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Research article

Nonlinear analysis of a nonlinear modified KdV equation under Atangana Baleanu Caputo derivative

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Abstract: The focus of the current manuscript is to provide a theoretical and computational analysis of the new nonlinear time-fractional (2+1)-dimensional modified KdV equation involving the Atangana-Baleanu Caputo ($\mathcal{ABC}$) derivative. A systematic and convergent technique known as the Laplace Adomian decomposition method (LADM) is applied to extract a semi-analytical solution for the considered equation. The notion of fixed point theory is used for the derivation of the results related to the existence of at least one and unique solution of the mKdV equation involving under $\mathcal{ABC}$-derivative. The theorems of fixed point theory are also used to derive results regarding to the convergence and Picard's X-stability of the proposed computational method. A proper investigation is conducted through graphical representation of the achieved solution to determine that the $\mathcal{ABC}$ operator produces better dynamics of the obtained analytic soliton solution. Finally, 2D and 3D graphs are used to compare the exact solution and approximate solution. Also, a comparison between the exact solution, solution under Caputo-Fabrizio, and solution under the $\mathcal{ABC}$ operator of the proposed equation is provided through graphs, which reflect that $\mathcal{ABC}$-operator produces better dynamics of the proposed equation than the Caputo-Fabrizio one.

Keywords: Atangana-Baleanu fractional operator; fixed point theory; Laplace Adomian decomposition
Mathematics Subject Classification: 35R11

1. Introduction

Korteweg and de-Vries developed the classical KdV equation in 1895 as a nonlinear PDE to investigate the waves that occurs on the surfaces of shallow water. Many studies have been conducted on this exactly solvable model. Many scholars have proposed novel applications of the classical KdV equation, such as acoustic waves that produces in a plasma in ion form, and acoustic waves which are produces on a crystal lattice. In [1], the classical KdV equation is as follows

$$\mathcal{U}_t + \beta_1 \mathcal{U}_{xxx} + \beta_2 \mathcal{U}\mathcal{U}_x = 0. \tag{1.1}$$

Different variations of Eq (1.1) have been published in the literature, including [2, 3]. In literature [2], various efficient methods have been used to get solitons solutions of different kinds of KdV equations. Wang [3] used a quadratic function Ansatz to obtain lump solutions for the (2+1)-dimensional KdV equation. In [4–6], several systematic techniques were utiltized to investigate the different types of KdV equations. Wang and Kara introduced a new 2D-mKdV in 2019 [7]. The new (2+1)-dimensional mKdV equation is given by

$$f_t = 6f^2 f_x - 6f^2 f_y + f_{xxx} - f_{yyy} - 3f_{xxy} + 3f_{xyy}. \tag{1.2}$$

After 1695, the non-integer order or fractional-order derivative (FOD) was described as a simple academic generalization of the classical derivative. A FOD is an operator that extends the order of differentiation from Natural numbers (N) to a set of real numbers (R) or even to a set of complex numbers (C). Fractional calculus has emerged as one of the most effective methods for describing long-memory processes over the last decade. Engineers and physicists, as well as pure mathematicians, are interested in such models. The models that are represented by differential equations with fractional-order derivatives are the most interesting [8–10]. Their evolutions are much more complicated than in the classical integer-order case, and deciphering the underlying principle is a difficult task. There are many fractional operators regarding to the kernels involved in the integration. The most popular fractional operator is Caputo-Liouville which is based on power-law kernel, but this kernel has issue about the singularity of the kernel. To tackle the limitation of derivative operators with power-law kernels, new types of nonlocal FOD have been introduced in recent literature. For example, in literature [11], Caputo and Fabrizio (CF) introduced a FOD that is focused on the exponential kernel. However, the CF derivative, on the other hand, has some issues with the kernel's locality. In 2016, Atangana and Baleanu constructed an updated version of FOD that is based on the Mittag-Leffler function [12]. This derivative solves the issues of locality and singularity. The FOD of Atangana-Baleanu in Caputo sense ($\mathcal{ABC}$) accurately describes the memory. The $\mathcal{ABC}$ operator's most significant applications are available in [13–17].

Certain important techniques have been used to solve fractional order differential equations (FODEs). Some of these includes the homotopy perturbation method (HPM), Laplace Adomian decomposition method (LADM) fractional operational matrix method (FOMM), homotopy analysis method (HAM) and many more [18–22]. In contrast to these techniques, the Laplace-Adomian decomposition method (LADM) is an important tool for solving non linear FODEs. The ADM and the Laplace transformation are two essential methods that are combined in LADM. Furthermore, unlike a Runge-Kutta process, LADM does not require a predefine size declaration. Every numerical or analytical approach has its own set of benefits and drawbacks. For instance, discretization of data is used in collocation techniques that require extra memory and a longer operation. Since the Laplace-Adomian approach has less parameters than all other methods, it is a useful tool that does not necessitate discretion or linearization [23]. With the aid of LADM, the smoke model was successfully solved in [24]. LADM was utilized by the authors to solve third order dispersive PDE defined by FOD in [22].

Inspired by above literature, in this paper, we study Eq (1.2) under $\mathcal{ABC}$-operator. We use LADM to solve the proposed equation. Consider Eq (1.2) under $\mathcal{ABC}$-operator as

$$^{\mathcal{ABC}}D_t^{\Psi} f_t = 6f^2 f_x - 6f^2 f_y + f_{xxx} - f_{yyy} - 3f_{xxy} + 3f_{xyy}. \tag{1.3}$$

2. Preliminaries

Definition 2.1. *[12] Let $0 < \Psi \leq 1$ and $f(t) \in \mathcal{H}^1$. Then $\mathcal{ABC}$ FOD of order Ψ is expressed as*

$$^{\mathcal{ABC}}D_t^{\Psi} f(t) = \frac{c(\Psi)}{(1-\Psi)} \int_0^t f'(\eta) E_{\Psi}\left(-\frac{(t-\eta)^{\Psi}\Psi}{(1-\Psi)}\right) d\eta,$$

where $c(\Psi)$ is the normalization function such that $c(0) = c(1) = 1$. The symbol E_{Ψ} denotes the Mittag-Leffler kernel which is defined as:

$$\mathbb{E}_{\Psi}(t) = \sum_{k=0}^{\infty} \frac{t^k}{\Gamma(\Psi k + 1)}.$$

Definition 2.2. *[12] Let $0 < \Psi \leq 1$ and $f(t) \in \mathcal{H}^1(0,T)$. Then $\mathcal{AB}$ fractional integral of order Ψ is defined as*

$$^{\mathcal{AB}}I_t^{\Psi} f(t) = \frac{(1-\Psi)}{c(\Psi)} f(t) + \frac{(\Psi)}{c(\Psi)\Gamma(\Psi)} \int_0^t (t-\eta)^{\Psi-1} f'(\eta)\, d\eta.$$

Definition 2.3. *[12] The formula for the Laplace transform of $\mathcal{ABC}$ FOD of $f(t)$ is defined by*

$$\mathcal{L}\left[^{\mathcal{ABC}}\mathcal{D}_0^{\Psi} f(t)\right] = \frac{c(\Psi)}{s^{\Psi}(1-\Psi)+\Psi}\left[s^{\Psi}\mathcal{L}[f(t)] - s^{\Psi-1}u(0)\right].$$

Theorem 2.4. *[25] Let $\mathbb{H}$ be a Banach space and $\mathbf{X} : \mathbb{H} \to \mathbb{H}$ be a mapping. Then $\mathbf{X}$ is said to be Picard's $\mathbf{X}$-stable, if $\forall\ \xi, m \in \mathbb{H}$,*

$$\left\|\mathbf{X}_{\xi} - \mathbf{X}_m\right\| \leq a\left\|\xi - \mathbf{X}_{\xi}\right\| + b\left\|\xi - m\right\|,$$

where $a \geq 0$, and $b \in [0,1]$. Further, $\mathbf{X}$ has a fixed point.

