AIMS Mathematics

AIMS Mathematics, 7(7): 13383–13401.
DOI: 10.3934/math.2022739
Received: 20 March 2022
Revised: 20 April 2022
Accepted: 04 May 2022
Published: 17 May 2022

http://www.aimspress.com/journal/Math

Research article

Analysis of HIV/AIDS model with Mittag-Leffler kernel

Muhammad Mannan Akram[1], Muhammad Farman[2], Ali Akgül[3,*], Muhammad Umer Saleem[4], Aqeel Ahmad[5], Mohammad Partohaghigh[6] and Fahd Jarad[7,8]

[1] Department of Mathematics and statistics, University of Lahore, Lahore-54590, Pakistan
[2] Department of Mathematics, Khawaja Fareed University of Engineering and Information Technology, Rahim Yar Khan, Pakistan
[3] Art and Science Faculty, Department of Mathematics, Siirt University, 56100 Siirt Turkey
[4] Department of Mathematics, University of Education, Lahore, Pakistan
[5] Department of Mathematics, Ghazi University, D.G. Khan, Pakistan
[6] Department of Mathematics, Clarkson University, USA
[7] Department of Mathematics, Çankaya University, 06790 Etimesgut, Ankara, Turkey
[8] Department of Medical Research, China Medical University Hospital, China Medical University, Taichung, Taiwan

* **Correspondence:** Email: aliakgul00727@gmail.com.

Abstract: Recently different definitions of fractional derivatives are proposed for the development of real-world systems and mathematical models. In this paper, our main concern is to develop and analyze the effective numerical method for fractional order HIV/ AIDS model which is advanced approach for such biological models. With the help of an effective techniques and Sumudu transform, some new results are developed. Fractional order HIV/AIDS model is analyzed. Analysis for proposed model is new which will be helpful to understand the outbreak of HIV/AIDS in a community and will be helpful for future analysis to overcome the effect of HIV/AIDS. Novel numerical procedures are used for graphical results and their discussion.

Keywords: Sumudu transform; uniqueness; stability analysis; fixed-point theorem
Mathematics Subject Classification: 37C75, 93B05, 65L07

1. Introduction

Biomathematics is basically the theoretical analysis of mathematical models and abstraction of living organism to investigate the principle that governs the structure development and behavior of system [1]. HIV contaminates the enthusiastic cells and tissues of the human immune system. This infection in the absence of antiretroviral treatment (ART), medication treatment that evades or moderates the infection, develop rapidly. Generally, HIV is diffused from perinatal or blood diffusion and sexual transmission. The symptoms of HIV at initial stage may incorporate joint agony, fever, muscle throbs, chills, sore throat, broadened organs, sweats (especially during the evening), a red rash, shortcoming, tiredness and inadvertent weight reduction thrush [2–4]. The HIV plague is perceived as the plainest debacle in current era. Regardless of advances in the biomedical front to the mind-boggling standard of the individuals who require it the treatment remains inaccessible and the plague keeps on spreading [5]. NSFD techniques by Mickens [6] are practical for numerical mix of differential conditions logically [7]. Effect by changing fractional order on the disease spread is also studied in some models. HIV fractional order models have continuously been under discussion of researchers due to the dynamics of HIV epidemics [8–14]. The fractional order model that involves integration and transects differentiation with the help of fractional calculus can also help to understand better the explanation of real-world problems than ordinary derivatives [15,16]. Based on the power law, fractional derivative idea was introduced by Riemann Liouville. The new fractional derivative by utilizing the exponential kernel is proposed by Atangana [17,18]. Non-singular kernel fractional derivative that includes the trigonometric and exponential function related problems [19–22] shows some related approaches for the models of epidemic. Recently a numerical scheme to solve the nonlinear fractional differential equation has been presented [28,29]. The proposed outbreak of this virus which effectively catches the time line for the COVID-19 disease conceptual model [23–25] is under discussion too nowadays.

The feasible and accurate technique for obtaining numerical solutions for a class of partial integro-differential equations of fractional order in Hilbert space within appropriate kernel functions is studied in [30]. The solution methodology lies in generating an infinite conformable series solution with reliable wave pattern by minimizing the residual error functions and its related PDE's are analyzed in [31–33]. The multistep generalized differential transform method is applied to solve the fractional-order multiple chaotic FitzHugh-Nagumo (FHN) neurons model [34]. Investigation of a novel fractional-order mathematical model that explains the behavior of COVID-19 in Ethiopia has been studied in [35]. The transmission of influenza has been explained by analyzing a diffusive epidemic model in [36]. The analysis of general fractional order system is investigated under ABC fractional order derivative [37].

In this paper, Section 2 consists of some basic fractional order derivative which is helpful to solve the epidemiological model. Sections 3 and 4 consist of generalized form of the model with Atangana-Baleanu in Caputo sense using Sumudu transforms, uniqueness and stability analysis of the model. A new technique with exponential decay kernel and Mittag-Leffler kernel respectively has been given in Section 5. Results and conclusion are discussed in Section 6 and Section 7 respectively.

2. Basic definitions

Definition 2.1. Atangana-Baleanu in Caputo sense (ABC) is given by [18]:

$$ {}^{ABC}_{\ a}D^{\alpha}_{\tau}\big(\phi(\tau)\big) = \frac{AB(\alpha)}{n-\alpha}\int_a^{\tau}\frac{d^n}{dw^n}f(w)E_{\alpha}\left\{-\alpha\frac{(\tau-w)^{\alpha}}{n-\alpha}\right\}dw,\ n-1<\alpha<n, \tag{1} $$

where E_{α} is Mittag-Leffler function, $AB(\alpha)$ is normalization function and $AB(0) = AB(1) = 1$. The Laplace transform is obtained by:

$$ [{}^{ABC}_{\ a}D^{\alpha}_{\tau}\phi(\tau)](s) = \frac{AB(\alpha)}{1-\alpha}\frac{s^{\alpha}L[\phi(\tau)](s)-s^{\alpha-1}\phi(0)}{s^{\alpha}+\frac{\alpha}{1-\alpha}}. \tag{2} $$

By using Sumudu transform (ST) for (1), we obtain

$$ ST[{}^{ABC}_{\ 0}D^{\alpha}_{\tau}\phi(\tau)](s) = \frac{B(\alpha)}{1-\alpha}\left\{\alpha\Gamma(\alpha+1)E_{\alpha}\left(-\frac{1}{1-\alpha}w^{\alpha}\right)\right\}\times\left[ST\big(\phi(t)\big)-\phi(0)\right]. \tag{3} $$

Definition 2.2. Atangana-Baleanu fractional integral of a function $\phi(t)$ of order α is given by:

$$ {}^{ABC}_{\ a}I^{\alpha}_{\tau}\big(\phi(\tau)\big) = \frac{1-\alpha}{B-\alpha}\phi(\tau) + \frac{\alpha}{B(\alpha)\Gamma(\alpha)}\int_a^{\tau}\phi(s)(\tau-s)^{\alpha-1}ds. \tag{4} $$

3. Fractional order HIV/AIDS model

In this section, we consider the HIV/AIDS epidemic model proposed by Huo et al. [26] with a treatment compartment. By transforming the model given in [26] into Mittag-Leffler kernel with Atangana-Baleanu Caputo derivative is given in the following equations:

$$ \begin{aligned} {}^{ABC}_{\ 0}D^{\alpha}_{t}S &= \Lambda - \beta IS - \mu_1 S - dS, \\ {}^{ABC}_{\ 0}D^{\alpha}_{t}I &= \beta IS + \alpha_1 T - dI - k_1 I - k_2 I, \\ {}^{ABC}_{\ 0}D^{\alpha}_{t}A &= k_1 I - (\delta_1 + d)A + \alpha_2 T, \\ {}^{ABC}_{\ 0}D^{\alpha}_{t}T &= k_2 I - \alpha_1 T - (d + \delta_2 + \alpha_2)T, \\ {}^{ABC}_{\ 0}D^{\alpha}_{t}R &= \mu_1 S - dR, \end{aligned} \tag{5} $$

