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Research article

Novel stability criterion for linear system with two additive time-varying delays using general integral inequalities

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Abstract: The stability analysis strategy for continuous linear system with two additive time-varying delays is proposed in this paper. First, for the purpose of analysis, the novel Lyapunov-Krasovskii functional (LKF) consisting of integral terms based on the first-order derivative of the system state is constructed. Second, the derivative of LKF is estimated by utilizing the Wirtinger-based integral inequality and extended reciprocally convex matrix inequality. The delay-dependent stability criterions are established in terms of linear matrix inequalities (LMIs) framework. The results show that the system performances are improved based on both enlarging the maximum allowable upper bound of the time-delays and reducing the number of decision variables. Furthermore, the conservatism of obtained delay-dependent stability criterion is reduced. Finally, a numerical simulation is given to demonstrate the effectiveness of obtained theoretical results.

Keywords: linear system; time-varying delays; stability; Lyapunov-Krasovskii functional (LKF); linear matrix inequalities (LMIs)
Mathematics Subject Classification: 34D20, 34K20, 93D05

1. Introduction

Stability is always one of the most important factors in control system design. However, due to the network communication, the occurrences of all kinds of time-delays in control systems are inevitable, see [1]. It has been demonstrated that time-delays are the main issues that affect the stability of systems in [2]. Recently, the results of time-delays in a system may result in the improvement rather than deterioration of the system performance are obtained in [3]. Therefore, all kinds of time-delays, especially the single time-delay should be taken into account and sophisticated analysis in order to guarantee the stability of system.

As for the time-delays system, the Lyapunov stability theory is usually employed to analyze the stability. Different methods of constructing suitable Lyapunov-Krasovskii functional (LKF) have been reported in the existing literature, for example, the simple LKF approach without delay decomposition is established in [4], the delay-product-type LKF is constructed in [5, 6], and so on. Because of the above reasons, more and more literature has considered the construction of LKF. Due to some LKFs containing multiple integral terms were constructed, these multiple integral terms were treated by model transformation approaches [7], free-weight-matrix methods [8–10] and so on. Recently, there have been some researches on estimating the integral terms through integral matrix inequalities [11], as well as a great number of matrix inequalities are established, such as Jensen's inequality [12], Wirtinger-based inequality [13], Bessel-Legendre inequality [14], auxiliary function based inequalities [15–18], double integral inequality [19], extended reciprocally convex quadratic inequality [20], relaxed quadratic function negative determination lemma [21] and quadratic function [22], etc. Thus, many stability criteria based on linear matrix inequalities (LMIs) are obtained, most of which are based on various LKFs [23–27]. These difficulties are between the estimation of multiple integral terms from the derivatives of LKFs. The development of new methods for this problem has traditional been an important consideration. Furthermore, when using these matrix inequalities to analyze the stability of time-varying delay system, an additional technique is applied to handle the estimated terms, for example, free-weighting matrix [28], reciprocally convex approach [29], extended reciprocally convex matrix inequality [30]. As a consequence, the reciprocally convex matrix inequality and integral matrix inequality are the most commonly prevalent methods to estimate the derivatives of LKFs.

On the one hand, with the deep research for time-delay systems contain one single time-delay, the application is hard to meet the requirements in networked control and long-range communication. The research of stability is mostly carried out in single time-delay systems and neural networks. On the other hand, compared with the single time-delay system, the continuous system with additive time-varying delay essentially requires can obtain high-precision performance. What is more, because of network transmission conditions, the properties of these two time-delays may not be identical. It is not reasonable to combine them together, hence they should be treated separately. The study of system with additive time-varying delay was first carried out by [31]. Up to now, several important and interesting results concerning the stability of a system with additive time-varying delay have been published in the literatures [32–44] and references therein.

Inspired by the above existing results, this paper provides further investigate the stability analysis strategy for continuous linear system with two additive time-varying delays is proposed in this paper. The major contributions of the paper are summarized as:

(1) The novel LKF consisting of integral terms based on the first-order derivative of the system state is constructed to guarantee that the stability region can be enhanced.
(2) The derivative of LKF is estimated by utilizing the Wirtinger-based integral inequality and extended reciprocally convex matrix inequality. The delay-dependent stability criterions are established via LMIs framework.
(3) The performances are improved based on both the maximum allowable upper bound of the time-varying delays are enlarged and the number of decision variables is reduced. Furthermore, the conservatism of obtained criterion is reduced.

Notation: Throughout this note, the superscript A^T (respectively, A^{-1}) represents the transpose (respectively, the inverse) of matrix A. $\mathbb{R}^n$ (respectively, $\mathbb{R}^{n\times m}$) denotes the set of $n \times 1$ real column vectors (respectively, the set of all $n \times m$ real matrices). $P > 0$ (respectively, $P \geq 0$) means that matrix

P is a positive (respectively, semi-positive) definite matrix. I_n (respectively, $0_{n,m}$) stands for the identity matrix in $\mathbb{R}^{n\times n}$ (respectively, the zero matrix in $\mathbb{R}^{n\times m}$). $\mathbb{S}^n$ (respectively, $\mathbb{S}^n_+$) represents the set of symmetric matrices (respectively, positive definite matrices) in $\mathbb{R}^{n\times n}$. In addition, diag$\{\cdots\}$ stands for the (block) diagonal matrix and $*$ denotes symmetric terms in a block symmetric matrix. Shorthand notations, $\|\cdot\|$ refers to the Euclidean vector norm, col$\{x_1,\cdots,x_p\}$ is the column block matrix with entries $x_1,\cdots,x_p\in\mathbb{R}^{n\times m}$, i.e., $\mathrm{col}\{x_1,\cdots,x_p\}=[x_1^T,\cdots,x_p^T]^T$.

