Mathematics

http://www.aimspress.com/journal/Math

AIMS Mathematics, 6(8): 8466–8476.
DOI: 10.3934/math.2021491
Received: 27 April 2021
Accepted: 19 May 2021
Published: 02 June 2021

Research article

Numerous graph energies of regular subdivision graph and complete graph

Imrana Kousar[1], Saima Nazeer[1], Abid Mahboob[2], Sana Shahid[1] and Yu-Pei Lv[3,*]

[1] Department of Mathematics, Lahore College for Women University, Lahore-Pakistan
[2] Department of Mathematics, Division of Science and Technology, University of Education, Lahore-Pakistan
[3] Department of Mathematics, Huzhou University, Huzhou 313000, China

* **Correspondence:** Email: peipei@zjhu.edu.cn.

Abstract: The graph energy $E(G)$ of a simple graph G is sum of its absolute eigenvalues where eigenvalues of adjacency matrix $A(G)$ are referred as eigenvalues of graph G. Depends upon eigenvalues of different graph matrices, several graph energies has been observed recently such as maximum degree energy, Randić energy, sum-connectivity energy etc. Depending on the definition of a graph matrix, the graph energy can be easily determined. This article contains upper bounds of several graph energies of s-regular subdivision graph $S(G)$. Also various graph energies of complete graph are mentioned in this article.

Keywords: graph; graph energy; Zagreb index; Randić index; graph matrices
Mathematics Subject Classification: 14H50, 14H20, 32S15

1. Introduction

Consider a simple connected graph $G = (V(G), E(G))$ having $|V(G)| = p$ vertices and $E(G) = q$ edges. Number of edges in the neighborhood of a vertex x in a graph G is named as degree of that vertex and is denoted by d_x or $d(x)$. If the number of edges in the neighborhood of each vertex in a graph are same say s then graph is said to be a s-regular graph.

Adjacency matrix is a $p \times p$ matrix having entries a_{xy} such that

$$a_{xy} = \begin{cases} 1, & \text{if } u_x u_y \in E(G) \\ 0, & \text{otherwise} \end{cases}$$

The eigenvalues of a graph are actually eigenvalues of its $A(G)$. The set which is constructed from eigenvalues of G with their multiplicities is known as spectrum of G.

In Mathematics, the graph energy was firstly introduced by Ivan Gutman in 1978. Graph energy is built upon eigenvalues of $A(G)$. It is sum of absolute values of elements of spectrum of G. For a p-vertex graph G with eigenvalues β_k in non-increasing order for $k = 1, 2, 3, ..., p$,

$$E(G) = \sum_{k=1}^{p} |\beta_k|. \tag{1.1}$$

The impression which is stated in Eq (1.1) is associated with computational chemistry. If, in a conjugated hydrocarbon system, the eigenvalues of a molecular graph are $\alpha_1, \alpha_2, \alpha_3, ..., \alpha_p$ and are in non-increasing order. Then Hückel molecular orbital approximation calculated the total Π-electron energy E_Π as

$$E_\Pi = p\gamma + \delta \left[2 \sum_{k=1}^{\frac{p}{2}} \alpha_k \right]$$

for p is even and

$$E_\Pi = p\gamma + \delta \left[\alpha_{\frac{k+1}{2}} + 2 \sum_{k=1}^{\frac{p-1}{2}} \alpha_k \right]$$

for p is odd with γ and δ are constants.

A large number of research papers have been published on graph energy. The thesis of Siraj [1] contains some elementary determinations of graph energy.

This paper include upper bounds of different graph energies of subdivision graph $S(G)$ of s-regular graph G containing p vertices and q edges. Also various graph energies of complete graph are explored in this paper.

Based on eigenvalues of different graph matrices, several energies of a graph have been such as maximum degree energy, seidel energy, sum-connectivity energy etc. These energies depends upon eigenvalues of their corresponding energy matrices, see [2–4].

2. Energies of s-regular subdivision graph

First we define subdivision graph.

Definition 2.1 (Subdivision graph)**.** The subdivision graph $S(G)$ of a graph G is acquire by dividing each edge of G into two edges with the help of a vertex of degree 2 on every edge. Thus $|V(S(G))| = |V(G)| + |E(G)|$ and $|E(S(G))| = 2|E(G)|$. The graph of subdivision of cycle C_4 is shown in Figure 1.

