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Research article

Non-parametric hypothesis testing to model some cancers based on goodness of fit

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Abstract: By observing the failure behavior of the recorded survival data, we aim to compare the different processing approaches or the effectiveness of the devices or systems applied in this non-parametric statistical test. We'll apply the proposed strategy of used better than aged in Laplace (UBAL) transform order, which assumes that the data used in the test will either behave as UBAL Property or exponential behavior. If the survival data is UBAL, it means that the suggested treatment strategy is effective, whereas if the data is exponential, the recommended treatment strategy has no negative or positive effect on patients, as indicated in the application section. To guarantee the test's validity, we calculated the suggested test's power in both censored and uncensored data, as well as its efficiency, compared the results to other tests, and then applied the test to a variety of real data.

Keywords: testing hypothesis; right censored data; exponential; Weibull; Gamma; Makeham and linear failure rate (LFR) distributions; UBA and UBAL classes of life distributions; medical data
Mathematics Subject Classification: 62G10, 62G20, 62N05

1. Introduction

Failure occurs when a unit or component fails to perform its needed function. The analysis of survival dataset failure behavior entails identifying whether the data exhibit a UBAL, or a constant failure rate. The two primary characteristics of the exponential distribution are: The memoryless property and the constant rate of failure property. The exponential distribution is the most important member of the life distribution classes due to these two characteristics. We now have a dataset with two claims: First, that the data are exponential, and second, that the data are UBAL. A statistical test is required to support one of the two hypotheses or claims, indicating which one is correct. The classification of life probability distributions has recently aided in the creation of novel high-efficiency statistical tests.

Several categories of life distributions have been studied to model data with different aging aspects. There are numerous definitions for various life distributions, like the IFR, IFRA, Navarro and Pellerey [1], Bryson and Siddiqui [2], Barlow and Proschan [3], Esary et al. [4] and Navarro J. [5]. Many researchers have discussed various aging classifications, such as NBUC and NWUC were introduced by Cao and Wang [6]. Fernandez-Ponce et al. [7] have also looked into the multivariate NBU. Furthermore, Ahmad [8] looked at UBA and UBAE. The Laplace order for UBA has been explored by Abu Youssef et al. [9].

The implications of the common classes of life distributions, which include the majority of well-known classes such as IFR, UBA, UBAE, and UBAL, are discussed as follows:

$$\begin{array}{ccccc} \text{IFR [1]} & \Rightarrow & \text{UBA [8]} & \Rightarrow & \text{UBAL [9]} \\ & & \Downarrow & & \\ & & \text{UBAE [8]} & & \end{array}$$

If $0 < \mu(\infty) < \infty$ and for all $x, t \geq 0$, Ahmad [8] defined the life distribution of used better than aged (UBA) as:

$$\bar{F}(t)e^{-\frac{x}{\mu(\infty)}} \leq \bar{F}(x+t), \qquad x, t \geq 0,$$

and used better than the aged in expectation (UBAE):

$$\mu(t) \geq \mu(\infty),$$

where

$$\bar{F}_t(x) = \begin{cases} \dfrac{\bar{F}(x+t)}{\bar{F}(t)} & \bar{F}(t) > 0 \\ 0 & \bar{F}(t) = 0 \end{cases},$$

and

$$\mu = E(X) = \int_0^\infty \bar{F}(u)du\,, \mu(t) = E(X_t) = \frac{\int_t^\infty \bar{F}(u)du}{\bar{F}(t)}.$$

Definition:

We said that F has used better than aged in the Laplace (UBAL) transform order property if $0 < \mu(\infty) < \infty$, $\forall x, t \geq 0$,

$$\int_0^\infty \bar{F}(x+t)\, e^{-sx}dx \geq \bar{F}(t)\, \frac{\mu(\infty)}{1+s\mu(\infty)}, \;\; s \geq 0, \tag{1.1}$$

for more details, see Abu Youssef and Bakr [10].