with initial conditions

$$ I(0) = I_0, S(0) = S_0, A(0) = A_0, R(0) = R_0, T(0) = T_0. \tag{6} $$

Here susceptible patients is S(t), I(t) is infectious HIV-positive individuals, $A(t)$ is the number of people with full-blown AIDS, T(t) is the total number of people being treated with ARV and R(t) is recovered populations. Λ is the rate of recruitment of susceptible individuals into the population, β represents the interaction rate between susceptible individuals and infectious individuals, μ_1 is the rate at which susceptible individuals change their sexual behaviors per unit time, d is the natural death rate, α_1 is the rate at which treated individuals leave T(t) compartment, k_1 is the rate at which people leave the infectious class and become people with full-blown AIDS, k_2 is the rate at which people with HIV are treated, δ_1 and δ_2 are the disease-induced death rates for people in A(t) and T(t) compartments, respectively. α_2 represents the rate at which treated individuals leave the treated class and enter the AIDS compartment $A(t)$. By putting left hand side equal to zero, we get disease free and endemic equilibrium point. Disease-free equilibrium point is given as:

$$ E^* = (S^*, I^*, A^*, T^*, R^*) = \left(\frac{\Lambda}{\mu_1 + d}, 0,0,0, \frac{\Lambda\mu_1}{d(d+\mu_1)}\right) $$

and EEP is given as:

$$S^0=\frac{\Lambda}{\beta I^0+\mu_1+d},\qquad I^0=\frac{(R_0-1)(\mu_1+d)}{\beta},\qquad A^0=\frac{k_1I^0+\alpha_2T^0}{d+\delta_1},$$

$$T^0=\frac{k_2I^0}{\alpha_1+d+\delta_2+\alpha_2},\qquad R^0=\frac{\mu_1\Lambda}{d(\beta I^0+\mu_1+d)}.$$

Reproductive number of the system [27] is given as:

$$R_0=\frac{\beta\Lambda(d+\delta_2+\alpha_1+\alpha_2)}{(\mu_1+d)(d+k_1+k_2)(d+\delta_2+\alpha_1+\alpha_2)-\alpha_1k_2}.$$

4. Mittag-Leffler kernel with Atangana-Baleanu Caputo derivative

Applying Mittag-Leffler kernel with Atangana-Baleanu Caputo derivative on system (5), we get

$$\begin{aligned}
&\frac{B(\alpha)\alpha\Gamma(\alpha+1)}{1-\alpha}E_\alpha\left(-\frac{1}{1-\alpha}w^\alpha\right)ST\{S(t)-S(0)\}=ST[\Lambda-\beta IS-\mu_1S-dS],\\
&\frac{B(\alpha)\alpha\Gamma(\alpha+1)}{1-\alpha}E_\alpha\left(-\frac{1}{1-\alpha}w^\alpha\right)ST\{I(t)-I(0)\}=ST[\beta IS+\alpha_1T-dI-k_1I-k_2I],\\
&\frac{B(\alpha)\alpha\Gamma(\alpha+1)}{1-\alpha}E_\alpha\left(-\frac{1}{1-\alpha}w^\alpha\right)ST\{A(t)-A(0)\}=ST[k_1I-(\delta_1+d)A+\alpha_2T],\\
&\frac{B(\alpha)\alpha\Gamma(\alpha+1)}{1-\alpha}E_\alpha\left(-\frac{1}{1-\alpha}w^\alpha\right)ST\{T(t)-T(0)\}=ST[k_2I-\alpha_1T-(d+\delta_2+\alpha_2)T],\\
&\frac{B(\alpha)\alpha\Gamma(\alpha+1)}{1-\alpha}E_\alpha\left(-\frac{1}{1-\alpha}w^\alpha\right)ST\{R(t)-R(0)\}=ST[\mu_1S-dR].
\end{aligned} \tag{7}$$

Rearranging the above equations yields:

$$\begin{aligned}
&ST\big(S(t)\big)=S(0)+\frac{1-\alpha}{B(\alpha)\alpha\Gamma(\alpha+1)E_\alpha\left(-\frac{1}{1-\alpha}w^\alpha\right)}\times ST[\Lambda-\beta IS-\mu_1S-dS],\\
&ST\big(I(t)\big)=I(0)+\frac{1-\alpha}{B(\alpha)\alpha\Gamma(\alpha+1)E_\alpha\left(-\frac{1}{1-\alpha}w^\alpha\right)}\times ST[\beta IS+\alpha_1T-dI-k_1I-k_2I],\\
&ST\big(A(t)\big)=A(0)+\frac{1-\alpha}{B(\alpha)\alpha\Gamma(\alpha+1)E_\alpha\left(-\frac{1}{1-\alpha}w^\alpha\right)}\times ST[k_1I-(\delta_1+d)A+\alpha_2T], (07)\\
&ST\big(T(t)\big)=T(0)+\frac{1-\alpha}{B(\alpha)\alpha\Gamma(\alpha+1)E_\alpha\left(-\frac{1}{1-\alpha}w^\alpha\right)}\times ST[k_2I-\alpha_1T-(d+\delta_2+\alpha_2)T],\\
&ST\big(R(t)\big)=R(0)+\frac{1-\alpha}{B(\alpha)\alpha\Gamma(\alpha+1)E_\alpha\left(-\frac{1}{1-\alpha}w^\alpha\right)}\times ST[\mu_1S-dR].
\end{aligned}$$

Using inverse transform on (7) gives

$$S(t) = S(0) + ST^{-1}\left[\frac{1-\alpha}{B(\alpha)\alpha\Gamma(\alpha+1)E_\alpha\left(-\frac{1}{1-\alpha}w^\alpha\right)} \times ST\{\Lambda - \beta IS - \mu_1 S - dS\}\right],$$

$$I(t) = I(0) + ST^{-1}\left[\frac{1-\alpha}{B(\alpha)\alpha\Gamma(\alpha+1)E_\alpha\left(-\frac{1}{1-\alpha}w^\alpha\right)} \times ST\{\beta IS + \alpha_1 T - dI - k_1 I - k_2 I\}\right],$$

$$A(t) = A(0) + ST^{-1}\left[\frac{1-\alpha}{B(\alpha)\alpha\Gamma(\alpha+1)E_\alpha\left(-\frac{1}{1-\alpha}w^\alpha\right)} \times ST\{k_1 I - (\delta_1 + d)A + \alpha_2 T\}\right],$$

$$T(t) = T(0) + ST^{-1}\left[\frac{1-\alpha}{B(\alpha)\alpha\Gamma(\alpha+1)E_\alpha\left(-\frac{1}{1-\alpha}w^\alpha\right)} \times ST\{k_2 I - \alpha_1 T - (d + \delta_2 + \alpha_2)T\}\right],$$

$$R(t) = R(0) + ST^{-1}\left[\frac{1-\alpha}{B(\alpha)\alpha\Gamma(\alpha+1)E_\alpha\left(-\frac{1}{1-\alpha}w^\alpha\right)} \times ST\{\mu_1 S - dR\}\right].$$

We next obtain the following recursive formula:

$$S_{n+1}(t) = S_n(0) + ST^{-1}\left[\frac{1-\alpha}{B(\alpha)\alpha\Gamma(\alpha+1)E_\alpha\left(-\frac{1}{1-\alpha}w^\alpha\right)} \times ST\{\Lambda - \beta I_n S_n - \mu_1 S_n - dS_n\}\right],$$

$$I_{n+1}(t) = I_n(0) + ST^{-1}\left[\frac{1-\alpha}{B(\alpha)\alpha\Gamma(\alpha+1)E_\alpha\left(-\frac{1}{1-\alpha}w^\alpha\right)} \times ST\{\beta I_n S_n + \alpha_1 T_n - dI_n - k_1 I_n - k_2 I_n\}\right],$$

$$A_{n+1}(t) = A_n(0) + ST^{-1}\left[\frac{1-\alpha}{B(\alpha)\alpha\Gamma(\alpha+1)E_\alpha\left(-\frac{1}{1-\alpha}w^\alpha\right)} \times ST\{k_1 I_n - (\delta_1 + d)A_n + \alpha_2 T_n\}\right],$$

$$T_{n+1}(t) = T_n(0) + ST^{-1}\left[\frac{1-\alpha}{B(\alpha)\alpha\Gamma(\alpha+1)E_\alpha\left(-\frac{1}{1-\alpha}w^\alpha\right)} \times ST\{k_2 I_n - \alpha_1 T_n - (d + \delta_2 + \alpha_2)T_n\}\right],$$

$$R_{n+1}(t) = R_n(0) + ST^{-1}\left[\frac{1-\alpha}{B(\alpha)\alpha\Gamma(\alpha+1)E_\alpha\left(-\frac{1}{1-\alpha}w^\alpha\right)} \times ST\{\mu_1 S_n - dR_n\}\right]. \quad (8)$$