3. Lipschitz condition holds for $\mathcal{ABC}$ derivative

Theorem 3.1. *[12] The $\mathcal{ABC}$ derivative holds the following Lipschitz type condition for $0 < F < \infty$.*

$$\left\|^{\mathcal{ABC}}D_t^{\Psi} f(t) - {}^{\mathcal{ABC}}D_t^{\Psi} g(t)\right\| \leq F\left\|f(t) - g(t)\right\|.$$

Proof. By using definition of $\mathcal{ABC}$, we have

$$\begin{aligned}
&\left\| {}^{\mathcal{ABC}}D_t^{\Psi} f(t) - {}^{\mathcal{ABC}}D_t^{\Psi} g(t) \right\| \\
=&\left\| \frac{c(\Psi)}{(1-\Psi)} \int_0^t f'(\eta) E_{\Psi}\left(-\frac{(t-\eta)^{\Psi}\Psi}{(1-\Psi)}\right) d\eta - \frac{c(\Psi)}{(1-\Psi)} \int_0^t g'(\eta) E_{\Psi}\left(-\frac{(t-\eta)^{\Psi}\Psi}{(1-\Psi)}\right) d\eta \right\| \\
=&\left\| \frac{c(\Psi)}{(1-\Psi)} \left[\int_0^t f'(\eta) E_{\Psi}\left(-\frac{(t-\eta)^{\Psi}\Psi}{(1-\Psi)}\right) d\eta - \int_0^t g'(\eta) E_{\Psi}\left(-\frac{(t-\eta)^{\Psi}\Psi}{(1-\Psi)}\right) d\eta \right] \right\|.
\end{aligned}$$

Using the Lipschitz condition for the first order derivative, we can find a small positive constant ρ_1 such that

$$\begin{aligned}
&\left\| {}^{\mathcal{ABC}}D_t^{\Psi} f(t) - {}^{\mathcal{ABC}}D_t^{\Psi} g(t) \right\| \\
=&\left\| \frac{c(\Psi)\rho_1}{(1-\Psi)} E_{\Psi}\left(-\frac{\Psi t^{\Psi}}{(1-\Psi)}\right) \left[\int_0^t f'(\eta) d\eta - \int_0^t g'(\eta) d\eta \right] \right\| \\
\leq&\frac{c(\Psi)\rho_1}{(1-\Psi)} E_{\Psi}\left(-\frac{\Psi t^{\Psi}}{(1-\Psi)}\right) \left\| \int_0^t f'(\eta) d\eta - \int_0^t g'(\eta) d\eta \right\| \\
\leq&F \left\| \int_0^t f'(\eta) d\eta - \int_0^t g'(\eta) d\eta \right\| \\
\leq&F \left\| f(t) - g(t) \right\|,
\end{aligned}$$

where $F = \frac{c(\Psi)\rho_1}{(1-\Psi)} E_{\Psi}\left(-\frac{\Psi t^{\Psi}}{(1-\Psi)}\right)$. Thus Lipschitz condition holds for $\mathcal{ABC}$ derivative. □

4. Existence theory

Let

$$\Phi(x,y,t;f) = 6f^2 f_x - 6f^2 f_y + f_{xxx} - f_{yyy} - 3f_{xxy} + 3f_{xyy}, \tag{4.1}$$

Eq (1.3) can be written as

$$ {}^{\mathcal{ABC}}D_t^{\Psi} f(x,y,t) \;=\; \Phi(x,y,t;f). \tag{4.2}$$

Applying the $\mathcal{ABC}$ FOI to Eq (4.2), we have

$$f(x,y,t) - f(x,y,0) = \frac{(1-\Psi)}{c(\Psi)}\Phi(x,y,t;f) + \frac{(\Psi)}{c(\Psi)\Gamma(\Psi)} \int_0^t (t-\eta)^{\Psi-1} \Phi(x,y,t;f)\, d\eta.$$

First we have to verify that the Lipschitz condition holds for the kernel $\Phi(x,y,t;f)$. For this, let us take two bounded functions, f and g, i.e., $\|f\| \leq \varDelta_1$, and $\|g\| \leq \varDelta_2$ where $\varDelta_1, \varDelta_2 > 0$, and consider

$$
\begin{aligned}
&\|\Phi(x,y,t;f)-\Phi(x,y,t;g)\| \\
=&\begin{cases} \Big\|\left(6f^2f_x-6g^2g_x\right)-\left(6f^2f_y-6g^2g_y\right)+\left(f_{xxx}-g_{xxx}\right)-\left(f_{yyy}-g_{yyy}\right) \\ -\left(3f_{xxy}-3g_{xxy}\right)+\left(3f_{xyy}-3g_{xyy}\right)\Big\| \end{cases} \\
=&\begin{cases} \Big\|2\frac{\partial}{\partial x}\left(f^3-g^3\right)-2\frac{\partial}{\partial y}\left(f^3-g^3\right)+\frac{\partial^3}{\partial x^3}(f-g)-\frac{\partial^3}{\partial y^3}(f-g) \\ -3\frac{\partial^3}{\partial y\partial x^2}(f-g)+3\frac{\partial^3}{\partial y^2\partial x}(f-g)\Big\| \end{cases} \\
\leq&\begin{cases} 2\left\|\frac{\partial}{\partial x}\left(f^3-g^3\right)\right\|+2\left\|\frac{\partial}{\partial y}\left(f^3-g^3\right)\right\|+\left\|\frac{\partial^3}{\partial x^3}(f-g)\right\|+\left\|\frac{\partial^3}{\partial y^3}(f-g)\right\| \\ +3\left\|\frac{\partial^3}{\partial y\partial x^2}(f-g)\right\|+3\left\|\frac{\partial^3}{\partial y^2\partial x}(f-g)\right\|. \end{cases}
\end{aligned}
$$

Since f and g are bounded functions. Therefore, their partial derivatives satisfy the Lipschitz conditions and there exists non-negative constants $\mathbb{K},\mathbb{L},\mathbb{M},\mathbf{N},\mathbb{O},\mathbb{P}$ such that

$$
\begin{aligned}
&\|\Phi(x,y,t;f)-\Phi(x,y,t;g)\| \\
\leq&\begin{cases} 2\mathbb{K}\left\|\left(f^3-g^3\right)\right\|+2\mathbb{L}\left\|\left(f^3-g^3\right)\right\|+\mathbb{M}\|(f-g)\|+\mathbf{N}\|(f-g)\| \\ +3\mathbb{O}\|(f-g)\|+3\mathbb{P}\|(f-g)\| \end{cases} \\
\leq&\begin{cases} (2\mathbb{K}+2\mathbb{L})\left(f^2+fg+g^2\right)\|(f-g)\|+\mathbb{M}\|(f-g)\|+\mathbf{N}\|(f-g)\| \\ +3\mathbb{O}\|(f-g)\|+3\mathbb{P}\|(f-g)\| \end{cases} \\
=&\left((2\mathbb{K}+2\mathbb{L})\left(\varDelta_1^2+\varDelta_1\varDelta_2+\varDelta_2^2\right)+\mathbb{M}+\mathbf{N}+3\mathbb{O}+3\mathbb{P}\right)\|(f-g)\| \\
=&\kappa\|(f-g)\|.
\end{aligned}
$$

