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Research article

Inference of fuzzy reliability model for inverse Rayleigh distribution

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Abstract: In this paper, the question of inference of the reliability parameter of fuzzy stress strength $R_F = P\ (Y < X)$ is attached to the difference between stress and strength values when X and Y are independently distributed from inverse Rayleigh random variables. Including fuzziness in the stress-strength interference enables researchers to make more sensitive and precise analyses about the underlying systems. The maximum product of the spacing method for the reliability of fuzzy stress intensity inference has been introduced. As classical estimation methods and Bayesian estimation methods are used to estimate the reliability parameter R_F, the maximum product of spacing and maximum likelihood estimation methods is used. The maximum product of spacing under fuzzy reliability of stress strength model is introducing in this paper. Markov Chain Monte Carlo approach is used to obtain Bayesian estimators of traditional and fuzzy reliability of stress strength for inverse Rayleigh model by using the Metropolis-Hastings algorithm. Using an extensive Monte Carlo simulation analysis, the outputs of the fuzzy reliability and traditional reliability estimators are contrasted. Finally, for example, and to verify the efficiency of the proposed estimators, a genuine data application is used.

Keywords: fuzzy set; inverse Rayleigh distribution; reliability stress-strength; maximum likelihood; maximum product of spacing; Bayesian

1. Introduction

The stress-strength reliability model is significant in reliability literature, medical, economic, and engineering applications where systems or specialized units may be exposed to randomly occurring environmental stresses such as pressure, temperature, and humidity. In this case, the survival of the system depends on its resistance. During World War II, it was found that some of the equipment such as radar and communication systems failed to work efficiently when they were used in an environment different from the environment for which they were designed. For that, experts started to consider the effects of environmental conditions while evaluating the reliability of equipment.

The computations of fuzzy reliability have been addressed by [1], where if X and Y are independent but not identical random variables in distribution. The idea behind fuzzy reliability is that including fuzziness in the stress-strength interference enables researchers to make more sensitive and precise analyses about the underlying systems of life reliability and the system becomes more stable and reliable when the difference $(x-y)$ gets larger. The advantages of the fuzzy stress-strength reliability model over the traditional stress-strength reliability model are in considering the randomness in reliability engineering and the fuzziness of operating time. For more information see [1–5].

The fuzzy reliability $R_F = P\ (Y < X)$ is defined as

$$R_F = \iint_{y<x} \mu_{A(y)}(x)\ dF_Y(y)\ dF_X(x) \tag{1}$$

where $A(y) = \{x: y < x\}$ is a fuzzy set and $\mu_{A(y)}(x)$ is an appropriate membership function on $A(y)$; that is assumed increasing on the difference $(x-y)$ (readers are encouraged to read [5,6] who used the definition of the fuzzy stress-strength model to estimate $R_F = (Y < X)$, when X and Y were independent inverse exponential random variables).

The probability that the system is strong enough to overcome the stress imposed on it is defined as system reliability. Traditional reliability $R = P\ (Y < X)$ may be equally explained as the region under the receiver operating characteristic (ROC) for diagnostic test or biomarkers, see reference [7]. The ROC curve is exceedingly used in medical, biological, economic, and health service research, to evaluate the reliability and distinguish between two groups of subjects, generally non-satisfied and satisfied subjects. The research conducted on the traditional stress-strength reliability model focuses on computing, calculating, and estimating the reliability of different stress and strength distributions. For example, reference [8] estimated the traditional reliability of the stress-strength model for a generalized exponential distribution with three parameters. Confidence intervals estimation of traditional reliability of stress-strength model for generalized Pareto distribution has been discussed by [9]. The stress-strength model of a generalized logistic distribution has been studied by [10]. Reference [11] estimated the R when X and Y independent Lindley populations. In 2020, [12] and others discussed the estimation of R when X and Y are independent exponentiated Pareto random variables when samples are selected using some ranked set sampling designs. Reference [13] presented a comprehensive review of the traditional reliability of the stress-strength model. In 2021, [14] estimated the traditional stress strength reliability by the use of the MPS estimation method.

In our study, we used failure times in insulating fluid between two electrodes subjected to a voltage of 34 kV and 36 kV as an application and for illustrative purposes. These failure times were randomly observed and there was no reliable information available. Moreover, the failure times model may be difficult to measure due to the complexity of the action of electrodes. So, we used stress-strength reliability model in the presence of fuzziness.

In this article, estimation of fuzzy stress-strength reliability model $R_F = P\ (Y < X)$, when X and Y are independent but not symmetrically distributed inverse Rayleigh random variables, is discussed. The product of the spacing method was presented to infer the reliability of fuzzy stress strength by using different methods. The proposed estimators are obtained using the maximum likelihood estimation method (MLE) and the maximum product of the spacing estimation method (MPS) as well as Bayesian estimation when prior distributions are assumed exponential. Besides, a Monte Carlo simulation study is made to analyze and compare the performance of the different estimators. A real data application is conducted for illustration purposes and to test the estimated functions of the reliability parameter R_F. Finally, the paper is concluded.

2. The stress strength model

An increase in the values of $x - y$ can be thought of equivalently as the increase in the difference of $\frac{1}{y^2} - \frac{1}{x^2}$. With such consideration, the membership function can be redefined as

$$\mu^{\star}_{A(y)}(x) = M^{\star}\left(\frac{1}{y^2} - \frac{1}{x^2}\right) = 1 - exp\left[-k\ \left(\frac{1}{y^2} - \frac{1}{x^2}\right)\right], x > y, \quad (2)$$

where $k > 0$.

