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Research article

Certain subclass of analytic functions based on q-derivative operator associated with the generalized Pascal snail and its applications

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Abstract: By the principle of differential subordination and the q-derivative operator, we introduce the q-analog $\mathcal{SP}^q_{snail}(\lambda;\alpha,\beta,\gamma)$ of certain class of analytic functions associated with the generalized Pascal snail. Firstly, we obtain the coefficient estimates and Fekete-Szegö functional inequalities for this class. Meanwhile, we also estimate the corresponding symmetric Toeplitz determinant. Secondly, for all the above results we provide the corresponding results for the reduced classes $\mathcal{SP}^q_{snail}(\alpha,\beta,\gamma)$ and $\mathcal{RP}^q_{snail}(\alpha,\beta,\gamma)$. Thirdly, we characterize the Bohr radius problems for the function class $\mathcal{SP}^q_{snail}(\alpha,\beta,\gamma)$. Lastly, we establish certain results for some new subclasses of functions defined by the neutrosophic Poisson distribution series.

Keywords: Fekete-Szegö inequality; symmetric Toeplitz determinant; analytic function; univalent function; q-derivative operator; Pascal snail; Bohr radius; neutrosophic Poisson distribution
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1. Introduction

Recall that the complex valued mapping $\mathcal{L}_{\alpha,\beta,\gamma} : \mathbb{D} \to \mathbb{C}$ is defined by

$$\begin{aligned}\mathcal{L}_{\alpha,\beta,\gamma}(z) &= \frac{(2-2\gamma)z}{(1-\alpha z)(1-\beta z)} \\ &= \sum_{n=1}^{\infty} M_n(\alpha,\beta,\gamma)z^n,\end{aligned} \tag{1.1}$$

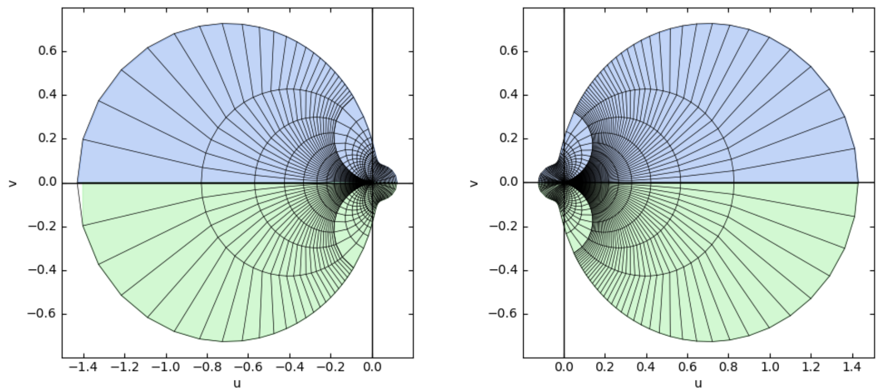
where

$$M_n(\alpha,\beta,\gamma) := M_n = \begin{cases} (2-2\gamma)\sum_{n=1}^{\infty}\left(\frac{\alpha^n-\beta^n}{\alpha-\beta}\right)z^n, & \text{if } \alpha \neq \beta, \\ (2-2\gamma)\sum_{n=1}^{\infty} n\alpha^{n-1}z^n, & \text{if } \alpha = \beta, \end{cases} \tag{1.2}$$

for $0 \leq \alpha, \beta \leq 1$ ($\alpha\beta \neq \pm 1$) and $0 \leq \gamma < 1$, whose image is in the domain $\Delta(\alpha,\beta,\gamma)$ with the boundary

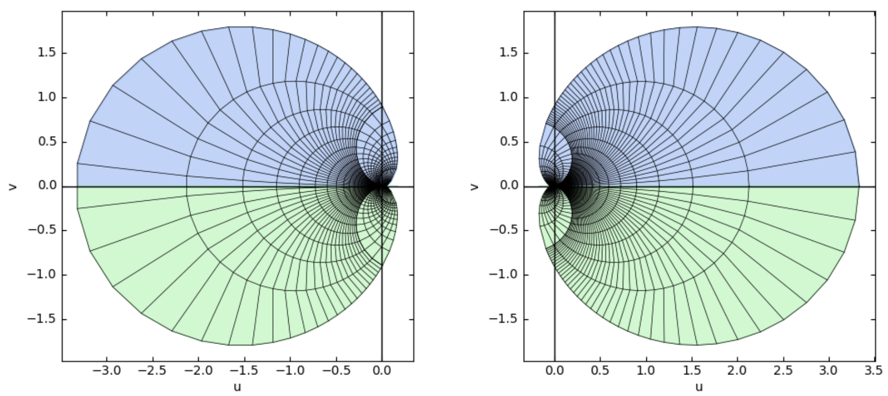
$$\partial\Delta(\alpha,\beta,\gamma) = \left\{ w = u + vi \in \mathbb{C} : \frac{[2(1-\gamma)u + (\alpha+\beta)(u^2+v^2)]^2}{(1+\alpha\beta)^2} = (u^2+v^2)^2 - \frac{4(1-\gamma)^2v^2}{(1-\alpha\beta)^2} \right\}, \tag{1.3}$$

that is called the generalized Pascal snail [16] (see Figures 1 and 2). It is well known that Pascal snail is the inversion of conic sections with respect to a focus. Note that $M_1 = 2-2\gamma$ and $M_2 = (2-2\gamma)(\alpha+\beta)$.



(a) $\alpha = -0.93,\ \beta = 0.4,\ \gamma = 0.93$ (b) $\alpha = -0.4,\ \beta = 0.93,\ \gamma = 0.93$

Figure 1. The image of $\mathbb{D}$ under $\mathcal{L}_{\alpha,\beta,\gamma}(z)$.



(a) $\alpha = -0.4,\ \beta = -0.93,\ \gamma = 0.93$ (b) $\alpha = 0.93,\ \beta = 0.4,\ \gamma = 0.93$

Figure 2. The image of $\mathbb{D}$ under $\mathcal{L}_{\alpha,\beta,\gamma}(z)$.

Define by $\mathcal{A}$ the class of analytic functions f which are expanded with the Taylor-Maclaurin's series

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n \tag{1.4}$$

in the open unit disk $\mathbb{D} = \{z \in \mathbb{C} :| z |< 1\}$. Further, if $f \in \mathcal{A}$ is univalent in $\mathbb{D}$, then we denote such class of functions by $\mathcal{S}$. For two analytic functions F and G in $\mathbb{D}$, if there has a Schwarz function $\omega \in \Omega$ with $\omega(0) = 0$ and $| \omega(z) |< 1$ for $z \in \mathbb{D}$, such that $F(z) = G(\omega(z))$, then F is subordinate to G in $\mathbb{D}$, i.e. $F \prec G$. In addition, if $G \in \mathcal{S}$, then there exists the following equivalent relation [20]:

$$F \prec G \Leftrightarrow F(0) = G(0) \text{ and } F(\mathbb{D}) \subset G(\mathbb{D}).$$

Besides, if $\omega(z) = z$, then F is majorized by G in $\mathbb{D}$, i.e. $F \ll G$.

