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Research article

Decision support system based on fuzzy credibility Dombi aggregation operators and modified TOPSIS method

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Abstract: The operational law plays an important role in the aggregation operator for group decision system. The aggregation information has high influence in aggregating group decision information. Therefore, the main objective of the proposed work is to develop some operational laws as aggregation operator for fuzzy credibility numbers based on Dombi norms. Dombi operations can benefit from the best operational parameter flexibility. To the best of our knowledge, Dombi operations have so far not been used in for fuzzy credibility numbers (FCNs). Using these Dombi t-norm and t-conorm to define some different fuzzy credibility aggregation operators. i.e., fuzzy credibility Dombi weighted averaging (FCDWA) operator, fuzzy credibility Dombi ordered weighted averaging (FCDOWA) operator, fuzzy credibility Dombi hybrid weighted averaging (FCDHWA) operator. Next, we used TOPSIS method procedure for multi-attribute grouped decision-making (MAGDM). Finally, we provided an example, as well as a discussion of the comparative result analysis, to ensure that their findings are credible and practical.

Keywords: fuzzy credibility number; Dombi operation; aggregation operators; TOPSIS method; decision making problem
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1. Introduction

The utmost acceptable gear for proper reasoning, computing and modeling are crisp, deterministic and precise in character where the meaning of crisp is dichotomous, which means the dual logic statement. In common dual logic contain two term states, that is true or false, in which central term is ignored [27]. A tremendous role is played in present research situation by multi-attribute group decision-making system (MAGDM) [8]. Expert examines and choose alternatives using MAGDM based on priorities. Assuming a chance of resolving that alternative decision is to be considered [10]. Selection of robots for the a manufacturing plant is a MAGDM problem which is to be committed through non-programmed decision-making technique which involve an extended time period of treaty with the company. This type of decision-making group comprises of many decision analysts such as research, economic, development and engineering. Actually, solo decision maker attentions might not be likely. In DM process the final result might be altered by the prominence level of the expert. In the selection and valuation of robots effected the development of a multi-functional team on buying the firms with efficiency. In DM a key subject is the representation of attribute value and which arises due crisp numbers. It is problematic in some cases, to show the attribute to use of crisp set at this time the decision-maker has a choice and theory of fuzzy set can be functional in numerous pitches that are engineering, management and social-sciences to determine the issues of DM, which include ambiguity and imprecision in the data. In DM problem the utilization of fuzzy set theory has a tremendous role. Fuzzy set theory has tremendous importance in decision-making problem. This issue is resolved by Zadeh in 1965 when developed the knowledge of fuzzy set [26]. An impression of the membership degree on interval $[0, 1]$ is associated with fuzzy set. The theory of fuzzy set has many evident properties. We deal with the problem of decision making in fuzzy set theory, that discriminate the stuff into more than one suitable group of a certain universe which has been examined. In the theory of fuzzy set, many absences were determined by Atanasov [1]. It is noticed that the idea of negative membership degree sometime arises, which is an important fact in arranging the whole suggested outcome and design of the problem. Intuitionistic fuzzy set (IFS) familiarized this king of degree as an auxiliary of accurate values. The IFS is presented in order of pairs which are considered by the membership of positive and negative degree following the condition, that the addition of both the function is to be less than or equal to one [16]. The intuitionistic fuzzy set is extended to two major models that are Neutrosophic and Pythagorean. Furthermore, an uncertainty need human judgment and vague assessment decision matrix is to be required. Fuzzy evaluation values might be given by decision makers with the support of a certain degree of credibility with respect to unlike attributes. Yahy et al. [24] defined an analysis of medical diagnosis based on fuzzy credibility Dombi Bonferroni mean operator. Expert are more aware with certain attributes, but not with all the criteria. Three types of decision response take place when supposition occurs that is: No, yes and refusal. In all of these responses "refusal" is the most refined which might not be obtainable via the common fuzzy sets [25] and IFS [1]. Cuong [5] applied picture fuzzy set (PFS) as a recent vital model near the computational brainpower problems to overcome such issues, which are eminent in presenting the degree of negative, neutral and positive membership. Qiyas et al. [20] proposed a decision support system based on CoCoSo method with the picture fuzzy Information. Some complication occurs in PFS than researchers presented Spherical fuzzy set (SFS) [6], where SFS is the generalized form of PFS and IFS. A trapezoidal cubic fuzzy numbers are a new idea and its application are given by Fahmi et al. [21].

A new operation defined by Dombi in 1982 [10], known as Dombi's t-norm and t-conorm with the operation of parameters with a priority of variability. To get a benefit of Dombi operations, Liu [13] used Dombi operations to IFSs and introduced MAGDM problem utilizing Dombi Bonferroni mean operator in the atmosphere of IF information. Chen and Ye [14] bring together the multi-attribute group DM problem by means of Dombi aggregation operators in a single-valued Neutrosophic data. In a position of multiple options, generally it is the desire of humans to make calculated decisions. Qiyas et al. [17] defined linguistic picture fuzzy Dombi aggregation operators and their application in MAGDM problem. Khan et al. [12] developed the concept of Pythagorean fuzzy Dombi aggregation operators and discussed their application in decision support system. Ayub et al. [2] suggested the idea of Cubic fuzzy Heronian mean Dombi aggregation operators and studied their application on MADM problem.

In case of multiple alternatives with multiple criteria; scientifically it is to be developed with the help of analytical and numerical methods. One numerical technique of MCDM is TOPSIS method which is a simple mathematical model due to this it is widely applicable. Zeng et al. [28] proposed an extended version of linguistic picture fuzzy TOPSIS method and its applications in enterprise resource planning systems. Naeem et al. [15] extended TOPSIS method based on the entropy measure and probabilistic hesitant fuzzy information and discussed their application in decision support system. Furthermore, it depends on support of computers, it is a very appropriate everyday technique. This method is practiced for the last three decades [4] and there are many papers on its applications [9]. Qiyas and Abdullah [18] defined Sine trigonometric Spherical fuzzy aggregation operators and their application in decision support system using TOPSIS, VIKOR methods. Qiyas et al. [19] defined the generalized interval-valued picture fuzzy linguistic induced hybrid operator and TOPSIS method for linguistic group decision-making.

From the above discussion, we are obliged that the FCNs has an operative consistency to reveal the debate and possible item which appears in real-life problems. From the above discussion inspired me as well as compiled me to make some contribution to fuzzy aggregation. Therefore, the following is a list of the main study novelties:

(1) To use Dombi t-norm and Dombi t-conorm, which can give DMs more options, to define some new operation laws for FCNs.

(2) To originate fuzzy credibility Dombi arithmetic aggregation operators, including fuzzy credibility Dombi weighted averaging (FCDWA), fuzzy credibility Dombi ordered weighted averaging (FCDOWA) and fuzzy credibility Dombi hybrid averaging (FCDHA) operators. Further, some basic properties like idempotency, monotonicity, boundedness, and some limiting cases of these operators are also investigated.

(3) To suggest an entropy measure for fuzzy credibility data that can be used to determine the criteria's unknown weights.

(4) To develop an MCGDM model based on the proposed fuzzy credibility Dombi operator and modified TOPSIS method to handle the fuzzy credibility decision problems.

(5) The application of the advised strategy is illustrated through a real-world problem involving the enterprise resource planning problem.

The paper is designed as followed: Introduction is contained in the order of the first section. The preliminaries are contained in the order of the second section. Third section contains the definitions Dombi operation for the FCNs and developed fuzzy credibility Dombi weighted averaging (FCDWA) operator, fuzzy credibility Dombi ordered weighted averaging (FCDOWA) operator, fuzzy credibility

Dombi hybrid weighted averaging (FCDHWA) operator. To create a fuzzy credibility MAGDM approach, we utilized the defined operators in section four. In order of section five, effects of parameters the decision-making results are analyzed. In order of section six, a comparative analysis is done, in which we have demonstrated the effectiveness of the anticipated method of procedure. Finally, in section seven, we have placed a conclusion.

2. Preliminaries

Some important definitions are discussed here for convinced in forward study.

Definition 2.1. [9] Let a set of FCNs on W is defined as:

$$\Upsilon = \{(w, \mu_i(w), \xi_i(w)|w \in W\}, \tag{2.1}$$

for all $\mu_i : W \to [0,1], \xi_i : W \to [0,1]$, given that $\mu_i|\xi_i$ are the membership degree and fuzzy credibility numbers respectively, then $(w, \mu_i(w), \xi_i(w))$.

Definition 2.2. [23] Let $\Upsilon_1 = (\mu_1, \xi_1)$ and $\Upsilon_2 = (\mu_2, \xi_2)$ are two fuzzy credibility numbers. Then, their relations are defined as follows:

(1) $\Upsilon_1 = \{\langle w, \mu_{\Upsilon_1}(w), \xi_{\Upsilon_1}(w)\rangle | w \in W\}$,

(2) $\Upsilon_1 \wedge \Upsilon_2 = \{\langle w, \min\{\mu_{\Upsilon_1}(w), \mu_{\Upsilon_2}(w)\}, \max\{\xi_{\Upsilon_1}(w), \xi_{\Upsilon_2}(w)\}\rangle | w \in W\}$,

(3) $\Upsilon_1 \vee \Upsilon_2 = \{\langle w, \max\{\mu_{\Upsilon_1}(w), \mu_{\Upsilon_2}(w)\}, \min\{\xi_{\Upsilon_1}(w), \xi_{\Upsilon_2}(w)\} | w \in W\}$,

(4) $\Upsilon_1 \oplus \Upsilon_2 = \{\langle \mu_{\Upsilon_1}(w) + \mu_{\Upsilon_2}(w) - \mu_{\Upsilon_1}(w)\mu_{\Upsilon_2}(w), \xi_{\Upsilon_1}(w) + \xi_{\Upsilon_2}(w) - \xi_{\Upsilon_1}(w)\xi_{\Upsilon_2}(w)\rangle\}$,

(5) $\Upsilon_1 \otimes \Upsilon_2 = \{\langle \mu_{\Upsilon_1}(w)\mu_{\Upsilon_2}(w), \xi_{\Upsilon_1}(w)\xi_{\Upsilon_2}(w)\rangle\}$,

(6) $\psi\Upsilon_1 = \{1 - (1 - \mu_1(w)^{\psi}), 1 - (1 - \xi_1(w)^{\psi})\}$,

(7) $\Upsilon_1^{\psi} = \{\mu_1(w)^{\psi}, \xi_1(w)^{\psi}\}$.

Definition 2.3. [11] Let $\Upsilon_i = (\mu_i, \xi_i)$ be a FCNs. Then, the score function $E(\Upsilon_i)$ is described:

$$E(\Upsilon_i) = [\mu_i\xi_i + (\mu_i + \xi_i)/2]/2 \text{ for } (\Upsilon_i) \in [0,1] \tag{2.2}$$

Definition 2.4. [6] Let μ and ξ are two numbers from real numbers $\mathbb{R}$, i.e., $\mu, \xi \in \mathbb{R}$. Then, Dombi's t-norm and t-conorm are defined with the assistance of an expression such that,

$$\begin{aligned} Dom(\mu, \xi) &= \frac{1}{1 + \left\{\left(\frac{1-\mu}{\mu}\right)^{\varpi} + \left(\frac{1-\xi}{\xi}\right)^{\varpi}\right\}^{1/\varpi}} \\ Dom^{\mathbb{C}}(\mu, \xi) &= 1 - \frac{1}{1 + \left\{\left(\frac{\mu}{1-\mu}\right)^{\varpi} + \left(\frac{\xi}{1-\xi}\right)^{\varpi}\right\}^{1/\varpi}} \end{aligned} \tag{2.3}$$

where $\varpi \geq 1$ and $(w, y) \in [0,1] * [0,1]$.