Let X and Y be two independent inverse Rayleigh random variables with scale parameters λ_1 and λ_2, respectively. The inverted Rayleigh $[\,IR(\lambda)]$ distribution has the following cumulative distribution function (CDF) and probability density function (PDF) for $x > 0$:

$$F(x) = exp\left[-\frac{\lambda}{x^2}\right] \text{ and } f(x) = \frac{2\lambda}{x^3} exp\left[-\frac{\lambda}{x^2}\right] \quad (3)$$

respectively, where $\lambda > 0$ is a scale parameter. The traditional reliability of the stress-strength model for inverse Rayleigh distribution was studied and calculated to be $R = \lambda_2/(\lambda_1 + \lambda_2)$ (see Kotz et al. [9]). Therefore, the fuzzy reliability of stress-strength $R_F = P(Y \prec X)$ is given by

$$R_F = P\ (Y < X) = \int_0^{\infty}\int_y^{\infty}\left(1 - exp\left[-k\left(\frac{1}{y^2} - \frac{1}{x^2}\right)\right]\right)\left(\frac{2\lambda_1}{x^3} exp\left[-\frac{\lambda_1}{x^2}\right]\right)\left(\frac{2\lambda_2}{y^3} exp\left[-\frac{\lambda_2}{y^2}\right]\right) dxdy$$

$$= \left(1 - \frac{\lambda_1}{\lambda_1 + k}\right)\left(\frac{\lambda_2}{\lambda_1 + \lambda_2}\right) = \left(\frac{k}{\lambda_1 + k}\right) R \quad (4)$$

The traditional reliability R is always greater than the fuzzy reliability R_F, and as $k \to \infty, R_F \to R$. Figure 1 shows different values for R when λ_1 and λ_2 changes simultaneously, Figure 2 shows fuzzy reliably values for different values of the constant k and when also λ_1 and λ_2 changes simultaneously.

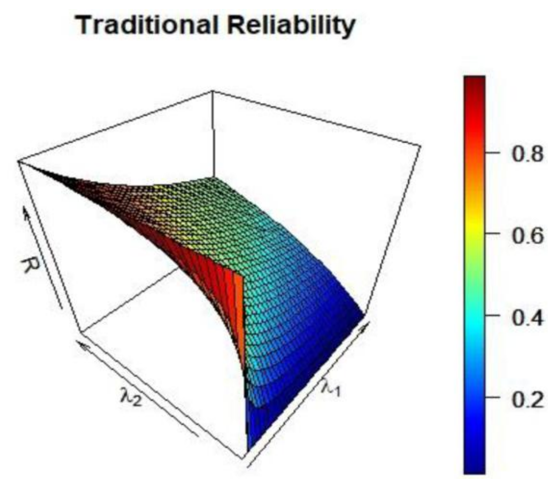

Figure 1. Values for traditional reliability parameter when λ_1 and λ_2 changes simultaneously.