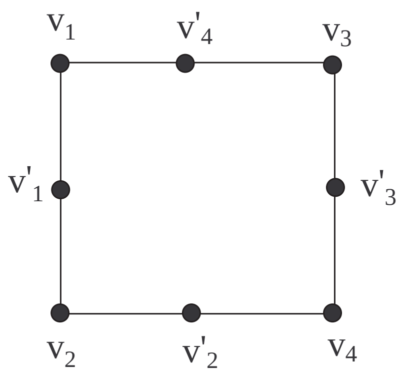

Figure 1. Subdivision of cycle C_4.

2.1. Degree energies

In this section, we present bounds of maximum degree energy, minimum degree energy, Randić energy, sum-connectivity energy and first and second Zagreb energies. Firstly, we define these energies.

Definition 2.2 (Maximum degree energy)**.** [5] The maximum degree energy E_M of a simple graph G is define as the sum of the absolute eigenvalues of its maximum degree matrix $M(G)$ where $M(G)$ has (i, j)th entry $max(d_j, d_j)$ if $v_i v_j \in E(G)$ and 0 elsewhere.

Definition 2.3 (Minimum degree energy)**.** [6] The minimum degree energy E_m of a simple connected graph G is define as the sum of the absolute eigenvalues of minimum degree matrix $m(G)$ of a graph G where (i, j)th entry of $m(G)$ is $min(d_i, d_j)$ if $v_i v_j \in E(G)$ and 0 otherwise.

Definition 2.4 (Randić energy)**.** [7] The randić energy E_R of a simple connected graph G is the sum of the absolute eigenvalues of the randić matrix $R(G)$ where if $v_i v_j \in E(G)$ then (i, j)th entry of $R(G)$ is $\frac{1}{\sqrt{d_i d_j}}$ and 0 elsewhere.

Definition 2.5 (Sum-connectivity energy)**.** [8] The sum-connectivity energy E_{SC} of a simple connected graph G is define as the sum of the absolute eigenvalues of the sum-connectivity matrix $SC(G)$ where (i, j)th entry of $SC(G)$ is $\frac{1}{\sqrt{d_i + d_j}}$ if $v_i v_j \in E(G)$ and 0 otherwise.

Definition 2.6 (First Zagreb energy)**.** [9] The first Zagreb energy ZE_1 of a simple connected graph G is define as the sum of the absolute eigenvalues of first Zagreb matrix $Z^{(1)}(G)$ of G where $Z^{(1)}(G)$ has (i, j)th entry $d_i + d_j$ if $v_i v_j \in E(G)$ and 0 otherwise.

Definition 2.7 (Second Zagreb energy)**.** [9] The second Zagreb energy ZE_2 of a simple connected graph G is define as the sum of the absolute eigenvalues of second Zagreb matrix $Z^{(2)}(G)$ of G where $Z^{(2)}(G)$ has (i, j)th entry $d_i.d_j$ if $v_i v_j \in E(G)$ and 0 otherwise.

In the following theorem, we give bounds of all above defined degree energies;

Theorem 2.8. *Let p and q be vertices and edges of a regular graph G. Then*

1. *for maximum degree energy, we have* $E_M(S(G)) \leq 2s\sqrt{2pq}$;
2. *for minimum degree energy, we have* $E_m(S(G)) \leq 4\sqrt{2pq}$,
3. *for Randić energy, we have* $E_R(S(G)) \leq \sqrt{\frac{4pq}{s}}$,
4. *for sum-connectivity energy, we have* $E_{SC}(S(G)) \leq 2\sqrt{\frac{2pq}{2+s}}$,
5. *for first Zagreb energy, we have* $ZE_1(S(G)) \leq 2(s+2)\sqrt{2pq}$,
6. *for second Zagreb energy, we have* $ZE_2(S(G)) \leq 4s\sqrt{2pq}$.