2. System description

The following lemmas will be applied in this paper.

Lemma 2.1 (Wirtinger-based integral inequality [13]). *For any positive definite matrix $R\in\mathbb{S}^n_+$, and a differentiable vector function x in $[a,b]\to\mathbb{R}^n$. Then the following single integral inequality holds*

$$(b-a)\int_a^b \dot{x}^T(s)R\dot{x}(s)\,\mathrm{d}s \geq \chi^T\mathrm{diag}\{R,3R\}\chi,$$

where $\chi=\mathrm{col}\{\chi_1,\chi_2\}$, with $\chi_1=x(b)-x(a),\chi_2=x(b)+x(a)-\frac{2}{b-a}\int_a^b x(s)\,\mathrm{d}s$.

Lemma 2.2 ([45]). *For any positive definite matrix $R\in\mathbb{S}^n_+$, two scalars a and b such that $a<b$, and a vector function x in $[a,b]\to\mathbb{R}^n$. Then the following double integral inequality holds*

$$\frac{(b-a)^2}{2}\int_a^b\int_\theta^b x^T(s)Rx(s)\,\mathrm{d}s\,\mathrm{d}\theta \geq \Big(\int_a^b\int_\theta^b x(s)\,\mathrm{d}s\,\mathrm{d}\theta\Big)^T R\Big(\int_a^b\int_\theta^b x(s)\,\mathrm{d}s\,\mathrm{d}\theta\Big)$$

Lemma 2.3 (Extended reciprocally convex matrix inequality [30]). *For a real scalar $\lambda\in(0,1)$, positive definite matrices $Z_1,Z_2\in\mathbb{S}^n_+$, and any two matrices $X_1,X_2\in\mathbb{R}^{n\times n}$. Then the following matrix inequality holds*

$$\begin{bmatrix}\frac{Z_1}{\lambda} & 0\\ 0 & \frac{Z_2}{1-\lambda}\end{bmatrix}\geq\begin{bmatrix}Z_1+(1-\lambda)Y_1 & (1-\lambda)X_1+\lambda X_2\\ * & Z_2+\lambda Y_2\end{bmatrix},$$

where $Y_1=Z_1-X_2Z_2^{-1}X_2^T$ and $Y_2=Z_2-X_1^TZ_1^{-1}X_1$.

3. Main results

In this section, the linear systems with two additive time-varying delays is presented at first. Then, the Wirtinger-based integral inequality and extended reciprocally convex matrix inequality are introduced to handle the derivative of the novel LKF. Finally, the delay-dependent stability criterions are established via LMIs and some discussions are carried out to show its advantages.

In this paper, we consider the following continuous linear system with two additive time-varying delays:

$$\begin{cases}\dot{x}(t)=Ax(t)+A_dx(t-d_1(t)-d_2(t)),\\ x(t)=\varphi(t),\forall t\in[-h,0],\end{cases}\tag{3.1}$$

where $x(t)\in\mathbb{R}^n$ is the system state vector, $\varphi(t)$ is the continuous initial vector function, $A\in\mathbb{R}^{n\times n}$ and $A_d\in\mathbb{R}^{n\times n}$ are constant matrices, $d_i(t)(i=1,2)$ are two additive time-varying delays differentiable functions.

Assumption 3.1. *For two positive scalars $h_i(i = 1, 2)$ and two scalars $\mu_i(i = 1, 2)$, the time-delay functions $d_i(t)(i = 1, 2)$ satisfying $0 \le d_i(t) \le h_i$, $\dot{d}_i(t) \le \mu_i < 1(i = 1, 2)$.*

Let us denote $d(t) = d_1(t) + d_2(t), h = h_1 + h_2, \mu = \mu_1 + \mu_2$.

Before introducing the main result, for simplicity, the vector notations are defined as follows:

$$\xi(t) = \operatorname{col}\Bigg\{ \underbrace{x(t)}_{\xi_1(t)},\ \underbrace{x(t-d_1(t))}_{\xi_2(t)},\ \underbrace{x(t-d(t))}_{\xi_3(t)},\ \underbrace{x(t-d_1(t)-h_2)}_{\xi_4(t)},\ \underbrace{x(t-h)}_{\xi_5(t)},$$
$$\underbrace{\int_{t-d(t)}^{t} \frac{x(s)}{d(t)}\,\mathrm{d}s}_{\xi_6(t)},\ \underbrace{\int_{t-h}^{t-d(t)} \frac{x(s)}{h-d(t)}\,\mathrm{d}s}_{\xi_7(t)}\Bigg\},$$

$$\eta(t) = \operatorname{col}\left\{x(t),\ \int_{t-h}^{t} x(s)\,\mathrm{d}s\right\} = \operatorname{col}\{\xi_1(t),\ d(t)\xi_6(t) + (h-d(t))\xi_7(t)\},$$
$$\dot{\eta}(t) = \operatorname{col}\{\dot{x}(t),\ x(t) - x(t-h)\} = \operatorname{col}\{A\xi_1(t) + A_d\xi_3(t),\ \xi_1(t) - \xi_5(t)\}.$$