The major aim of this research is to address the issue of comparing $H_0 : F$ is exponential to $H_1 :$

F is the greatest class of life distribution UBAL. The following is how the paper is structured: In Section 2, we provide a test statistic for complete data based on the goodness of fit technique, Monte Carlo critical values are simulated for different sample sizes, and power estimates are produced and presented. The test statistic for censored data is obtained in Section 3. Finally, in Section 4, we go through some examples of how the suggested statistical test can be used in practice.

2. Testing complete data

A random sample of F is represented by $X_1, X_2, \ldots, X_n$. We develop a test statistic to test the null hypothesis $H_0 : F$ is exponential ($F(t) = \beta e^{-\beta t}$), vs $H_1 : F$ is UBAL. Many writers have addressed non-parametric testing for classes of life distributions (see Fernandez-Ponce and Rodrıguez-Grinolo [11]; Abu-Youssef et al. [9]; Mahmoud et al. [12]; Abu-Youssef et al. [13] and Abu-Youssef et al. [14]. According to (1.1) and without loss of generality, we assume $\mu(\infty)$ is known and equal one; the measure of departure based on the goodness of fit approach can be stated as;

$$\delta(s) = E\left[\int_0^\infty e^{-sx}\bar{F}(x+t)\,dx - \frac{1}{1+s}\bar{F}(t)\right]$$

$$= \int_0^\infty \left[\int_0^\infty \bar{F}(x+t)\,e^{-sx}\,dx - \frac{1}{1+s}\bar{F}(t)\right] dF_0(t). \tag{2.1}$$

It's worth noting that under H_0: $\delta(S) = 0$ and under H_1: $\delta(s) > 0$.

The test statistic of the proposed test for the UBAL class is given by the following theorem.

Theorem 2.1.

Suppose X be a UBAL random variable with distribution function F, then we'll build the test statistic using the goodness of fit approach as,

$$\delta(s) = \frac{1}{(1-s)}\left[\frac{1}{s}(1-\varphi) + \frac{2}{(1+s)}\left(\int_0^\infty e^{-x}dF(x) - 1\right)\right], \tag{2.2}$$

where $\varphi(s) = \int_0^\infty e^{-sx}dF(x)$.

Proof.

$$\delta(s) = \int_0^\infty \left[\int_0^\infty e^{-sx}\bar{F}(x+t)\,dx - \frac{1}{1+s}\bar{F}(t)\right] dF_0(t).$$

We can take $F_0(x) = 1 - e^{-x}, x \geq 0$, then

$$\delta(s) = \int_0^\infty \int_0^\infty e^{-t-su}\bar{F}(u+t)\,du dt - \frac{1}{1+s}\int_0^\infty \bar{F}(t)e^{-t}dt$$

$$= I_1 - I_2.$$

Where,

$$I_1 = \int_0^\infty \int_0^\infty e^{-su} e^{-t} \bar{F}(u+t)\,dudt$$
$$= \int_0^\infty \int_t^\infty e^{-s(x-t)} e^{-t} \bar{F}(x)\,dxdt$$
$$= \int_0^\infty \int_0^t e^{-s(t-x)} e^{-t} \bar{F}(t)\,dxdt$$
$$= \frac{1}{s}\int_0^\infty (1-e^{-st}) e^{-t} \bar{F}(t)\,dt$$
$$= \frac{1}{1-s}\left[\frac{1}{s}\big(1-\varphi(s)\big) - 1 + \int_0^\infty e^{-t} dF(t)\right]. \quad (2.3)$$

And,

$$I_2 = \frac{1}{1+s}\int_0^\infty \bar{F}(t) dF_0(t) = \frac{1}{1+s}\left[1 - \int_0^\infty e^{-t} dF(t)\right]. \quad (2.4)$$

From Eqs (2.3) and (2.4), we obtain (2.2).

The statistic's empirical estimator can be calculated as follows:

$$\hat{\delta}_n(s) = \frac{1}{n(1-s)} \sum_i \left\{\frac{1}{s}(1-e^{-sX_i}) - \frac{2}{(1+s)}(1-e^{-X_i})\right\}, \quad (2.5)$$

and the corresponding invariant test statistic can be found as:

$$\hat{\Delta}_n(s) = \frac{\hat{\delta}_n(s)}{\bar{X}} = \frac{1}{n\bar{X}} \sum_i \left\{\frac{1}{(1-s)}\left(\frac{1}{s}(1-e^{-sX_i}) - \frac{2}{(1+s)}(1-e^{-X_i})\right)\right\}. \quad (2.6)$$

The asymptotic normality of the demonstrated statistic in (2.2) is illustrated in the next theorem.

