1) $\rho\chi = \Big(\Lambda(\sigma(1-(1-\frac{\Lambda^{-1}(c_{\phi_1},\varrho_1)}{\sigma})^{\rho})), \ldots, \Lambda(\sigma(1-(1-\frac{\Lambda^{-1}(c_{\phi_m},\varrho_m)}{\sigma})^{\rho}))\Big), \rho > 0,$

(2) $\chi^{\rho} = \Big(\Lambda(\sigma(\frac{\Lambda^{-1}(c_{\phi_1},\varrho_1)}{\sigma})^{\rho}), \ldots, \Lambda(\sigma(\frac{\Lambda^{-1}(c_{\phi_m},\varrho_m)}{\sigma})^{\rho})\Big), \rho > 0,$

(3) $$\chi_1 \oplus \chi_2 = (\Lambda(\sigma(\frac{\Lambda^{-1}(c_{\phi_1^1},\varrho_1^1)}{\sigma} + \frac{\Lambda^{-1}(c_{\phi_1^2},\varrho_1^2)}{\sigma} - \frac{\Lambda^{-1}(c_{\phi_1^1},\varrho_1^1)}{\sigma}.\frac{\Lambda^{-1}(c_{\phi_1^2},\varrho_1^2)}{\sigma})), \ldots,$$
$$(\Lambda(\sigma(\frac{\Lambda^{-1}(c_{\phi_m^1},\varrho_m^1)}{\sigma} + \frac{\Lambda^{-1}(c_{\phi_m^2},\varrho_m^2)}{\sigma} - \frac{\Lambda^{-1}(c_{\phi_m^1},\varrho_m^1)}{\sigma}.\frac{\Lambda^{-1}(c_{\phi_m^2},\varrho_m^2)}{\sigma}))),$$

(4) $\chi_1 \otimes \chi_2 = (\Lambda(\sigma(\frac{\Lambda^{-1}(c_{\phi_1^1},\varrho_1^1)}{\sigma}.\frac{\Lambda^{-1}(c_{\phi_1^2},\varrho_1^2)}{\sigma})), \ldots, \Lambda(\sigma(\frac{\Lambda^{-1}(c_{\phi_m^1},\varrho_m^1)}{\sigma}.\frac{\Lambda^{-1}(c_{\phi_m^2},\varrho_m^2)}{\sigma}))).$

Definition 2.10. Let $\chi_1 = ((c_{\phi_1^1}, \varrho_1^1), (c_{\phi_2^1}, \varrho_2^1), \ldots, (c_{\phi_m^1}, \varrho_m^1))$, and $\chi_2 = ((c_{\phi_1^2}, \varrho_1^2), (c_{\phi_2^2}, \varrho_2^2), \ldots, (c_{\phi_m^2}, \varrho_m^2))$, be two 2TL$m$F sets. Then the normalized Euclidean distance between χ_1 and χ_2 is defined as

$$\mathbb{D}(\chi_1, \chi_2) = \sqrt{\frac{1}{m}\Bigg[\bigg(\frac{\Lambda^{-1}(c_{\phi_1^1},\varrho_1^1)}{\sigma} - \frac{\Lambda^{-1}(c_{\psi_1^2},\varrho_1^2)}{\sigma}\bigg)^2 + \ldots + \bigg(\frac{\Lambda^{-1}(c_{\phi_m^1},\varrho_m^1)}{\sigma} - \frac{\Lambda^{-1}(c_{\phi_m^2},\varrho_m^2)}{\sigma}\bigg)^2\Bigg]}.$$

3. Structure of 2TLmF-TOPSIS method for MAGDM

In this section, we construct a mathematical tool to interact with the 2TLmF TOPSIS approach for multi-attribute group decision making (MAGDM). In this approach, we consider $\mathfrak{R} = \{\mathfrak{R}_1, \mathfrak{R}_2, \ldots, \mathfrak{R}_v\}$ the set of decision makers who are recruited for decision-making. where each expert ($\mathfrak{R}_u$, u=1,2,...,v) assigns a suitable rating to every attribute by observing its impact on alternative and the ratings values must be in the form of 2TLmF numbers. The developed 2TLmF-TOPSIS method is used to evaluate the most desirable alternative which is closest to the positive ideal solution (PIS) and farthest from negative ideal solution (NIS). Let $F = \{F_1, F_2, \ldots, F_k\}$ be the set of alternatives against the 2TLmF information from which the best one is selected based on some attributes denoted as, $\zeta = \{\zeta_1, \zeta_2, \ldots, \zeta_t\}$. let q, $(q = 1, 2, \ldots, m)$ be the number of poles according to m characteristics and $(c_{\phi_q}(x), \varrho_q(x))$, $q = 1, 2, \ldots, m$ be the number of membership values to each pole. The weight vector for attributes is represented as, $\varphi = \{\varphi_1, \varphi_2, \ldots, \varphi_t\}$, where $\varphi \in [0, 1]$ and $\Sigma_{p=1}^{t}\varphi_p = 1$. The main ambition is the evaluation of most desirable alternative as the solution of this MAGDM problem. We present the proposed 2TLmF-TOPSIS method step by step as follows:

Step 1: In this step, a group of experts are responsible for assessing 2TLm-polar fuzzy information of k distinct alternatives, and the appropriate ratings of alternatives are determined according to all experts that are evaluated in terms of m various qualities. Tabular depiction of 2TLmF decision matrix, $\mathbb{Z}^{(u)} = (z_{ij}^{(u)})_{k\times t} = ((c^{(u)}_{\phi_1^{ij}}, \varrho_1^{ij^{(u)}}), (c^{(u)}_{\phi_2^{ij}}, \varrho_2^{ij^{(u)}}), \ldots, (c^{(u)}_{\phi_m^{ij}}, \varrho_m^{ij^{(u)}}))_{k\times t}$, of predicted ratings under the group of u decision-makers according to their choice is given below:

$$\mathbb{Z}^{(u)} = \begin{array}{cc} & \begin{array}{cccc} \zeta_1 & \zeta_2 & \cdots & \zeta_t \end{array} \\ \begin{array}{c} F_1 \\ F_2 \\ \vdots \\ F_k \end{array} & \begin{pmatrix} ((c^{(u)}_{\phi_1^{11}}, \varrho_1^{11^{(u)}}), (c^{(u)}_{\phi_2^{11}}, \varrho_2^{11^{(u)}}), \ldots, (c^{(u)}_{\phi_m^{11}}, \varrho_m^{11^{(u)}})) & ((c^{(u)}_{\phi_1^{12}}, \varrho_1^{12^{(u)}}), (c^{(u)}_{\phi_2^{12}}, \varrho_2^{12^{(u)}}), \ldots, (c^{(u)}_{\phi_m^{12}}, \varrho_m^{12^{(u)}})) & \cdots & ((c^{(u)}_{\phi_1^{1t}}, \varrho_1^{1t^{(u)}}), (c^{(u)}_{\phi_2^{1t}}, \varrho_2^{1t^{(u)}}), \ldots, (c^{(u)}_{\phi_m^{1t}}, \varrho_m^{1t^{(u)}})) \\ ((c^{(u)}_{\phi_1^{21}}, \varrho_1^{21^{(u)}}), (c^{(u)}_{\phi_2^{21}}, \varrho_{u2}^{21^{(u)}}), \ldots, (c^{(u)}_{\phi_m^{21}}, \varrho_m^{21^{(u)}})) & ((c^{(u)}_{\phi_1^{22}}, \varrho_1^{22^{(u)}}), (c^{(u)}_{\phi_2^{22}}, \varrho_2^{22^{(u)}}), \ldots, (c^{(u)}_{\phi_m^{22}}, \varrho_m^{22^{(u)}})) & \cdots & ((c^{(u)}_{\phi_1^{2t}}, \varrho_1^{2t^{(u)}}), (c^{(u)}_{\phi_2^{2t}}, \varrho_2^{2t^{(u)}}), \ldots, (c^{(u)}_{\phi_m^{2t}}, \varrho_m^{2t^{(u)}})) \\ \vdots & \vdots & \vdots & \vdots \\ ((c^{(u)}_{\phi_1^{k1}}, \varrho_1^{k1^{(u)}}), (c^{(u)}_{\phi_2^{k1}}, \varrho_2^{k1^{(u)}}), \ldots, (c^{(u)}_{\phi_m^{k1}}, \varrho_m^{k1^{(u)}})) & ((c^{(u)}_{\phi_1^{k2}}, \varrho_1^{k2^{(u)}}), (c^{(u)}_{\phi_2^{k2}}, \varrho_2^{k2^{(u)}}), \ldots, (c^{(u)}_{\phi_m^{k2}}, \varrho_m^{k2^{(u)}})) & \cdots & ((c^{(u)}_{\phi_1^{kt}}, \varrho_1^{kt^{(u)}}), (c^{(u)}_{\phi_2^{kt}}, \varrho_2^{kt^{(u)}}), \ldots, (c^{(u)}_{\phi_m^{kt}}, \varrho_m^{kt^{(u)}})) \end{pmatrix} \end{array}.$$

Step 2: The weight vector for DMs denoted as $\varpi = \{\varpi_1, \varpi_1, \ldots, \varpi_v\}$, depending on their importance and weight vector satisfying $\varpi_u \in [0, 1]$, $\sum_{u=1}^{v} \varpi_u = 1$. By utilizing the group decision making given in matrix, $\mathbb{Z}^{(u)}$ we drive the aggregated 2TL*m*F decision matrix $\mathbb{Z} = (z_{ij})_{k\times t} = ((c_{\phi_1^{ij}}, \varrho_1^{ij}), (c_{\phi_2^{ij}}, \varrho_2^{ij}), \ldots, (c_{\phi_m^{ij}}, \varrho_m^{ij}))_{k\times t}$, by using 2TL*m*F weighted average operator (2TL*m*FWA)defined as

$$z_{ij} = 2TLmFWA_{\varpi}(z_{ij}^{(1)}, z_{ij}^{(2)}, \ldots, z_{ij}^{(v)}),$$

$$= \varpi_1 z_{ij}^{(1)} \oplus \varpi_2 z_{ij}^{(2)} \oplus \ldots \oplus \varpi_n z_{ij}^{(v)},$$

$$= \left(\Lambda(\sigma(1 - \prod_{u=1}^{v} (1 - \frac{\Lambda^{-1}(c^{(u)}_{\phi_1^{ij}}, \varrho_1^{ij^{(u)}})}{\sigma})^{\varpi_u})), \Lambda(\sigma(1 - \prod_{u=1}^{v} (1 - \frac{\Lambda^{-1}(c^{(u)}_{\phi_2^{ij}}, \varrho_2^{ij})}{\sigma})^{\varpi_u})), \ldots, \Lambda(\sigma(1 - \prod_{u=1}^{v} (1 - \frac{\Lambda^{-1}(c^{(u)}_{\phi_m^{ij}}, \varrho_m^{ij})}{\sigma})^{\varpi_u})) \right).$$

The aggregated 2TL*m*F decision matrix which obtained by using the 2TL*m*FWA operator can be displayed as:

$$\mathbb{Z} = \begin{array}{cc} & \begin{array}{cccc} \zeta_1 & \zeta_2 & \cdots & \zeta_t \end{array} \\ \begin{array}{c} F_1 \\ F_2 \\ \vdots \\ F_k \end{array} & \begin{pmatrix} ((c_{\phi_1^{11}}, \varrho_1^{11}), (c_{\phi_2^{11}}, \varrho_2^{11}), \ldots, (c_{\phi_m^{11}}, \varrho_m^{11})) & ((c_{\phi_1^{12}}, \varrho_1^{12}), (c_{\phi_2^{12}}, \varrho_2^{12}), \ldots, (c_{\phi_m^{12}}, \varrho_m^{12})) & \cdots & ((c_{\phi_1^{1t}}, \varrho_1^{1t}), (c_{\phi_2^{1t}}, \varrho_2^{1t}), \ldots, (c_{\phi_m^{1t}}, \varrho_m^{1t})) \\ ((c_{\phi_1^{21}}, \varrho_1^{21}), (c_{\phi_2^{21}}, \varrho_2^{21}), \ldots, (c_{\phi_m^{21}}, \varrho_m^{21})) & ((c_{\phi_1^{22}}, \varrho_1^{22}), (c_{\phi_2^{22}}, \varrho_2^{22}), \ldots, (c_{\phi_m^{22}}, \varrho_m^{22})) & \cdots & ((c_{\phi_1^{2t}}, \varrho_1^{2t}), (c_{\phi_2^{2t}}, \varrho_2^{2t}), \ldots, (c_{\phi_m^{2t}}, \varrho_m^{2t})) \\ \vdots & \vdots & \vdots & \vdots \\ ((c_{\phi_1^{k1}}, \varrho_1^{k1}), (c_{\phi_2^{k1}}, \varrho_2^{k1}), \ldots, (c_{\phi_m^{k1}}, \varrho_m^{k1})) & ((c_{\phi_1^{k2}}, \varrho_1^{k2}), (c_{\phi_2^{k2}}, \varrho_2^{k2}), \ldots, (c_{\phi_m^{k2}}, \varrho_m^{k2})) & \cdots & ((c_{\phi_1^{kt}}, \varrho_1^{kt}), (c_{\phi_2^{kt}}, \varrho_2^{kt}), \ldots, (c_{\phi_m^{kt}}, \varrho_m^{kt})) \end{pmatrix} \end{array}.$$

Step 3: All of the attributes may not be equally significant. The decision-makers assigned the weights to each attribute based on their relevance for the alternatives. Let $\varphi = \{\varphi_1, \varphi_2, \ldots, \varphi_t\}$, be the weight vector for attributes, it must satisfy the normality condition as, $\Sigma_{p=1}^{t} \varphi_p = 1$ where $\varphi \in [0, 1]$.

Step 4: Compute the aggregated weighted 2TL*m*F decision matrix.

$$\hat{\mathbb{Z}} = (\hat{z}_{ij})_{k\times t} = ((\hat{c}_{\phi_1^{ij}}, \hat{\varrho}_1^{ij}), (\hat{c}_{\phi_2^{ij}}, \hat{\varrho}_2^{ij}), \ldots, (\hat{c}_{\phi_m^{ij}}, \hat{\varrho}_m^{ij}))_{k\times t},$$

where,

$$\hat{z_{ij}} = z_{ij} \otimes \varphi_p,$$

$$= \left(\Lambda(\sigma(1 - (1 - \frac{\Lambda^{-1}(c_{\phi_1^{ij}}, \varrho_1^{ij})}{\sigma})^{\varphi_1})), \ldots, \Lambda(\sigma(1 - (1 - \frac{\Lambda^{-1}(c_{\phi_m^{ij}}, \varrho_m^{ij})}{\sigma})^{\varphi_n})) \right), \ \varphi_p > 0, \ p = 1, 2, \ldots, t.$$

The 2TLmF aggregated weighted decision matrix can be evaluated as

$$\hat{\mathbb{Z}} = \begin{array}{c} \\ F_1 \\ F_2 \\ \vdots \\ F_k \end{array} \begin{array}{c} \begin{array}{cccc} \zeta_1 & \zeta_2 & \cdots & \zeta_t \end{array} \\ \begin{pmatrix} ((\hat{c}_{\phi_1^{11}}, \hat{\varrho}_1^{11}), (\hat{c}_{\phi_2^{11}}, \hat{\varrho}_2^{11}), \ldots, (\hat{c}_{\phi_m^{11}}, \hat{\varrho}_m^{11})) & ((\hat{c}_{\phi_1^{12}}, \hat{\varrho}_1^{12}), (\hat{c}_{\phi_2^{12}}, \hat{\varrho}_2^{12}), \ldots, (\hat{c}_{\phi_m^{12}}, \hat{\varrho}_m^{12})) & \cdots & ((\hat{c}_{\phi_1^{1t}}, \hat{\varrho}_1^{1t}), (\hat{c}_{\phi_2^{1t}}, \hat{\varrho}_2^{1t}), \ldots, (\hat{c}_{\phi_m^{1t}}, \hat{\varrho}_m^{1t})) \\ ((\hat{c}_{\phi_1^{21}}, \hat{\varrho}_1^{21}), (\hat{c}_{\phi_2^{21}}, \hat{\varrho}_2^{21}), \ldots, (\hat{c}_{\phi_m^{21}}, \hat{\varrho}_m^{21})) & ((\hat{c}_{\phi_1^{22}}, \hat{\varrho}_1^{22}), (\hat{c}_{\phi_2^{22}}, \hat{\varrho}_2^{22}), \ldots, (\hat{c}_{\phi_m^{22}}, \hat{\varrho}_m^{22})) & \cdots & ((\hat{c}_{\phi_1^{2t}}, \hat{\varrho}_1^{2t}), (\hat{c}_{\phi_2^{2t}}, \hat{\varrho}_2^{2t}), \ldots, (\hat{c}_{\phi_m^{2t}}, \hat{\varrho}_m^{2t})) \\ \vdots & \vdots & \vdots & \vdots \\ ((\hat{c}_{\phi_1^{k1}}, \hat{\varrho}_1^{k1}), (\hat{c}_{\phi_2^{k1}}, \hat{\varrho}_2^{k1}), \ldots, (\hat{c}_{\phi_m^{k1}}, \hat{\varrho}_m^{k1})) & ((\hat{c}_{\phi_1^{k2}}, \hat{\varrho}_1^{k2}), (\hat{c}_{\phi_2^{k2}}, \hat{\varrho}_2^{k2}), \ldots, (\hat{c}_{\phi_m^{k2}}, \hat{\varrho}_m^{k2})) & \cdots & ((\hat{c}_{\phi_1^{kt}}, \hat{\varrho}_1^{kt}), (\hat{c}_{\phi_2^{kt}}, \hat{\varrho}_2^{kt}), \ldots, (\hat{c}_{\phi_m^{kt}}, \hat{\varrho}_m^{kt})) \end{pmatrix} \end{array}.$$

