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Research article

Some inequalities on Bazilevič class of functions involving quasi-subordination

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Abstract: Quasi-subordination which is an extension of the majorization and subordination principle, has been used to define a subclass of Bazilevič functions of complex order. Various classes of analytic functions that map unit disc onto conic domains and some classes of special functions are studied in dual. Inequalities for the initial Taylor-Maclaurin coefficients and unified solution of Fekete-Szegö problem for subclasses of analytic functions related to various conic regions are our main results. Our main results have many applications which are presented in the form of corollaries.

Keywords: analytic functions; Bazilevič functions; starlike functions; subordination; Fekete-Szegö problem; coefficient inequalities; q-calculus
Mathematics Subject Classification: 30C45

1. Introduction

Bazilevič [3] introduced the class $\mathcal{B}(\alpha, \zeta, g)$ of functions which is defined by the integral

$$f(z) = \left\{ \frac{\alpha}{1+\zeta^2} \int_0^z \left[p(\eta) - i\zeta\right] \eta^{-\left(1+\frac{i\alpha\zeta}{1+\zeta^2}\right)} [\eta]^{\frac{\alpha}{1+\zeta^2}} \left[g(\eta)\right]^{\left(\frac{\alpha}{1+\zeta^2}\right)} d\eta \right\}^{\frac{1+i\zeta}{\alpha}},$$

where $p \in \mathcal{P}$, the class of analytic function with positive real part and $g \in \mathcal{S}^*$, the well-known class of starlike function. The numbers $\alpha > 0$ and ζ are real and all powers are chosen so that it remains single-valued. Apart from the fact that $\mathcal{B}(\alpha, \zeta, g)$ is univalent, we have little or no information on these family of functions. But if we simplify, for example, letting $\zeta = 0$ and $g(z) = z$ we get the well-known class $\mathcal{B}(\alpha)$ which is given by

$$\Re\left(\left(\frac{z}{f(z)}\right)^{1-\alpha} f'(z)\right) > 0,$$

where $f \in \mathcal{A}$ is the class of functions having a Taylor series expansion of the form

$$f(z) = z + \sum_{k=2}^{\infty} a_k z^k \quad (z \in \mathbb{U} = \{z : |z| < 1\}). \tag{1.1}$$

Let $0 \leq \eta < 1$, $\mathcal{S}^*(\eta)$ and $C(\eta)$ symbolize the classes of starlike functions of order η and convex functions of order η, respectively. Let $\mathcal{S}^*(\eta, \vartheta)$ (see [9]) denote the class of functions $f \in \mathcal{A}$ satisfying the inequality

$$\eta < \Re\left(\frac{zf'(z)}{f(z)}\right) < \vartheta, \quad (0 \leq \eta < 1 < \vartheta;\ z \in \mathbb{U}). \tag{1.2}$$

Robertson [18] introduced quasi-subordination unifying the concept of subordination and majorization. For analytic functions f and g in $\mathbb{U}$, f is quasi-subordinate to g in $\mathbb{U}$, denoted by $f \prec_q g$, if there exist a Schwarz function w and an analytic function ϕ satisfying $|\phi(z)| < 1$ and $f(z) = \phi(z)g(w(z))$ in $\mathbb{U}$. If $\phi(z) = 1$, quasi-subordination reduces to subordination. If we let $w(z) = z$, then quasi-subordination reduces to the concept of majorization.

For $f \in \mathcal{A}$ given by (1.1) and $0 < q < 1$, the Jackson's q-derivative operator or q-difference operator is defined by (see [1, 2])

$$\mathfrak{D}_q f(z) = \begin{cases} f'(0), & \text{if } z = 0, \\ \frac{f(z)-f(qz)}{(1-q)z}, & \text{if } z \neq 0. \end{cases} \tag{1.3}$$

From (1.3), we can easily see that $\mathfrak{D}_q f(z) = 1 + \sum_{k=2}^{\infty} [k]_q a_k z^{k-1}$ $(z \neq 0)$, where the q-integer number $[k]_q$ is defined by

$$[k]_q = \frac{1 - q^k}{1 - q} \tag{1.4}$$

and note that $\lim_{q \to 1^-} \mathfrak{D}_q f(z) = f'(z)$. Notations and symbols play an very important role in the study of q-calculus. Throughout this paper, we let $([k]_q)_n = [k]_q[k+1]_q[k+2]_q \cdots [k+n-1]_q$. Let q-analogue incomplete beta function $\chi(z)$ (see [19]) is defined by

$$\chi(z) = z + \sum_{k=2}^{\infty} \frac{\left([b]_q\right)_{k-1}}{\left([c]_q\right)_{k-1}} z^k, \tag{1.5}$$

where $(b \in \mathbb{C},\ c \in \mathbb{C} \setminus \mathbb{Z}_0^- = \{\cdots, -2, -1, 0, 1, \cdots\})$.