Lately, the study of the q-calculus has riveted the rigorous consecration of researchers. The great attention is due to its gains in many areas of mathematics and physics. The significance of the q-derivative operator $\mathcal{D}_q$ is quite evident by its applications in the study of several subclasses of analytic functions. Initially, in 1990, Ismail et al. [14] gave the idea of q-starlike functions. Nevertheless, a firm base of the usage of the q-calculus in the context of geometric function theory was efficiently established (refer to the work by Purohit and Raina [24]), and the use of the generalized basic (or q-) hypergeometric functions in geometric function theory was made by Srivastava (see [25] for more details). After that, the extraordinary studies have been done by many mathematicians, which offer a significant part in the encroachment of geometric function theory (see [27–31, 33]).

For $f \in \mathcal{A}$, its q-derivative or the q-difference $\mathcal{D}_q f(z)$ is given by

$$\mathcal{D}_q f(z) = 1 + \sum_{n=2}^{\infty} [n]_q a_n z^{n-1}, \quad 0 < q < 1,$$

where the q-derivative operator $\mathcal{D}_q f(z)$ (refer to [15]) of f is defined by

$$\mathcal{D}_q f(z) := \begin{cases} \frac{f(z)-f(qz)}{(1-q)z}, & z \neq 0,\ 0 < q < 1, \\ f'(0), & z = 0, \end{cases}$$

provided that $f'(0)$ exists, and the q-number $[n]_q$ is exactly $[\varsigma]_q$ when $\varsigma = n \in \mathbb{N}$, with

$$[\varsigma]_q = \begin{cases} \frac{1-q^{\varsigma}}{1-q}, & \text{for } \varsigma \in \mathbb{C}, \\ \sum_{k=0}^{\varsigma-1} q^k, & \text{for } \varsigma = n \in \mathbb{N}. \end{cases}$$

Note that $\mathcal{D}_q f(z) \to f'(z)$ when $q \to 1^-$, where f' is the ordinary derivative of f.

For $f \in \mathcal{A}$, we recall the symmetric determinant matrix $\mathcal{T}_j(n)$ given in [9] as below:

$$\mathcal{T}_j(n) = \begin{vmatrix} a_n & a_{n+1} & \cdots & a_{n+j-1} \\ a_{n+1} & a_n & \cdots & a_{n+j} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n+j-1} & a_{n+j} & \cdots & a_n \end{vmatrix}.$$

Here, we also point out that

$$\mathcal{T}_2(2) = \begin{vmatrix} a_2 & a_3 \\ a_3 & a_2 \end{vmatrix} = a_2^2 - a_3^2$$

and

$$\mathcal{T}_3(1) = \begin{vmatrix} 1 & a_2 & a_3 \\ a_2 & 1 & a_2 \\ a_3 & a_2 & 1 \end{vmatrix} = 1 + 2a_2^2 a_3 - 2a_2^2 - a_3^2.$$

Let the image of the analytic function $g(z) = \sum_{n=0}^{\infty} b_n z^n$ in $\mathbb{D}$ belong to $\Omega \subseteq \mathbb{D}$. In 1914, Harald Bohr [10] showed that the inequality $\sum_{n=0}^{\infty} |b_n z^n| \leq 1$ in the disc $\mathbb{D}_\delta = \{z||z| \leq \delta\}$ with $\delta \geq \frac{1}{6}$. Because of the works of Weiner, Riesz and Schur, we know that $r^* = \frac{1}{3}$ is best number and called Bohr radius that named after Niels Bohr, the founder of quantum theory [23]. Further, the corresponding inequality (the called Bohr inequality) can be denoted by

$$d(\sum_{n=0}^{\infty} |a_n||z|^n, |a_0|) \leq d(f(0), \partial f(\mathbb{D})),$$

for $|z| < r_\Omega$ with respect to the Euclidean distance d. The largest radius r_Ω is called the Bohr radius for the corresponding class. For more details for the Bohr radius and Bohr inequality, we can refer to [2, 6, 7, 12, 17].

Recently, Kanas and Masih [16] considered the analytic representation of the domain by a generalized Pascal snail. Besides, Allu and Halder [5] investigated Bohr radius for certain classes of starlike and convex univalent functions (also refer to [8, 11]). Thomas et al. [9, 34] studied the symmetric Toeplitz determinants for starlike and close-to-convex functions. Stimulated by recent studies [1, 3, 4, 18, 25, 26], we introduce and study certain new subclasses of analytic functions associated with the generalized Pascal snail involving q-derivative operator, and obtain the corresponding upper estimates of the initial coefficients a_2 and a_3 of Taylors series and Fekete-Szegö functional inequalities for functions of the new subclasses given in Definition 1.1. In addition, we characterize the Bohr radius problems for certain reduced version of this class. Srivastava and Porwal [32] ever investigated the coefficient inequalities of Poisson distribution series in conic domain related to uniformly convex, k-spiralike and starlike functions. Along this line, Oladipo [21] estimated the bound on the first few coefficients and classical Fekete-Szegö problem for Poisson and neutrosophic Poisson distribution series connected with Chebyshev polynomials. As an application, all our results are almost generalized into the new class related with the neutrosophic Poisson distribution series.

Now, by term of the unified subordination technique by Ma-Mind [19], we introduce the following subclass of analytic and univalent functions associated with the generalized Pascal snail and q-derivative operator.

Definition 1.1. Let $\mathcal{L}_{\alpha,\beta,\gamma}$ be given by (1.1). A function $f \in \mathcal{A}$ is said to be in the class $\mathcal{SP}^q_{snail}(\lambda; \alpha, \beta, \gamma)$ if the following subordination:

$$\frac{z\mathcal{D}_q f(z)}{(1-\lambda) f(z) + \lambda z} \prec 1 + \mathcal{L}_{\alpha,\beta,\gamma}(z) \tag{1.5}$$

holds for $z \in \mathbb{D}$, where $0 \leq \lambda \leq 1$.

Note that by specializing the parameter λ, we get the reduced classes below:

(1) $\mathcal{SP}^{q}_{snail}(0;\alpha,\beta,\gamma) \equiv \mathcal{SP}^{q}_{snail}(\alpha,\beta,\gamma) = \{f \in \mathcal{A} : \frac{z\mathcal{D}_q f(z)}{f(z)} \prec 1 + \mathcal{L}_{\alpha,\beta,\gamma}(z)\}$,

(2) $\mathcal{SP}^{q}_{snail}(1;\alpha,\beta,\gamma) \equiv \mathcal{RP}^{q}_{snail}(\alpha,\beta,\gamma) = \{f \in \mathcal{A} : \mathcal{D}_q f(z) \prec 1 + \mathcal{L}_{\alpha,\beta,\gamma}(z)\}$.

