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Research article

The integer part of nonlinear forms with prime variables

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Abstract: In this paper, we discuss problems that integer part of nonlinear forms with prime variables represent primes infinitely. We prove that under suitable conditions there exist infinitely many primes p_j, p such that $[\lambda_1 p_1^2 + \lambda_2 p_2^2 + \lambda_3 p_3^k] = p$ and $[\lambda_1 p_1^3 + \cdots + \lambda_4 p_4^3 + \lambda_5 p_5^k] = p$ with $k \geq 2$ and $k \geq 3$ respectively, which improve the author's earlier results.

Keywords: prime variables; Diophantine approximation; Davenport-Heilbronn method
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1. Introduction

Let $[x]$ be the greatest integer not exceeding x. For any natural number k, an interesting question is whether there exist $s = s(k)$ and infinitely many primes $p_1, \cdots, p_s, p$ such that

$$[\lambda_1 p_1^k + \cdots + \lambda_s p_s^k] = p,$$

where $\lambda_1, \cdots, \lambda_s$ are real non-zero numbers and at least one of $\lambda_i/\lambda_j (1 \leq i < j \leq s)$ is irrational. Following the work of Danicic [1] for the linear case $k = 1$ with s=2, Li and Wang [2] made progress for the quadratic case $k = 2$ with $s = 3$, and Li and Su [3] for the cubic case $k = 3$ with $s = 5$.

In 1988, Srinivasan [4] established one result being of form $[\lambda_1 p_1 + \lambda_2 p_2^k] = p$. Inspired by Srinivasan's conclusion, in this paper, we prove two more general and sharper results as follows.

Theorem 1.1. Suppose that $\lambda_1, \lambda_2, \lambda_3$ are positive real numbers, at least one of $\lambda_i/\lambda_j (1 \leq i < j \leq 3)$ is irrational, and positive integer $k \geq 2$, then there exist infinitely many primes p_1, p_2, p_3, p such that

$$[\lambda_1 p_1^2 + \lambda_2 p_2^2 + \lambda_3 p_3^k] = p.$$

Theorem 1.2. Suppose that $\lambda_1, \cdots, \lambda_5$ are positive real numbers, at least one of $\lambda_i/\lambda_j (1 \le i < j \le 5)$ is irrational, and positive integer $k \ge 3$, then there exist infinitely many primes $p_1, \cdots, p_5, p$ such that

$$[\lambda_1 p_1^3 + \cdots + \lambda_4 p_4^3 + \lambda_5 p_5^k] = p.$$

Here we only give the proof of Theorem 1.2, since Theorem 1.1 can be proved similarly. In Section 2, we give the outline of the proof of Theorem 1.2. In Sections 3 and 4, we restrict our attention to the neighbourhood of the origin and the intermediate region, respectively. In Section 5, we consider the trivial region and complete the proof of Theorem 1.2.

Throughout the paper, we use standard notations in number theory. In particular, δ stands for a sufficiently small positive number, ε is an arbitrarily small positive number, ν is positive real number, and N is a sufficiently large real number.

2. Outline of the method

The basic method builds on the modification of the Hardy-Littlewood circle method first introduced by Davenport and Heilbronn. Denote

$$K_\nu(\alpha) = \nu(\frac{\sin \pi\nu\alpha}{\pi\nu\alpha})^2,$$

for $\nu > 0$ and $\alpha \ne 0$. By continuity, we define $K_\nu(0) = \nu$. Then we have

$$K_\nu(\alpha) \ll \min(\nu, \nu^{-1}|\alpha|^{-2}), \tag{2.1}$$

$$\int_{-\infty}^{+\infty} e(\alpha y) K_\nu(\alpha) d\alpha = \max(0, 1 - \nu^{-1}|y|). \tag{2.2}$$

Since at least one of the ratios $\lambda_i/\lambda_j (1 \le i < j \le 5)$ is irrational, we may assume that λ_1/λ_2 is irrational, and for other cases, one may deal with them similarly. For λ_1/λ_2 is irrational, there are infinitely many pairs of integers q, a with

$$|\lambda_1/\lambda_2 - a/q| \le q^{-2}, (a, q) = 1, q > 0, a \ne 0.$$

We choose q to be large in terms of $\lambda_1, \cdots, \lambda_5$, and make the following definitions.