And the solution of (8) is provided by

$$S(t) = \lim_{n\to\infty} S_n(t), \quad I(t) = \lim_{n\to\infty} I_n(t), \quad A(t) = \lim_{n\to\infty} A_n(t),$$

$$T(t) = \lim_{n\to\infty} T_n(t), \quad R(t) = \lim_{n\to\infty} R_n(t).$$

Theorem 4.1. Let $(X, |.|)$ be a Banach space and H a self-map of Xsatisfying

$$\|H_x - H_r\| \leq \theta\|X - H_x\| + \theta\|x - r\|,$$

for all $x, r \in X$, where $0 \leq \theta < 1$. Suppose that H is Picard H-Stable. Let us consider Eq (8), and we get

$$S_{n+1}(t)=S_n(0)+ST^{-1}\left[\frac{1-\alpha}{B(\alpha)\alpha\Gamma(\alpha+1)E_\alpha\left(-\frac{1}{1-\alpha}w^\alpha\right)}\times ST\{\Lambda-\beta I_nS_n-\mu_1S_n-dS_n\}\right],$$

$$I_{n+1}(t)=I_n(0)+ST^{-1}\left[\frac{1-\alpha}{B(\alpha)\alpha\Gamma(\alpha+1)E_\alpha\left(-\frac{1}{1-\alpha}w^\alpha\right)}\times ST\{\beta I_nS_n+\alpha_1T_n-dI_n-k_1I_n-k_2I_n\}\right],$$

$$A_{n+1}(t)=A_n(0)+ST^{-1}\left[\frac{1-\alpha}{B(\alpha)\alpha\Gamma(\alpha+1)E_\alpha\left(-\frac{1}{1-\alpha}w^\alpha\right)}\times ST\{k_1I_n-(\delta_1+d)A_n+\alpha_2T_n\}\right],$$

$$T_{n+1}(t)=T_n(0)+ST^{-1}\left[\frac{1-\alpha}{B(\alpha)\alpha\Gamma(\alpha+1)E_\alpha\left(-\frac{1}{1-\alpha}w^\alpha\right)}\times ST\{k_2I_n-\alpha_1T_n-(d+\delta_2+\alpha_2)T_n\}\right],$$

$$R_{n+1}(t)=R_n(0)+ST^{-1}\left[\frac{1-\alpha}{B(\alpha)\alpha\Gamma(\alpha+1)E_\alpha\left(-\frac{1}{1-\alpha}w^\alpha\right)}\times ST\{\mu_1S_n-dR_n\}\right],$$

where $\frac{1-\alpha}{B(\alpha)\alpha\Gamma(\alpha+1)E_\alpha\left(-\frac{1}{1-\alpha}w^\alpha\right)}$ is the fractional Lagrange multiplier.

Theorem 4.2.

$$\begin{aligned}
K[S_{n+1}(t)]&=S_{(n+1)}(t)\\
&=S_n(t)+ST^{-1}\left[\frac{1-\alpha}{B(\alpha)\alpha\Gamma(\alpha+1)E_\alpha\left(-\frac{1}{1-\alpha}w^\alpha\right)}\times ST\{\Lambda-\beta I_nS_n-\mu_1S_n-dS_n\}\right],\\
K[I_{n+1}(t)]&=I_{(n+1)}(t)\\
&=I_n(t)+ST^{-1}\left[\frac{1-\alpha}{B(\alpha)\alpha\Gamma(\alpha+1)E_\alpha\left(-\frac{1}{1-\alpha}w^\alpha\right)}\times ST\{\beta I_nS_n+\alpha_1T_n-dI_n-k_1I_n-k_2I_n\}\right],\\
K[A_{n+1}(t)]&=A_{(n+1)}(t)\\
&=A_n(t)+ST^{-1}\left[\frac{1-\alpha}{B(\alpha)\alpha\Gamma(\alpha+1)E_\alpha\left(-\frac{1}{1-\alpha}w^\alpha\right)}\times ST\{k_1I_n-(\delta_1+d)A_n+\alpha_2T_n\}\right],\\
K[T_{n+1}(t)]&=T_{(n+1)}(t)\\
&=T_n(t)+ST^{-1}\left[\frac{1-\alpha}{B(\alpha)\alpha\Gamma(\alpha+1)E_\alpha\left(-\frac{1}{1-\alpha}w^\alpha\right)}\times ST\{k_2I_n-\alpha_1T_n-(d+\delta_2+\alpha_2)T_n\}\right],\\
K[R_{n+1}(t)]&=R_{(n+1)}(t)\\
&=R_n(t)+ST^{-1}\left[\frac{1-\alpha}{B(\alpha)\alpha\Gamma(\alpha+1)E_\alpha\left(-\frac{1}{1-\alpha}w^\alpha\right)}\times ST\{\mu_1S_n-dR_n\}\right]
\end{aligned} \tag{9}$$

where K be a self-map.

Proof. Using triangular inequality property with norm yields:

$$\begin{aligned}&\|K[S_n(t)] - K[S_m(t)]\|\\ &\leq \|S_n(t) - S_m(t)\|\\ &+ ST^{-1}\left[\frac{1-\alpha}{B(\alpha)\alpha\Gamma(\alpha+1)E_\alpha\left(-\frac{1}{1-\alpha}w^\alpha\right)}\right.\\ &\left.\times ST\{\Lambda + \beta\|(I_nS_n - I_mS_m)\| + \mu_1\|(S_n - S_m)\| + d\|(S_n - S_m)\|\}\right],\end{aligned}$$

$$\begin{aligned}&\|K[I_n(t)] - K[I_m(t)]\|\\ &\leq \|I_n(t) - I_m(t)\|\\ &+ ST^{-1}\left[\frac{1-\alpha}{B(\alpha)\alpha\Gamma(\alpha+1)E_\alpha\left(-\frac{1}{1-\alpha}w^\alpha\right)}\right.\\ &\times ST\{\beta\|(I_nS_n - I_mS_m)\| + \alpha_1\|(T_n - T_m)\| + d\|(I_n - I_m)\| + k_1\|(I_n - I_m)\|\\ &\left.+ k_2\|(I_n - I_m)\|\}\right],\end{aligned}$$

$$\begin{aligned}&\|K[A_n(t)] - K[A_m(t)]\|\\ &\leq \|A_n(t) - A_m(t)\|\\ &+ ST^{-1}\left[\frac{1-\alpha}{B(\alpha)\alpha\Gamma(\alpha+1)E_\alpha\left(-\frac{1}{1-\alpha}w^\alpha\right)}\right.\\ &\left.\times ST\{k_1\|(I_n - I_m)\| + (\delta_1 + d)\|(A_n - A_m)\| + \alpha_2\|(T_n - T_m)\|\}\right],\end{aligned}$$

$$\begin{aligned}&\|K[T_n(t)] - K[T_m(t)]\|\\ &\leq \|T_n(t) - T_m(t)\|\\ &+ ST^{-1}\left[\frac{1-\alpha}{B(\alpha)\alpha\Gamma(\alpha+1)E_\alpha\left(-\frac{1}{1-\alpha}w^\alpha\right)}\right.\\ &\left.\times ST\{k_2\|(I_n - I_m)\| + \alpha_1\|(T_n - T_m)\| + (d + \delta_2 + \alpha_2)\|(T_n - T_m)\|\}\right],\end{aligned}$$

$$\begin{aligned}&\|K[R_n(t)] - K[R_m(t)]\| \leq \|R_n(t) - R_m(t)\|\\ &+ST^{-1}\left[\frac{1-\alpha}{B(\alpha)\alpha\Gamma(\alpha+1)E_\alpha\left(-\frac{1}{1-\alpha}w^\alpha\right)} \times ST\{\mu_1\|S_n - S_m\| + d\|(R_n - R_m)\|\}\right]. \quad (10)\end{aligned}$$

It's satisfied the condition given in Theorem 4.1, when

$$\theta = (0,0,0,0,0),$$

$$\theta = \begin{cases} \|S_n(t) - S_m(t)\| \times \left\|-\left(S_n(t) - S_m(t)\right)\right\| + \Lambda - \beta\|(I_nS_n - I_mS_m)\| \\ -\mu_1\|(S_n - S_m)\| - d\|(S_n - S_m)\| \\ \times \left\|\left(I_n(t) - I_m(t)\right)\right\| \times \left\|-\left(I_n(t) - I_m(t)\right)\right\| + \beta\|(I_nS_n - I_mS_m)\| \\ +\alpha_1\|(T_n - T_m)\| - d\|(I_n - I_m)\| \\ -k_1\|(I_n - I_m)\| - k_2\|(I_n - I_m)\| \\ \times \|A_n(t) - A_m(t)\| \times \left\|-\left(A_n(t) - A_m(t)\right)\right\| + k_1\|(I_n - I_m)\| \\ -(\delta_1 + d)\|(A_n - A_m)\| + \alpha_2\|(T_n - T_m)\| \\ \times \|T_n(t) - T_m(t)\| \times \left\|-\left(T_n(t) - T_m(t)\right)\right\| \\ +k_2\|(I_n - I_m)\| - \alpha_1\|(T_n - T_m)\| \\ -(d + \delta_2 + \alpha_2)\|(T_n - T_m)\| \\ \times \|R_n(t) - R_m(t)\| \times \|R_n(t) - R_m(t)\| \\ +\mu_1\|S_n - S_m\| - d\|(R_n - R_m)\| \end{cases}.$$

Hence, it's stable.