Denote by $\mathcal{P}$ the class of all analytic and univalent functions $h(z)$ of the following form:

$$h(z) = 1 + \sum_{n=1}^{\infty} c_n z^n, \quad z \in \mathbb{D}, \tag{1.6}$$

satisfying $\Re\,[h(z)] > 0$ and $h(0) = 1$. To proceed our results, we are ready for some indispensable Lemmas given below.

Lemma 1.1. *[13] Let $h(z) \in \mathcal{P}$. Then the sharp estimates*

$$| c_n | \leq 2, \ n \in \mathbb{N},$$

are true. In particular, the equality holds for all n for the following function:

$$h(z) = \frac{1+z}{1-z} = 1 + \sum_{n=1}^{\infty} 2z^n.$$

Lemma 1.2. *[19] If $h(z) \in \mathcal{P}$, then, for any complex $\kappa \in \mathbb{C}$,*

$$|c_2 - \kappa c_1^2| \leq 2 \max\{1, |2\kappa - 1|\},$$

and the sharp result holds for the functions

$$h(z) = \frac{1+z}{1-z} \quad or \quad h(z) = \frac{1+z^2}{1-z^2}, \quad z \in \mathbb{D}.$$

Lemma 1.3. *[19] Assume that the function $h(z) \in \mathcal{P}$ and $\kappa \in \mathbb{R}$. Then*

$$| c_2 - \kappa c_1^2 | \leq \begin{cases} -4\kappa + 2, & if \ \kappa \leq 0, \\ 2, & if \ 0 \leq \kappa \leq 1, \\ 4\kappa - 2, & if \ \kappa \geq 1. \end{cases}$$

For $\kappa < 0$ or $\kappa > 1$, the inequality holds literally if and only if $h(z) = \frac{1+z}{1-z}$ or one of its rotations. If $0 < \kappa < 1$, the inequality holds literally if and only if $h(z) = \frac{1+z^2}{1-z^2}$ or one of its rotations. In particular, if $\kappa = 0$, then the sharp result holds for the following function:

$$h(z) = \left(\frac{1}{2} + \frac{\eta}{2}\right)\frac{1+z}{1-z} + \left(\frac{1}{2} - \frac{\eta}{2}\right)\frac{1-z}{1+z}, \quad 0 \leq \eta \leq 1,$$

or one of its rotations. If $\kappa = 1$, then the sharp result holds for the following function:

$$\frac{1}{h(z)} = \left(\frac{1}{2} + \frac{\eta}{2}\right)\frac{1+z}{1-z} + \left(\frac{1}{2} - \frac{\eta}{2}\right)\frac{1-z}{1+z}, \quad 0 \leq \eta \leq 1,$$

or one of its rotations. If $0 < \kappa < 1$, then the upper bound is sharp as follows:

$$|c_2 - \kappa c_1^2| + \kappa |c_1|^2 \leq 2, \quad 0 < \kappa \leq \frac{1}{2},$$

and

$$|c_2 - \kappa c_1^2| + (1-\kappa)|c_1|^2 \leq 2, \quad \frac{1}{2} < \kappa < 1.$$

2. Coefficient bounds and Fekete-Szegö inequalities for $f \in \mathcal{SP}^q_{snail}(\lambda; \alpha, \beta, \gamma)$

Denote the function $h \in \mathcal{P}$ by

$$h(z) = \frac{1 + u(z)}{1 - u(z)} = 1 + \sum_{n=1}^{\infty} c_n z^n.$$

Then, from (1.6) we derive that

$$u(z) = \frac{h(z) - 1}{h(z) + 1} = \frac{c_1}{2} z + \left(\frac{c_2}{2} - \frac{c_1^2}{4}\right) z^2 + \left(\frac{c_3}{2} - \frac{c_2 c_1}{2} + \frac{c_1^3}{8}\right) z^3 + \ldots, \quad z \in \mathbb{D}p, \tag{2.1}$$

such that $u(z) \in \Omega$. By (1.1) and (2.1), we imply that

$$\begin{aligned}\mathcal{L}_{\alpha,\beta,\gamma}(u(z)) =& \frac{M_1 c_1}{2} z + \left[\frac{M_1 c_2}{2} + \frac{(M_2 - M_1) c_1^2}{4}\right] z^2 \\ &+ \left[\frac{M_1 c_3}{2} + \frac{(M_2 - M_1) c_2 c_1}{2} + \frac{(M_1 - 2M_2 + M_3) c_1^3}{8}\right] z^3 + \ldots, \quad z \in \mathbb{D}.\end{aligned} \tag{2.2}$$

Throughout our study unless otherwise stated we note that

$$M_1 = 2 - 2\gamma > 0 \quad \text{a}nd \quad M_2 = (2 - 2\gamma)(\alpha + \beta).$$

Now, we characterize the functional estimates for the class $\mathcal{SP}^q_{snail}(\lambda; \alpha, \beta, \gamma)$ and establish the next theorems for the coefficient bounds and the corresponding Feteke-Szegö problems.

Theorem 2.1. *If $f(z)$ given by (1.4) belongs to the class $\mathcal{SP}^q_{snail}(\lambda; \alpha, \beta, \gamma)$, then*

$$|a_2| \leq \frac{M_1}{[2]_q + \lambda - 1} \tag{2.3}$$

and

$$|a_3| \quad \leq \quad \frac{M_1 + |M_2 - M_1|}{[3]_q + \lambda - 1} + \frac{(1 - \lambda) M_1^2}{([3]_q + \lambda - 1)([2]_q + \lambda - 1)}. \tag{2.4}$$

Proof. Assume that $f \in \mathcal{SP}^q_{snail}(\lambda; \alpha, \beta, \gamma)$. Then, there exists a Schwarz function $u(z) \in \Omega$ so that

$$\frac{z \mathcal{D}_q f(z)}{(1 - \lambda) f(z) + \lambda z} = 1 + \mathcal{L}_{\alpha,\beta,\gamma}(u(z)). \tag{2.5}$$

Since

$$\frac{z \mathcal{D}_q f(z)}{(1 - \lambda) f(z) + \lambda z} = 1 + \left([2]_q + \lambda - 1\right) a_2 z + [\left([3]_q + \lambda - 1\right) a_3 - (1 - \lambda)([2]_q + \lambda - 1) a_2^2] z^2 + \cdots \tag{2.6}$$

for $f \in \mathcal{A}$, combing (2.5) and (2.6), with (2.2) we get that

$$\frac{M_1 c_1}{2} = \left([2]_q + \lambda - 1\right) a_2$$

and

$$\frac{M_1c_2}{2}+\frac{(M_2-M_1)c_1^2}{4}=\Big([3]_q+\lambda-1\Big)a_3-(1-\lambda)([2]_q+\lambda-1)a_2^2,$$

such that

$$a_2=\frac{M_1c_1}{2([2]_q+\lambda-1)} \tag{2.7}$$

and

$$a_3=\frac{2M_1c_2+(M_2-M_1)c_1^2}{4([3]_q+\lambda-1)}+\frac{(1-\lambda)M_1^2c_1^2}{4([3]_q+\lambda-1)([2]_q+\lambda-1)}. \tag{2.8}$$

By Lemma 1.1 we assert that Theorem 2.1 holds true. □

By taking $\lambda=0$ in Theorem 2.1, we deduce the corollary below.

