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Research article

Finite-time anti-synchronization for delayed inertial neural networks via the fractional and polynomial controllers of time variable

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Abstract: This paper focuses on the finite-time anti-synchronization for a class of delayed master-slave inertial neural networks. By means of using the property of quadratic inequality of one variable and designing the fractional and polynomial controllers of time variable, two sufficient conditions to assure the finite-time anti-synchronization for the master-slave delayed inertial neural networks are established. Our controllers designed related to time variable t and the study method on the finite-time anti-synchronization are different from these in the existing papers.

Keywords: drive-response delayed inertial neural networks; finite-time anti-synchronization; quadratic inequality of one variable; the fractional and polynomial controllers of time variable
Mathematics Subject Classification: 34K24

1. Introduction

As a very important neural network system, the inertial neural network model was first proposed by Babcock and Westervelt [1]. In view of its important application background in biology and engineering [2], for example, the surface layer of hair cells can be achieved in the semicircular canal of some membrane animals, which consists of an equivalent integrated circuit containing an inductor [3, 4], thus, it is very crucial to add an inertial term to the nervous system. Further, the inertial term can be regarded as a powerful tool for inducing bifurcation and chaos [5, 6]. Consequently, the study of dynamical behavior of the inertial delayed neural networks is very important. Recently, the dynamic behaviors of inertial neural networks have received wide attention [7–17].

Synchronization of neural networks has been extensively discussed in recent years in view of their potential applications in image process, secure communication, information science and many other fields [10, 12]. In practice, it also appears another prevailing phenomenon in symmetrical oscillators, anti-synchronization, which means that the state vector of synchronized systems have the same absolute values but opposite signs. It was stated that the application of anti-synchronization to lasers provides a new way to generate pulses with special forms; and its application to communication systems can strengthen the security and secrecy by the transform of the synchronization and anti-synchronization continuously in the process of digital signal transmission. As a result, the research of anti-synchronization for delayed neural networks is of very great importance in both theory and application. So far, the finite-time anti-synchronization for delayed neural networks has been investigated by some researchers [18–27]. In [18], the finite-time anti-synchronization of neural networks with time-varying delays were concerned, by combining the Holder inequality and other techniques, a sufficient condition to assure the finite-time anti-synchronization for the considered drive-response neural networks was gained. In [19], by applying integral inequality method, two criteria to ensure the finite-time anti-synchronization for the master-slave neural networks discussed in [18] were established. In [20], by mean of the inequality skills used in [18], the criteria to assure the finite-time anti-synchronization for the discussed master-slave were achieved. In [21], the master-slave finite-time anti-synchronization for memristive bidirectional associative memory neural networks (MBAMNNS) was discussed, by employing some inequality skills and constructing an appropriate Lyapunov function, some anti-synchronization criteria were derived. In [22], the finite-time anti-synchronization control of memristive neural networks with stochastic perturbations was studied by using the linear matrix inequality method. In [23], the finite-time anti-synchronization of time-varying delayed neural networks was investigated, by employing some differential inequalities and finite-time stability theory, some novel effective finite-time anti-synchronization criteria were derived based on the Lyapunov function method. In [24], the finite-time anti-synchronization of the multi-weighted coupled neural networks with and without coupling delays was analyzed, by utilizing Lyapunov functional approach and some inequality skills, several anti-synchronization criteria were put forward for the considered networks.

To the best of our knowledge, up to now, the finite-time anti-synchronization (or finite-time synchronization) has been extensively studied mainly by applying the following four classes of study approaches: (1) Some finite-time stability theorems were used to study the finite-time anti-synchronization (or finite-time synchronization)[23, 28, 29]; (2) Algebraic inequality approaches were used to investigate finite-time the anti-synchronization (or finite-time synchronization)[17,18, 21, 24]; (3) Linear matrix inequality approaches were applied to studying the finite-time anti-synchronization (or finite-time synchronization)[22]; (4) Integral inequality approaches were used to investigate the finite-time anti-synchronization (or finite-time synchronization)[19, 25–27]. On the other hand, up to until, in almost papers which studied the synchronization, the controllers designed only have been independent of the time variable t, a.e, the designed controllers are only the functions of the error variables $e_i(t)$.

Inspired by the above analysis, we will attempt to study the finite-time anti-synchronization of the master-slave delayed inertial neural networks by employing the quadratic inequality of one variable under the fractional and polynomial controllers of time variable t. By applying the quadratic inequality of one variable (see Lemma 2.1), the differential inequalities (3.7) and (3.20) (see (3.7) and (3.20) In the proofs of Theorem 3.1 and Theorem 3.2) are obtained. Then integrating two differential inequalities give two sufficient conditions on the finite-time anti-synchronization for the master-slave neural networks. Our results of the finite-time anti-synchronization are more concise and easily verified than these obtained in the existing papers [18–29]. Designing the the fractional and polynomial controllers of time variable t, our results obtained are more objective and practical than these obtained in [18–33] on the finite-time anti-synchronization of the master-slave systems. The main contributions of this paper are the following aspects: (1) For the first place, by using the behavior of quadratic inequality of one variable, more concise and easily verified criteria of the finite-time anti-synchronization for the delayed master-slave neural networks are put forward; (2) By using the fractional and polynomial controllers of time variable, the more objective and practical criteria of finite-time anti-synchronization for the master-slave neural networks are given.

2. Preliminaries

Consider the following delayed inertial neural networks:

$$\frac{d^2x_r(t)}{dt^2} = -a_r\frac{dx_r(t)}{dt} - b_rx_r(t) + F_r(x'(t), x(t)), r = 1, 2, \cdots, \hat{n}, \tag{2.1}$$

where

$$F_r(x'(t), x(t)) \quad = \quad \sum_{j=1}^{\hat{n}} c_{rj}f_j(x_j(t)) + \sum_{j=1}^{\hat{n}} d_{rj}f_j(x_j(qt)) + \sum_{j=1}^{\hat{n}} f_{rj}f_j(x'_j(qt)) + \hat{I}_r,$$

$x(t) = (x_1(t), x_2(t), \cdots, x_{\hat{n}}(t))^T \in R^{\hat{n}}$, $x_r(t)$ represents the states of the rth neuron at time t; the second derivatives are called inertial terms of system (2.1); $a_r > 0, b_r > 0$ they denote the rates with which the ith neuron will reset its potential to the resting state in isolation when disconnected from the network and external inputs; c_{rj}, d_{rj}, f_{rj} are constants, denoting the connection weights; $\hat{I}_r$ denotes external inputs of the rth neurons, f_j is the activation function; $qt = t - \tau(t), \tau(t) \geq 0, \tau'(t) \leq \tau < 1$.