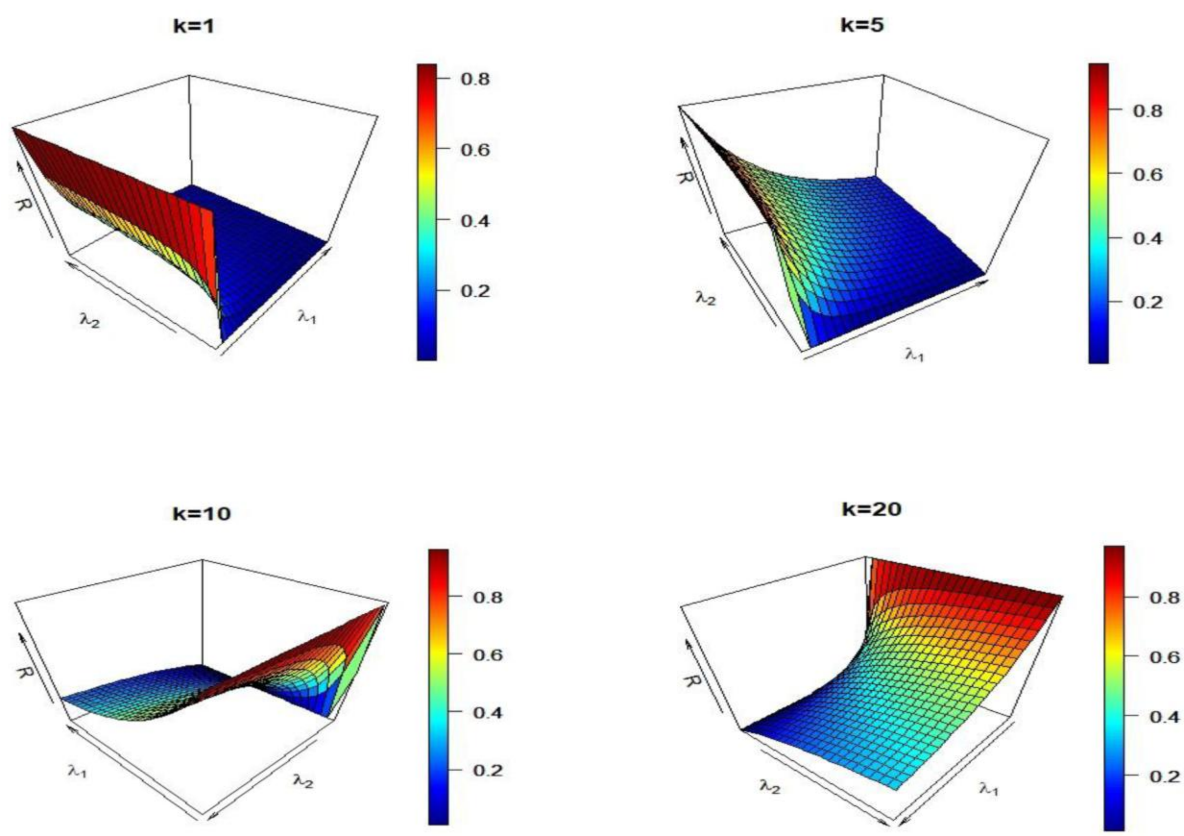

Figure 2. Values for fuzzy reliability parameter for different values of k when λ_1 and λ_2 changes simultaneously**.**

3. Inference of stress-strength model

In this section, the two methods (MLE and MPS) of estimation are used to estimate the fuzzy reliability parameter R_F. Let $(X_1, X_2, \dots, X_n)$ and $(Y_1, Y_2, \dots, Y_m)$ be two independent random samples from IR (λ_1) and IR (λ_2), respectively.

3.1. Likelihood estimation

The joint likelihood function of the IR distribution for the stress-strength model can be written as

$$L(\lambda_1, \lambda_2) = \prod_{i=1}^{n} f(x_i; \lambda_1) \prod_{j=1}^{m} f(y_j; \lambda_2), \tag{5}$$

and the log-likelihood function is given as

$$\begin{aligned} l(\Omega) &= \ln L\,(\lambda_1, \lambda_2) \\ &= n\ln(2\lambda_1) - 3\sum_{i=1}^{n}\ln(x_i) - \lambda_1\sum_{i=1}^{n}x_i^2 + m\ln(2\lambda_2) - 3\sum_{i=1}^{m}\ln(y_i) - \lambda_2\sum_{i=1}^{m}y_i^2. \end{aligned} \tag{6}$$

The normal equations for unknown parameters λ_1, λ_2, are obtained by differentiating (6) partially concerning the parameters λ_1, λ_2, and equate them to zero. The estimators $\hat{\lambda}_1, \hat{\lambda}_2$ can be obtained as a solution to the following equation:

$$\frac{\partial l(\lambda_1, \lambda_2)}{\partial \lambda_j} = \frac{\aleph_j}{\lambda_j} - \sum_{i=1}^{\aleph_j} g_{ji}^2\,; j = 1,2,$$

where $g_{ji} = (x_i, y_i)$, $\aleph_j = (n, m)$. The estimate by using MLE is $\hat{\lambda}_j = \frac{\aleph_j}{\sum_{i=1}^{\aleph_j} g_{ji}^2}$. By using the invariance property of MLE, traditional reliability R and the fuzzy reliability R_F of IR distribution for the stress-strength model are obtained by using MLE's as following

$$\hat{R} = \frac{\hat{\lambda}_2}{\hat{\lambda}_1 + \hat{\lambda}_2}, \qquad \hat{R}_F = \frac{k}{\hat{\lambda}_1 + k}\hat{R}.$$

3.2. Maximum product of spacing estimation

The maximum product spacing for stress-strength model is denoted as following.

$$GS(\lambda_1, \lambda_2) = \left(\prod_{i=1}^{n+1} \mathrm{D}_i(x_i; \lambda_1)\right)^{\frac{1}{n+1}} \left(\prod_{i=1}^{m+1} \mathrm{D}_i(y_j; \lambda_2)\right)^{\frac{1}{m+1}}, \tag{7}$$

such that $\sum_i \mathrm{D}_i(g_{ji}; \lambda_j) = 1$, where

$$\mathrm{D}_i(\mathcal{g}_{ji};\lambda_j)=\begin{cases} D_1=F(\mathcal{g}_{\mathrm{j}1};\lambda_j) \\ D_i=F(\mathcal{g}_{\mathrm{j}i};\lambda_j)-F(\mathcal{g}_{\mathrm{j}(i-1)};\lambda_j);\ i=2\ldots s_j, j=1,2. \\ D_{n+1}=1-F(\mathcal{g}_{\mathrm{j}n};\lambda_j)) \end{cases} \tag{8}$$

The natural logarithm of the product spacing function of the exponential distribution for stress-strength model is denoted as following

$$gs(\Omega)=\ln GS\,(\lambda_1,\lambda_2)=\frac{1}{n+1}\left[-\frac{\lambda_1}{x_1^2}+\ln\left(1-e^{-\frac{\lambda_1}{x_n^2}}\right)+\sum_{i=2}^{n}\ln\left(e^{-\frac{\lambda_1}{x_i^2}}-e^{-\frac{\lambda_1}{x_{i-1}^2}}\right)\right] + \frac{1}{m+1}\left[-\frac{\lambda_2}{y_1^2}+\ln\left(1-e^{-\frac{\lambda_2}{y_m^2}}\right)+\textstyle\sum_{i=2}^{m}\ln\,(e^{-\frac{\lambda_2}{y_i^2}}-e^{-\frac{\lambda_2}{y_{i-1}^2}})\right]. \tag{9}$$

