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Research article

New complex wave structures to the complex Ginzburg-Landau model

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Abstract: Int his paper, we study and analysis the complex Ginzburg-Landau model or CGL model to obtain some new solitary wave structures through the modified (G'/G)-expansion method. Those solutions can explain through hyperbolic, trigonometric, and rational functions. The graphical design makes the dynamics of the equations noticeable. Herein, we state that the examined method is important, powerful, and significant in performing numerous solitary wave structures of various nonlinear wave models following in physics and engineering as well.

Keywords: modified (G'/G)-expansion method; the complex Ginzburg-Landau model; solitary wave solutions
Mathematics Subject Classification: 35A20, 35A24, 35A25, 35B10, 70K50

1. Introduction

The nonlinear wave models have been excited about the observation of numerous scientists in different areas. Everybody can express various natural phenomena. Moreover, they describe the dynamics of these phenomena and determine the physical application of these models. Several scientists have been endeavored to get different procedures that able to make the closed-form wave and solitary wave solutions of these equations. Some crucial processes as the extended mapping method [1], the extended direct algebraic sech method [2], the extended modified mapping method [3], the Sech-tanh method [4], the direct algebraic function method [5], the (G'/G)-expansion scheme [6–10], the finite series Jacobi elliptic cosine function Ansatz [11, 12], the modified auxiliary equation method [13, 14], the generalized unified method [15], the generalized exponential function [16], the general bilinear form [17], the reproducing kernel Hilbert space method [18], the residual power series method [19], the $\exp(-\phi(\xi)$-expansion method [20–22], the variation of parameters method (VPM) [23], the traditional homotopy perturbation method (HPM) [24, 25], the optimal Galerkin-homotopy asymptotic method (OGHAM) [26], the Laplace variational iterative method [27, 28], the improved $\tan(\phi(\xi)/2)$-expansion method [29], the Sumudu homotopy perturbation method [30], the sine-Gordon expansion method [31], the Riccati-Bernoulli sub-ODE method [32], the improve $\tan(\phi(\xi))$-expansion method [33, 34], the reproducing kernel method [35], a systematic calculative algorithm [36], the extended trial equation method (ETEM) [37] and many more. The paper applied the modified $(\frac{G\prime}{G})$-expansion method [38] to derive the different type of solitary wave structures for the complex Ginzburg-Landau model [14, 16]. The complex Ginzburg-Landau model can be represented as

$$iW_t + s_1 W_{xx} + s_2 f(|W|^2)W - \frac{s_3}{|W|^2 W^*}\{2|W|^2(|W|^2)_{xx} - ((|W|^2)_x)^2\} - s_4 W = 0, \tag{1.1}$$

where x is the non-dimensional distance along the fiber, t is the time in dimensionless form, s_1, s_2, s_3, s_4 are the group of velocity dispersion parameters and the function $f(|W|^2)$ is a k-times continuously differentiable real-valued algebraic function, $k = 1, 2, ...$, respectively. Authors of [39] used the generalized logistic equation method for Kerr law and dual power law Schrödinger equations, and obtained the exact optical solitons to the perturbed nonlinear Schrödinger equation with dual-power law of nonlinearity [40], and also the dynamical behavior of mixed type lump solutions on the (3+ 1)-dimensional generalized Kadomtsev-Petviashvili-Boussinesq equation has been investigated in [41].

This investigation proposes to acquire new solitary wave solutions to the the complex Ginzburg-Landau model via the modified $(\frac{G\prime}{G})$-expansion method. The synopsis of this paper shown below. In Section 2, we mentioned the algorithm of the modified $(\frac{G\prime}{G})$-expansion method. In Section 3, new solitary wave solutions of the studied equation is formulated. In Section 4, result and discussions are given.

2. Outline of the modified $(\frac{G\prime}{G})$-expansion method

The modified $(\frac{G\prime}{G})$-expansion method [38] is summarized as follows:

$$\mathrm{P}(u, u_x, u_{xx}, u_t, u_{tt}, u_{xt}, ...) = 0, \tag{2.1}$$

where $u = u(x, t)$, $u_x = \frac{\partial u}{\partial x}$, $u_{xx} = \frac{\partial^2 u}{\partial x^2}$ and P is a polynomial in $u(x, t)$ and their partial derivatives, in which the nonlinear terms and biggest order derivatives are involved.