The initial conditions of system (2.1) are

$$x_r(s) = \phi^x_r(s), \frac{dx_r(s)}{dt} = \psi^x_r(s), s \in [-\alpha, 0],$$

where $\phi^x_r(s), \psi^x_r(s)$ are real-valued bounded continuous functions on $[-\alpha, 0], \alpha = \max_{t\in R}\{\tau(t)\}$.

If we refer to system (2.1) as the master system, then the slave system is expressed as follows:

$$\frac{d^2u_r(t)}{dt^2} = -a_r\frac{du_r(t)}{dt} - b_ru_r(t) + F_r(u'(t), u(t)) + \hat{I}_r + v_r(t), r = 1, 2, \cdots, \hat{n},$$

where $v_r(t)$ is the controller to design later.

The initial conditions of system (2.2) are

$$u_r(s) = \phi^u_r(s), \frac{du_r(s)}{dt} = \psi^u_r(s), s \in [-\alpha, 0],$$

where $\phi^u_r(s), \psi^u_r(s)$ are real-valued continuous functions on $[-\alpha, 0]$.

Let $w_r(t) = u_r(t) + x_r(t)$. Then we can get the following error system of (2.1) and (2.2) for $r = 1, 2, \cdots, \hat{n}$:

$$\frac{d^2w_r(t)}{dt^2} = -a_r\frac{dw_r(t)}{dt} - b_rw_r(t) + F_r(u'(t), u(t)) + F_r(x'(t), x(t)) + 2\hat{I}_r + v_r(t), \tag{2.2}$$

where

$$\begin{aligned}
& F_r(u'(t), u(t)) + F_r(x'(t), x(t)) \\
= & \sum_{j=1}^{\hat{n}} c_{rj}[f_j(u_j(t)) + f_j(x_j(t))] + \sum_{j=1}^{\hat{n}} d_{rj}[f_j(u_j(qt)) + f_j(x_j(qt))] + \sum_{j=1}^{\hat{n}} f_{rj}[f_j(u'_j(qt)) \\
& + f_j(x'_j(qt))].
\end{aligned}$$

Assumption 1. The activation function f_j is odd, and there exists a constant $L_j \geq 0$ such that

$$|f_j(w) - f_j(z)| \leq L_j|w - z|, j = 1, 2, \cdots, \hat{n}, w, z \in R.$$

Definition 2.1. Master system (2.1) and slave system (2.2) are said to achieve the finite-time anti-synchronization, if there exists a constant $\bar{T} > 0$, which depends on the initial conditions of the system (2.1) and system (2.2), such that for $r = 1, 2, \cdots, \hat{n}$,

$$\lim_{t \to \bar{T}} |u_r(t) + x_r(t)| = 0, |u_r(t) + x_r(t)| = 0, t \geq \bar{T}.$$

Lemma 2.1. [quadratic inequality of one variable] If $a < 0, b^2 < 4ac$, then $ax^2 + bx + c < 0, x \in R$.
Proof. The inequality is well known and its proof is omitted.

3. Main results

In this section, two novel sufficient conditions on the finite-time anti-synchronization for drive-response delayed inertial neural networks (2.1) and (2.2) are derived by applying quadratic inequality of one variable under the fractional and polynomial controllers.

The controllers in system (2.3) are designed as follows :

$$\begin{aligned}
& v_r(t) \\
= & -[w'_r(t)]^{-1}\Big[\xi_1 w_r^2(t) + \beta_0 + c_0 + 2c_1 t + 3c_2 t^2 + 4c_3 t^3 + \cdots + (\hat{k} + 1)c_{\hat{k}} t^{\hat{k}}\Big], w'_r(t) \neq 0,
\end{aligned} \tag{3.1}$$

and

$$v_r(t) = sign[w'_r(t)]\Big[\frac{b}{(t + a)^2} + \beta_1 + \beta_2 w_r^2(t) + \beta_3[w'_r(t)]^2 - \beta_4 - \beta_5\Big], \tag{3.2}$$

where $sign[w'_r(t)] = \begin{cases} 1, w'_r(t) > 0, \\ -1, w'_r(t) < 0, \\ 0, w'_r(t) = 0, \end{cases}$ $a > 0, b < 0, \beta_1 < 0, \xi_1 > 0, \beta_2 < 0, \beta_3 < 0, \beta_4 > 0, \beta_5 > 0, \hat{k}$ is a positive integer, $t \geq 0, \beta_0 > 0, c_1 > 0, c_2 > 0, \cdots, c_{\hat{k}} > 0$ with $c_0 - \hat{I}_r^2 < 0, \beta_4 > \hat{I}_r, \beta_5 > \hat{I}_r, b < -aM(0), M(0) = \sum_{r=1}^{\hat{n}}(|w_r(0)| + |w'_r(0)|) + \frac{1}{1-\tau}\sum_{r=1}^{\hat{n}}\sum_{j=1}^{\hat{n}} L_j\Big(|d_{rj}|\int_{-\tau(0)}^{0}|w'_j(s)|ds + |f_{rj}|\int_{-\tau(0)}^{0}|w'_j(s)|ds\Big)$.

Theorem 3.1. Under Assumption 1, the master system (2.1) and slave system (2.2) can gain the finite-time anti-synchronization under the controller (3.1) when the following conditions are satisfied for

$r = 1, 2, \cdots, \hat{n}$:
(m_1)

$$\xi_1 > 0.5\sum_{j=1}^{\hat{n}}(|c_{rj}| + \frac{|d_{rj}|}{1-\tau})L_r$$

(m_2) There exists a constant $\gamma_r > 0$ such that

$$\begin{aligned}&(\gamma_r - b_r)^2\\ < \;& 4\Big[1 - a_r + 0.5\sum_{j=1}^{\hat{n}}(|c_{rj}| + \frac{|d_{rj}|}{1-\tau})L_j + \sum_{j=1}^{\hat{n}}\frac{|f_{rj}|L_j}{1-\tau}\Big]\Big[0.5\sum_{j=1}^{\hat{n}}(|c_{rj}| + \frac{|d_{rj}|}{1-\tau})L_r - \xi_1\Big],\end{aligned}$$

where, the finite-time $t^* = \max\{t_1, t_2\}, t_2 = \frac{K(0)}{\hat{k}\beta_0}, t_1$ is the only positive real root of the equation $-\hat{I}_r^2 + c_0 + c_1 t + c_2 t^2 + \cdots + c_{\hat{n}} t^{\hat{n}} = 0$.