Let

$$\kappa=\left((2\mathbb{K}+2\mathbb{L})\left(\varDelta_1^2+\varDelta_1\varDelta_2+\varDelta_2^2\right)+\mathbb{M}+\mathbf{N}+3\mathbb{O}+3\mathbb{P}\right),$$

thus

$$\|\Phi(x,y,t;f)-\Phi(x,y,t;g)\| \quad\leq\quad \kappa\|f-g\|.$$

For further analysis, we make an iterative scheme as

$$f_{\xi+1}(x,y,t)=\frac{(1-\varPsi)}{c(\varPsi)}\Phi\left(x,y,t;f_\xi\right)+\frac{(\varPsi)}{c(\varPsi)\Gamma(\varPsi)}\int_0^t(t-\eta)^{\varPsi-1}\Phi\left(x,y,\eta;f_\xi\right)d\eta,$$

where $f_0(x,y,t)=f(x,y,0)$. Now the difference between two consecutive terms can be taken as

$$
\begin{aligned}
e_\xi(x,y,t)=&f_\xi(x,y,t)-f_{\xi-1}(x,y,t) \\
=&\frac{(1-\varPsi)}{c(\varPsi)}\left[\Phi\left(x,y,t;f_{\xi-1}\right)-\Phi\left(x,y,t;f_{\xi-2}\right)\right] \\
&+\frac{(\varPsi)}{c(\varPsi)\Gamma(\varPsi)}\int_0^t(t-\eta)^{\varPsi-1}\left[\Phi\left(x,y,\eta;f_{\xi-1}\right)-\Phi\left(x,y,\eta;f_{\xi-2}\right)\right]d\eta.
\end{aligned}
$$

Also, we have

$$f_\xi(x,y,t)=\sum_{k=0}^{\xi}e_k(x,y,t), \tag{4.3}$$

with $f_{-1}=0$.

Theorem 4.1. *Assume that $f(x,y,t)$ is bounded a function. Then*

$$\|e_\xi(x,y,t)\|\le\left(\frac{(1-\Psi)}{c(\Psi)}\kappa+\frac{\kappa t^{\Psi}}{c(\Psi)\Gamma(\Psi)}\right)^{\xi}\|f(x,y,0)\|. \tag{4.4}$$

Proof. Consider

$$e_\xi(x,y,t)=f_\xi(x,y,t)-f_{\xi-1}(x,y,t). \tag{4.5}$$

The Eq (4.5) gets the form under norm as

$$\left\|e_\xi(x,y,t)\right\|=\left\|f_\xi(x,y,t)-f_{\xi-1}(x,y,t)\right\|.$$

To get the required result, we use the concept of mathematical induction. For $\xi=1$, one can get

$$\begin{aligned}\|e_1(x,y,t)\|&=\|f_1(x,y,t)-f_0(x,y,t)\|\\&\le\frac{(1-\Psi)}{c(\Psi)}\|\Phi(x,y,t;f_0)-\Phi(x,y,t;f_{-1})\|\\&+\frac{\Psi}{c(\Psi)\Gamma(\Psi)}\int_0^t(t-\eta)^{\Psi-1}\|\Phi(x,y,\eta;f_0)-\Phi(x,y,\eta;f_{-1})\|\,d\eta\\&\le\frac{(1-\Psi)}{c(\Psi)}\kappa\|f_0-f_{-1}\|+\frac{\Psi}{c(\Psi)\Gamma(\Psi)}\kappa\int_0^t(t-\eta)^{\Psi-1}\|f_0-f_{-1}\|\,d\eta\\&=\frac{(1-\Psi)}{c(\Psi)}\kappa\|f(x,y,0)\|+\frac{\Psi}{c(\Psi)\Gamma(\Psi)}\kappa\|f(x,y,0)\|\int_0^t(t-\eta)^{\Psi-1}d\eta\\&=\frac{(1-\Psi)}{c(\Psi)}\kappa\|f(x,y,0)\|+\frac{\Psi}{c(\Psi)\Gamma(\Psi)}\kappa\|f(x,y,0)\|\,t\\&=\left(\frac{(1-\Psi)}{c(\Psi)}\kappa+\frac{\kappa t^{\Psi}}{c(\Psi)\Gamma(\Psi)}\right)\|f(x,y,0)\|.\end{aligned}$$

Now assume that the result is true for $\xi-1$, i.e.,

$$\left\|e_{\xi-1}(x,y,t)\right\|\le\left(\frac{(1-\Psi)}{c(\Psi)}\kappa+\frac{\kappa t^{\Psi}}{c(\Psi)\Gamma(\Psi)}\right)^{\xi-1}\|f(x,y,0)\|. \tag{4.6}$$

Next, we have to show that

$$\left\|e_\xi(x,y,t)\right\|\le\left(\frac{(1-\Psi)}{c(\Psi)}\kappa+\frac{\kappa t^{\Psi}}{c(\Psi)\Gamma(\Psi)}\right)^{\xi}\|f(x,y,0)\|. \tag{4.7}$$

To get the result (4.4), consider

$$
\begin{aligned}
\left\|e_{\xi}(x,y,t)\right\| &= \left\|f_{\xi}(x,y,t) - f_{\xi-1}(x,y,t)\right\| \\
&\leq \frac{(1-\Psi)}{c(\Psi)}\left\|\Phi\left(x,y,t;f_{\xi-1}\right) - \Phi\left(x,y,t;f_{\xi-2}\right)\right\| \\
&+ \frac{\Psi}{c(\Psi)\Gamma(\Psi)}\int_0^t (t-\eta)^{\Psi-1}\left\|\Phi\left(x,y,\eta;f_{\xi-1}\right) - \Phi\left(x,y,\eta;f_{\xi-2}\right)\right\| d\eta \\
&\leq \frac{(1-\Psi)}{c(\Psi)}\kappa\left\|f_{\xi-1} - f_{\xi-2}\right\| + \frac{\Psi}{c(\Psi)\Gamma(\Psi)}\kappa\int_0^t (t-\eta)^{\Psi-1}\left\|f_{\xi-1} - f_{\xi-2}\right\| d\eta \\
&= \frac{(1-\Psi)}{c(\Psi)}\kappa\left\|e_{\xi-1}\right\| + \frac{\kappa t^{\Psi-1}}{c(\Psi)\Gamma(\Psi)}\left\|e_{\xi-1}\right\| \\
&= \left(\frac{(1-\Psi)}{c(\Psi)}\kappa + \frac{\kappa t^{\Psi-1}}{c(\Psi)\Gamma(\Psi)}\right)\left\|e_{\xi-1}\right\| \\
&= \left(\frac{(1-\Psi)}{c(\Psi)}\kappa + \frac{\kappa t^{\Psi-1}}{c(\Psi)\Gamma(\Psi)}\right)\left(\frac{(1-\Psi)}{c(\Psi)}\kappa + \frac{\kappa t^{\Psi-1}}{c(\Psi)\Gamma(\Psi)}\right)^{\xi-1}\|f(x,y,0)\| \\
&= \left(\frac{(1-\Psi)}{c(\Psi)}\kappa + \frac{\kappa t^{\Psi-1}}{c(\Psi)\Gamma(\Psi)}\right)^{\xi}\|f(x,y,0)\|.
\end{aligned}
$$

This ends the proof. □

Theorem 4.2. *If the following relation holds at $t = t_0 \geq 0$, where*

$$
0 \leq \left(\tfrac{(1-\Psi)}{c(\Psi)}\kappa + \tfrac{\kappa t_0^{\Psi-1}}{c(\Psi)\Gamma(\Psi)}\right) < 1. \tag{4.8}
$$

Then at least one solution of the new 2D KdV equation under the $\mathcal{ABC}$ fractional derivative exists.