Lately, the study of the $q-$ *calculus* has riveted the rigorous consecration of researchers. The great attention is because of its gains in many areas of mathematics and physics. The significance of the $q-$ *derivative* operator $\mathfrak{D}_q$ is quite evident by its applications in the study of several subclasses of analytic functions. Initially, in the year 1990, Ismail *et al.* [5] gave the idea of $q-$ *starlike* functions. Nevertheless, a firm base of the usage of the $q-$ *calculus* in the context of Geometric Function Theory was efficiently established, and the use of the generalized basic (or $q-$) hypergeometric functions in Geometric Function Theory was made by Srivastava (see, for details, [23]).The study of geometric function theory in dual with quantum calculus was initiated by Srivastava ([24], also see [25]). After that, extraordinary studies have been done by many mathematicians, which offer a significant part in the encroachment of Geometric Function Theory. In particular, Srivastava *et al*. [26–32] also considered some function classes of $q-$ *starlike* functions related with conic region and focussed upon the classes of $q-$ *starlike* functions related with the Janowski functions from several different aspects. Inspired by aforementioned works on $q-$ *calculus* we now define the q-analogue of the function which maps $\mathbb{U}$ onto a conic region. Let

$$\psi(z)=\frac{(1+q)z}{2+(1-q)z}+\sqrt[3]{1+\left(\frac{(1+q)z}{2+(1-q)z}\right)^{3}}. \tag{1.6}$$

The function ψ defined by (1.6) is the q-analogue of $h(z)=z+\sqrt[3]{1+z^3}$ which maps the unit disc onto a leaf-like shaped region which is analytic and univalent. For details of functions mapping unit disc onto a leaf-like domain, refer to [20].

For functions $f\in\mathcal{A}$ given by (1.1) and $h\in\mathcal{A}$ of the form

$$h(z)=z+\sum_{k=2}^{\infty}\Gamma_k z^k, \tag{1.7}$$

the Hadamard product (or convolution) is defined by

$$H(z)=(f*h)(z)=z+\sum_{k=2}^{\infty}a_k\Gamma_k z^k. \tag{1.8}$$

We now introduce the following class of functions.

Definition 1.1. For $-\frac{\pi}{2}<\xi<\frac{\pi}{2}$, $0\leq\lambda\leq 1$, $t\geq 0$, $\gamma\in\mathbb{C}\setminus\{0\}$ and $H=f*h$ defined as in (1.8), let $\mathcal{B}_{\lambda}^{t}(\gamma;\psi)$ be the class of functions defined by

$$\frac{1}{\gamma}\left[(1+i\tan\xi)\frac{z^{1-t}[\mathfrak{D}_q(H(z))]}{[(1-\lambda)H(z)+\lambda z]^{1-t}}-i\tan\xi-1\right]\prec_q\psi(z)-1, \tag{1.9}$$

where $\psi\in\mathcal{P}$ and has a series expansion of the form

$$\psi(z)=1+A_1z+A_2z^2+A_3z^3+\cdots\qquad(A_1\neq 0;\ z\in\mathbb{U}). \tag{1.10}$$

Remark 1.1. Several well-known classes can be seen as special case of $\mathcal{B}_{\lambda}^{t}(\gamma;\psi)$ (see [7, 15, 17, 22]). Here we highlight only the recent works which are associated with a conic region.

1. If we let $\lambda=t=0$ and $h(z)=E_rh(z)=\frac{\sqrt{\pi z}}{2}erh(\sqrt{z})=z+\sum_{k=2}^{\infty}\frac{(-1)^{k-1}}{(2k-1)(k-1)!}z^k$, in the Definition 1.1, where the function $erh(\sqrt{z})$ is defined by

$$erh(z)=\frac{2}{\sqrt{\pi}}\int_0^z\exp(-t^2)\,dt, \tag{1.11}$$

then the class $\mathcal{B}_{\lambda}^{t}(\gamma;\psi)$ reduces to class $\widetilde{\mathcal{ES}}_{q,\gamma}^{\xi}(\psi)$ introduced by Ramachandran et al. [16].

2. It can be easily seen that, with the choice of $h = \chi$ as in (1.5) we have

$$\left[\lim_{q\to 1^-} \mathcal{B}_\lambda^0(1;\,\psi)\right]_{\alpha=0,\,\psi=z+\sqrt{1+z^2}} = \mathcal{ML}_b^c\,(t;\,\psi)\,,$$

where $\mathcal{ML}_b^c\,(t;\,\psi)$ is the class recently introduced and studied by Murugusundaramoorthy and Bulboacă [12].

3. If $h(z) = z + \sum_{k=2}^{\infty} z^k$, $\phi(z) = 1$, $\alpha = t = 0$, $\lambda = \gamma = 1$ and ψ is of the form (1.6), then $\mathcal{B}_\lambda^t(\gamma;\,\psi)$ reduces to

$$\mathcal{R}(\psi) = \left\{ f \in \mathcal{S} :\ \mathfrak{D}_q f(z) \prec \frac{(1+q)z}{2+(1-q)z} + \sqrt[3]{1+\left(\frac{(1+q)z}{2+(1-q)z}\right)^3} \right\},$$

where $\mathcal{S}$ is the class of all univalent functions in $\mathcal{A}$. The class $\mathcal{R}(\psi)$ was recently introduced by Khan et al. [8]. Further, we note that $\lim_{q\to 1^-} \mathcal{R}(\psi) = \mathcal{R}(h)$, where $h(z) = z + \sqrt[3]{1+z^3}$, the class of functions recently studied by Priya and Sharma [13].