Theorem 3.1. *Consider system (3.1) with two additive time-varying delays subject to Assumption 3.1. For given scalars $h_i(i = 1, 2)$ and $\mu_i(i = 1, 2)$, system (3.1) is asymptotically stable if there exist symmetric positive-definite matrices $P \in \mathbb{S}_+^{2n}$, $Q_i \in \mathbb{S}_+^n(i = 1, 2, 3, 4)$, $Z \in \mathbb{S}_+^n, U \in \mathbb{S}_+^n$, and two matrices $X_1, X_2 \in \mathbb{R}^{2n\times 2n}$ such that the following LMI conditions*

$$\Omega_1 = \begin{bmatrix} \Psi_{[1,0]} - \Psi_2 - \Psi_3 & E_1^T X_2 \\ * & -\widetilde{Z} \end{bmatrix} < 0, \tag{3.2}$$

$$\Omega_2 = \begin{bmatrix} \Psi_{[1,h]} - \Psi_2 - \Psi_4 & E_2^T X_1^T \\ * & -(\widetilde{Z} + \widetilde{U}) \end{bmatrix} < 0, \tag{3.3}$$

are satisfied, where

$$\begin{aligned}
\Psi_{[1,d(t)]} &= \Pi_2^T P \Pi_{[1,d(t)]} + \Pi_{[1,d(t)]}^T P \Pi_2 + e_1^T(Q_1 + Q_2 + Q_3 + Q_4)e_1 \\
&\quad - (1-\mu_1)(e_2^T Q_1 e_2 + e_4^T Q_3 e_4) - e_5^T Q_4 e_5 - (1-\mu)e_3^T Q_2 e_3 + \overline{A}^T\left(h^2 Z + \frac{h^2}{2}U\right)\overline{A}, \\
\Psi_2 &= 2E_3^T U E_3 + 2E_4^T U E_4, \\
\Psi_3 &= \begin{bmatrix} E_1 \\ E_2 \end{bmatrix}^T \begin{bmatrix} 2\widetilde{Z} + \widetilde{U} & X_1 \\ * & \widetilde{Z} \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \end{bmatrix}, \\
\Psi_4 &= \begin{bmatrix} E_1 \\ E_2 \end{bmatrix}^T \begin{bmatrix} \widetilde{Z} & X_2 \\ * & 2\widetilde{Z} \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \end{bmatrix}, \\
\Pi_{[1,d(t)]} &= \operatorname{col}\{e_1,\ d(t)e_6 + (h-d(t))e_7\}, \\
\Pi_2 &= \operatorname{col}\left\{\overline{A},\ e_1 - e_5\right\}, \\
\widetilde{U} &= \operatorname{diag}\{U,\ 3U\}, \quad \widetilde{Z} = \operatorname{diag}\{Z,\ 3Z\}, \\
E_1 &= \operatorname{col}\{e_1 - e_3,\ e_1 + e_3 - 2e_6\}, \\
E_2 &= \operatorname{col}\{e_3 - e_5,\ e_3 + e_5 - 2e_7\}, \\
E_3 &= e_1 - e_6, \quad E_4 = e_3 - e_7,
\end{aligned}$$

$$e_i = [\ \underbrace{0_{n,n} \quad \cdots \quad 0_{n,n}}_{i-1} \quad I_n \quad \underbrace{0_{n,n} \quad \cdots \quad 0_{n,n}}_{7-i}\ ]^T, (i = 1, 2, \cdots, 7),$$

$$\overline{A} = [\ A \quad 0_{n,n} \quad A_d \quad 0_{n,n} \quad 0_{n,n} \quad 0_{n,n} \quad 0_{n,n}\ ].$$

Proof. First, construct the following LKF candidate contains four parts:

$$V(t) = V_1(t) + V_2(t) + V_3(t) + V_4(t), \tag{3.4}$$

with

$$V_1(t) = \eta^T(t)P\eta(t), \tag{3.5}$$

$$\begin{aligned} V_2(t) = &\int_{t-d_1(t)}^{t} x^T(s)Q_1x(s)\,\mathrm{d}s + \int_{t-d(t)}^{t} x^T(s)Q_2x(s)\,\mathrm{d}s \\ &+ \int_{t-(d_1(t)+h_2)}^{t} x^T(s)Q_3x(s)\,\mathrm{d}s + \int_{t-h}^{t} x^T(s)Q_4x(s)\,\mathrm{d}s, \end{aligned} \tag{3.6}$$

$$V_3(t) = h\int_{-h}^{0}\int_{t+s}^{t} \dot{x}^T(r)Z\dot{x}(r)\,\mathrm{d}r\,\mathrm{d}s, \tag{3.7}$$

$$V_4(t) = \int_{-h}^{0}\int_{s}^{0}\int_{t+\theta}^{t} \dot{x}^T(u)U\dot{x}(u)\,\mathrm{d}u\,\mathrm{d}\theta\,\mathrm{d}s. \tag{3.8}$$

Next, it is not difficult to obtain that the time derivative of $V(t)$ along the trajectory of system (3.1) can be computed as follows:

$$\dot{V}(t) = \dot{V}_1(t) + \dot{V}_2(t) + \dot{V}_3(t) + \dot{V}_4(t), \tag{3.9}$$

where

$$\dot{V}_1(t) = \dot{\eta}^T(t)P\eta(t) + \eta^T(t)P\dot{\eta}(t) = \xi^T(t)\left(\Pi_2^T P\Pi_{[1,d(t)]} + \Pi_{[1,d(t)]}^T P\Pi_2\right)\xi(t), \tag{3.10}$$

$$\begin{aligned} \dot{V}_2(t) = &x^T(t)Q_1x(t) - (1 - \dot{d}_1(t))x^T(t - d_1(t))Q_1x(t - d_1(t)) \\ &+ x^T(t)Q_2x(t) - (1 - \dot{d}(t))x^T(t - d(t))Q_2x(t - d(t)) \\ &+ x^T(t)Q_3x(t) - (1 - \dot{d}_1(t))x^T(t - (d_1(t) + h_2))Q_3x(t - (d_1(t) + h_2)) \\ &+ x^T(t)Q_4x(t) - x^T(t - h)Q_4x(t - h), \end{aligned} \tag{3.11}$$