Proof. With the help of Eq (4.3), we have

$$
\left\|f_{\xi}(x,y,0)\right\| \leq \sum_{k=0}^{\xi}\|e_k(x,y,t)\| \leq \sum_{k=0}^{\xi}\left(\left(\frac{(1-\Psi)}{c(\Psi)}\kappa + \frac{\kappa t^{\Psi-1}}{c(\Psi)\Gamma(\Psi)}\right)^k\|f(x,y,0)\|\right),
$$

for $t = t_0$, we get

$$
\left\|f_{\xi}(x,y,0)\right\| \leq \|f(x,y,0)\|\sum_{k=0}^{\xi}\left(\frac{(1-\Psi)}{c(\Psi)}\kappa + \frac{\kappa t_0^{\Psi-1}}{c(\Psi)\Gamma(\Psi)}\right)^k.
$$

From the above relation, we can say that

$$
\lim_{\xi\to\infty}\left\|f_{\xi}(x,y,0)\right\| \leq \|f(x,y,0)\|\lim_{\xi\to\infty}\sum_{k=0}^{\xi}\left(\frac{(1-\Psi)}{c(\Psi)}\kappa + \frac{\kappa t_0^{\Psi-1}}{c(\Psi)\Gamma(\Psi)}\right)^k.
$$

Since

$$
0 \leq \left(\tfrac{(1-\Psi)}{c(\Psi)}\kappa + \tfrac{\kappa t_0^{\Psi-1}}{c(\Psi)\Gamma(\Psi)}\right) < 1, \tag{4.9}
$$

this implies that sequence $f_\xi(x,y,t)$ is convergent and therefore the sequence is bounded for each ξ. Further, assume that

$$\mathbf{R}_\xi(x,y,t) = f(x,y,t) - f_\xi(x,y,t).$$

Since $f_\xi(x,y,t)$ is bounded. It follows that for $\lambda > 0$, we have $\left\|f_\xi(x,y,t)\right\| \leq \lambda$. After simple manipulation like we did in Theorems 4.1 and 4.2, we obtain

$$\left\|\mathbf{R}_\xi(x,y,t)\right\| \leq \left(\frac{(1-\Psi)}{c(\Psi)}\kappa + \frac{\kappa t_0^{\Psi-1}}{c(\Psi)\Gamma(\Psi)}\right)^{\xi+1} \lambda.$$

Using Eq (4.9), one can get

$$\lim_{\xi\to\infty} \left\|\mathbf{R}_\xi(x,y,t)\right\| = 0,$$

it follows that $\lim_{\xi\to\infty} f_\xi(x,y,t) = f(x,y,t)$. This finish the proof. □

Theorem 4.3. *If the inequality (4.8) holds at $t = t_0 \geq 0$. Then the unique solution of proposed equation exists.*

Proof. On contrary, suppose that there are two solutions f and g of the proposed equation such that $f \neq g$. Now

$$\begin{aligned} & f(x,y,t) - g(x,y,t) \\ = \ & \frac{(1-\Psi)}{c(\Psi)}\left[\Phi(x,y,t;f) - \Phi(x,y,t;g)\right] + \frac{\Psi}{c(\Psi)\Gamma(\Psi)} \\ & \times\left[\int_0^t (t-\eta)^{\Psi-1}\left[\Phi(x,y,\eta;f) - \Phi(x,y,\eta;g)\right] d\eta\right]. \end{aligned}$$

Taking norm both side

$$\begin{aligned} & \|f(x,y,t) - g(x,y,t)\| \\ \leq \ & \frac{(1-\Psi)}{c(\Psi)}\|\Phi(x,y,t;f) - \Phi(x,y,t;g)\| + \frac{\Psi}{c(\Psi)\Gamma(\Psi)} \\ & \times\left[\int_0^t (t-\eta)^{\Psi-1}\|\Phi(x,y,\eta;f) - \Phi(x,y,\eta;g)\| d\eta\right] \\ \leq \ & \frac{(1-\Psi)}{c(\Psi)}\kappa\|f-g\| + \frac{\Psi}{c(\Psi)\Gamma(\Psi)}\int_0^t \kappa(t-\eta)^{\Psi-1}\|f-g\| d\eta \\ \leq \ & \left(\frac{(1-\Psi)}{c(\Psi)}\kappa + \frac{\Psi}{c(\Psi)\Gamma(\Psi)}\kappa\right)\|f-g\|\int_0^t (t-\eta)^{\Psi-1} d\eta \\ = \ & \left(\frac{(1-\Psi)}{c(\Psi)}\kappa + \frac{\kappa t^{\Psi}}{c(\Psi)\Gamma(\Psi)}\right)\|f-g\|, \end{aligned}$$

but

$$0 \leq \left(\frac{(1-\Psi)}{c(\Psi)}\kappa + \frac{\kappa t^{\Psi}}{c(\Psi)\Gamma(\Psi)}\right) < 1.$$

Using the above inequality, we achieve

$$\|f(x,y,t) - g(x,y,t)\| = 0,$$

thus, our supposition is wrong. Hence, the solution is unique. □

5. Solution of Eq (1.2)

In this section, we briefly discuss the solution of the model by applying Ansatz method. For this purpose, we will consider a test function as

$$f(x,y,t) = \beta_0 + \beta_1 sech\,(b_1x + b_2y + b_3t)\,. \tag{5.1}$$

By putting above equation into classical form of the model, we obtain

$$\begin{aligned} &6\beta_0^2a_1 - 6\beta_0^2b_2 + b_1^3 - 3b_1^2b_2 + 3b_2^2b_1 - b_2^3 - b_3 = 0,\\ &12\beta_0\beta_1b_1 - 12\beta_0b_1b_2 = 0,\\ &6\beta_1^2b_1 - 6\beta_1^2b_2 - 6b_1^3 + 18b_1^2b_2 - 18b_2^2b_1 + 6b_2^3 = 0, \end{aligned}$$

solution becomes as

$$\beta_0 = 0, \beta_1 = \mp b_1 \pm b_2, b_3 = b_1^3 - 3b_1^2b_2 + 3b_2^2b_1 - b_2^3,$$

solution of classical model becomes as

$$f_{1,2}(x,y,t) = (\mp b_1 \pm b_2)\, sech\left(b_1x + b_2y + \left(b_1^3 - 3b_1^2b_2 + 3b_2^2b_1 - b_2^3\right)t\right).$$

For $b_1 = 1$ and $b_2 = -1$, the above solution becomes

$$f_{1,2}(x,y,t) = \mp\frac{4\exp(x-y+8t)}{1+\exp(2x-2y+16t)}. \tag{5.2}$$

6. Solution of Eq (1.3)

In this section, to obtain analytic solution we applying Laplace transform (LT) on both sides of equations $f(x,y,t)$ is the source term. Subject to the initial condition $f(x,y,0) = f_0(x,y,0)$. On utilizing LT, one can get

$$\mathcal{L}\left[{}^{\mathcal{ABC}}D_t^{\Psi}f\right] = \mathcal{L}\left[6f^2f_x - 6f^2f_y + f_{xxx} - f_{yyy} - 3f_{xxy} + 3f_{xyy}\right],$$

$$\mathcal{L}[f(x,y,t)] = \frac{f(x,y,0)}{s} + \left[\frac{s(1-\Psi)+\Psi}{c(\Psi)}\mathcal{L}\left[6f^2f_x - 6f^2f_y\right] + f_{xxx} - f_{yyy} - 3f_{xxy} + 3f_{xyy}\right]. \tag{6.1}$$