Proof. Without loss of generalization, we assume that $w_r'(t) \neq 0, r = 1, 2, \cdots, \hat{n}$. If $w_r'(t) = 0$, then $w_r(t)$=constant. In the case, by letting $\phi_r^u(s) + \phi_r^x(s) = 0, s \in [-\alpha, 0]$, where $\phi_r^x(s)$ and $\phi_r^u(s)$ are respectively the initial conditions of the solution $x_r(s)$ of system (2.1) and the solution $u_r(s)$ of system (2.2), the proof of Theorem 3.1 can be finished.

Introduce a Lyapunov functional as follows:

$$K(t) = K_1(t) + K_2(t),$$

where

$$K_1(t) = \frac{1}{2}\sum_{r=1}^{\hat{n}}\Big[\frac{dw_r(t)}{dt}\Big]^2 + \frac{1}{2}\sum_{r=1}^{\hat{n}}\gamma_r w_r^2(t),$$

$$K_2(t) = \frac{1}{1-\tau}\sum_{r=1}^{\hat{n}}\sum_{j=1}^{\hat{n}}\int_{t-\tau(t)}^{t}|f_{rj}|L_j|w_j'(s)|ds + \frac{1}{1-\tau}\sum_{r=1}^{\hat{n}}\sum_{j=1}^{\hat{n}}\int_{t-\tau(t)}^{t}|d_{rj}|L_j|w_j(s)|ds.$$

Calculating the derivatives of $K_1(t)$ along the solution of system (2.3), one has based on the Assumption 1 as follows:

$$\begin{aligned}&K_1'(t)\\ =\;& \sum_{r=1}^{\hat{n}}\Big(w_r'(t)w_r''(t) + \gamma_r w_r(t)w_r'(t)\Big)\\ =\;& \sum_{r=1}^{\hat{n}}\Big\{w_r'(t)\Big(-a_r w_r'(t) + (\gamma_r - b_r)w_r(t) + \sum_{j=1}^{\hat{n}}c_{rj}[f_j(u_j(t)) + f_j(x_j(t))] + \sum_{j=1}^{\hat{n}}d_{rj}\times\\ & [f_j(u_j(qt)) + f_j(x_j(qt))] + \sum_{j=1}^{\hat{n}}f_{rj}[f_j(u_j'(qt)) + f_j(x_j'(qt))] + 2\hat{I}_r + v_r(t)\Big)\Big\}\\ \leq\;& \sum_{r=1}^{\hat{n}}\Big\{w_r'(t)\Big(-a_r w_r'(t) + (\gamma_r - b_r)w_r(t) + \sum_{j=1}^{\hat{n}}|c_{rj}||f_j(u_j(t)) + f_j(-[-x_j(t)])| + \sum_{j=1}^{\hat{n}}|d_{rj}|\\ & \times|f_j(u_j(qt)) + f_j(-[-x_j(qt)])| + \sum_{j=1}^{\hat{n}}|f_{rj}||f_j(u_j'(qt)) + f_j(-[-x_j'(qt)])| + 2\hat{I}_r + v_r(t)\Big)\Big\}\end{aligned}$$

$$\leq \sum_{r=1}^{\hat{n}} \Big\{ w_r'(t)\Big(- a_r w_r'(t) + (\gamma_r - b_r)w_r(t) + \sum_{j=1}^{\hat{n}} |c_{rj}|L_j|w_j(t)| + \sum_{j=1}^{\hat{n}} |d_{rj}||w_j(qt)|L_j + \sum_{j=1}^{\hat{n}} |f_{rj}||w_j'(qt)|L_j + 2\hat{I}_r\Big) - \Big[\xi_1 w_r^2(t) + \beta_0 + c_0 + 2c_1 t + 3c_2 t^2 + \cdots + (\hat{k}+1)c_{\hat{k}} t^{\hat{k}}\Big]. \quad (3.3)$$

At the same time, one has

$$\begin{aligned} &K_2'(t) \\ = \;& \frac{1}{1-\tau}\sum_{r=1}^{\hat{n}}\sum_{j=1}^{\hat{n}} |f_{rj}|L_j\Big[|w_j'(t)| - (1-\tau'(t))|w_j'(t-\tau(t))|\Big] + \frac{1}{1-\tau}\sum_{r=1}^{\hat{n}}\sum_{j=1}^{\hat{n}} |d_{rj}| \\ & \times L_j\Big[|w_j(t)| - (1-\tau'(t))|w_j(t-\tau(t))|\Big] \\ \leq \;& \frac{1}{1-\tau}\sum_{r=1}^{\hat{n}}\sum_{j=1}^{\hat{n}} |f_{rj}|L_j\Big[|w_j'(t)| - (1-\tau)|w_j'(t-\tau(t))|\Big] + \frac{1}{1-\tau}\sum_{r=1}^{\hat{n}}\sum_{j=1}^{\hat{n}} |d_{rj}| \\ & \times L_j\Big[|w_j(t)| - (1-\tau)|w_j(t-\tau(t))|\Big]. \end{aligned} \quad (3.4)$$