$$\begin{aligned} \dot{V}_3(t) = &h\int_{-h}^{0}(\dot{x}^T(t)Z\dot{x}(t) - \dot{x}^T(t + s)Z\dot{x}(t + s))\,\mathrm{d}s \\ = &h^2\dot{x}^T(t)Z\dot{x}(t) - h\int_{-h}^{0}\dot{x}^T(t + s)Z\dot{x}(t + s)\,\mathrm{d}s \\ = &h^2\dot{x}^T(t)Z\dot{x}(t) - h\int_{t-h}^{t}\dot{x}^T(r)Z\dot{x}(r)\,\mathrm{d}r \\ = &h^2\dot{x}^T(t)Z\dot{x}(t) - h\int_{t-h}^{t}\dot{x}^T(s)Z\dot{x}(s)\,\mathrm{d}s, \end{aligned} \tag{3.12}$$

$$\dot{V}_4(t) = \int_{-h}^{0}\int_{s}^{0}(\dot{x}^T(t)U\dot{x}(t) - \dot{x}^T(t + \theta)U\dot{x}(t + \theta))\,\mathrm{d}\theta\,\mathrm{d}s$$

$$
\begin{aligned}
&= \int_{-h}^{0}\int_{s}^{0} \dot{x}^T(t)U\dot{x}(t)\,\mathrm{d}\theta\,\mathrm{d}s - \int_{-h}^{0}\int_{s}^{0} \dot{x}^T(t+\theta)U\dot{x}(t+\theta)\,\mathrm{d}\theta\,\mathrm{d}s \\
&= \int_{-h}^{0}(-s)\dot{x}^T(t)U\dot{x}(t)\,\mathrm{d}s - \int_{-h}^{0}\int_{t+s}^{t} \dot{x}^T(r)U\dot{x}(r)\,\mathrm{d}r\,\mathrm{d}s \\
&=\frac{h^2}{2}\dot{x}^T(t)U\dot{x}(t) - \int_{-h}^{-d(t)}\int_{t+\theta}^{t} \dot{x}^T(s)U\dot{x}(s)\,\mathrm{d}s\,\mathrm{d}\theta - \int_{-d(t)}^{0}\int_{t+\theta}^{t} \dot{x}^T(s)U\dot{x}(s)\,\mathrm{d}s\,\mathrm{d}\theta \\
&=\frac{h^2}{2}\dot{x}^T(t)U\dot{x}(t) - \int_{-h}^{-d(t)}\int_{t+\theta}^{t-d(t)} \dot{x}^T(s)U\dot{x}(s)\,\mathrm{d}s\,\mathrm{d}\theta \\
&\quad - \int_{-h}^{-d(t)}\int_{t-d(t)}^{t} \dot{x}^T(s)U\dot{x}(s)\,\mathrm{d}s\,\mathrm{d}\theta - \int_{-d(t)}^{0}\int_{t+\theta}^{t} \dot{x}^T(s)U\dot{x}(s)\,\mathrm{d}s\,\mathrm{d}\theta.
\end{aligned} \tag{3.13}
$$

We apply Lemma 2.1 to estimate the lower bound of the single integral terms on the right side of equality (3.12), we obtain

$$
\begin{aligned}
& h\int_{t-h}^{t} \dot{x}^T(s)Z\dot{x}(s)\,\mathrm{d}s \\
=& h\left[\int_{t-h}^{t-d(t)} \dot{x}^T(s)Z\dot{x}(s)\,\mathrm{d}s + \int_{t-d(t)}^{t} \dot{x}^T(s)Z\dot{x}(s)\,\mathrm{d}s\right] \\
\geq& \frac{h}{h-d(t)}\begin{bmatrix} \xi_3(t)-\xi_5(t) \\ \xi_3(t)+\xi_5(t)-2\xi_7(t) \end{bmatrix}^T \begin{bmatrix} Z & 0 \\ 0 & 3Z \end{bmatrix}\begin{bmatrix} \xi_3(t)-\xi_5(t) \\ \xi_3(t)+\xi_5(t)-2\xi_7(t) \end{bmatrix} \\
&+ \frac{h}{d(t)}\begin{bmatrix} \xi_1(t)-\xi_3(t) \\ \xi_1(t)+\xi_3(t)-2\xi_6(t) \end{bmatrix}^T \begin{bmatrix} Z & 0 \\ 0 & 3Z \end{bmatrix}\begin{bmatrix} \xi_1(t)-\xi_3(t) \\ \xi_1(t)+\xi_3(t)-2\xi_6(t) \end{bmatrix} \\
=& \xi^T(t)\left(\frac{h}{h-d(t)}E_2^T\widetilde{Z}E_2 + \frac{h}{d(t)}E_1^T\widetilde{Z}E_1\right)\xi(t).
\end{aligned} \tag{3.14}
$$

By applying Lemma 2.2, the lower bound of the first double integral terms on the right side of equality (3.13) can be estimated as

$$
\begin{aligned}
& \int_{-h}^{-d(t)}\int_{t+\theta}^{t-d(t)} \dot{x}^T(s)U\dot{x}(s)\,\mathrm{d}s\,\mathrm{d}\theta \\
\geq& \frac{2}{(h-d(t))^2}\left[\int_{-h}^{-d(t)}\int_{t+\theta}^{t-d(t)} \dot{x}(s)\,\mathrm{d}s\,\mathrm{d}\theta\right]^T U\left[\int_{-h}^{-d(t)}\int_{t+\theta}^{t-d(t)} \dot{x}(s)\,\mathrm{d}s\,\mathrm{d}\theta\right] \\
=& \frac{2}{(h-d(t))^2}\left[\int_{-h}^{-d(t)} (x(t-d(t))-x(t+\theta))\,\mathrm{d}\theta\right]^T U\left[\int_{-h}^{-d(t)} (x(t-d(t))-x(t+\theta))\,\mathrm{d}\theta\right] \\
=& 2\left[x(t-d(t)) - \int_{t-h}^{t-d(t)}\frac{x(s)}{h-d(t)}\,\mathrm{d}s\right]^T U\left[x(t-d(t)) - \int_{t-h}^{t-d(t)}\frac{x(s)}{h-d(t)}\,\mathrm{d}s\right] \\
=& 2\left(\xi_3(t)-\xi_7(t)\right)^T U\left(\xi_3(t)-\xi_7(t)\right) \\
=& \xi^T(t)\left(2E_4^TUE_4\right)\xi(t).
\end{aligned} \tag{3.15}
$$