Theorem 2.2.

Using the theory of U-statistics According to Lee [15], the statistic $\delta(s)$ has the following characteristics:

As $n \to \infty$, $\sqrt{n}(\hat{\Delta}_n(s) - \delta(s))$ is asymptotically normal with $\mu_0 = 0$ and variance $\sigma^2(s)$, where

$$\sigma^2(s) = var\left\{\frac{1}{(1-s)}\left[\frac{1}{s}(1-\varphi) + \frac{2}{(1+s)}\left(\int_0^\infty e^{-x} dF(x) - 1\right)\right]\right\}.$$

The variance in H_0 is calculated as follows

$$\sigma_0^2(s) = \frac{2}{3(1+s)^2(2+s)(1+2s)}.$$

Proof.

By derived direct calculations, we can get μ_0 as:

$$\mu_0 = \int_0^\infty \left(\frac{1}{(1-s)}\left\{\frac{1}{s}(1-e^{-sx}) + \frac{2}{(1+s)}(e^{-x}-1)\right\}\right) dx = 0,$$

as well as the variance

$$\sigma^2(s) = var\left(\frac{1}{(1-s)}\left[\frac{1}{s}(1-\varphi) + \frac{2}{(1+s)}\left(\int_0^\infty e^{-x} dF(x) - 1\right)\right]\right)$$

$$= E\left(\frac{1}{(1-s)}\left[\frac{1}{s}(1-\ \varphi) + \frac{2}{(1+s)}\left(\int_0^\infty e^{-x} dF(x) - 1\right)\right]\right)^2.$$

The variance under H_0 is given by

$$\sigma_0^2(s) = \frac{2}{3(2+s)(1+s)^2(1+2s)}.$$

2.1. Relative efficiency

We can compare our test to some other known classes to determine the quality of the suggested test technique. We use the test $\hat{\Delta}(2)$ proposed by Mahmoud, et al. [12] for the (RNBUL) class of life distribution and δ_{F_n} presented Mahmoud and Abdul Alim [16] for (NBUFR) class of life distribution. The Pitman asymptotic relative efficiency PARE is then used to make comparisons. In this case, we'll use the following options:

(i) Linear failure rate family (LFR):

$$\bar{F}_1(x) = e^{-x-\frac{x^2}{2}\theta},\ \theta, x \geq 0. \tag{2.7}$$

(ii) Weibull family:

$$\bar{F}_2(x) = e^{-x^\theta},\ \theta \geq 1,, x \geq 0. \tag{2.8}$$

(iii) Makeham family:

$$\bar{F}_2(x) = e^{-x-\theta(x+e^{-x}-1)}, \theta, x \geq 0. \tag{2.9}$$

It's worth noting that H_0 (the exponential distribution) is achieved at $\theta = 0$ in (i & iii) and $\theta = 1$ in (ii). The asymptotic efficiency of the Pitman (PAE) of $\delta(s)$ as $s = 0.01\ and\ s = 0.1$ is equal to

$$PAE(\delta(0.01)) = \frac{1}{\sigma_0(0.01)}\left|\frac{1}{0.0099}\int_0^\infty e^{-0.01x} d\bar{F}'_{\theta_0}(x) - \frac{2}{0.9999}\int_0^\infty e^{-x}\, d\bar{F}'_{\theta_0}(x)\right|,$$

$$PAE(\delta(0.1)) = \frac{1}{\sigma_0(0.1)}\left|\frac{1}{0.09}\int_0^\infty e^{-0.1x} d\bar{F}'_{\theta_0}(x) - \frac{2}{0.99}\int_0^\infty e^{-x}\, d\bar{F}'_{\theta_0}(x)\right|,$$

where $\bar{F}'_{\theta_0}(x) = \frac{d}{d\theta}\bar{F}_\theta(u)\Big|_{\theta\to\theta_0}$. This leads to:

(i) PAE in case of the linear failure rate distribution:

$$\text{PAE}\left(\hat{\delta}(0.01)\right)=\frac{1}{\sigma_0(0.01)}\left|\frac{1}{0.0099}\int_0^\infty e^{-0.01x}d\left(\frac{-x^2}{2}e^{-x}\right)+\frac{2}{0.9999}\int_0^\infty e^{-x}d\left(\frac{-x^2}{2}e^{-x}\right)\right|=1.29.$$

$$\text{PAE}\left(\hat{\delta}(0.1)\right)=\frac{1}{\sigma_0(0.1)}\left|\frac{1}{0.09}\int_0^\infty e^{-0.1x}d\left(\frac{-x^2}{2}e^{-x}\right)+\frac{2}{0.99}\int_0^\infty e^{-x}d\left(\frac{-x^2}{2}e^{-x}\right)\right|=1.25.$$

(ii) PAE in case of the Weibull distribution:

$$\text{PAE}\left(\hat{\delta}(0.01)\right)=\frac{1}{\sigma_0(0.01)}\left|\frac{1}{0.0099}\int_0^\infty e^{-0.01x}d(-x\ln|x|\,e^{-x})+\frac{2}{0.9999}\int_0^\infty e^{-x}d(-x\ln|x|\,e^{-x})\right|=0.96.$$

$$\text{PAE}\left(\hat{\delta}(0.1)\right)=\frac{1}{\sigma_0(0.1)}\left|\frac{1}{0.09}\int_0^\infty e^{-0.1x}d(-x\ln|x|\,e^{-x})+\frac{2}{0.99}\int_0^\infty e^{-x}d(-x\ln|x|\,e^{-x})\right|=0.94.$$

(iii) PAE in case of the Makeham distribution.

$$\text{PAE}\left(\hat{\delta}(0.01)\right)=\frac{1}{\sigma_0(0.01)}\left|\frac{1}{0.0099}\int_0^\infty e^{-0.01x}d((1-x-e^{-x})\,e^{-x})+\frac{2}{0.9999}\int_0^\infty e^{-x}d((1-x-e^{-x})\,e^{-x})\right|=0.86.$$

$$\text{PAE}\left(\hat{\delta}(0.1)\right)=\frac{1}{\sigma_0(0.1)}\left|\frac{1}{0.09}\int_0^\infty e^{-0.1x}d((1-x-e^{-x})\,e^{-x})+\frac{2}{0.99}\int_0^\infty e^{-x}d((1-x-e^{-x})\,e^{-x})\right|=0.77.$$

Table 1 summarizes the direct computations of PAE of of $\hat{\Delta}(2)$, δ_{F_n} and our $\delta(0.01)$ *and* $\delta(0.1)$. The efficiencies in the table clearly illustrate that our test performs well for F_1, F_2 and F_3.

Table 1. PAE of $\hat{\Delta}(2)$, δ_{F_n} and $\delta(0.01)$ *and* $\delta(0.1)$.

Distribution	$\hat{\Delta}(2)$	δ_{F_n}	$\delta(0.01)$	$\delta(0.1)$
LFR	0.915	0.217	1.29	1.25
Weibull	0.618	0.050	0.96	0.94
Makeham	0.172	0.144	0.86	0.77

PARE's of $\delta(0.01)$ and $\delta(0.1)$ concerning $\hat{\Delta}(2)$ and δ_{F_n} whose PAE are listed in Table 1 are shown in Table 2.

Table 2. PARE of $\delta(0.01)$ and $\delta(0.1)$ concerning $\delta(0.01)$ *and* $\delta(0.1)$.

Distribution	$e(\delta(0.01),\hat{\Delta}(2))$	$e(\delta(0.1),\hat{\Delta}(2))$	$e(\delta(0.01),\delta_{F_n})$	$e(\delta(0.1),\delta_{F_n})$
LFR	1.40	1.37	5.94	5.76
Weibull	1.55	1.52	19.2	18.8
Makeham	5	4.48	5.97	5.35

Table 2 shows that for F_1, F_2 and F_3, the statistics $\delta(0.01)$ and $\delta(0.1)$ perform well. For all of the scenarios discussed above, it outperforms both $\hat{\Delta}(2)$ and δ_{F_n}.