Corollary 2.1. *If $f(z)$ given by (1.4) belongs to the class $\mathcal{SP}^q_{snail}(\alpha,\beta,\gamma)$, then*

$$|a_2|\leq\frac{M_1}{[2]_q-1}$$

and

$$|a_3|\leq\frac{M_1+|M_2-M_1|}{[3]_q-1}+\frac{M_1^2}{([3]_q-1)([2]_q-1)}.$$

By taking $\lambda=1$ in Theorem 2.1, we deduce the corollary below.

Corollary 2.2. *If $f(z)$ given by (1.4) belongs to the class $\mathcal{RP}^q_{snail}(\alpha,\beta,\gamma)$, then*

$$|a_2|\leq\frac{M_1}{[2]_q}$$

and

$$|a_3|\leq\frac{M_1+|M_2-M_1|}{[3]_q}+\frac{M_1^2}{[3]_q[2]_q}.$$

Theorem 2.2. *If $f(z)$ given by (1.4) belongs to the class $\mathcal{SP}^q_{snail}(\lambda;\alpha,\beta,\gamma)$, then*

$$|a_3-\mu a_2^2|\leq\frac{M_1\max\{1,|2\rho-1|\}}{[3]_q+\lambda-1} \tag{2.9}$$

holds for $\mu\in\mathbb{C}$, where

$$\rho=\frac{\mu([3]_q+\lambda-1)M_1}{2([2]_q+\lambda-1)^2}-\frac{M_2-M_1}{2M_1}-\frac{(1-\lambda)M_1}{2([2]_q+\lambda-1)}.$$

Proof. For $\mu \in \mathbb{C}$, by using (2.7) and (2.8), we infer that

$$a_3 - \mu a_2^2 = \frac{M_1}{2([3]_q + \lambda - 1)}\left(c_2 - \rho c_1^2\right), \tag{2.10}$$

where

$$\rho = \frac{\mu([3]_q + \lambda - 1)M_1}{2([2]_q + \lambda - 1)^2} - \frac{M_2 - M_1}{2M_1} - \frac{(1-\lambda)M_1}{2([2]_q + \lambda - 1)}.$$

Hence, we apply Lemma 1.2 to Eq (2.10) and show that Theorem 2.2 holds true. □

Corollary 2.3. *If $f(z)$ given by (1.4) belongs to the class $\mathcal{SP}_{snail}^{q}(\alpha,\beta,\gamma)$, then*

$$|a_3 - \mu a_2^2| \leq \frac{M_1 \max\{1, |2\rho - 1|\}}{[3]_q - 1}$$

holds for $\mu \in \mathbb{C}$, where

$$\rho = \frac{\mu([3]_q - 1)M_1}{2([2]_q - 1)^2} - \frac{M_2 - M_1}{2M_1} - \frac{M_1}{2([2]_q - 1)}.$$

Corollary 2.4. *If $f(z)$ given by (1.4) belongs to the class $\mathcal{RP}_{snail}^{q}(\alpha,\beta,\gamma)$, then*

$$|a_3 - \mu a_2^2| \leq \frac{M_1 \max\{1, |2\rho - 1|\}}{[3]_q}$$

holds for $\mu \in \mathbb{C}$, where

$$\rho = \frac{\mu[3]_q M_1}{2[2]_q^2} - \frac{M_2 - M_1}{2M_1}.$$

If we let $\mu \in \mathbb{R}$, then we are based on the proof of Theorem 2.2 and Lemma 1.3 to establish the Fekete-Szegö functional inequality for $\mathcal{SP}_{snail}^{q}(\lambda;\alpha,\beta,\gamma)$.

Theorem 2.3. *For $\mu \in \mathbb{R}$, if $f(z) \in \mathcal{A}$ belongs to the class $\mathcal{SP}_{snail}^{q}(\lambda;\alpha,\beta,\gamma)$, then*

$$|a_3 - \mu a_2^2| \leq \begin{cases} \frac{M_1(-2\rho+1)}{[3]_q+\lambda-1}, & \mu \leq \aleph_1, \\ \frac{M_1}{[3]_q+\lambda-1}, & \aleph_1 \leq \mu \leq \aleph_2, \\ \frac{M_1(2\rho-1)}{[3]_q+\lambda-1}, & \mu \geq \aleph_2, \end{cases}$$

where ρ is the same as in Theorem 2.2,

$$\aleph_1 = \frac{([2]_q + \lambda - 1)^2(M_2 - M_1)}{([3]_q + \lambda - 1)M_1^2} + \frac{(1-\lambda)([2]_q + \lambda - 1)}{[3]_q + \lambda - 1}$$

and

$$\aleph_2 = \frac{([2]_q + \lambda - 1)^2(M_2 + M_1)}{([3]_q + \lambda - 1)M_1^2} + \frac{(1-\lambda)([2]_q + \lambda - 1)}{[3]_q + \lambda - 1}.$$

In addition, we fix

$$\aleph_3 = \frac{([2]_q + \lambda - 1)^2 M_2}{([3]_q + \lambda - 1)M_1^2} + \frac{(1-\lambda)([2]_q + \lambda - 1)}{[3]_q + \lambda - 1}.$$

Then, each of the following inequalities holds:
(A) For $\mu \in [\aleph_1, \aleph_3]$,

$$|a_3 - \mu a_2^2| + \frac{2\rho([2]_q + \lambda - 1)^2}{([3]_q + \lambda - 1)M_1}|a_2|^2 \leq \frac{M_1}{[3]_q + \lambda - 1};$$

(B) For $\mu \in [\aleph_3, \aleph_2]$,

$$|a_3 - \mu a_2^2| + \frac{2(1-\rho)([2]_q + \lambda - 1)^2}{([3]_q + \lambda - 1)M_1}|a_2|^2 \leq \frac{M_1}{[3]_q + \lambda - 1}.$$

Proof. For $\mu \in \mathbb{R}$ and ρ in Theorem 2.2, if we let $\rho \leq 0$, then we know that

$$\frac{\mu([3]_q + \lambda - 1)M_1}{2([2]_q + \lambda - 1)^2} - \frac{M_2 - M_1}{2M_1} - \frac{(1-\lambda)M_1}{2([2]_q + \lambda - 1)} \leq 0$$

and obtain

$$\mu \leq \frac{([2]_q + \lambda - 1)^2(M_2 - M_1)}{([3]_q + \lambda - 1)M_1^2} + \frac{(1-\lambda)([2]_q + \lambda - 1)}{[3]_q + \lambda - 1} := \aleph_1.$$