In a completely analogous argument, the second double integral terms in (3.13) being estimated through Lemma 2.1 can be expressed as

$$\begin{aligned}
&\int_{-h}^{-d(t)}\int_{t-d(t)}^{t}\dot{x}^T(s)U\dot{x}(s)\,\mathrm{d}s\,\mathrm{d}\theta\\
=&(h-d(t))\int_{t-d(t)}^{t}\dot{x}^T(s)U\dot{x}(s)\,\mathrm{d}s\\
\geq&\frac{h-d(t)}{d(t)}\begin{bmatrix}\xi_1(t)-\xi_3(t)\\ \xi_1(t)+\xi_3(t)-2\xi_6(t)\end{bmatrix}^T\begin{bmatrix}U&0\\0&3U\end{bmatrix}\begin{bmatrix}\xi_1(t)-\xi_3(t)\\ \xi_1(t)+\xi_3(t)-2\xi_6(t)\end{bmatrix}\\
=&\xi^T(t)\left(\left(\frac{h}{d(t)}-1\right)E_1^T\widetilde{U}E_1\right)\xi(t).
\end{aligned}\tag{3.16}$$

Similarly, the last double integral term in (3.13) being estimated through Lemma 2.2 leads to

$$\begin{aligned}
&\int_{-d(t)}^{0}\int_{t+\theta}^{t}\dot{x}^T(s)U\dot{x}(s)\,\mathrm{d}s\,\mathrm{d}\theta\\
\geq&\frac{2}{d^2(t)}\left[\int_{-d(t)}^{0}\int_{t+\theta}^{t}\dot{x}(s)\,\mathrm{d}s\,\mathrm{d}\theta\right]^T U\left[\int_{-d(t)}^{0}\int_{t+\theta}^{t}\dot{x}(s)\,\mathrm{d}s\,\mathrm{d}\theta\right]\\
=&\frac{2}{d^2(t)}\left[\int_{-d(t)}^{0}(x(t)-x(t+\theta))\,\mathrm{d}\theta\right]^T U\left[\int_{-d(t)}^{0}(x(t)-x(t+\theta))\,\mathrm{d}\theta\right]\\
=&\frac{2}{d^2(t)}\left[d(t)x(t)-\int_{-d(t)}^{0}x(t+\theta)\,\mathrm{d}\theta\right]^T U\left[d(t)x(t)-\int_{-d(t)}^{0}x(t+\theta)\,\mathrm{d}\theta\right]\\
=&2\left[x(t)-\int_{t-d(t)}^{t}\frac{x(s)}{d(t)}\,\mathrm{d}s\right]^T U\left[x(t)-\int_{t-d(t)}^{t}\frac{x(s)}{d(t)}\,\mathrm{d}s\right]\\
=&2\left(\xi_1(t)-\xi_6(t)\right)^T U\left(\xi_1(t)-\xi_6(t)\right)\\
=&\xi^T(t)\left(2E_3^TUE_3\right)\xi(t).
\end{aligned}\tag{3.17}$$

Furthermore, on the ground of the equations (3.10)-(3.13) and inequalities (3.14)-(3.17), the upper bound of $\dot{V}(t)$ can be estimated as the following form

$$\begin{aligned}
\dot{V}(t)\leq&\xi^T(t)\left[\Psi_{[1,d(t)]}-\Psi_2+E_1^T\widetilde{U}E_1\right]\xi(t)\\
&-\xi^T(t)\left[\frac{h}{d(t)}E_1^T(\widetilde{Z}+\widetilde{U})E_1+\frac{h}{h-d(t)}E_2^T\widetilde{Z}E_2\right]\xi(t).
\end{aligned}\tag{3.18}$$

Then, by applying Lemma 2.3, for matrices $X_1, X_2\in\mathbb{R}^{2n\times 2n}$, the following inequality can be stated

$$\xi^T(t)\left[\frac{h}{d(t)}E_1^T(\widetilde{Z}+\widetilde{U})E_1+\frac{h}{h-d(t)}E_2^T\widetilde{Z}E_2-E_1^T\widetilde{U}E_1\right]\xi(t)\geq\xi^T(t)\Psi_{[5,d(t)]}\xi(t),\tag{3.19}$$

where

$$\begin{aligned}
\Psi_{[5,d(t)]}=&E_1^T(\widetilde{Z}+\widetilde{U})E_1+E_2^T\widetilde{Z}E_2-E_1^T\widetilde{U}E_1\\
&+\frac{h-d(t)}{h}\begin{bmatrix}E_1\\E_2\end{bmatrix}^T\begin{bmatrix}\widetilde{Z}+\widetilde{U}-X_2\widetilde{Z}^{-1}X_2 & X_1\\ * & 0\end{bmatrix}\begin{bmatrix}E_1\\E_2\end{bmatrix}
\end{aligned}$$

$$+\frac{d(t)}{h}\begin{bmatrix} E_1 \\ E_2 \end{bmatrix}^T \begin{bmatrix} 0 & X_2 \\ * & \widetilde{Z} - X_1^T(\widetilde{Z}+\widetilde{U})^{-1}X_1 \end{bmatrix}\begin{bmatrix} E_1 \\ E_2 \end{bmatrix}.$$