$$L = \log N,\ [N^{1-8\delta}] = q,\ \tau = N^{-1+\delta},\ Q = (|\lambda_1|^{-1} + |\lambda_2|^{-1})N^{1-\delta},\ P = N^{6\delta},\ T = T_1^3 = T_2^k = N^{\frac{1}{3}},$$

$$S_i(\alpha) = \sum_{(\delta N)^{\frac{1}{3}} \le p \le N^{\frac{1}{3}}} e(\lambda_i p^3 \alpha) \log p, \quad i = 1, \cdots, 4,$$

$$S_5(\alpha) = \sum_{(\delta N)^{\frac{1}{k}} \le p \le N^{\frac{1}{k}}} e(\lambda_5 p^k \alpha) \log p, \quad S_0(\alpha) = \sum_{p \le N} (\log p) e(\alpha p).$$

By (2.2),

$$J(\mathbb{R}) \ =: \ \int_{-\infty}^{+\infty} \prod_{i=1}^{5} S_i(\alpha) S_0(-\alpha) e(-\frac{1}{2}\alpha) K_{\frac{1}{2}}(\alpha) d\alpha$$

$$\leq L^6 \sum_{\substack{|\lambda_1 p_1^3+\cdots+\lambda_4 p_4^3+\lambda_5 p_5^k - p - \frac{1}{2}|<\frac{1}{2} \\ (\delta N)^{\frac{1}{3}} \leq p_1,\cdots,p_4 \leq N^{\frac{1}{3}}, (\delta N)^{\frac{1}{k}} \leq p_5 \leq N^{\frac{1}{k}}, p \leq N}} 1$$

$$=: L^6 \mathcal{N}(N).$$

Thus it suffices to establish a positive lower bound for $J(\mathbb{R})$. In order to estimate $J(\mathbb{R})$, we split $(-\infty, +\infty)$ into three parts $\mathfrak{C} = \{\alpha \in \mathbb{R} : |\alpha| \leq \tau\}$, $\mathfrak{D} = \{\alpha \in \mathbb{R} : \tau < |\alpha| \leq P\}$, $\mathfrak{c} = \{\alpha \in \mathbb{R} : |\alpha| > P\}$, traditionally named the neighbourhood of the origin, the intermediate region, and the trivial region.

Thus

$$J(\mathbb{R}) = J(\mathfrak{C}) + J(\mathfrak{D}) + J(\mathfrak{c}). \tag{2.3}$$

In the following sections, we compute the integrals in the neighbourhood of the origin, the intermediate region, and the trivial region, respectively.

3. The neighbourhood of the origin

In this section, we evaluate the contribution from the neighbourhood of the origin and give a low bound.

Let $\rho = \beta + i\gamma$ be the zeros of the Riemann zeta function and C be a positive constant. By Lemma 5 of [5], one has

$$S_0(\alpha) = \int_1^N e(y\alpha)dy - \sum_{|\gamma|\leq T, \beta \geq \frac{2}{3}} \sum_{n \leq N} n^{\rho-1} e(n\alpha) + O((1+|\alpha|N)N^{\frac{2}{3}} L^C)$$

$$=: I_0(\alpha) - J_0(\alpha) + B_0(\alpha),$$

$$S_i(\alpha) = \int_{(\delta N)^{\frac{1}{3}}}^{N^{\frac{1}{3}}} e(\lambda_i y^3 \alpha)dy - \sum_{|\gamma|\leq T_1, \beta \geq \frac{2}{3}} \sum_{(\delta N)^{\frac{1}{3}} \leq n \leq N^{\frac{1}{3}}} n^{\rho-1} e(\lambda_i n^3 \alpha) + O((1+|\alpha|N)N^{\frac{2}{9}} L^C)$$

$$=: I_i(\alpha) - J_i(\alpha) + B_i(\alpha), i = 1, \cdots, 4,$$

$$S_5(\alpha) = \int_{(\delta N)^{\frac{1}{k}}}^{N^{\frac{1}{k}}} e(\lambda_5 y^k \alpha)dy - \sum_{|\gamma|\leq T_2, \beta \geq \frac{2}{3}} \sum_{(\delta N)^{\frac{1}{k}} \leq n \leq N^{\frac{1}{k}}} n^{\rho-1} e(\lambda_5 n^k \alpha) + O((1+|\alpha|N)N^{\frac{2}{3k}} L^C)$$