Based on (3.3) and (3.4), one obtain

$$\begin{aligned} &K'(t) \\ \leq \;& \sum_{r=1}^{\hat{n}} \Big\{ w_r'(t)\Big(- a_r w_r'(t) + (\gamma_r - b_r)w_r(t) + \sum_{j=1}^{\hat{n}}(|c_{rj}| + \frac{|d_{rj}|}{1-\tau})L_j|w_j(t)| + \sum_{j=1}^{\hat{n}} \frac{|f_{rj}|}{1-\tau} \times \\ & |w_j'(t)|L_j + 2\hat{I}_r\Big) - \Big[\xi_1 w_r^2(t) + \beta_0 + c_0 + 2c_1 t + 3c_2 t^2 + \cdots + c_{\hat{k}} t^{\hat{k}}\Big]\Big\}, \end{aligned}$$

from which, by means of using $ab \leq 0.5(a^2 + b^2)$, it follows that

$$\begin{aligned} &K'(t) \\ \leq \;& \sum_{r=1}^{\hat{n}} \Big\{ \Big[0.5\sum_{j=1}^{\hat{n}}(|c_{rj}| + \frac{|d_{rj}|}{1-\tau})L_r - \xi_1\Big] w_r^2(t) + (\gamma_r - b_r)w_r'(t)w_r(t) + \Big[1 - a_r + 0.5\times \\ & \sum_{j=1}^{\hat{n}}(|c_{rj}| + \frac{|d_{rj}|}{1-\tau})L_j + \sum_{j=1}^{\hat{n}} \frac{|f_{rj}|L_j}{1-\tau}\Big][w_r'(t)]^2\Big\} + \hat{n}\Big(\hat{I}_r^2 - \Big[\beta_0 + c_0 + 2c_1 t + 3c_2 t^2 + \cdots + \\ & (\hat{k}+1)c_{\hat{k}} t^{\hat{k}}\Big]\Big). \end{aligned} \quad (3.5)$$

According to Lemma 2.1, in view of (m_1) and (m_2), one has

$$\begin{aligned} &\Big[0.5\sum_{j=1}^{\hat{n}}(|c_{rj}| + \frac{|d_{rj}|}{1-\tau})L_r - \xi_1\Big] w_r^2(t) + (\gamma_r - b_r)w_r'(t)w_r(t) + \Big[1 - a_r + 0.5 \times \\ & \sum_{j=1}^{\hat{n}}(|c_{rj}| + \frac{|d_{rj}|}{1-\tau})L_j + \sum_{j=1}^{\hat{n}} \frac{|f_{rj}|L_j}{1-\tau}\Big][w_r'(t)]^2 < 0. \end{aligned} \quad (3.6)$$

Substituting (3.6) into (3.5) yields

$$K'(t) \le \hat{k}\Big(\hat{I}_r^2 - \Big[c_0 + 2c_1 t + 3c_2 t^2 + \cdots + (\hat{k}+1)c_{\hat{k}} t^{\hat{k}}\Big]\Big) - \hat{k}\beta_0. \tag{3.7}$$

Integrating (3.7) over $[0, t]$ yields

$$K(t) \le K(0) - \hat{k}\beta_0 t + \hat{k}t\Big[\hat{I}_r^2 - \Big[c_0 + c_1 t + c_2 t^2 + \cdots + c_{\hat{k}} t^{\hat{k}}\Big]. \tag{3.8}$$

Let

$$F(t) = -\hat{I}_r^2 + \Big[c_0 + c_1 t + c_2 t^2 + \cdots + c_{\hat{n}} t^{\hat{n}}\Big], t \ge 0.$$

Then

$$F(0) = c_0 - \hat{I}_r^2 < 0, \lim_{t\to\infty} F(t) = +\infty > 0.$$

So there exists a point $t_1 > 0$ such that $F(t_1) = 0$. Letting

$$-\hat{I}_r^2 + \Big[c_0 + c_1 t + c_2 t^2 + \cdots + c_{\hat{n}} t^{\hat{n}}\Big] = (t - t_1)[b_{\hat{n}-1} t^{\hat{n}-1} + b_{\hat{n}-2} t^{\hat{n}-2} + \cdots + b_1 t + b_0], \tag{3.9}$$

one has

$$\begin{cases} b_{\hat{n}-1} = c_{\hat{n}}, \\ b_{\hat{n}-2} - t_1 b_{\hat{n}-1} = c_{\hat{n}-1}, \\ b_{\hat{n}-3} - t_1 b_{\hat{n}-2} = c_{\hat{n}-2}, \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ b_1 - b_2 t_1 = c_2, \\ b_0 - b_1 t_1 = c_1, \\ -t_1 b_0 = c_0 - \hat{I}_r^2. \end{cases},$$

As a result

$$\begin{cases} b_{\hat{n}-1} = c_{\hat{n}} > 0, \\ b_{\hat{n}-2} = c_{\hat{n}-1} + t_1 b_{\hat{n}-1} > 0, \\ b_{\hat{n}-3} = c_{\hat{n}-2} + t_1 b_{\hat{n}-2} > 0, \\ \cdots\cdots\cdots\cdots\cdots\cdots \\ b_1 = c_2 + t_1 b_2 > 0, \\ b_0 = c_1 + b_1 t_1 > 0, \\ c_0 - \hat{I}_r^2 = -t_1 b_0 < 0. \end{cases} \tag{3.10}$$

Since $b_i > 0, i = 0, 1, \cdots, b_{\hat{n}-1}$, by (3.9) and (3.10), it follows that the equation

$$-\hat{I}_r^2 + \Big[c_0 + c_1 t + c_2 t^2 + \cdots + c_{\hat{n}} t^{\hat{n}}\Big] = 0,$$

namely the equation

$$(t - t_1)[b_{\hat{n}-1} t^{\hat{n}-1} + b_{\hat{n}-2} t^{\hat{n}-2} + \cdots + b_1 t + b_0] = 0$$

has only positive real root $t = t_1$ and when $t \ge t_1$

$$-\hat{I}_r^2 + \Big[c_0 + c_1 t + c_2 t^2 + \cdots + c_{\hat{n}} t^{\hat{n}}\Big]$$

$$= \quad (t-t_1)[b_{\hat{n}-1}t^{\hat{n}-1} + b_{\hat{n}-2}t^{\hat{n}-2} + \cdots + b_1 t + b_0] > 0.$$

That is when $t \geq t_1$

$$\begin{aligned} & -\hat{k}t\Big\{-\hat{I}_r^2 + \Big[c_0 + c_1 t + c_2 t^2 + \cdots + c_{\hat{n}} t^{\hat{n}}\Big] \\ = \quad & -(t-t_1)[b_{\hat{n}-1}t^{\hat{n}-1} + b_{\hat{n}-2}t^{\hat{n}-2} + \cdots + b_1 t + b_0]\Big\} < 0. \end{aligned} \tag{3.11}$$