Step 5: Calculate the 2TLmF positive ideal solution ($2TL\text{m}FP_{IS}$) and 2TLmF negative ideal solution ($2TL\text{m}FN_{IS}$) as

$$\begin{aligned} 2TL\text{m}FP_{IS} &= ((\hat{c}_{\phi_1^{ij}}, \hat{\varrho}_1^{ij})^+, (\hat{c}_{\phi_2^{ij}}, \hat{\varrho}_2^{ij})^+, \ldots, (\hat{c}_{\phi_m^{ij}}, \hat{\varrho}_m^{ij})^+), \\ &= \left\{\left(max_i(\hat{c}_{\phi_q^{ij}}, \hat{\varrho}_q^{ij})|j \in I\right), \left(min_i(\hat{c}_{\phi_q^{ij}}, \hat{\varrho}_q^{ij})|j \in J\right)|i = 1, 2, \ldots, k\right\}, q = 1, 2, \ldots, m \end{aligned}$$

and

$$\begin{aligned} 2TL\text{m}FN_{IS} &= ((\hat{c}_{\phi_1^{ij}}, \hat{\varrho}_1^{ij})^-, (\hat{c}_{\phi_2^{ij}}, \hat{\varrho}_2^{ij})^-, \ldots, (\hat{c}_{\phi_m^{ij}}, \hat{\varrho}_m^{ij})^-), \\ &= \left\{\left(min_i(\hat{c}_{\phi_q^{ij}}, \hat{\varrho}_q^{ij})|j \in I\right), \left(max_i(\hat{c}_{\phi_q^{ij}}, \hat{\varrho}_q^{ij})|j \in J\right)|i = 1, 2, \ldots, k\right\}, q = 1, 2, \ldots, m \end{aligned}$$

where I and J are the benefit and cost criterions respectively.

Step 6: Compute the separation of each alternative l_k, $(i = 1, 2, \ldots, k)$ from $2TL\text{m}FP_{IS}$ and $2TL\text{m}FN_{IS}$, respectively, by using normalized Euclidean distance formula given as follows:

$$\hat{\mathbb{D}}(l_i, 2TL\text{m}FP_{IS}) = \sqrt{\frac{1}{mt}\sum_{j=1}^{t}\left[\left(\frac{\Lambda^{-1}(c_{\phi_1^{ij}}, \varrho_1^{ij})}{\sigma} - \frac{\Lambda^{-1}(\hat{c}_{\phi_1^{ij}}, \hat{\varrho}_1^{ij})^+}{\sigma}\right)^2 + \ldots + \left(\frac{\Lambda^{-1}(c_{\phi_m^{ij}}, \varrho_m^{ij})}{\sigma} - \frac{\Lambda^{-1}(\hat{c}_{\phi_m^{ij}}, \hat{\varrho}_m^{ij})^+}{\sigma}\right)^2\right]},$$

$$\hat{\mathbb{D}}(l_i, 2TL\text{m}FN_{IS}) = \sqrt{\frac{1}{mt}\sum_{j=1}^{t}\left[\left(\frac{\Lambda^{-1}(c_{\phi_1^{ij}}, \varrho_1^{ij})}{\sigma} - \frac{\Lambda^{-1}(\hat{c}_{\phi_1^{ij}}, \hat{\varrho}_1^{ij})^-}{\sigma}\right)^2 + \ldots + \left(\frac{\Lambda^{-1}(c_{\phi_m^{ij}}, \varrho_m^{ij})}{\sigma} - \frac{\Lambda^{-1}(\hat{c}_{\phi_m^{ij}}, \hat{\varrho}_m^{ij})^-}{\sigma}\right)^2\right]}.$$

Step 7: Compute the relative 2TLmF closeness index of each alternative which is defined as

$$\hat{\mathbb{E}}_i = \frac{\hat{\mathbb{D}}(l_i, 2TLmFN_{IS})}{\hat{\mathbb{D}}(l_i, 2TLmFP_{IS}) + \hat{\mathbb{D}}(l_i, 2TLmFN_{IS})}, i = 1, 2, \ldots, k.$$

Step 8: Rank the objects based on their index values. The alternative which have the highest 2TLmF closeness index will be the desired alternative.

4. Application

In this section, we apply the 2TLmF-TOPSIS method to select the meritable manager for the project of highway construction and best textile industry which, transparently elaborate the proposed model.

4.1. Numerical Example I: Selection of highway construction project manager

Construction project management needs varieties of skills including the ability to interface with a diverse range of agencies and groups of people to lead the project from idea to building. A manager needs to follow the management principles during each phase of management and plays a crucial role to make project planning, coordination, budgeting, and supervision of the construction project. Construction planning is a challenging task in the management of projects. As the manager supervises the project from an initial and makes decisions to define, the bidding process, contractor selection, and project delivery method. Highway construction project managers are responsible for all aspects related to building materials. They must look with engineers and architects to develop plans, especially determining labor and material cost also, responsible for ensuring the project must complete within budget and on time. Generally, project management is the resources management over the life cycle of a project through different tools and methodologies to control cost, time and quality, etc. The main objective is to develop a systematic method for identifying the best among the candidates in the process of construction project manager selection. Let $F = \{F_1, F_2, F_3, F_4, F_5, F_6\}$ be the set of managers as an alternatives. The proposed work is examined by using a case study in a project-based organization for selecting the most suitable project construction manager, in which six candidates under four different criteria $\zeta = \{\zeta_1, \zeta_2, \zeta_3, \zeta_4\}$ are evaluated and prioritized by decision-makers, $\Re = \{\Re_1, \Re_2, \Re_3\}$ with weight vector $\varpi = \{0.3571, 0.3455, 0.2974\}$. The hired experts express their assessments using linguistic terms as, $\wp$={c_0=extremely poor, c_1=very poor, c_2=poor, c_3=fair, c_4=good, c_5=very good, c_6=extremely good}. However, if we continue to use these linguistic terms, we may end up using the same linguistic term for various alternatives, making it impossible to rank the options to draw conclusions. Therefore, we solve this problem using zero-symbol translation, which transforms linguistic term data into 2TL data and allows us to rank alternatives based on symbol translation. The decision-maker select the best one manager under the following criteria:
ζ_1: Basic requirements.
ζ_2: Management Skills.
ζ_3: Project management skills.
ζ_4:Interpersonal skills.
Each criterion has been divided into four components to form a 2TL4-polar fuzzy set.

- **Basic requirements**: To select the best manager for a construction project, the decision-makers must have a look at the basic needs for project management. Thus to make the 2TL4-polar fuzzy set, we consider four factors of basic requirements as qualification, experience, communication skills, and computer skills.
- **Management skills**: Construction management is a complex issue, so deal with management skills involve the allocation of resources, cost management, risk management, and human resources. Here we take the four factors of management skills as time management, budget management, resources management, and Quality management.
- **Project management skills**: From the initial phase of the project, it is the responsibility of the project manager to plan the entire process. staffing, creating benchmarks is the essential duty of the project manager. It is used to evaluate and regulate project health. To make a 2TL4-polar fuzzy set we consider four characteristics of Project management skills like planning, organizing,

controlling, and monitoring progress

- **Interpersonal skills**: Interpersonal skills play a crucial role in project management skills. The manager can eye on their team and address the issues that might arise. Take four factors as problem-solving, decision-making, team development, and leadership skills. The subdivision of criterion is elaborated in Figure 1.

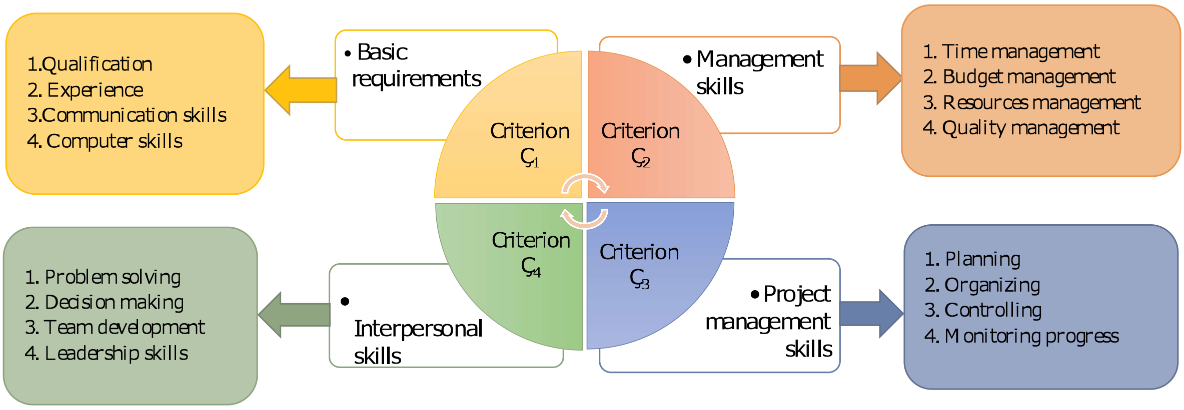

Figure 1. Representation of criterion subdivision for 2TL4-polar fuzzy set.

We apply our proposed 2TL*m*F-TOPSIS method to solve MAGDM problem.

(1) The 2TL*m*F preference ratings of decision makers $\mathfrak{R}_1$, $\mathfrak{R}_2$ and $\mathfrak{R}_3$ are arranged in Tables 1, 2 and 3, respectively.

Table 1. 2TL4F decision matrix provided by first expert $\mathfrak{R}_1$.

Alternatives	Ç_1	Ç_2	Ç_3	Ç_4
F_1	$((c_4,0),(c_2,0),(c_3,0),(c_4,0))$	$((c_4,0),(c_3,0),(c_4,0),(c_5,0))$	$((c_3,0),(c_5,0),(c_5,0),(s_4,0))$	$((c_4,0),(c_3,0),(c_3,0),(c_4,0))$
F_2	$((c_4,0),(c_5,0),(c_5,0),(c_4,0))$	$((c_5,0),(c_4,0),(c_5,0),(c_4,0))$	$((c_4,0),(c_5,0),(c_5,0),(c_4,0))$	$((c_5,0),(c_6,0),(c_4,0),(c_3,0))$
F_3	$((c_5,0),(c_6,0),(c_5,0),(c_5,0))$	$((c_5,0),(c_5,0),(c_6,0),(c_4,0))$	$((c_4,0),(c_5,0),(c_5,0),(c_5,0))$	$((c_6,0),(c_5,0),(c_6,0),(c_5,0))$
F_4	$((c_4,0),(c_3,0),(c_5,0),(c_3,0))$	$((c_3,0),(c_5,0),(c_4,0),(c_4,0))$	$((c_5,0),(c_2,0),(c_3,0),(c_5,0))$	$((c_3,0),(c_4,0),(c_4,0),(c_4,0))$
F_5	$((c_4,0),(c_3,0),(c_3,0),(c_3,0))$	$((c_4,0),(c_3,0),(c_4,0),(c_3,0))$	$((c_3,0),(c_4,0),(c_3,0),(c_4,0))$	$((c_4,0),(c_3,0),(c_4,0),(c_4,0))$
F_6	$((c_5,0),(c_5,0),(c_6,0),(c_5,0))$	$((c_3,0),(c_4,0),(c_5,0),(c_4,0))$	$((c_6,0),(c_4,0),(c_5,0),(c_5,0))$	$((c_6,0),(c_5,0),(c_6,0),(c_5,0))$

Table 2. 2TL4F decision matrix provided by second expert $\mathfrak{R}_2$.

Alternatives	Ç_1	Ç_2	Ç_3	Ç_4
F_1	$((c_4,0),(c_4,0),(c_3,0),(c_4,0))$	$((c_4,0),(c_3,0),(c_5,0),(c_4,0))$	$((c_3,0),(c_4,0),(c_5,0),(s_5,0))$	$((c_4,0),(c_3,0),(c_4,0),(c_3,0))$
F_2	$((c_5,0),(c_4,0),(c_5,0),(c_4,0))$	$((c_4,0),(c_5,0),(c_4,0),(c_5,0))$	$((c_4,0),(c_5,0),(c_4,0),(c_5,0))$	$((c_4,0),(c_5,0),(c_5,0),(c_4,0))$
F_3	$((c_5,0),(c_6,0),(c_5,0),(c_5,0))$	$((c_5,0),(c_4,0),(c_6,0),(c_4,0))$	$((c_5,0),(c_6,0),(c_5,0),(c_6,0))$	$((c_5,0),(c_6,0),(c_5,0),(c_5,0))$
F_4	$((c_4,0),(c_5,0),(c_4,0),(c_3,0))$	$((c_4,0),(c_5,0),(c_5,0),(c_4,0))$	$((c_4,0),(c_3,0),(c_4,0),(c_3,0))$	$((c_4,0),(c_3,0),(c_4,0),(c_4,0))$
F_5	$((c_3,0),(c_4,0),(c_3,0),(c_5,0))$	$((c_5,0),(c_4,0),(c_3,0),(c_4,0))$	$((c_4,0),(c_5,0),(c_4,0),(c_5,0))$	$((c_3,0),(c_4,0),(c_3,0),(c_5,0))$
F_6	$((c_6,0),(c_4,0),(c_6,0),(c_5,0))$	$((c_4,0),(c_5,0),(c_4,0),(c_3,0))$	$((c_6,0),(c_5,0),(c_5,0),(c_6,0))$	$((c_6,0),(c_5,0),(c_4,0),(c_6,0))$

Table 3. 2TL4F decision matrix provided by third expert $\mathfrak{R}_3$.

Alternatives	Ç$_1$	Ç$_2$	Ç$_3$	Ç$_4$
F_1	$((c_5,0),(c_3,0),(c_4,0),(c_3,0))$	$((c_4,0),(c_3,0),(c_4,0),(c_4,0))$	$((c_3,0),(c_5,0),(c_5,0),(s_5,0))$	$((c_4,0),(c_3,0),(c_5,0),(c_4,0))$
F_2	$((c_4,0),(c_5,0),(c_4,0),(c_4,0))$	$((c_5,0),(c_4,0),(c_5,0),(c_5,0))$	$((c_4,0),(c_4,0),(c_5,0),(c_6,0))$	$((c_3,0),(c_5,0),(c_3,0),(c_4,0))$
F_3	$((c_5,0),(c_6,0),(c_5,0),(c_4,0))$	$((c_4,0),(c_5,0),(c_6,0),(c_5,0))$	$((c_4,0),(c_6,0),(c_5,0),(c_6,0))$	$((c_4,0),(c_5,0),(c_6,0),(c_5,0))$
F_4	$((c_4,0),(c_3,0),(c_3,0),(c_5,0))$	$((c_5,0),(c_4,0),(c_4,0),(c_3,0))$	$((c_4,0),(c_3,0),(c_4,0),(c_3,0))$	$((c_4,0),(c_3,0),(c_3,0),(c_5,0))$
F_5	$((c_4,0),(c_3,0),(c_4,0),(c_5,0))$	$((c_4,0),(c_3,0),(c_3,0),(c_4,0))$	$((c_3,0),(c_4,0),(c_4,0),(c_5,0))$	$((c_3,0),(c_4,0),(c_4,0),(c_5,0))$
F_6	$((c_6,0),(c_4,0),(c_5,0),(c_6,0))$	$((c_4,0),(c_5,0),(c_5,0),(c_5,0))$	$((c_6,0),(c_5,0),(c_6,0),(c_5,0))$	$((c_6,0),(c_4,0),(c_5,0),(c_6,0))$

(2) The aggregated 2TL4F decision matrix as given in Table 4.