Theorem 4.3. The special solution of Eq (5) using the iteration method is unique singular solution.

Proof. Take into consideration the following Hilbert space $H = L^2\left((p,q) \times (0,T)\right)$ which can be defined as

$$h: (p,q) \times (0,T) \to \mathbb{R}, \qquad \iint ghdgdh < \infty.$$

Considering the following operator, we have

$$\theta(0,0,0,0,0), \theta = \begin{cases} \Lambda - \beta IS - \mu_1 S - dS, \\ \beta IS + \alpha_1 T - dI - k_1 I - k_2 I, \\ k_1 I - (\delta_1 + d)A + \alpha_2 T, \\ k_2 I - \alpha_1 T - (d + \delta_2 + \alpha_2)T, \\ \mu_1 S - dR. \end{cases}$$

By using

$$P\left((S_{11} - S_{12}, I_{21} - I_{22}, A_{31} - A_{32}, T_{41} - T_{42}, R_{51} - R_{52}), (V_1, V_2, V_3, V_4, V_5)\right).$$

Where

$$(S_{11} - S_{12}, I_{21} - I_{22}, A_{31} - A_{32}, T_{41} - T_{42}, R_{51} - R_{52}),$$

we have

$$\{\Lambda - \beta(I_{21} - I_{22})(S_{11} - S_{12}) - \mu_1(S_{11} - S_{12}) - d(S_{11} - S_{12})\}$$
$$\leq \Lambda\|V_1\| + \beta\|I_{21} - I_{22}\|\|S_{11} - S_{12}\|\|V_1\|$$
$$+\mu_1\|S_{11} - S_{12}\|\|V_1\| + d\|S_{11} - S_{12}\|\|V_1\|,$$

$$\begin{Bmatrix} \beta(I_{21}-I_{22})(S_{11}-S_{12})+\alpha_1(T_{41}-T_{42}) \\ -d(I_{21}-I_{22})-k_1(I_{21}-I_{22})-k_2(I_{21}-I_{22}) \end{Bmatrix}$$

$$\leq \beta\|I_{21}-I_{22}\|\|S_{11}-S_{12}\|\|V_2\|+\alpha_1\|T_{41}-T_{42}\|\|V_2\|$$

$$+d\|I_{21}-I_{22}\|\|V_2\|+k_1\|I_{21}-I_{22}\|\|V_2\|+k_2\|I_{21}-I_{22}\|\|V_2\|,$$

$$\{k_1(I_{21}-I_{22})-(\delta_1+d)(A_{31}-A_{32})+\alpha_2(T_{41}-T_{42})\}$$

$$\leq k_1\|I_{21}-I_{22}\|\|V_3\|+(\delta_1+d)\|A_{31}-A_{32}\|\|V_3\|+\alpha_2\|T_{41}-T_{42}\|\|V_3\|,$$

$$\{k_2(I_{21}-I_{22})-\alpha_1(T_{41}-T_{42})-(d+\delta_2+\alpha_2)(T_{41}-T_{42})\}$$

$$\leq k_2\|I_{21}-I_{22}\|\|V_4\|+\alpha_1\|T_{41}-T_{42}\|\|V_4\|+(d+\delta_2+\alpha_2)\|T_{41}-T_{42}\|\|V_4\|,$$

$$\{\mu_1(S_{11}-S_{12})-d(R_{51}-R_{52})\}\leq \mu_1\|S_{11}-S_{12}\|\|V_5\|+d\|R_{51}-R_{52}\|\|V_5\|.$$

For convergence solution, we have

$$\|S-S_{11}\|,\|S-S_{12}\|\leq \frac{\chi_{e_1}}{\varpi},$$

$$\|I-I_{21}\|,\|I-I_{22}\|\leq \frac{\chi_{e_2}}{\varsigma},$$

$$\|A-A_{31}\|,\|A-A_{32}\|\leq \frac{\chi_{e_3}}{\upsilon},$$

$$\|T-T_{41}\|,\|T-T_{42}\|\leq \frac{\chi_{e_4}}{\kappa},$$

and

$$\|R-R_{51}\|,\|R-R_{52}\|\leq \frac{\chi_{e_5}}{\varrho}.$$

Where

$$\varpi=5(\Lambda+\beta\|I_{21}-I_{22}\|\|S_{11}-S_{12}\|+\mu_1\|S_{11}-S_{12}\|+d\|S_{11}-S_{12}\|)\|V_1\|,$$

$$\varsigma=5(\beta\|I_{21}-I_{22}\|\|S_{11}-S_{12}\|+\alpha_1\|T_{41}-T_{42}\|+d\|I_{21}-I_{22}\|+k_1\|I_{21}-I_{22}\| \\ +k_2\|I_{21}-I_{22}\|)\|V_2\|,$$

$$\upsilon=5(k_1\|I_{21}-I_{22}\|+(\delta_1+d)\|A_{31}-A_{32}\|+\alpha_2\|T_{41}-T_{42}\|)\|V_3\|,$$

$$\kappa=5(k_2\|I_{21}-I_{22}\|+\alpha_1\|T_{41}-T_{42}\|+(d+\delta_2+\alpha_2)\|T_{41}-T_{42}\|)\|V_4\|,$$

$$\varrho=5(\mu_1\|S_{11}-S_{12}\|+d\|R_{51}-R_{52}\|)\|V_5\|.$$

But it is obvious that

$$(\Lambda+\beta\|I_{21}-I_{22}\|\|S_{11}-S_{12}\|+\mu_1\|S_{11}-S_{12}\|+d\|S_{11}-S_{12}\|)\neq 0,$$

$$(\beta\|I_{21}-I_{22}\|\|S_{11}-S_{12}\|+\alpha_1\|T_{41}-T_{42}\|+d\|I_{21}-I_{22}\|+k_1\|I_{21}-I_{22}\|+k_2\|I_{21}-I_{22}\|)\neq 0,$$

$$(k_1\|I_{21}-I_{22}\|+(\delta_1+d)\|A_{31}-A_{32}\|+\alpha_2\|T_{41}-T_{42}\|)\neq 0,$$

$$(k_2\|I_{21}-I_{22}\|+\alpha_1\|T_{41}-T_{42}\|+(d+\delta_2+\alpha_2)\|T_{41}-T_{42}\|)\neq 0,$$

$$(\mu_1\|S_{11}-S_{12}\|+d\|R_{51}-R_{52}\|)\neq 0.$$

Where $\|V_1\|,\ \|V_2\|,\ \|V_3\|,\ \|V_4\|,\ \|V_5\|\neq 0$.

Therefore, we have

$$\|S_{11}-S_{12}\|=0,\ \|I_{21}-I_{22}\|=0,\ \|A_{31}-A_{32}\|=0,$$
$$\|T_{41}-T_{42}\|=0,\qquad \|R_{51}-R_{52}\|=0.$$

Which yields that

$$S_{11}=S_{12},\ I_{21}=I_{22},\ A_{31}=A_{32},\ T_{41}=T_{42},\ R_{51}=R_{52}.$$

We get required results. Hence, it's proved.