Proof. Let the incidence matrix of G is $C(G)$. Note that the degree matrix of the subdivision graph $S(G)$ can be stated as:

$$M(S(G)) = \begin{bmatrix} 0I_p & tC(G) \\ tC^T(G) & 0I_q \end{bmatrix}. \tag{2.1}$$

1. By taking $t = s$ in Eq (2.1), we have following computations for the maximum degree energy of the subdivision graph $S(G)$;

$$E_M(S(G)) = \sum_{j=1}^{p+q} \left| \alpha_i \begin{bmatrix} 0I_p & sC(G) \\ sC^T(G) & 0I_q \end{bmatrix} \right|$$
$$= s\Big(\sum_{j=1}^{p+q} \alpha_j \begin{bmatrix} 0I_p & C(G) \\ C^T(G) & 0I_q \end{bmatrix} \Big).$$

As in [12] $CC^T = L^+(G)$, we have

$$\sum_{j=1}^{p+q} \nu_j \begin{bmatrix} 0I_p & C(G) \\ C^T(G) & 0I_q \end{bmatrix} = 2\sum_{j=1}^{p} \sqrt{\nu_j^+(G)}.$$

where $L^+(G)$ is signless Laplacian matrix and ν_j^+ are eigenvalues of $L^+(G)$. Thus by Cauchy Schawaz inequality

$$\sum_{j=1}^{p} \sqrt{\nu_j^+(G)} \leq \sqrt{p\sum_{j=1}^{p} \nu_j^+(G)} = \sqrt{2pq}.$$

Hence,

$$E_M(S(G)) \leq 2s\sqrt{2pq}.$$

2. By taking $t = 2$ in Eq (2.1), we have following computations for the minimum degree energy of the subdivision graph $S(G)$;

$$E_m(S(G)) = \sum_{j=1}^{p+q} \left| \nu_j \begin{bmatrix} 0I_p & 2C(G) \\ 2C^T(G) & 0I_q \end{bmatrix} \right|.$$

Since,

$$\sum_{j=1}^{p+q} \left| \nu_j \begin{bmatrix} 0I_p & 2C(G) \\ 2C^T(G) & 0I_q \end{bmatrix} \right| \leq 4\sqrt{2pq}.$$

Therefore,

$$E_m(S(G)) \leq 4\sqrt{2pq}.$$

3. By taking $t = \frac{1}{\sqrt{2s}}$ in Eq (2.1), we have following computations for the Randić energy of the subdivision graph $S(G)$;

$$E_R(S(G)) = \sum_{j=1}^{p+q} \left| \rho_j \begin{pmatrix} 0I_p & \frac{1}{\sqrt{2s}}[C(G)] \\ \frac{1}{\sqrt{2s}}[C^T(G)] & 0I_q \end{pmatrix} \right|$$
$$= \frac{1}{\sqrt{2s}} \sum_{j=1}^{p+q} \left| \rho_j \begin{pmatrix} 0I_p & [C(G)] \\ [C^T(G)] & 0I_q \end{pmatrix} \right|.$$

As

$$\sum_{j=1}^{p+q} \left| \rho_j \begin{bmatrix} 0I_p & C(G) \\ [C^T(G)] & 0I_q \end{bmatrix} \right| \leq 2\sqrt{2pq}.$$

Therefore,

$$E_R(S(G)) \leq \frac{1}{\sqrt{2s}}.2\sqrt{2pq} = \sqrt{\frac{4pq}{s}}.$$

4. By taking $t = \frac{1}{\sqrt{2+s}}$ in Eq (2.1), we have following computations for the sum-connectivity energy of the subdivision graph $S(G)$;

$$E_{SC}(S(G)) = \frac{1}{\sqrt{2+s}} \sum_{j=1}^{p+q} \left| \eta_j \begin{bmatrix} 0I_p & C(G) \\ C^T(G) & 0I_q \end{bmatrix} \right|.$$

As

$$\sum_{j=1}^{p+q} \left| \eta_j \begin{bmatrix} 0I_p & C(G) \\ C^T(G) & 0I_q \end{bmatrix} \right| \leq 2\sqrt{2pq}.$$

Hence,

$$E_{SC}(S(G)) \leq \frac{1}{\sqrt{2+s}} 2\sqrt{2pq} = 2\sqrt{\frac{2pq}{2+s}}.$$