Table 4. Aggregated 2TL4-polar decision matrix by using 2TL*m*FWA operator.

Alternatives	Ç$_1$	Ç$_2$
F_1	$((c_4,0.37256),(c_3,0.11000),(c_3,0.34079),(c_4,-0.2563))$	$((c_4,0.00000),(c_3,0.00000),(c_4,0.42592),(c_4,0.43853))$
F_2	$((c_4,0.42592),(c_5,-0.2705),(c_5,-0.2289),(c_4,0.00000))$	$((c_5,-0.2705),(c_4,0.42592),(c_5,-0.2705),(c_5,-0.2808))$
F_3	$((c_5,0.00000),(c_6,0.00001),(c_5,0.00000),(c_5,-0.2289))$	$((c_5,-0.2289),(c_5,-0.2705),(c_6,0.00000),(c_4,0.37256))$
F_4	$((c_4,0.00000),(c_4,-0.0524),(c_4,0.23842),(c_4,-0.1638))$	$((c_4,0.11901),(c_5,-0.2289),(c_4,0.42592),(c_4,-0.2563))$
F_5	$((c_4,-0.3007),(c_3,0.39215),(c_3,0.34079),(c_5,0.00000))$	$((c_4,0.42592),(c_3,0.39215),(c_3,0.40439),(c_4,-0.3115))$
F_6	$((c_6,0.00000),(c_4,0.43853),(c_6,0.00000),(c_6,0.00000))$	$((c_4,-0.3115),(c_5,-0.2808),(c_5,-0.2705),(c_4,0.12783))$

Alternatives	Ç$_3$	Ç$_4$
F_1	$((c_3,0.00000),(c_5,-0.2705),(c_5,0.00000),(c_5,-0.2808))$	$((c_4,0.00000),(c_3,0.00000),(c_4,0.11901),(c_4,-0.3007))$
F_2	$((c_4,0.00000),(c_5,-0.22892),(c_5,-0.2705),(c_6,0.00000))$	$((c_4,0.23842),(c_6,0.00000),(c_4,0.22420),(c_4,-0.3115))$
F_3	$((c_4,0.42592),(c_6,0.00000),(c_5,0.00001),(c_6,0.00000))$	$((c_6,0.00001),(c_6,0.00000),(c_6,0.00000),(c_5,0.00000))$
F_4	$((c_4,0.43853),(c_3,-0.3245),(c_4,-0.3115),(c_4,-0.0264))$	$((c_4,-0.3115),(c_3,0.40439),(c_4,-0.2563),(c_4,0.37256))$
F_5	$((c_3,0.39215),(c_4,0.39215),(c_4,-0.3115),(c_5,-0.2808))$	$((c_3,0.00000),(c_4,0.00000),(c_3,0.34079),(c_5,-0.2808))$
F_6	$((c_6,0.00000),(c_5,-0.2808),(c_6,0.00000),(c_6,0.00000))$	$((c_6,0.00000),(c_5,-0.2289),(c_6,0.00000),(c_6,0.00000))$

(3) Suppose the decision makers assign the following weights to the each attribute.

$$\varphi=(\varphi_1,\varphi_2,\varphi_3,\varphi_4)=(0.3,0.2,0.4,0.1).$$

(4) The aggregated weighted 2TL4F decision matrix as given in Table 5.

Table 5. Aggregated weighted 2TL4-polar decision matrix.

Alternatives	Ç$_1$	Ç$_2$
F_1	$((c_2,-0.05655),(c_1,0.18079),(c_1,0.29963),(c_2,-0.47430))$	$((c_1,0.18355),(c_1,-0.22330),(c_1,0.40880),(c_1,0.41618))$
F_2	$((c_2,-0.01617),(c_2,0.23376),(c_2,0.27124),(c_2,-0.31533))$	$((c_2,-0.39867),(c_1,0.40880),(c_2,-0.3986),(c_2,-0.40575))$
F_3	$((c_2,0.49485),(c_6,0.00000),(c_2,0.494851),(c_2,0.271240))$	$((c_2,-0.36944),(c_2,-0.39867),(c_6,0.00000),(c_1,0.37808))$
F_4	$((c_2,-0.3153),(c_2,-0.3489),(c_2,-0.15409),(c_2,-0.41848))$	$((c_1,0.24228),(c_2,-0.36944),(c_1,0.40880),(c_1,0.06597))$
F_5	$((c_1,0.499431),(c_1,0.32705),(c_1,0.29963),(c_2,0.49485))$	$((c_1,0.40880),(c_1,-0.07898),(c_1,-0.07421),(c_1,0.04203))$
F_6	$((c_6,0.00000),(c_2,-0.006501),(c_6,0.00000),(c_6,0.00000))$	$((c_1,0.04203),(c_2,-0.40575),(c_2,-0.39867),(c_1,0.24676))$

Alternatives	Ç$_3$	Ç$_4$
F_1	$((c_1,0.45285),(c_3,-0.22472),(c_3,0.06984),(c_3,-0.23511))$	$((c_1,-0.37575),(c_0,0.40180),(c_1,-0.34287),(c_1,-0.45158))$
F_2	$((c_2,0.13363),(c_3,-0.18200),(c_3,-0.22472),(c_6,0.00000))$	$((c_1,-0.30794),(c_6,0.00000),(c_1,-0.31221),(c_1,-0.45415))$
F_3	$((c_2,0.486812),(c_6,0.00000),(c_3,0.06984),(c_6,0.00000))$	$((c_6,0.00000),(c_6,0.00001),(c_6,0.00000),(c_1,-0.01575))$
F_4	$((c_2,0.48681),(c_6,0.00000),(c_3,0.069841),(c_6,0.00000))$	$((c_6,0.00000),(c_6,0.00001),(c_6,0.00000),(c_1,-0.01575))$
F_5	$((c_2,-0.29935),(c_2,0.48681),(c_2,-0.0969),(c_3,-0.2351))$	$((c_0,0.40180),(c_1,-0.37575),(c_0,0.4689),(c_1,-0.14145))$
F_6	$((c_6,0.00000),(c_3,-0.23511),(c_6,0.00000),(c_6,0.00000))$	$((c_6,0.00000),(c_1,-0.12022),(c_6,0.00000),(c_6,0.00000))$

(6) The 2TL4F positive ideal solution ($2TL4FP_{IS}$) is given as

$$2TL4FP_{IS} = \Bigg\{\Big((c_6, 0.00000), (c_6, 0.00000), (c_6, 0.00000), (c_6, 0.00000)\Big),\\ \Big((c_2, -0.3694), (c_2, -0.3694), (c_6, 0.0000), (c_2, -0.4057)\Big),\\ \Big((c_6, 0.00000), (c_6, 0.00000), (c_6, 0.00000), (c_6, 0.00000)\Big),\\ \Big((c_6, 0.0000), (c_6, 0.00000), (c_6, 0.0000), (c_6, 0.00000)\Big)\Bigg\}.$$

(7) The 2TL4F negative ideal colution ($2TL4FN_{IS}$) is given below:

$$2TL4FN_{Ic} = \Bigg\{\Big((c_1, 0.49943), (c_1, 0.18079), (c_1, 0.29963), (c_2, -0.47430)\Big),\\ \Big((c_1, 0.04203), (c_1, -0.22330), (c_1, -0.07421), (c_1, 0.04203)\Big),\\ \Big((c_1, 0.45285), (c_1, 0.26210), (c_2, -0.09690), (c_2, 0.11324)\Big),\\ \Big((c_0, 0.40180), (c_0, 0.40180), (c_0, 0.46890), (c_1, -0.45415)\Big)\Bigg\}.$$

(8) The 2TL4F distances of each alternatives from $2TL4FP_{IS}$ and $2TL4FN_{IS}$ are given in Table 6.

Table 6. 2TL4F distances of alternatives from $2TL4FP_{IS}$ and $2TL4FN_{IS}$.

$2TL4FP_{IS}$	$2TL4FN_{IS}$
$\hat{\mathbb{D}}(f_1, 2TL4FP_{IS})$=0.687770	$\hat{\mathbb{D}}(f_1, 2TL4FN_{IS})$=0.090819
$\hat{\mathbb{D}}(f_2, 2TL4FP_{IS})$=0.594904	$\hat{\mathbb{D}}(f_2, 2TL4FN_{IS})$=0.306214
$\hat{\mathbb{D}}(f_3, 2TL4FP_{IS})$=0.383263	$\hat{\mathbb{D}}(f_3, 2TL4FN_{IS})$=0.569076
$\hat{\mathbb{D}}(f_4, 2TL4FP_{IS})$=0.697161	$\hat{\mathbb{D}}(f_4, 2TL4FN_{IS})$=0.068759
$\hat{\mathbb{D}}(f_5, 2TL4FP_{IS})$=0.695246	$\hat{\mathbb{D}}(f_5, 2TL4FN_{IS})$=0.075106
$\hat{\mathbb{D}}(f_6, 2TL4FP_{IS})$=0.354904	$\hat{\mathbb{D}}(f_6, 2TL4FN_{IS})$=0.605143

(9) Relative 2TL4F closeness coefficients are given in Table 7.

Table 7. Relative closeness index.

$\hat{\mathbb{E}}_i$	Closeness index values
$\hat{\mathbb{E}}_1$	0.116646
$\hat{\mathbb{E}}_2$	0.339816
$\hat{\mathbb{E}}_3$	0.597555
$\hat{\mathbb{E}}_4$	0.089773
$\hat{\mathbb{E}}_5$	0.097496
$\hat{\mathbb{E}}_6$	0.630326

(10) Ranking of objects based on their index values.

$$F_6 > F_3 > F_2 > F_1 > F_5 > F_4.$$

Thus, F_6 is a best alternative.

4.2. Numerical Example II: Selection of the best textile industry

The industry is considered as the fundamental element for the economic development of any country, especially the textile industry contributes significantly to the country gross domestic products (GDP's), exports as well as employment. The textile industry is concerned, with the design, production, and distribution of yarn and clothing. It is the largest manufacturing industry, in fact the backbone of Pakistan's economy. In 1947, there were only 6 spinning factories in Pakistan, which have now grown to figure 503. Pakistan is the world's fourth-largest cotton grower, with the third-largest spinning capacity in Asia after China and India, 5 percent of worldwide spinning capacity, and the eighth-largest exporter of textile products. There are currently 1,221 ginning units, 442 spinning units, 124 big spinning units, and 425 small spinning units manufacturing textiles. This sector generates 9.5 percent of GDP and employs around 15 million people or roughly 30 percent of the country's 49 million workers. The development of a textile industry that makes full use of Pakistan's enormous cotton resources has been a priority area for the industrialization of Pakistan, which was formerly one of the world's leading producers of cotton. The main ambition is to develop a systematic scheme for identifying the best textile industry by using the proposed approach. Let $\mathbb{L} = \{\mathbb{L}_1, \mathbb{L}_2, \mathbb{L}_3, \mathbb{L}_4, \mathbb{L}_5, \mathbb{L}_6\}$ be the set of alternatives as an textile industries. The decision-makers, $\aleph = \{\aleph_1, \aleph_2, \aleph_3\}$ with weight vector $\breve{\omega} = \{0.391, 0.362, 0.247\}$ express their assessments using linguistic terms as, $\hat{\wp}$=\{$\hbar_0$=very un-preferable , $\hbar_1$= un-preferable, $\hbar_2$=medium un-preferable, $\hbar_3$=medium, $\hbar_4$= medium preferable, $\hbar_5$=preferable, $\hbar_6$=very preferable, $\hbar_7$=very very preferable, $\hbar_8$=extremely preferable\}.

The decision-maker select the best textile industry under the following criterion:
$\mathbb{T}_1$: Financial Condition and Performance,
$\mathbb{T}_2$: Industrial Information,
$\mathbb{T}_3$: ESG Unification.
Each criterion has been divided into six components to form a 2TL6-polar fuzzy set.

- **Financial condition and performance**: The high financial performance is attractive for investors. Financial analysis not only helps you to understand your company's financial conditions, helping you to understand its creditworthiness, profitability, and ability to generate wealth but will also provide you with a more in-depth look at how well it operates internally. It plays a crucial role to evaluate economic trends and setting financial policies to build long-term plans for business activity. Financial reporting and taxation frameworks apply to textile companies like other sectors. These include budgeting and forecasting, financing, and treasury options. To make a 2TL6-polar fuzzy set we take Gross profit margin, net profit margin, inventory turnover, return on assets, return on equity, debt-to-equity ratio.
- **Industrial information**: In the evaluation of industrial development, industrial development information is an important factor. we take six factors of industrial information as industrial history, innovation capability, availability of resources or technology options, service offering, fabric care or fabric selection, new sustainable policies.

- **ESG unification**: The ESG integration is the collection of economic, social, and governmental factors which plays pivotal role for the industrial burgeoning. To make a 2TL6-polar fuzzy set, we consider six factors of ESG as energy-efficient designs, pollution prevention, regulatory standards, renewable energy sources, waste treatment and hazardous engineering processes.The subdivision of criterion is elaborated in Figure 2.

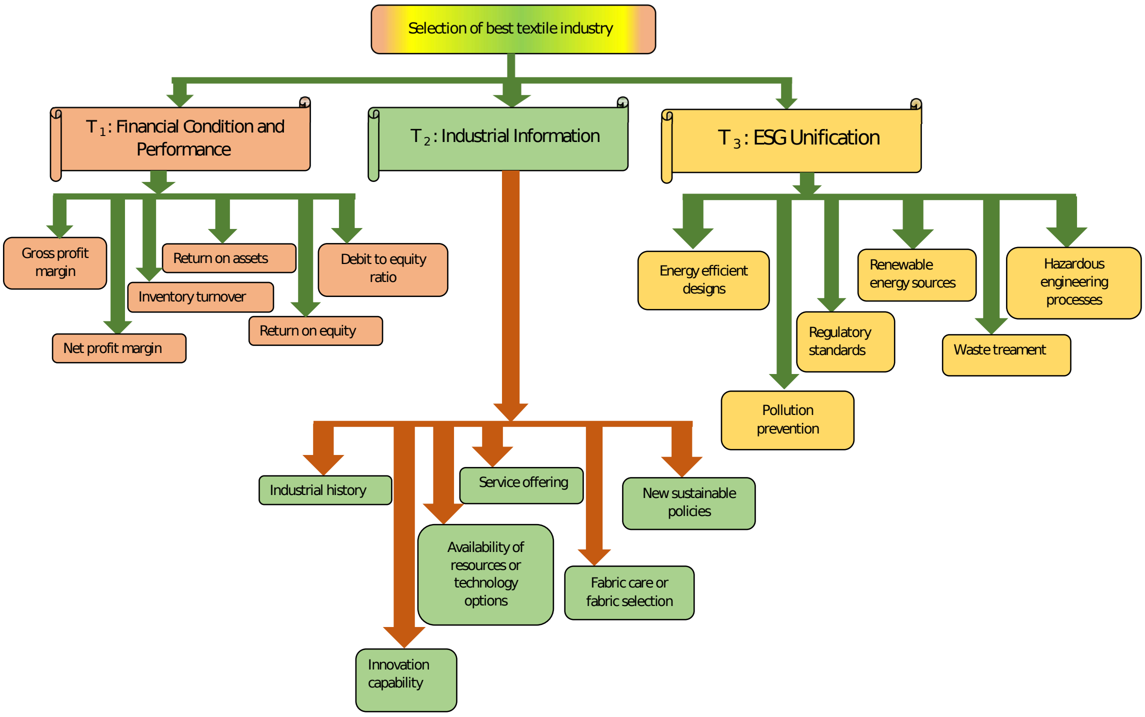

Figure 2. Representation of criterion subdivision for 2TL6F set.

(1) The 2TLmF preference ratings of decision makers $\aleph_1$, $\aleph_2$ and $\aleph_3$ are arranged in Tables 8, 9 and 10, respectively.

Table 8. 2TL6F decision matrix provided by first expert $\aleph_1$.