Fuzzy credibility Dombi operation laws

Definition 2.5. Let us we have two FCNs $\Upsilon_1 = (\mu_1, \xi_1)$ and $\Upsilon_2 = (\mu_2, \xi_2)$, $\varpi \geq 1, \psi > 0$. Then, Dombi's t-norm and t-conorm operation for FCNs are defined as:

$$\Upsilon_1 \oplus \Upsilon_2 = \left(\begin{array}{c} 1 - \frac{1}{1+\left\{\left(\frac{\mu_1}{1-\mu_1}\right)^{\varpi}+\left(\frac{\mu_2}{1-\mu_2}\right)^{\varpi}\right\}^{1/\varpi}}, \\ 1 - \frac{1}{1+\left\{\left(\frac{\xi_1}{1-\xi_1}\right)^{\varpi}+\left(\frac{\xi_2}{1-\xi_2}\right)^{\varpi}\right\}^{1/\varpi}} \end{array} \right)$$

$$\Upsilon_1 \otimes \Upsilon_2 = \left(\frac{1}{1+\left\{\left(\frac{1-\mu_1}{\mu_1}\right)^{\varpi}+\left(\frac{1-\mu_2}{\mu_2}\right)^{\varpi}\right\}^{1/\varpi}}, \frac{1}{1+\left\{\left(\frac{1-\xi_1}{\xi_1}\right)^{\varpi}+\left(\frac{1-\xi_2}{\xi_2}\right)^{\varpi}\right\}^{1/\varpi}} \right)$$

$$\psi.\Upsilon_1 = \left(1 - \frac{1}{1+\left\{\psi\left(\frac{\mu_1}{1-\mu_1}\right)^{\varpi}\right\}^{1/\varpi}}, 1 - \frac{1}{1+\left\{\psi\left(\frac{\xi_1}{1-\xi_1}\right)^{\varpi}\right\}^{1/\varpi}} \right)$$

$$\Upsilon_1^{-\psi} = \left(\frac{1}{1+\left\{\psi\left(\frac{1-\mu_1}{\mu_1}\right)^{\varpi}\right\}^{1/\varpi}}, \frac{1}{1+\left\{\psi\left(\frac{1-\xi_1}{\xi_1}\right)^{\varpi}\right\}^{1/\varpi}} \right).$$

Theorem 2.1. Let $\Upsilon = (\mu, \xi)$ and $\Upsilon_1 = (\mu_1, \xi_1)$, $\Upsilon_2 = (\mu_2, \xi_2)$ be two FCNs. Then, we have the following equations:

(1) $\Upsilon_1 \oplus \Upsilon_2 = \Upsilon_2 \oplus \Upsilon_1$;
(2) $\Upsilon_1 \otimes \Upsilon_2 = \Upsilon_2 \otimes \Upsilon_1$;
(3) $\psi(\Upsilon_1 \oplus \Upsilon_2) = \psi\Upsilon_1 \oplus \psi\Upsilon_2, \psi > 0$;
(4) $(\Upsilon_1 \otimes \Upsilon_2)^{\psi} = \Upsilon_1^{\psi} \otimes \Upsilon_2^{\psi}$;
(5) $\psi_1\Upsilon \oplus \psi_2\Upsilon = (\psi_1 \oplus \psi_2)\Upsilon$;
(6) $\Upsilon^{\psi_1} \otimes \Upsilon^{\psi_2} = \Upsilon^{(\psi_1 \otimes \psi_2)}$.

For these three FCNs Υ, Υ_1 and Υ_2 where $\psi, \psi_1, \psi_2 > 0$, we obtain

$$\Upsilon_1 \oplus \Upsilon_2 = \left(1 - \frac{1}{1+\left\{\left(\frac{\mu_1}{1-\mu_1}\right)^{\varpi}+\left(\frac{\mu_2}{1-\mu_2}\right)^{\varpi}\right\}^{1/\varpi}}, 1 - \frac{1}{1+\left\{\left(\frac{\xi_1}{1-\xi_1}\right)^{\varpi}+\left(\frac{\xi_2}{1-\xi_2}\right)^{\varpi}\right\}^{1/\varpi}} \right)$$
$$\left(1 - \frac{1}{1+\left\{\left(\frac{\mu_2}{1-\mu_2}\right)^{\varpi}+\left(\frac{\mu_1}{1-\mu_1}\right)^{\varpi}\right\}^{1/\varpi}}, 1 - \frac{1}{1+\left\{\left(\frac{\xi_2}{1-\xi_2}\right)^{\varpi}+\left(\frac{\xi_1}{1-\xi_1}\right)^{\varpi}\right\}^{1/\varpi}} \right)$$
$$= \Upsilon_2 \oplus \Upsilon_1.$$

$$\Upsilon_1 \otimes \Upsilon_2 = \left(\frac{1}{1+\left\{\left(\frac{1-\mu_1}{\mu_1}\right)^{\varpi}+\left(\frac{1-\mu_2}{\mu_2}\right)^{\varpi}\right\}^{1/\varpi}}, \frac{1}{1+\left\{\left(\frac{1-\xi_1}{\xi_1}\right)^{\varpi}+\left(\frac{1-\xi_2}{\xi_2}\right)^{\varpi}\right\}^{1/\varpi}} \right)$$
$$= \left(\frac{1}{1+\left\{\left(\frac{1-\mu_2}{\mu_2}\right)^{\varpi}+\left(\frac{1-\mu_1}{\mu_1}\right)^{\varpi}\right\}^{1/\varpi}}, \frac{1}{1+\left\{\left(\frac{1-\xi_2}{\xi_2}\right)^{\varpi}+\left(\frac{1-\xi_1}{\xi_1}\right)^{\varpi}\right\}^{1/\varpi}} \right)$$
$$= \Upsilon_2 \otimes \Upsilon_1.$$

Let $m = 1 - \frac{1}{1+\left\{\left(\frac{\mu_1}{1-\mu_1}\right)^{\varpi}+\left(\frac{\mu_2}{1-\mu_2}\right)^{\varpi}\right\}^{1/\varpi}}$, then we have $\frac{m}{1-m} = \left\{\left(\frac{\mu_1}{1-\mu_1}\right)^{\varpi}+\left(\frac{\mu_2}{1-\mu_2}\right)^{\varpi}\right\}^{1/\varpi}$ and $\left(\frac{m}{1-m}\right)^{\varpi} = \left(\frac{\mu_1}{1-\mu_1}\right)^{\varpi}+\left(\frac{\mu_2}{1-\mu_2}\right)^{\varpi}$.

Using above terms, we get

$$
\begin{aligned}
\psi(\Upsilon_1 \oplus \Upsilon_2) &= \psi\left(1-\frac{1}{1+\left\{\left(\frac{\mu_1}{1-\mu_1}\right)^{\varpi}+\left(\frac{\mu_2}{1-\mu_2}\right)^{\varpi}\right\}^{1/\varpi}}, 1-\frac{1}{1+\left\{\left(\frac{\xi_1}{1-\xi_1}\right)^{\varpi}+\left(\frac{\xi_2}{1-\xi_2}\right)^{\varpi}\right\}^{1/\varpi}}\right) \\
&= \left(1-\frac{1}{1+\left\{\psi\left(\frac{\mu_1}{1-\mu_1}\right)^{\varpi}+\psi\left(\frac{\mu_2}{1-\mu_2}\right)^{\varpi}\right\}^{1/\varpi}}, 1-\frac{1}{1+\left\{\psi\left(\frac{\xi_1}{1-\xi_1}\right)^{\varpi}+\psi\left(\frac{\xi_2}{1-\xi_2}\right)^{\varpi}\right\}^{1/\varpi}}\right) \\
\psi\Upsilon_1 \oplus \psi\Upsilon_2 &= \left(1-\frac{1}{1+\left\{\psi\left(\frac{\mu_1}{1-\mu_1}\right)^{\varpi}\right\}^{1/\varpi}}, 1-\frac{1}{1+\left\{\psi\left(\frac{\xi_1}{1-\xi_1}\right)^{\varpi}\right\}^{1/\varpi}}\right) \\
&\oplus \left(1-\frac{1}{1+\left\{\psi\left(\frac{\mu_2}{1-\mu_2}\right)^{\varpi}\right\}^{1/\varpi}}, 1-\frac{1}{1+\left\{\psi\left(\frac{\xi_2}{1-\xi_2}\right)^{\varpi}\right\}^{1/\varpi}}\right) \\
&= \left(1-\frac{1}{1+\left\{\psi\left(\frac{\mu_1}{1-\mu_1}\right)^{\varpi}+\psi\left(\frac{\mu_2}{1-\mu_2}\right)^{\varpi}\right\}^{1/\varpi}}, 1-\frac{1}{1+\left\{\psi\left(\frac{\xi_1}{1-\xi_1}\right)^{\varpi}+\psi\left(\frac{\xi_2}{1-\xi_2}\right)^{\varpi}\right\}^{1/\varpi}}\right) \\
&= \psi(\Upsilon_1 \oplus \Upsilon_2)
\end{aligned}
$$

$$
\begin{aligned}
(\Upsilon_1 \otimes \Upsilon_2)^{\psi} &= \left(\frac{1}{1+\left\{\left(\frac{1-\xi_1}{\xi_1}\right)^{\varpi}+\left(\frac{1-\xi_2}{\xi_2}\right)^{\varpi}\right\}^{1/\varpi}}, \frac{1}{1+\left\{\left(\frac{1-\xi_1}{\xi_1}\right)^{\varpi}+\left(\frac{1-\xi_2}{\xi_2}\right)^{\varpi}\right\}^{1/\varpi}}\right)^{\psi} \\
&= \left(\frac{1}{1+\left\{\psi\left(\frac{1-\xi_1}{\xi_1}\right)^{\varpi}+\psi\left(\frac{1-\xi_2}{\xi_2}\right)^{\varpi}\right\}^{1/\varpi}}, \frac{1}{1+\left\{\psi\left(\frac{1-\xi_1}{\xi_1}\right)^{\varpi}+\psi\left(\frac{1-\xi_2}{\xi_2}\right)^{\varpi}\right\}^{1/\varpi}}\right) \\
&= \left(\frac{1}{1+\left\{\psi\left(\frac{1-\xi_1}{\xi_1}\right)^{\varpi}\right\}^{1/\varpi}}, \frac{1}{1+\left\{\psi\left(\frac{1-\xi_1}{\xi_1}\right)^{\varpi}\right\}^{1/\varpi}}\right) \\
&\otimes \left(\frac{1}{1+\left\{\psi\left(\frac{1-\xi_2}{\xi_2}\right)^{\varpi}\right\}^{1/\varpi}}, \frac{1}{1+\left\{\psi(\left(\frac{1-\xi_2}{\xi_2}\right)^{\varpi}\right\}^{1/\varpi}}\right) \\
&= \Upsilon_1^{\psi} \otimes \Upsilon_2^{\psi}
\end{aligned}
$$

$$
\begin{aligned}
\psi_1\Upsilon \oplus \psi_2\Upsilon &= \left(1-\frac{1}{1+\left\{\psi_1\left(\frac{\mu}{1-\mu}\right)^{\varpi}\right\}^{1/\varpi}}, 1-\frac{1}{1+\left\{\psi_1\left(\frac{\xi}{1-\xi}\right)^{\varpi}\right\}^{1/\varpi}}\right) \\
&\oplus \left(1-\frac{1}{1+\left\{\psi_2\left(\frac{\mu}{1-\mu}\right)^{\varpi}\right\}^{1/\varpi}}, 1-\frac{1}{1+\left\{\psi_2\left(\frac{\xi}{1-\xi}\right)^{\varpi}\right\}^{1/\varpi}}\right) \\
&= \left(1-\frac{1}{1+\left\{(\psi_1+\psi_2)\left(\frac{\mu}{1-\mu}\right)^{\varpi}\right\}^{1/\varpi}}, 1-\frac{1}{1+\left\{(\psi_1+\psi_2)\left(\frac{\xi}{1-\xi}\right)^{\varpi}\right\}^{1/\varpi}}\right) \\
&= (\psi_1 \oplus \psi_2)\Upsilon
\end{aligned}
$$

$$
\begin{aligned}
\Upsilon^{\psi_1} \otimes \Upsilon^{\psi_2} &= \left(\frac{1}{1+\left\{\psi_1\left(\frac{1-\mu}{\mu}\right)^{\varpi}\right\}^{1/\varpi}}, \frac{1}{1+\left\{\psi_1\left(\frac{1-\xi}{\xi}\right)^{\varpi}\right\}^{1/\varpi}}\right) \\
&\otimes \left(\frac{1}{1+\left\{\psi_2\left(\frac{1-\mu}{\mu}\right)^{\varpi}\right\}^{1/\varpi}}, \frac{1}{1+\left\{\psi_2\left(\frac{1-\xi}{\xi}\right)^{\varpi}\right\}^{1/\varpi}}\right) \\
&= \left(\frac{1}{1+\left\{(\psi_1+\psi_2)\left(\frac{1-\mu}{\mu}\right)^{\varpi}\right\}^{1/\varpi}}, \frac{1}{1+\left\{(\psi_1+\psi_2)\left(\frac{1-\xi}{\xi}\right)^{\varpi}\right\}^{1/\varpi}}\right) \\
&= \Upsilon^{(\psi_1+\psi_2)}.
\end{aligned}
$$

3. Fuzzy credibility Dombi averaging (FCDA) operators

In this section, we proposed some aggregation operators using defined operational laws.