The approximate solution is represented by

$$f(x,y,t) = \sum_{\xi=0}^{\infty} f_\xi(x,y,t), \tag{6.2}$$

and the nonlinear term is represented by Adomain polynomials, i.e., $G(f) = f^2 = \sum_{\xi=0}^{\infty}\mathscr{A}_\xi$, where $\mathscr{A}_\xi$ is defined as follows for any $\xi = 0,1,2,\cdots$

$$\mathscr{A}_\xi = \frac{1}{\Gamma(\xi+1)}\frac{d^\xi}{d\lambda^\xi}\left[G\left(\sum_{\xi=0}^{\infty}\left(\lambda_\xi f_\xi\right)\right)\right]_{\lambda=0}.$$

Using Eq (6.2), we obtain

$$\mathcal{L}\left[\sum_{\xi=0}^{\infty} f_\xi(x,y,t)\right] = \frac{f(x,y,0)}{s} + \frac{s(1-\Psi)+\Psi}{c(\Psi)}\mathcal{L}\left[6\sum_{\xi=0}^{\infty}\mathscr{A}_\xi \sum_{\xi=0}^{\infty}\frac{\partial}{\partial x}f_\xi - 6\sum_{\xi=0}^{\infty}\mathscr{A}_\xi \sum_{\xi=0}^{\infty}\frac{\partial}{\partial y}f_\xi \right.$$
$$\left. + \sum_{\xi=0}^{\infty}\left(\frac{\partial^3}{\partial x^3}f_\xi - \frac{\partial^3}{\partial y^3}f_\xi - 3\frac{\partial^3}{\partial x^2\partial y}f_\xi + 3\frac{\partial^3}{\partial y\partial x^2}f_\xi\right)\right].$$

The following can be obtain by comparing terms

$$\begin{aligned}
\mathcal{L}[f_0(x,y,t)] &= \frac{f(x,y,0)}{s},\\
\mathcal{L}[f_1(x,y,t)] &= \frac{s^\Psi(1-\Psi)+\Psi)}{s^\Psi c(\Psi)}\mathcal{L}\Big[6\mathscr{A}_0 f_{0_x} - 6\mathscr{A}_0 f_{0_y} + f_{0_{xxx}}\\
&\quad + f_{0_{yyy}} - 3f_{0_{xxy}} + 3f_{0_{xyy}}\Big],\\
\mathcal{L}[f_2(x,y,t)] &= \frac{s^\Psi(1-\Psi)+\Psi)}{s^\Psi c(\Psi)}\mathcal{L}\Big[6\mathscr{A}_1 f_{1_x} - 6\mathscr{A}_1 f_{1_y} + f_{1_{xxx}}\\
&\quad + f_{1_{yyy}} - 3f_{1_{xxy}} + 3f_{1_{xyy}}\Big],\\
&\vdots\\
\mathcal{L}\left[f_{\xi+1}(x,y,t)\right] &= \frac{s^\Psi(1-\Psi)+\Psi)}{s^\Psi c(\Psi)}\mathcal{L}\Big[6\mathscr{A}_\xi f_{\xi_x} - 6\mathscr{A}_\xi f_{\xi_y} + f_{\xi_{xxx}}\\
&\quad + f_{\xi_{yyy}} - 3f_{\xi_{xxy}} + 3f_{\xi_{xyy}}\Big].
\end{aligned}$$

Applying $\mathcal{L}^{-1}$, we get

$$\begin{aligned}
f_0(x,y,t) &= \mathcal{L}^{-1}\left[\frac{f(x,y,0)}{s}\right],\\
f_1(x,y,t) &= \mathcal{L}^{-1}\left[\frac{s^\Psi(1-\Psi)+\Psi)}{s^\Psi c(\Psi)}\mathcal{L}\Big[6\mathscr{A}_0 f_{0_x} - 6\mathscr{A}_0 f_{0_y} + f_{0_{xxx}}\right.\\
&\quad \left. + f_{0_{yyy}} - 3f_{0_{xxy}} + 3f_{0_{xyy}}\Big]\right]\\
f_2(x,y,t) &= \mathcal{L}^{-1}\left[\frac{s^\Psi(1-\Psi)+\Psi)}{s^\Psi c(\Psi)}\mathcal{L}\Big[6\mathscr{A}_1 f_{1_x} - 6\mathscr{A}_1 f_{1_y} + f_{1_{xxx}}\right.\\
&\quad \left. + f_{1_{yyy}} - 3f_{1_{xxy}} + 3f_{1_{xyy}}\Big]\right]\\
&\vdots\\
f_{\xi+1}(x,y,t) &= \mathcal{L}^{-1}\left[\frac{s^\Psi(1-\Psi)+\Psi)}{s^\Psi c(\Psi)}\mathcal{L}\Big[6\mathscr{A}_\xi f_{\xi_x} - 6\mathscr{A}_\xi f_{\xi_y} + f_{\xi_{xxx}}\right.\\
&\quad \left. + f_{\xi_{yyy}} - 3f_{\xi_{xxy}} + 3f_{\xi_{xyy}}\Big]\right].
\end{aligned}$$

The required series solution is given as

$$f(x,y,t) = \sum_{\xi=0}^{\infty} f_\xi(x,y,t). \tag{6.3}$$

Here, we present two special cases of the proposed equation.

6.1. Case A

For the first case, we take the initial condition as

$$f(x,y,0) = -4\frac{\exp(x+y)}{1+\exp(2(x-y))}.$$

Using the detail procedure as discussed above, we achieve

$$\begin{aligned} f_0(x,y,t) =& f(x,y,0) = -4\frac{\exp(x+y)}{1+\exp(2(x-y))}, \\ f_1(x,y,t) =& \mathcal{L}^{-1}\left[\frac{s^{\Psi}(1-\Psi)+\Psi}{s^{\Psi}c(\Psi)}\mathcal{L}\left[6f_0^2\frac{\partial}{\partial x}f_0 - 6f_0^2\frac{\partial}{\partial y}f_0 + \frac{\partial^3}{\partial x^3}f_0 - \frac{\partial^3}{\partial y^3}f_0 \right.\right.\\ & \left.\left. -3\frac{\partial^2}{\partial x^2}(\frac{\partial}{\partial y}f_0) + 3\frac{\partial}{\partial x}(\frac{\partial^2}{\partial y^2}f_0)\right]\right], \end{aligned}$$

using Mathematica, we obtain

$$f_1(x,y,t) = \left(1-\Psi+\frac{\Psi t^{\Psi}}{\Gamma(\Psi+1)}\right)\left(32t\exp(x+y)\frac{(-\exp(2x)+\exp(2y))}{(\exp(2x)+\exp(2y))^2\,c\,(\Psi)}\right),$$

similarly, other terms can be calculated with the help of Mathematica. The required series solution is given as

$$\begin{cases} f(x,y,t) & = -4\frac{\exp(x+y)}{1+\exp(2(x-y))} + 32t\left(1-\Psi+\frac{\Psi t^{\Psi}}{\Gamma(\Psi+1)}\right) \\ & \times\left(\exp(x+y)\frac{(-\exp(2x)+\exp(2y))}{(\exp(2x)+\exp(2y))^2 c(\Psi)}\right) + \dots \end{cases} \tag{6.4}$$