Alternatives	$\mathbb{T}_1$	$\mathbb{T}_2$	$\mathbb{T}_3$
$\mathbb{L}_1$	$((\hbar_4,0),(\hbar_4,0),(\hbar_5,0),(\hbar_4,0),(\hbar_5,0),(\hbar_4,0))$	$((\hbar_3,0),(\hbar_4,0),(\hbar_5,0),(\hbar_6,0),(\hbar_5,0),(\hbar_6,0))$	$((\hbar_5,0),(\hbar_4,0),(\hbar_5,0),(\hbar_3,0),(\hbar_5,0),(\hbar_4,0))$
$\mathbb{L}_2$	$((\hbar_5,0),(\hbar_6,0),(\hbar_4,0),(\hbar_5,0),(\hbar_7,0),(\hbar_6,0))$	$((\hbar_4,0),(\hbar_5,0),(\hbar_4,0),(\hbar_6,0),(\hbar_5,0),(\hbar_7,0))$	$((\hbar_6,0),(\hbar_5,0),(\hbar_7,0),(\hbar_5,0),(\hbar_7,0),(\hbar_6,0))$
$\mathbb{L}_3$	$((\hbar_4,0),(\hbar_2,0),(\hbar_5,0),(\hbar_4,0),(\hbar_3,0),(\hbar_5,0))$	$((\hbar_2,0),(\hbar_4,0),(\hbar_5,0),(\hbar_3,0),(\hbar_5,0),(\hbar_3,0))$	$((\hbar_4,0),(\hbar_3,0),(\hbar_5,0),(\hbar_4,0),(\hbar_3,0),(\hbar_5,0))$
$\mathbb{L}_4$	$((\hbar_5,0),(\hbar_7,0),(\hbar_4,0),(\hbar_5,0),(\hbar_4,0),(\hbar_7,0))$	$((\hbar_5,0),(\hbar_4,0),(\hbar_6,0),(\hbar_8,0),(\hbar_4,0),(\hbar_5,0))$	$((\hbar_7,0),(\hbar_5,0),(\hbar_4,0),(\hbar_6,0),(\hbar_5,0),(\hbar_6,0))$
$\mathbb{L}_5$	$((\hbar_5,0),(\hbar_6,0),(\hbar_5,0),(\hbar_4,0),(\hbar_4,0),(\hbar_5,0))$	$((\hbar_5,0),(\hbar_6,0),(\hbar_4,0),(\hbar_5,0),(\hbar_5,0),(\hbar_4,0))$	$((\hbar_6,0),(\hbar_7,0),(\hbar_6,0),(\hbar_5,0),(\hbar_6,0),(\hbar_5,0))$
$\mathbb{L}_6$	$((\hbar_5,0),(\hbar_5,0),(\hbar_6,0),(\hbar_7,0),(\hbar_5,0),(\hbar_6,0))$	$((\hbar_5,0),(\hbar_6,0),(\hbar_5,0),(\hbar_4,0),(\hbar_6,0),(\hbar_7,0))$	$((\hbar_6,0),(\hbar_5,0),(\hbar_5,0),(\hbar_6,0),(\hbar_7,0),(\hbar_6,0))$

Table 9. 2TL6F decision matrix provided by second expert $\aleph_2$.

Alternatives	$\mathbb{T}_1$	$\mathbb{T}_2$	$\mathbb{T}_3$
$\mathbb{L}_1$	$((\hbar_4,0),(\hbar_3,0),(\hbar_5,0),(\hbar_4,0),(\hbar_5,0),(\hbar_4,0))$	$((\hbar_3,0),(\hbar_5,0),(\hbar_5,0),(\hbar_6,0),(\hbar_6,0),(\hbar_5,0))$	$((\hbar_5,0),(\hbar_4,0),(\hbar_5,0),(\hbar_3,0),(\hbar_6,0),(\hbar_4,0))$
$\mathbb{L}_2$	$((\hbar_5,0),(\hbar_6,0),(\hbar_5,0),(\hbar_4,0),(\hbar_7,0),(\hbar_5,0))$	$((\hbar_5,0),(\hbar_4,0),(\hbar_5,0),(\hbar_6,0),(\hbar_5,0),(\hbar_5,0))$	$((\hbar_6,0),(\hbar_4,0),(\hbar_7,0),(\hbar_5,0),(\hbar_6,0),(\hbar_7,0))$
$\mathbb{L}_3$	$((\hbar_4,0),(\hbar_5,0),(\hbar_3,0),(\hbar_4,0),(\hbar_5,0),(\hbar_4,0))$	$((\hbar_3,0),(\hbar_5,0),(\hbar_4,0),(\hbar_3,0),(\hbar_5,0),(\hbar_3,0))$	$((\hbar_4,0),(\hbar_3,0),(\hbar_5,0),(\hbar_4,0),(\hbar_3,0),(\hbar_3,0))$
$\mathbb{L}_4$	$((\hbar_4,0),(\hbar_5,0),(\hbar_6,0),(\hbar_5,0),(\hbar_5,0),(\hbar_7,0))$	$((\hbar_5,0),(\hbar_7,0),(\hbar_6,0),(\hbar_8,0),(\hbar_4,0),(\hbar_5,0))$	$((\hbar_6,0),(\hbar_5,0),(\hbar_7,0),(\hbar_6,0),(\hbar_5,0),(\hbar_5,0))$
$\mathbb{L}_5$	$((\hbar_6,0),(\hbar_5,0),(\hbar_5,0),(\hbar_4,0),(\hbar_5,0),(\hbar_6,0))$	$((\hbar_4,0),(\hbar_6,0),(\hbar_4,0),(\hbar_7,0),(\hbar_6,0),(\hbar_5,0))$	$((\hbar_6,0),(\hbar_7,0),(\hbar_6,0),(\hbar_5,0),(\hbar_7,0),(\hbar_6,0))$
$\mathbb{L}_6$	$((\hbar_6,0),(\hbar_7,0),(\hbar_5,0),(\hbar_4,0),(\hbar_6,0),(\hbar_7,0))$	$((\hbar_5,0),(\hbar_6,0),(\hbar_5,0),(\hbar_7,0),(\hbar_6,0),(\hbar_7,0))$	$((\hbar_5,0),(\hbar_4,0),(\hbar_5,0),(\hbar_6,0),(\hbar_5,0),(\hbar_4,0))$

Table 10. 2TL6F decision matrix provided by third expert $\aleph_3$.

Alternatives	$\mathbb{T}_1$	$\mathbb{T}_2$	$\mathbb{T}_3$
$\mathbb{L}_1$	$((\hbar_3,0),(\hbar_4,0),(\hbar_5,0),(\hbar_3,0),(\hbar_5,0),(\hbar_5,0))$	$((\hbar_4,0),(\hbar_5,0),(\hbar_4,0),(\hbar_5,0),(\hbar_5,0),(\hbar_6,0))$	$((\hbar_5,0),(\hbar_6,0),(\hbar_5,0),(\hbar_4,0),(\hbar_3,0),(\hbar_5,0))$
$\mathbb{L}_2$	$((\hbar_6,0),(\hbar_5,0),(\hbar_6,0),(\hbar_4,0),(\hbar_6,0),(\hbar_4,0))$	$((\hbar_4,0),(\hbar_5,0),(\hbar_6,0),(\hbar_5,0),(\hbar_6,0),(\hbar_5,0))$	$((\hbar_6,0),(\hbar_5,0),(\hbar_6,0),(\hbar_6,0),(\hbar_5,0),(\hbar_7,0))$
$\mathbb{L}_3$	$((\hbar_5,0),(\hbar_4,0),(\hbar_3,0),(\hbar_6,0),(\hbar_4,0),(\hbar_7,0))$	$((\hbar_2,0),(\hbar_6,0),(\hbar_4,0),(\hbar_6,0),(\hbar_7,0),(\hbar_6,0))$	$((\hbar_6,0),(\hbar_4,0),(\hbar_3,0),(\hbar_5,0),(\hbar_6,0),(\hbar_5,0))$
$\mathbb{L}_4$	$((\hbar_7,0),(\hbar_5,0),(\hbar_6,0),(\hbar_4,0),(\hbar_5,0),(\hbar_4,0))$	$((\hbar_6,0),(\hbar_7,0),(\hbar_5,0),(\hbar_4,0),(\hbar_5,0),(\hbar_5,0))$	$((\hbar_5,0),(\hbar_7,0),(\hbar_5,0),(\hbar_6,0),(\hbar_5,0),(\hbar_7,0))$
$\mathbb{L}_5$	$((\hbar_4,0),(\hbar_5,0),(\hbar_4,0),(\hbar_5,0),(\hbar_4,0),(\hbar_6,0))$	$((\hbar_5,0),(\hbar_6,0),(\hbar_7,0),(\hbar_6,0),(\hbar_5,0),(\hbar_4,0))$	$((\hbar_4,0),(\hbar_6,0),(\hbar_5,0),(\hbar_6,0),(\hbar_5,0),(\hbar_6,0))$
$\mathbb{L}_6$	$((\hbar_5,0),(\hbar_4,0),(\hbar_5,0),(\hbar_4,0),(\hbar_5,0),(\hbar_7,0))$	$((\hbar_5,0),(\hbar_6,0),(\hbar_5,0),(\hbar_4,0),(\hbar_5,0),(\hbar_6,0))$	$((\hbar_5,0),(\hbar_6,0),(\hbar_5,0),(\hbar_7,0),(\hbar_5,0),(\hbar_4,0))$

(2) The aggregated 2TL6F decision matrix by using 2TL*m*FWA operator as given in Table 11.

Table 11. Aggregated 2TL6-polar decision matrix.

Alternatives	$\mathbb{T}_1$
$\mathbb{L}_1$	$((\hbar_4,-0.226654),(\hbar_4,-0.336520),(\hbar_5,0.000000),(\hbar_4,-0.226654),(\hbar_5,0.000000),(\hbar_4,0.274366))$
$\mathbb{L}_2$	$((\hbar_5,0.285894),(\hbar_6,-0.210673),(\hbar_5,-0.037230),(\hbar_4,0.425552),(\hbar_7,-0.186736),(\hbar_5,0.251286))$
$\mathbb{L}_3$	$((\hbar_4,0.274366),(\hbar_4,-0.223610),(\hbar_4,-0.094747),(\hbar_5,-0.370587),(\hbar_4,0.066999),(\hbar_5,0.461950))$
$\mathbb{L}_4$	$((\hbar_5,0.461950),(\hbar_6,0.047608),(\hbar_5,0.377395),(\hbar_5,-0.220928),(\hbar_5,-0.357162),(\hbar_7,-0.408344))$
$\mathbb{L}_5$	$((\hbar_5,0.218775),(\hbar_5,0.439825),(\hbar_5,-0.220928),(\hbar_4,0.274366),(\hbar_4,0.395606),(\hbar_6,-0.3435901))$
$\mathbb{L}_6$	$((\hbar_5,0.409544),(\hbar_6,-0.164029),(\hbar_5,0.439825),(\hbar_6,-0.326240),(\hbar_5,0.409544),(\hbar_7,-0.311302))$
Alternatives	$\mathbb{T}_2$
$\mathbb{L}_1$	$((\hbar_3,0.268125),(\hbar_5,-0.357162),(\hbar_5,-0.220928),(\hbar_6,-0.210673),(\hbar_5,0.409544),(\hbar_6,-0.316194))$
$\mathbb{L}_2$	$((\hbar_4,0.395606),(\hbar_5,-0.329270),(\hbar_5,-0.037230),(\hbar_6,-0.210673),(\hbar_5,0.285894),(\hbar_6,0.047608))$
$\mathbb{L}_3$	$((\hbar_2,0.383217),(\hbar_5,-0.037230),(\hbar_4,0.425552),(\hbar_4,0.012700),(\hbar_6,-0.287032),(\hbar_4,0.012700))$
$\mathbb{L}_4$	$((\hbar_5,0.2858941),(\hbar_6,0.2804870),(\hbar_6,-0.210673),(\hbar_8,0.000000),(\hbar_4,0.274366),(\hbar_5,0.000000))$
$\mathbb{L}_5$	$((\hbar_5,-0.3292700),(\hbar_6,0.000000),(\hbar_5,0.1597852),(\hbar_6,0.1764870),(\hbar_5,0.409544),(\hbar_4,0.395606))$
$\mathbb{L}_6$	$((\hbar_5,0.000000),(\hbar_6,0.000000),(\hbar_5,0.000000),(\hbar_6,-0.421666),(\hbar_6,-0.210673),(\hbar_7,-0.186736))$
Alternatives	$\mathbb{T}_3$
$\mathbb{L}_1$	$((\hbar_5,0.000000),(\hbar_5,-0.370587),(\hbar_5,0.000000),(\hbar_3,0.268125),(\hbar_5,0.061180),(\hbar_4,0.274366))$
$\mathbb{L}_2$	$((\hbar_6,0.0000),(\hbar_5,-0.329270),(\hbar_7,-0.186736),(\hbar_6,-0.343590),(\hbar_6,0.047608),(\hbar_7,-0.311302))$
$\mathbb{L}_3$	$((\hbar_5,-0.370587),(\hbar_3,0.268125),(\hbar_5,-0.403438),(\hbar_4,0.274366),(\hbar_4,0.012700),(\hbar_4,0.390637))$
$\mathbb{L}_4$	$((\hbar_6,0.314138),(\hbar_6,-0.287032),(\hbar_6,-0.255560),(\hbar_6,0.000000),(\hbar_5,0.000000),(\hbar_6,0.048266))$
$\mathbb{L}_5$	$((\hbar_6,-0.373473),(\hbar_7,-0.186736),(\hbar_6,-0.210673),(\hbar_5,0.28589),(\hbar_6,0.279907),(\hbar_6,-0.34359))$
$\mathbb{L}_6$	$((\hbar_5,0.439825),(\hbar_5,-0.011997),(\hbar_5,0.000000),(\hbar_6,0.314706),(\hbar_6,0.047608),(\hbar_5,-0.050403))$

(3) Suppose the decision makers assign the following weights to the each attribute.

$$\Im=(\Im_1,\Im_2,\Im_3)=(0.421,0.234,0.345).$$

(4) The aggregated weighted 2TL6F decision matrix as given in Table 12.

Table 12. Aggregated weighted 2TL6-polar decision matrix.

Alternatives	$\mathbb{T}_1$
$\mathbb{L}_1$	$((\hbar_2,-0.115524),(\hbar_2,-0.18195),(\hbar_3,-0.293673),(\hbar_2,-0.115524),(\hbar_3,-0.293673),(\hbar_2,0.20084))$
$\mathbb{L}_2$	$((\hbar_3,-0.075115),(\hbar_3,0.344847),(\hbar_3,-0.321232),(\hbar_2,0.301112),(\hbar_4,0.417449),(\hbar_3,-0.102260))$
$\mathbb{L}_3$	$((\hbar_2,0.200849),(\hbar_2,-0.113670),(\hbar_2,-0.034436),(\hbar_2,0.440277),(\hbar_2,0.067088),(\hbar_3,0.066175))$
$\mathbb{L}_4$	$((\hbar_3,0.066175),(\hbar_4,-0.417918),(\hbar_3,-0.002367),(\hbar_3,-0.454427),(\hbar_2,0.449610),(\hbar_4,0.149691))$
$\mathbb{L}_5$	$((\hbar_3,-0.127580),(\hbar_3,0.048114),(\hbar_3,-0.454427),(\hbar_2,0.200849),(\hbar_2,0.281061),(\hbar_3,0.229001))$
$\mathbb{L}_6$	$((\hbar_3,0.023540),(\hbar_3,0.386453),(\hbar_3,0.048114),(\hbar_3,0.0481141),(\hbar_3,0.0235401),(\hbar_4,0.26369810))$
Alternatives	$\mathbb{T}_2$
$\mathbb{L}_1$	$((\hbar_1,-0.074971),(\hbar_1,0.471033),(\hbar_2,-0.465982),(\hbar_2,0.079128),(\hbar_2,-0.144647),(\hbar_2,0.014171))$
$\mathbb{L}_2$	$((\hbar_1,0.361565),(\hbar_1,0.483766),(\hbar_2,-0.377738),(\hbar_2,0.079128),(\hbar_2,-0.212059),(\hbar_2,0.248785))$
$\mathbb{L}_3$	$((\hbar_1,-0.364563),(\hbar_2,-0.377738),(\hbar_1,0.425552),(\hbar_1,0.202866),(\hbar_2,0.031892),(\hbar_1,0.202866))$
$\mathbb{L}_4$	$((\hbar_2,-0.212059),(\hbar_2,0.417203),(\hbar_2,0.079128),(\hbar_8,0.00000),(\hbar_1,0.309974),(\hbar_2,-0.35935810))$
$\mathbb{L}_5$	$((\hbar_1,0.483766),(\hbar_2,0.216270),(\hbar_2,-0.278430),(\hbar_2,0.339958),(\hbar_2,-0.144647),(\hbar_1,0.361565))$
$\mathbb{L}_6$	$((\hbar_2,-0.359358),(\hbar_2,0.216270),(\hbar_2,-0.359358),(\hbar_2,-0.048527),(\hbar_2,0.07912),(\hbar_3,-0.11877))$
Alternatives	$\mathbb{T}_3$
$\mathbb{L}_1$	$((\hbar_2,0.296640),(\hbar_2,0.062790),(\hbar_2,0.296640),(\hbar_1,0.325628),(\hbar_2,0.3370380),(\hbar_2,-0.1459351))$
$\mathbb{L}_2$	$((\hbar_3,0.041169),(\hbar_2,0.088000),(\hbar_4,-0.141676),(\hbar_3,-0.237609),(\hbar_3,0.082215),(\hbar_4,-0.286781))$
$\mathbb{L}_3$	$((\hbar_2,0.062790),(\hbar_1,0.325628),(\hbar_2,0.042890),(\hbar_2,-0.145935),(\hbar_2,-0.291556),(\hbar_2,-0.079074))$
$\mathbb{L}_4$	$((\hbar_3,0.325044),(\hbar_3,-0.193652),(\hbar_3,-0.168883),(\hbar_3,0.041169),(\hbar_2,0.296640),(\hbar_3,0.082786))$
$\mathbb{L}_5$	$((\hbar_3,-0.26055),(\hbar_4,-0.141676),(\hbar_3,-0.133161),(\hbar_2,0.490335),(\hbar_3,0.29251),(\hbar_3,-0.237609))$
$\mathbb{L}_6$	$((\hbar_3,-0.399790),(\hbar_2,0.2887811),(\hbar_2,0.296640),(\hbar_3,0.3255870),(\hbar_3,0.082215),(\hbar_2,0.263761))$