3.1. Fuzzy credibility Dombi weighted averaging (FCDWA) operator

With the help of the above operational laws, we can define fuzzy credibility Dombi weighted averaging aggregation operator in this section.

Definition 3.1. Let $\Upsilon_i = (\mu_i, \xi_i)(i = 1, ..., n)$ be a FCNs. Then, the fuzzy credibility Dombi weighted averaging (FCDWA) operator is a function $\Upsilon^n \to \Upsilon$, such as

$$FCDWA_\rho(\Upsilon_1, \Upsilon_2, ..., \Upsilon_n) = \bigoplus_{i=1}^{n} (\rho_i \Upsilon_i), \tag{3.1}$$

where $\rho = (\rho_1, \rho_2, ..., \rho_n)^T$ be the weight vector of $\rho_i(i = 1, ..., n)$ with $\rho_i \succ 0$ and $\sum_{i=1}^{n} \rho_i = 1$.

Theorem 3.1. Let $\Upsilon_i = (\mu_i, \xi_i)(i = 1, ..., n)$ be the set of FCNs. Then, the aggregated value by using the fuzzy credibility Dombi weighted averaging (FCDWA) operator is also a FCNs, defined as

$$\begin{aligned} FCDWA_\rho(\Upsilon_1, \Upsilon_2, ..., \Upsilon_n) &= \bigoplus_{i=1}^{n} (\rho_i \Upsilon_i) \\ &= \left(1 - \frac{1}{1+\left\{ \sum_{i=1}^{n} \rho_i \left(\frac{\mu_i}{1-\mu_i} \right)^{\varpi} \right\}^{1/\varpi}}, 1 - \frac{1}{1+\left\{ \sum_{i=1}^{n} \rho_i \left(\frac{\xi_i}{1-\xi_i} \right)^{\varpi} \right\}^{1/\varpi}} \right), \end{aligned} \tag{3.2}$$

where $\rho = (\rho_1, \rho_2, ..., \rho_n)^T$ be the weight vector of $\rho_i(i = 1, ..., n)$ with $\rho_i \succ 0$ and $\sum_{i=1}^{n} \rho_i = 1$.

Proof. This theorem is proved by using the method of mathematical induction.

Let $n = 2$, bases on the operations of FCNs, we get results in left hand side $FCDWA_\rho(\Upsilon_1, \Upsilon_2) = \Upsilon_1 \oplus \Upsilon_2 = (\mu_1, \xi_1) \oplus (\mu_2, \xi_2)$ for right hand side we get

$$\begin{aligned} &\left(1 - \frac{1}{1+\left\{ \rho_1 \left(\frac{\mu_1}{1-\mu_1} \right)^{\varpi} + \rho_2 \left(\frac{\mu_2}{1-\mu_2} \right)^{\varpi} \right\}^{1/\varpi}}, 1 - \frac{1}{1+\left\{ \rho_1 \left(\frac{\xi_1}{1-\xi_1} \right)^{\varpi} + \rho_2 \left(\frac{\xi_2}{1-\xi_2} \right)^{\varpi} \right\}^{1/\varpi}} \right) \\ &= \left(1 - \frac{1}{1+\left\{ \sum_{i=1}^{2} \rho_i \left(\frac{\mu_i}{1-\mu_i} \right)^{\varpi} \right\}^{1/\varpi}}, 1 - \frac{1}{1+\left\{ \sum_{i=1}^{2} \rho_i \left(\frac{\xi_i}{1-\xi_i} \right)^{\varpi} \right\}^{1/\varpi}} \right), \end{aligned}$$

show that it is true for $n = 2$.

Now, for $n = k$, we have

$$\begin{aligned} FCDWA_\rho(\Upsilon_1, \Upsilon_2, ..., \Upsilon_k) &= \bigoplus_{i=1}^{n} (\rho_i \Upsilon_i) \\ &= \left(1 - \frac{1}{1+\left\{ \sum_{i=1}^{k} \rho_i \left(\frac{\mu_i}{1-\mu_i} \right)^{\varpi} \right\}^{1/\varpi}}, 1 - \frac{1}{1+\left\{ \sum_{i=1}^{k} \rho_i \left(\frac{\xi_i}{1-\xi_i} \right)^{\varpi} \right\}^{1/\varpi}} \right). \end{aligned}$$

For $n = k + 1$, then we have,

$$\begin{aligned} FCDWA_\rho(\Upsilon_1, \Upsilon_2, ..., \Upsilon_k, \Upsilon_{k+1}) &= \bigoplus_{i=1}^{n} (\rho_i \Upsilon_i) \oplus (\rho_{k+1} \Upsilon_{k+1}) \\ &= \left(1 - \frac{1}{1+\left\{ \sum_{i=1}^{k} \rho_i (\frac{\mu_i}{1-\mu_i})^{\varpi} \right\}^{1/\varpi}}, 1 - \frac{1}{1+\left\{ \sum_{i=1}^{k} \rho_i \left(\frac{\xi_i}{1-\xi_i} \right)^{\varpi} \right\}^{1/\varpi}} \right) \\ &\oplus \left(1 - \frac{1}{1+\left\{ \rho_{k+1} \left(\frac{\mu_{k+1}}{1-\mu_{k+1}} \right)^{\varpi} \right\}^{1/\varpi}}, 1 - \frac{1}{1+\left\{ \rho_{k+1} \left(\frac{\xi_{k+1}}{1-\xi_{k+1}} \right)^{\varpi} \right\}^{1/\varpi}} \right) \end{aligned}$$

$$=\left(1-\frac{1}{1+\left\{\sum_{i=1}^{k+1}\rho_i\left(\frac{\mu_i}{1-\mu_i}\right)^{\varpi}\right\}^{1/\varpi}}, 1-\frac{1}{1+\left\{\sum_{i=1}^{k+1}\rho_i\left(\frac{\xi_i}{1-\xi_i}\right)^{\varpi}\right\}^{1/\varpi}}\right).$$

Thus, the result is true for $n = k + 1$. As a result of the above proof, it is clear that it is true for any n.

Theorem 3.2. (Idempotency) Let $\Upsilon_i = (\mu_i, \xi_i)$ be a set of FCNs are all identical where $(i = 1, ..., n)$ such as $\Upsilon_i = \Upsilon$, $\forall i$. Then,

$$FCDWA_\rho(\Upsilon_1, \Upsilon_2, ..., \Upsilon_n) = \Upsilon. \quad (3.3)$$

Proof. Since $\Upsilon_i = (\mu_i, \xi_i) = \Upsilon$, where $(i = 1, ..., n)$. Then, we have,

$$FCDWA_\rho(\Upsilon_1, \Upsilon_2, ..., \Upsilon_i) = \bigoplus_{i=1}^{n}(\rho_i\Upsilon_i)$$

$$=\left(1-\frac{1}{1+\left\{\sum_{i=1}^{n}\rho_i\left(\frac{\mu_i}{1-\mu_i}\right)^{\varpi}\right\}^{1/\varpi}}, 1-\frac{1}{1+\left\{\sum_{i=1}^{n}\rho_i\left(\frac{\xi_i}{1-\xi_i}\right)^{\varpi}\right\}^{1/\varpi}}\right)$$

$$=\left(1-\frac{1}{1+\left\{\left(\frac{\mu}{1-\mu}\right)^{\varpi}\right\}^{1/\varpi}}, 1-\frac{1}{1+\left\{\sum_{i=1}^{n}\rho_i\left(\frac{\xi}{1-\xi}\right)^{\varpi}\right\}^{1/\varpi}}\right)$$

$$=\left(1-\frac{1}{1+\frac{\mu}{1-\mu}}, 1-\frac{1}{1+\frac{\xi}{1-\xi}}\right)$$

$$=(\mu, \xi)$$

$$=\Upsilon.$$

thus, $FCDWA_\rho(\Upsilon_1, \Upsilon_2, ..., \Upsilon_n) = \Upsilon$ holds.

Theorem 3.3. (Boundedness) Suppose $\Upsilon_i = (\mu_i, \xi_i)(i = 1, ..., n)$ be a set of FCNs and $\Upsilon^- = \min(\Upsilon_1, \Upsilon_2, ..., \Upsilon_n)$ and $\Upsilon^+ = \max(\Upsilon_1, \Upsilon_2, ..., \Upsilon_n)$. Then,

$$\Upsilon^- \leq FCDWA_\rho(\Upsilon_1, \Upsilon_2, ..., \Upsilon_i) \leq \Upsilon^+. \quad (3.4)$$

Proof. Let $\Upsilon_i = (\mu_i, \xi_i)(i = 1, ..., n)$ be a number of FCNs. Let $\Upsilon^- = \min(\Upsilon_1, \Upsilon_2, ..., \Upsilon_i) = (\mu^-, \xi^-)$ and $\Upsilon^+ = \max(\Upsilon_1, \Upsilon_2, ..., \Upsilon_n) = (\mu^+, \xi^+)$. we have, $\Upsilon^- = \min(\mu_i), \xi^- = \max(\xi_i), \mu^+ = \max(\mu_i), \xi^+ = \min(\xi_i)$. Hence, we have the subsequent inequalities,

$$1-\frac{1}{1+\left\{\sum_{i=1}^{k+1}\rho_i\left(\frac{\mu^-}{1-\mu^-}\right)^{\varpi}\right\}^{1/\varpi}} \leq 1-\frac{1}{1+\left\{\sum_{i=1}^{n}\rho_i\left(\frac{\mu}{1-\mu}\right)^{\varpi}\right\}^{1/\varpi}} \leq 1-\frac{1}{1+\left\{\sum_{i=1}^{n}\rho_i\left(\frac{\mu^+}{1-\mu^+}\right)^{\varpi}\right\}^{1/\varpi}},$$

$$1-\frac{1}{1+\left\{\sum_{i=1}^{n}\rho_i\left(\frac{\xi^-}{1-\xi^-}\right)^{\varpi}\right\}^{1/\varpi}} \leq 1-\frac{1}{1+\left\{\sum_{i=1}^{n}\rho_i\left(\frac{\xi}{1-\xi}\right)^{\varpi}\right\}^{1/\varpi}} \leq 1-\frac{1}{1+\left\{\sum_{i=1}^{n}\rho_i\left(\frac{\xi^+}{1-\xi^+}\right)^{\varpi}\right\}^{1/\varpi}}.$$

Therefore, $\Upsilon^- \leq FCDWA_\rho(\Upsilon_1, \Upsilon_2, ..., \Upsilon_n) \leq \Upsilon^+$.