To obtain the normal equations for the unknown parameters, we partially differentiate Eq (8) concerning the parameters λ_1, λ_2, and equate them with zero. The estimators $\hat{\lambda}_1, \hat{\lambda}_2$ can be obtained as a solution of the following equations.

$$\frac{\partial\, gs(\lambda_1,\lambda_2)}{\partial\lambda_j}=\frac{1}{\aleph_j+1}\left[-\frac{1}{\mathcal{g}_{j1}^2}+\frac{\frac{1}{\mathcal{g}_{jn}^2}e^{-\frac{\lambda_j}{\mathcal{g}_{jn}^2}}}{1-e^{-\frac{\lambda_j}{\mathcal{g}_{jn}^2}}}+\sum_{i=2}^{\aleph_j}\frac{\frac{1}{\mathcal{g}_{ji}^2}e^{-\frac{\lambda_j}{\mathcal{g}_{ji}^2}}-\frac{1}{\mathcal{g}_{j\,(i-1)}^2}e^{-\frac{\lambda_j}{\mathcal{g}_{j\,(i-1)}^2}}}{e^{-\frac{\lambda_j}{\mathcal{g}_{ji}^2}}-e^{-\frac{\lambda_j}{\mathcal{g}_{j\,(i-1)}^2}}}\right].$$

The above nonlinear equations can't be solved analytically to find $\hat{\lambda}_1, \hat{\lambda}_2$ of λ_1, λ_2. So, by using optimization algorism as conjugate-gradient or Newton-Raphson optimization methods, the estimators of λ_1, λ_2 are obtained. Using the invariance property of MPS estimators of the MPS's which have been discussed by [13–18] and have concluded that it is the same as that of MLE, traditional reliability R and fuzzy reliability R_F for IR distribution for the stress-strength model can be computed.

4. Bayesian estimation

Assume that the parameters λ_1 and λ_2 are random variables with exponential prior distributions [11] with density function given by

$$\pi_1(\lambda_1)\propto e^{-b_1\,\lambda_1} \quad \text{and} \quad \pi_2(\lambda_2)\propto e^{-b_2\,\lambda_2},$$

where $b_1>0$ and $b_2>0$ are hyperparameters of the prior distributions of λ_1 and λ_2. The posterior distributions of λ_1 and λ_2 are obtained by combining the likelihood function with the prior distributions of both λ_1 and λ_2 and are found to be

$$\pi_1(\lambda_1|\mathrm{x})\propto\lambda_1^n\exp\left[-\lambda_1\left(\mathrm{b}_1+\sum_{i=1}^{n}x_i^{-2}\right)\right]\sim gamma\left(n+1,\left(\mathrm{b}_1+\sum_{i=1}^{n}x_i^{-2}\right)\right),$$

and

$$\pi_1(\lambda_1|x) = \frac{(b_1 + \sum_{i=1}^{n} x_i^{-2})^{n+1}}{\Gamma(n+1)} \lambda_1^n \exp\left[-\lambda_1\left(b_1 + \sum_{i=1}^{n} x_i^{-2}\right)\right].$$

Also,

$$\pi_2(\lambda_2|y) \propto \lambda_2^m \exp\left[-\lambda_2\left(b_2 + \sum_{i=1}^{m} y_i^{-2}\right)\right] \sim gamma\left(m+1, \left(b_2 + \sum_{i=1}^{m} y_i^{-2}\right)\right),$$

and

$$\pi_2(\lambda_2|y) = \frac{(b_2 + \sum_{i=1}^{m} y_i^{-2})^{m+1}}{\Gamma(m+1)} \lambda_2^m \exp\left[-\lambda_2\left(b_2 + \sum_{i=1}^{m} y_i^{-2}\right)\right].$$

Now, let $T_1 = b_1 + \sum_{i=1}^{n} x_i^{-2}$ and $T_2 = b_2 + \sum_{i=1}^{m} y_i^{-2}$. After some manipulations and simplifications, we have that $\pi_1(\lambda_1 \backslash \boldsymbol{u}) \sim Gamma(n+1, T_1)$ and $\pi_2(\lambda_2 \backslash \boldsymbol{v}) \sim Gamma(m+1, T_2)$ respectively. The joint posterior distribution of λ_1 and λ_2 is then given by

$$\pi(\lambda_1, \lambda_2 |\, x, y) = \frac{T_1^{n+1}\, T_2^{m+1}}{\Gamma(n+1)\Gamma(m+1)} \lambda_1^n\, \lambda_2^m \exp[-(\lambda_1 T_1 + \lambda_2 T_2)].$$

Let $S = \lambda_1 + \lambda_2$ and since $R = \frac{\lambda_2}{\lambda_1 + \lambda_2}$, then using standard transformation techniques, the joint posterior pdf of R and S will be

$$\pi(r, s |\, x, y) = \frac{T_1^{n+1}\, T_2^{m+1}}{\Gamma(n+1)\Gamma(m+1)} r^m\, (1-r)^n\, s^{n+m+2} \exp\left[-s\big((1-r)T_1 + rT_2\big)\right].$$

Under squared error loss function, the Bayes estimate $\tilde{R}_F$ is the expected value of R_F and is given by,

$$\tilde{R}_F = E(R_F|x, y) = \int_0^1 \int_0^\infty R_F\, \pi(r, s |\, x, y)\, ds\, dr$$

$$\tilde{R}_F = A \int_0^1 \int_0^\infty \frac{kr}{s(1-r)+k} r^{m+1}\, (1-r)^n\, s^{n+m+3} \exp\left[-s\big((1-r)T_1 + rT_2\big)\right] ds\, dr, \qquad (10)$$

where $A = \frac{T_1^{n+1}\, T_2^{m+1}}{\Gamma(n+1)\Gamma(m+1)}$.