Hence, substituting (3.19) into (3.18), the upper bound of $\dot{V}(t)$ is estimated as

$$\dot{V}(t) \leq \xi^T(t)\left[\Psi_{[1,d(t)]} - \Psi_2 - \Psi_{[5,d(t)]}\right]\xi(t). \tag{3.20}$$

Finally, by applying convex combination technique, $\Psi_{[1,d(t)]} - \Psi_2 - \Psi_{[5,d(t)]} \leq 0$ holds if the following two inequalities hold

$$\Psi_{[1,0]} - \Psi_2 - \Psi_{[5,0]} < 0, \tag{3.21}$$

$$\Psi_{[1,h]} - \Psi_2 - \Psi_{[5,h]} < 0. \tag{3.22}$$

Therefore, we conclude that inequality (3.21) and (3.22) are equivalent to (3.2) and (3.3) in term of Schur complement, respectively. Thus, if (3.2) and (3.3) hold, then, for a sufficient small scalar $\varepsilon > 0$, $\dot{V}(t) \leq -\varepsilon\|x(t)\|^2$ holds, which ensures that system (3.1) with two additive time-varying delays is asymptotically stable. □

Remark 3.1. *Let $d_2(t) = 0$, the system (3.1) with two additive time-varying delays becomes the system that we see in the previous article [30], etc. The result of the system (3.1) is further improved the boundness of h_1 when $d_2(t) = 0$. This paper provides the LKF method, which obtains less conservatism of stability criterion at the cost of less computational burden and decision variables. It is worth noting that the different LKF (3.4) are introduced in Theorem 3.1 to reduce the conservatism for system (3.1). Compared with the inequalities developed in article [36], this paper utilized the extended reciprocally convex matrix inequality to deal with the time-varying delays, which reduce the estimated gap for derivatives of LKFs.*

By analogous methods, we have the following result.

For convenience, some vector notations are denoted as follows:

$$\widehat{\xi}(t) = \mathrm{col}\left\{\underbrace{x(t)}_{\widehat{\xi}_1(t)},\ \underbrace{x(t-d(t))}_{\widehat{\xi}_2(t)},\ \underbrace{x(t-h)}_{\widehat{\xi}_3(t)},\ \underbrace{\int_{t-d(t)}^{t}\frac{x(s)}{d(t)}\,\mathrm{d}s}_{\widehat{\xi}_4(t)},\ \underbrace{\int_{t-h}^{t-d(t)}\frac{x(s)}{h-d(t)}\,\mathrm{d}s}_{\widehat{\xi}_5(t)}\right\},$$

$$\widehat{\eta}(t) = \mathrm{col}\left\{x(t),\ \int_{t-h}^{t}x(s)\,\mathrm{d}s\right\} = \mathrm{col}\left\{\widehat{\xi}_1(t),\ d(t)\widehat{\xi}_4(t) + (h-d(t))\widehat{\xi}_5(t)\right\},$$

$$\dot{\widehat{\eta}}(t) = \mathrm{col}\left\{\dot{x}(t),\ x(t) - x(t-h)\right\} = \mathrm{col}\left\{A\widehat{\xi}_1(t) + A_d\widehat{\xi}_2(t),\ \widehat{\xi}_1(t) - \widehat{\xi}_3(t)\right\}.$$

Corollary 3.1. *Consider system (3.1) with two additive time-varying delays subject to Assumption 3.1. For given scalars $h_i (i = 1, 2)$ and $\mu_i (i = 1, 2)$, system (3.1) is asymptotically stable if there exist symmetric positive-definite matrices $P \in \mathbb{S}_+^{2n}$, $Q \in \mathbb{S}_+^n$, $Z \in \mathbb{S}_+^n$, $U \in \mathbb{S}_+^n$, and two matrices $X_1, X_2 \in \mathbb{R}^{2n\times 2n}$, such that the following LMI conditions*

$$\widehat{\Omega}_1 = \begin{bmatrix} \widehat{\Psi}_{[1,0]} - \widehat{\Psi}_2 - \widehat{\Psi}_3 & \widehat{E}_1^T X_2 \\ * & -\widetilde{Z} \end{bmatrix} < 0,$$

$$\widehat{\Omega}_2 = \begin{bmatrix} \widehat{\Psi}_{[1,h]} - \widehat{\Psi}_2 - \widehat{\Psi}_4 & \widehat{E}_2^T X_1^T \\ * & -(\widetilde{Z} + \widetilde{U}) \end{bmatrix} < 0,$$