Because when $t \geq t_2 = \frac{K(0)}{\hat{k}\beta_0}$

$$K(0) - \hat{k}\beta_0 t < 0, \tag{3.12}$$

Then letting $t^* = \max\{t_1, t_2\}$, it follows that when $t \geq t^*$, the following two inequalities hold:

$$\begin{aligned} & -\hat{k}t\Big\{-\hat{I}_r^2 + \Big[c_0 + c_1 t + c_2 t^2 + \cdots + c_{\hat{n}} t^{\hat{n}}\Big] \\ = \quad & (t-t_1)[b_{\hat{n}-1}t^{\hat{n}-1} + b_{\hat{n}-2}t^{\hat{n}-2} + \cdots + b_1 t + b_0]\Big\} < 0 \end{aligned} \tag{3.13}$$

and

$$K(0) - \hat{k}\beta_0 t < 0. \tag{3.14}$$

Substituting (3.13) into (3.14) into (3.8), it follows that when $t \geq t^*$

$$0 \leq K(t) \leq 0.$$

Consequently, $\lim\limits_{t\to t^*} K_1(t) = 0, K_1(t) = 0, t \geq t^*$.

Namely, $\lim\limits_{t\to t^*} |u_r(t) + x_r(t)| = 0, |u_r(t) + x_r(t)| = 0, t \geq t^*$. This finishes the proof of Theorem 3.1.

Theorem 3.2. Assume that Assumption 1 holds. Then the master system (2.1) and the slave system (2.2) can reach the finite-time anti-synchronization under the controller (3.2) when the following inequalities hold:

(l_1)

$$\Big(1 - a_r + \sum_{j=1}^{\hat{n}} \frac{|f_{rj}|}{1-\tau} L_j\Big)^2 < 4\beta_3(|\hat{I}_r| - \beta_5)$$

(l_2)

$$\Big[b_r + \sum_{j=1}^{\hat{n}} L_r(|c_{jr}| + \frac{|d_{jr}|}{1-\tau})\Big]^2 < 4\beta_2(|\hat{I}_r| - \beta_4),$$

where, the finite-time $T^* = \frac{\beta_1 a + \sqrt{\beta_1^2 a^2 + 4\beta_1 b}}{-2\beta_1}$.

Proof. Introduce a Lyapunov functional as follows:

$$M(t) = M_1(t) + M_2(t),$$

where

$$M_1(t) = \sum_{r=1}^{\hat{n}} [|w_r(t)| + |w_r'(t)|],$$

$$M_2(t) = \frac{1}{1-\tau}\sum_{r=1}^{\hat{n}}\sum_{j=1}^{\hat{n}} L_j\Big(|d_{rj}|\int_{t-\tau(t)}^{t}|w_j(s)|ds + |f_{rj}|\int_{t-\tau(t)}^{t}|w_j'(s)|ds\Big).$$

Calculating the derivatives of $M_1(t)$ along the solution of system (2.3), one has based on the Assumption 1:

$$\begin{aligned}
&M_1'(t)\\
= &\sum_{r=1}^{\hat{n}}\Big(sign[w_r'(t)]w_r''(t) + sign[w_r(t)]w_r'(t)\Big)\\
= &\sum_{r=1}^{\hat{n}}\Big\{sign[w_r'(t)]\Big(-a_r w_r'(t) - b_r w_r(t) + \sum_{j=1}^{\hat{n}} c_{rj}[f_j(u_j(t)) + f_j(x_j(t))] + \sum_{j=1}^{\hat{n}} d_{rj}\times\\
&[f_j(u_j(qt)) + f_j(x_j(qt))] + \sum_{j=1}^{\hat{n}} f_{rj}[f_j(u_j'(qt)) + f_j(x_j'(qt))] + 2\hat{I}_r + v_r(t)\Big) + sign[w_r(t)]\\
&\times w_r'(t)\Big\}\\
\le &\sum_{r=1}^{\hat{n}}\Big\{(1-a_r)|w_r'(t)| + b_r|w_r(t)| + \sum_{j=1}^{\hat{n}}|c_{rj}||f_j(u_j(t)) + f_j(-[-x_j(t)])| + \sum_{j=1}^{\hat{n}}|d_{rj}|\\
&\times|f_j(u_j(qt)) + f_j(-[-x_j(qt)])| + \sum_{j=1}^{\hat{n}}|f_{rj}||f_j(u_j'(qt)) + f_j(-[-x_j'(qt)])| + 2|\hat{I}_r| - \beta_1\\
&+\frac{b-a}{(t+a)^2} + \beta_2 w_r^2(t) + \beta_3[w_r'(t)]^2\Big\}\\
\le &\sum_{r=1}^{\hat{n}}\Big\{(1-a_r)|w_r'(t)| + b_r|w_r(t)| + \sum_{j=1}^{\hat{n}}|c_{rj}|L_j|w_j(t)| + \sum_{j=1}^{\hat{n}}|d_{rj}||w_j(qt)|L_j + \sum_{j=1}^{\hat{n}}|f_{rj}|\times\\
&|w_j'(qt)|L_j + 2|\hat{I}_r| - \beta_1 + \frac{b-a}{(t+a)^2} + \beta_2 w_r^2(t) + \beta_3[w_r'(t)]^2 - \beta_4 - \beta_5\Big].
\end{aligned} \tag{3.15}$$

On the other hand, we have

$$\begin{aligned}
&M_2'(t)\\
= &\frac{1}{1-\tau}\sum_{r=1}^{\hat{n}}\sum_{j=1}^{\hat{n}} L_j\Big(|d_{rj}||w_j(t)| - (1-\tau'(t))|d_{rj}||w_j(qt)| + |f_{rj}||w_j'(t)| - (1-\tau'(t))\times\\
&|f_{rj}||w_j'(qt)|\Big)\\
\le &\frac{1}{1-\tau}\sum_{r=1}^{\hat{n}}\sum_{j=1}^{\hat{n}} L_j\Big(|d_{rj}||w_j(t)| - (1-\tau)|d_{rj}||w_j(qt)| + |f_{rj}||w_j'(t)| - (1-\tau)|f_{rj}|\times\\
&|w_j'(qt)|\Big).
\end{aligned} \tag{3.16}$$