The above integrals are hard to obtain, so numerical technique is used to evaluate the Bayes estimate. Markov Chain Monte Carlo (MCMC) approach is used to obtain Bayesian estimators. An important sub-class of the MCMC techniques is Gibbs's sampling and more general Metropolis within Gibbs samplers see [24,25]. The Metropolis-Hastings algorithm, together with the Gibbs sampling, are the two most popular examples of an MCMC method. It's similar to acceptance-rejection sampling, and the Metropolis-Hastings algorithm considers that to each iteration of the algorithm, a candidate value can be generated from the IR distributions. We use the Metropolis-Hastings within Gibbs sampling steps to generate random samples from conditional posterior densities of λ_1, λ_2, and R_F. For more information, see [19–23].

5. Simulation study

In this section, we provide a numerical comparison using the Monte Carlo simulation algorithm. We explain our algorithm through an application of fuzzy and traditional stress-strength models by different estimation methods. In this current simulation, we will compare MLE, MPS, and Bayesian estimation methods based on traditional and fuzzy stress-strength measures for estimating the parameter of IR distribution. The comparison is made through bias and mean squared errors (MSE) of the different measures. The simulations are made using the R program for several combinations of the parameters and m, n, and k.

Simulation Algorithm:

We build our model by creating all simulation controls. At this point, we must follow the following steps in order:

Step 1: Suppose different values of the parameters vector of IR distribution.

Step 2: Choose the different sample size of strength
$n = 30, 50, 80, 100, 150\ 200$ and different sample size of stress
$m = 20, 40, 90, 110, 120$ and 150 respectively.

Step 3: Generate the sample random values of IR distribution by using quantile function in equation

$$x_i = \lambda_1 \sqrt{\frac{-1}{\ln(u_i)}}; 0 < u < 1 \text{ and } y_i = \lambda_2 \sqrt{\frac{-1}{\ln(v_i)}};\ 0 < v < 1.$$

Step 4: Solve differential equations for each estimation method. To obtain the estimators of the parameters for IR distribution, we calculate $\hat{\lambda}_1, \hat{\lambda}_2$, R_{F1} when k = 1; and R_{F2} when k = 5.

Step 5: Repeat this experiment (L-1) iterations. In each experiment, the parameter values are the same. The generated random values are certainly varying from experiment to experiment even though the sample size is not changed. In the end, we have L-values of mean and MSE, and we restricted the number of repeats in this experiment to 10,000. Take the averages of these values and call them Monte Carlo estimates.

After completing the treatment stage, simulated outcomes are listed in Tables 1–4, Figure 3, and the following observations were observed:

- The Bias and MSE decrease as sample sizes increase for all estimates.
- For fixed values of λ_1, the biases and MSE's of estimates of parameters are increasing with λ_2 increasing.
- For fixed values of λ_1, the biases and MSE's of estimates of parameters are increasing with λ_2 increasing, but the estimate of R for fuzzy stress strength and tradition are decreasing in approximately most situations.
- For fixed values of λ_2, the bias and MSE of estimates of λ_1, R for fuzzy stress strength and tradition are decreasing with λ_1 increasing, but the estimates of λ_2 are increasing, in approximately most situations.
- The MPS method is found to be superior to the MLE and Bayesian methods in most cases.
- From the observed results of reliability, we note the efficiency of the fuzzy stress strength is over traditional stress strength in most situations according to Bias and MSE.

- In fuzzy stress strength, the efficiency is better with decreased values of k according to Bias and MSE.

Table 1. Bias and MSE for MLE, MPS and Bayesian of IR parameters under the stress-strength model when $\lambda_1 = 0.5$ and $\lambda_2 = 0.5$.