are satisfied, where

$$\begin{aligned}
\widehat{\Psi}_{[1,d(t)]} &= \widehat{\Pi}_2^T P \widehat{\Pi}_{[1,d(t)]} + \widehat{\Pi}_{[1,d(t)]}^T P \widehat{\Pi}_2 + \widehat{e}_1^T Q \widehat{e}_1 - \widehat{e}_5^T Q \widehat{e}_5 + \widehat{A}^T \left(h^2 Z + \frac{h^2}{2} U \right) \widehat{A}, \\
\widehat{\Psi}_2 &= 2\widehat{E}_3^T U \widehat{E}_3 + 2\widehat{E}_4^T U \widehat{E}_4, \\
\widehat{\Psi}_3 &= \begin{bmatrix} \widehat{E}_1 \\ \widehat{E}_2 \end{bmatrix}^T \begin{bmatrix} 2\widetilde{Z} + \widetilde{U} & X_1 \\ * & \widetilde{Z} \end{bmatrix} \begin{bmatrix} \widehat{E}_1 \\ \widehat{E}_2 \end{bmatrix}, \\
\widehat{\Psi}_4 &= \begin{bmatrix} \widehat{E}_1 \\ \widehat{E}_2 \end{bmatrix}^T \begin{bmatrix} \widetilde{Z} & X_2 \\ * & 2\widetilde{Z} \end{bmatrix} \begin{bmatrix} \widehat{E}_1 \\ \widehat{E}_2 \end{bmatrix}, \\
\widehat{\Pi}_{[1,d(t)]} &= \mathrm{col}\left\{ \widehat{e}_1,\ d(t)\widehat{e}_4 + (h - d(t))\widehat{e}_5 \right\}, \\
\widehat{\Pi}_2 &= \mathrm{col}\left\{ \widehat{A},\ \widehat{e}_1 - \widehat{e}_3 \right\}, \\
\widehat{E}_1 &= \mathrm{col}\left\{ \widehat{e}_1 - \widehat{e}_2,\ \widehat{e}_1 + \widehat{e}_2 - 2\widehat{e}_4 \right\}, \\
\widehat{E}_2 &= \mathrm{col}\left\{ \widehat{e}_2 - \widehat{e}_3,\ \widehat{e}_2 + \widehat{e}_3 - 2\widehat{e}_5 \right\}, \\
\widehat{E}_3 &= \widehat{e}_1 - \widehat{e}_4, \quad \widehat{E}_4 = \widehat{e}_2 - \widehat{e}_5, \\
\widehat{e}_i &= [\ \underbrace{0_{n,n} \quad \cdots \quad 0_{n,n}}_{i-1} \quad I_n \quad \underbrace{0_{n,n} \quad \cdots \quad 0_{n,n}}_{5-i}\ ]^T, (i = 1, 2, \cdots, 5), \\
\widehat{A} &= [\ A \quad A_d \quad 0_{n,n} \quad 0_{n,n} \quad 0_{n,n}\ ],
\end{aligned}$$

with the definitions of $\widetilde{U}$ and $\widetilde{Z}$ are the same as previous.

Proof. Similarly, we also construct the following LKF candidate contains four parts:

$$\widetilde{V}(t) = \widetilde{V}_1(t) + \widetilde{V}_2(t) + \widetilde{V}_3(t) + \widetilde{V}_4(t),$$

with

$$\begin{aligned}
\widetilde{V}_1(t) &= \widehat{\eta}^T(t) P \widehat{\eta}(t), \\
\widetilde{V}_2(t) &= \int_{t-h}^{t} x^T(s) Q x(s)\, \mathrm{d}s, \\
\widetilde{V}_3(t) &= h \int_{-h}^{0} \int_{t+s}^{t} \dot{x}^T(r) Z \dot{x}(r)\, \mathrm{d}r\, \mathrm{d}s, \\
\widetilde{V}_4(t) &= \int_{-h}^{0} \int_{s}^{0} \int_{t+\theta}^{t} \dot{x}^T(u) U \dot{x}(u)\, \mathrm{d}u\, \mathrm{d}\theta\, \mathrm{d}s.
\end{aligned}$$

Since the process of proof is the same as Theorem 3.1, so the paper skips the proof here. The derivative of LKF can be shown negative definite while the conditions of Corollary 3.1 are satisfied. □

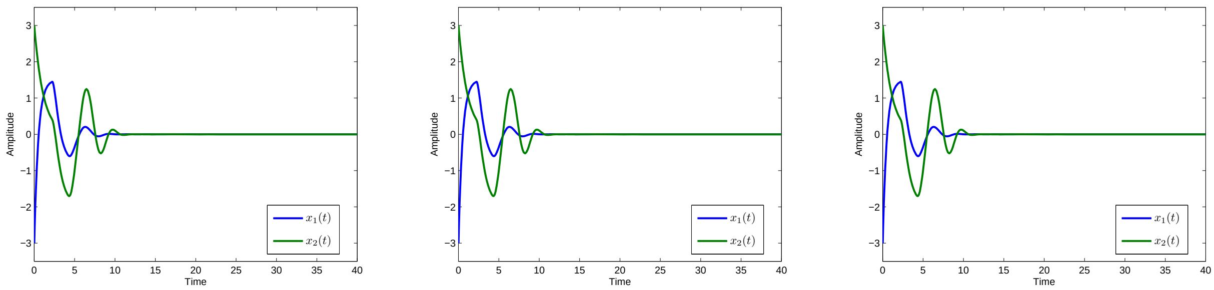
Remark 3.2. *Applying LMIs along with the extended reciprocally convex matrix inequality, a tighter bound constraint on the negative definite derivative with respect to time for the LKFs is established. It* *is a remarkable fact that Theorem 3.1 is used to find the maximum upper bound of time-varying delays, however, the Corollary 3.1 deals with the same system when omitting the first three terms associated with the LKF (3.6). In the same way, the Corollary 3.1 has lower computational burden and less conservatism than those previous article [35].*

4. Evaluation and comparison results

A numerical example is used to demonstrate the effectiveness of proposed matrix inequality. The result shows that the given conditions in this paper are better than other existing methods.