Use the transformation:

$$\mathrm{u} = u(x, t) = u(\xi), \xi = k(x - Vt + \xi_0), \tag{2.2}$$

where k, ξ_0 and V are a constant. From Eq 2.1 and Eq 2.2, we find:

$$\mathrm{R}(u, ku', k^2u'', -kVu', k^2V^2u'', -k^2V^2u'', ...) = 0. \tag{2.3}$$

- **Step 1:** Calculate m through the balance rule on Eq 2.3.
- **Step 2:** We consider the modified $(\frac{G\prime}{G})$-expansion method:

$$u(\xi) = \sum_{i=-m}^{m} A_i F^i, \tag{2.4}$$

where $F = (\frac{G^{'}}{G} + \frac{\lambda}{2})$, $|A_{-m}| + |A_m| \neq 0$ and $G = G(\xi)$ satisfies the equation:

$$G^{''} + \lambda G^{'} + \mu G = 0, \tag{2.5}$$

where $A_i(\pm 1, \pm 2, ..., \pm m)$, λ and μ are free constants. From the Eq 2.5, after some manipulation we find:

$$F' = h - F^2, \tag{2.6}$$

where $h = \frac{\lambda^2 - 4\mu}{4}$ and h is calculated by λ and μ. So, F now satisfies the Riccati like equation 2.6. It is found that the Riccati like equation 2.6 admits several types of solutions (see Appendix for details).
- **Step 3:** Applying Eq 2.4 into Eq 2.3 and Eq 2.6, collecting all terms with the same order of F together. Equating each coefficient of this polynomial to zero, yields a set of algebraic equations which can be solved to find the values of $A_i(\pm 1, \pm 2, ..., \pm m)$, λ and μ with the help of MAPLE.

3. Solitary wave solutions for the CGL model

Let us consider:

$$iW_t + s_1 W_{xx} + s_2 f(|W|^2)W - \frac{s_3}{|W|^2 W^*}\{2|W|^2(|W|^2)_{xx} - ((|W|^2)_x)^2\} - s_4 W = 0. \tag{3.1}$$

The constants η is the phase component, a is the wave number, b is the frequency, r is the phase constant and c is the velocity of the above model, respectively. Plugging $W(x,t) = e^{i\eta}V(\xi)$, $\xi = x - ct$, $\eta = -ax + bt + r$ into the Eq 3.1. Equation 3.1 separates the imaginary, and real parts through the above transformation:

$$-(2as_1 + c)V' = 0, \tag{3.2}$$

$$-(s_1 a^2 + s_4 + b)V + s_2 f(|V|^2)V + (s_1 - 4s_3)V'' = 0. \tag{3.3}$$

The Eq 3.2 provides $c = -2as_1$. For researching the Kerr-law nonlinearity of Eq 3.1, we put $f(|V|^2) = V^2$ and Eq 3.3 becomes:

$$V^3 + PV + RV'' = 0, \tag{3.4}$$

where $P = -\frac{s_1 a^2 + s_4 + b}{s_2}$ and $R = -\frac{s_1 - 4s_3}{s_2}$. In accordance with the rule of the modified $(\frac{G\prime}{G})$-expansion method [38], Equation 3.4 gives:

$$U(\xi) = A_1 F(\xi) + A_0 + A_{-1}F^{-1}(\xi), \tag{3.5}$$

where the coefficients A_0, A_1 and A_{-1} are constants. By Eq 3.5 and Eq 3.4 and then equating each coefficients of F^i to zeros, we get:

- The first set:

$$P = \frac{1}{2}R\lambda^2 - 2R\mu, g = \pm\sqrt{-2R}, A_0 = 0, A_1 = g, A_{-1} = 0.$$