		MLE		MPS		Bayesian	
n, m		Bias	MSE	Bias	MSE	Bias	MSE
30, 20	λ_1	0.01688	0.009853	-0.01175	0.008694	0.01667	0.009923
	λ_2	0.02254	0.012176	-0.01042	0.010402	0.02298	0.012269
	R	0.00191	0.004646	-0.00010	0.004668	0.00222	0.004652
	R_{F1}	0.00082	0.003707	0.00574	0.003819	0.00108	0.003709
	R_{F2}	0.00123	0.004530	0.00173	0.004564	0.00152	0.004536
50, 40	λ_1	0.01460	0.005687	-0.00528	0.005089	0.01421	0.005724
	λ_2	0.01605	0.006444	-0.00743	0.005703	0.01554	0.006470
	R	0.00037	0.002765	-0.00140	0.002770	0.00032	0.002777
	R_{F1}	-0.00101	0.002248	0.00214	0.002275	-0.00096	0.002259
	R_{F2}	-0.00034	0.002712	-0.00034	0.002716	-0.00036	0.002724
80, 90	λ_1	0.00467	0.003394	-0.00915	0.003286	0.00454	0.003400
	λ_2	0.00465	0.002857	-0.00793	0.002753	0.00475	0.002900
	R	0.00025	0.001457	0.00088	0.001462	0.00035	0.001479
	R_{F1}	0.00032	0.001266	0.00382	0.001296	0.00043	0.001280
	R_{F2}	0.00015	0.001461	0.00187	0.001470	0.00026	0.001482
100, 110	λ_1	0.00709	0.002738	-0.00456	0.002591	0.00702	0.002759
	λ_2	0.00432	0.002318	-0.00647	0.002250	0.00435	0.002311
	R	-0.00120	0.001109	-0.00079	0.001115	-0.00113	0.001116
	R_{F1}	-0.00144	0.000968	0.00140	0.000980	-0.00138	0.000976
	R_{F2}	-0.00143	0.001113	-0.00011	0.001118	-0.00136	0.001121
150, 120	λ_1	0.00495	0.001869	-0.00344	0.001793	0.00502	0.001872
	λ_2	0.00500	0.002185	-0.00504	0.002103	0.00476	0.002177
	R	-0.00012	0.000925	-0.00095	0.000927	-0.00026	0.000930
	R_{F1}	-0.00052	0.000753	0.00078	0.000756	-0.00063	0.000757
	R_{F2}	-0.00034	0.000906	-0.00041	0.000906	-0.00048	0.000911
200, 150	λ_1	0.00143	0.001289	-0.00518	0.001280	0.00146	0.001289
	λ_2	-0.00026	0.001603	-0.00862	0.001624	-0.00023	0.001609
	R	-0.00101	0.000743	-0.00190	0.000745	-0.00101	0.000745
	R_{F1}	-0.00051	0.000586	0.00036	0.000587	-0.00052	0.000586
	R_{F2}	-0.00091	0.000721	-0.00118	0.000721	-0.00091	0.000722

Table 2. Bias and MSE for MLE, MPS, and Bayesian of IR parameters under the stress-strength model when $\lambda_1 = 0.5$ and $\lambda_2 = 2$.

		MLE		MPS		Bayesian	
n, m		Bias	MSE	Bias	MSE	Bias	MSE
30, 20	λ_1	0.01722	0.009963	-0.01153	0.008799	0.01663	0.009996
	λ_2	0.09455	0.209672	-0.03808	0.178814	0.08884	0.200672
	R	-0.00244	0.002011	-0.00378	0.002049	-0.00251	0.001967
	R_{F1}	-0.00430	0.003224	0.00483	0.003249	-0.00415	0.003192
	R_{F2}	-0.00377	0.002483	-0.00125	0.002477	-0.00377	0.002441
50, 40	λ_1	0.00929	0.005053	-0.01045	0.004713	0.00944	0.005122
	λ_2	0.02895	0.104113	-0.06328	0.098289	0.02630	0.101853
	R	-0.00321	0.001175	-0.00435	0.001196	-0.00342	0.001176
	R_{F1}	-0.00357	0.001841	0.00262	0.001843	-0.00374	0.001854
	R_{F2}	-0.00376	0.001436	-0.00220	0.001430	-0.00396	0.001442
80, 90	λ_1	0.00908	0.003693	-0.00466	0.003437	0.00905	0.003681
	λ_2	0.02371	0.046319	-0.02677	0.044176	0.02369	0.046968
	R	-0.00193	0.000621	-0.00159	0.000619	-0.00193	0.000622
	R_{F1}	-0.00318	0.001212	0.00187	0.001202	-0.00318	0.001210
	R_{F2}	-0.00268	0.000834	-0.00057	0.000821	-0.00269	0.000834
100, 110	λ_1	0.00456	0.002584	-0.00704	0.002496	0.00453	0.002569
	λ_2	0.01642	0.034435	-0.02646	0.033520	0.01529	0.034530
	R	-0.00092	0.000489	-0.00064	0.000486	-0.00102	0.000495
	R_{F1}	-0.00127	0.000913	0.00303	0.000916	-0.00132	0.000915
	R_{F2}	-0.00124	0.000645	0.00055	0.000637	-0.00132	0.000651
150, 120	λ_1	0.00318	0.001967	-0.00528	0.001923	0.00302	0.001941
	λ_2	0.01613	0.035520	-0.02435	0.034584	0.01556	0.035056
	R	-0.00057	0.000409	-0.00110	0.000412	-0.00056	0.000406
	R_{F1}	-0.00079	0.000700	0.00186	0.000704	-0.00073	0.000694
	R_{F2}	-0.00079	0.000515	-0.00016	0.000515	-0.00076	0.000511
200, 150	λ_1	0.00140	0.001126	-0.00524	0.001124	0.00142	0.001134
	λ_2	0.02209	0.028304	-0.01175	0.027012	0.02163	0.028342
	R	0.00057	0.000293	0.00001	0.000294	0.00053	0.000295
	R_{F1}	0.00031	0.000442	0.00230	0.000448	0.00027	0.000445
	R_{F2}	0.00042	0.000349	0.00079	0.000350	0.00038	0.000351

Table 3. Bias and MSE for MLE, MPS and Bayesian of IR parameters under the stress-strength model when $\lambda_1 = 2$ and $\lambda_2 = 0.5$.