Similarly, we get that for $\rho \geq 1$,

$$\mu \geq \frac{([2]_q + \lambda - 1)^2(M_2 + M_1)}{([3]_q + \lambda - 1)M_1^2} + \frac{(1-\lambda)([2]_q + \lambda - 1)}{[3]_q + \lambda - 1} := \aleph_2,$$

and for $\rho = \frac{1}{2}$,

$$\mu = \frac{([2]_q + \lambda - 1)^2 M_2}{([3]_q + \lambda - 1)M_1^2} + \frac{(1-\lambda)([2]_q + \lambda - 1)}{[3]_q + \lambda - 1} := \aleph_3.$$

Therefore, together with Lemma 1.3 and Eqs (2.7) and (2.10), we can prove that Theorem 2.3 holds true. □

Corollary 2.5. *For $\mu \in \mathbb{R}$, if $f(z) \in \mathcal{A}$ belongs to the class $\mathcal{SP}^q_{snail}(\alpha, \beta, \gamma)$, then*

$$|a_3 - \mu a_2^2| \leq \begin{cases} \frac{M_1(-2\rho+1)}{[3]_q - 1}, & \mu \leq \aleph_1, \\ \frac{M_1}{[3]_q - 1}, & \aleph_1 \leq \mu \leq \aleph_2, \\ \frac{M_1(2\rho-1)}{[3]_q - 1}, & \mu \geq \aleph_2, \end{cases}$$

where ρ is the same as in Corollary 2.3,

$$\aleph_1 = \frac{([2]_q - 1)^2(M_2 - M_1)}{([3]_q - 1)M_1^2} + \frac{[2]_q - 1}{[3]_q - 1}$$

and

$$\aleph_2 = \frac{([2]_q - 1)^2(M_2 + M_1)}{([3]_q - 1)M_1^2} + \frac{[2]_q - 1}{[3]_q - 1}.$$

In addition, we fix

$$\aleph_3 = \frac{([2]_q - 1)^2 M_2}{([3]_q - 1)M_1^2} + \frac{[2]_q - 1}{[3]_q - 1}.$$

Then, each of the following inequalities holds:
(A) For $\mu \in [\aleph_1, \aleph_3]$,

$$|a_3 - \mu a_2^2| + \frac{2\rho([2]_q - 1)^2}{([3]_q - 1)M_1}|a_2|^2 \leq \frac{M_1}{[3]_q - 1};$$

(B) For $\mu \in [\aleph_3, \aleph_2]$,

$$|a_3 - \mu a_2^2| + \frac{2(1-\rho)([2]_q - 1)^2}{([3]_q - 1)M_1}|a_2|^2 \leq \frac{M_1}{[3]_q - 1}.$$

Corollary 2.6. *For $\mu \in \mathbb{R}$, if $f(z) \in \mathcal{A}$ belongs to the class $\mathcal{RP}^q_{snail}(\alpha, \beta, \gamma)$, then*

$$|a_3 - \mu a_2^2| \leq \begin{cases} \frac{M_1(-2\rho+1)}{[3]_q}, & \mu \leq \aleph_1, \\ \frac{M_1}{[3]_q}, & \aleph_1 \leq \mu \leq \aleph_2, \\ \frac{M_1(2\rho-1)}{[3]_q}, & \mu \geq \aleph_2, \end{cases}$$

where ρ is the same as in Corollary 2.4,

$$\aleph_1 = \frac{[2]_q^2(M_2 - M_1)}{[3]_q M_1^2} \quad \text{and} \quad \aleph_2 = \frac{[2]_q^2(M_2 + M_1)}{[3]_q M_1^2}.$$

In addition, we put

$$\aleph_3 = \frac{[2]_q^2 M_2}{[3]_q M_1^2}.$$

Then, each of the following inequalities holds:
(A) For $\mu \in [\aleph_1, \aleph_3]$,

$$|a_3 - \mu a_2^2| + \frac{2\rho[2]_q^2}{[3]_q M_1}|a_2|^2 \leq \frac{M_1}{[3]_q};$$

(B) For $\mu \in [\aleph_3, \aleph_2]$,

$$|a_3 - \mu a_2^2| + \frac{2(1-\rho)[2]_q^2}{[3]_q M_1}|a_2|^2 \leq \frac{M_1}{[3]_q}.$$

3. Symmetric Toeplitz determinants for $\mathcal{SP}^q_{snail}(\lambda;\alpha,\beta,\gamma)$

Now we pay attention to the symmetric Toeplitz determinants $T_2(2)$ and $T_3(1)$ for the class $\mathcal{SP}^q_{snail}(\lambda;\alpha,\beta,\gamma)$. From (2.7) and (2.8), in view of Lemma 1.1, we easily obtain the theorem below.

Theorem 3.1. *If $f(z)$ given by (1.4) belongs to the class $\mathcal{SP}^q_{snail}(\lambda;\alpha,\beta,\gamma)$, then*

$$|\,\mathcal{T}_2(2)\,| \;\leq\; \frac{M_1^2}{([2]_q+\lambda-1)^2}+\left[\frac{M_1+|M_2-M_1|}{[3]_q+\lambda-1}+\frac{(1-\lambda)M_1^2}{([3]_q+\lambda-1)([2]_q+\lambda-1)}\right]^2.$$

Corollary 3.1. *If $f(z)$ given by (1.4) belongs to the class $\mathcal{SP}^q_{snail}(\alpha,\beta,\gamma)$, then*

$$|\,\mathcal{T}_2(2)\,| \;\leq\; \frac{M_1^2}{([2]_q-1)^2}+\left[\frac{M_1+|M_2-M_1|}{[3]_q-1}+\frac{M_1^2}{([3]_q-1)([2]_q-1)}\right]^2.$$

Corollary 3.2. *If $f(z)$ given by (1.4) belongs to the class $\mathcal{RP}^q_{snail}(\alpha,\beta,\gamma)$, then*

$$|\,\mathcal{T}_2(2)\,| \;\leq\; \frac{M_1^2}{[2]_q^2}+\frac{[M_1+|M_2-M_1|]^2}{[3]_q^2}.$$

Based on the proof of Theorem 2.2 and Lemma 1.2, we consider the symmetric determinant $T_3(1)$ for $\mathcal{SP}^q_{snail}(\lambda;\alpha,\beta,\gamma)$ and establish the next theorem.