(5) The 2TL6F positive ideal solution ($2TL6FP_{IS}$) is given as

$$\begin{aligned}
&2TL6FP_{IS} = \\
&\Bigg\{\Bigg((\hbar_3,0.066175),(\hbar_4,-0.417918),(\hbar_3,0.048114),\\
&(\hbar_3,0.243903),(\hbar_4,0.417449),(\hbar_4,0.263698)\Bigg),\\
&\Bigg((\hbar_2,-0.212059),(\hbar_2,0.417203),(\hbar_2,0.079128),\\
&(\hbar_8,0.000000),(\hbar_2,0.079128),(\hbar_3,-0.118772)\Bigg),\\
&\Bigg((\hbar_3,0.325044),(\hbar_4,-0.141676),(\hbar_4,-0.141676),\\
&(\hbar_3,0.325587),(\hbar_3,0.292511),(\hbar_4,-0.286781)\Bigg)\Bigg\}.
\end{aligned}$$

(6) The 2TL6F negative ideal solution ($2TL6FN_{IS}$) is

$$2TL6FN_{IS} = \Bigg\{\Bigg((\hbar_2, -0.115524), (\hbar_2, -0.181952), (\hbar_2, -0.034436),$$

$$(\hbar_2, -0.115524), (\hbar_2, 0.067088), (\hbar_2, 0.200849)\Bigg),$$

$$\Bigg((\hbar_1, -0.364563), (\hbar_1, 0.471033), (\hbar_1, 0.374512),$$

$$(\hbar_1, 0.202866), (\hbar_1, 0.309974), (\hbar_1, 0.202866)\Bigg),$$

$$\Bigg((\hbar_2, 0.062790), (\hbar_1, 0.325628), (\hbar_2, 0.042890),$$

$$(\hbar_1, 0.325628), (\hbar_2, -0.291556), (\hbar_2, -0.145935)\Bigg)\Bigg\}.$$

(7) The 2TL6F distances of each alternatives from $2TL6FP_{IS}$ and $2TL6FN_{IS}$ are given in Table 13.

Table 13. 2TL6F distances of alternatives from $2TL6FP_{IS}$ and $2TL6FN_{IS}$.

$2TL6FP_{IS}$	$2TL6FN_{IS}$
$\hat{\mathfrak{D}}(l_1, 2TL6FP_{IS})$=0.240336	$\hat{\mathfrak{D}}(l_1, 2TL6FN_{IS})$=0.057793
$\hat{\mathfrak{D}}(l_2, 2TL6FP_{IS})$=0.193977	$\hat{\mathfrak{D}}(l_2, 2TL6FN_{IS})$=0.147393
$\hat{\mathfrak{D}}(l_3, 2TL6FP_{IS})$=0.268528	$\hat{\mathfrak{D}}(l_3, 2TL6FN_{IS})$=0.041564
$\hat{\mathfrak{D}}(l_4, 2TL6FP_{IS})$=0.093810	$\hat{\mathfrak{D}}(l_4, 2TL6FN_{IS})$=0.243444
$\hat{\mathfrak{D}}(l_5, 2TL6FP_{IS})$=0.197382	$\hat{\mathfrak{D}}(l_5, 2TL6FN_{IS})$=0.129373
$\hat{\mathfrak{D}}(l_6, 2TL6FP_{IS})$=0.200682	$\hat{\mathfrak{D}}(l_6, 2TL6FN_{IS})$=0.146986

(8) Relative 2TL6F closeness coefficients are given in Table 14.

Table 14. Relative closeness index.

$\hat{\mathfrak{C}}_i$	Closeness index values
$\hat{\mathfrak{C}}_1$	0.193852
$\hat{\mathfrak{C}}_2$	0.431768
$\hat{\mathfrak{C}}_3$	0.134039
$\hat{\mathfrak{C}}_4$	0.721839
$\hat{\mathfrak{C}}_5$	0.395933
$\hat{\mathfrak{C}}_6$	0.422776

(9) Ranking of objects based on their index values.

$$\mathbb{L}_4 > \mathbb{L}_2 > \mathbb{L}_6 > \mathbb{L}_5 > \mathbb{L}_1 > \mathbb{L}_3.$$

Thus, $\mathbb{L}_4$ is a best alternative.

5. Another mathematical approach for MAGDM

5.1. Structure of 2TLmF ELECTRE-I method

In this section, we develop another mathematical technique, to deal with multi-attribute group decision making (MAGDM), namely, the 2TL*m*F ELECTRE-I approach comprising 2TL*m*F information. In this approach, a group of experts is selected to choose the best alternative. Each expert ($\Re_u$, u=1,2,...,v) has the right of assigning the 2TL*m*F number to each alternative with respect to criteria. Let $F = \{F_1, F_2, \ldots, F_k\}$ be the set of alternatives against the 2TL*m*F information from which the best one is selected based on some attributes denoted as, $\zeta = \{\zeta_1, \zeta_2, \ldots, \zeta_t\}$. Let q, $(q = 1, 2, \ldots, m)$ be the number of poles according to m characteristics and $(c_{\phi_q}(x), \varrho_q(x))$, $q = 1, 2, \ldots, m$ be the number of membership values to each pole. The weight vector for attributes is represented by, $\varphi = \{\varphi_1, \varphi_2, \ldots, \varphi_t\}$, where $\varphi \in [0, 1]$, the weights are given by the experts must satisfy the normality condition as, $\Sigma_{j=1}^{t} \varphi_j = 1$.

The main target is the selection of the most desirable alternative as the solution to this MAGDM problem. We describe the 2TL*m*F ELECTRE-1 method step by step as follows.

In 2TL*m*F ELECTRE-1 method, **(Step 1–Step 4)** are same as in 2TL*m*F-TOPSIS method, already described in Section 3.

Step 5: The core concept behind the ELECTRE-1 method is to compare the distinct alternatives in pairs, the 2TL*m*FNs are compared by using the score and accuracy function. In case when two alternatives have the same score function, than we calculate accuracy function. The 2TL*m*F concordance sets are constructed as defined below:

$$\mathbb{M}_{rs} = \Big\{1 \le j \le k : (\hat{c}_{\phi_q^{rj}}, \hat{\varrho}_q^{rj}) \ge (\hat{c}_{\phi_q^{sj}}, \hat{\varrho}_q^{sj}); r \ne s; r, s = 1, 2, \ldots, k\Big\}, q = 1, 2, \ldots, m$$

where,

$$(\hat{c}_{\phi_q^{ij}}, \hat{\varrho}_q^{ij}) = \Lambda\Bigg(\frac{\sigma}{m}\Bigg(\frac{\Lambda^{-1}(c_{\phi_{ij}^1}, \varrho_{ij}^1)}{\sigma} + \frac{\Lambda^{-1}(c_{\phi_{ij}^2}, \varrho_{ij}^2)}{\sigma} + \ldots + \frac{\Lambda^{-1}(c_{\phi_{ij}^2}, \varrho_{ij}^2)}{\varrho}\Bigg)\Bigg).$$

Step 6: The 2TL*m*F discordance sets are the complementary subsets of 2TL*m*F concordance sets. The 2TL*m*F discordance sets are constructed as

$$\mathbb{N}_{rs} = \Big\{1 \le j \le k : (\hat{c}_{\phi_q^{rj}}, \hat{\varrho}_q^{rj}) \le (\hat{c}_{\phi_q^{sj}}, \hat{\varrho}_q^{sj}); r \ne s; r, s = 1, 2, \ldots, k\Big\}, q = 1, 2, \ldots, m$$

where,

$$(\hat{c}_{\phi_q^{ij}}, \hat{\varrho}_q^{ij}) = \Lambda\Bigg(\frac{\sigma}{m}\Bigg(\frac{\Lambda^{-1}(c_{\phi_{ij}^1}, \varrho_{ij}^1)}{\sigma} + \frac{\Lambda^{-1}(c_{\phi_{ij}^2}, \varrho_{ij}^2)}{\sigma} + \ldots + \frac{\Lambda^{-1}(c_{\phi_{ij}^2}, \varrho_{ij}^2)}{\varrho}\Bigg)\Bigg).$$

Step 7: The 2TL*m*F concordance indices p_{rs}'s are calculated by means of weights assigned to the concordance indicators involved in the corresponding concordance sets as

$$p_{rs} = \textstyle\sum_{j \in \mathbb{P}_{rs}} \varphi_j, \forall r, s.$$

Step 8: The 2TL*m*F concordance matrix P can be obtained by arranging all the 2TL*m*F concordance indices p_{rs}'s as below:

$$\mathbb{P} = \begin{pmatrix} - & p_{12} & \cdots & p_{1(k-1)} & p_{1k} \\ p_{21} & - & \cdots & p_{2(k-2)} & p_{2k} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ p_{(k-1)1} & p_{(k-2)2} & \cdots & - & p_{(k-1)k} \\ p_{k1} & p_{k2} & \cdots & p_{(k-1)k} & - \end{pmatrix}.$$

Step 9: On contrary to the 2TL*m*F concordance indices, the 2TL*m*F discordance indices q_{rs}'s are the extent in which one alternative is inferior to other. The 2TL*m*F discordance indices are obtained by using normalized Euclidean distance $d(\hat{z}_{rj}, \hat{z}_{sj})$, as defined below:

$$q_{rs} = \frac{max_{j\in\mathbb{Q}_{rs}}(d(\hat{z}_{rj}, \hat{z}_{sj}))}{max_j(d(\hat{z}_{rj}, \hat{z}_{sj}))},$$

$$= \frac{max_{j\in\mathbb{Q}_{rs}} \sqrt{\frac{1}{m}\left[\left(\frac{\Lambda^{-1}(\hat{c}_{\phi_1^{rj}}, \hat{\varrho}_1^{rj})}{\sigma} - \frac{\Lambda^{-1}(\hat{c}_{\psi_1^{sj}}, \hat{\varrho}_1^{sj})}{\sigma}\right)^2 + \ldots + \left(\frac{\Lambda^{-1}(\hat{c}_{\phi_m^{rj}}, \hat{\varrho}_m^{rj})}{\sigma} - \frac{\Lambda^{-1}(\hat{c}_{\psi_m^{sj}}, \hat{\varrho}_m^{sj})}{\sigma}\right)^2\right]}}{max_j \sqrt{\frac{1}{m}\left[\left(\frac{\Lambda^{-1}(\hat{c}_{\psi_1^{rj}}, \hat{\varrho}_1^{rj})}{\sigma} - \frac{\Lambda^{-1}(\hat{c}_{\psi_1^{sj}}, \hat{\varrho}_1^{sj})}{\sigma}\right)^2 + \ldots + \left(\frac{\Lambda^{-1}(\hat{c}_{\psi_m^{rj}}, \hat{\varrho}_m^{rj})}{\sigma} - \frac{\Lambda^{-1}(\hat{c}_{\psi_m^{sj}}, \hat{\varrho}_m^{sj})}{\sigma}\right)^2\right]}}, \quad \forall \text{ r,s.}$$

Step 10: The 2TL*m*F discordance matrix $\mathbb{Q}$ can be obtained by arranging all the 2TL*m*F discordance indices q_{rs}'s as below:

$$\mathbb{Q} = \begin{pmatrix} - & q_{12} & \cdots & q_{1(k-1)} & q_{1k} \\ q_{21} & - & \cdots & q_{2(k-2)} & q_{2k} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ q_{(k-1)1} & q_{(k-2)2} & \cdots & - & q_{(k-1)k} \\ q_{k1} & q_{k2} & \cdots & q_{k(k-1)} & - \end{pmatrix}.$$

Step 11: Now, we evaluate the 2TL*m*F concordance level $\bar{p}$ and 2TL*m*F discordance level $\bar{q}$. The 2TL*m*F concordance level and the 2TL*m*F discordance level are obtained by calculating average of 2TL*m*F concordance and 2TL*m*F discordance indices as defined below:

$$\bar{p} = \frac{1}{k(k-1)} \sum_{\substack{r=1 \\ r\neq s}}^{k} \sum_{\substack{s=1 \\ r\neq s}}^{k} p_{rs},$$

$$\bar{q} = \frac{1}{k(k-1)} \sum_{\substack{r=1 \\ r\neq s}}^{k} \sum_{\substack{s=1 \\ r\neq s}}^{k} q_{rs}.$$

Step 12: According to concordance and discordance level, the 2TL*m*F concordance dominance matrix and 2TLmF discordance dominance matrix are constructed as

$$\Upsilon = \begin{pmatrix} - & \gamma_{12} & \cdots & \gamma_{1(k-1)} & \gamma_{1k} \\ \gamma_{21} & - & \cdots & \gamma_{2(k-1)} & \gamma_{2k} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \gamma_{(k-1)1} & \gamma_{(k-2)2} & \cdots & - & \gamma_{(k-1)k} \\ \gamma_{k1} & \gamma_{k2} & \cdots & \gamma_{k(k-1)} & - \end{pmatrix},$$

where,

$$\gamma_{rs} = \begin{cases} 1, & p_{rs} \geq \bar{p}, \\ 0, & p_{rs} \leq \bar{p}. \end{cases}$$

$$\eta = \begin{pmatrix} - & \eta_{12} & \cdots & \eta_{1(k-1)} & \eta_{1k} \\ \eta_{21} & - & \cdots & \eta_{1(k-2)} & \eta_{2k} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \eta_{(k-1)1} & \eta_{(k-2)2} & \cdots & - & \eta_{(k-1)k} \\ \eta_{k1} & \eta_{k2} & \cdots & \eta_{k(k-1)} & - \end{pmatrix},$$

where,

$$\eta_{rs} = \begin{cases} 1, & q_{rs} \leq \bar{q}, \\ 0, & q_{rs} \geq \bar{q}. \end{cases}$$

Step 13: The 2TLmF concordance and discordance dominance matrices are combined to obtained the aggregated outranking Boolean matrix that offers more accurate information on the outranking relationship and superiority of one alternative over another. Assemble the aggregated outranking boolean matrix as follows:

$$\tau = \begin{pmatrix} - & \tau_{12} & \cdots & \tau_{1(k-1)} & \tau_{1k} \\ \tau_{21} & - & \cdots & \tau_{1(k-2)} & \tau_{2k} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \tau_{(k-1)1} & \tau_{(k-2)2} & \cdots & - & \tau_{(k-1)k} \\ \tau_{k1} & \tau_{k2} & \cdots & \tau_{k(k-1)} & - \end{pmatrix},$$

where,

$$\tau_{ij} = \gamma_{ij} \times \eta_{ij}, (i, j = 1, 2, 3, \ldots, k, i \neq j).$$

Step 14: Lastly, our target is to ranked the alternatives by using aggregated outranking Boolean matrix τ information. For convenience, we can visualized the outranking relations by using directed graph. A directed edge is exist from alternative F_r to F_s if and only if $\tau_{rs} = 1$.

Then there exist the following three situations as described below:

- For a unique directed edge from F_r to F_s implies that F_r is preferred over F_s.

- Existence of directed edge from F_r to F_s and F_s to F_r implies that F_r and F_s are indifferent.
- If there is no edge between F_r to F_s implies that F_r and F_s both are incomparable.

The representation of relations in an outranking decision graph is shown in Figure 3.