In view of (3.15) and (3.16), one has

$$M'(t)$$

$$\leq \sum_{r=1}^{\hat{n}} \Big\{(1-a_r)|w_r'(t)| + b_r|w_r(t)| + \sum_{j=1}^{\hat{n}} L_j[|c_{rj}| + \frac{|d_{rj}|}{1-\tau}]|w_j(t)| + \sum_{j=1}^{\hat{n}} \frac{|f_{rj}|}{1-\tau} L_j|w_j'(t)|$$
$$+2|\hat{I}_r| - \beta_1 + \frac{b-a}{(t+a)^2} + \beta_2 w_r^2(t) + \beta_3[w_r'(t)]^2 - \beta_4 - \beta_5\Big\}$$

$$= \sum_{r=1}^{\hat{n}} \Big\{\beta_2|w_r(t)|^2 + \Big[b_r + \sum_{j=1}^{\hat{n}} L_r(|c_{jr}| + \frac{|d_{jr}|}{1-\tau})\Big]|w_r(t)| + (|\hat{I}_r| - \beta_4) + \beta_3|w_r'(t)|^2 + (1-a_r$$
$$+\sum_{j=1}^{\hat{n}} \frac{|f_{rj}|}{1-\tau} L_j)|w_r'(t)| + (|\hat{I}_r| - \beta_5) - \beta_1 + \frac{b-a}{(t+a)^2}\Big\} \tag{3.17}$$

In view of (l_1) and (l_2), according Lemma 2.1, one has

$$\beta_2|w_r(t)|^2 + \Big[b_r + \sum_{j=1}^{\hat{n}} L_r(|c_{jr}| + \frac{|d_{jr}|}{1-\tau})\Big]|w_r(t)| + (|\hat{I}_r| - \beta_4) < 0 \tag{3.18}$$

and

$$\beta_3|w_r'(t)|^2 + (1 - a_r + \sum_{j=1}^{\hat{n}} \frac{|f_{rj}|}{1-\tau} L_j)|w_r'(t)| + (|\hat{I}_r| - \beta_5) < 0. \tag{3.19}$$

Substituting (3.18) and (3.19) into (3.17) yields

$$M'(t) \leq \beta_1 + \frac{b}{(t+a)^2}. \tag{3.20}$$

Integrating (3.20) over $[0, t]$ yields

$$\begin{aligned} M(t) &\leq M(0) + \beta_1 t + b\int_0^t \frac{ds}{(s+a)^2} \\ &= M(0) + \beta_1 t - \frac{b}{t+a} + \frac{b}{a} \\ &\leq \beta_1 t - \frac{b}{t+a}. \end{aligned} \tag{3.21}$$

Letting $\beta_1 t - \frac{b}{t+a} \leq 0$, then $-\beta_1 t^2 - \beta_1 t + b > 0$. Consequently

$$t \geq T^* = \frac{\beta_1 a + \sqrt{\beta_1^2 a^2 + 4\beta_1 b}}{-2\beta_1}.$$

Thus when $t \geq \frac{\beta_1 a + \sqrt{\beta_1^2 a^2 + 4\beta_1 b}}{-2\beta_1}$, we have by (3.21)

$$0 \leq M(t) \leq 0.$$

Namely,

$$\lim_{t\to T^*}|u_r(t)+x_r(t)|=0, |u_r(t)-x_r(t)|=0, t\geq T^*.$$

The proof of Theorem 3.2 is finished.

Remark 1. In [19–21, 23–25], the integral inequality is used to study the finite-time synchronization (anti-synchronization), in [29], the finite-time stability theory is used to study the finite-time synchronization, but in our paper, without applying above study approaches, the quadratic inequality of one variable is used to study the finite-time anti-synchronization for the master inertial neural networks and the slave inertial neural networks. Hence, our approach of finite-time synchronization for master-slave neural networks is different from these in the existing papers.

Remark 2. In our paper, by designing different controllers from those in existing papers [16–17,19–21, 23–25, 28–31], namely, by designing the polynomial and fractional controllers, two novel criteria ensuring the finite-time anti-synchronization for the master system (2.1) and the slave system (2.2) are established. Hence, our results on the finite-time synchronization for the master-slave neural networks are novel.

Remark 3. It is true that the controller (3.1) contain many parameters ($\hat{k}+2$ parameters), but (3.1) is only designed in theory. In practice, when we take $\hat{k}=2$, then the controller (3.1) only contain 5 parameters. By designing these 5 parameters, we can establish the sufficient condition of the finite-time anti-synchronization for system (2.1) and system (2.2) by putting more large ξ_1 and more small γ_r-b_r(see (m_1) and (m_2) in Theorem 3.1).

Remark 4. In many papers which studied the stability and synchronization of inertial neural networks, the results were obtained on the stability and synchronization for discussed inertial neural networks by transforming the discussed inertial neural networks described with a second order differential equations into the new system described with first order differential equations. Thus to show the stability or synchronization, a Lyapunov functional of $w_i(t)$ has to be constructed. In this paper, since without transforming the discussed inertial neural networks described with a second order differential equations into the new system described with first order differential equations, to finish showing the finite-time anti-synchronization, a Lyapunov functional of $w_i(t)$ and $w_i'(t)$ is constructed in each Theorem. By constructing such Lyapunov functionals, without transforming processing and complicated computation, the more concise and easily verified criteria on the finite-time anti-synchronization are acquired.