5. Numerical scheme

We consider the following non-linear fractional ordinary equation [28,29].

$$
\begin{aligned}
S(t)-S(0)&=\frac{(1-\alpha)}{ABC(\alpha)}\{\Lambda-\beta I(t)S(t)-\mu_1S(t)-dS(t)\}\\
&\quad+\frac{\alpha}{\Gamma(\alpha)\times ABC(\alpha)}\int_0^t\{\Lambda-\beta I(\tau)S(\tau)-\mu_1S(\tau)-dS(\tau)\}(t-\tau)^{\alpha-1}d\tau,\\
I(t)-I(0)&=\frac{(1-\alpha)}{ABC(\alpha)}\{\beta I(t)S(t)+\alpha_1T(t)-dI(t)-k_1I(t)-k_2I(t)\}\\
&\quad+\frac{\alpha}{\Gamma(\alpha)\times ABC(\alpha)}\int_0^t\{\beta I(\tau)S(\tau)+\alpha_1T(\tau)-dI(\tau)-k_1I(\tau)\\
&\quad-k_2I(\tau)\}(t-\tau)^{\alpha-1}d\tau,\\
A(t)-A(0)&=\frac{(1-\alpha)}{ABC(\alpha)}\{k_1I(t)-(\delta_1+d)A(t)+\alpha_2T(t)\}\\
&\quad+\frac{\alpha}{\Gamma(\alpha)\times ABC(\alpha)}\int_0^t\{k_1I(\tau)-(\delta_1+d)A(\tau)+\alpha_2T(\tau)\}(t-\tau)^{\alpha-1}d\tau, \qquad (11)\\
T(t)-T(0)&=\frac{(1-\alpha)}{ABC(\alpha)}\{k_2I(t)-\alpha_1T(t)-(d+\delta_2+\alpha_2)T(t)\}\\
&\quad+\frac{\alpha}{\Gamma(\alpha)\times ABC(\alpha)}\int_0^t\{k_2I(\tau)-\alpha_1T(\tau)-(d+\delta_2+\alpha_2)T(\tau)\}(t-\tau)^{\alpha-1}d\tau,\\
R(t)-R(0)&=\frac{(1-\alpha)}{ABC(\alpha)}\{\mu_1S(t)-dR(t)\}+\frac{\alpha}{\Gamma(\alpha)\times ABC(\alpha)}\int_0^t\{\mu_1S(\tau)-dR(\tau)\}(t-\tau)^{\alpha-1}d\tau.
\end{aligned}
$$

At a given point $t_{n+1},\ n=0,1,2,3,\ldots,$ the above equation is reformulated as

$$
\begin{aligned}
&S(t_{n+1})-S(0)\\
&=\frac{(1-\alpha)}{ABC(\alpha)}\{\Lambda-\beta I(t_n)S(t_n)-\mu_1S(t_n)-dS(t_n)\}\\
&+\frac{\alpha}{\Gamma(\alpha)\times ABC(\alpha)}\int_0^{t_{n+1}}\{\Lambda-\beta I(\tau)S(\tau)-\mu_1S(\tau)-dS(\tau)\}(t_{n+1}-\tau)^{\alpha-1}d\tau,
\end{aligned}
$$

$$I(t_{n+1})-I(0)$$
$$=\frac{(1-\alpha)}{ABC(\alpha)}\{\beta I(t_n)S(t_n)+\alpha_1 T(t_n)-dI(t_n)-k_1 I(t_n)-k_2 I(t_n)\}$$
$$+\frac{\alpha}{\Gamma(\alpha)\times ABC(\alpha)}\int_0^{t_{n+1}}\{\beta I(\tau)S(\tau)+\alpha_1 T(\tau)-dI(\tau)-k_1 I(\tau)$$
$$-k_2 I(\tau)\}(t_{n+1}-\tau)^{\alpha-1}d\tau,$$

$$A(t_{n+1})-A(0)$$
$$=\frac{(1-\alpha)}{ABC(\alpha)}\{k_1 I(t_n)-(\delta_1+d)A(t_n)+\alpha_2 T(t_n)\}$$
$$+\frac{\alpha}{\Gamma(\alpha)\times ABC(\alpha)}\int_0^{t_{n+1}}\{k_1 I(\tau)-(\delta_1+d)A(\tau)+\alpha_2 T(\tau)\}(t_{n+1}-\tau)^{\alpha-1}d\tau,$$

$$T(t_{n+1})-T(0)$$
$$=\frac{(1-\alpha)}{ABC(\alpha)}\{k_2 I(t_n)-\alpha_1 T(t_n)-(d+\delta_2+\alpha_2)T(t_n)\}$$
$$+\frac{\alpha}{\Gamma(\alpha)\times ABC(\alpha)}\int_0^{t_{n+1}}\{k_2 I(\tau)-\alpha_1 T(\tau)-(d+\delta_2+\alpha_2)T(\tau)\}(t_{n+1}-\tau)^{\alpha-1}d\tau,$$

$$R(t_{n+1})-R(0)$$
$$=\frac{(1-\alpha)}{ABC(\alpha)}\{\mu_1 S(t_n)-dR(t_n)\}$$
$$+\frac{\alpha}{\Gamma(\alpha)\times ABC(\alpha)}\int_0^{t_{n+1}}\{\mu_1 S(\tau)-dR(\tau)\}(t_{n+1}-\tau)^{\alpha-1}d\tau.$$

Also, we have

$$S(t_{n+1})-S(0)$$
$$=\frac{(1-\alpha)}{ABC(\alpha)}\{\Lambda-\beta I(t_n)S(t_n)-\mu_1 S(t_n)-dS(t_n)\}$$
$$+\frac{\alpha}{\Gamma(\alpha)\times ABC(\alpha)}\sum_{j=0}^{n}\int_{t_j}^{t_{j+1}}\{\Lambda-\beta I(\tau)S(\tau)-\mu_1 S(\tau)-dS(\tau)\}(t_{n+1}-\tau)^{\alpha-1}d\tau,$$

$$I(t_{n+1})-I(0)$$
$$=\frac{(1-\alpha)}{ABC(\alpha)}\{\beta I(t_n)S(t_n)+\alpha_1 T(t_n)-dI(t_n)-k_1 I(t_n)-k_2 I(t_n)\}$$
$$+\frac{\alpha}{\Gamma(\alpha)\times ABC(\alpha)}\sum_{j=0}^{n}\int_{t_j}^{t_{j+1}}\{\beta I(\tau)S(\tau)+\alpha_1 T(\tau)-dI(\tau)-k_1 I(\tau)$$
$$-k_2 I(\tau)\}(t_{n+1}-\tau)^{\alpha-1}d\tau,$$

$$A(t_{n+1})-A(0)$$
$$=\frac{(1-\alpha)}{ABC(\alpha)}\{k_1 I(t_n)-(\delta_1+d)A(t_n)+\alpha_2 T(t_n)\}$$
$$+\frac{\alpha}{\Gamma(\alpha)\times ABC(\alpha)}\sum_{j=0}^{n}\int_{t_j}^{t_{j+1}}\{k_1 I(\tau)-(\delta_1+d)A(\tau)+\alpha_2 T(\tau)\}(t_{n+1}-\tau)^{\alpha-1}d\tau, \quad (12)$$

$$T(t_{n+1}) - T(0) = \frac{(1-\alpha)}{ABC(\alpha)}\{k_2 I(t_n) - \alpha_1 T(t_n) - (d+\delta_2+\alpha_2)T(t_n)\} + \frac{\alpha}{\Gamma(\alpha)\times ABC(\alpha)}\sum_{j=0}^{n}\int_{t_j}^{t_{j+1}}\{k_2 I(\tau) - \alpha_1 T(\tau) - (d+\delta_2+\alpha_2)T(\tau)\}(t_{n+1}-\tau)^{\alpha-1}d\tau,$$

$$R(t_{n+1}) - R(0) = \frac{(1-\alpha)}{ABC(\alpha)}\{\mu_1 S(t_n) - dR(t_n)\} + \frac{\alpha}{\Gamma(\alpha)\times ABC(\alpha)}\sum_{j=0}^{n}\int_{t_j}^{t_{j+1}}\{\mu_1 S(\tau) - dR(\tau)\}(t_{n+1}-\tau)^{\alpha-1}d\tau.$$