Using the values of the first set and Eq 3.5 into Eq 3.4, we have:

$$W_1(x,t) = e^{i(-ax+bt+r)}[\frac{2g}{\sqrt{\lambda^2-4\mu}} \times coth\{\frac{\sqrt{\lambda^2-4\mu}}{2}(x+2as_1t)\}],$$

$$W_2(x,t) = e^{i(-ax+bt+r)}[\frac{2g}{\sqrt{\lambda^2-4\mu}} \times tanh\{\frac{\sqrt{\lambda^2-4\mu}}{2}(x+2as_1t)\}],$$

$$W_3(x,y,z,t) = e^{i(-ax+bt+r)} \times (x+2as_1t),$$

$$W_4(x,t) = e^{i(-ax+bt+r)}[-\frac{2g}{\sqrt{4\mu-\lambda^2}} \times cot\{\frac{\sqrt{4\mu-\lambda^2}}{2}(x+2as_1t)\}],$$

$$W_5(x,t) = e^{i(-ax+bt+r)}[\frac{2g}{\sqrt{4\mu-\lambda^2}} \times tan\{\frac{\sqrt{4\mu-\lambda^2}}{2}(x+2as_1t)\}].$$

- The second set:

$$P = \frac{1}{2}R\lambda^2 - 2R\mu - \frac{3}{2}gh\lambda^2 + 6gh\mu, h = \pm\sqrt{\frac{-2}{R}}, A_0 = 0, A_1 = g, A_{-1} = \frac{1}{2}h(\lambda^2-4\mu).$$

Similarly, we get:

$$W_6(x,t) = e^{i(-ax+bt+r)}[\frac{g\sqrt{\lambda^2-4\mu}}{2} \times tanh\{\frac{\sqrt{\lambda^2-4\mu}}{2}(x+2as_1t)\} + h\sqrt{\lambda^2-4\mu} \times coth\{\frac{\sqrt{\lambda^2-4\mu}}{2}(x+2as_1t)\}],$$

$$W_7(x,t) = e^{i(-ax+bt+r)}[\frac{g\sqrt{\lambda^2-4\mu}}{2} \times coth\{\frac{\sqrt{\lambda^2-4\mu}}{2}(x+2as_1t)\} + h\sqrt{\lambda^2-4\mu} \times tanh\{\frac{\sqrt{\lambda^2-4\mu}}{2}(x+2as_1t)\}],$$

$$W_8(x,y,z,t) = e^{i(-ax+bt+r)}[g \times \frac{1}{x+2as_1t} + \frac{1}{2}h(\lambda^2-4\mu) \times (x+2as_1t)],$$

$$W_9(x,t) = e^{i(-ax+bt+r)}[\frac{-g\sqrt{4\mu-\lambda^2}}{2} \times tan\{\frac{\sqrt{4\mu-\lambda^2}}{2}(x+2as_1t)\} - h\sqrt{4\mu-\lambda^2} \times cot\{\frac{\sqrt{4\mu-\lambda^2}}{2}(x+2as_1t)\}],$$

$$W_{10}(x,t) = e^{i(-ax+bt+r)}[\frac{g\sqrt{4\mu-\lambda^2}}{2} \times cot\{\frac{\sqrt{4\mu-\lambda^2}}{2}(x+2as_1t)\}$$
$$+ h\sqrt{4\mu-\lambda^2} \times tan\{\frac{\sqrt{4\mu-\lambda^2}}{2}(x+2as_1t)\}].$$

- The third set:

$$P = \frac{1}{2}R\lambda^2 - 2R\mu, A_0 = 0, A_1 = 0, A_{-1} = \frac{1}{2}h(\lambda^2 - 4\mu).$$

Similarly, we find:

$$W_{11}(x,t) = e^{i(-ax+bt+r)}[h\sqrt{\lambda^2-4\mu} \times coth\{\frac{\sqrt{\lambda^2-4\mu}}{2}(x+2as_1t)\}],$$

$$W_{12}(x,t) = e^{i(-ax+bt+r)}[h\sqrt{\lambda^2-4\mu} \times tanh\{\frac{\sqrt{\lambda^2-4\mu}}{2}(x+2as_1t)\}],$$