4. If $h(z) = z + \sum_{k=2}^{\infty} \frac{([2]_q)_{k-1}}{([1]_q)_{k-1}} z^k$, $\phi(z) = 1$, $\alpha = \lambda = t = 0$, $\gamma = 1 + 0i$, $q \to 1^-$ and for a choice of ψ, we have

$$\frac{(zf'(z))'}{f'(z)} \prec \frac{(A+1)\kappa(z)-(A-1)}{(B+1)\kappa(z)-(B-1)} \qquad \left(\kappa(z) = 1 + \frac{2}{\pi}\log\left[\frac{1+\sqrt{z}}{1-\sqrt{z}}\right]^2\right),$$

where $-1 \leq B < A \leq 1$. The class $UP[A,\,B]$ of all functions satisfying the above subordination condition was introduced and studied by Malik *et al.* [10].

2. Prelimnaries

In this section, we state the results that would be used to establish our main results which can be found in the standard text on univalent function theory.

Lemma 2.1. *[4] If the function $f \in \mathcal{A}$ given by (1.1) and $g(w)$ given by*

$$g(w) = w + \sum_{k=2}^{\infty} b_k w^k \tag{2.1}$$

are inverse functions, then for $k \geq 2$

$$b_k = \frac{(-1)^{k+1}}{k!} \begin{vmatrix} ka_2 & 1 & 0 & \cdots & 0 \\ 2ka_3 & (k+1)a_2 & 2 & \cdots & 0 \\ 3ka_4 & (2k+1)a_3 & (k+2)a_2 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & (k-2) \\ (k-1)ka_k & [k(k-2)+1]\,a_{k-1} & [k(k-3)+2]\,a_{k-2} & \cdots & (2k-2)a_2 \end{vmatrix}. \tag{2.2}$$

Remark 2.1. The elements of the determinant in (2.2) are given by

$$\Theta_{ij} = \begin{cases} [(i-j+1)k+j-1]\,a_{i-j+2}, & \text{if } i+1 \geq j \\ 0, & \text{if } i+1 < j. \end{cases}$$

Lemma 2.2. *[14] If $p(z) = 1 + \sum_{k=1}^{\infty} p_k z^k \in \mathcal{P}$, then $|p_k| \leq 2$ for all $k \geq 1$, and the inequality is sharp for $p(z) = p_1(z) = \frac{1+z}{1-z}$.*

Lemma 2.3. *[11] Let $p \in \mathcal{P}$ and also let v be a complex number. Then*

$$|p_2 - vp_1^2| \leq 2 \ \max\{1, |2v - 1|\}. \tag{2.3}$$

The result is sharp for functions given by

$$p(z) = p_2(z) = \frac{1+z^2}{1-z^2}, \qquad p(z) = p_1(z) = \frac{1+z}{1-z}.$$

3. Coefficients estimates for functions in $\mathcal{B}_\lambda^t(\gamma;\, \psi)$

Hereafter, unless otherwise mentioned we assume that

$$-\frac{\pi}{2} < \xi < \frac{\pi}{2}, \ \ 0 \leq \lambda \leq 1, \ \ t \geq 0 \ \text{ and } \ q \in (0,\ 1).$$

Also let $g = f^{-1}$ defined by $f^{-1}(f(z)) = z = f(f^{-1}(z))$ be inverse of f and

$$g(w) = f^{-1}(w) = w + \sum_{k=2}^{\infty} b_k w^k \quad (|w| < r_0;\ r_0 > \frac{1}{4}). \tag{3.1}$$

The class of all functions in $\mathcal{B}_\lambda^t(\gamma;\, \psi)$ is not univalent, so the inverse is not guaranteed. However, there exist an inverse function in some small disk with center at $w = 0$ depending on the parameters involved.

Let $\phi(z) = d_0 + d_1 z + d_2 z^2 + \cdots (d_0 \neq 0)$ and $|d_0| \leq 1$. Further, in general, Γ_k's are the respective coefficients of z^k in the power series expansion of h given by (1.7).

Theorem 3.1. *If the function $f(z)$ given by (1.1) and $g(w)$ given by (2.1) are inverse functions and if $f \in \mathcal{B}_\lambda^t(\gamma;\, \psi)$ with $\psi(z) = 1 + A_1 z + A_2 z^2 + A_3 z^3 + \cdots$, $(A_1 \neq 0;\ z \in \mathbb{U})$, then the estimates of the inverse coefficients of f are*

$$|b_2| \leq \frac{|\gamma||A_1|}{|\Gamma_2| \sec\xi \left[(t+q) + \lambda(1-t)\right]} \tag{3.2}$$

and

$$|b_3| \leq \frac{|A_1||\gamma|}{\sec\xi \left[q(1+q) + t(1-\lambda) + \lambda\right] |\Gamma_3|} \left[\left|\frac{d_1}{d_0}\right| + \max\{1;\ |2v - 1|\}\right]. \tag{3.3}$$

with

$$v = \frac{1}{2}\left(1 - \frac{A_2}{A_1} + A_1 \frac{\gamma d_0 (1-\lambda)(t-1)[t + 2q + \lambda(2-t)]}{2(1 + i\tan\xi)[t + q + \lambda(1-t)]^2} + \frac{2A_1 d_0 \gamma \left[q(1+q) + t(1-\lambda) + \lambda\right] \Gamma_3}{\Gamma_2^2 (1 + i\tan\xi)[t + q + \lambda(1-t)]^2}\right). \tag{3.4}$$