2.2. Power estimates

At a significance level of 0.05, Table 3 will be utilized to evaluate the power of the proposed test. For the Weibull; LFR, and Gamma distributions, these powers were estimated using 10000 simulated samples with n=10, 20, and 30.

Table 3. Powers estimates at $\alpha = 0.05$.

Distribution	n	$\theta = 2$	$\theta = 3$	$\theta = 4$
Weibull	10	0.9998	1	1
	20	1	1	1
	30	1	1	1
LFR	10	0.9988	1	1
	20	1	1	1
	30	1	1	1
Gamma	10	0.9441	0.9995	1
	20	0.9924	1	1
	30	0.9987	1	1

As demonstrated in Table 3, our test has high powers for the Weibull, LFR, and Gamma families.

2.3. Critical values

Here, we use 10000 simulations with sample sizes n=10(5)100 from the standard exponential distribution to calculate the test statistic of our test test $\hat{\Delta}_n(s)$ as $s = 0.01$ *and* $s = 0.1$ given in (2.6) for some selected values s.

The asymptotic normality of our test improves as s decreases, as shown in Table 4.

Table 4. The upper percentile points of $\hat{\delta}_n(s)$.

	$\hat{\boldsymbol{\delta}}_{\boldsymbol{n}}(0.01)$			$\hat{\boldsymbol{\delta}}_{\boldsymbol{n}}(0.1)$		
n	90%	95%	99%	90%	95%	99%
5	0.222947	0.296991	0.416736	0.190928	0.250687	0.331011
10	0.175661	0.234867	0.328956	0.148956	0.191851	0.266253
15	0.152844	0.198555	0.282422	0.129411	0.164337	0.222148
20	0.136991	0.180429	0.255117	0.111626	0.143976	0.200654
25	0.12162	0.157556	0.223607	0.103156	0.132082	0.18402

Continued on next page

	$\hat{\boldsymbol{\delta}}_{\boldsymbol{n}}(0.01)$			$\hat{\boldsymbol{\delta}}_{\boldsymbol{n}}(0.1)$		
n	90%	95%	99%	90%	95%	99%
30	0.112775	0.14715	0.211407	0.0963509	0.122499	0.169919
35	0.10628	0.136184	0.193097	0.0845552	0.109254	0.159572
39	0.10213	0.13368	0.184562	0.083692	0.10837	0.150329
40	0.102546	0.133687	0.186511	0.0836914	0.107069	0.15120
41	0.096624	0.125063	0.178481	0.0801545	0.10443	0.14379
45	0.095567	0.122137	0.174346	0.078977	0.100291	0.141727
50	0.0933263	0.119181	0.167259	0.075482	0.0966459	0.132828
55	0.0883399	0.113484	0.162532	0.0716097	0.0924242	0.127282
60	0.0845056}	0.109896	0.156001	0.0709048	0.0905189	0.123108
65	0.0800721	0.106347	0.149221	0.0674512	0.0854014	0.119576
70	0.079694	0.102598	0.147153	0.0655145	0.0847923	0.11628
75	0.0781665	0.0990352	0.138235	0.0634726	0.0803639	0.112566
80	0.0750521	0.0960944	0.13506	0.0623859	0.0801786	0.110811
85	0.0709399	0.0906362	0.12933	0.0593002	0.0768853	0.102688
90	0.0704061	0.0898579	0.125016	0.0579873	0.0741982	0.102586
95	0.0689002	0.0886083	0.124733	0.0555379	0.0718737	0.0998331
100	0.068162	0.0866082	0.123173	0.054814	0.0702883	0.0990065

3. Testing of censored data

In this section, a test statistic is provided to compare H_0 and H_1 using data that has been randomly right-censored.