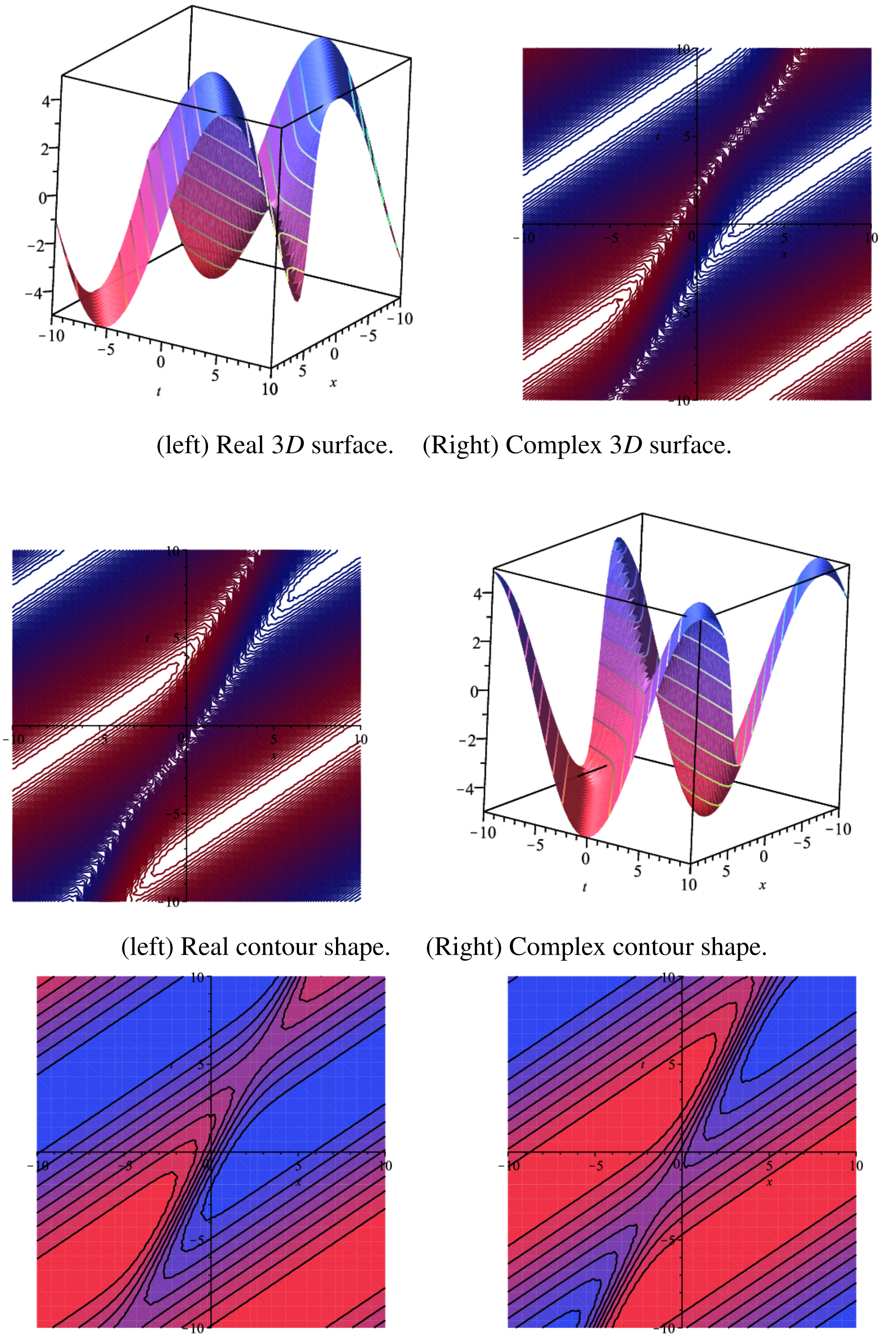
$$W_{13}(x,y,z,t) = e^{i(-ax+bt+r)}[\frac{1}{2}h(\lambda^2-4\mu) \times (x-ct)],$$

$$W_{14}(x,t) = e^{i(-ax+bt+r)}[-h\sqrt{4\mu-\lambda^2} \times cot\{\frac{\sqrt{4\mu-\lambda^2}}{2}(x+2as_1t)\}],$$

$$W_{15}(x,t) = e^{i(-ax+bt+r)}[h\sqrt{4\mu-\lambda^2} \times tan\{\frac{\sqrt{4\mu-\lambda^2}}{2}(x+2as_1t)\}].$$