5. By taking $t = s+2$ in Eq (2.1), we have following computations for the first Zagreb energy of the subdivision graph $S(G)$;

$$ZE_1(S(G)) = (s+2) \sum_{j=1}^{p+q} \left| \nu_j \begin{bmatrix} 0I_p & C(G) \\ C^T(G) & 0I_q \end{bmatrix} \right| \leq (s+2).2\sqrt{2pq}.$$

as

$$\sum_{j=1}^{p+q} \left| \nu_j \begin{bmatrix} 0I_p & C(G) \\ C^T(G) & 0I_q \end{bmatrix} \right| \leq 2\sqrt{2pq}.$$

Hence,

$$ZE_1(S(G)) \leq 2(s+2)\sqrt{2pq}.$$

6. By taking $t = 2s$ in Eq (2.1), we have following computations for the second Zagreb energy energy of the subdivision graph $S(G)$;

$$ZE_2(S(G)) = 2s \sum_{j=1}^{p+q} \left| z_j \begin{bmatrix} 0I_p & C(G) \\ C^T(G) & 0I_q \end{bmatrix} \right| \leq 2s[2\sqrt{2pq}].$$

where

$$\sum_{j=1}^{p+q} \left| z_j \begin{bmatrix} 0I_p & C(G) \\ C^T(G) & 0I_q \end{bmatrix} \right| \leq [2\sqrt{2pq}]$$

Hence,

$$ZE_2(S(G)) \leq 4s\sqrt{2pq}.$$

□

2.2. *Seidel energy*

Definition 2.9 (Seidel energy)**.** [10] The Seidel energy E_{SE} of a simple connected graph G as the sum of the absolute eigenvalues of the seidel matrix $SE(G)$ of G. Here $SE(G) = [s_{ij}]$ where

$$s_{ij} = \begin{cases} -1, & \text{if } v_i \text{ and } v_j \text{ are adjacent and } i \neq j \\ 1, & \text{if } v_i \text{ and } v_j \text{ are non adjacent and } i \neq j \\ 0, & \text{if i=j} \end{cases}$$

Theorem 2.10. *For an s-regular* (p, q) *graph G,*

$$E_{SE}(S(G)) \leq 2(p+q) + 2s\sqrt{pq} - 4.$$

Proof. Let $u_1, u_2, u_3, ..., u_p$ be vertices of an s-regular graph G and let u'_j for $1 \leq j \leq q$ be vertices that are added at all edges of G to gain $S(G)$. Note that $SE(S(G))$ is given as:

$$SE(S(G)) = \begin{bmatrix} J_p - I_p & E_{p\times q} \\ E_{q\times p} & J_q - I_q \end{bmatrix}.$$

where $E_{p\times q} = [e_{pq}]$ such that

$$e_{pq} = \begin{cases} -1, & \text{if } v_p \text{ and } v_q \text{ are adjacent} \\ 1, & \text{Otherwise} \end{cases}$$

Therefore

$$E_{SE}(S(G)) \leq \sum_{j=1}^{p} \left| v_j[J_p - I_q] \right| + \sum_{j=1}^{p+q} \left| v_j \begin{bmatrix} 0I_p & E_{p\times q} \\ E_{q\times p} & 0I_q \end{bmatrix} \right| + \sum_{j=1}^{q} \left| v_j[J_q - I_q] \right|.$$

As

$$\sum_{j=1}^{p} \left| v_j[J_p - I_p] \right| \leq 2(p-1),$$

$$\sum_{j=1}^{q}\left|\nu_j[J_q - I_q]\right| \leq 2(q-1),$$

and

$$\sum_{j=1}^{p+q}\left|\nu_j\begin{bmatrix} 0I_p & E_{p\times q} \\ E_{q\times p} & 0I_q \end{bmatrix}\right| \leq 2s\sqrt{pq}.$$

Hence,

$$\begin{aligned} E_{SE}(S(G)) &\leq 2(p-1) + 2s\sqrt{pq} + 2(q-1), \\ &= 2(p+q) + 2s\sqrt{pq} - 4. \end{aligned}$$

□

2.3. Degree sum energy

Definition 2.11 (Degree sum energy). [11] The degree sum energy E_{DS} of a simple connected graph G is define as the sum of the absolute eigenvalues of the degree sum matrix $DS(G)$ of G where $DS(G)$ has (i, j)th entry $d_i + d_j$ if $i \neq j$ and 0 otherwise.