Let the test statistic written as follows:

$$\delta_c(s) = \frac{1}{(1-s)} \sum_{j=1}^{n} \prod_{k=1}^{j-1} \left(\frac{1}{s}\left(1 - \hat{\varphi}(s)\right) + \frac{2}{(1+s)}(\Psi - 1) \right), \tag{3.1}$$

where

$$\hat{\varphi}(s) = \sum_{m=1}^{n} e^{-sZ_{(m)}} \left(\prod_{p=1}^{m-2} C_p^{I_p} - \prod_{p=1}^{m-1} C_p^{I_p} \right),$$

$$\Psi = \sum_{m=1}^{n} e^{sZ_{(m)}} \left(\prod_{p=1}^{m-2} C_p^{I_p} - \prod_{p=1}^{m-1} C_p^{I_p} \right) \, and \, C_m = \frac{n-m}{n-m+1}, \qquad t \in [0, z_{(m)}].$$

Again, based on 10000 simulated and sample sizes n=5(5)100 from the standard exponential distribution in Table (5) below, the 90%, 95% and 99% percentage points of the test statistic in (3.1) are simulated for some selected values s.

Table 5. The upper percentile points of $\hat{\delta}_c(s)$.

	$\hat{\boldsymbol{\delta}}_{\boldsymbol{n}}(0.01)$			$\hat{\boldsymbol{\delta}}_{\boldsymbol{n}}(0.1)$		
n	90%	95%	99%	90%	95%	99%
5	79.1722	99.0099	99.0099	7.24026	9.09091	9.09091
10	58.5798	66.7518	82.4772	5.30214	6.06734	7.54631
15	48.3237	55.6325	69.5771	4.34842	5.05073	6.38021
20	41.8856	48.071	59.9788	3.69852	4.28524	5.42593
25	37.4361	43.4792	54.1946	3.35664	3.90187	4.99075
30	34.2465	39.6075	50.708	3.09701	3.61357	4.69185
35	31.8667	36.4782	46.2339	2.84254	3.34405	4.29118
40	29.906	34.817	44.2144	2.6558	3.08495	3.97099
45	28.031	32.6912	42.216	2.47709	2.86345	3.72305
50	26.5686	30.8355	40.6515	2.34039	2.73995	3.48289
51	26.2765	30.713	40.2296	2.3204	2.72125	3.40845
55	25.321	29.3385	37.4214	2.24602	2.64514	3.37032
60	24.4339	28.3712	36.8932	2.13922	2.49681	3.19249
61	24.2339	28.3127	35.9142	2.09573	2.44195	3.13856
65	23.3836	7.2437	34.2578	2.02729	2.3679	3.07914
70	22.5253	26.2706	33.526	1.98388	2.31408	2.90431
75	21.8598	25.6862	32.4598	1.9148	2.23591	2.90803
80	20.927	24.4351	30.753	1.84222	2.15769	2.75052
85	20.3111	23.9109	30.7706	1.76628	2.06116	2.66595
90	19.9521	23.3886	29.6384	1.72034	2.0335	2.60288
95	19.4658	22.4647	28.5529	1.70941	1.99008	2.54748
100	18.6863	21.6688	28.2181	1.63263	1.93282	2.48212

When s decreases, our test of $\hat{\delta}_c(s)$ behaves better in terms of asymptotic normality, as seen in Table 5.

3.1. Power estimates

The powers estimate of the proposed test $\hat{\delta}$ will be carried out in Table 6 at the significant level $\alpha = 0.05$. These powers are estimated for Weibull, LFR and Gamma distributions based on 10000 simulated samples for sizes $n = 10,\ 20$ and 30.

Table 6. Powers estimates at $\alpha = 0.05$.

n	θ	Distribution		
		Weibull	LFR	Gamma
10	1	0.9504	0.9532	0.9537
	2	0.9516	0.9534	0.9551
	3	0.9521	0.9534	0.9570
20	1	0.9487	0.940	0.9465
	2	0.950	0.945	0.9468
	3	0.9516	0.950	0.9469
30	1	0.950	0.9511	0.9541
	2	0.9523	0.9581	0.9545
	3	0.9591	0.9587	0.9549

Our test has good powers for the Weibull, LFR, and Gamma families, as shown in Table 6.

4. Applications

To demonstrate the utility of the conclusions in this study, we apply them to various real data sets.

Application 1: Case of complete data.

Example 1: Analyze the data in Abouammoh et al. [17], which show the ages (in years) of 40 patients aged with blood cancer (leukemia) in one of Saudi Arabia's health ministry hospitals.