4. Results and discussions

Ma et al. [38] have introduced a method which is called the modified (G'/G)-expansion approach to derive for solitary wave solutions of nonlinear wave models, where $G = G(\xi)$ satisfies $G''(\xi) + \lambda G'(\xi) + \mu G(\xi) = 0$, where λ and μ are arbitrary constants and $u(\xi) = \sum_{i=-m}^{m} A_i(\frac{G'}{G} + \frac{\lambda}{2})^i$ be the anstaz equation of nonlinear wave models. We apply the modified (G'/G)-expansion process on the CGL model and provided fifteen solitary wave solutions. Osman et al. [14] studied CGL model to derive only ten solitary wave solutions through the modified auxiliary equation method. If $B = 2$ and $4\alpha\sigma - B^2 = \lambda^2 - 4\mu$, the solutions of Eqs. (3.7), (3.8), (3.9), (3.10), and (3.11) in [14] are similar solutions W_{11}, W_{12}, W_{13}, W_{14} and W_{15}. And solutions W_1, W_2, W_3, W_4, W_5, W_6, W_7, W_8, W_9 and W_{10} are all new solitary wave solutions. Osman et al. [14] only derived trigonometric and hyperbolic solutions but failed to achieve the rational ones. The rational function solutions are vital not only for physics but also for the areas of sciences and engineering. Moreover, hyperbolic solutions are useful for analyzing the modulus instability in plasma physics. Comparison between two methods, the modified (G'/G)-expansion process is provided more solitary wave solutions rather than the modified auxiliary equation method. Finally, the newly method successfully implemented to derive new solitary wave solutions to the CGL model. The graph is an important tool for information and to demonstrate the solutions to the problems lucidly. When making the computation in daily life, we need a fundamental knowledge of building the application of graphs. Accordingly, the graphical performances of few got solutions are drawn in the Figures 1, 2 and 3, respectively. We expressed Figure 1, 2 and 3, respectively for few of the derived solutions to display more of properties for the recommended model. The representation of the examined process gives the accuracy and influence of this procedure and also the capacity for implementing various nonlinear wave models.

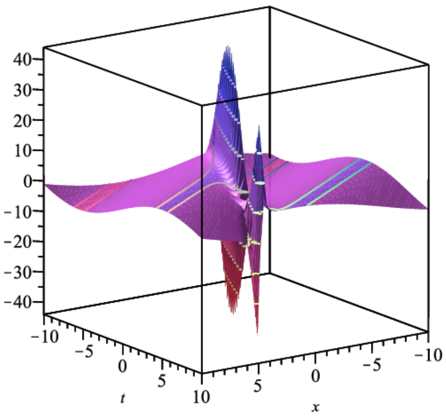


(left) Real $3D$ surface. (Right) Complex $3D$ surface.

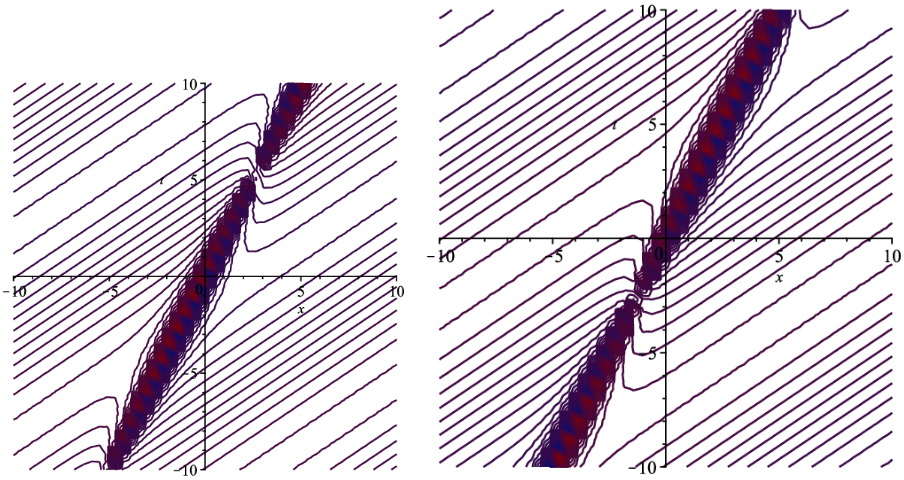
(left) Real contour shape. (Right) Complex contour shape.