Theorem 3.4. (Monotonicity) Let $\Upsilon_i = (\mu_i, \xi_i)$ where $(i = 1, ..., n)$ be a number of FCNs, if $\Upsilon_i \leq \Upsilon_i^{'}$, $\forall i$. Then,

$$FCDWA_\rho(\Upsilon_1, \Upsilon_2, ..., \Upsilon_n) \leq FCDWA_\rho(\Upsilon_1^{'}, \Upsilon_2^{'}, ..., \Upsilon_n^{'}). \quad (3.5)$$

3.2. Fuzzy credibility Dombi ordered weighted averaging (FCDOWA) operator

Definition 3.2. Let $\Upsilon_i = (\mu_i, \xi_i)(i = 1, ..., n)$ be a set of FCNs. Then, the fuzzy credibility Dombi ordered weighted averaging operator (FCDOWAO) of dimension n is a function $FCDOWA : \Upsilon^n \to \Upsilon$, such as

$$FCDOWA_{\rho}(\Upsilon_1, \Upsilon_2, ..., \Upsilon_n) = \bigoplus_{i=1}^{n}(\rho_i \Upsilon_i), \tag{3.6}$$

with the corresponding weight vector $\rho = (\rho_1, \rho_2, ..., \rho_n)^T$ of $\rho_i|(i = 1, ..., n)$ with $\rho_i \succ 0$ and $\sum_{i=1}^{n} \rho_i = 1$. Where the permutation $(\varepsilon(1), ..., \varepsilon(n))$ of $(i = 1, ..., n)$, for which $\Upsilon_{\varepsilon(i-1)} \geq \Upsilon_{\varepsilon(i)}, \forall i = 1, ..., n$.

Theorem 3.5. Let $\Upsilon_i = (\mu_i, \xi_i)(i = 1, ..., n)$ be a set of FCNs. Then, fuzzy credibility Dombi ordered weighted averaging operator (FCDOWAO) of dimension i and function $FCDOWA : \Upsilon^n \rightarrow \Upsilon$, such that

$$\begin{aligned} FCDOWA_{\rho}(\Upsilon_1, \Upsilon_2, ..., \Upsilon_n) &= \bigoplus_{i=1}^{n}(\rho_i \Upsilon_i) \\ &= \left(1 - \frac{1}{1+\left\{\sum_{i=1}^{n}\rho_i\left(\frac{\mu_{\varepsilon(i)}}{1-\mu_{\varepsilon(i)}}\right)^{\varpi}\right\}^{1/\varpi}}, 1 - \frac{1}{1+\left\{\sum_{i=1}^{n}\rho_i\left(\frac{\xi_{\varepsilon(i)}}{1-\xi_{\varepsilon(i)}}\right)^{\varpi}\right\}^{1/\varpi}}\right), \end{aligned} \tag{3.7}$$

with the corresponding weight vector $\rho = (\rho_1, \rho_2, ..., \rho_n)^T$ of $\rho_i|(i = 1, ..., n)$ with $\rho_i \succ 0$ and $\sum_{i=1}^{n} \rho_i = 1$. Where the permutation $(\varepsilon(1), ..., \varepsilon(n))$ of $(i = 1, ..., n)$, for which $\Upsilon_{\varepsilon(i-1)} \geq \Upsilon_{\varepsilon(i)}(i = 1, ..., n)$.

Theorem 3.6. (Idempotency) Let $\Upsilon_i(i = 1, ..., n)$ are identical, i.e., $\Upsilon_i = \Upsilon$ for all n. Then,

$$FCDOWA_{\rho}(\Upsilon_1, \Upsilon_2, ..., \Upsilon_n) = \Upsilon. \tag{3.8}$$

Theorem 3.7. (Boundedness) Let $\Upsilon_i = (\mu_i, \xi_i)(i = 1, ..., n)$ be a number of FCNs and $\Upsilon^- = \min(\Upsilon_1, \Upsilon_2, ..., \Upsilon_n)$ and $\Upsilon^+ = \max(\Upsilon_1, \Upsilon_2, ..., \Upsilon_n)$. Then,

$$\Upsilon^- \leq FCDOWA_{\rho}(\Upsilon_1, \Upsilon_2, ..., \Upsilon_n) \leq \Upsilon^+. \tag{3.9}$$

Theorem 3.8. (Monotonicity) Let $\Upsilon_i = (\mu_i, \xi_i)(i = 1, ..., n)$ be a number of FCNs, if $\Upsilon_i \leq \Upsilon_i^{'}$ for all i. Then,

$$FCDOWA_{\rho}(\Upsilon_1, \Upsilon_2, ..., \Upsilon_n) \leq FCDOWA_{\rho}(\Upsilon_1^{'}, \Upsilon_2^{'}, ..., \Upsilon_n^{'}). \tag{3.10}$$

Theorem 3.9. (Commutivity) Let $\Upsilon_i(i = 1, ..., n)$ be a set of FCNs. Then,

$$FCDOWA_{\rho}(\Upsilon_1, \Upsilon_2, ..., \Upsilon_n) = FCDOWA_{\rho}(\Upsilon_1^{'}, \Upsilon_2^{'}, ..., \Upsilon_n^{'}), \tag{3.11}$$

where $\Upsilon_i^{'}(i = 1, ..., n)$ is any permutation of $\Upsilon_i(i = 1, ..., n)$.

3.3. Fuzzy credibility Dombi hybrid weighted averaging (FCDHWA) operator

Definition 3.3. Let $\Upsilon_i = (\mu_i, \xi_i)(i = 1, ..., n)$ be a set of FCNs. Then, the fuzzy credibility Dombi hybrid weighted averaging operator (FCDHWAO) of dimension n and function $FCDHWA : \Upsilon^n \rightarrow \Upsilon$ with correlated weight vector $\rho = (\rho_1, \rho_2, ..., \rho_n)|\rho_i \succ 0$ and $\sum_{i=1}^{n} \rho_i = 1$. Therefore, FCDHWA operator can be evaluated as:

$$\begin{aligned} FCDHWA_{\rho}(\Upsilon_1, \Upsilon_2, ..., \Upsilon_n) &= \bigoplus_{i=1}^{n}(\rho_i \Upsilon^*_{i(\varepsilon)}) \\ &= \left(1 - \frac{1}{1+\left\{\sum_{i=1}^{n}\rho_i\left(\frac{\mu^*_{\varepsilon(i)}}{1-\mu^*_{\varepsilon(i)}}\right)^{\varpi}\right\}^{1/\varpi}}, 1 - \frac{1}{1+\left\{\sum_{i=1}^{n}\rho_i(\left(\frac{\xi^*_{\varepsilon(i)}}{1-\xi^*_{\varepsilon(i)}}\right)^{\varpi}\right\}^{1/\varpi}}\right), \end{aligned} \tag{3.12}$$

with the corresponding weight vector $\rho = (\rho_1, \rho_2, ..., \rho_n)^T$ of $\rho_i(i = 1, ..., n)|\rho_i \succ 0$ and $\sum_{i=1}^{n} \rho_i = 1$. Where $\Upsilon_{i(\varepsilon)}$ is the i^{th} biggest weighted credibility fuzzy values $\Upsilon_i^*(\Upsilon_i^* = n\rho_i\Upsilon_i, i = 1, ..., n)$ and $\rho = (\rho_1, \rho_2, ..., \rho_n)^T$ be the weight vector of Υ_i with $\rho_i \succ 0$ and $\sum_{i=1}^{n} \rho_i = 1$, where the balancing coefficient is n.

4. Entropy measure for fuzzy credibility numbers

In this section, we propose the generalized and weighted generalized distance measure of fuzzy credibility sets (FCSs) respectively. Subsequently, making the utilization of the generalized distance measure, we proposed the fuzzy credibility entropy measure for FCNs to measure the fuzziness of FCNs.

4.1. Distance measure of fuzzy credibility numbers

Definition 4.1. [3] Let for any $\Upsilon = \{\Upsilon_1, \Upsilon_2, ..., \Upsilon_n\}, G = \{G_1, G_2, ..., G_n\}$ belong to FCN given that $\Upsilon_w = \{\mu_{\Upsilon_w}(w), \xi_{\Upsilon_w}(w)\}$ and $G_w = \{\mu_{G_w}(w), \xi_{G_w}(w)\}\,(w = 1, ..., n)$. Then, Υ and G are the two points in which the generalized distance measure (GDM) is defined $\forall\phi > 0 \in \mathbb{R}$.

$$d_g(\Upsilon, G) = \left(\frac{1}{2n}\sum_{w=1}^{n}\left(\left|(\mu_{\Upsilon_w})^2 - (\mu_{G_w})^2\right|^{\phi} + \left|(\xi_{\Upsilon_w})^2 - (\xi_{G_w})^2\right|^{\phi}\right)\right)^{\frac{1}{\phi}}. \tag{4.1}$$

Definition 4.2. [3] Let for any $\Upsilon = \{\Upsilon_1, \Upsilon_2, ..., \Upsilon_n\}, G = \{G_1, G_2, ..., G_n\}$ belong to FCNs, given that $\Upsilon_w = \{\mu_{\Upsilon_w}(w), \xi_{\Upsilon_w}(w)\}$ and $G_w = \{\mu_{G_w}(w), \xi_{G_w}(w)\}\,(w = 1, ..., n)$. Then, Υ and G are the two points in which the weighted generalized distance measure (WGDM) between Υ and G is defined $\forall\phi > 0 \in \mathbb{R}$.

$$d_{wg}(\Upsilon, G) = \left(\frac{1}{2n}\sum_{w=1}^{n}\omega_w\left(\left|(\mu_{\Upsilon_w})^2 - (\mu_{G_w})^2\right|^{\phi} + \left|(\xi_{\Upsilon_w})^2 - (\xi_{G_w})^2\right|^{\phi}\right)\right)^{\frac{1}{\phi}}, \tag{4.2}$$

where $\omega_w(w = 1, ..., n)$ stands for weight with condition that $\omega_w \geq 0$ and $\sum_{w=1}^{n}\omega_w = 1$.

Definition 4.3. [3] Let $\Upsilon_w = \{\mu_{\Upsilon_w}(w), \xi_{\Upsilon_w}(w)\}\,(w = 1, ..., n)$ belong to FCNs. Then, the GDM defined in previous definition reduced as follows:

$$d_{wg}(\Upsilon_1, \Upsilon_2) = \left(\frac{1}{2}\left(\left|(\mu_{\Upsilon_1})^2 - (\mu_{\Upsilon_2})^2\right|^{\phi} + \left|(\xi_{\Upsilon_1})^2 - (\xi_{\Upsilon_2})^2\right|^{\phi}\right)\right)^{\frac{1}{\phi}}, \phi > 0\,(\in \mathbb{R}). \tag{4.3}$$

For any two $\Upsilon_1, \Upsilon_2 \in FCN$, the following properties must be satisfied by the given GDMs.

(1) $1 \geq d(\Upsilon_1, \Upsilon_2) \geq 0$,

(2) $d(\Upsilon_1, \Upsilon_2) \Leftrightarrow \Upsilon_1 = \Upsilon_2 = 1$,

(3) $d(\Upsilon_2, \Upsilon_1) = d(\Upsilon_1, \Upsilon_2)$.

4.2. Fuzzy credibility entropy measure

In this portion, we proposed an entropy measure for FCNs using distance measure and the concept of Guo and Song [7].

Definition 4.4. [3] Let for any $\Upsilon = \{\Upsilon_1, \Upsilon_2, ..., \Upsilon_n\} \in FCNs$, given that $\Upsilon_w = \{\mu_{\Upsilon_w}(w), \xi_{\Upsilon_w}(w)\}$ is FCNs $\forall\,(w = 1, ..., n)$. Then, for FCNs entropy measure is defined as:

$$E(\Upsilon) = \frac{1}{n}\sum_{w=1}^{n}\left[\{1 - (d\,(\Upsilon_w, \Upsilon_w^c))\}\,\frac{1 + (v_{\Upsilon_w})^2}{2}\right], \tag{4.4}$$

where the degree of hesitancy is $v_{\Upsilon_w} = \sqrt[1]{1 - \mu^2_{\Upsilon_w} - \xi^2_{\Upsilon_w}}$.