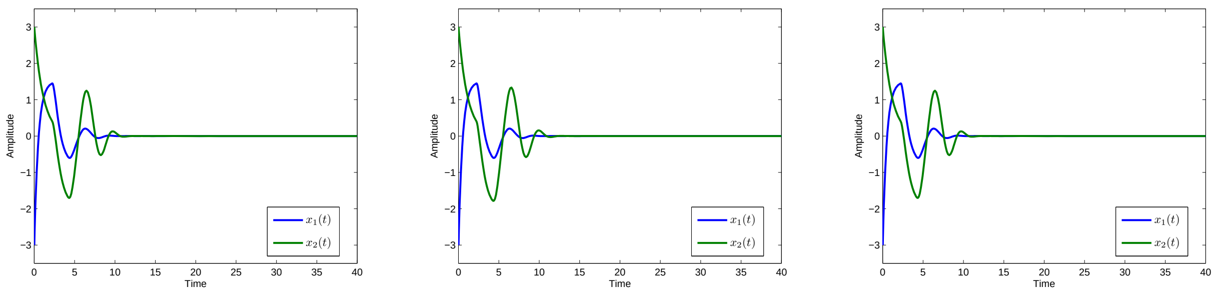
Example 4.1. *Consider system (3.1) with the following parameters:*

$$A = \begin{bmatrix} -2 & 0 \\ 0 & -0.9 \end{bmatrix}, \quad A_d = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix}.$$



(a) $h_1 = 1$ and $h_2 = 1.781$. (b) $h_1 = 1.2$ and $h_2 = 1.581$. (c) $h_1 = 1.5$ and $h_2 = 1.281$.

Figure 1. The dynamical behavior of systems (3.1).



(a) $h_2 = 0.3$ and $h_1 = 2.481$. (b) $h_2 = 0.4$ and $h_1 = 2.381$. (c) $h_2 = 0.5$ and $h_1 = 2.281$.

Figure 2. The dynamical behavior of systems (3.1).

For given h_1 or h_2, the Table 1 shows the allowable upper bounds of $d_2(t)$ and $d_1(t)$ that guarantee the asymptotic stability of system (3.1), respectively, and reported on the typical literatures is listed. For $h_1 = 1.0, h_2 = 1.781$, the time-varying functions $d_1(t) = 0.8 + 0.2\sin(5\mu_1 t), d_2(t) = 1.581 + 0.2\sin(5\mu_2 t)$. For $h_1 = 1.2, h_2 = 1.581$, the time-varying functions $d_1(t) = 1.0 + 0.2\sin(5\mu_1 t), d_2(t) = 1.381 + 0.2\sin(5\mu_2 t)$. For $h_1 = 1.5, h_2 = 1.281$, the time-varying functions $d_1(t) = 1.3 + 0.2\sin(5\mu_1 t), d_2(t) = 1.081 + 0.2\sin(5\mu_2 t)$. For $h_2 = 0.3, h_1 = 2.481$, the time-varying functions $d_1(t) = 2.281 + 0.2\sin(5\mu_1 t), d_2(t) = 0.1 + 0.2\sin(5\mu_2 t)$. For $h_2 = 0.4, h_1 = 2.381$, the time-varying functions $d_1(t) = 2.181 + 0.2\sin(5\mu_1 t), d_2(t) = 0.2 + 0.2\sin(5\mu_2 t)$. For $h_2 = 0.5, h_1 = 2.281$, the time-varying functions $d_1(t) = 2.081 + 0.2\sin(5\mu_1 t), d_2(t) = 0.3 + 0.2\sin(5\mu_2 t)$. The initial value of system state is $[-3, 3]^T$, the state trajectories of system (3.1) are given in Figures 1 and 2, respectively. We can see that the proposed stability criterion in Theorem 3.1 has the less conservatism than the results of literatures [32–38, 41–44], with the reasonable in the relatively small number of decision variables and the larger upper bounds of time-delays.

Table 1. Calculated bounds of time-delays for different cases $\mu_1 = 0.1$ and $\mu_2 = 0.8$.

Method	Bound h_2 for given h_1			Bound h_1 for given h_2			Number of decision variables
	h_1=1	h_1=1.2	h_1=1.5	h_2=0.3	h_2=0.4	h_2=0.5	
[32]	0.873	0.673	0.373	1.573	1.473	1.373	$40n^2 + 5n$
[33]	0.988	0.845	0.675	2.332	2.069	1.842	$5n^2 + 3n$
[34]	0.982	0.782	0.482	1.682	1.582	1.482	$63n^2 + 8n$
[35]	0.983	0.849	0.671	2.329	2.065	1.863	$8.5n^2 + 5.5n$
Corollary 1 of [36]	1.190	1.001	0.749	2.434	2.140	1.904	$28.5n^2 + 7.5n$
Theorem 1 of [37]	1.233	1.035	0.752	2.441	2.145	1.912	$68.5n^2 + 3.5n$
Theorem 1 of [38]	0.872	0.672	0.371	1.572	1.472	1.372	$32n^2 + 10n$
Corollary 1 of [41]	1.075	0.824	0.416	1.827	1.727	1.626	$24n^2 + 6n$
Theorem 1 of [41]	1.163	0.965	0.669	1.875	1.773	1.671	$32n^2 + 10n$
Theorem 4 of [42]	1.243	1.043	0.759	2.444	2.147	1.916	$226.5n^2 + 6.5n$
Corollary 5 of [42]	1.243	1.043	0.752	2.443	2.145	1.912	$114n^2 + 4n$
Theorem 1 of [43]	0.9999	0.9725	0.6807	1.8804	1.7798	1.6759	$202n^2 + 25n$
Theorem 1 of [44]	1.2136	1.0137	0.7137	1.9137	1.8137	1.7136	$29n^2 + 7n$
Theorem 3.1	1.781	1.581	1.281	2.481	2.381	2.281	$13n^2 + 4n$

5. Conclusions

This paper has investigated the stability problem of a continuous linear system with two additive time-varying delays. The appropriate LKF consisting of integral terms based on the first-order derivative of the system state is proposed. The new stability conditions are obtained in the forms of LMIs by utilizing the Wirtinger-based integral inequality and extended reciprocally convex matrix inequality. The numerical example has been given to show the advantage of conservative and computational boundaries over the comparative results in recent articles.

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Conflict of interest

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