(left) Real filled region shape. (Right) Complex filled region shape.

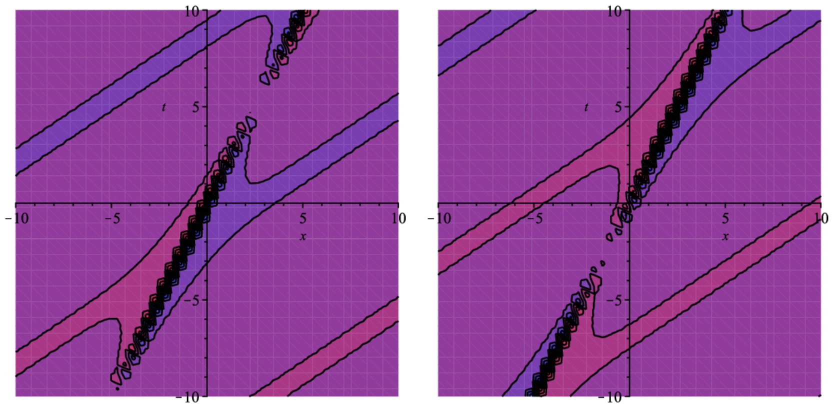
Figure 1. Graphical description of the answer in $W_1(x, t)$ under the values $a = 0.22$, $b = 0.3$, $r = 0.5$, $c = 0.5$, $R = -10$, $\mu = 1$, $\lambda = 3$ and $t = 0.01$ for $2D$ graphics.



(left) Real $3D$ surface. (Right) Complex $3D$ surface.

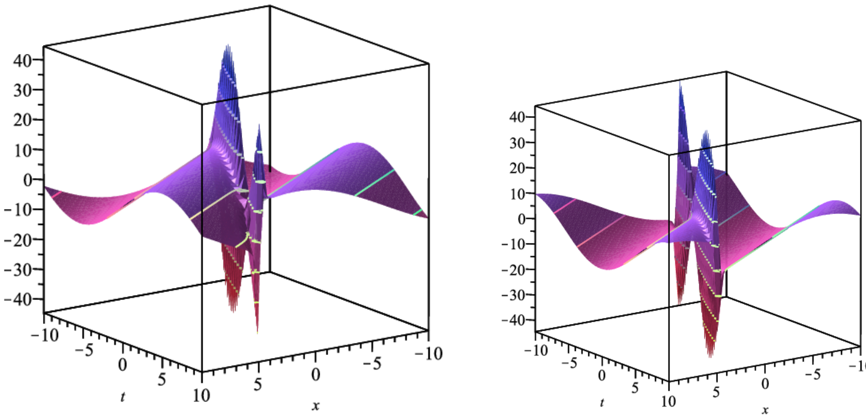


(left) Real contour shape. (Right) Complex contour shape.