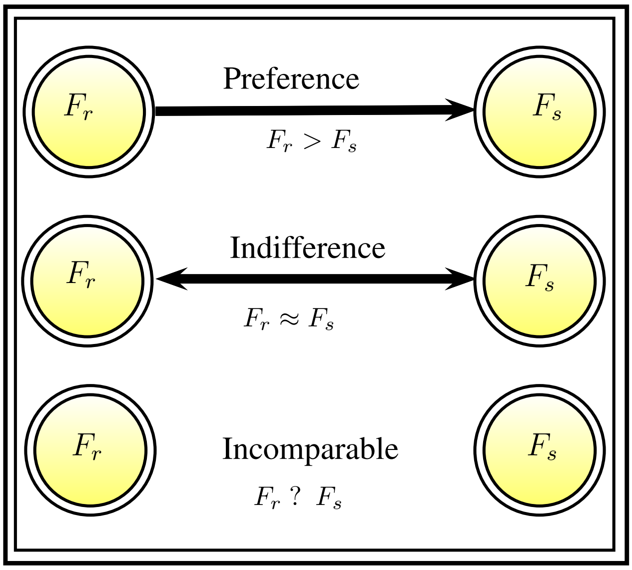

Figure 3. Graphically representation of relations in an outranking decision graph.

Step 15: The decision graph allows for the partial ordering of the alternatives and eliminates the less favorable alternatives. On the other hand, the linear ranking order of the alternatives reduces the partial ordering dilemma that can be evaluated by employing net outranking indices to strengthen the proposed 2TL*m*F ELECTRE-I technique.

Let $\{\overbrace{\Theta_i}, i = 1, 2, \ldots, k\}$ be the concordance outranking relation derived with the help of concordance matrix $\mathbb{P}$ and the following Eq (5.1):

$$\overbrace{\Theta_i} = \sum_{\substack{r=1\\r\neq i}}^{k} p_{ir} - \sum_{\substack{s=1\\s\neq i}}^{k} p_{is}. \tag{5.1}$$

The discordance outranking relation $\{\underbrace{\Theta_i}, i = 1, 2, \ldots, k\}$ computed by using discordance matrix $\mathbb{Q}$ and Eq (5.2) as given below:

$$\underbrace{\Theta_i} = \sum_{\substack{r=1\\r\neq s}}^{k} p_{ir} - \sum_{\substack{r=1\\r\neq s}}^{k} p_{is}. \tag{5.2}$$

Thus, for the final linear ranking order, the net outranking index is defined as follows in Eq (5.3).

$$\Xi = \overbrace{\Theta_i} - \underbrace{\Theta_i}. \tag{5.3}$$

The alternative with the maximum net outranking index is considered the most suitable choice. We describe our proposed methods, namely, 2TLmF-TOPSIS method and 2TL*m*F ELECRTE-1 method, in a flowchart as shown in Figure 4.

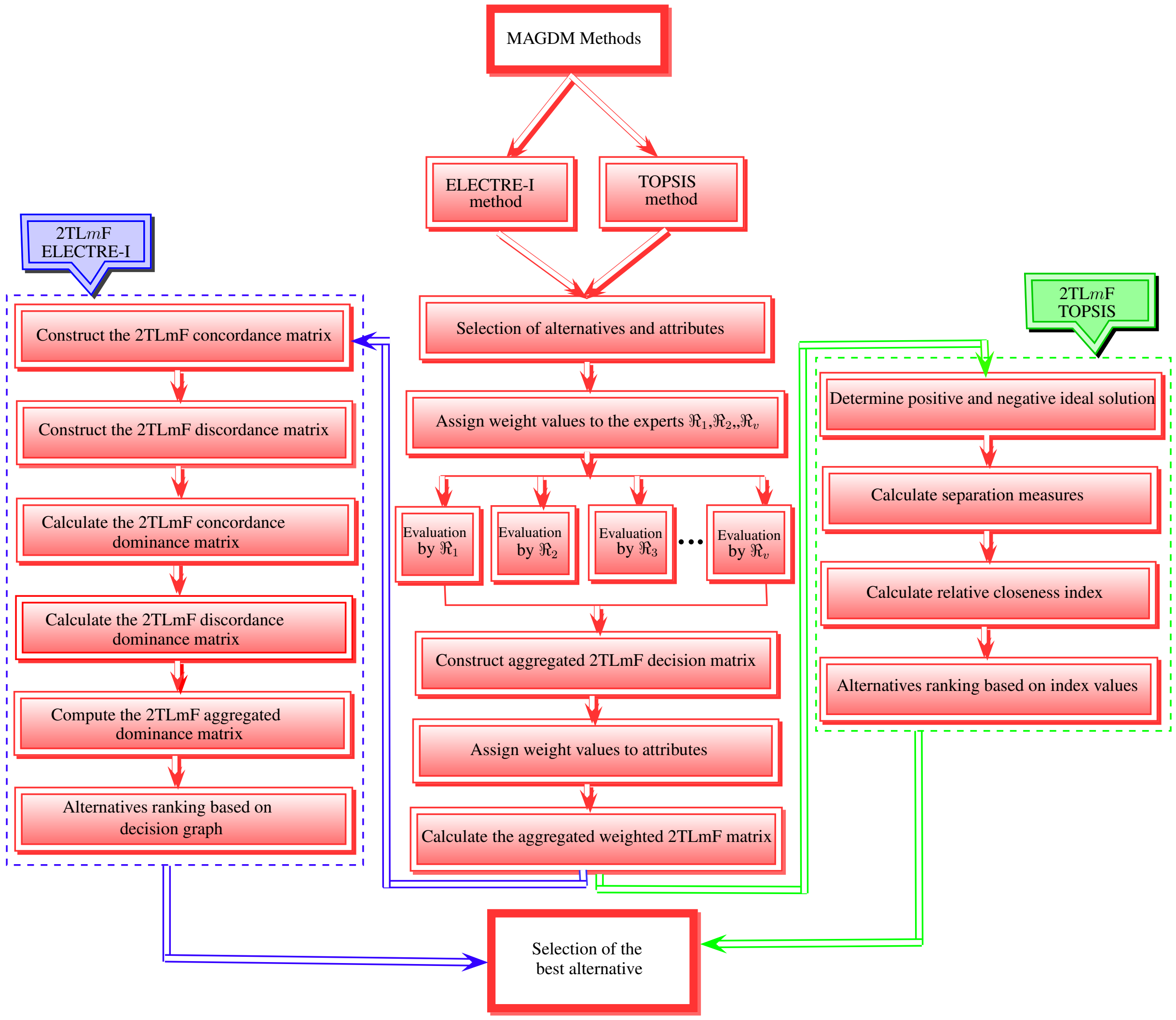

Figure 4. Flowchart for 2TLmF ELECTRE-I and 2TLmF-TOPSIS method.

6. Application

In this section, we apply the proposed 2TLmF ELECTRE-I method to the same MAGDM problems, whose case studies related to construction and industry field are already discussed in Section 3.

6.1. Numerical Example I: Selection of highway construction project manager

(1) In this subsection, we apply proposed 2TLmF ELECTRE-I method to the "Selection of Highway Construction Project Manager" as already 2TLmF-TOPSIS method is applied on it in Section 3. In 2TLmF ELECTRE-I method, **(Step 1–Step 4)** are same as already discussed in 2TLmF-TOPSIS method.

In order to construct the 2TL4F concordance and 2TL4F discordance set, the superiority and inferiority of the alternatives are checked by using 2TL4F score values as displayed in Table 15.

Table 15. 2TL4F score values.

Alternatives	$Ç_1$	$Ç_2$	$Ç_3$	$Ç_4$
F_1	$(c_1, 0.487394)$	$(c_1, 0.196308)$	$(c_3, -0.48428)$	$(c_1, -0.44210)$
F_2	$(c_2, 0.043372)$	$(c_2, -0.44857)$	$(c_3, 0.431724)$	$(c_2, -0.01857)$
F_3	$(c_3, 0.315239)$	$(c_3, -0.34750)$	$(c_4, 0.389165)$	$(c_5, -0.25393)$
F_4	$(c_2, -0.30922)$	$(c_1, 0.336906)$	$(c_2, -0.05586)$	$(c_1, -0.41972)$
F_5	$(c_2, -0.34475)$	$(c_1, 0.074408)$	$(c_2, 0.213861)$	$(c_1, -0.41162)$
F_6	$(c_5, -0.00162)$	$(c_1, 0.371090)$	$(c_5, 0.191220)$	$(c_5, -0.28005)$

Step 5: The 2TL4F concordance set is given in Table 16.

Table 16. 2TL4F concordance set representation.

k	1	2	3	4	5	6
M_{1k}	–	$\emptyset$	$\emptyset$	$\{3\}$	$\{2, 3\}$	$\emptyset$
M_{2k}	$\{1, 2, 3, 4\}$	–	$\emptyset$	$\{1, 2, 3, 4\}$	$\{1, 2, 3, 4\}$	$\{2\}$
M_{3k}	$\{1, 2, 3, 4\}$	$\{1, 2, 3, 4\}$	–	$\{1, 2, 3, 4\}$	$\{1, 2, 3, 4\}$	$\{2, 4\}$
M_{4k}	$\{1, 2, 4\}$	$\emptyset$	$\emptyset$	–	$\{1, 2\}$	$\emptyset$
M_{5k}	$\{1, 4\}$	$\emptyset$	$\emptyset$	$\{3, 4\}$	–	$\emptyset$
M_{6k}	$\{1, 2, 3, 4\}$	$\{1, 2, 3, 4\}$	$\{1, 3\}$	$\{1, 2, 3, 4\}$	$\{1, 2, 3, 4\}$	–

Note: $\emptyset$ indicates an empty set.

Step 6: The 2TL4F discordance set is given in Table 17.

Table 17. 2TL4F discordance set representation.

k	1	2	3	4	5	6
N_{1k}	–	$\{1, 2, 3, 4\}$	$\{1, 2, 3, 4\}$	$\{1, 2, 4\}$	$\{1, 4\}$	$\{1, 2, 3, 4\}$
N_{2k}	$\emptyset$	–	$\{1, 2, 3, 4\}$	$\emptyset$	$\emptyset$	$\{1, 3, 4\}$
N_{3k}	$\emptyset$	$\emptyset$	–	$\emptyset$	$\emptyset$	$\{1, 3\}$
N_{4k}	$\{3\}$	$\{1, 2, 3, 4\}$	$\{1, 2, 3, 4\}$	–	$\{3, 4\}$	$\{1, 2, 3, 4\}$
N_{5k}	$\{2, 3\}$	$\{1, 2, 3, 4\}$	$\{1, 2, 3, 4\}$	$\{1, 2\}$	–	$\{1, 2, 3, 4\}$
N_{6k}	$\emptyset$	$\{2\}$	$\{2, 4\}$	$\emptyset$	$\emptyset$	–

Step 7: The 2TL4F-concordance matrix is computed as given below:

$$\mathbb{P} = \begin{pmatrix} - & 0 & 0 & 0.4 & 0.6 & 0 \\ 1 & - & 0 & 1 & 1 & 0.2 \\ 1 & 1 & - & 1 & 1 & 0.3 \\ 0.6 & 0 & 0 & - & 0.5 & 0 \\ 0.4 & 0 & 0 & 0.5 & - & 0 \\ 1 & 0.8 & 0.7 & 1 & 1 & - \end{pmatrix}.$$

Step 8: The 2TL4F concordance level is: $\bar{p} = 0.5001$.
Step 9: The normalized Euclidean distances $d(\hat{z}_{rj}, \hat{z}_{sj})$ are arranged in Table 18, where the entries $\hat{z}_{rj}$, $\hat{z}_{sj}$ are from aggregated weighted 2TL4F decision matrix as arranged in Table 5.

Table 18. Normalized Euclidean distances for 2TL4F data $d(\hat{z}_{rj}, \hat{z}_{sj})$.

	$\hat{z}_{11}$	$\hat{z}_{21}$	$\hat{z}_{31}$	$\hat{z}_{41}$	$\hat{z}_{51}$	$\hat{z}_{61}$		$\hat{z}_{12}$	$\hat{z}_{22}$	$\hat{z}_{32}$	$\hat{z}_{42}$	$\hat{z}_{52}$	$\hat{z}_{62}$
$\hat{z}_{11}$	–	0.020029	0.070153	0.010664	0.01494	0.106889	$\hat{z}_{12}$	–	0.011135	0.065085	0.009263	0.01494	0.012061
$\hat{z}_{21}$	–	–	0.053503	0.010941	0.022640	0.096937	$\hat{z}_{22}$	–	–	0.061226	0.009763	0.014138	0.009500
$\hat{z}_{31}$	–	–	–	0.062834	0.068473	0.102559	$\hat{z}_{32}$	–	–	–	0.064142	0.071325	0.061664
$\hat{z}_{41}$	–	–	–	–	0.015663	0.103487	$\hat{z}_{42}$	–	–	–	–	0.012148	0.004630
$\hat{z}_{51}$	–	–	–	–	–	0.103076	$\hat{z}_{52}$	–	–	–	–	–	0.014473
$\hat{z}_{61}$	–	–	–	–	–	–	$\hat{z}_{62}$	–	–	–	–	–	–
	$\hat{z}_{13}$	$\hat{z}_{23}$	$\hat{z}_{33}$	$\hat{z}_{43}$	$\hat{z}_{53}$	$\hat{z}_{63}$		$\hat{z}_{14}$	$\hat{z}_{24}$	$\hat{z}_{34}$	$\hat{z}_{44}$	$\hat{z}_{54}$	$\hat{z}_{64}$
$\hat{z}_{13}$	–	0.046101	0.065046	0.031574	0.017043	0.087542	$\hat{z}_{14}$	–	0.077759	0.131008	0.003306	0.006669	0.129839
$\hat{z}_{23}$	–	–	0.044653	0.059610	0.047148	0.069929	$\hat{z}_{24}$	–	–	0.104477	0.076727	0.074959	0.147221
$\hat{z}_{33}$	–	–	–	0.086642	0.069149	0.077820	$\hat{z}_{34}$	–	–	–	0.131658	0.132380	0.099549
$\hat{z}_{43}$	–	–	–	–	0.022224	0.094621	$\hat{z}_{44}$	–	–	–	–	0.003528	0.129726
$\hat{z}_{53}$	–	–	–	–	–	0.094006	$\hat{z}_{54}$	–	–	–	–	–	0.130609
$\hat{z}_{63}$	–	–	–	–	–	–	$\hat{z}_{64}$	–	–	–	–	–	–

Step 10: The 2TL4F-discordance matrix is computed as given below:

$$\mathbb{Q} = \begin{pmatrix} - & 1 & 1 & 0.406777 & 0.876840 & 1 \\ 0 & - & 1 & 0 & 0 & 1 \\ 0 & 0 & - & 0 & 0 & 1 \\ 1 & 1 & 1 & - & 1 & 1 \\ 1 & 1 & 1 & 0.704784 & - & 1 \\ 0 & 0.064534 & 0.570654 & 0 & 0 & - \end{pmatrix}.$$

Step 11: The 2TL4F discordance level is: $\bar{q} = 0.60078$.
Step 12: By using concordance and discordance level, the 2TL4F-concordance dominance and 2TL4F-discordance dominance matrix are evaluated as:

$$\Upsilon = \begin{pmatrix} - & 0 & 0 & 0 & 1 & 0 \\ 1 & - & 0 & 1 & 1 & 0 \\ 1 & 1 & - & 1 & 1 & 0 \\ 1 & 0 & 0 & - & 1 & 0 \\ 0 & 0 & 0 & 1 & - & 0 \\ 1 & 1 & 1 & 1 & 1 & - \end{pmatrix},$$

$$\eta = \begin{pmatrix} - & 0 & 0 & 1 & 0 & 0 \\ 1 & - & 0 & 1 & 1 & 0 \\ 1 & 1 & - & 1 & 1 & 0 \\ 0 & 0 & 0 & - & 0 & 0 \\ 0 & 0 & 0 & 0 & - & 0 \\ 1 & 1 & 1 & 1 & 1 & - \end{pmatrix}.$$

Step 13: The 2TL4F aggregated dominance matrix is evaluated as

$$\tau = \begin{pmatrix} - & 0 & 0 & 0 & 0 & 0 \\ 1 & - & 0 & 1 & 1 & 0 \\ 1 & 1 & - & 1 & 1 & 0 \\ 0 & 0 & 0 & - & 0 & 0 \\ 0 & 0 & 0 & 0 & - & 0 \\ 1 & 1 & 1 & 1 & 1 & - \end{pmatrix}.$$

Step 14: The alternatives ranking is shown in Figure 5. Thus, F_6 is outranking over all other alternatives, F_1, F_2, F_3, F_4, F_5. Where the partial preferences of the alternatives are pinned up in Table 19.

Table 19. Decision graph exploration.