In the two situations of $\hat{\Delta}_n(0.01)$ and $\hat{\Delta}_n(0.1)$ as $n = 40$, we calculate the statistic in (2.6) $\hat{\Delta}_n(0.01) = 0.42$ and $\hat{\Delta}_n(0.1) = 0.35$, which are both higher than the corresponding critical value in Table 4. As a result, we infer that this set of data seems to have the UBAL property rather than the exponential characteristic.

Example 2: Take, for example, the data in Mahmoud et al. [12], which represent 39 liver cancer patients from Egypt's Ministry of Health's Elminia Cancer Center 2000.

In the two situations of $\hat{\Delta}_n(0.01)$ and $\hat{\Delta}_n(0.1)$ as $n = 39$, we calculate the statistic in (2.6) $\hat{\Delta}_n(0.01) = 0.68$ and $\hat{\Delta}_n(0.1) = 0.16$, which are both higher than the critical value in Table 4. As a result, we infer that this set of data seems to have the UBAL property rather than the exponential characteristic.

Example 3: This data set from Abu-Youssef and Silvana Gerges [18] shows the survival times (in years) of 43 patients with a specific kind of leukemia diagnosis.

In the two situations of $\hat{\Delta}_n(0.01)$ and $\hat{\Delta}_n(0.1)$ as $n = 43$, we calculate the statistic in (2.6) $\hat{\Delta}_n(0.01) = 0.098$ and $\hat{\Delta}_n(0.1) = 0.0097$, which are both smaller than the critical value in Table 4. As a result, we infer that this set of data seems to have the exponential characteristic property rather than the UBAL.

Application 2: Case of censored data.

Example 1: In this application, we use the data from Mahmoud et al. [12], which reflects the ages (in days) of 51 liver cancer patients from the Elminia cancer center Ministry of health Egypt, who began the medical investigation in the year 2000. In the investigation, only 39 patients are watched (right-censored), while the remaining 11 are dropped (missing from the investigation).

In the two situations of $\hat{\Delta}_n(0.1, 0.2)$ and $\hat{\Delta}_n(0.5, 5)$as $n = 51$, we calculate the statistic in (3.1) $\hat{\Delta}_n(0.01) = 44.9$ and $\hat{\Delta}_n(0.1) = 8.42$, which are both higher than the critical value in Table 5. As a result, we infer that this set of data seems to have the UBAL property rather than the exponential characteristic.

Example 2: Consider the data in Kamran Abbas et al. [19] and in Lee and Wolfe [20], the survival times, in weeks, of 61 patients with inoperable lung cancer treated with cyclophosphamide. There are 33 uncensored observations and 28 censored observations, representing the patients whose treatment was terminated because of a devolving condition.

In the two situations of $\hat{\Delta}_n(0.01)$ and $\hat{\Delta}_n(0.1)$ as $n = 61$, we calculate the statistic in (3.1) $\hat{\Delta}_n(0.01) = 28.4$ and $\hat{\Delta}_n(0.1) = 6.87$, which are both higher than the critical value in Table 5. As a result, we infer that this set of data seems to have the UBAL property rather than the exponential characteristic.

5. Conclusions

In this paper, a non-parametric testing for the UBAL based on goodness of fit is developed in both complete and censored cases. The percentage points of the proposed statistics are simulated. The efficacies of our developed tests are compared to Mahmoud, et al. [12] for the (RNBUL) class of life distribution and δ_{F_n} presented by Mahmoud and Abdul Alim [16] based on Pitman asymptotic relative efficiency using some well-known life distributions; namely, Linear failure rate family (LFR), Makeham and Weibull family. Finally, the findings of the paper are applied to some medical real data sets.

Appendix

Notations and abbreviations.

IFR	Increasing failure rate.
IFRA	Increasing failure rate average.
NBU	New better than used.
NB(W)UC	New better (worse) than used in a convex ordering.
UBA	Used better than age.
UBAE	Used better than age in expectation.
UBAL	Used better than age in Laplace transform.

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Conflict of interest

The authors declare there is no conflict of interest.

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