Theorem 3.2. *If $f(z)\in\mathcal{A}$ belongs to the class $\mathcal{SP}^q_{snail}(\lambda;\alpha,\beta,\gamma)$, then*

$$|T_3(1)|\leq\left(1+\frac{M_1\max\{1,|2\rho-1|\}}{[3]_q+\lambda-1}\right)\times\left[1+\frac{M_1+|M_2-M_1|}{[3]_q+\lambda-1}+\frac{(1-\lambda)M_1^2}{([3]_q+\lambda-1)([2]_q+\lambda-1)}\right]$$

holds, where

$$\rho=\frac{([3]_q+\lambda-1)M_1}{([2]_q+\lambda-1)^2}-\frac{M_2-M_1}{2M_1}-\frac{(1-\lambda)M_1}{2([2]_q+\lambda-1)}.$$

Proof. According to the definition of symmetric Toeplitz determinant, we remark that

$$|\mathcal{T}_3(1)|=|(1+a_3-2a_2^2)(1-a_3)|\leq(1+|a_3-2a_2^2|)(1+|a_3|). \tag{3.1}$$

Therefore, with (2.8) we apply Theorem 2.2 into the inequality (3.1) to ensure Theorem 3.2 is true. □

Corollary 3.3. *If $f(z)\in\mathcal{A}$ belongs to the class $\mathcal{SP}^q_{snail}(\alpha,\beta,\gamma)$, then*

$$|T_3(1)|\leq\left(1+\frac{M_1\max\{1,|2\rho-1|\}}{[3]_q-1}\right)\times\left[1+\frac{M_1+|M_2-M_1|}{[3]_q-1}+\frac{M_1^2}{([3]_q-1)([2]_q-1)}\right]$$

holds, where

$$\rho=\frac{([3]_q-1)M_1}{([2]_q-1)^2}-\frac{M_2-M_1}{2M_1}-\frac{M_1}{2([2]_q-1)}.$$

Corollary 3.4. *If $f(z) \in \mathcal{A}$ belongs to the class $\mathcal{RP}^q_{snail}(\alpha,\beta,\gamma)$, then*

$$|T_3(1)| \leq \left(1 + \frac{M_1 \max\{1, |2\rho - 1|\}}{[3]_q}\right) \times \left(1 + \frac{M_1 + |M_2 - M_1|}{[3]_q}\right)$$

holds, where

$$\rho = \frac{[3]_q M_1}{[2]_q^2} - \frac{M_2 - M_1}{2M_1}.$$

Similarly, from Theorem 2.3 and Lemma 1.3, we also estimate the symmetric determinant $T_3(1)$ for $\mathcal{SP}^q_{snail}(\lambda;\alpha,\beta,\gamma)$.

Theorem 3.3. *If $f(z) \in \mathcal{A}$ belongs to the class $\mathcal{SP}^q_{snail}(\lambda;\alpha,\beta,\gamma)$, then*

$$|T_3(1)| \leq \begin{cases} \left(1 + \frac{M_1(-2\rho+1)}{[3]_q+\lambda-1}\right) \times \left[1 + \frac{M_1+|M_2-M_1|}{[3]_q+\lambda-1} + \frac{(1-\lambda)M_1^2}{([3]_q+\lambda-1)([2]_q+\lambda-1)}\right], & \Xi \leq M_2 - M_1, \\ \left(1 + \frac{M_1}{[3]_q+\lambda-1}\right) \times \left[1 + \frac{M_1+|M_2-M_1|}{[3]_q+\lambda-1} + \frac{(1-\lambda)M_1^2}{([3]_q+\lambda-1)([2]_q+\lambda-1)}\right], & M_2 - M_1 \leq \Xi \leq M_2 + M_1, \\ \left(1 + \frac{M_1(2\rho-1)}{[3]_q+\lambda-1}\right) \times \left[1 + \frac{M_1+|M_2-M_1|}{[3]_q+\lambda-1} + \frac{(1-\lambda)M_1^2}{([3]_q+\lambda-1)([2]_q+\lambda-1)}\right], & M_2 + M_1 \leq \Xi, \end{cases}$$

where ρ is the same as in Theorem 2.2, and

$$\Xi = \frac{2([3]_q + \lambda - 1)M_1^2}{([2]_q + \lambda - 1)^2} - \frac{(1-\lambda)M_1^2}{[2]_q + \lambda - 1}.$$

Proof. Assume that $\mu = 2$ in Theorem 2.3. Then, we derive that

$$|a_3 - 2a_2^2| \leq \begin{cases} \frac{M_1(-2\rho+1)}{[3]_q+\lambda-1}, & 2 \leq \aleph_1, \\ \frac{M_1}{[3]_q+\lambda-1}, & \aleph_1 \leq 2 \leq \aleph_2, \\ \frac{M_1(2\rho-1)}{[3]_q+\lambda-1}, & 2 \geq \aleph_2, \end{cases}$$

where $\aleph_i$ $(i = 1, 2)$ are the same as in Theorem 2.3. If $2 \leq \aleph_1$, then we infer that

$$M_2 - M_1 \geq \frac{2([3]_q + \lambda - 1)M_1^2}{([2]_q + \lambda - 1)^2} - \frac{(1-\lambda)M_1^2}{[3]_q + \lambda - 1} := \Xi.$$

Similarly, we see that $\aleph_1 \leq 2 \leq \aleph_2$ and $2 \geq \aleph_2$ are equivalent to $M_2 - M_1 \leq \Xi \leq M_2 + M_1$ and $\Xi \geq M_2 + M_1$, respectively. Moreover, together with (2.8) and the inequality (3.1) we complete the proof of Theorem 3.3. □

Corollary 3.5. *If $f(z) \in \mathcal{A}$ belongs to the class $\mathcal{SP}_{snail}(\alpha,\beta,\gamma)$,*

$$|T_3(1)| \leq \begin{cases} \left(1 + \frac{M_1(-2\rho+1)}{[3]_q-1}\right) \times \left[1 + \frac{M_1+|M_2-M_1|}{[3]_q-1} + \frac{M_1^2}{([3]_q-1)([2]_q-1)}\right], & \Xi \leq M_2 - M_1, \\ \left(1 + \frac{M_1}{[3]_q-1}\right) \times \left[1 + \frac{M_1+|M_2-M_1|}{[3]_q-1} + \frac{M_1^2}{([3]_q-1)([2]_q-1)}\right], & M_2 - M_1 \leq \Xi \leq M_2 + M_1, \\ \left(1 + \frac{M_1(2\rho-1)}{[3]_q-1}\right) \times \left[1 + \frac{M_1+|M_2-M_1|}{[3]_q-1} + \frac{M_1^2}{([3]_q-1)([2]_q-1)}\right], & M_2 + M_1 \leq \Xi, \end{cases}$$

where ρ is the same as in Corollary 2.3, and

$$\Xi = \frac{2([3]_q - 1)M_1^2}{([2]_q - 1)^2} - \frac{M_1^2}{[2]_q - 1}.$$

Corollary 3.6. *If $f(z) \in \mathcal{A}$ belongs to the class $\mathcal{RP}^q_{snail}(\alpha,\beta,\gamma)$, then*

$$|T_3(1)| \leq \begin{cases} \left(1+\frac{M_1(-2\rho+1)}{[3]_q}\right)\times\left(1+\frac{M_1+|M_2-M_1|}{[3]_q}\right), & \frac{2[3]_qM_1^2}{[2]_q^2} \leq M_2-M_1, \\ \left(1+\frac{M_1}{[3]_q}\right)\times\left(1+\frac{M_1+|M_2-M_1|}{[3]_q}\right), & M_2-M_1 \leq \frac{2[3]_qM_1^2}{[2]_q^2} \leq M_2+M_1, \\ \left(1+\frac{M_1(2\rho-1)}{[3]_q}\right)\times\left(1+\frac{M_1+|M_2-M_1|}{[3]_q}\right), & M_2+M_1 \leq \frac{2[3]_qM_1^2}{[2]_q^2}, \end{cases}$$

where ρ is the same as in Corollary 2.4.