4. Numerical test

Letting $x_r'(t)=y_r(t), u_r(t)=z_r(t)$, then the delayed inertial neural networks (2.1) and (2.2) reduce to respectively the master system

$$\begin{cases} x_r'(t)=y_r(t) \\ y_r'(t)=-a_ry_r(t)-b_rx_r(t)+\sum\limits_{j=1}^{2}\Big[c_{rj}f_j(x_j(t))+d_{rj}f_j(x_j(t-\tau(t))) \\ \qquad +f_{rj}f_j(y_j(t-\tau(t)))+\hat{I}_r\Big] \end{cases} \tag{4.1}$$

and the slave system

$$\begin{cases} u_r'(t) = z_r(t) \\ z_r'(t) = -a_r z_r(t) - b_r u_r(t) + \sum\limits_{j=1}^{2} \Big[c_{rj} f_j(u_j(t)) + d_{rj} f_j(u_j(t-\tau(t))) \\ \quad + f_{rj} f_j(z_j(t-\tau(t))) + v_r(t) + \hat{I}_r\Big], \end{cases} \tag{4.2}$$

Example 1. Consider the neural networks (4.1) and (4.2) with following controllers:

$$v_r(t) = -[w_r'(t)]^{-1}\Big[\xi_1 w_r^2(t) + \beta_0 + c_0 + 2c_1 t + 3c_2 t^2 + 4c_3 t^3 + \cdots + (\hat{k}+1)c_{\hat{k}} t^{\hat{k}}\Big] \tag{4.3}$$

where $r = 2, \beta_0 = 1.5, \xi_1 = 10, a_1 = 25, a_2 = 30, b_1 = 2.1, b_2 = 0.8,\ \gamma_1 = 2, \gamma_2 = 1, c_0 = 1, c_1 = 0.5, c_2 = 0.3, c_3 = 0.1, c_4 = 0.2, c_5 = 1, c_6 = 0.35, c_7 = 0.4, c_8 = 0.2, c_9 = 0.25, c_{10} = 0.15, \hat{k} = 10, \hat{I}_1 = 7, \hat{I}_2 = 5$, with $c_0 - \hat{I}_r^2 < 0$, $\tau(t) = 0.2t + 1, \tau = 0.4, 0.2 = \tau'(t) < \tau < 1, f_r(x) = 0.2x$,

$$\begin{pmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{pmatrix} = \begin{pmatrix} 2 & 3 \\ 1 & 2 \end{pmatrix}, \begin{pmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 3 & 1 \end{pmatrix}, \begin{pmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{pmatrix} = \begin{pmatrix} 4 & 1 \\ 2 & 3 \end{pmatrix}.$$

Therefore $L_1 = L_2 = 0.2$,

$$10 = \xi_1 > 0.5 \sum_{j=1}^{\hat{n}} (|c_{1j}| + \frac{|d_{1j}|}{1-\tau}) L_1 = 1,$$

$$10 = \xi_1 > 0.5 \sum_{j=1}^{\hat{n}} (|c_{2j}| + \frac{|d_{2j}|}{1-\tau}) L_2 = 0.9667,$$

$$\begin{aligned} 0.01 &= (\gamma_1 - b_1)^2 < 4\Big[1 - a_1 + 0.5 \sum_{j=1}^{\hat{n}} (|c_{1j}| + \frac{|d_{1j}|}{1-\tau}) L_j + \sum_{j=1}^{\hat{n}} \frac{|f_{1j}| L_j}{1-\tau}\Big] \times \\ &\quad \Big[0.5 \sum_{j=1}^{\hat{n}} (|c_{1j}| + \frac{|d_{1j}|}{1-\tau}) L_1 - \xi_1\Big] \\ &= 798. \end{aligned}$$

$$\begin{aligned} 0.04 &= (\gamma_2 - b_2)^2 < 4\Big[1 - a_2 + 0.5 \sum_{j=1}^{\hat{n}} (|c_{2j}| + \frac{|d_{2j}|}{1-\tau}) L_j + \sum_{j=1}^{\hat{n}} \frac{|f_{2j}| L_j}{1-\tau}\Big] \times \\ &\quad \Big[0.5 \sum_{j=1}^{\hat{n}} (|c_{2j}| + \frac{|d_{2j}|}{1-\tau}) L_2 - \xi_1\Big] \\ &= 982.8267. \end{aligned}$$

The all conditions are satisfied. Based on Theorem 3.1, the system (4.1) and system (4.2) are finite-time anti-synchronization. In the existing papers, the controllers designed are not related to the time variable t, hence, the result in the example cannot verified with these theorems in the existing papers [18–29].

Letting $x_1(0) = -1.12, x_2(0) = -2.1, y_1(0) = 50, y_2(0) = 50, u_1(0) = 1, u_2(0) = 2, z_1(0) = -45, z_2(0) = -45,$ then the finite-time anti-synchronization diagrams can be seen in Figures 1–3.

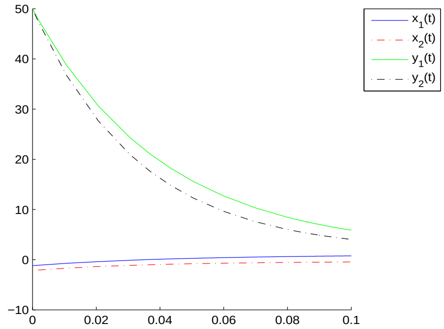

Figure 1. Curves of the $x_r(t)$, $y_r(t)$.

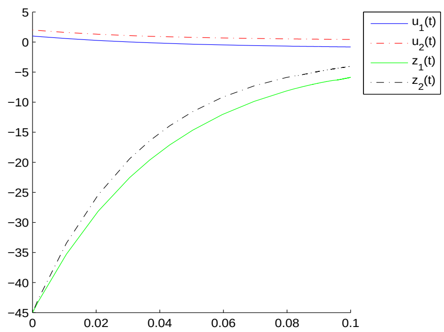

Figure 2. Curves of the $u_r(t)$, $z_r(t)$.

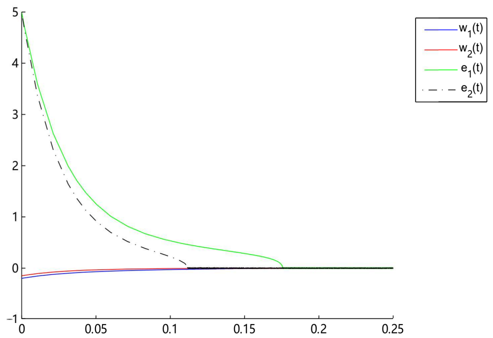

Figure 3. Curves of the $w_r(t)$, $e_r(t)$.