		MLE		MPS		Bayesian	
n, m		Bias	MSE	Bias	MSE	Bias	MSE
30, 20	λ_1	0.06763	0.155282	-0.04768	0.137440	0.06555	0.156610
	λ_2	0.02046	0.011668	-0.01262	0.010028	0.02030	0.011591
	R	0.00402	0.001864	0.00273	0.001846	0.00420	0.001887
	R_{F1}	0.00203	0.000444	0.00421	0.000483	0.00216	0.000452
	R_{F2}	0.00298	0.001319	0.00439	0.001348	0.00317	0.001340
50, 40	λ_1	0.04891	0.094691	-0.02992	0.086752	0.04807	0.094583
	λ_2	0.01049	0.006881	-0.01254	0.006335	0.01044	0.006899
	R	0.00135	0.001199	0.00026	0.001189	0.00140	0.001200
	R_{F1}	0.00076	0.000280	0.00215	0.000295	0.00079	0.000280
	R_{F2}	0.00092	0.000848	0.00173	0.000856	0.00097	0.000847
80, 90	λ_1	0.03999	0.054062	-0.01579	0.050003	0.04016	0.053777
	λ_2	0.00592	0.002720	-0.00667	0.002597	0.00561	0.002704
	R	-0.00002	0.000582	0.00040	0.000586	-0.00013	0.000580
	R_{F1}	-0.00007	0.000147	0.00130	0.000155	-0.00012	0.000147
	R_{F2}	-0.00027	0.000431	0.00116	0.000440	-0.00035	0.000430
100, 110	λ_1	0.00977	0.042263	-0.03626	0.041625	0.01012	0.042315
	λ_2	0.00263	0.002330	-0.00810	0.002294	0.00291	0.002340
	R	0.00112	0.000520	0.00137	0.000522	0.00118	0.000522
	R_{F1}	0.00084	0.000130	0.00197	0.000137	0.00085	0.000131
	R_{F2}	0.00107	0.000383	0.00219	0.000391	0.00111	0.000385
150, 120	λ_1	0.00724	0.028570	-0.02618	0.028321	0.00687	0.028620
	λ_2	0.00732	0.002145	-0.00283	0.002016	0.00731	0.002161
	R	0.00236	0.000382	0.00182	0.000379	0.00240	0.000386
	R_{F1}	0.00108	0.000090	0.00165	0.000093	0.00110	0.000091
	R_{F2}	0.00184	0.000271	0.00214	0.000274	0.00188	0.000274
200, 150	λ_1	0.00595	0.020483	-0.02065	0.020369	0.00619	0.020443
	λ_2	0.00547	0.001682	-0.00295	0.001608	0.00545	0.001685
	R	0.00171	0.000301	0.00116	0.000300	0.00168	0.000302
	R_{F1}	0.00077	0.000068	0.00118	0.000070	0.00075	0.000068
	R_{F2}	0.00133	0.000210	0.00148	0.000212	0.00130	0.000210

Table 4. Bias and MSE for MLE, MPS, and Bayesian of IR parameters under the stress-strength model when $\lambda_1 = 2$ and $\lambda_2 = 2$.

		MLE		MPS		Bayesian	
n, m		Bias	MSE	Bias	MSE	Bias	MSE
30, 20	λ_1	0.06224	0.176286	-0.05263	0.157645	0.06459	0.177161
	λ_2	0.09727	0.203480	-0.03537	0.172121	0.09340	0.199105
	R	0.00357	0.004826	0.00162	0.004858	0.00292	0.004771
	R_{F1}	0.00261	0.001646	0.00843	0.001815	0.00227	0.001636
	R_{F2}	0.00246	0.004133	0.00686	0.004254	0.00188	0.004101
50, 40	λ_1	0.03017	0.082788	-0.04820	0.078270	0.03007	0.083022
	λ_2	0.06224	0.113595	-0.03145	0.101059	0.06146	0.113486
	R	0.00315	0.002556	0.00139	0.002559	0.00306	0.002556
	R_{F1}	0.00181	0.000863	0.00564	0.000929	0.00178	0.000862
	R_{F2}	0.00220	0.002158	0.00494	0.002204	0.00214	0.002157
80, 90	λ_1	0.02923	0.051774	-0.02658	0.048799	0.03008	0.052007
	λ_2	0.01866	0.047300	-0.03152	0.045476	0.01861	0.047487
	R	-0.00118	0.001537	-0.00050	0.001532	-0.00129	0.001538
	R_{F1}	-0.00039	0.000552	0.00294	0.000576	-0.00047	0.000553
	R_{F2}	-0.00131	0.001370	0.00200	0.001378	-0.00142	0.001371
100, 110	λ_1	0.02007	0.043336	-0.02620	0.041573	0.02040	0.043196
	λ_2	0.01477	0.035742	-0.02818	0.034901	0.01559	0.035898
	R	-0.00043	0.001201	-0.00003	0.001200	-0.00038	0.001204
	R_{F1}	0.00010	0.000455	0.00281	0.000473	0.00010	0.000455
	R_{F2}	-0.00048	0.001100	0.00215	0.001108	-0.00046	0.001101
150, 120	λ_1	0.02229	0.027613	-0.01152	0.026372	0.02199	0.027322
	λ_2	0.01364	0.036194	-0.02658	0.035347	0.01402	0.036164
	R	-0.00135	0.000946	-0.00218	0.000952	-0.00127	0.000944
	R_{F1}	-0.00084	0.000300	0.00075	0.000305	-0.00080	0.000298
	R_{F2}	-0.00157	0.000781	-0.00045	0.000783	-0.00151	0.000778
200, 150	λ_1	0.01240	0.020863	-0.01433	0.020372	0.01221	0.020684
	λ_2	0.01492	0.025875	-0.01865	0.025209	0.01504	0.025844
	R	0.00016	0.000705	-0.00069	0.000707	0.00020	0.000703
	R_{F1}	0.00002	0.000232	0.00122	0.000237	0.00004	0.000231
	R_{F2}	-0.00011	0.000594	0.00064	0.000597	-0.00007	0.000592

Figure 3. Relative Efficiency for different measures.