By using above equation, we have generalized form as:

$$S_{n+1} = S_0 + \frac{(1-\alpha)}{ABC(\alpha)}\{\Lambda - \beta I(t_n)S(t_n) - \mu_1 S(t_n) - dS(t_n)\} + \frac{\alpha}{\Gamma(\alpha)\times ABC(\alpha)}\sum_{j=0}^{n}\left(\frac{\{\Lambda - \beta I_j S_j - \mu_1 S_j - dS_j\}}{h} \times \int_{t_j}^{t_{j+1}}(\tau - t_{j-1})(t_{n+1}-\tau)^{\alpha-1}d\tau - \frac{\{\Lambda - \beta I_{j-1}S_{j-1} - \mu_1 S_{j-1} - dS_{j-1}\}}{h}\times\int_{t_j}^{t_{j+1}}(\tau - t_j)(t_{n+1}-\tau)^{\alpha-1}d\tau\right),$$

$$I_{n+1} = I_0 + \frac{(1-\alpha)}{ABC(\alpha)}\{\beta I(t_n)S(t_n) + \alpha_1 T(t_n) - dI(t_n) - k_1 I(t_n) - k_2 I(t_n)\} + \frac{\alpha}{\Gamma(\alpha)\times ABC(\alpha)}\sum_{j=0}^{n}\left(\frac{\{\beta I_j S_j + \alpha_1 T_j - dI_j - k_1 I_j - k_2 I_j\}}{h} \times \int_{t_j}^{t_{j+1}}(\tau - t_{j-1})(t_{n+1}-\tau)^{\alpha-1}d\tau - \frac{\{\beta I_{j-1}S_{j-1} + \alpha_1 T_{j-1} - dI_{j-1} - k_1 I_{j-1} - k_2 I_{j-1}\}}{h} \times \int_{t_j}^{t_{j+1}}(\tau - t_j)(t_{n+1}-\tau)^{\alpha-1}d\tau\right),$$

$$A_{n+1} = A_0 + \frac{(1-\alpha)}{ABC(\alpha)}\{k_1 I(t_n) - (\delta_1 + d)A(t_n) + \alpha_2 T(t_n)\} + \frac{\alpha}{\Gamma(\alpha)\times ABC(\alpha)}\sum_{j=0}^{n}\left(\frac{\{k_1 I_j - (\delta_1+d)A_j + \alpha_2 T_j\}}{h}\times\int_{t_j}^{t_{j+1}}(\tau - t_{j-1})(t_{n+1}-\tau)^{\alpha-1}d\tau - \frac{\{k_1 I_{j-1} - (\delta_1+d)A_{j-1} + \alpha_2 T_{j-1}\}}{h}\times\int_{t_j}^{t_{j+1}}(\tau - t_j)(t_{n+1}-\tau)^{\alpha-1}d\tau\right), \quad (13)$$

$$T_{n+1} = T_0 + \frac{(1-\alpha)}{ABC(\alpha)}\{k_2 I(t_n) - \alpha_1 T(t_n) - (d+\delta_2+\alpha_2)T(t_n)\}$$
$$+ \frac{\alpha}{\Gamma(\alpha)\times ABC(\alpha)}\sum_{j=0}^{n}\Bigg(\frac{\{k_2 I_j - \alpha_1 T_j - (d+\delta_2+\alpha_2)T_j\}}{h}\times\int_{t_j}^{t_{j+1}}(\tau - t_{j-1})(t_{n+1}-\tau)^{\alpha-1}d\tau$$
$$- \frac{\{k_2 I_{j-1} - \alpha_1 T_{j-1} - (d+\delta_2+\alpha_2)T_{j-1}\}}{h}\times\int_{t_j}^{t_{j+1}}(\tau - t_j)(t_{n+1}-\tau)^{\alpha-1}d\tau\Bigg),$$

$$R_{n+1} = R_0 + \frac{(1-\alpha)}{ABC(\alpha)}\{\mu_1 S(t_n) - dR(t_n)\}$$
$$+ \frac{\alpha}{\Gamma(\alpha)\times ABC(\alpha)}\sum_{j=0}^{n}\Bigg(\frac{\{\mu_1 S_j - dR_j\}}{h}\times\int_{t_j}^{t_{j+1}}(\tau - t_{j-1})(t_{n+1}-\tau)^{\alpha-1}d\tau$$
$$- \frac{\{\mu_1 S_{j-1} - dR_{j-1}\}}{h}\times\int_{t_j}^{t_{j+1}}(\tau - t_j)(t_{n+1}-\tau)^{\alpha-1}d\tau\Bigg).$$

Thus, we get

$$S_{n+1} = S_0 + \frac{(1-\alpha)}{ABC(\alpha)}\{\Lambda - \beta I(t_n)S(t_n) - \mu_1 S(t_n) - dS(t_n)\}$$
$$+ \frac{\alpha}{ABC(\alpha)}\sum_{j=0}^{n}\Bigg(\frac{h^\alpha\{\Lambda - \beta I_j S_j - \mu_1 S_j - dS_j\}}{\Gamma(\alpha+2)}$$
$$\times\{(n+1-j)^\alpha(n-j+2+\alpha) - (n-j)^\alpha(n-j+2+2\alpha)\}$$
$$- \frac{h^\alpha\{\Lambda - \beta I_{j-1}S_{j-1} - \mu_1 S_{j-1} - dS_{j-1}\}}{\Gamma(\alpha+2)}\times\{(n+1-j)^{\alpha+1} - (n-j)^\alpha(n-j+1+\alpha)\}\Bigg),$$

$$I_{n+1} = I_0 + \frac{(1-\alpha)}{ABC(\alpha)}\{\beta I(t_n)S(t_n) + \alpha_1 T(t_n) - dI(t_n) - k_1 I(t_n) - k_2 I(t_n)\}$$
$$+ \frac{\alpha}{ABC(\alpha)}\sum_{j=0}^{n}\Bigg(\frac{h^\alpha\{\beta I_j S_j + \alpha_1 T_j - dI_j - k_1 I_j - k_2 I_j\}}{\Gamma(\alpha+2)}$$
$$\times\{(n+1-j)^\alpha(n-j+2+\alpha) - (n-j)^\alpha(n-j+2+2\alpha)\}$$
$$- \frac{h^\alpha\{\beta I_{j-1}S_{j-1} + \alpha_1 T_{j-1} - dI_{j-1} - k_1 I_{j-1} - k_2 I_{j-1}\}}{\Gamma(\alpha+2)}$$
$$\times\{(n+1-j)^{\alpha+1} - (n-j)^\alpha(n-j+1+\alpha)\}\Bigg),$$

$$A_{n+1} = A_0 + \frac{(1-\alpha)}{ABC(\alpha)}\{k_1 I(t_n) - (\delta_1+d)A(t_n) + \alpha_2 T(t_n)\}$$
$$+ \frac{\alpha}{ABC(\alpha)}\sum_{j=0}^{n}\left(\begin{array}{c}\frac{h^\alpha\{k_1 I_j - (\delta_1+d)A_j + \alpha_2 T_j\}}{\Gamma(\alpha+2)}\times\left\{\begin{array}{c}(n+1-j)^\alpha(n-j+2+\alpha)\\ -(n-j)^\alpha(n-j+2+2\alpha)\end{array}\right\}\\ -\frac{h^\alpha\{k_1 I_{j-1} - (\delta_1+d)A_{j-1} + \alpha_2 T_{j-1}\}}{\Gamma(\alpha+2)}\\ \times\{(n+1-j)^{\alpha+1} - (n-j)^\alpha(n-j+1+\alpha)\}\end{array}\right), \quad (14)$$

$$
\begin{aligned}
T_{n+1} = T_0 + \frac{(1-\alpha)}{ABC(\alpha)}\{k_2 I(t_n) - \alpha_1 T(t_n) - (d+\delta_2+\alpha_2)T(t_n)\} \\
+ \frac{\alpha}{ABC(\alpha)}\sum_{j=0}^{n}\Bigg(\frac{h^{\alpha}\{k_2 I_j - \alpha_1 T_j - (d+\delta_2+\alpha_2)T_j\}}{\Gamma(\alpha+2)} \\
\times \{(n+1-j)^{\alpha}(n-j+2+\alpha) - (n-j)^{\alpha}(n-j+2+2\alpha)\} \\
- \frac{h^{\alpha}\{k_2 I_{j-1} - \alpha_1 T_{j-1} - (d+\delta_2+\alpha_2)T_{j-1}\}}{\Gamma(\alpha+2)} \\
\times \{(n+1-j)^{\alpha+1} - (n-j)^{\alpha}(n-j+1+\alpha)\}\Bigg),
\end{aligned}
$$

$$
\begin{aligned}
R_{n+1} = R_0 + \frac{(1-\alpha)}{ABC(\alpha)}\{\mu_1 S(t_n) - dR(t_n)\} \\
+ \frac{\alpha}{ABC(\alpha)}\sum_{j=0}^{n}\Bigg(\frac{h^{\alpha}\{\mu_1 S_j - dR_j\}}{\Gamma(\alpha+2)} \\
\times \{(n+1-j)^{\alpha}(n-j+2+\alpha) - (n-j)^{\alpha}(n-j+2+2\alpha)\} \\
- \frac{h^{\alpha}\{\mu_1 S_{j-1} - dR_{j-1}\}}{\Gamma(\alpha+2)} \times \{(n+1-j)^{\alpha+1} - (n-j)^{\alpha}(n-j+1+\alpha)\}\Bigg).
\end{aligned}
$$