Remark 6.1. *When we put $\Psi = 1$ in Eq (6.4), the solution rapidly converges to the exact classical solution, i.e.,*

$$f(x,y,t) = -4\frac{\exp(x-y+8t)}{1+\exp(2(x-y+8t))}. \tag{6.5}$$

6.2. Case B

For the first case, we take the initial condition as

$$f(x,y,0) = 4\frac{\exp(x+y)}{1+\exp(2(x-y))}.$$

Using the detail procedure as discussed above, we get

$$\begin{aligned} f_0(x,y,t) =& f(x,y,0) = 4\frac{\exp(x+y)}{1+\exp(2(x-y))}, \\ f_1(x,y,t) =& \mathcal{L}^{-1}\left[\frac{s^{\Psi}(1-\Psi)+\Psi}{s^{\Psi}c(\Psi)}\mathcal{L}\left[6f_0^2\frac{\partial}{\partial x}f_0 - 6f_0^2\frac{\partial}{\partial y}f_0 + \frac{\partial^3}{\partial x^3}f_0 - \frac{\partial^3}{\partial y^3}f_0 \right.\right.\\ & \left.\left. -3\frac{\partial^2}{\partial x^2}(\frac{\partial}{\partial y}f_0) + 3\frac{\partial}{\partial x}(\frac{\partial^2}{\partial y^2}f_0)\right]\right], \end{aligned}$$

using Mathematica, we obtain

$$f_1(x,y,t) = \left(1-\Psi+\frac{\Psi t^{\Psi}}{\Gamma(\Psi+1)}\right)\left(-32t\exp(x+y)\frac{(-\exp(2x)+\exp(2y))}{(\exp(2x)+\exp(2y))^2\,c\,(\Psi)}\right),$$

similarly, other terms can be calculated with the help of Mathematica. The required series solution is given as

$$\begin{cases} f(x,y,t) & = 4\frac{\exp(x+y)}{1+\exp(2(x-y))} - 32t\left(1-\Psi+\frac{\Psi t^{\Psi}}{\Gamma(\Psi+1)}\right) \\ & \times\left(\exp(x+y)\frac{(-\exp(2x)+\exp(2y))}{\left(\exp(2x)+\exp(2y)\right)^2 c(\Psi)}\right)+\ldots \end{cases} \tag{6.6}$$

Remark 6.2. *When we put $\Psi = 1$ in Eq (6.6), the solution rapidly converges to the exact classical solution, i.e.,*

$$f(x,y,t) = 4\frac{\exp(x-y+8t)}{1+\exp\left(2(x-y+8t)\right)}. \tag{6.7}$$

7. Convergence and stability analysis

Here, we derive some results regarding to the convergence and stability of the proposed scheme with the help of functional analysis. The convergence of the proposed scheme of the is presented in the following theorem.

Theorem 7.1. *Let $\mathbb{H}$ be a Banach space and $\mathrm{T}:\mathbb{H}\to\mathbb{H}$ be an operator. Suppose that f be the exact solution of the proposed equation. If $\exists\ \varpi$ such that $0\le\varpi<1$ and $\left\|f_{\xi+1}\right\|\le\varpi\left\|f_{\xi}\right\|$, $\forall\ \xi\in\mathbb{N}\cup\{0\}$, then the approximate solution $\sum_{\xi=0}^{\infty}f_{\xi}$ converges to the exact solution f.*

Proof. We construct a series as

$$\begin{aligned} \boldsymbol{S}_0 &= f_0, \\ \boldsymbol{S}_1 &= f_0+f_1, \\ \boldsymbol{S}_2 &= f_0+f_1+f_2, \\ &\vdots \\ \boldsymbol{S}_{\xi} &= f_0+f_1+\cdots f_{\xi}. \end{aligned}$$

We want to prove that the sequence $\left\{\boldsymbol{S}_{\xi}\right\}_{\xi=0}^{\infty}$ is a Cauchy sequence in $\mathbb{H}$. Let us consider

$$\begin{aligned} \left\|\boldsymbol{S}_{\xi+1}-\boldsymbol{S}_{\xi}\right\| &= \left\|f_{\xi}\right\| \\ &\le \varpi\left\|f_{\xi}\right\| \\ &\le \varpi^2\left\|f_{\xi-1}\right\| \\ &\le \varpi^3\left\|f_{\xi-2}\right\| \\ &\vdots \\ &\le \varpi^{\xi+1}\left\|f_0\right\|. \end{aligned}$$

Now for every $\xi, m \in \mathbb{N}$, we have

$$\begin{aligned}\left\|S_\xi - S_m\right\| &= \left\|\left(S_\xi - S_{\xi-1}\right) + \left(S_{\xi-1} - S_{\xi-2}\right) + \cdots + (S_{m+1} - S_m)\right\| \\ &\leq \left\|S_\xi - S_{\xi-1}\right\| + \left\|S_{\xi-1} - S_{\xi-2}\right\| + \cdots + \|S_{m+1} - S_m\| \\ &\leq \varpi^{\xi}\|f_0\| + \varpi^{\xi-1}\|f_0\| + \cdots + \varpi^{m+1}\|f_0\| \\ &\leq \left(\varpi^{\xi+1} + \varpi^{\xi+2} + \cdots\right)\|f_0\| = \frac{\varpi^{\xi+1}}{1-\varpi}\|f_0\|.\end{aligned}$$

Now, $\lim_{\xi,m\to\infty}\left\|S_\xi - S_m\right\| = 0$. This implies that $\left\{S_\xi\right\}_{\xi=0}^{\infty}$ is Cauchy sequence in $\mathbb{H}$. So $\exists\ f \in \mathbb{H}$ such that $\lim_{\xi\to\infty} S_\xi = f$. This ends the proof. □

Next, we present the Picard's **X**-stability of the proposed scheme in the following theorem.

Theorem 7.2. *Let* **X** *be a self-mapping which is defined as*

$$\begin{aligned}\mathbf{X}\left(f_\xi(x,t)\right) =& f_{\xi+1}(x,t) \\ =& f_\xi(x,t) + \mathcal{L}^{-1}\left[\frac{s^{\Psi}(1-\Psi)+\Psi)}{s^{\Psi}c(\Psi)}\mathcal{L}\left[6f_\xi^2 f_{\xi_x} - 6f_\xi^2 f_{\xi_y} + f_{\xi_{xxx}} + f_{\xi_{yyy}} - 3f_{\xi_{xxy}} + 3f_{\xi_{xyy}}\right]\right]. \quad (7.1)\end{aligned}$$

The iteration is **X***-stable in* $\mathbb{L}^1(a,b)$, *if the condition*

$$\left\|(2\mathbb{K}+2\mathbb{L})\left(\Phi_1^2 + \Phi_1\Phi_2 + \Phi_2\right)\aleph_1 + \mathbb{M}\aleph_2 - \mathbf{N}\aleph_3 + 3\mathbb{O}\aleph_4 + 3\mathbb{P}\aleph_5\right\| < 1, \quad (7.2)$$

is satisfied.