Theorem 4.1. Let for any $\Upsilon = \{\Upsilon_1, \Upsilon_2, ..., \Upsilon_n\}, G = \{G_1, G_2, ..., G_n\}$ belong to FCNs, given that $\Upsilon_w = \{\mu_{\Upsilon_w}(w), \xi_{\Upsilon_w}(w)\} \,|\, G_w = \{\mu_{G_w}(w), \xi_{G_w}(w)\}, \forall\, (w = 1, ..., n)$. Then, $E(\Upsilon)$ and $E(G)$ be the entropy measure which fulfills the following properties:

(1) $E(\Upsilon) \leq E(G)$ if $\Upsilon \leq G$, then, $\mu_\Upsilon(w) \leq \mu_G(w), \xi_\Upsilon(w) \leq \xi_G(w), \forall w \in \psi$.

(2) $E(\Upsilon) \leq E(\Upsilon^c)$.

Proof. (1) Let $\Upsilon \leq G$. then, $\mu_\Upsilon(w) \leq \mu_G(W), \xi_\Upsilon(w) \leq \xi_G(w), \forall w \in \psi$.

To show that $E(G) - E(\Upsilon) \geq 0$, we have

$$\begin{aligned} E(G) - E(\Upsilon) &= \frac{1}{2n}\sum_{w=1}^{n}\left[(1 - \left|\mu_\Upsilon^2 - \xi_\Upsilon^2\right|)\left(2 - \mu_\Upsilon^2 - \xi_\Upsilon^2\right) - \left(1 - \left|\mu_G^2 - \xi_G^2\right|\right)\left(2 - \mu_G^2 - \xi_G^2\right)\right] \\ &= \frac{1}{2n}\sum_{w=1}^{n}\left[(1 + (\mu_\Upsilon^2 - \xi_\Upsilon^2))\left(2 - \mu_\Upsilon^2 - \xi_\Upsilon^2\right) - \left(1 + (\mu_G^2 - \xi_G^2)\right)\left(2 - \mu_G^2 - \xi_G^2\right)\right] \\ &= \frac{1}{2n}\sum_{w=1}^{n}\left[(2 - \mu_\Upsilon^2 - \xi_\Upsilon^2 - 3\xi_\Upsilon^2) - \left(2 - \mu_G^2 - \xi_G^2 - 3\xi_G^2\right)\right] \\ &= \frac{1}{2n}\sum_{w=1}^{n}\left[\left(\mu_\Upsilon^2 - \mu_\Upsilon^2\right) + \left(\xi_G^2 - \xi_\Upsilon^2\right) + \left(3\xi_G^2 - 3\xi_\Upsilon^2\right)\right] \geq 0, \end{aligned}$$

all the powers are even, then implies that

$$E(G) - E(\Upsilon) \geq 0.$$

(2) Since, we have

$$\begin{aligned} E(\Upsilon) &= \frac{1}{n}\sum_{w=1}^{n}\left[\{1 - (d\,(\Upsilon_w, \Upsilon_w^c))\}\,\frac{1 + (\nu_{\Upsilon_w})^2}{2}\right] \\ &= \frac{1}{n}\sum_{w=1}^{n}\left[\left\{1 - \left|\mu_\Upsilon^2 - \xi_\Upsilon^2\right|\right\}\frac{\left(2 - \mu_\Upsilon^2 - \xi_\Upsilon^2\right)}{2}\right] \\ &= \frac{1}{n}\sum_{w=1}^{n}\left[\left\{1 - \left|\xi_\Upsilon^2 - \mu_\Upsilon^2\right|\right\}\frac{\left(2 - \xi_\Upsilon^2 - \mu_\Upsilon^2\right)}{2}\right] \\ &= E(\Upsilon^c). \end{aligned}$$

5. Fuzzy credibility TOPSIS method

5.1. MAGDM approach for fuzzy credibility numbers

In this section, we proposed an approach for MAGDM problem with the fuzzy credibility information. The problems of MAGDM can also be addressed in the decision matrix form where the columns and rows represents the attribute/alternatives respectively. That's why, decision-matrix is represented by $D_{n\times m}$. A set $\{\Re_1, \Re_2, ..., \Re_n\}$ is considered which represents n alternatives and $\{\Im_1, \Im_2, ..., \Im_m\}$ represents m criteria/attributes. $\rho_j \in [0, 1]$ be the unknown weight vector of m criteria/attributes such that $\sum_{j=1}^{m} \rho_j = 1$. Suppose $D^{(k)} = \left[\Upsilon_{ij}^{(k)}\right]_{n\times m} = \left\langle \mu_{\Upsilon_{ij}}^{(k)}, \xi_{\Upsilon_{ij}}^{(k)} \right\rangle_{n\times m,}$ $k \in 1, ..., e$, denotes the fuzzy credibility Dombi decision matrix, where the degree of alternatives is represented by μ_{ij} and $\Im_i$ be the attribute contemplated by the DMs, the degree of the alternative is represented by ξ_{ij} which is neutral for the criteria $\Im_i$ considered by DMs.

$$D^{(k)}_{n\times m} = \begin{matrix} \Re_1 \\ \Re_2 \\ \Re_3 \\ \vdots \\ \Re_i \end{matrix} \begin{pmatrix} \Im_1 & \Im_2 & \Im_2 & \cdots & \Im_m \\ \left(\mu^{(k)}_{\Upsilon_{11}}, \xi^{(k)}_{\Upsilon_{11}}\right) & \left(\mu^{(k)}_{\Upsilon_{12}}, \xi^{(k)}_{\Upsilon_{12}}\right) & \left(\mu^{(k)}_{\Upsilon_{13}}, \xi^{(k)}_{\Upsilon_{13}}\right) & \cdots & \left(\mu^{(k)}_{\Upsilon_{1m}}, \xi^{(k)}_{\Upsilon_{1m}}\right) \\ \left(\mu^{(k)}_{\Upsilon_{21}}, \xi^{(k)}_{\Upsilon_{21}}\right) & \left(\mu^{(k)}_{\Upsilon_{22}}, \xi^{(k)}_{\Upsilon_{22}}\right) & \left(\mu^{(k)}_{\Upsilon_{23}}, \xi^{(k)}_{\Upsilon_{23}}\right) & \cdots & \left(\mu^{(k)}_{\Upsilon_{2m}}, \xi^{(k)}_{\Upsilon_{2m}}\right) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \left(\mu^{(k)}_{\Upsilon_{n1}}, \xi^{(k)}_{\Upsilon_{n1}}\right) & \left(\mu^{(k)}_{\Upsilon_{n2}}, \xi^{(k)}_{\Upsilon_{n2}}\right) & \left(\mu^{(k)}_{\Upsilon_{n3}}, \xi^{(k)}_{\Upsilon_{n3}}\right) & \cdots & \left(\mu^{(k)}_{\Upsilon_{nm}}, \xi^{(k)}_{\Upsilon_{nm}}\right) \end{pmatrix}.$$

It should be noted that all the data about the weights of DM and attributes are known in the context of DM.

5.2. *TOPSIS method for fuzzy credibility numbers*

This procedure contains fourteen steps. For the proof of fuzzy credibility numbers "MAGDM" problem utilizing the "TOPSIS" method, for which the following steps of the procedure are developed.

Step 1. Only two types of attributes are there in MAGDM problem, one is the cost type and the other is benefit type. If the attribute is cost type, we convert it to benefit type using the following equations.

$$N^k_{ij} = \begin{cases} \left(\mu_{ij}, \xi_{k_{ij}}\right) \text{ if attribute is benefit type,} \\ \left(\xi_{k_{ij}}, \mu_{k_{ij}}\right) \text{ if attribute is cost type.} \end{cases} \tag{5.1}$$

For fuzzy credibility Dombi ordered weighted averaging (FCDOWA) operator decision matrix are arranged in descending order with the help of score function.

$$N^k_{ij} = \begin{cases} \left(\mu_{k_{\varepsilon(ij)}}, \xi_{k_{\varepsilon(ij)}}\right) \text{ if attribute is benefit type,} \\ \left(\xi_{k_{\varepsilon(ij)}}, \mu_{k_{\varepsilon(ij)}}\right) \text{ if attribute is cost type.} \end{cases} \tag{5.2}$$

For fuzzy credibility Dombi hybrid weighted averaging (FCDHWA) operator decision matrix sorted by the product of number of attributes weight and respective value of DM.

$$N^k_{ij} = \begin{cases} \left(\mu^*_{k_{\varepsilon(ij)}}, \xi^*_{k_{\varepsilon(ij)}}\right) \text{ if attribute is benefit type,} \\ \left(\xi^*_{k_{\varepsilon(ij)}}, \mu^*_{k_{\varepsilon(ij)}}\right) \text{ if attribute is cost type.} \end{cases} \tag{5.3}$$

Step 2. The opinion of each single decision matrix is close to the group decision ideal solution (GDIS) and as result, the computation of best GDIS is done by taking average of all the outlook of each single decision matrix. Here in this step, we take fuzzy credibility weighted average of alternatives value to calculate the GDIS. The corresponding criteria which is specified via the decision matrix with in view of the same weight of decision matrix values as follows:

Here GDIS is denoted by GD.

$$GD = \begin{pmatrix} D_{11} & D_{12} & \cdots & D_{1i} \\ D_{21} & D_{22} & \cdots & D_{2i} \\ \vdots & \vdots & \ddots & \vdots \\ D_{m1} & D_{m2} & \cdots & D_{mi} \end{pmatrix},$$

where

$$D_{ij} = \sum_{k=1}^{\psi} \frac{1}{\psi} N_{ij}^{k} = \left\{ 1 - \frac{1}{1 + \left\{ \psi \left(\frac{\mu_1}{1-\mu_1} \right)^{\varpi} \right\}^{1/\varpi}}, 1 - \frac{1}{1 + \left\{ \psi \left(\frac{\xi_1}{1-\xi_1} \right)^{\varpi} \right\}^{1/\varpi}} \right\}. \tag{5.4}$$

Step 3. Computed the group right and left ideal solution GRIS, GLIS as follows:
Here GRIS, GLIS is denoted by GR and GL, respectively.

$$GR = \begin{pmatrix} R_{11} & R_{12} & \cdots & R_{1n} \\ R_{21} & R_{22} & \cdots & R_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ R_{m1} & R_{m2} & \cdots & R_{mn} \end{pmatrix}$$

where

$$R_{ij} = \left(\left(N_{ij}^{k} \right) : \max_{k} \left[Sc \left(N_{ij}^{k} \right) \right] \right), \tag{5.5}$$

and

$$GL = \begin{pmatrix} L_{11} & L_{12} & \cdots & L_{1n} \\ L_{21} & L_{22} & \cdots & L_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ L_{m1} & L_{m2} & \cdots & L_{mn} \end{pmatrix}$$

where

$$L_{ij} = \left(\left(N_{ij}^{k} \right) : \min_{k} \left[Sc \left(N_{ij}^{k} \right) \right] \right). \tag{5.6}$$

Step 4. Computed the distance of the decision matrix N_{ij}^{k} to GD, GR and GL. The distance are presented symbolically as DGD, DGR and DGL, respectively. Where,

$$DGD_i^k = \left(\frac{1}{2n} \sum_{u=1}^{n} \left(\left| (\mu_{i_{ij}^k})^2 - (\mu_{D_{ij}})^2 \right|^{\phi} + \left| (\xi_{i_{ij}^k})^2 - (\xi_{D_{ij}})^2 \right|^{\phi} \right) \right)^{\frac{1}{\phi}}, \tag{5.7}$$

$$DGR_i^k = \left(\frac{1}{2n} \sum_{u=1}^{n} \left(\left| (\mu_{i_{ij}^k})^2 - (\mu_{R_{ij}})^2 \right|^{\phi} + \left| (\xi_{i_{ij}^k})^2 - (\xi_{R_{ij}})^2 \right|^{\phi} \right) \right)^{\frac{1}{\phi}}, \tag{5.8}$$

$$DGL_i^k = \left(\frac{1}{2n} \sum_{u=1}^{n} \left(\left| (\mu_{i_{ij}^k})^2 - (\mu_{L_{ij}})^2 \right|^{\phi} + \left| (\xi_{i_{ij}^k})^2 - (\xi_{L_{ij}})^2 \right|^{\phi} \right) \right)^{\frac{1}{\phi}}, \tag{5.9}$$

for $i = 1, ..., m$ and $k = 1, ..., n$.