Proof. Let $f \in \mathcal{B}_\lambda^t(\gamma;\, \psi)$. Then by the definition of quasi-subordination, there is a function $w(z)$ such that

$$\frac{1}{\gamma}\left[(1 + i\tan\xi)\frac{z^{1-t}[\mathfrak{D}_q(H(z))]}{[(1-\lambda)H(z) + \lambda z]^{1-t}} - i\tan\xi - 1\right] = \phi(z)\left[\psi(w(z)) - 1\right].$$

Define the function p by

$$p(z) = 1 + p_1 z + p_2 z^2 + \cdots = \frac{1 + w(z)}{1 - w(z)} \prec \frac{1+z}{1-z} \quad (z \in \mathbb{U}). \tag{3.5}$$

We can note that $p(0) = 1$ and $p \in \mathcal{P}$ (see Lemma 2.2). Using (3.5), it is easy to see that

$$w(z) = \frac{p(z) - 1}{p(z) + 1} = \frac{1}{2}\left[p_1 z + \left(p_2 - \frac{p_1^2}{2}\right)z^2 + \left(p_3 - p_1 p_2 + \frac{p_1^3}{4}\right)z^3 + \cdots\right].$$

So we have

$$\begin{aligned} &\frac{1}{\gamma}\left[(1 + i\tan\xi)\frac{z^{1-t}[\mathfrak{D}_q(H(z))]}{[(1-\lambda)H(z) + \lambda z]^{1-t}} - i\tan\xi - 1\right] \\ &= 1 + \frac{1}{2}A_1 d_0 p_1 z + \left[d_0\left(\frac{1}{2}A_1\left(p_2 - \frac{p_1^2}{2}\right) + \frac{1}{4}A_2 p_1^2\right) + \frac{d_1 A_1 p_1}{2}\right]z^2 + \cdots . \end{aligned} \tag{3.6}$$

The left hand side of (3.6) will be

$$\begin{aligned} \frac{z^{1-t}[\mathfrak{D}_q(H(z))]}{[(1-\lambda)H(z) + \lambda z]^{1-t}} &= 1 + [(t+q) + \lambda(1-t)]\,\Gamma_2 a_2 z + \{[q(1+q) + t(1-\lambda) + \lambda]\,\Gamma_3 a_3 \\ &+ \left[\frac{(1-\lambda)(t-1)}{2}(t + 2q + \lambda(2-t))\Gamma_2^2 a_2^2\right]\Big\} z^2 \\ &+ \Big\{\left[q^3 + q(1+q) + t(1-\lambda) + \lambda\right]\Gamma_4 a_4 \\ &+ (1-\lambda)(t-1)\,[q(q+2) + \lambda(t+2) + t]\,\Gamma_2\Gamma_3 a_2 a_3 \\ &+ \frac{(t-1)(t-2)[(t+3q) + \lambda(3-t)](1-\lambda)^2}{6}\Gamma_2^3 a_2^3\Big\} z^3 + \cdots . \end{aligned} \tag{3.7}$$

where Γ_k's are the corresponding coefficients from the power series expansion of h, which may be real or complex.

By using (3.6) and (3.7), we have

$$a_2 = \frac{\gamma A_1 d_0 p_1}{2\Gamma_2(1 + i\tan\xi)\,[(t+q) + \lambda(1-t)]}, \tag{3.8}$$

$$\begin{aligned} a_3 = &\frac{A_1 d_0 \gamma}{2(1 + i\tan\xi)\,[q(1+q) + t(1-\lambda) + \lambda]\,\Gamma_3}\left[p_2 - \frac{1}{2}\left(1 - \frac{A_2}{A_1}\right.\right. \\ &\left.\left. + A_1\frac{\gamma d_0(1-\lambda)(t-1)[t + 2q + \lambda(2-t)]}{2(1 + i\tan\xi)[t + q + \lambda(1-t)]^2}\right)p_1^2 + \frac{d_1 p_1}{d_0}\right]. \end{aligned} \tag{3.9}$$

From (2.2), we see that $b_2 = -a_2$. Hence, applying Lemma 2.3 in (3.8), we have (3.2).

Also from (2.2), we have

$$\begin{aligned} b_3 = \frac{(-1)^4}{3!}\begin{vmatrix} 3a_2 & 1 \\ 6a_3 & 4a_2 \end{vmatrix} = 2a_2^2 - a_3 = &\frac{\gamma^2 A_1^2 d_0^2 p_1^2}{2\Gamma_2^2(1 + i\tan\xi)^2\,[(t+q) + \lambda(1-t)]^2} \\ &- \frac{A_1 d_0 \gamma}{2(1 + i\tan\xi)\,[q(1+q) + t(1-\lambda) + \lambda]\,\Gamma_3} \\ \left[p_2 - \frac{1}{2}\left(1 - \frac{A_2}{A_1} + A_1\frac{\gamma d_0(1-\lambda)(t-1)[t + 2q + \lambda(2-t)]}{2(1 + i\tan\xi)[t + q + \lambda(1-t)]^2}\right)p_1^2 + \frac{d_1 p_1}{d_0}\right] \end{aligned}$$

$$= \frac{-A_1 d_0 \gamma}{2(1+i\tan\xi)\left[q(1+q)+t(1-\lambda)+\lambda\right]\Gamma_3}\left[p_2 - \frac{1}{2}\left(1-\frac{A_2}{A_1}\right.\right.$$
$$\left.\left.+A_1\frac{\gamma d_0(1-\lambda)(t-1)[t+2q+\lambda(2-t)]}{2(1+i\tan\xi)[t+q+\lambda(1-t)]^2}+\frac{2A_1 d_0\gamma\left[q(1+q)+t(1-\lambda)+\lambda\right]\Gamma_3}{\Gamma_2^2(1+i\tan\xi)[t+q+\lambda(1-t)]^2}\right)p_1^2+\frac{d_1 p_1}{d_0}\right].$$