Proof. With the help of Banach contraction theorem, first we show that the mapping **X** possesses a unique fixed point. For this, assume that the bounded iterations for $(\xi, m) \in \mathbb{N}\times\mathbb{N}$. Let $\Phi_1, \Phi_2 > 0$ such that $\left\|f_\xi\right\| \leq \Phi_1$, and $\|f_m\| \leq \Phi_2$. Consider

$$\begin{aligned}&\mathbf{X}\left(f_\xi(x,t)\right) - \mathbf{X}\left(f_m(x,t)\right) \\ =& f_\xi(x,t) - f_m(x,t) + \mathcal{L}^{-1}\left[\frac{s^{\Psi}(1-\Psi)+\Psi)}{s^{\Psi}c(\Psi)}\mathcal{L}\left[2f_\xi^2 f_{\xi_x} - 2f_\xi^2 f_{\xi_y} + f_{\xi_{xxx}} + f_{\xi_{yyy}} - 3f_{\xi_{xxy}} + 3f_{\xi_{xyy}}\right]\right] \\ &- \mathcal{L}^{-1}\left[\frac{s^{\Psi}(1-\Psi)+\Psi)}{s^{\Psi}c(\Psi)}\ \mathcal{L}\left[2f_m^2 f_{m_x} - 2f_m^2 f_{m_y} + f_{m_{xxx}} + f_{m_{yyy}} - 3f_{m_{xxy}} + 3f_{m_{xyy}}\right]\right] \\ =& f_\xi(x,t) - f_m(x,t) + \mathcal{L}^{-1}\left[\frac{s^{\Psi}(1-\Psi)+\Psi)}{s^{\Psi}c(\Psi)}\mathcal{L}\left[2\mathbb{K}\left(f_\xi^3 - f_m^3\right) - 6\mathbb{L}\left(f_\xi^3 - f_m^3\right)\right.\right. \\ &\left.\left.+\mathbb{M}\left(f_\xi - f_m\right) - \mathbf{N}\left(f_\xi - f_m\right) - 3\mathbb{O}\left(f_\xi - f_m\right) + 3\mathbb{P}\left(f_\xi - f_m\right)\right]\right].\end{aligned}$$

Now, using triangle inequality, we have

$$\begin{aligned}&\left\|\mathbf{X}\left(f_\xi(x,t)\right) - \mathbf{X}\left(f_m(x,t)\right)\right\| \\ =& \left\|f_\xi(x,t) - f_m(x,t)\right\| + \left\|\mathcal{L}^{-1}\left[\frac{s^{\Psi}(1-\Psi)+\Psi)}{s^{\Psi}c(\Psi)}\mathcal{L}\left[2\mathbb{K}\left(f_\xi^3 - f_m^3\right)\right.\right.\right. \\ &\left.\left.\left.-2\mathbb{L}\left(f_\xi^3 - f_m^3\right) + \mathbb{M}\left(f_\xi - f_m\right) - \mathbf{N}\left(f_\xi - f_m\right) - 3\mathbb{O}\left(f_\xi - f_m\right) + 3\mathbb{P}\left(f_\xi - f_m\right)\right]\right]\right\|.\end{aligned}$$

Using boundedness of f_ξ and f_m, we have

$$\begin{aligned} & \left\| \mathbf{X}\left(f_\xi(x,t)\right) - \mathbf{X}\left(f_m(x,t)\right) \right\| \\ \leq & \left[(2\mathbb{K} + 2\mathbb{L}) \left(\Phi_1^2 + \Phi_1 \Phi_2 + \Phi_2 \right) \aleph_1 + \mathbb{M} \aleph_2 - \mathbf{N} \aleph_3 \right. \\ & \left. + 3\mathbb{O}\aleph_4 + 3\mathbb{P}\aleph_5 \right] \left\| f_\xi - f_m \right\|, \end{aligned}$$

where $\aleph_i,\ i = 1,2,3,4,5$, are functions obtained from $\mathcal{L}^{-1}\left[\frac{s^\Psi(1-\Psi)+\Psi)}{s^\Psi c(\Psi)}\mathcal{L}[*]\right]$. Using assumption (7.2), the mapping $\mathbf{X}$ fulfills the contraction condition. Hence by Banach fixed point result, $\mathbf{X}$ has a unique fixed point. Also, the mapping $\mathbf{X}$ satisfies the condition of Theorem 2.4 with

$$\begin{aligned} a &= 0, \\ b &= (2\mathbb{K} + 2\mathbb{L}) \left(\Phi_1^2 + \Phi_1 \Phi_2 + \Phi_2 \right) \aleph_1 + \mathbb{M} \aleph_2 - \mathbf{N} \aleph_3 + 3\mathbb{O}\aleph_4 + 3\mathbb{P}\aleph_5. \end{aligned}$$

Thus, the mapping $\mathbf{X}$ fulfills all conditions of Picard's $\mathbf{X}$-stable. Hence our proposed scheme is Picard's $\mathbf{X}$-stable. □

8. Discussion and conclusions

Thanks to the Mittag-Leffler kernel, which solves the singularity and locality problems with the Caputo and Caputo-Fabrizio FOD kernels. Since $\mathcal{ABC}$-derivative is based on the Mittag-Leffler kernel, it has recently become popular for investigating the dynamics of a mathematical model that governs a physical process. We use $\mathcal{ABC}$-derivative to investigate the soliton solution of the new modified KdV equation in (2+1) dimension in the current paper. Since the presence of a solution is essential for the study of a model, we have deduced some results using fixed point theory that guarantees at least one solution and the unique solution of the proposed equation. There are several techniques for solving FDEs, but among the analytical methods, LADM is the most effective and accurate. Under $\mathcal{ABC}$-derivative, we used the LADM to obtain the solution of the proposed equation. Graphs of the solution are used to observe the method's convergence. The exact solution and the approximate solution obtained with the aid of LADM are in good agreement (See Figures 1 and 2). The dynamics of the solution under the $\mathcal{ABC}$-derivative have been investigated. We can see from Figures 3 and 4 that the fractional-order solution curves are approaching the integer-order curve when fractional-order equals 1. Figure 5 shows that the $\mathcal{ABC}$-derivative solution curve is much closer to the exact solution than the Caputo-Fabrizio derivative curve. As a result, the proposed model is better than Caputo-Fabrizio's. Figures 6–9 denote the graphical representation of solution obtained in the Case B. The considered equation will be studied under more generalized fractional operators in the next paper.

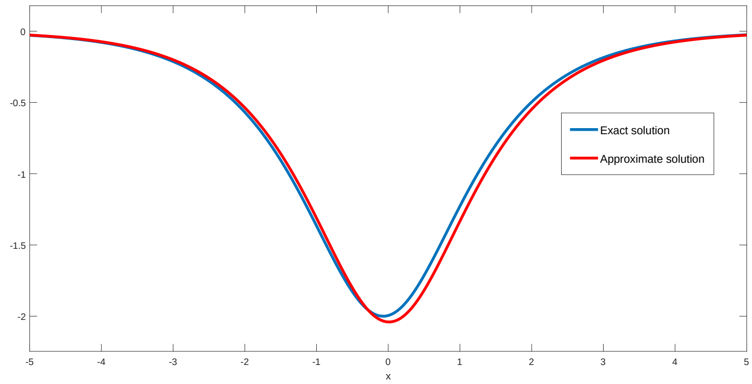

Figure 1. Two dimensional dynamics of exact and approximate solutions.

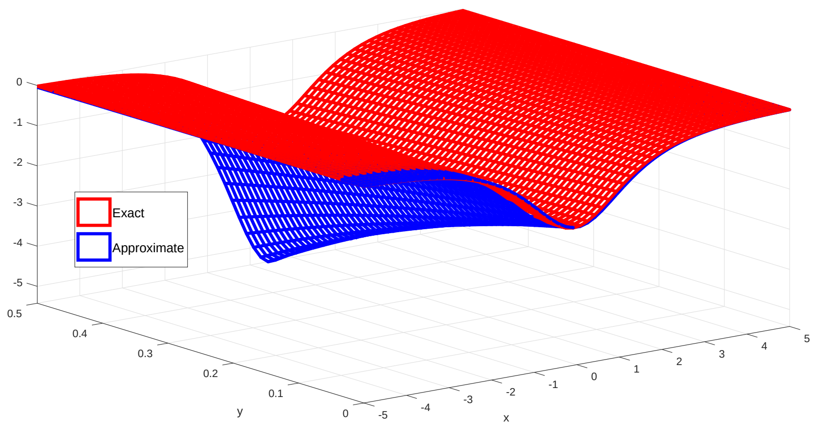

Figure 2. Dynamics of exact and approximate solutions in 3D.