Theorem 2.12. *For a s-regular graph G having p and q as order and size respectively,*

$$E_{DS}(S(G)) \leq 4s(p-1) + 2(s+2)\sqrt{pq} + 8(q-1).$$

Proof. Let G has vertices $w_1, w_2, w_3, ..., w_p$ and w'_j for $1 \leq j \leq q$ be the vertices that are added at every edge of G to acquire $S(G)$. Then

$$DS(S(G)) = \begin{array}{c} \\ w_1 \\ w_2 \\ w_3 \\ \vdots \\ w_p \\ w'_1 \\ w'_2 \\ w'_3 \\ \vdots \\ w'_q \end{array}\begin{array}{c} \begin{array}{cccccccccc} w_1 & w_2 & w_3 & ... & w_p & w'_1 & w'_2 & w'_3 & ... & w'_q \end{array} \\ \begin{pmatrix} 0 & 2s & 2s & ... & 2s & s+2 & s+2 & s+2 & ... & s+2 \\ 2s & 0 & 2s & ... & 2s & s+2 & s+2 & s+2 & ... & s+2 \\ 2s & 2s & 0 & ... & 2s & s+2 & s+2 & s+2 & ... & s+2 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 2s & 2s & 2s & ... & 0 & s+2 & s+2 & s+2 & ... & s+2 \\ s+2 & s+2 & s+2 & ... & s+2 & 0 & 4 & 4 & ... & 4 \\ s+2 & s+2 & s+2 & ... & s+2 & 4 & 0 & 4 & ... & 4 \\ s+2 & s+2 & s+2 & ... & s+2 & 4 & 4 & 0 & ... & 4 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ s+2 & s+2 & s+2 & ... & s+2 & 4 & 4 & 4 & ... & 0 \end{pmatrix} \end{array}.$$

Or

$$DS(S(G)) = \begin{bmatrix} 2s[J_p - I_p] & (s+2)[J_{p\times q}] \\ (s+2)[J_{q\times p}] & 4[J_q - I_q] \end{bmatrix}.$$

Therefore

$$\begin{aligned} E_{DS}(S(G)) \leq \sum_{j=1}^{p}\left|\mu_j\begin{bmatrix} 2s[J_p - I_p] & 0J_{p\times q} \\ 0J_{q\times p} & 0[J_q - I_q] \end{bmatrix}\right| + \sum_{j=1}^{p+q}\left|\mu_j\begin{bmatrix} 0I_p & (s+2)J_{p\times q} \\ (s+2)J_{q\times p} & 0[J_q - I_q] \end{bmatrix}\right| \\ + \sum_{j=1}^{q}\left|\mu_j\begin{bmatrix} 0I_p & 0J_{p\times q} \\ 0J_{q\times p} & 4[J_q - I_q] \end{bmatrix}\right| \end{aligned}$$

$$E_{DS}(S(G)) \leq 2s\sum_{j=1}^{p}\left|\mu_j\begin{bmatrix}[J_p-I_p] & 0J_{p\times q}\\ 0J_{q\times p} & 0[J_q-I_q]\end{bmatrix}\right| + (s+2)\sum_{j=1}^{p+q}\left|\mu_j\begin{bmatrix}0I_p & J_{p\times q}\\ J_{q\times p} & 0[J_q-I_q]\end{bmatrix}\right| + 4\sum_{j=1}^{q}\left|\mu_j\begin{bmatrix}0I_p & 0J_{p\times q}\\ 0J_{q\times p} & [J_q-I_q]\end{bmatrix}\right|$$

Hence,

$$\begin{aligned}E_{DS}(S(G)) &\leq 2s[2(n-1)] + (s+2)[2\sqrt{pq}] + 4[2(q-1)],\\ &= 4s(p-1) + 2(s+2)\sqrt{pq} + 8(q-1).\end{aligned}$$

□

2.4. Degree square sum energy

Definition 2.13 (Degree square sum energy)**.** [12] The degree square sum energy E_{DSS} of a simple connected graph G is define as the sum of the absolute eigenvalues of the degree square sum matrix $DSS(G)$ of G where $DSS(G)$ has (i,j)th entry $d_i^2 + d_j^2$ if $i \neq j$ and 0 otherwise.