This completes the proof of the Theorem 3.1. □

Theorem 3.2. *If the function f given by (1.1) and g given by (2.1) are inverse functions and if $f \in \mathcal{A}$ satisfies the inequality*

$$\eta < \Re\left(1+\frac{1}{\gamma}\left[(1+i\tan\xi)\frac{z^{1-t}[\mathfrak{D}_q\{(f*h)(z)\}]}{[(1-\lambda)\{(f*h)(z)\}+\lambda z]^{1-t}} - i\tan\xi - 1\right]\right) < \vartheta, \qquad (0 \le \eta < 1 < \vartheta), \tag{3.10}$$

then the estimates of the inverse coefficients of f satisfying the inequality (3.10) are

$$|b_2| \le \frac{2|\gamma|(\vartheta-\eta)\sin\left(\frac{\pi(1-\eta)}{\vartheta-\eta}\right)}{\pi|\Gamma_2|\sec\xi\left[(t+q)+\lambda(1-t)\right]}$$

and

$$|b_3| \le \frac{2|\gamma|(\vartheta-\eta)\sin\left(\frac{\pi(1-\eta)}{\vartheta-\eta}\right)}{\pi\sec\xi\left[q(1+q)+t(1-\lambda)+\lambda\right]|\Gamma_3|}\max\left\{1;\ \frac{2(\vartheta-\eta)}{\pi}\cos\left(\frac{\pi(1-\eta)}{\vartheta-\eta}\right)\left|1+\frac{\vartheta-\eta}{n\pi}i\right.\right.$$
$$\left.\left.\tan\left(\frac{\pi(1-\eta)}{\vartheta-\eta}\right)\left(\frac{\gamma d_0(1-\lambda)(t-1)[t+2q+\lambda(2-t)]}{2(1+i\tan\xi)[t+q+\lambda(1-t)]^2}+\frac{2d_0\gamma\left[q(1+q)+t(1-\lambda)+\lambda\right]\Gamma_3}{\Gamma_2^2(1+i\tan\xi)[t+q+\lambda(1-t)]^2}\right)\right|\right\}.$$

Proof. From the equivalent subordination condition proved by Kuroki and Owa in [9], we may rewrite the conditions (3.10) in the form

$$\begin{aligned}1+\frac{1}{\gamma}\left[(1+i\tan\xi)\frac{z^{1-t}[\mathfrak{D}_q\{(f*h)(z)\}]}{[(1-\lambda)\{(f*h)(z)\}+\lambda z]^{1-t}} - i\tan\xi - 1\right] \prec \\ 1+\frac{\vartheta-\eta}{\pi}i\log\left(\frac{1-e^{2\pi i((1-\eta)/(\vartheta-\eta))}z}{1-z}\right).\end{aligned} \tag{3.11}$$

Further, we note that

$$\Psi(z) = 1+\frac{\vartheta-\eta}{\pi}i\log\left(\frac{1-e^{2\pi i((1-\eta)/(\vartheta-\eta))}z}{1-z}\right) \tag{3.12}$$

maps $\mathbb{U}$ onto a convex domain conformally and is of the form

$$\Psi(z) = 1+\sum_{n=1}^{\infty}A_n z^n$$

where

$$A_n = \frac{\vartheta-\eta}{n\pi}i\left(1-e^{2n\pi i((1-\eta)/(\vartheta-\eta))}\right).$$

Substituting the values of A_1, A_2, $d_0 = 1$ and $d_1 = 0$ in Theorem 3.1, we have the assertion of the theorem. □

If we let $h(z) = z + \sum_{n=2}^{\infty} z^n$, $t = \lambda = 0$ and $q \to 1^-$ in Theorem 3.2, we get the following result obtained by Sim and Kwon [21].

Corollary 3.3. *[21] If the function $f(z)$ given by (1.1) and $g(w)$ given by (2.1) are inverse functions and if $f \in \mathcal{S}^*(\eta, \vartheta)$, then*

$$|b_2| \leq \frac{2(\vartheta - \eta)}{\pi} \sin\left(\frac{\pi(1-\eta)}{\vartheta - \eta}\right)$$

and

$$|b_3| \leq \frac{2(\vartheta - \eta)}{\pi} \sin\left(\frac{\pi(1-\eta)}{\vartheta - \eta}\right) \max\left\{1; \left|\frac{1}{2} - 3\frac{\vartheta - \eta}{\pi} i + \left(\frac{1}{2} + 3\frac{\vartheta - \eta}{\pi} i\right) e^{2\pi i \frac{1-\eta}{\vartheta - \eta}}\right|\right\}.$$