$$=: I_5(\alpha) - J_5(\alpha) + B_5(\alpha).$$

Lemma 3.1. We have

$$S_0(\alpha) \ll N, \quad I_0(\alpha) \ll \min(N, |\alpha|^{-1}), \quad \int_{-\frac{1}{2}}^{\frac{1}{2}} |I_0(\alpha)|^2 d\alpha \ll N,$$

$$\int_{-\frac{1}{2}}^{\frac{1}{2}} |J_0(\alpha)|^2 d\alpha \ll N \exp(-L^{\frac{1}{5}}), \quad \int_{-\tau}^{\tau} |B_0(\alpha)|^2 d\alpha \ll N^{\frac{1}{3}+4\delta}, \quad \int_{-\frac{1}{2}}^{\frac{1}{2}} |S_0(\alpha)|^2 d\alpha \ll NL.$$

Proof. These can be deduced from Lemmas 6–8 of [5].

From Lemma 8 of [6], we can deduce the following Lemmas 3.2 and 3.3.

Lemma 3.2. For $i = 1, \cdots, 4$, we have

$$S_i(\alpha) \ll N^{\frac{1}{3}}, \quad I_i(\alpha) \ll N^{\frac{1}{3}} \min(1, N^{-1}|\alpha|^{-1}), \int_{-\frac{1}{2}}^{\frac{1}{2}} |I_i(\alpha)|^2 d\alpha \ll N^{-\frac{1}{3}},$$

$$\int_{-\frac{1}{2}}^{\frac{1}{2}} |J_i(\alpha)|^2 d\alpha \ll N^{-\frac{1}{3}} \exp(-L^{\frac{1}{5}}), \int_{-\tau}^{\tau} |B_i(\alpha)|^2 d\alpha \ll N^{-\frac{1}{3}} \exp(-L^{\frac{1}{5}}), \int_{-\tau}^{\tau} |S_i(\alpha)|^2 d\alpha \ll N^{-\frac{1}{3}}.$$

Lemma 3.3. We have

$$S_5(\alpha) \ll N^{\frac{1}{k}}, \quad I_5(\alpha) \ll N^{\frac{1}{k}} \min(1, N^{-1}|\alpha|^{-1}),$$

$$\int_{-\frac{1}{2}}^{\frac{1}{2}} |I_5(\alpha)|^2 d\alpha \ll N^{\frac{2}{k}-1}, \quad \int_{-\frac{1}{2}}^{\frac{1}{2}} |J_5(\alpha)|^2 d\alpha \ll N^{\frac{2}{k}-1} \exp(-L^{\frac{1}{5}}),$$

$$\int_{-\tau}^{\tau} |B_5(\alpha)|^2 d\alpha \ll N^{\frac{2}{k}-1} \exp(-L^{\frac{1}{5}}), \quad \int_{-\tau}^{\tau} |S_5(\alpha)|^2 d\alpha \ll N^{\frac{2}{k}-1}.$$

Lemma 3.4. We have

$$\int_{\mathfrak{C}} |\prod_{i=1}^{5} S_i(\alpha) S_0(-\alpha) - \prod_{i=1}^{5} I_i(\alpha) I_0(-\alpha)| K_{\frac{1}{2}}(\alpha) d\alpha \ll N^{\frac{4}{3}+\frac{1}{k}} L^{-1}. \tag{3.1}$$

Proof. Obviously,

$$\begin{aligned}
& \prod_{i=1}^{5} S_i(\alpha) S_0(-\alpha) - \prod_{i=1}^{5} I_i(\alpha) I_0(-\alpha) \\
= \; & (S_1(\alpha) - I_1(\alpha)) \prod_{i=2}^{5} S_i(\alpha) S_0(-\alpha) + I_1(\alpha)(S_2(\alpha) - I_2(\alpha)) \prod_{i=3}^{5} S_i(\alpha) S_0(-\alpha) \\
& + \cdots + \prod_{i=1}^{4} I_i(\alpha)(S_5(\alpha) - I_5(\alpha)) S_0(-\alpha) + \prod_{i=1}^{5} I_i(\alpha)(S_0(-\alpha) - I_0(-\alpha)).
\end{aligned}$$