Theorem 2.14. *For an s-regular graph G*

$$E_{DSS}(S(G)) \leq 4s^2(p-1) + 2(s^2+4)\sqrt{pq} + 16(q-1).$$

Proof. Let for $1 \leq j \leq p$, u_j be vertices of G and u'_k for $1 \leq k \leq q$ be the vertices that are added in G to get $S(G)$. Note that the degree square sum matrix of $S(G)$ is denoted by $DSS(S(G))$ and is given as:

$$\begin{array}{c|cccccccccc} & u_1 & u_2 & u_3 & \dots & u_p & u'_1 & u'_2 & u'_3 & \dots & u'_q\\ \hline u_1 & 0 & 2s^2 & 2s^2 & \dots & 2s^2 & s^2+4 & s^2+4 & s^2+4 & \dots & s^2+4\\ u_2 & 2s^2 & 0 & 2s^2 & \dots & 2s^2 & s^2+4 & s^2+4 & s^2+4 & \dots & s^2+4\\ u_3 & 2s^2 & 2s^2 & 0 & \dots & 2s^2 & s^2+4 & s^2+4 & s^2+4 & \dots & s^2+4\\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots\\ u_p & 2s^2 & 2s^2 & 2s^2 & \dots & 0 & s^2+4 & s^2+4 & s^2+4 & \dots & s^2+4\\ u'_1 & s^2+4 & s^2+4 & s^2+4 & \dots & s^2+4 & 0 & 8 & 8 & \dots & 8\\ u'_2 & s^2+4 & s^2+4 & s^2+4 & \dots & s^2+4 & 8 & 0 & 8 & \dots & 8\\ u'_3 & s^2+4 & s^2+4 & s^2+4 & \dots & s^2+4 & 8 & 8 & 0 & \dots & 8\\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots\\ u'_q & s^2+4 & s^2+4 & s^2+4 & \dots & s^2+4 & 8 & 8 & 8 & \dots & 0\end{array}.$$

Or

$$DSS(S(G)) = \begin{bmatrix}2s^2[J_p-I_p] & (s^2+4)[J_{p\times q}]\\ (s^2+4)[J_{q\times p}] & 8[J_q-I_q]\end{bmatrix}.$$

Therefore

$$E_{DSS}(S(G)) \leq \sum_{j=1}^{p}\left|\mu_j\begin{bmatrix}2s^2[J_p-I_p] & 0J_{p\times q}\\ 0J_{q\times p} & 0[J_q-I_q]\end{bmatrix}\right| + \sum_{j=1}^{p+q}\left|\mu_j\begin{bmatrix}0I_p & (s^2+4)J_{p\times q}\\ (s^2+4)J_{q\times p} & 0[J_q-I_q]\end{bmatrix}\right| + \sum_{j=1}^{q}\left|\mu_j\begin{bmatrix}0I_p & 0J_{p\times q}\\ 0J_{q\times p} & 8[J_q-I_q]\end{bmatrix}\right|$$

Or

$$E_{DSS}(S(G)) \leq 2s^2 \sum_{j=1}^{p} \left| \mu_j \begin{bmatrix} [J_p - I_p] & 0J_{p\times q} \\ 0J_{q\times p} & 0[J_q - I_q] \end{bmatrix} \right| + (s^2+4) \sum_{j=1}^{p+q} \left| \mu_j \begin{bmatrix} 0I_p & J_{p\times q} \\ J_{q\times p} & 0[J_q - I_q] \end{bmatrix} \right| + 8 \sum_{j=1}^{q} \left| \mu_j \begin{bmatrix} 0I_p & 0J_{p\times q} \\ 0J_{q\times p} & [J_q - I_q] \end{bmatrix} \right|$$

Hence,

$$\begin{aligned} E_{DSS}(S(G)) &\leq 2s^2[2(p-1)] + (s^2+4)[2\sqrt{pq}] + 8[2(q-1)], \\ &= 4s^2(p-1) + 2(s^2+4)\sqrt{pq} + 16(q-1). \end{aligned}$$

□

3. Energies of complete graph

A complete graph denoted by K_p is a simple undirected graph in which every pair of distinct vertices is connected by a unique edge. K_5 is shown in Figure 2.