Theorem 3.4. *Let the function*

$$\mathcal{F}(z) = \frac{\sqrt{\pi z}}{2} erh(\sqrt{z}) * f(z) = z + \sum_{k=2}^{\infty} \frac{(-1)^{k-1}}{(2k-1)(k-1)!} a_k z^k$$

satisfy

$$\frac{1}{\gamma}\left[(1 + i\tan\xi)\frac{z^{1-t}[\mathfrak{D}_q(\mathcal{F}(z))]}{[(1-\lambda)\mathcal{F}(z) + \lambda z]^{1-t}} - i\tan\xi - 1\right] \prec_q \psi(z) - 1,$$

then

$$|a_2| \leq \frac{3|\gamma||A_1|}{\sec\xi\,[(t+q) + \lambda(1-t)]}$$

and

$$|a_3| \leq \frac{10|A_1||\gamma|}{\sec\xi\,[q(1+q) + t(1-\lambda) + \lambda]}\left[\left|\frac{d_1}{d_0}\right| + \max\{1; |2\varrho - 1|\}\right],$$

where

$$\varrho = \frac{1}{2}\left(1 - \frac{A_2}{A_1} + A_1 \frac{\gamma d_0(1-\lambda)(t-1)[t + 2q + \lambda(2-t)]}{2(1 + i\tan\xi)[t + q + \lambda(1-t)]^2}\right).$$

Proof. Fixing $\Gamma_k = \frac{(-1)^{k-1}}{(2k-1)(k-1)!}$ in (3.8) and (3.9), we can prove the assertion of the theorem by applying Lemma 2.3. □

If we let $t = 0$ and $\lambda = 0$ in Theorem 3.4, we have the following result obtained by Ramachandran et al. [16].

Corollary 3.5. *[16] If the function f of the form (1.1) belongs to $\widetilde{\mathcal{ES}}^{\xi}_{q,\gamma}(\psi)$, then*

$$|a_2| \leq \frac{3|\gamma||A_1|}{\sec\xi\left(1 - [2]_q\right)}$$

and

$$|a_3| \leq \frac{10|\gamma|}{\sec\xi\left([3]_q - 1\right)}\left[A_1 + \max\left\{A_1; \left|\frac{\gamma A_1^2}{(1 + i\tan\xi)(1 - [2]_q)}\right| + |A_2|\right\}\right].$$

Remark 3.1. Some subordination results for the well-known Janowski class with the function κ defined by

$$\kappa(z) = 1 + \frac{2}{\pi^2}\left(\log\frac{1+\sqrt{z}}{1-\sqrt{z}}\right)^2 \qquad (z \in \mathbb{U}) \tag{3.13}$$

was recently studied by Malik *et al*. [10].

Theorem 3.6. *Suppose that* $f \in \mathcal{B}_\lambda^t(\gamma;\psi)$ *with* $\psi(z)$ *of the form*

$$\psi(z) = \frac{(A+1)\kappa(z) + (A-1)}{(B+1)\kappa(z) + (B-1)},$$

where $-1 \leq B < A \leq 1$ *and* $\kappa(z)$ *is defined as in(3.13), then the estimates of the inverse coefficients of* f *are*

$$|b_2| \leq \frac{4|\gamma|(A-B)}{\pi^2|\Gamma_2|\sec\xi\,[(t+q)+\lambda(1-t)]}$$

and

$$|b_3| \leq \frac{4(A-B)|\gamma|}{\pi^2\sec\xi\,[q(1+q)+t(1-\lambda)+\lambda]\,|\Gamma_3|}\max\left\{1;\ \left|\left(\frac{4(B+1)}{\pi^2}-\frac{2}{3}\right)+\left(\frac{4(A-B)}{\pi^2}\right)\right.\right.$$
$$\left.\left.\left(\frac{\gamma d_0(1-\lambda)(t-1)[t+2q+\lambda(2-t)]}{2(1+i\tan\xi)[t+q+\lambda(1-t)]^2}+\frac{2d_0\gamma\,[q(1+q)+t(1-\lambda)+\lambda]\,\Gamma_3}{\Gamma_2^2(1+i\tan\xi)[t+q+\lambda(1-t)]^2}\right)\right|\right\}.$$

Proof. Following the steps as in Theorem 1 of [8], we get

$$\psi(z) = 1 + \frac{4(A-B)}{\pi^2}z + \frac{8(A-B)}{3\pi^2}\left[1-\frac{6(B+1)}{\pi^2}\right]z^2 + \cdots. \tag{3.14}$$

Now replacing A_1, A_2 and A_3 in Theorem 3.1 with the corresponding coefficients of the series given in (3.14), we have the assertion of the theorem. □

If we let $h(z) = z + \sum_{k=2}^{\infty}\frac{([2]_q)_{k-1}}{([1]_q)_{k-1}}z^k$, $\phi(z) = 1$, $\lambda = t = 0$ and $q \to 1^-$ in Theorem 3.1 we have the following result.

Corollary 3.7. *[10] Suppose that* $f \in UP[A,B]\,(-1 \leq B < A \leq 1)$*, then*

$$|b_2| \leq \frac{2(A-B)}{\pi^2}$$

and

$$|b_3| \leq \frac{4(A-B)}{6\pi^2}.$$

For a choice of the parameter $h(z) = z + \sum_{k=2}^{\infty} z^k$, $\phi(z) = 1$, $\lambda = 1$, $\xi = t = 0$, $\gamma = 1 + 0i$ and for a appropriate choice of ψ in the Theorem 3.1, we get the following result.