4. Bohr radius problems for $\mathcal{SP}_{snail}(\alpha,\beta,\gamma)$

Next we study the Bohr radius problems for $\mathcal{SP}_{snail}(\alpha,\beta,\gamma)$. Here, we following the methods of Allu and Halder [5]. Define the following function $\hbar \in \mathcal{S}$ by

$$\frac{z\mathcal{D}_q\hbar(z)}{\hbar(z)} = 1 + \mathcal{L}_{\alpha,\beta,\gamma}(z). \tag{4.1}$$

Note that $\hbar$ is the same role as Kobe function for the class $\mathcal{SP}_{snail}(\alpha,\beta,\gamma)$. Now, without proof we state our results as follows.

Theorem 4.1. *If $f(z)$ given by (1.4) belongs to the class $\mathcal{SP}^q_{snail}(\alpha,\beta,\gamma)$ and $1+\mathcal{L}_{\alpha,\beta,\gamma}(z)$ is in the Hardy class $\mathcal{H}^2$ of analytic functions in $\mathbb{D}$, then*

$$|z| + \sum_{n=2}^{\infty} |a_n||z|^n \leq d(0, \partial f(\mathbb{D})) \tag{4.2}$$

for $|z| < \max\{r^*, 1/3\}$, where r^* is the smallest positive solution of

$$\hbar(r) + \hbar(-1) = 0$$

in $(0,1)$, and $\hbar(z)$ is defined by (4.1) with

$$\hbar(r) = r\exp\left(\sum_{n=1}^{\infty} M_n(\alpha,\beta,\gamma)\frac{r^n}{n}\right).$$

In this case, the class $\mathcal{SP}^q_{snail}(\alpha,\beta,\gamma)$ is said to satisfy the Bohr phenomenon.

5. Application to functions defined by neutrosophic Poisson distribution series

From now on, by letting $\wp_N(z)$ as the neutrosophic Poisson distribution series, we study the following problems. As is well known that the classical probability distributions only deal with specified data and specified parameter values, while the neutrosophic probability distribution is deeply concerned with some more general and clear ones. In fact, neutrosophic Poisson distribution of a discrete variable ξ is a classical Poisson distribution of x with the imprecise parameter value. A variable ξ is said to have the neutrosophic Poisson distribution if its probability with the value $k \in \mathbb{N}^* = \mathbb{N}\cup\{0\}$ is

$$NP(\xi = k) = \frac{(m_N)^k}{k!}e^{-m_N},$$

where the distribution parameter m_N is the expected value and the variance, that is to say, $NE(x) = NV(x) = m_N$ for the neutrosophic statistical number $N = d + I$ (refer to [22] and the references cited). Define a power series whose coefficients are probabilities of neutrosophic Poisson distribution by

$$\Phi(m_N, z) = z + \sum_{n=2}^{\infty} \frac{(m_N)^{n-1}}{(n-1)!} e^{-m_N} z^n, \ \ z \in \mathbb{D}.$$

For $f \in \mathcal{A}$, we take the convolution operator $*$ to introduce the linear operator $\mathfrak{N} : \mathcal{A} \to \mathcal{A}$ defined by

$$\begin{aligned} \mathfrak{N} f(z) &= \Phi(m_N, z) * f(z) = z + \sum_{n=2}^{\infty} \frac{(m_N)^{n-1}}{(n-1)!} e^{-m_N} a_n z^n \\ &= z + \sum_{n=2}^{\infty} E(m_N, n) a_n z^n, \end{aligned} \tag{5.1}$$

where

$$E_n := E(m_N, n) = \frac{(m_N)^{n-1}}{(n-1)!} e^{-m_N}.$$

Specially

$$E_2 := m_N e^{-m_N}, \quad E_3 := \frac{(m_N)^2}{2} e^{-m_N}.$$

Referring to Definition 1.1, now we introduce the new class associated with the neutrosophic Poisson distribution series.

Definition 5.1. Let $\mathcal{L}_{\alpha,\beta,\gamma}$ be given by (1.1). For $0 \leq \lambda \leq 1$, a function $f \in \mathcal{A}$ is said to be in the class $\mathcal{NSP}^q_{snail}(\lambda; \alpha, \beta, \gamma)$ if the following subordination

$$\frac{z\mathcal{D}_q[\mathfrak{N} f(z)]}{(1-\lambda)\mathfrak{N} f(z) + \lambda z} \prec 1 + \mathcal{L}_{\alpha,\beta,\gamma}(z) \tag{5.2}$$

holds for $z \in \mathbb{D}$, where $\mathfrak{N} f(z)$ is given by (5.1).

As the similar as Definition 1.1, we denote that

$$\mathcal{NSP}^q_{snail}(0; \alpha, \beta, \gamma) = \mathcal{NSP}^q_{snail}(\alpha, \beta, \gamma)$$

and

$$\mathcal{NSP}^q_{snail}(1; \alpha, \beta, \gamma) = \mathcal{NRP}^q_{snail}(\alpha, \beta, \gamma).$$

By applying Theorems 2.1–2.3, we can deduce the theorems below.