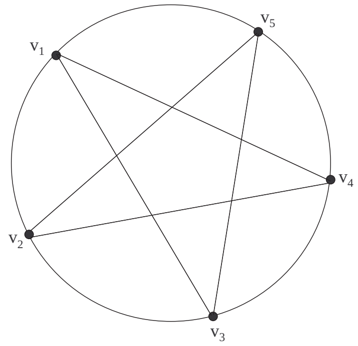

Figure 2. K_5.

We have following trivial results about energies of complete graphs.

Theorem 3.1. *For the complete graph K_p, the maximum degree energy is*

$$E_M(K_p) = 2(p-1)^2.$$

Theorem 3.2. *For the complete graph K_p, the minimum degree energy is*

$$E_m(K_p) = 2(p-1)^2.$$

Theorem 3.3. *For the complete graph K_p, the Randić energy is*

$$E_R(K_p) = 2.$$

Theorem 3.4. *For the complete graph K_p, the Seidel energy is*

$$E_{SE}(K_p) = 2(p-1) = 2(p-1)).$$

Theorem 3.5. *For the complete graph K_p, the sum-connectivity energy is*

$$E_{SC}(K_p) = \sqrt{2(p-1)}.$$

Theorem 3.6. *For the complete graph K_p, the degree sum energy is*

$$E_{DS}(K_p) = 4(p-1)^2.$$

Theorem 3.7. *For the complete graph K_p, the degree square sum energy is*

$$E_{DSS}(K_p) = 4(p-1)^3.$$

Theorem 3.8. *For the complete graph K_p, the first Zagreb energy is*

$$ZE_1(K_p) = 4(p-1)^2.$$

Theorem 3.9. *For the complete graph K_p, the second Zagreb energy is*

$$ZE_2(K_p) = 2(p-1)^3.$$

4. Conclusions

In this paper we gave bounds of maximum degree energy, Randić energy, sum-connectivity energy etc of s-regular subdivision graph $S(G)$. Also various graph energies of complete graph are mentioned in this article. In future, we aim to study graph energies for the other families of graphs.

Conflict of interest

Authors do not have any competing interests.

References

1. M. A. Sriraj, *Some studies on energy of graphs,* Ph. D. Thesis, Univ. Mysore, Mysore, India, 2014.
2. M. S. Ahmad, W. Nazeer, S. M. Kang, M. Imran, W. Gao, Calculating degree-based topological indices of dominating David derived networks, *Open Phys.*, **15** (2017), 1015–1021.
3. A. Farooq, M. Habib, A. Mahboob, W. Nazeer, S. M. Kang, Zagreb polynomials and redefined zagreb indices of dendrimers and polyomino chains, *Open Chem.,* **17** (2019), 1374–1381.
4. Y. C. Kwun, A. Farooq, W. Nazeer, Z. Zahid, S. Noreen, S. M. Kang, Computations of the M-polynomials and degree-based topological indices for dendrimers and polyomino chains, *Int. J. Anal. Chem.,* **2018** (2018), 1709073.
5. C. Adiga, M. Smitha, On maximum degree energy of a graph, *Int. J. Contemp. Math. Sci.*, **4** (2009), 385–396.
6. C. Adiga, C. S. Swamy, Bounds on the largest of minimum degree eigenvalues of graphs, *Int. Math. Forum*, **5** (2010), 1823–1831.

7. K. C. Das, S. Sorguna, K. Xu, On randic energy of graphs, *Math. Commun. Math. Comput. Chem.*, **72** (2014), 227–238.

8. B. Zhou, N. Trinajstic, On the sum-connectivity matrix and sum-connectivity energy of (molecular) graphs, *Acta Chim. Slov.*, **57** (2010), 518–523.

9. N. J. Rad, A. Jahanbani, I. Gutman, Zagreb energy and Zagreb estrada index of graphs, *Math. Commun. Math. Comput. Chem.*, **79** (2018), 371–386.

10. P. Nageswari, P. B. Sarasija, Seidel energy and its bounds, *Int. J. Math. Anal.*, **8** (2014), 2869–2871.

11. S. M. Hosamani, H. S. Ramane, On degree sum energy of a graph, *Eur. J. Pure Appl. Math.*, **9** (2016), 340–345.

12. B. Basavanagoud, E. Chitra, Degree square sum energy of graphs, *Int. J. Math. Appl.*, **6** (2018), 193–205.