Example 2. Consider the master system (4.1) and the slave system (4.2) with controllers (3.2) as follows :

$$v_r(t) = sign[w_r'(t)]\Big[\frac{b}{(t+a)^2} + \beta_1 + \beta_2 w_r^2(t) + \beta_3 [w_r'(t)]^2 - \beta_4 - \beta_5\Big], \tag{4.4}$$

where $r = 2, \beta_1 = -2, \beta_2 = -10, \beta_3 = -8, \beta_4 = 5, \beta_5 = 7.5, a_1 = 1, a_2 = 1.2, b_1 = 0.2, b_2 = 0.3,$ $a = 1, b = -30, \hat{I}_1 = 1, \hat{I}_2 = 1.5,$ with $5 = \beta_4 > \hat{I}_r, 7.5 = \beta_5 > \hat{I}_r,$ $\tau(t) = 0.3t, \tau = 0.5, 0.3 = \tau'(t) < \tau < 1, f_r(x) = -0.1x.$

$$\begin{pmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{pmatrix} = \begin{pmatrix} 0.3 & 0.1 \\ 0.2 & 0.4 \end{pmatrix}, \begin{pmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{pmatrix} = \begin{pmatrix} 0.2 & 0.3 \\ 0.1 & 0.1 \end{pmatrix}, \begin{pmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{pmatrix} = \begin{pmatrix} 0.4 & 0.2 \\ 0.3 & 0.2 \end{pmatrix}.$$

Therefore $L_1 = L_2 = 0.1$,

$$0.0144 = \Big(1 - a_1 + \sum_{j=1}^{\hat{n}} \frac{|f_{1j}|}{1-\tau} L_j\Big)^2 < 4\beta_3(\hat{I}_1 - \beta_5) = 208$$

$$0.0100 = \Big(1 - a_2 + \sum_{j=1}^{\hat{n}} \frac{|f_{2j}|}{1-\tau} L_j\Big)^2 < 4\beta_3(\hat{I}_2 - \beta_5) = 192$$

$$0.5776 = \Big[b_1 + \sum_{j=1}^{\hat{n}} L_1(|c_{j1}| + \frac{|d_{j1}|}{1-\tau})\Big]^2 < 4\beta_2(|\hat{I}_1| - \beta_4) = 160$$

$$0.7744 = \Big[b_2 + \sum_{j=1}^{\hat{n}} L_2(|c_{j2}| + \frac{|d_{j2}|}{1-\tau})\Big]^2 < 4\beta_2(|\hat{I}_2| - \beta_4) = 140.$$

Let $w_1(s) = u_1(s) + x_1(s) = s - 1 + 2s + 1 = 3s, w_2(s) = u_2(s) + x_2(s) = 2s - 3 + s + 3 = 3s, w'_j(s) = (3s)' = 3, w_j(0) = 0, x_1(0) = 1, x_2(0) = 3, y_1(0) = 1, y_2(0) = 2, u_1(0) = -1, u_2(0) = -3, z_1(0) = 1, z_2(0) = -1.$

Then

$$\begin{aligned} M(0) &= \sum_{r=1}^{\hat{n}} (|w_r(0)| + |w'_r(0)|) + \frac{1}{1-\tau} \sum_{r=1}^{\hat{n}} \sum_{j=1}^{\hat{n}} L_j\Big(|d_{rj}| \int_{-\tau(0)}^{0} |w'_j(s)| ds + |f_{rj}| \int_{-\tau(0)}^{0} |w'_j(s)| ds\Big) \\ &= 6, \end{aligned}$$

so $-30 = b < -aM(0) = -6$. By Theorem 3.2, the system (4.1) and system (4.2) are finite-time anti-synchronization. Since the controllers of time variable t are designed, while in the existing papers [17, 23–27], the controllers related to the time variable t were not designed, thus the results in the example cannot be verified with the theorems in [19, 25–29].

The finite-time anti-synchronization diagrams can be seen in Figures 4–6.

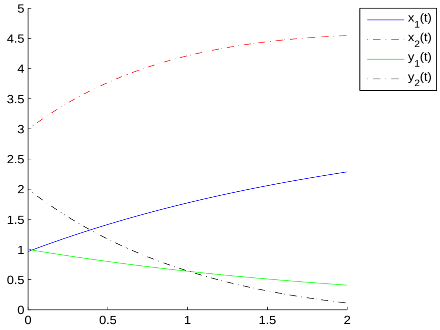

Figure 4. Curves of the $x_r(t)$, $y_r(t)$.

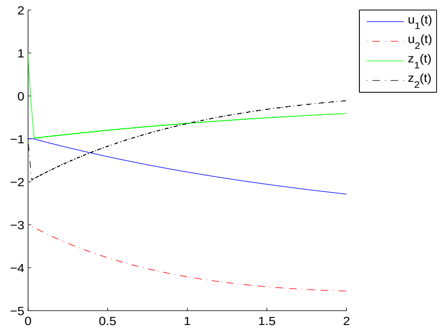

Figure 5. Curves of the $u_r(t)$, $z_r(t)$.

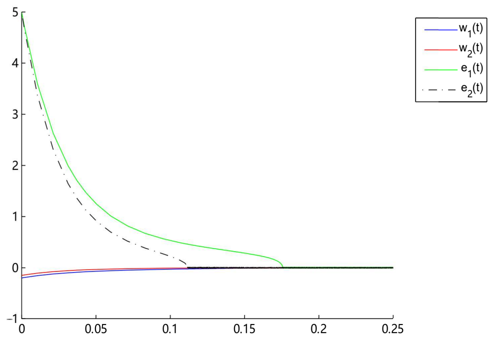

Figure 6. Curves of the $w_r(t)$, $e_r(t)$.

5. Conclusions

This paper discusses the finite-time anti-synchronization for the master-slave delayed inertial neural networks. Without making the variable transformation, the inertial system was analyzed directly. By making use of the quadratic inequality of one variable under the fractional and polynomial controllers, two novel sufficient conditions are obtained to ensure the finite-time anti-synchronization between the master system and the slave system. Applying the quadratic inequality of one variable and the fractional and polynomial controllers of the time variable t, our results obtained are more concise and easily verified and more objective and practical than these in the existing papers [16–31]. Our future works are using the more quadratic inequality to discuss the finite-time anti-synchronization for the master-slave delayed inertial neural networks, there are many problems in this field that deserve further study.

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Conflict of interest

The authors declare that there is no conflict of interests regarding the publication of this paper.

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