By Lemmas 3.1–3.3, we have

$$\begin{aligned}
& \int_{\mathfrak{C}} |(S_1(\alpha) - I_1(\alpha)) \prod_{i=2}^{5} S_i(\alpha) S_0(-\alpha)| K_{\frac{1}{2}}(\alpha) d\alpha \\
\ll \; & N^{\frac{5}{3}+\frac{1}{k}} \left(\int_{-\tau}^{\tau} |B_1(\alpha)|^2 d\alpha + \int_{-\tau}^{\tau} |J_1(\alpha)|^2 d\alpha \right)^{\frac{1}{2}} \left(\int_{-\tau}^{\tau} |S_2(\alpha)|^2 d\alpha \right)^{\frac{1}{2}} \\
\ll \; & N^{\frac{4}{3}+\frac{1}{k}} L^{-1},
\end{aligned}$$

$$\int_{\mathfrak{C}} |\prod_{i=1}^{4} I_i(\alpha)(S_5(\alpha) - I_5(\alpha)) S_0(-\alpha)| K_{\frac{1}{2}}(\alpha) d\alpha$$

$$
\begin{aligned}
&\ll N^{\frac{4}{3}}\left(\int_{-\tau}^{\tau}|B_5(\alpha)|^2d\alpha+\int_{-\tau}^{\tau}|J_5(\alpha)|^2d\alpha\right)^{\frac{1}{2}}\left(\int_{-\tau}^{\tau}|S_0(-\alpha)|^2d\alpha\right)^{\frac{1}{2}}\\
&\ll N^{\frac{4}{3}+\frac{1}{k}}L^{-1},
\end{aligned}
$$

$$
\begin{aligned}
&\int_{\mathfrak{C}}|\prod_{i=1}^{5}I_i(\alpha)(S_0(-\alpha)-I_0(-\alpha))|K_{\frac{1}{2}}(\alpha)d\alpha\\
\ll\ & N^{\frac{4}{3}}\left(\int_{-\tau}^{\tau}|I_5(\alpha)|^2d\alpha\right)^{\frac{1}{2}}\left(\int_{-\tau}^{\tau}|B_0(-\alpha)|^2d\alpha+\int_{-\tau}^{\tau}|J_0(-\alpha)|^2d\alpha\right)^{\frac{1}{2}}\\
\ll\ & N^{\frac{4}{3}+\frac{1}{k}}L^{-1}.
\end{aligned}
$$

The argument for other terms are similar, and the proof of Lemma 3.4 is concluded.

Lemma 3.5. We have

$$
\int_{|\alpha|>N^{-1+\delta}}|\prod_{i=1}^{5}I_i(\alpha)I_0(-\alpha)|K_{\frac{1}{2}}(\alpha)d\alpha\ll N^{(\frac{4}{3}+\frac{1}{k})(1-\delta)}, \tag{3.2}
$$

and

$$
\int_{-\infty}^{+\infty}\prod_{i=1}^{5}I_i(\alpha)I_0(-\alpha)e(-\frac{1}{2}\alpha)K_{\frac{1}{2}}(\alpha)d\alpha\gg N^{\frac{4}{3}+\frac{1}{k}}. \tag{3.3}
$$

Proof. For $\alpha\neq 0$, we have

$$
I_i(\alpha)\ll|\alpha|^{-\frac{1}{3}},\ i=1,\cdots,4,\quad I_5(\alpha)\ll|\alpha|^{-\frac{1}{k}},\quad I_0(-\alpha)\ll|\alpha|^{-1},
$$

thus the left hand of (3.2)

$$
\ll\int_{|\alpha|>N^{-1+\delta}}|\alpha|^{-\frac{7}{3}-\frac{1}{k}}d\alpha\ll N^{(\frac{4}{3}+\frac{1}{k})(1-\delta)}.
$$

The proof of (3.3) is similar to (36) in [5], we omit the details.