6. Application of real data

The numerical results of tradition and fuzzy stress-strength reliability estimation of the IR distribution for real data are presented in this section.

Two real stress and strength data sets contained times to breakdown down an insulating fluid between electrodes recorded at different voltages; these data have been discussed by [26]. Data I and data II as presented in Table 4, are the failure times (in minutes) are presented, which are for an insulating fluid between two electrodes subject to a voltage of 34 kV (data set I) and 36 kV (data set II). Table 6 provides information about the estimated parameters of the IR model and the corresponding traditional and fuzzy reliability measures

Table 5. Data sets of times to breakdown down an insulating fluid between electrodes recorded at different voltages.

Data I	0.19	0.78	0.96	1.31	2.78	3.16	4.15	4.67	4.85	6.5
	7.35	8.01	8.27	12.06	31.75	32.52	33.91	36.71	72.89	-
Data II	0.35	0.59	0.96	0.99	1.69	1.97	2.07	2.58	2.71	2.9
	3.67	3.99	5.35	13.77	25.50	-	-	-	-	-

Table 6. Tradition and the fuzzy stress-strength estimation of the IR distribution.

	λ_1	λ_2	R	R_{F1}	R_{F2}
MLE	0.60363	1.02982	0.63046	0.39315	0.56254
MPS	0.55815	0.89220	0.61516	0.39480	0.55339
Bayesian	0.64494	1.12664	0.63595	0.38661	0.56330

The graph of MCMC estimates for λ_1 and λ_2 using the MH algorithm are the plotting of estimates, histogram of estimates, and convergence of estimates are shown in Figure 3. In Figure 4, we note the convergence of MCMC estimates for λ_1 and λ_2 in the first quartile iteration.

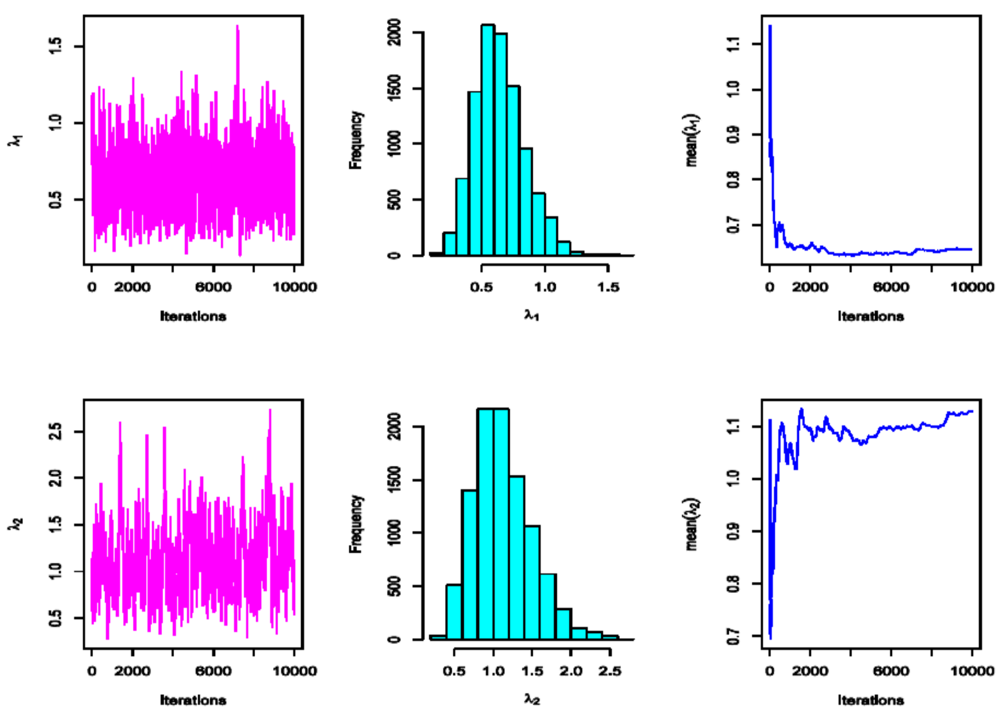

Figure 4. Convergence of MCMC estimation of λ_1 and λ_2.

7. Conclusions

The new approach of estimating fuzzy stress-strength reliability $R_F = P\ (Y < X)$ is getting much attention because of the properties of R_F, which makes the analysis more sensitive and more reliable. Also, when the study results are not known completely, the use of traditional methods may be misleading, and the need for new approaches that can handle such situations is very important. In this paper, the stress and strength variables were distributed as inverted Rayleigh distribution. It can be noted that different membership functions will provide different measures of R_F. It is also noted that the MPS method is superior to the MLE and Bayesian methods in most cases.

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Conflict of interest

The authors declare that they have no conflict of interest to report regarding the present study.

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