Alternatives	Submissive alternatives	Incomparable alternatives
F_1	–	F_4, F_5
F_2	F_1, F_4, F_5	–
F_3	F_1, F_2, F_4, F_5	–
F_4	–	F_1, F_5
F_5	–	F_1, F_4
F_6	F_1, F_2, F_3, F_4, F_5	–

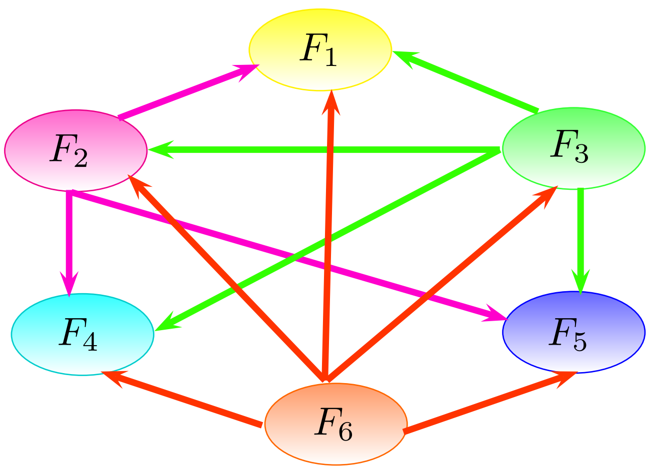

Figure 5. Directed graph for alternatives ranking of Example I.

Step 15: The concordance $\overbrace{\Theta_i}$ and discordance $\underbrace{\Theta_i}$ outranking coefficients are displayed in Table 20, with net Ξ outranking index. Table 20, represents that F_6 is the best alternative with linear ranking order of alternatives, $F_6 > F_3 > F_2 > F_1 > F_5 > F_4$.

Table 20. Outranking indices of the alternatives.

Alternatives	Concordance outranking index	Discordance outranking index	Net outranking index	Ranking
F_1	-3.001	2.2836	-5.2836	4
F_2	1.390	-1.0645	2.4645	3
F_3	3.570	-3.9706	7.5706	2
F_4	-2.761	3.8884	-6.6884	6
F_5	-3.200	2.8279	-6.0279	5
F_6	4.001	-3.9648	7.9648	1

6.2. Numerical Example II: Selection of the best textile industry

(2) We now used our proposed 2TL*m*F ELECTRE-I method for the "Selection of the Best Textile Industry" as already 2TL*m*F-TOPSIS method is applied on it in Section 3.

In 2TL*m*F ELECTRE-I method **(Step 1–Step 4)** are same as in 2TL*m*F-TOPSIS method.

In order to construct the 2TL6F concordance and 2TL6F discordance set, every pair of the alternatives are compared by using 2TL6F score values as displayed in Table 21.

Table 21. 2TL4F score values.

Alternatives	$\mathbb{T}_1$	$\mathbb{T}_2$	$\mathbb{T}_3$
$\mathbb{L}_1$	$(\hbar_2, 0.200083)$	$(\hbar_2, -0.35354)$	$(\hbar_2, 0.028800)$
$\mathbb{L}_2$	$(\hbar_3, 0.094133)$	$(\hbar_2, -0.23609)$	$(\hbar_3, 0.090886)$
$\mathbb{L}_3$	$(\hbar_2, 0.271047)$	$(\hbar_1, 0.344972)$	$(\hbar_2, -0.18087)$
$\mathbb{L}_4$	$(\hbar_3, 0.131794)$	$(\hbar_3, -0.12751)$	$(\hbar_3, -0.10281)$
$\mathbb{L}_5$	$(\hbar_3, -0.30383)$	$(\hbar_2, -0.1702)$	$(\hbar_3, 0.001640)$
$\mathbb{L}_6$	$(\hbar_3, 0.331541)$	$(\hbar_2, 0.068230)$	$(\hbar_3, -0.35713)$

Step 5: The 2TL6F concordance set is given in Table 22.

Table 22. 2TL6F concordance set representation.

k	1	2	3	4	5	6
$\mathfrak{M}_{1k}$	–	$\emptyset$	$\{2,3\}$	$\emptyset$	$\emptyset$	$\emptyset$
$\mathfrak{M}_{2k}$	$\{1,2,3\}$	–	$\{1,2,3\}$	$\{3\}$	$\{1,3\}$	$\{3\}$
$\mathfrak{M}_{3k}$	$\{1\}$	$\emptyset$	–	$\emptyset$	$\emptyset$	$\emptyset$
$\mathfrak{M}_{4k}$	$\{1,2,3\}$	$\{1,2\}$	$\{1,2,3\}$	–	$\{1,2\}$	$\{2,3\}$
$\mathfrak{M}_{5k}$	$\{1,2,3\}$	$\{2\}$	$\{1,2,3\}$	$\{3\}$	–	$\{3\}$
$\mathfrak{M}_{6k}$	$\{1,2,3\}$	$\{1,2\}$	$\{1,2,3\}$	$\{1\}$	$\{1,2\}$	–

Step 6: The 2TL6F discordance set is given in Table 23.

Table 23. 2TL6F discordance set representation.

k	1	2	3	4	5	6
$\mathfrak{N}_{1k}$	$-$	$\{1,2,3\}$	$\{1\}$	$\{1,2,3\}$	$\{1,2,3\}$	$\{1,2,3\}$
$\mathfrak{N}_{2k}$	$\emptyset$	$-$	$\emptyset$	$\{1,2\}$	$\{2\}$	$\{1,2\}$
$\mathfrak{N}_{3k}$	$\{2,3\}$	$\{1,2,3\}$	$-$	$\{1,2,3\}$	$\{1,2,3\}$	$\{1,2,3\}$
$\mathfrak{N}_{4k}$	$\emptyset$	$\{3\}$	$\emptyset$	$-$	$\{3\}$	$\{1\}$
$\mathfrak{N}_{5k}$	$\emptyset$	$\{1,3\}$	$\emptyset$	$\{1,2\}$	$-$	$\{1,2\}$
$\mathfrak{N}_{6k}$	$\emptyset$	$\{3\}$	$\emptyset$	$\{2,3\}$	$\{3\}$	$-$

Step 7: The 2TL6F concordance matrix is calculated as given below:

$$\mathfrak{P} = \begin{pmatrix} - & 0 & 0.579 & 0 & 0 & 0 \\ 1 & - & 1 & 0.345 & 0.766 & 0.345 \\ 0.421 & 0 & - & 0 & 0 & 0 \\ 1 & 0.655 & 1 & - & 0.655 & 0.579 \\ 1 & 0.234 & 1 & 0.345 & - & 0.345 \\ 1 & 0.655 & 1 & 0.421 & 0.655 & - \end{pmatrix}.$$

Step 8: The 2TL6F concordance level is: $\hat{\hat{p}} = 0.5001$.

Step 9: The normalized Euclidean distances $d(\breve{z}_{rj}, \breve{z}_{sj})$ are arranged in Table 24, where the entries $\breve{z}_{rj}$, $\breve{z}_{sj}$ are from aggregated weighted 2TL6F decision matrix as arranged in Table 12.

Step 10: The 2TL6F-discordance matrix is computed as given below:

$$\mathfrak{Q} = \begin{pmatrix} - & 1 & 1 & 1 & 1 & 1 \\ 0 & - & 0 & 1 & 0.521245 & 0.975527 \\ 0.865896 & 1 & - & 1 & 1 & 1 \\ 0 & 0.272636 & 0 & - & 0.295279 & 0.149646 \\ 0 & 1 & 0 & 1 & - & 0.903347 \\ 0 & 1 & 0 & 1 & 1 & - \end{pmatrix}.$$

Step 11: The 2TL6F discordance level is: $\hat{\hat{q}} = 0.632785$.

Step 12: By using concordance and discordance level, the 2TL6F-concordance dominance and 2TL6F-discordance dominance matrix are evaluated as:

$$\beth = \begin{pmatrix} - & 0 & 1 & 0 & 0 & 0 \\ 1 & - & 0 & 0 & 1 & 0 \\ 0 & 0 & - & 0 & 0 & 0 \\ 1 & 1 & 1 & - & 1 & 1 \\ 1 & 0 & 1 & 0 & - & 0 \\ 1 & 1 & 1 & 0 & 1 & - \end{pmatrix},$$

$$¥ = \begin{pmatrix} - & 0 & 0 & 0 & 0 & 0 \\ 1 & - & 1 & 0 & 1 & 0 \\ 0 & 0 & - & 0 & 0 & 0 \\ 1 & 1 & 1 & - & 1 & 1 \\ 1 & 0 & 1 & 0 & - & 0 \\ 1 & 0 & 1 & 0 & 0 & - \end{pmatrix}.$$

Table 24. Normalized Euclidean distances for 2TL6F data $d(\breve{z}_{rj}, \breve{z}_{sj})$.

	$\breve{z}_{11}$	$\breve{z}_{21}$	$\breve{z}_{31}$	$\breve{z}_{41}$	$\breve{z}_{51}$	$\breve{z}_{61}$		$\breve{z}_{12}$	$\breve{z}_{22}$	$\breve{z}_{32}$	$\breve{z}_{42}$	$\breve{z}_{52}$	$\breve{z}_{62}$		$\breve{z}_{13}$	$\breve{z}_{23}$	$\breve{z}_{33}$	$\breve{z}_{43}$	$\breve{z}_{53}$	$\breve{z}_{63}$
$\breve{z}_{11}$	–	0.135030	0.074299	0.152189	0.100168	0.162004	$\breve{z}_{12}$	–	0.025925	0.064335	0.312211	0.060308	0.070286	$\breve{z}_{13}$	–	0.153685	0.059102	0.128600	0.133460	0.112571
$\breve{z}_{21}$	–	–	0.150797	0.121593	0.111715	0.112332	$\breve{z}_{22}$	–	–	0.081155	0.310060	0.060833	0.053906	$\breve{z}_{23}$	–	–	0.167385	0.084533	0.116708	0.115150
$\breve{z}_{31}$	–	–	–	0.125217	0.076831	0.135916	$\breve{z}_{32}$	–	–	–	0.358565	0.081366	0.11196	$\breve{z}_{33}$	–	–	–	0.139868	0.170580	0.119144
$\breve{z}_{41}$	–	–	–	–	0.062955	0.047672	$\breve{z}_{42}$	–	–	–	–	0.291691	0.318564	$\breve{z}_{43}$	–	–	–	–	0.086130	0.079839
$\breve{z}_{51}$	–	–	–	–	–	0.089844	$\breve{z}_{52}$	–	–	–	–	–	0.081354	$\breve{z}_{53}$	–	–	–	–	–	0.0994578
$\breve{z}_{61}$	–	–	–	–	–	–	$\breve{z}_{62}$	–	–	–	–	–	–	$\breve{z}_{63}$	–	–	–	–	–	–

Step 13: The 2TL6F aggregated dominance matrix is evaluated as

$$\daleth = \begin{pmatrix} - & 0 & 0 & 0 & 0 & 0 \\ 1 & - & 1 & 0 & 1 & 0 \\ 0 & 0 & - & 0 & 0 & 0 \\ 1 & 1 & 1 & - & 1 & 1 \\ 1 & 0 & 1 & 0 & - & 0 \\ 1 & 0 & 1 & 0 & 0 & - \end{pmatrix}.$$

Step 14: The alternatives ranking is shown in Figure 6. Thus, $\mathbb{L}_4$ is outranking over all other alternatives, $\mathbb{L}_1, \mathbb{L}_2, \mathbb{L}_3, \mathbb{L}_5, \mathbb{L}_6$. Where the partial preferences of the alternatives are pinned up in Table 25.

Table 25. Decision graph exploration.

Alternatives	Submissive alternatives	Incomparable alternatives
$\mathbb{L}_1$	–	$\mathbb{L}_3$
$\mathbb{L}_2$	$\mathbb{L}_1, \mathbb{L}_3, \mathbb{L}_5$	$\mathbb{L}_6$
$\mathbb{L}_3$	–	$\mathbb{L}_1$
$\mathbb{L}_4$	$\mathbb{L}_1, \mathbb{L}_2, \mathbb{L}_3, \mathbb{L}_5, \mathbb{L}_6$	–
$\mathbb{L}_5$	$\mathbb{L}_1, \mathbb{L}_3$	$\mathbb{L}_6$
$\mathbb{L}_6$	$\mathbb{L}_1, \mathbb{L}_3$	$\mathbb{L}_2, \mathbb{L}_5$

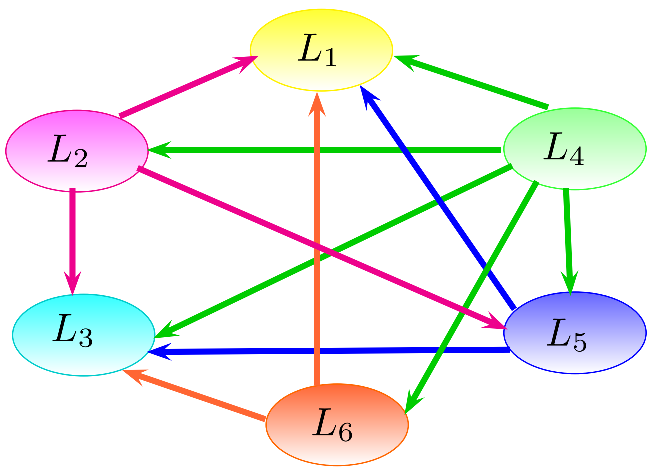

Figure 6. Directed graph for alternatives ranking of Example II.

Step 15: The concordance $\overbrace{\Omega_i}$ and discordance $\underbrace{\mho_i}$ outranking coefficients are displayed in Table 26, with net $\pounds$ outranking index. Table 26 represents that $\mathbb{L}_6$ is the best alternative with linear ranking order of alternatives, $\mathbb{L}_4 > \mathbb{L}_2 > \mathbb{L}_6 > \mathbb{L}_5 > \mathbb{L}_1 > \mathbb{L}_3$.

Table 26. Outranking indices of the alternatives.

Alternatives	Concordance outranking index	Discordance outranking index	Net outranking index	Ranking
$\mathbb{L}_1$	-3.842	4.13410	-7.9761	5
$\mathbb{L}_2$	1.912	-1.0645	3.6878	2
$\mathbb{L}_3$	-4.158	3.8658	-8.0238	6
$\mathbb{L}_4$	2.778	3.8884	7.0604	1
$\mathbb{L}_5$	0.848	-0.9131	1.7611	4
$\mathbb{L}_6$	2.462	-1.0285	3.4905	3

7. Comparative analysis

7.1. Comparison of the proposed methods

In this subsection, we present comparison study of our proposed methods, namely, 2TL*m*F-TOPSIS and 2TL*m*F ELECTRE-I. The 2TL*m*F-TOPSIS technique is, based on the concept of finding an alternative, that is nearest to the 2TLPIS and far away from the 2TLNIS, while an outranking 2TL*m*F ELECTRE-I method is based on the pairwise comparison of the alternatives. The 2TL*m*F ELECTRE-I method sometimes provides two optimal solutions at once because it only considers the benefit criterion, but the 2TL*m*F-TOPSIS method provides a single alternative because we believe that both benefits and cost standards. Based on two case studies related to construction and industrial fields by using the same descriptive 2TL*m*F set, the comparison of proposed methods is described below:

(1) For "Selection of Highway Construction Project Manager" the comparison between 2TL*m*F-TOPSIS and 2TL*m*F ELECTRE-I methods is displayed in Table 27. This comparison shows that F_6 is the most desirable alternative by using the 2TLmF ELECTRE-1 and 2TLmF TOPSIS methods, order of ranking is given in Table 27. Comparison graph of proposed methods for the selection of highway construction project manager is displayed in Figure 7.

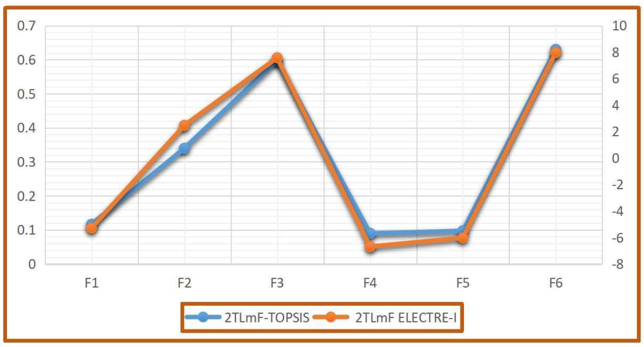

Figure 7. Comparison of proposed methods in case of manager selection.

Table 27. Alternative ranking order.