Combining (3.1), (3.2) and (3.3), we get

$$
J(\mathfrak{C})\gg N^{\frac{4}{3}+\frac{1}{k}}. \tag{3.4}
$$

4. The intermediate region

The goal of this section is to estimate the integral $J(\mathfrak{D})$.

Lemma 4.1. We have

$$
\int_{-\infty}^{+\infty}|S_i(\alpha)|^8K_{\frac{1}{2}}(\alpha)d\alpha\ll N^{\frac{5}{3}+\frac{1}{3}\varepsilon},\ i=1,\cdots,4, \tag{4.1}
$$

$$
\int_{-\infty}^{+\infty}|S_5(\alpha)|^{2^k}K_{\frac{1}{2}}(\alpha)d\alpha\ll N^{\frac{1}{k}2^k-1+\varepsilon}, \tag{4.2}
$$

$$
\int_{-\infty}^{+\infty}|S_0(-\alpha)|^2K_{\frac{1}{2}}(\alpha)d\alpha\ll NL. \tag{4.3}
$$

Proof. By (2.1) and Hua's inequality,

$$\int_{-\infty}^{+\infty} |S_5(\alpha)|^{2^k} K_{\frac{1}{2}}(\alpha)d\alpha \ll \sum_{m=-\infty}^{+\infty} \int_m^{m+1} |S_5(\alpha)|^{2^k} K_{\frac{1}{2}}(\alpha)d\alpha$$

$$\ll \sum_{m=0}^{1} \int_m^{m+1} |S_5(\alpha)|^{2^k} d\alpha + \sum_{m=2}^{+\infty} m^{-2} \int_m^{m+1} |S_5(\alpha)|^{2^k} d\alpha \ll N^{\frac{1}{k}2^k-1+\varepsilon}.$$

The proofs of (4.1) and (4.3) are similar.

Lemma 4.2. Suppose $\varepsilon > 0$ is given. Let $f(x)$ be a real valued polynomial in x of degree $k \geq 2$. Suppose α is the leading coefficient of f and there are integers a, q such that $|q\alpha - a| < q^{-1}$ with $(a, q) = 1$. Then we have

$$\sum_{p \leq X} (\log p)e(f(p)) \ll X^{1+\varepsilon}(q^{-1} + X^{-\frac{1}{2}} + qX^{-k})^{4^{1-k}}.$$

Proof. This is Theorem 1 of [7].

Lemma 4.3. For every real number $\alpha \in \mathfrak{D}$, let $W(\alpha) = \min(|S_1(\alpha)|, |S_2(\alpha)|)$, then

$$W(\alpha) \ll N^{\frac{1}{3}-\frac{1}{16}\delta+\varepsilon}.$$

Proof. The proof is similar to Lemma 9 in [3]. In $\alpha \in \mathfrak{D}$, we know that at least one j, $P < q_j \ll Q$, and Lemma 4.3 can be established.

Using Hölder's inequality, we have

$$\begin{aligned} J(\mathfrak{D}) \ll\ & \max_{\alpha\in\mathfrak{D}} |W(\alpha)|^{\frac{1}{2^{k-3}}} \left(\int_{-\infty}^{+\infty} |S_1(\alpha)|^8 K_{\frac{1}{2}}(\alpha)d\alpha\right)^{\frac{1}{8}-\frac{1}{2^k}} \prod_{i=2}^{4} \left(\int_{-\infty}^{+\infty} |S_i(\alpha)|^8 K_{\frac{1}{2}}(\alpha)d\alpha\right)^{\frac{1}{8}} \\ & \cdot \left(\int_{-\infty}^{+\infty} |S_5(\alpha)|^{2^k} K_{\frac{1}{2}}(\alpha)d\alpha\right)^{\frac{1}{2^k}} \left(\int_{-\infty}^{+\infty} |S_0(-\alpha)|^2 K_{\frac{1}{2}}(\alpha)d\alpha\right)^{\frac{1}{2}} \\ & + \max_{\alpha\in\mathfrak{D}} |W(\alpha)|^{\frac{1}{2^{k-3}}} \left(\int_{-\infty}^{+\infty} |S_2(\alpha)|^8 K_{\frac{1}{2}}(\alpha)d\alpha\right)^{\frac{1}{8}-\frac{1}{2^k}} \prod_{i=1,3,4} \left(\int_{-\infty}^{+\infty} |S_i(\alpha)|^8 K_{\frac{1}{2}}(\alpha)d\alpha\right)^{\frac{1}{8}} \\ & \cdot \left(\int_{-\infty}^{+\infty} |S_5(\alpha)|^{2^k} K_{\frac{1}{2}}(\alpha)d\alpha\right)^{\frac{1}{2^k}} \left(\int_{-\infty}^{+\infty} |S_0(-\alpha)|^2 K_{\frac{1}{2}}(\alpha)d\alpha\right)^{\frac{1}{2}}. \end{aligned}$$