6. Results and discussion

The mathematical analysis of epidemic HIV/AIDS model with non-linear occurrence is studied to notice the sound effects of the fractional parameters. Following initial conditions and parameter values [26] are used for simulations:

$$\Lambda = 0.55, \quad \beta = 0.03, \quad d = 0.0196, \quad k_1 = 0.15, \quad k_2 = 0.35, \quad \alpha_1 = 0.08,$$

$$\alpha_2 = 0.03, \quad \delta_1 = 0.0909, \quad \delta_2 = 0.0667, \quad \mu_1 = 0.03, \quad S(0) = 35,$$

$$I(0) = 24, \quad A(0) = 15, \quad T(0) = 8, \quad R(0) = 0.$$

Numerical solutions are obtained for different values by using ABC derivative according to steady state. The graphs of the approximate solutions against different fractional order φ are provided in Figures 1–5. In Figures 1–5, we observe that behavior of S(t), A(t) and R(t) start increasing by decreasing the fractional values while behavior of infected I(t) and T(t) start decreasing by decreasing fractional values which approaches to our steady state. It is easily observed that susceptible individual rise after certain time while both HIV infected and AIDS infected individual start decreasing after some rise due to treatment. Also in Figure 5, the recovered individual starts increasing rapidly due to treatment for different fractional values. Observation has been made at different fractional values according to given parameters to check the effect of fractional order model. Solutions for all compartments come to our desired accuracy and more reliable by decreasing fractional values. The simulations clearly show that we can obtain better approximation to control the disease by using fractional derivative as compared to classical derivative.

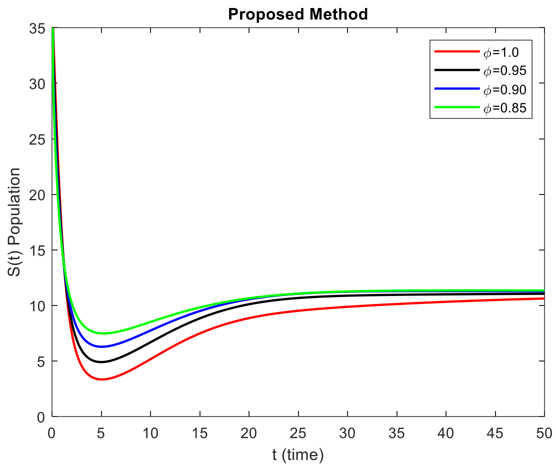

Figure 1. Numerical solution of S(t) population with fractional order.

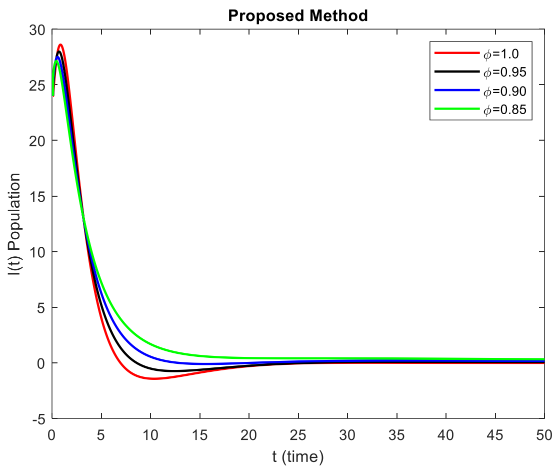

Figure 2. Numerical solution of I(t) population with fractional order.

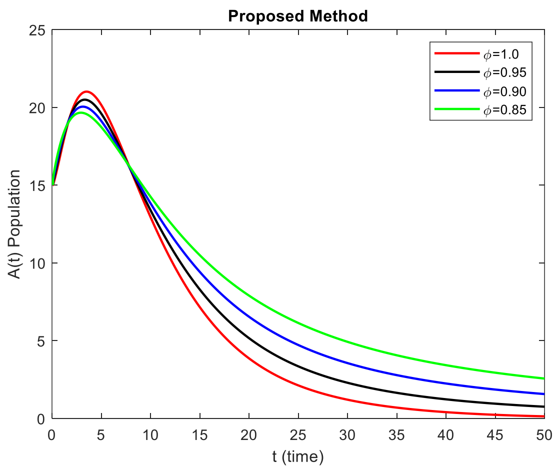

Figure 3. Numerical solution of A(t) population with fractional order.

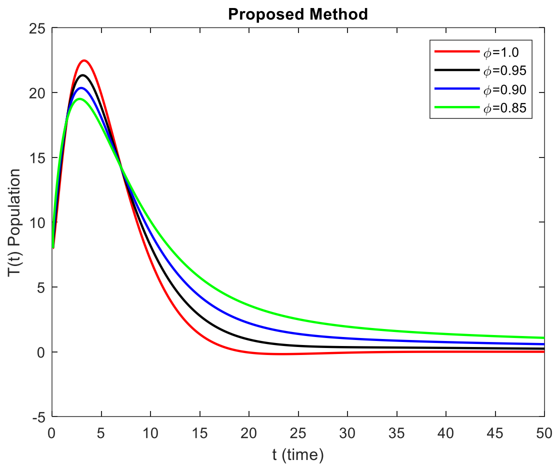

Figure 4. Numerical solution of T(t) population with fractional order.

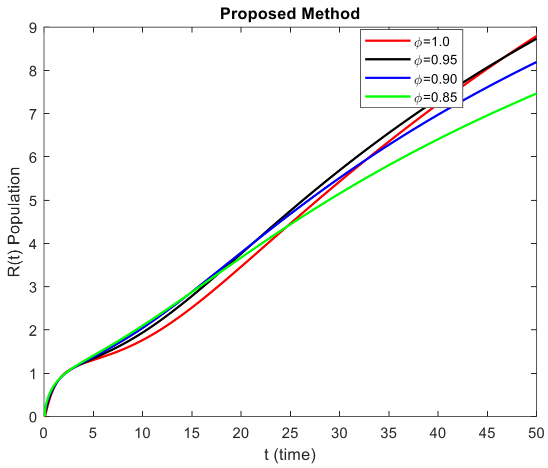

Figure 5. Numerical solution of R(t) population with fractional order.

7. Conclusions

In this article, a new scheme with Mittag-Leffler law has been studied for HIV/AIDS with an antiretroviral treatment compartment. The existence and uniqueness of the solutions of the model has been proved by using iterative method and fixed-point theory. Advanced numerical approximation is used with non-singular and non-local kernel to solve for this kind of fractional order system. The advanced developed numerical technique converges to exact solution, also provides reliable and efficient results with large step size h which is mixture of the two-step Lagrange polynomial and the fundamental theorem of fractional calculus. We obtained very effective results for the proposed model. Simulation has been made to check the actual behavior of the HIV/AIDS and effect of treatment as we see rapid increase in recovered individual due to treatment. These results will be very helpful for the future study of HIV/AIDS and for control strategies with fractional operators.

Conflicts of interest

The authors declare that they have no conflicts of interest to report regarding the present study.