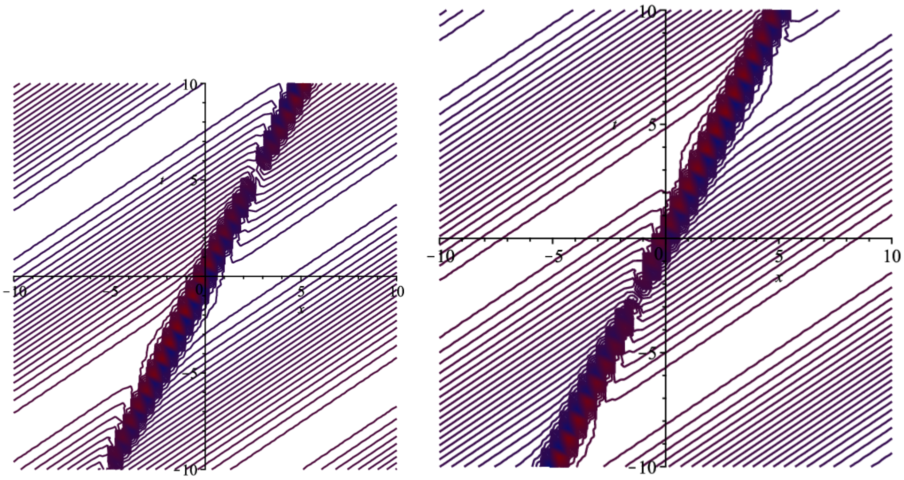


(left) Real filled region shape. (Right) Complex filled region shape.

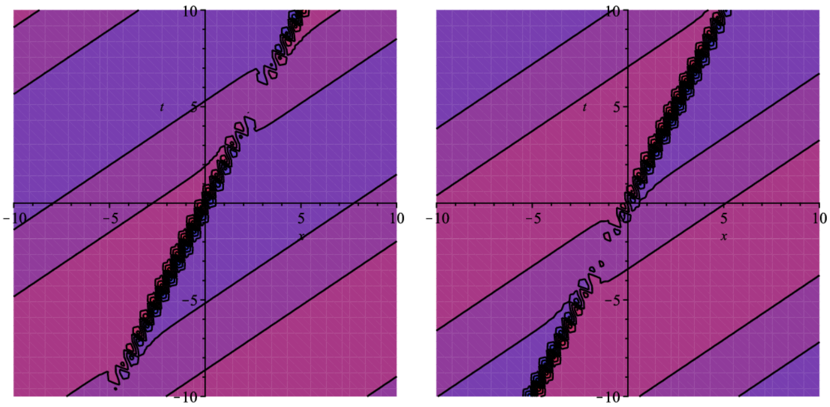
Figure 2. Graphical description of the answer in $W_1(x, t)$ under the values $a = 0.22$, $b = 0.3$, $r = 0.5$, $c = 0.5$, $R = -10$, $\mu = 1$, $\lambda = 3$ and $t = 0.01$ for $2D$ graphics.



(left) Real $3D$ surface. (Right) Complex $3D$ surface.



(left) Real contour shape. (Right) Complex contour shape.



(left) Real filled region shape. (Right) Complex filled region shape.

Figure 3. Graphical description of the answer in $W_6(x,t)$ under the values $a = 0.22$, $b = 0.3$, $r = 0.5$, $c = 0.5$, $R = -10$, $\mu = 1$, $\lambda = 3$ and $t = 0.01$ for $2D$ graphics.

5. Conclusions

The solution of every PDE is always utilized for understanding the system and various phenomena described by it. The modified G'/G -expansion method is helpful to obtain the solutions in the form of hyperbolic and trigonometric forms which are exact and helpful in understanding the fractional forms of it. Finally, a transformation is used to draw a soliton solution of Eq (3.1) by the use of Maple software. So, this gives the efficient applications of modified G'/G-expansion for fractional PDEs.

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Conflict of interest

The authors declare that there is no conflict of interest regarding the publication of this paper.

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Appendix

The solutions of equation 2.6 are:

- If $h > 0$, then

$$F = \sqrt{h} tanh(\sqrt{h}\xi), \tag{5.1a}$$

$$F = \sqrt{h} coth(\sqrt{h}\xi). \tag{5.1b}$$

- If $h = 0$, then

$$F = \frac{1}{\xi}. \tag{5.2}$$

- If $h < 0$, then

$$F = -\sqrt{-h}tan(\sqrt{-h}\xi), \tag{5.3a}$$

$$F = \sqrt{-h}cot(\sqrt{-h}\xi). \tag{5.3b}$$

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