Hence, by Lemmas 4.1 and 4.3, we have

$$J(\mathfrak{D}) \ll N^{\frac{4}{3}+\frac{1}{k}-\frac{1}{2^{k+1}}\delta+\varepsilon}. \tag{4.4}$$

5. The trivial region and completion of the proof

In this section, we consider the contribution from the trivial region, and then establish Theorem 1.2.

Lemma 5.1. Let $V(\alpha) = \sum e(\alpha f(x_1, \cdots, x_m))$, where f is any real function and the summation is over any finite set of values of $x_1, \cdots, x_m$. Then, for any $A > 4$, we have

$$\int_{|\alpha|>A} |V(\alpha)|^2 K_\nu(\alpha) d\alpha \leq \frac{16}{A} \int_{-\infty}^{\infty} |V(\alpha)|^2 K_\nu(\alpha) d\alpha.$$

Proof. This is Lemma 2 of [8].

By Lemmas 5.1, 4.1 and Schwarz's inequality,

$$\begin{aligned}
& \int_{\mathfrak{c}} |\prod_{i=1}^{5} S_i(\alpha) S_0(-\alpha)| K_{\frac{1}{2}}(\alpha) d\alpha \\
\ll\ & \frac{1}{P} \int_{-\infty}^{+\infty} |\prod_{i=1}^{5} S_i(\alpha) S_0(-\alpha)| K_{\frac{1}{2}}(\alpha) d\alpha \\
\ll\ & N^{-6\delta} \max_{\alpha\in\mathbb{R}} |S_5(\alpha)| \prod_{i=1}^{4} \left(\int_{-\infty}^{+\infty} |S_i(\alpha)|^8 K_{\frac{1}{2}}(\alpha) d\alpha \right)^{\frac{1}{8}} \left(\int_{-\infty}^{+\infty} |S_0(\alpha)|^2 K_{\frac{1}{2}}(\alpha) d\alpha \right)^{\frac{1}{2}} \\
\ll\ & N^{\frac{4}{3}+\frac{1}{k}-6\delta+\varepsilon}.
\end{aligned}$$

Thus, we have

$$J(\mathfrak{c}) \ll N^{\frac{4}{3}+\frac{1}{k}-6\delta+\varepsilon}. \tag{5.1}$$

Combining (2.3), (3.4), (4.4) and (5.1), we get

$$J(\mathbb{R}) \gg N^{\frac{4}{3}+\frac{1}{k}}, \quad \mathcal{N}(N) \gg N^{\frac{4}{3}+\frac{1}{k}} L^{-6},$$

i.e., under the conditions of Theorem 1.2,

$$|\lambda_1 p_1^3 + \cdots + \lambda_4 p_4^3 + \lambda_5 p_5^k - p - \frac{1}{2}| < \frac{1}{2} \tag{5.2}$$

has infinitely many primes solutions $p_1, \cdots, p_5, p$.

By (5.2), we have

$$p < \lambda_1 p_1^3 + \cdots + \lambda_4 p_4^3 + \lambda_5 p_5^k < p + 1,$$

and

$$[\lambda_1 p_1^3 + \cdots + \lambda_4 p_4^3 + \lambda_5 p_5^k] = p.$$

This proves Theorem 1.2.

6. Conclusions

In this work, using the circle method, we have established two theorems that integer part of nonlinear forms with prime variables represent primes infinitely. The results presented in this article are new and improve the author's earlier results.

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