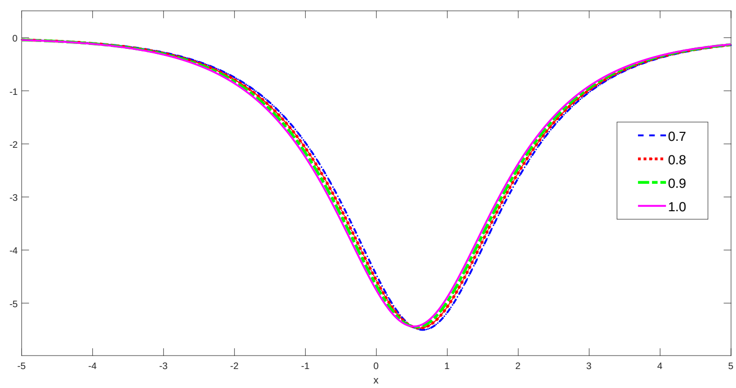

Figure 3. Dynamics of approximate solution for different fractional orders in 2D.

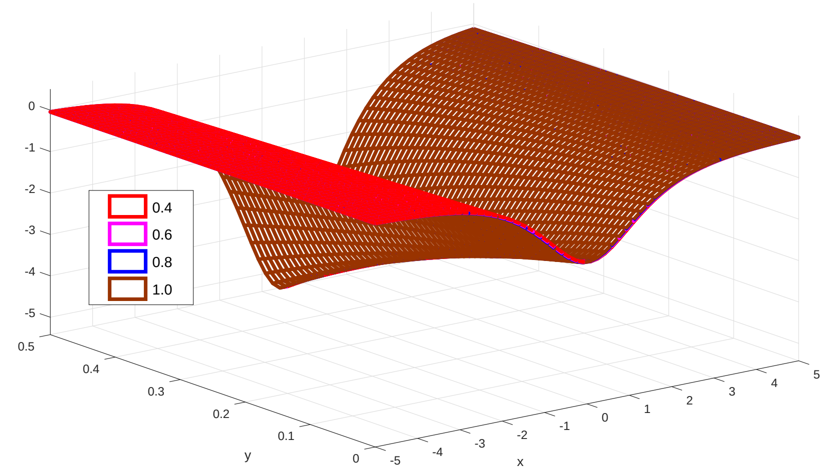

Figure 4. Dynamics of approximate solution for different fractional orders in 3D.

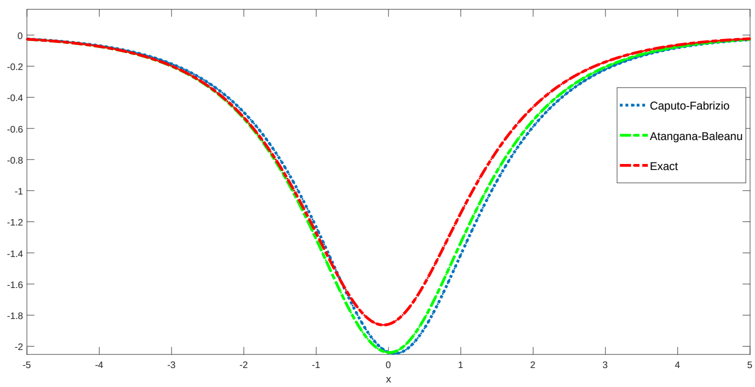

Figure 5. Comparison between solution curves of exact solution, ABC-solution and Caputo-Fabrizio solution.

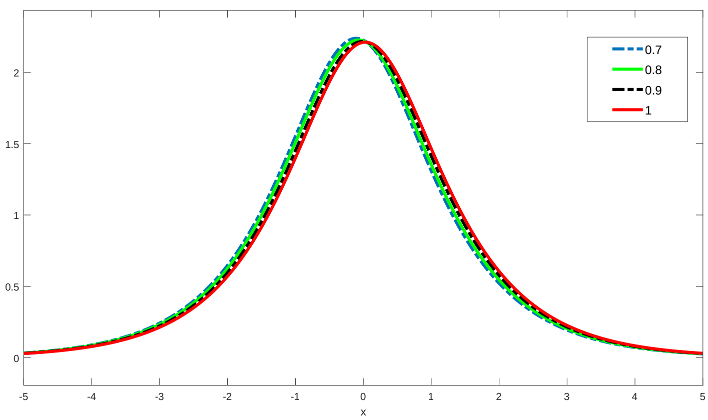

Figure 6. Dynamics of approximate solution in case B for different fractional orders in 2D.

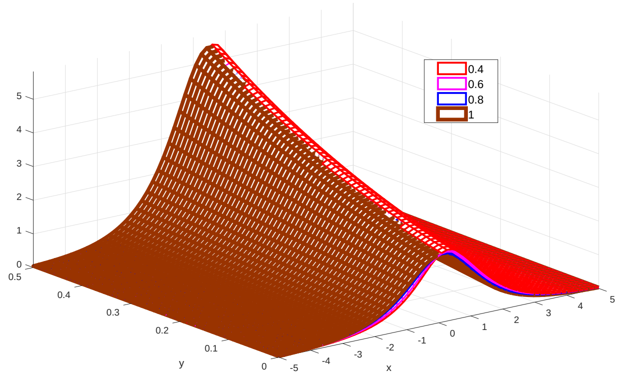

Figure 7. Dynamics of approximate solution case B for different fractional orders in 3D.

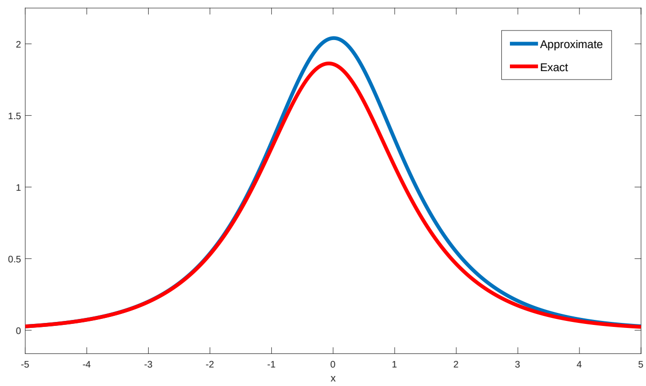

Figure 8. Two dimensional dynamics of exact and approximate solutions in case B.

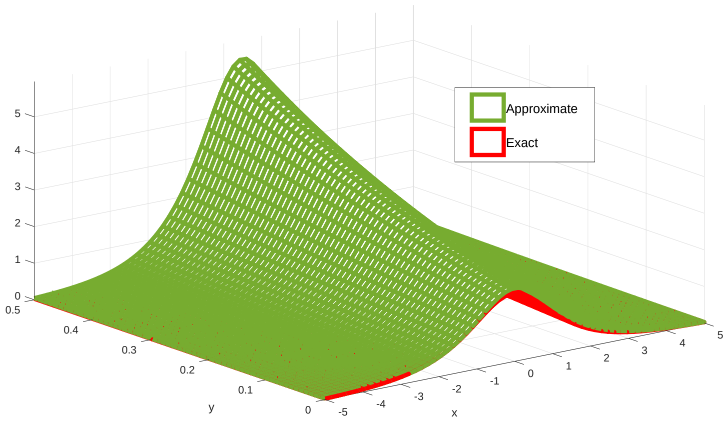

Figure 9. Dynamics of exact and approximate solutions in 3D in case B.

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Conflict of interest

In this research, there are no conflicts of interest.

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