Step 5. Computed the closeness indices (CI), using the following Yue [22] model.

$$CI^k = \frac{\sum_{i=1}^{m} DGR_i^k + \sum_{i=1}^{m} DGL_i^k}{\sum_{i=1}^{m} DGD_i^k + \sum_{i=1}^{m} DGR_i^k + \sum_{i=1}^{m} DGL_i^k}, \tag{5.10}$$

for $k = 1, ..., n$.

Step 6. Calculate decision matrices weights,

$$w^k = \frac{CI^k}{\sum\limits_{k=1}^{e} CI^k}. \tag{5.11}$$

Step 7. Here, in this step, the attributes weight are computed by means of the proposed fuzzy credibility entropy measure, for this the revised group decision ideal solution (RGDIS) is calculated as follows:

$$R_{ij} = \sum_{k=1}^{e} w^k N_{ij}^k = \left(1 - \frac{1}{1 + \left\{ w \left(\frac{\mu_1}{1-\mu_1} \right)^{\varpi} \right\}^{1/\varpi}}, 1 - \frac{1}{1 + \left\{ w \left(\frac{\xi_1}{1-\xi_1} \right)^{\varpi} \right\}^{1/\varpi}} \right). \tag{5.12}$$

Step 8. Each attribute is computed with the correspondence of fuzzy credibility entropy measure which follows as:

$$E(\Upsilon) = \frac{1}{n} \sum_{k=1}^{n} \left[\{1 - (d(\Upsilon_k, \Upsilon_k^c))\} \frac{1 + (v_{\Upsilon_k})^2}{2} \right] \tag{5.13}$$

$$E\left(\Upsilon_j\right) = E\left(Rv_{1j}, Rv_{2j}, ..., Rv_{mj}\right), j = 1, ..., n \tag{5.14}$$

Step 9. Computed the attribute weights by the which follows equation,

$$\phi\mathfrak{R}_j = \frac{1 - E\mathfrak{R}_j}{n - \sum\limits_{u=1}^{n} E\mathfrak{R}_j}, j = 1, ..., n \tag{5.15}$$

Step 10. Using attributes weight vector, to calculate the weighted normalized decision matrix are as follows:

$$DM(N)_{ij}^k = \sum_{u=1}^{n} \phi\mathfrak{R}_j N_{ij}^k = \left(1 - \frac{1}{1 + \left\{ \phi\mathfrak{R}_j \left(\frac{\mu_1}{1-\mu_1} \right)^{\varpi} \right\}^{1/\varpi}}, 1 - \frac{1}{1 + \left\{ \phi\mathfrak{R}_j \left(\frac{\xi_1}{1-\xi_1} \right)^{\varpi} \right\}^{1/\varpi}} \right), \tag{5.16}$$

for each $k = 1, ..., e$.

Step 11. Using the weighted normalized decision-metrics DM $(N)_{ij}^k$, which computes the PIS^k and NIS^k for each DMs as follows:

$$PD_{ij} = \left(\left(DM(N)_{ij}^k \right) : \max_k \left[Sc\left(DM(N)_{ij}^k \right) \right] \right), j = 1, ..., n; \tag{5.17}$$

$$ND_{ij} = \left(\left(DM(N)_{ij}^k \right) : \min_k \left[Sc\left(DM(N)_{ij}^k \right) \right] \right), j = 1, ..., n. \tag{5.18}$$

Step 12. Computed the WGDM from DM $(N)^k$ to PD^k and ND^k as follows:

$$DIS_i^{+k} = \left(\frac{1}{2n} \sum_{u=1}^{n} \phi\mathfrak{R}_j \left(\left| (\mu_{DM(i)^k})^2 - (\mu_{PD^k})^2 \right|^{\phi} + \left| (\xi_{DM(i)^k})^2 - (\xi_{PD^k})^2 \right|^{\phi} \right) \right)^{\frac{1}{\phi}}, \tag{5.19}$$

and

$$DIS_i^{-k} = \left(\frac{1}{2n} \sum_{u=1}^{n} \phi\mathfrak{R}_j \left(\left| (\mu_{DM(i)^k})^2 - (\mu_{iD^k})^2 \right|^{\phi} + \left| (\xi_{DM(i)^k})^2 - (\xi_{iD^k})^2 \right|^{\phi} \right) \right)^{\frac{1}{\phi}}, \tag{5.20}$$

$\forall i = 1, ..., m$.

Step 13. The revised closeness indices (RCIs) for every single decision matrix are calculated as follows:

$$RCI_i^k = \frac{DIS_i^{-k}}{DIS_i^{+k} + DIS_i^{-k}}. \tag{5.21}$$

Step 14. Calculate the final revised closeness indices (FRCI) by using the decision matrix as follows:

$$FRCI_i = \sum_{k=1}^{e} w^k.RCI_i^k. \tag{5.22}$$

The calculated FRCIs value is ranked by descending order, the finest alternative has lager value. In Figure 1, we show the algorithm of the TOPSIS method.

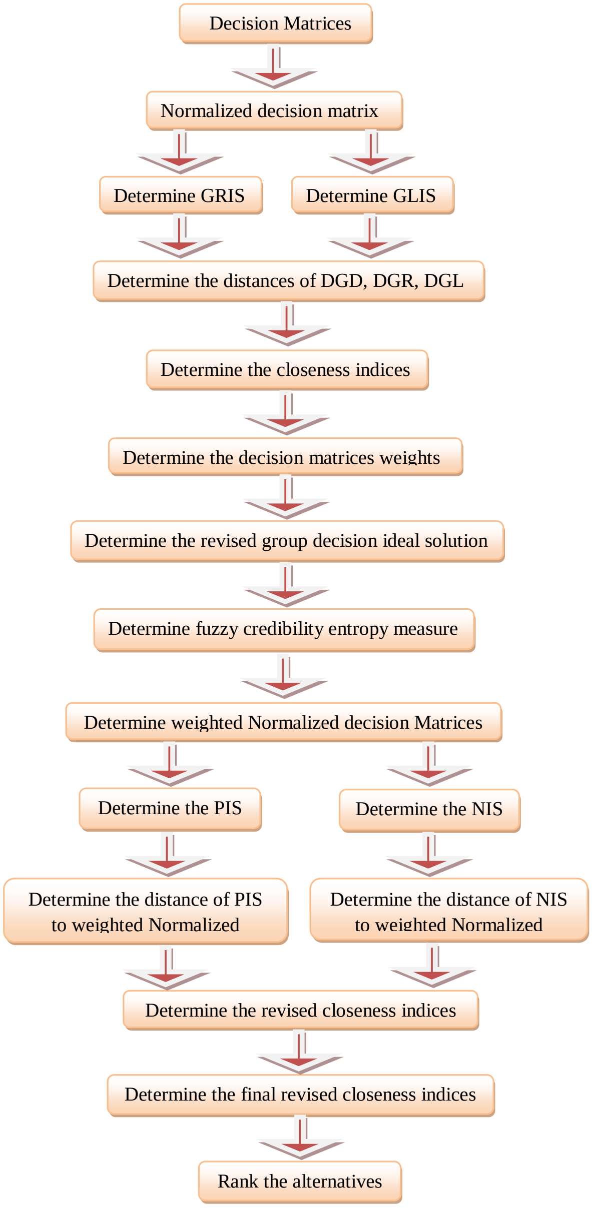

Figure 1. Algorithm of the TOPSIS method.

6. Fuzzy credibility Dombi TOPSIS method

In this section, a numerical method is used to select the best alternative, which was originally used to demonstrate the MAGDM method designation. The comparison of the proposed technique and the existing technique using fuzzy credibility information is used to demonstrate the characteristic and advantage of the proposed method.

6.1. Example

This Example is adopted from [22]. A company want to apply an "enterprise resource planning system" (ERPS). A group of three decision-makers are selected to choose the best alternative out of seven $\Re_i\,(i = 1, 2, ..., 7)$ for enterprise resource planning (ERP) vendors and systems. In order to find the credibility of the selected vendors and systems, all the candidates are evaluated under six attributes. They are Function, Technology, Strategic fitness, Vendors ability, Vendor financial status, and Vendor reputation which as, $\Im_1$, $\Im_2$, $\Im_3$, $\Im_4$, $\Im_5$ and $\Im_6$ respectively. The decision-making committee is required to utilized FCNs to express.

6.2. Using fuzzy credibility Dombi weighted averaging (FCDWA) operator

Table 1. Fuzzy credibility information given by expert E^1.

	$\Im_1$	$\Im_2$	$\Im_3$	$\Im_4$	$\Im_5$	$\Im_6$
$\Re_1$	$(0.84, 0.34)$	$(0.43, 0.39)$	$(0.67, 0.5)$	$(0.31, 0.21)$	$(0.4, 0.78)$	$(0.3, 0.71)$
$\Re_2$	$(0.6, 0.11)$	$(0.23, 0.35)$	$(0.72, 0.31)$	$(0.11, 0.25)$	$(0.53, 0.59)$	$(0.41, 0.82)$
$\Re_3$	$(0.79, 0.19)$	$(0.11, 0.21)$	$(0.71, 0.41)$	$(0.34, 0.25)$	$(0.39, 0.91)$	$(0.13, 0.51)$
$\Re_4$	$(0.63, 0.51)$	$(0.49, 0.33)$	$(0.61, 0.43)$	$(0.49, 0.37)$	$(0.13, 0.42)$	$(0.45, 0.59)$
$\Re_5$	$(0.57, 0.36)$	$(0.5, 0.15)$	$(0.7, 0.32)$	$(0.33, 0.44)$	$(0.29, 0.6)$	$(0.4, 0.65)$
$\Re_6$	$(0.4, 0.39)$	$(0.78, 0.91)$	$(0.3, 0.13)$	$(0.71, 0.51)$	$(0.84, 0.43)$	$(0.67, 0.31)$
$\Re_7$	$(0.53, 0.13)$	$(0.59, 0.42)$	$(0.41, 0.45)$	$(0.82, 0.59)$	$(0.34, 0.39)$	$(0.5, 0.21)$

Table 2. Fuzzy credibility information given by expert E^2.

	$\Im_1$	$\Im_2$	$\Im_3$	$\Im_4$	$\Im_5$	$\Im_6$
$\Re_1$	$(0.61, 0.15)$	$(0.16, 0.35)$	$(0.61, 0.35)$	$(0.55, 0.17)$	$(0.53, 0.62)$	$(0.47, 0.74)$
$\Re_2$	$(0.66, 0.11)$	$(0.43, 0.23)$	$(0.93, 0.08)$	$(0.02, 0.06)$	$(0.51, 0.77)$	$(0.09, 0.99)$
$\Re_3$	$(0.88, 0.09)$	$(0.05, 0.06)$	$(0.56, 0.17)$	$(0.43, 0.13)$	$(0.07, 0.89)$	$(0.44, 0.61)$
$\Re_4$	$(0.59, 0.32)$	$(0.24, 0.48)$	$(0.68, 0.53)$	$(0.34, 0.21)$	$(0.34, 0.51)$	$(0.39, 0.61)$
$\Re_5$	$(0.71, 0.31)$	$(0.35, 0.41)$	$(0.73, 0.44)$	$(0.22, 0.49)$	$(0.24, 0.69)$	$(0.21, 0.74)$
$\Re_6$	$(0.53, 0.07)$	$(0.62, 0.89)$	$(0.47, 0.44)$	$(0.74, 0.61)$	$(0.61, 0.16)$	$(0.61, 0.55)$
$\Re_7$	$(0.51, 0.34)$	$(0.77, 0.51)$	$(0.09, 0.39)$	$(0.99, 0.61)$	$(0.15, 0.35)$	$(0.35, 0.17)$

Table 3. Fuzzy credibility information given by expert E^3.