Corollary 3.8. *[8] Suppose that* $f \in \mathcal{A}$ *satisfies the condition*

$$\mathfrak{D}_q f(z) \prec \psi(z) = \frac{(1+q)z}{2+(1-q)z} + \sqrt[3]{1+\left(\frac{(1+q)z}{2+(1-q)z}\right)^3}.$$

Then

$$|a_2| \leq \frac{1}{2}$$

and

$$|a_3| \leq \frac{1+q}{2(1+q+q^2)}.$$

Remark 3.2. If we let $q \to 1^-$ in Corollary 3.8, we get the corresponding result of Priya and Sharma [13].

4. Fekete-Szegö problem for functions in $\mathcal{B}_\lambda^t(\gamma;\, \psi)$

The Fekete-Szegö problem which is related to the Bieberbach conjecture represents various geometric quantities. The motivation to provide a unified approach to the Fekete-Szegö problem and initial coefficients was from the study due to Kanas [6]. Note that Theorem 4.1 is a generalization of result obtained in [6].

Theorem 4.1. *Suppose $f(z) = z + a_2z^2 + a_3z^3 + \cdots \in \mathcal{B}_\lambda^t(\gamma;\, \psi)$ $(z \in \mathbb{U})$. Then, for any $\mu \in \mathbb{C}$*

$$\left|a_3 - \mu a_2^2\right| \leq \frac{|A_1||\gamma|}{\sec\xi\,[q(1+q) + t(1-\lambda) + \lambda]\,|\Gamma_3|}\left[\left|\frac{d_1}{d_0}\right| + \max\{1;\, |2\rho - 1|\}\right], \tag{4.1}$$

where ρ is given by

$$\rho = \frac{1}{2}\left[1 - \frac{A_2}{A_1} + A_1\left(\frac{\gamma d_0(1-\lambda)(t-1)[t+2q+\lambda(2-t)]}{2(1+i\tan\xi)[t+q+\lambda(1-t)]^2} - \frac{\mu\gamma d_0\,[q(1+q)+t(1-\lambda)+\lambda]\,\Gamma_3}{\Gamma_2^2(1+i\tan\xi)\,[(t+q)+\lambda(1-t)]^2}\right)\right]. \tag{4.2}$$

The inequalities are sharp for each μ.

Proof. Let $f \in \mathcal{B}_\lambda^t(\gamma;\, \psi)$, then in view of the Eqs (3.8) and (3.9), for $\mu \in \mathbb{C}$ we have

$$\begin{aligned}
\left|a_3 - \mu a_2^2\right| &= \left|\frac{A_1 d_0 \gamma}{2(1+i\tan\xi)\,[q(1+q)+t(1-\lambda)+\lambda]\,\Gamma_3}\left[p_2 - \frac{1}{2}\left(1 - \frac{A_2}{A_1}\right.\right.\right. \\
&\quad \left.\left. + A_1\frac{\gamma d_0(1-\lambda)(t-1)[t+2q+\lambda(2-t)]}{2(1+i\tan\xi)[t+q+\lambda(1-t)]^2}\right)p_1^2 + \frac{d_1p_1}{d_0}\right] - \left.\frac{\mu\gamma^2A_1^2d_0^2p_1^2}{4\Gamma_2^2(1+i\tan\xi)^2\,[(t+q)+\lambda(1-t)]^2}\right| \\
&= \left|\frac{A_1 d_0 \gamma}{2(1+i\tan\xi)\,[q(1+q)+t(1-\lambda)+\lambda]\,\Gamma_3}\left[p_2 - \frac{p_1^2}{2} + \frac{1}{2}p_1^2\left(\frac{A_2}{A_1}\right.\right.\right. \\
&\quad \left.\left.\left. - A_1\frac{\gamma d_0(1-\lambda)(t-1)[t+2q+\lambda(2-t)]}{2(1+i\tan\xi)[t+q+\lambda(1-t)]^2} - \frac{\mu\gamma A_1 d_0\,[q(1+q)+t(1-\lambda)+\lambda]\,\Gamma_3}{\Gamma_2^2(1+i\tan\xi)\,[(t+q)+\lambda(1-t)]^2}\right) + \frac{d_1p_1}{d_0}\right]\right| \\
&\leq \frac{|A_1||\gamma|}{2\sec\xi\,[q(1+q)+t(1-\lambda)+\lambda]\,|\Gamma_3|}\left[2 + 2\left|\frac{d_1}{d_0}\right| + \frac{1}{2}|p_1|^2\left(\left|\frac{A_2}{A_1}\right.\right.\right. \\
&\quad \left.\left.\left. - A_1\frac{\gamma d_0(1-\lambda)(t-1)[t+2q+\lambda(2-t)]}{2(1+i\tan\xi)[t+q+\lambda(1-t)]^2} - \frac{\mu\gamma A_1 d_0\,[q(1+q)+t(1-\lambda)+\lambda]\,\Gamma_3}{\Gamma_2^2(1+i\tan\xi)\,[(t+q)+\lambda(1-t)]^2}\right| - 1\right)\right].
\end{aligned} \tag{4.3}$$

Now if $\left|\frac{A_2}{A_1} - A_1\frac{\gamma d_0(1-\lambda)(t-1)[t+2q+\lambda(2-t)]}{2(1+i\tan\xi)[t+q+\lambda(1-t)]^2} - \frac{\mu\gamma A_1 d_0\left[q(1+q)+t(1-\lambda)+\lambda\right]\Gamma_3}{\Gamma_2^2(1+i\tan\xi)\left[(t+q)+\lambda(1-t)\right]^2}\right| \leq 1$ in (4.3), then

$$\left|a_3 - \mu a_2^2\right| \leq \frac{|A_1||\gamma|}{\sec\xi\left[q(1+q)+t(1-\lambda)+\lambda\right]|\Gamma_3|}\left[1+\left|\frac{d_1}{d_0}\right|\right]. \tag{4.4}$$