Theorem 5.1. *If $f(z)$ given by (1.4) belongs to the class $\mathcal{NSP}^q_{snail}(\lambda; \alpha, \beta, \gamma)$, then*

$$|a_2| \leq \frac{M_1}{([2]_q + \lambda - 1)E_2}, \tag{5.3}$$

$$|a_3| \leq \frac{M_1 + |M_2 - M_1|}{([3]_q + \lambda - 1)E_3} + \frac{(1-\lambda)M_1^2}{([3]_q + \lambda - 1)([2]_q + \lambda - 1)E_3} \tag{5.4}$$

and

$$|a_3 - \mu a_2^2| \leq \frac{M_1 \max\{1, |2\varrho - 1|\}}{([3]_q + \lambda - 1)E_3} \tag{5.5}$$

holds for $\mu \in \mathbb{C}$, *where*

$$\varrho = \frac{\mu([3]_q + \lambda - 1)M_1 E_3}{2([2]_q + \lambda - 1)^2 E_2^2} - \frac{M_2 - M_1}{2M_1} - \frac{(1-\lambda)M_1}{2([2]_q + \lambda - 1)}.$$

Theorem 5.2. *For* $\mu \in \mathbb{R}$, *if* $f(z) \in \mathcal{A}$ *belongs to the class* $\mathcal{NSP}^q_{snail}(\lambda; \alpha, \beta, \gamma)$, *then*

$$|a_3 - \mu a_2^2| \leq \begin{cases} \frac{M_1(-2\varrho+1)}{([3]_q+\lambda-1)E_3}, & \mu \leq \Upsilon_1, \\ \frac{M_1}{([3]_q+\lambda-1)E_3}, & \Upsilon_1 \leq \mu \leq \Upsilon_2, \\ \frac{M_1(2\varrho-1)}{([3]_q+\lambda-1)E_3}, & \mu \geq \Upsilon_2, \end{cases}$$

where ϱ *is the same as in Theorem 5.1,*

$$\Upsilon_1 = \frac{([2]_q + \lambda - 1)^2(M_2 - M_1)E_2^2}{([3]_q + \lambda - 1)M_1^2 E_3} + \frac{(1-\lambda)([2]_q + \lambda - 1)E_2^2}{([3]_q + \lambda - 1)E_3}$$

and

$$\Upsilon_2 = \frac{([2]_q + \lambda - 1)^2(M_2 + M_1)E_2^2}{([3]_q + \lambda - 1)M_1^2 E_3} + \frac{(1-\lambda)([2]_q + \lambda - 1)E_2^2}{([3]_q + \lambda - 1)E_3}.$$

In addition, we fix

$$\Upsilon_3 = \frac{([2]_q + \lambda - 1)^2 M_2 E_2^2}{([3]_q + \lambda - 1)M_1^2 E_3} + \frac{(1-\lambda)([2]_q + \lambda - 1)E_2^2}{([3]_q + \lambda - 1)E_3}.$$

Then, each of the following inequalities holds:
(A) For $\mu \in [\Upsilon_1, \Upsilon_3]$,

$$|a_3 - \mu a_2^2| + \frac{2\varrho([2]_q + \lambda - 1)^2 E_2^2}{([3]_q + \lambda - 1)M_1 E_3}|a_2|^2 \leq \frac{M_1}{([3]_q + \lambda - 1)E_3};$$

(B) For $\mu \in [\Upsilon_3, \Upsilon_2]$,

$$|a_3 - \mu a_2^2| + \frac{2(1-\varrho)([2]_q + \lambda - 1)^2 E_2^2}{([3]_q + \lambda - 1)M_1 E_3}|a_2|^2 \leq \frac{M_1}{([3]_q + \lambda - 1)E_3}.$$

Similarly, by applying Theorems 3.1–3.3, we can establish the theorems below.

Theorem 5.3. *If* $f(z)$ *given by (1.4) belongs to the class* $\mathcal{NSP}^q_{snail}(\lambda; \alpha, \beta, \gamma)$, *then*

$$|\,\mathcal{T}_2(2)\,| \;\leq\; \frac{M_1^2}{([2]_q + \lambda - 1)^2 E_2^2} + \left[\frac{M_1 + |M_2 - M_1|}{([3]_q + \lambda - 1)E_3} + \frac{(1-\lambda)M_1^2}{([3]_q + \lambda - 1)([2]_q + \lambda - 1)E_3}\right]^2.$$

Theorem 5.4. *If $f(z) \in \mathcal{A}$ belongs to the class $\mathcal{NSP}^q_{snail}(\lambda;\alpha,\beta,\gamma)$, then*

$$|T_3(1)| \leq \left(1+\frac{M_1\max\{1,|2\varrho-1|\}}{([3]_q+\lambda-1)E_3}\right) \times\left[1+\frac{M_1+|M_2-M_1|}{([3]_q+\lambda-1)E_3}+\frac{(1-\lambda)M_1^2}{([3]_q+\lambda-1)([2]_q+\lambda-1)E_3}\right]$$

holds, where

$$\varrho=\frac{([3]_q+\lambda-1)M_1E_3}{([2]_q+\lambda-1)^2E_2^2}-\frac{M_2-M_1}{2M_1}-\frac{(1-\lambda)M_1}{2([2]_q+\lambda-1)}.$$

Theorem 5.5. *If $f(z) \in \mathcal{A}$ belongs to the class $\mathcal{NSP}^q_{snail}(\lambda;\alpha,\beta,\gamma)$, then*

$$|T_3(1)| \leq \begin{cases} \left(1+\frac{M_1(-2\varrho+1)}{([3]_q+\lambda-1)E_3}\right)\times\left[1+\frac{M_1+|M_2-M_1|}{([3]_q+\lambda-1)E_3}+\frac{(1-\lambda)M_1^2}{([3]_q+\lambda-1)([2]_q+\lambda-1)E_3}\right], & \Pi \leq M_2-M_1, \\ \left(1+\frac{M_1}{([3]_q+\lambda-1)E_3}\right)\times\left[1+\frac{M_1+|M_2-M_1|}{([3]_q+\lambda-1)E_3}+\frac{(1-\lambda)M_1^2}{([3]_q+\lambda-1)([2]_q+\lambda-1)E_3}\right], & M_2-M_1 \leq \Pi \leq M_2+M_1, \\ \left(1+\frac{M_1(2\varrho-1)}{([3]_q+\lambda-1)E_3}\right)\times\left[1+\frac{M_1+|M_2-M_1|}{([3]_q+\lambda-1)E_3}+\frac{(1-\lambda)M_1^2}{([3]_q+\lambda-1)([2]_q+\lambda-1)E_3}\right], & M_2+M_1 \leq \Pi, \end{cases}$$

where ϱ is the same as in Theorem 5.4, and

$$\Pi=\frac{2([3]_q+\lambda-1)M_1^2E_3}{([2]_q+\lambda-1)^2E_2^2}-\frac{(1-\lambda)M_1^2}{[2]_q+\lambda-1}.$$

6. Conclusions

By involving the generalized Pascal snail and q-derivative operator, certain new subclass of analytic and univalent functions can be defined to improve the classical starlike functions. In our main results, for this class we obtain the corresponding the Fekete-Szegö functional inequalities and the symmetric Toeplitz determinants as well as the bound estimates of the coefficients a_2 and a_3. In addition, we characterize the Bohr radius problems for the reduced version of this class. Moreover, the above results are applied to the neutrosophic Poisson distribution series. Besides, some other problems like Hankel determinant, partial sum inequalities, and many more can be discussed for this class as the future work. In fact, we also replace the generalized Pascal snail by the other Limaçons. In the neutrosophic logic sense, other types of probability distributions, for example, exponential distributions, Bernoulli distributions and uniform distributions, can be studied in various classes of analytic functions.

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Conflict of interest

The authors declare no conflicts of interest.

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