Methods	Ranking	Best alternative
2TL*m*F TOPSIS	$F_6 > F_3 > F_2 > F_1 > F_5 > F_4$	F_6
2TL*m*F ELECTRE-1	$F_6 > F_3 > F_2 > F_1 > F_5 > F_4$	F_6

(2) For "Selection of the best textile industry" the comparison between 2TL*m*F-TOPSIS and 2TL*m*F ELECTRE-I methods is displayed in Table 28. This comparison shows that $\mathbb{L}_4$ is the most desirable alternative, order of ranking is given in Table 28. The comparative graph of proposed methods for the selection of best textile industry is displayed in Figure 8.

Table 28. Alternative ranking order.

Methods	Ranking	Best alternative
2TL*m*F TOPSIS	$\mathbb{L}_4 > \mathbb{L}_2 > \mathbb{L}_6 > \mathbb{L}_5 > \mathbb{L}_1 > \mathbb{L}_3$	$\mathbb{L}_4$
2TL*m*F ELECTRE-1	$\mathbb{L}_4 > \mathbb{L}_2 > \mathbb{L}_6 > \mathbb{L}_5 > \mathbb{L}_1 > \mathbb{L}_3$	$\mathbb{L}_4$

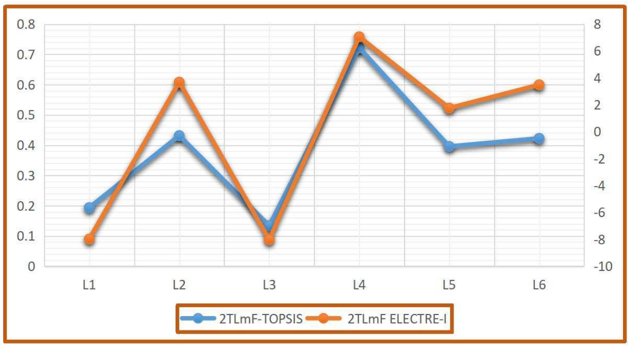

Figure 8. Comparison of proposed methods in case of textile industry selection.

7.2. *Comparison with existing techniques*

Recently, Akram et al. [28] developed the 2-tuple linguistic *m*-polar fuzzy Hamacher weighted average (2TL*m*FHWA) and 2TL*m*FHWG operators. In this subsection, we compare the proposed methods with the existing method [28] to check the applicability and versatility of the proposed models.

(1) For a comparative study between the proposed method and the existing method [28], i.e. 2TL*m*FHWA and 2TL*m*FHWG operators with parameters $\lambda = 3$, are employed to assemble the values given in Table 4. The values combined using the 2TL*m*FHWA and 2TL*m*FHWG operators are given in the Tables 29 and 30. The score values for each alternative using the Definition 2.7 are given in Tables 31 and 32. Therefore, according to the comparison of the existing model and the proposed model, F_6 is the most ideal alternative, ensuring the authenticity of our proposed study. The rank order of alternatives for existing and proposed work is shown in the Table 37.

Table 29. Assembled values ($\hat{\flat}_i$) by using the 2TL*m*FHWA operator.

$\hat{\flat}_i$	2TL*m*FHWA operator
$\hat{\flat}_1$	$((c_4, -0.237025), (c_4, -0.156577), (c_4, 0.405718), (c_4, 0.312066))$
$\hat{\flat}_2$	$((c_4, 0.3160501), (c_6, 0.000000), (c_5, -0.302145), (c_6, 0.000000))$
$\hat{\flat}_3$	$((c_6, 0.000000), (c_5, 0.2000000), (c_6, 0.000000), (c_5, 0.2000000))$
$\hat{\flat}_4$	$((c_4, 0.180325), (c_4, -0.352272), (c_4, 0.024654), (c_4, -0.06813))$
$\hat{\flat}_5$	$((c_4, -0.317807), (c_4, -0.09277), (c_3, 0.49582), (c_5, -0.350218))$
$\hat{\flat}_6$	$((c_6, 0.000000), (c_5, -0.354769), (c_6, 0.000000), (c_6, 0.000000))$

Table 30. Scores values for all the 2TL4F numbers $\hat{\flat}_i$.

Scores values	2TL*m*FHWA operator
$\mathbb{S}(\hat{\flat}_1)$	$(c_4, 0.081045)$
$\mathbb{S}(\hat{\flat}_2)$	$(c_5, 0.330680)$
$\mathbb{S}(\hat{\flat}_3)$	$(c_6, -0.40000)$
$\mathbb{S}(\hat{\flat}_4)$	$(c_4, -0.05385)$
$\mathbb{S}(\hat{\flat}_5)$	$(c_4, -0.06624)$
$\mathbb{S}(\hat{\flat}_6)$	$(c_6, -0.33869)$

Table 31. Assembled values ($\hat{\flat}_i$) by using the 2TL*m*FHWG operator.

$\hat{\flat}_i$	2TL*m*FHWG operator
$\hat{\flat}_1$	$((c_4, -0.303973), (c_4, -0.298562), (c_4, 0.2921963), (c_4, 0.2656442))$
$\hat{\flat}_2$	$((c_4, 0.2968202), (c_5, -0.181945), (c_5, -0.308545), (c_5, -0.066331))$
$\hat{\flat}_3$	$((c_5, -0.169367), (c_6, -0.240905), (c_5, 0.3103902), (c_5, 0.2236302))$
$\hat{\flat}_4$	$((c_4, 0.1669631), (c_4, -0.482343), (c_4, 0.0040604), (c_4, -0.074481))$
$\hat{\flat}_5$	$((c_4, -0.355479), (c_4, -0.140987), (c_3, 0.4912188), (c_5, -0.4038449))$
$\hat{\flat}_6$	$((c_6, -0.436916), (c_5, -0.359750), (c_6, -0.240905), (c_6, -0.352806))$

Table 32. Scores values for all the 2TL4F numbers $\hat{\flat}_i$.

Scores values	2TL*m*FHWG operator
$\mathbb{S}(\hat{\flat}_1)$	$(c_4, -0.011173)$
$\mathbb{S}(\hat{\flat}_2)$	$(c_5, -0.315000)$
$\mathbb{S}(\hat{\flat}_3)$	$(c_5, 0.2809367)$
$\mathbb{S}(\hat{\flat}_4)$	$(c_4, -0.096450)$
$\mathbb{S}(\hat{\flat}_5)$	$(c_4, -0.102273)$
$\mathbb{S}(\hat{\flat}_6)$	$(c_5, 0.4024051)$

(2) For a comparative study between the proposed method and the existing method [28], i.e. 2TL*m*FHWA and 2TL*m*FHWG operators with parameters $\lambda = 3$, are employed to assemble the values given in Table 11. The values assembled using the 2TL*m*FHWA and 2TL*m*FHWG operators are given in the Tables 33 and 34. The score values for each alternative using the Definition 2.7 are given in Tables 35 and 36. Therefore, according to the comparison of the existing model and the proposed model, $\mathbb{L}_4$ is the most ideal alternative, ensuring the authenticity of our proposed study. The rank order of alternatives for existing and proposed work is shown in the Table 37.

Table 33. Assembled values ($\tilde{\natural}_i$) by using the 2TL*m*FHWA operator.

$\tilde{\natural}_i$	2TL*m*FHWA operator
$\tilde{\natural}_1$	$((\hbar_4, 0.11334054), (\hbar_4, 0.2419176), (\hbar_5, -0.050780), (\hbar_4, 0.1573784), (\hbar_5, 0.1203015), (\hbar_5, -0.356388))$
$\tilde{\natural}_2$	$((\hbar_5, 0.3708851), (\hbar_5, 0.18080780), (\hbar_6, -0.241310), (\hbar_5, 0.2173407), (\hbar_6, 0.2731900), (\hbar_6, 0.0144995))$
$\tilde{\natural}_3$	$((\hbar_4, -0.007891), (\hbar_4, -0.0935551), (\hbar_4, 0.2726495), (\hbar_4, 0.367347), (\hbar_4, 0.4859191), (\hbar_5, -0.207669))$
$\tilde{\natural}_4$	$((\hbar_6, -0.249351), (\hbar_6, -0.003903), (\hbar_6, -0.393966), (\hbar_8, 0.0000000), (\hbar_5, -0.313300), (\hbar_6, 0.101870))$
$\tilde{\natural}_5$	$((\hbar_5, 0.24687350), (\hbar_6, 0.1279355), (\hbar_5, 0.2425176), (\hbar_5, 0.1416756), (\hbar_5, 0.3783680), (\hbar_5, 0.3944357))$
$\tilde{\natural}_6$	$((\hbar_5, 0.3282144), (\hbar_6, -0.390561), (\hbar_5, 0.1911016), (\hbar_6, -0.106643), (\hbar_6, -0.268146), (\hbar_6, 0.2560303))$

Table 34. Scores values for all the 2TL6F numbers $\tilde{\natural}_i$.

Scores values	2TL*m*FHWA operator
$\mathbb{S}(\tilde{\natural}_1)$	$(\hbar_5, -0.462371)$
$\mathbb{S}(\tilde{\natural}_2)$	$(\hbar_6, -0.364097)$
$\mathbb{S}(\tilde{\natural}_3)$	$(\hbar_4, 0.3027999)$
$\mathbb{S}(\tilde{\natural}_4)$	$(\hbar_6, 0.0235580)$
$\mathbb{S}(\tilde{\natural}_5)$	$(\hbar_5, 0.4219677)$
$\mathbb{S}(\tilde{\natural}_6)$	$(\hbar_6, -0.331667)$

Table 35. Assembled values ($\tilde{\natural}_i$) by using the 2TL*m*FHWG operator.

$\tilde{\natural}_i$	2TL*m*FHWG operator
$\tilde{\natural}_1$	$((\hbar_4, 0.0507263), (\hbar_4, 0.2128660), (\hbar_5, -0.051972), (\hbar_4, 0.0254261), (\hbar_5, 0.1163187), (\hbar_5, -0.407553))$
$\tilde{\natural}_2$	$((\hbar_5, 0.3161339), (\hbar_5, 0.1347207), (\hbar_6, -0.403355), (\hbar_5, 0.1588720), (\hbar_6, 0.192966), (\hbar_6, -0.067063))$
$\tilde{\natural}_3$	$((\hbar_4, -0.113670), (\hbar_4, -0.146005), (\hbar_4, 0.2602225), (\hbar_4, 0.359559), (\hbar_4, 0.4149561), (\hbar_5, -0.260478))$
$\tilde{\natural}_4$	$((\hbar_6, -0.286330), (\hbar_6, -0.013376), (\hbar_6, -0.400008), (\hbar_6, -0.034697), (\hbar_5, -0.32309), (\hbar_6, 0.030347))$
$\tilde{\natural}_5$	$((\hbar_5, 0.2282171), (\hbar_6, 0.0457900), (\hbar_5, 0.2126563), (\hbar_5, 0.0541447), (\hbar_5, 0.2693903), (\hbar_5, 0.3542517))$
$\tilde{\natural}_6$	$((\hbar_5, 0.3234717), (\hbar_6, -0.4208887), (\hbar_5, 0.1841108), (\hbar_6, -0.127622), (\hbar_6, -0.282073), (\hbar_6, 0.116531))$

Table 36. Scores values for all the 2TL6F numbers $\tilde{\natural}_i$.

Scores values	2TLmFHWG operator
$\mathbb{S}(\tilde{\natural}_1)$	$(\hbar_4, 0.49096840)$
$\mathbb{S}(\tilde{\natural}_2)$	$(\hbar_6, -0.4446209)$
$\mathbb{S}(\tilde{\natural}_3)$	$(\hbar_4, 0.25243047)$
$\mathbb{S}(\tilde{\natural}_4)$	$(\hbar_6, -0.3378598)$
$\mathbb{S}(\tilde{\natural}_5)$	$(\hbar_5, 0.36074173)$
$\mathbb{S}(\tilde{\natural}_6)$	$(\hbar_6, -0.3677451)$

Table 37. Comparison of proposed models with existing work.

Case study	Methods	Ranking	Best alternative
Project manager selection	2TLmFHWA [28]	$F_6 > F_3 > F_2 > F_1 > F_4 > F_5$	F_6
	2TLmFHWG [28]	$F_6 > F_3 > F_2 > F_1 > F_4 > F_5$	F_6
	2TLmF TOPSIS (proposed)	$F_6 > F_3 > F_2 > F_1 > F_5 > F_4$	F_6
	2TLmF ELECTRE-1 (proposed)	$F_6 > F_3 > F_2 > F_1 > F_5 > F_4$	F_6
Textile industry selection	2TLmFHWA [28]	$\mathbb{L}_4 > \mathbb{L}_6 > \mathbb{L}_2 > \mathbb{L}_5 > \mathbb{L}_1 > \mathbb{L}_3$	$\mathbb{L}_4$
	2TLmFHWG [28]	$\mathbb{L}_4 > \mathbb{L}_6 > \mathbb{L}_2 > \mathbb{L}_5 > \mathbb{L}_1 > \mathbb{L}_3$	$\mathbb{L}_4$
	2TLmF TOPSIS (proposed)	$\mathbb{L}_4 > \mathbb{L}_2 > \mathbb{L}_6 > \mathbb{L}_5 > \mathbb{L}_1 > \mathbb{L}_3$	$\mathbb{L}_4$
	2TLmF ELECTRE-1 (proposed)	$\mathbb{L}_4 > \mathbb{L}_2 > \mathbb{L}_6 > \mathbb{L}_5 > \mathbb{L}_1 > \mathbb{L}_3$	$\mathbb{L}_4$

Finally, the similar results of the proposed and existing methods illustrate the reliability and transparency of our proposed methods. In contrast, our proposed methods can quickly summarize mF and 2TL information without parameter selection. However, selecting different parameter values makes the prior art difficult. These limitations of existing work are being reduced, and we can investigate better resiliency in practical use by using our proposed MAGDM model. Therefore, the advanced decision model provides a new flexible measure for expert control of the 2TLmF-MAGDM problem.

8. Contribution and limitations

In this section, we describe the contribution and limitations of the proposed work. The major contribution of this research article is described as follows:

- The most important contribution of this study is the establishment of revolutionary strategies to efficiently manipulate 2TLmF information with great precision to the benefit of MAGDM.
- The proposed method 2TLmF-TOPSIS focuses on 2TLPIS, 2TLNIS, normalized Euclidean distance using 2TL information. The alternatives ranking is based on a relative closeness index.
- In another proposed method, 2TLmF ELECTRE-I, the salient concepts of concordance and discordance sets are defined by using 2TLmF score and 2TLmF accuracy values. The strategy uses outranking relationships and outranking graphs to indicate the best alternatives.
- The overall picture of the proposed technique is depicted by a flowchart, which thoroughly depicts the step-by-step approach of the proposed work.

- The practical application of the proposed methods 2TL*m*F-TOPSIS, 2TL*m*F-ELECTRE-I is demonstrated by using two case studies from the construction and industrial fields to consider the importance and effectiveness of the provided techniques.
- A comparative study is carried out with existing techniques, which provides transparency and validity of our proposed methods.

The limitations of this research article are described below:

- The proposed methods, i.e. 2TL*m*F-TOPSIS and 2TL*m*F ELECTRE-I only deal with 2TL*m*-polar membership values and cannot take into account the unfavorable and hesitant preferences of decision makers.
- The 2TL nature of multi-pole data makes the proposed method difficult because of the evaluation of symbolic translation at every step of calculations.

9. Conclusions

The simplicity of *m*-polar fuzzy sets has been discovered by many researchers in a short time since many real-world examples rely on multipolar information. However, in the issue at hand, the technique cannot handle 2-tuple linguistic information, which can lead to data loss. Therefore, the multi-functional 2TL*m*FSs are found to be more suitable to utilize the linguistic multi-polar data aptly owing to its captivating design and modern structure in complex decision-making scenarios. In this research paper, the popular theories and heuristics of the TOPSIS and ELECTRE-I methods has been exploited for constructing new group decision-making techniques, namely, 2TL*m*F-TOPSIS and 2TL*m*F-ELECTRE I methods, which are beneficial and convenient for decision-makers. In addition, the presented MAGDM strategies have been embedded in an elegant flow chart diagram. Furthermore, the 2TL*m*F-TOPSIS and 2TL*m*F-ELECTRE I methods have been implemented with detailed explanations to solve two real-life problems for the evaluation of suitable highway construction project manager and the best textile industry. The proposed techniques have been proved to be transparent and suitable for MAGDM, as these techniques successfully deal with ambiguous data on the properties of 2TL*m*FS. Ultimately, the ability of the proposed method to take over data qualitatively and quantitatively makes it superior to previous techniques, preventing data loss, and engaging researchers for future decision-making. In the future, we will try to extend our work to other variants of the ELECTRE family, such as ELECTRE-II, ELECTRE-III, and ELECTRE-IV, under the 2TL*m*FS.

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Conflict of interest

The authors declare that they have no conflicts of interest.

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