$\Re_1$	$(0.85, 0.25)$	$(0.14, 0.23)$	$(0.78, 0.38)$	$(0.29, 0.39)$	$(0.15, 0.88)$	$(0.18, 0.83)$
$\Re_2$	$(0.94, 0.04)$	$(0.39, 0.19)$	$(0.63, 0.18)$	$(0.48, 0.49)$	$(0.07, 0.61)$	$(0.35, 0.56)$
$\Re_3$	$(0.73, 0.13)$	$(0.19, 0.39)$	$(0.87, 0.35)$	$(0.41, 0.13)$	$(0.46, 0.88)$	$(0.18, 0.81)$
$\Re_4$	$(0.82, 0.12)$	$(0.55, 0.21)$	$(0.53, 0.33)$	$(0.46, 0.23)$	$(0.43, 0.63)$	$(0.47, 0.51)$
$\Re_5$	$(0.61, 0.33)$	$(0.28, 0.41)$	$(0.74, 0.34)$	$(0.37, 0.32)$	$(0.29, 0.63)$	$(0.14, 0.65)$
$\Re_6$	$(0.15, 0.46)$	$(0.88, 0.88)$	$(0.18, 0.18)$	$(0.83, 0.81)$	$(0.85, 0.14)$	$(0.78, 0.29)$
$\Re_7$	$(0.07, 0.43)$	$(0.61, 0.63)$	$(0.35, 0.47)$	$(0.56, 0.51)$	$(0.25, 0.23)$	$(0.38, 0.39)$

Step 1. In this step, we normalized the decision matrix given in Tables 4–6.

Table 4. Normalized information given by expert E^1.

	$\Im_1$	$\Im_2$	$\Im_3$	$\Im_4$	$\Im_5$	$\Im_6$
$\Re_1$	$(0.84, 0.34)$	$(0.39, 0.43)$	$(0.67, 0.5)$	$(0.21, 0.31)$	$(0.4, 0.78)$	$(0.71, 0.3)$
$\Re_2$	$(0.6, 0.11)$	$(0.35, 0.23)$	$(0.72, 0.31)$	$(0.25, 0.11)$	$(0.53, 0.59)$	$(0.82, 0.41)$
$\Re_3$	$(0.79, 0.19)$	$(0.21, 0.11)$	$(0.71, 0.41)$	$(0.25, 0.34)$	$(0.39, 0.91)$	$(0.51, 0.13)$
$\Re_4$	$(0.63, 0.51)$	$(0.33, 0.49)$	$(0.61, 0.43)$	$(0.37, 0.49)$	$(0.13, 0.42)$	$(0.59, 0.45)$
$\Re_5$	$(0.57, 0.36)$	$(0.15, 0.5)$	$(0.7, 0.32)$	$(0.44, 0.33)$	$(0.29, 0.6)$	$(0.65, 0.4)$
$\Re_6$	$(0.4, 0.39)$	$(0.91, 0.78)$	$(0.3, 0.13)$	$(0.51, 0.71)$	$(0.84, 0.43)$	$(0.31, 0.67)$
$\Re_7$	$(0.53, 0.13)$	$(0.42, 0.59)$	$(0.41, 0.45)$	$(0.59, 0.82)$	$(0.34, 0.39)$	$(0.21, 0.5)$

Table 5. Normalized information given by expert E^2.

	$\Im_1$	$\Im_2$	$\Im_3$	$\Im_4$	$\Im_5$	$\Im_6$
$\Re_1$	$(0.61, 0.15)$	$(0.35, 0.16)$	$(0.61, 0.35)$	$(0.17, 0.55)$	$(0.53, 0.62)$	$(0.74, 0.47)$
$\Re_2$	$(0.66, 0.11)$	$(0.23, 0.43)$	$(0.93, 0.08)$	$(0.06, 0.02)$	$(0.51, 0.77)$	$(0.99, 0.09)$
$\Re_3$	$(0.88, 0.09)$	$(0.06, 0.05)$	$(0.56, 0.17)$	$(0.13, 0.43)$	$(0.07, 0.89)$	$(0.61, 0.44)$
$\Re_4$	$(0.59, 0.32)$	$(0.48, 0.24)$	$(0.68, 0.53)$	$(0.21, 0.34)$	$(0.34, 0.51)$	$(0.61, 0.39)$
$\Re_5$	$(0.71, 0.31)$	$(0.41, 0.35)$	$(0.73, 0.44)$	$(0.49, 0.22)$	$(0.24, 0.69)$	$(0.74, 0.21)$
$\Re_6$	$(0.53, 0.07)$	$(0.89, 0.62)$	$(0.47, 0.44)$	$(0.61, 0.74)$	$(0.61, 0.16)$	$(0.55, 0.61)$
$\Re_7$	$(0.51, 0.34)$	$(0.51, 0.77)$	$(0.09, 0.39)$	$(0.61, 0.99)$	$(0.15, 0.35)$	$(0.17, 0.35)$

Table 6. Normalized information given by expert E^3.

	$\Im_1$	$\Im_2$	$\Im_3$	$\Im_4$	$\Im_5$	$\Im_6$
$\Re_1$	$(0.85, 0.25)$	$(0.23, 0.14)$	$(0.78, 0.38)$	$(0.39, 0.29)$	$(0.15, 0.88)$	$(0.83, 0.18)$
$\Re_2$	$(0.94, 0.04)$	$(0.19, 0.39)$	$(0.63, 0.18)$	$(0.49, 0.48)$	$(0.07, 0.61)$	$(0.56, 0.35)$
$\Re_3$	$(0.73, 0.13)$	$(0.39, 0.19)$	$(0.87, 0.35)$	$(0.13, 0.41)$	$(0.46, 0.88)$	$(0.81, 0.18)$
$\Re_4$	$(0.82, 0.12)$	$(0.21, 0.55)$	$(0.53, 0.33)$	$(0.23, 0.46)$	$(0.43, 0.63)$	$(0.51, 0.47)$
$\Re_5$	$(0.61, 0.33)$	$(0.41, 0.28)$	$(0.74, 0.34)$	$(0.32, 0.37)$	$(0.29, 0.63)$	$(0.65, 0.14)$
$\Re_6$	$(0.15, 0.46)$	$(0.88, 0.88)$	$(0.18, 0.18)$	$(0.81, 0.83)$	$(0.85, 0.14)$	$(0.29, 0.78)$
$\Re_7$	$(0.07, 0.43)$	$(0.63, 0.61)$	$(0.35, 0.47)$	$(0.51, 0.56)$	$(0.25, 0.23)$	$(0.39, 0.38)$

Step 2. In this step, we find the group decision ideal solution (GDIS) given in Table 7.

Table 7. Group decision ideal solution.

	$\Im_1$	$\Im_2$	$\Im_3$	$\Im_4$	$\Im_5$	$\Im_6$
$\Re_1$	(0.82, 0.80)	(0.34, 0.83)	(0.72, 0.61)	(0.29, 0.66)	(0.43, 0.28)	(0.78, 0.75)
$\Re_2$	(0.90, 0.94)	(0.28, 0.69)	(0.89, 0.88)	(0.37, 0.97)	(0.47, 0.36)	(0.98, 0.86)
$\Re_3$	(0.83, 0.88)	(0.27, 0.92)	(0.81, 0.76)	(0.19, 0.61)	(0.38, 0.11)	(0.73, 0.83)
$\Re_4$	(0.75, 0.82)	(0.38, 0.67)	(0.62, 0.60)	(0.29, 0.59)	(0.35, 0.51)	(0.58, 0.57)
$\Re_5$	(0.65, 0.67)	(0.37, 0.66)	(0.73, 0.65)	(0.43, 0.72)	(0.28, 0.37)	(0.69, 0.81)
$\Re_6$	(0.43, 0.89)	(0.90, 0.28)	(0.37, 0.83)	(0.73, 0.25)	(0.82, 0.83)	(0.44, 0.33)
$\Re_7$	(0.47, 0.80)	(0.55, 0.36)	(0.34, 0.57)	(0.58, 0.32)	(0.27, 0.71)	(0.29, 0.61)

Step 3. In this step, group decision right and left ideal solution is calculated given in Tables 8 and 9.

Table 8. Group decision right ideal solution.

	$\Im_1$	$\Im_2$	$\Im_3$	$\Im_4$	$\Im_5$	$\Im_6$
$\Re_1$	(0.84, 0.34)	(0.39, 0.43)	(0.67, 0.5)	(0.17, 0.55)	(0.53, 0.62)	(0.83, 0.18)
$\Re_2$	(0.94, 0.04)	(0.04, 0.23)	(0.72, 0.31)	(0.49, 0.48)	(0.51, 0.77)	(0.99, 0.09)
$\Re_3$	(0.79, 0.19)	(0.19, 0.39)	(0.87, 0.35)	(0.25, 0.34)	(0.46, 0.88)	(0.51, 0.13)
$\Re_4$	(0.63, 0.51)	(0.51, 0.33)	(0.68, 0.53)	(0.37, 0.49)	(0.43, 0.63)	(0.51, 0.47)
$\Re_5$	(0.71, 0.31)	(0.31, 0.41)	(0.73, 0.44)	(0.44, 0.33)	(0.29, 0.63)	(0.65, 0.14)
$\Re_6$	(0.4, 0.39)	(0.39, 0.88)	(0.47, 0.44)	(0.81, 0.83)	(0.84, 0.43)	(0.31, 0.67)
$\Re_7$	(0.51, 0.34)	(0.51, 0.77)	(0.41, 0.45)	(0.61, 0.99)	(0.34, 0.39)	(0.17, 0.35)

Table 9. Group decision left ideal solution.

	$\Im_1$	$\Im_2$	$\Im_3$	$\Im_4$	$\Im_5$	$\Im_6$
$\Re_1$	(0.61, 0.15)	(0.23, 0.14)	(0.61, 0.35)	(0.21, 0.31)	(0.15, 0.88)	(0.83, 0.18)
$\Re_2$	(0.6, 0.11)	(0.19, 0.39)	(0.63, 0.18)	(0.06, 0.02)	(0.07, 0.61)	(0.99, 0.09)
$\Re_3$	(0.73, 0.13)	(0.06, 0.05)	(0.56, 0.17)	(0.13, 0.41)	(0.07, 0.89)	(0.51, 0.13)
$\Re_4$	(0.82, 0.12)	(0.48, 0.24)	(0.53, 0.33)	(0.21, 0.34)	(0.13, 0.42)	(0.51, 0.47)
$\Re_5$	(0.57, 0.36)	(0.15, 0.5)	(0.7, 0.32)	(0.49, 0.22)	(0.29, 0.6)	(0.65, 0.14)
$\Re_6$	(0.53, 0.07)	(0.89, 0.92)	(0.18, 0.18)	(0.51, 0.71)	(0.61, 0.16)	(0.31, 0.67)
$\Re_7$	(0.07, 0.43)	(0.42, 0.95)	(0.09, 0.39)	(0.51, 0.56)	(0.25, 0.23)	(0.17, 0.35)

Step 4. In this step, we compute the distance of the decision matrix N_{ij}^k to GD, GR and GL. The distance are presented symbolically as DGD, DGR and DGL respectively. Table 10 shows the distance of GDIS.

Table 10. Distance of GDIS.

	E_1	E_2	E_3
Q_1	0.31202	0.3183	0.38247
Q_2	0.49903	0.51718	0.4999
Q_3	0.47445	0.47161	0.46183
Q_4	0.6056	0.22613	0.22391
Q_5	0.24517	0.28799	0.28317
Q_6	0.38349	0.389	0.44525
Q_7	0.27999	0.35531	0.23044

Table 11 shows the distance of GRIS.