References

1. C. S. Chou, A. Friedman, Introduction, In: *Introduction to mathematical biology*, Springer, 2016. https://doi.org/10.1007/978-3-319-29638-8_1
2. F. M. Barre-Sinoussi, J. C. Chermann, R. Rey, M. T. Nugeyre, S. Chamaret, J. Gruest, et al., Isolation of a T-lymphotropic retrovirus from a patient at risk for acquired immune deficiency syndrome (AIDS), *Science*, **220** (1983), 868–871. https://doi.org/10.1126/science.6189183
3. U. L. Abbas, R. M. Anderson, J. W. Mellors, Potential impact of antiretroviral chemoprophylaxis on HIV-1 transmission in resource-limited settings, *PLoS ONE*, **2** (2007), e875. https://doi.org/10.1371/journal.pone.0000875
4. Z. Sadegh, N. A. Miehran, A nonstandard finite difference scheme for solving fractional-order model of HIV-1 infection of CD_4^+ T-cells, *Iran. J. Math. Chem.*, **6** (2015), 169–184.
5. J. S. Cristiana, F. M. Delfim, Global stability for a HIV/AIDS modlers, *Commun. Fac. Sci. Univ. Ank. Ser.*, **67** (2018), 93–101.
6. R. E. Mickens, *Advances in the applications of nonstandard finite difference schemes*, Singapore: Wiley-Interscience, 2005. https://doi.org/10.1142/5884
7. R. E. Mickens, Calculation of denominator functions for nonstandard finite difference schemes for differential equations satisfying a positivity condition, *Numer. Meth. Part. Differ. Eq.*, **23** (2007), 672–691. https://doi.org/10.1002/num.20198
8. Y. Ding, H. Ye, A fractional-order differential equation model of HIV infection of CD_4^+ T-cells, *Math. Comput. Model*, **50** (2009), 386–392. https://doi.org/10.1016/j.mcm.2009.04.019
9. A. Gökdoğan, A. Yildirim, M. Merdan, Solving a fractional order model of HIV infection of CD_4^+ T cells, *Math. Comput. Model*, **54** (2011), 2132–2138. https://doi.org/10.1016/j.mcm.2011.05.022
10. M. M. Khader, The modeling dynamics of HIV and CD_4^+ T-cells during primary infection in fractional order: Numerical simulation, *Mediterr. J. Math.*, **15** (2018), 139. https://doi.org/10.1007/s00009-018-1178-9
11. A. Agila, D. Baleanu, R. Eid, B. Irfanoglu, Applications of the extended fractional Euler-Lagrange equations model to freely oscillating dynamical systems, *Rom. J. Phys.*, **61** (2016), 350–359.
12. P. K. Gupta, Local and global stability of fractional order HIV/AIDS dynamics model, In: D. Ghosh, D. Giri, R. Mohapatra, E. Savas, K. Sakurai, L. Singh, *Mathematics and computing*, International Conference on Mathematics and Computing, Communications in Computer and Information Science, **834** (2018) 141–148. https://doi.org/10.1007/978-981-13-0023-3_14
13. N. Özalp, E. Demirci, A fractional order SEIR model with vertical transmission, *Math. Comput. Model*, **54** (2011), 1–6. https://doi.org/10.1016/j.mcm.2010.12.051
14. M. Javidi, N. Nyamoradi, Numerical behavior of a fractional order HIV/AIDS epidemic model, *World J. Model Simul.*, **9** (2013), 139–149.
15. M. Caputo, M. Fabrizio, A new definition of fractional derivative without singular kernel, *Prog. Fract. Differ. Appl.*, **1** (2015), 73–85. http://dx.doi.org/10.12785/pfda/010201
16. J. Losada, J. J. Nieto, Properties of a new fractional derivative without singular kernel, *Prog. Fract. Differ. Appl.*, **1** (2015), 87–92. http://dx.doi.org/10.12785/pfda/010202
17. A. Atangana, B. S. T. Alkahtani, Analysis of the Keller-Segel model with a fractional derivative without singular kernel, *Entropy*, **17** (2015), 4439–4453. https://doi.org/10.3390/e17064439
18. A. Atangana, D. Baleanu, New fractional derivatives with nonlocal and non-singular kernel: Theory and application to heat transfer model, *arXiv*, 2016. https://doi.org/10.48550/arXiv.1602.03408

19. M. Farman, M. U. Saleem, M. F Tabassum, A. Ahmad, M. O. Ahmad, A linear control of composite model for glucose insulin glucagon pump, *Ain Shamas Eng. J.*, **10** (2019), 867–872. https://doi.org/10.1016/j.asej.2019.04.001
20. M. Farman, M. U. Saleem, A. Ahmad, S. Imtiaz, M. F. Tabassm, S. Akram, et al., A control of glucose level in insulin therapies for the development of artificial pancreas by Atangana Baleanu derivative, *Alex. Eng. J.*, **59** (2020), 2639–2648. https://doi.org/10.1016/j.aej.2020.04.027
21. M. Farman, A. Akgül, D. Baleanu, S. Imtiaz, A. Ahmad, Analysis of fractional order chaotic financial model with minimum interest rate impact, *Fractal Fract.*, **4** (2020), 43. https://doi.org/10.3390/fractalfract4030043
22. M. U. Saleem, M. Farman, A. Ahmad, H. Ehsan, M. O. Ahmad, A Caputo Fabrizio fractional order model for control of glucose in insulin therapies for diabetes, *Ain Shamas Eng. J.*, **11** (2020), 1309–1316. https://doi.org/10.1016/j.asej.2020.03.006
23. A. Hussain, S. Yaqoob, On a nonlinear fractional-order model of novel coronavirus (NCOV-2019) under AB-fractional derivative, *Authorea*, 2020. https://doi.org/10.22541/au.158739577.76215854
24. M. A. Khan, A. Atangana, Modeling the dynamics of novel coronavirus (2019-nCov) with fractional derivative, *Alex. Eng J.*, 2020. https://doi.org/10.1016/j.aej.2020.02.033
25. M. Amin, M. Farman, A. Akgül, R. T. Alqahtani, Effect of vaccination to control COVID-19 with Fractal-Fractional operator, *Alex. Eng. J.*, **61** (2022), 3551–3557. https://doi.org/10.1016/j.aej.2021.09.006
26. H. F. Huo, R. Chen, X. Y. Wang, Modelling and stability of HIV/AIDS epidemic model with treatment, *Appl. Math. Model*, **40** (2016), 6550–6559. https://doi.org/10.1016/j.apm.2016.01.054
27. E. J. Moore, S. Sirisubtawee, S. Koonprasert, A Caputo-Fabrizio fractional differential equation model for HIV/AIDS with treatment compartment, *Adv. Differ. Equ.*, **2019** (2019), 200. https://doi.org/10.1186/s13662-019-2138-9
28. A. Atangana, E. Bonyah, A. A. Elsadany, A fractional order optimal 4D chaotic financial model with Mittag-Leffler law, *Chin. J. Phys.*, **65** (2020), 38–53, https://doi.org/10.1016/j.cjph.2020.02.003
29. M. Toufik, A. Atangana, New numerical approximation of fractional derivative with non-local and non-singular kernel: Application to chaotic models, *Eur. Phys. J. Plus*, **132** (2017), 444. https://doi.org/10.1140/epjp/i2017-11717-0
30. M. Al-Smadi, O. A. Arqub, Computational algorithm for solving fredholm time-fractional partial integrodifferential equations of dirichlet functions type with error estimates, *Appl. Math. Comput.*, **342** (2019), 280–294. https://doi.org/10.1016/j.amc.2018.09.020
31. M. Al-Smadi, O. A. Arqub, S. Momani, Numerical computations of coupled fractional resonant Schrödinger equations arising in quantum mechanics under conformable fractional derivative sense, *Phys. Scr.*, **95** (2020), 075218.
32. M. Al-Smadi, O. A. Arqub, S. Hadid, An attractive analytical technique for coupled system of fractional partial differential equations in shallow water waves with conformable derivative, *Commun. Theor. Phys.*, **72** (2020), 085001.
33. M. Al-Smadi, O. A. Arqub, S. Hadid, Approximate solutions of nonlinear fractional Kundu-Eckhaus and coupled fractional massive Thirring equations emerging in quantum field theory using conformable residual power series method, *Phys. Scr.*, **95** (2020) 105205.
34. S. Momani, A. Freihat, M. Al-Smadi, Analytical study of fractional-order multiple chaotic Fitzhugh-Nagumo neurons model using multistep generalized differential transform method, *Abstr. Appl. Anal.*, **2014** (2014), 276279. https://doi.org/10.1155/2014/276279

35. H. Habenom, M. Aychluh, D. L. Suthar, Q. Al-Mdallal, S. D. Purohit, Modeling and analysis on the transmission of COVID -19 Pandemic in Ethiopia, *Alex. Eng. J.*, **61** (2022), 5323–5342. https://doi.org/10.1016/j.aej.2021.10.054
36. M. Asif, Z. A. Khan, N. Haider, Q. Al-Mdallal, Numerical simulation for solution of SEIR models by meshless and finite difference methods, *Chaos Soliton. Fract.*, **141** (2020), 110340. https://doi.org/10.1016/j.chaos.2020.110340
37. K. Shah, M. Arfan, A. Ullah, Q. Al-Mdallal, K. J. Ansari, T. Abdeljawad, Computational study on the dynamics of fractional order differential equations with applications, *Chaos Soliton. Fract.*, **157** (2022), 111955. https://doi.org/10.1016/j.chaos.2022.111955

AIMS Press