Table 11. Distance of GRIS.

	E_1	E_2	E_3
$\Re_1$	0.10273	0.11843	0.12577
$\Re_2$	0.19063	0.21565	0.4857
$\Re_3$	0.10345	0.15857	0.10007
$\Re_4$	0.08861	0.1025	0.13627
$\Re_5$	0.08277	0.05908	0.06748
$\Re_6$	0.16286	0.1884	0.13389
$\Re_7$	0.12753	0.06667	0.22532

Table 12 shows the distance of GLIS.

Table 12. Distance of GLIS.

	E_1	E_2	E_3
$\Re_1$	0.14808	0.16383	0.12666
$\Re_2$	0.14046	0.16849	0.26464
$\Re_3$	0.08845	0.0924	0.8898
$\Re_4$	0.13863	0.13212	0.12275
$\Re_5$	0.04613	0.0963	0.08414
$\Re_6$	0.13781	0.10184	0.22384
$\Re_7$	0.15763	0.22362	0.083

Table 13 shows the total distance of GDIS.

Table 13. Total distance of GDIS.

E_1	E_2	E_3
2.35471	2.56552	2.52697

Table 14 shows the total distance of GRIS.

Table 14. Total Distance of GRIS.

E_1	E_2	E_3
0.85858	0.90929	0.99737

Table 15 shows the total distance of GLIS.

Table 15. Total Distance of GLIS.

E_1	E_2	E_3
0.85720	0.97859	1.09401

Step 5. In this step, we compute the closeness indices (CI), given in Table 16.

Table 16. Closeness indices.

1	2	3
0.42152	0.42392	0.45284

Step 6. In this step, we calculate decision matrix weights, given in Table 17.

Table 17. Weights.

1	2	3
0.32467	0.32652	0.34880

Step 7. In this step, we find revised group decision ideal solution (RGDIS) is calculated in Table 18.

Table 18. Revised group decision ideal solution.

	$\mathfrak{I}_1$	$\mathfrak{I}_2$	$\mathfrak{I}_3$	$\mathfrak{I}_4$	$\mathfrak{I}_5$	$\mathfrak{I}_6$
$\mathfrak{R}_1$	$(0.82, 0.75)$	$(0.34, 0.80)$	$(0.72, 0.46)$	$(0.30, 0.55)$	$(0.43, 0.07)$	$(0.78, 0.69)$
$\mathfrak{R}_2$	$(0.90, 0.94)$	$(0.27, 0.59)$	$(0.89, 0.86)$	$(0.38, 0.97)$	$(0.47, 0.13)$	$(0.98, 0.83)$
$\mathfrak{R}_3$	$(0.83, 0.87)$	$(0.29, 0.92)$	$(0.81, 0.69)$	$(0.19, 0.47)$	$(0.38, 0.01)$	$(0.73, 0.79)$
$\mathfrak{R}_4$	$(0.75, 0.78)$	$(0.38, 0.55)$	$(0.62, 0.45)$	$(0.29, 0.43)$	$(0.35, 0.30)$	$(0.58, 0.40)$
$\mathfrak{R}_5$	$(0.65, 0.56)$	$(0.37, 0.54)$	$(0.73, 0.52)$	$(0.43, 0.63)$	$(0.28, 0.13)$	$(0.69, 0.77)$
$\mathfrak{R}_6$	$(0.43, 0.87)$	$(0.90, 0.07)$	$(0.37, 0.79)$	$(0.73, 0.05)$	$(0.82, 0.80)$	$(0.44, 0.10)$
$\mathfrak{R}_7$	$(0.47, 0.76)$	$(0.55, 0.13)$	$(0.34, 0.40)$	$(0.58, 0.10)$	$(0.27, 0.62)$	$(0.30, 0.46)$

Step 8. In this step, we computed fuzzy credibility entropy measure which are given in Table 19.

Table 19. Entropy measure.

$\mathfrak{I}_1$	$\mathfrak{I}_2$	$\mathfrak{I}_3$	$\mathfrak{I}_4$	$\mathfrak{I}_5$	$\mathfrak{I}_6$
0.22596	0.39935	0.37902	0.41512	0.37263	0.33033

Step 9. In this step, we computed the attribute weights which is given in Table 20.

Table 20. Attribute weights.

$\mathfrak{I}_1$	$\mathfrak{I}_2$	$\mathfrak{I}_3$	$\mathfrak{I}_4$	$\mathfrak{I}_5$	$\mathfrak{I}_6$
0.1996	0.15489	0.16014	0.15083	0.16185	0.17269

Step 11. In this step, we find the PIS^k and NIS^k for each DMs, which are given in Table 21.

Table 21. Positive ideal solution.

PD_1	$(0.70, 0.28)$	$(0.80, 0.04)$	$(0.51, 0.26)$	$(0.12, 0.55)$	$(0.68, 0.18)$	$(0.30, 0.54)$
PD_2	$(0.77, 0.67)$	$(0.76, 0.09)$	$(0.84, 0.65)$	$(0.02, 0.88)$	$(0.39, 0.46)$	$(0.98, 0.64)$
PD_3	$(0.88, 0.83)$	$(0.74, 0.02)$	$(0.73, 0.23)$	$(0.62, 0.03)$	$(0.70, 0.50)$	$(0.67, 0.44)$

In Table 22, we show the negative ideal solution.

Table 22. Negative ideal solution.

	$\mathfrak{I}_1$	$\mathfrak{I}_2$	$\mathfrak{I}_3$	$\mathfrak{I}_4$	$\mathfrak{I}_5$	$\mathfrak{I}_6$
ND_1	$(0.23, 0.05)$	$(0.07, 0.13)$	$(0.22, 0.16)$	$(0.19, 0.14)$	$(0.21, 0.02)$	$(0.16, 0.08)$
ND_2	$(0.32, 0.28)$	$(0.11, 0.17)$	$(0.04, 0.20)$	$(0.074, 0.11)$	$(0.03, 0.02)$	$(0.08, 0.24)$
ND_3	$(0.03, 0.21)$	$(0.10, 0.11)$	$(0.18, 0.15)$	$(0.06, 0.18)$	$(0.07, 0.02)$	$(0.15, 0.05)$

Step 12. In this step, we computed the WGDM from DM $(N)^k$ to PD^k and ND^k, show in Table 23.

Table 23. Revised closeness indices.

	$\mathfrak{R}_1$	$\mathfrak{R}_2$	$\mathfrak{R}_3$	$\mathfrak{R}_4$	$\mathfrak{R}_5$	$\mathfrak{R}_6$	$\mathfrak{R}_7$
D_1	0.42	0.40	0.41	0.20	0.23	0.52	0.19
D_2	0.23	0.68	0.36	0.16	0.26	0.35	0.09
D_3	0.43	0.54	0.44	0.35	0.263	0.39	0.13

Step 13. In this step, we find the revised closeness indices (RCIs) for every single decision matrix given in Table 24.

Table 24. Weighted revised closeness indices.

	$\mathfrak{R}_1$	$\mathfrak{R}_2$	$\mathfrak{R}_3$	$\mathfrak{R}_4$	$\mathfrak{R}_5$	$\mathfrak{R}_6$	$\mathfrak{R}_7$
D_1	0.14	0.13	0.13	0.06	0.07	0.17	0.06
D_2	0.08	0.22	0.12	0.05	0.08	0.11	0.03
D_3	0.15	0.19	0.15	0.12	0.09	0.14	0.05

Step 14. In this step, we find the closeness indices using the FCDWA operator given in Table 25.

Table 25. Final revised closeness indices.

$\Re_1$	$\Re_2$	$\Re_3$	$\Re_4$	$\Re_5$	$\Re_6$	$\Re_7$
0.36331	0.54029	0.40717	0.23801	0.24956	0.42058	0.13288

Here, $\Re_2$ is the finest alternatives.
In Table 26, we find closeness indices using different operators.

Table 26. Final revised closeness indices using different operators.

Operator	$\Re_1$	$\Re_2$	$\Re_3$	$\Re_4$	$\Re_5$	$\Re_6$	$\Re_7$
$FCDWA$	0.36331	0.54029	0.40717	0.23801	0.24956	0.42058	0.13288
$FCDOWA$	0.44697	0.55914	0.36619	0.27564	0.23239	0.43913	0.30253
$FCDHWA$	0.48535	0.5542	0.52324	0.32323	0.2986	0.34126	0.11599

Hence, $\Re_2$ is the finest alternative in operators.

7. Comparative analysis

This section includes a comparison analysis to demonstrate the benefits of the proposed technique. This comparison is made using the same data between the fuzzy credibility TOPSIS method based on the Entropy measure and the fuzzy credibility Dombi weighted averaging operator. In Table 27, we obtained the Entropy measure.

Table 27. Weights.

1	2	3
0.3431	0.33374	0.32316

In Table 28, we obtained the attribute weights.

Table 28. Attributes weights.

$\Im_1$	$\Im_2$	$\Im_3$	$\Im_4$	$\Im_5$	$\Im_6$
0.18185	0.15885	0.16134	0.16151	0.16766	0.16879

In Table 29, we obtained closeness indices.

Table 29. Final revised closeness indices.

$\Re_1$	$\Re_2$	$\Re_3$	$\Re_4$	$\Re_5$	$\Re_6$	$\Re_7$
0.49486	0.60946	0.44243	0.47084	0.46546	0.49824	0.31911

In a comparative analysis, $\Re_2$ is once again the best option.

We have obtained comparison of the proposed operators as well as a comparative analysis in Table 30, and the best option in all operators is $\Re_2$.

Table 30. Revised closeness indices.

Operator	$\Re_1$	$\Re_2$	$\Re_3$	$\Re_4$	$\Re_5$	$\Re_6$	$\Re_7$
Comparative Analysis	0.49486	0.60946	0.44243	0.47084	0.46546	0.49824	0.31911
FCDWA	0.36331	0.54029	0.40717	0.23801	0.24956	0.42058	0.13288
FCDOWA	0.44697	0.55914	0.36619	0.27564	0.23239	0.43913	0.30253
FCDHWA	0.48535	0.5542	0.52324	0.32323	0.2986	0.34126	0.11599

We have a comparison of the results of the proposed method in Table 31.

Table 31. Different operators and their ranking.

Operator	Ranking	Finest alternative
comparative Analysis	$\Re_2 > \Re_6 > \Re_1 > \Re_4 > \Re_5 > \Re_3 > \Re_7$	$\Re_2$
FCDWA	$\Re_2 > \Re_6 > \Re_3 > \Re_1 > \Re_5 > \Re_4 > \Re_7$	$\Re_2$
FCDOWA	$\Re_2 > \Re_1 > \Re_6 > \Re_3 > \Re_7 > \Re_4 > \Re_5$	$\Re_2$
FCDHWA	$\Re_2 > \Re_3 > \Re_1 > \Re_6 > \Re_4 > \Re_5 > \Re_7$	$\Re_2$

As a result, the proposed method for solving MAGDM problems is more accurate, effective, and generalize.

8. Conclusions

This study presents Dombi operations based on Dombi t-norm and Dombi t-conorm for fuzzy credibility numbers. Based on the defined fuzzy credibility Dombi operation laws, we proposed fuzzy credibility Dombi weighted averaging (FCDWA) operator, fuzzy credibility Dombi ordered weighted averaging (FCDOWA) operator and fuzzy credibility hybrid weighted averaging (FCDHWA) operator and investigated their properties. Further, we used the TOPSIS method on MAGDM. An illustrative example of the selection case of enterprise resource planning system and comparison with existing method were given to indicate the applicability and validity of the new method. Furthermore, our new techniques not only overcome the drawbacks of the existing techniques, but also are more extensive and more useful than the existing techniques when performing MAGDM problems in the setting of FCSs. The developed method would be more suitable for real applications in handling indeterminate information and inconsistent information in decision-making problems with fuzzy and credibility information.

In the future work, we will do some more contribution to fuzzy credibility numbers.

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Conflict of interest

The authors declare that they have no competing interests.

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