Further, if $\left|\frac{A_2}{A_1} - A_1\frac{\gamma d_0(1-\lambda)(t-1)[t+2q+\lambda(2-t)]}{2(1+i\tan\xi)[t+q+\lambda(1-t)]^2} - \frac{\mu\gamma A_1 d_0\left[q(1+q)+t(1-\lambda)+\lambda\right]\Gamma_3}{\Gamma_2^2(1+i\tan\xi)\left[(t+q)+\lambda(1-t)\right]^2}\right| \geq 1$ in (4.3), then

$$\begin{aligned}\left|a_3 - \mu a_2^2\right| \leq & \frac{|A_1||\gamma|}{\sec\xi\left[q(1+q)+t(1-\lambda)+\lambda\right]|\Gamma_3|}\left(\left|\frac{d_1}{d_0}\right| + \left|\frac{A_2}{A_1}\right.\right.\\ & \left.\left. -A_1\frac{\gamma d_0(1-\lambda)(t-1)[t+2q+\lambda(2-t)]}{2(1+i\tan\xi)[t+q+\lambda(1-t)]^2} - \frac{\mu\gamma A_1 d_0\left[q(1+q)+t(1-\lambda)+\lambda\right]\Gamma_3}{\Gamma_2^2(1+i\tan\xi)\left[(t+q)+\lambda(1-t)\right]^2}\right|\right).\end{aligned} \tag{4.5}$$

An examination of the proof shows equality for (4.4) holds if $p_1 = 0,\ p_2 = 2$. Equivalently, we have $p(z) = p_2(z) = \frac{1+z^2}{1-z^2}$ by Lemma 2.3. Therefore, the extremal function in $\mathcal{B}_\lambda^t(\gamma;\psi)$ is given by

$$\begin{aligned}&\frac{1}{\gamma}\left[(1+i\tan\xi)\frac{z^{1-t}[\mathfrak{D}_q(H(z))]}{[(1-\lambda)H(z)+\lambda z]^{1-t}} - i\tan\xi - 1\right]\\ &= \phi(z)\psi\left(\frac{p_2(z)-1}{p_2(z)+1}\right) = \phi(z)\left[\psi(z^2)-1\right] \quad (z\in\mathbb{U}).\end{aligned}$$

Similarly, equality for (4.5) holds if $p_2 = 2$. Equivalently, we have $p(z) = p_1(z) = \frac{1+z}{1-z}$ by Lemma 2.3. Therefore, the extremal function in $\mathcal{B}_\lambda^t(\gamma;\psi)$ $(z\in\mathbb{U})$ is given by

$$\frac{1}{\gamma}\left[(1+i\tan\xi)\frac{z^{1-t}[\mathfrak{D}_q(H(z))]}{[(1-\lambda)H(z)+\lambda z]^{1-t}} - i\tan\xi - 1\right] = \phi(z)\left[\psi(z)-1\right] \quad (z\in\mathbb{U}).$$

□

Corollary 4.2. *[16] Suppose $f(z) = z + a_2z^2 + a_3z^3 + \cdots \in \mathcal{B}_0^0(\gamma;\psi)$ $(z\in\mathbb{U})$. Then, for any $\mu\in\mathbb{C}$*

$$\left|a_3-\mu a_2^2\right| \leq \frac{10|\gamma|}{\sec\xi\left[q(1+q)\right]}\left[A_1 + \max\left\{A_1;\ \left|\frac{9\mu(q+1)-10}{10q(1+i\tan\xi)}\right|\gamma A_1^2 + |A_2|\right\}\right].$$

Proof. If we let

$$h(z) = E_r h(z) = \frac{\sqrt{\pi z}}{2}erh(\sqrt{z}) = z + \sum_{k=2}^{\infty}\frac{(-1)^{k-1}}{(2k-1)(k-1)!}z^k,$$

where *erh* is the error function (see [16]), and $\lambda = t = 0$ in Theorem 4.1 we can establish the assertion of the corollary. □

If we let $h(z) = z + \sum_{k=2}^{\infty}\frac{([b]_q)_{k-1}}{([c]_q)_{k-1}}z^k$, $\psi(z) = z + \sqrt{1+z^2}$, $\phi(z) = 1$ $\xi = t = 0$, $\gamma = 1 + 0i$ and $q\to 1^-$ in Theorem 4.1, we get the following.

Corollary 4.3. *[12] Let $f\in\mathcal{ML}_b^c(t;\psi)$ is of the form (1.1), then for any $\mu\in\mathbb{C}$, we have*

$$\left|a_3-\mu a_2^2\right| \leq \left|\frac{(c)_2}{(b)_2}\right|\frac{1}{2+t}\max\left\{1;\ \frac{|(t-3)(1+t)b(c+1)+2\mu(2+t)c(b+1)|}{2(1+t)^2|b(c+1)|}\right\}.$$

The inequalities are sharp for each μ.

If we let $\Gamma_k = 1, \xi = \lambda = t = 0, \phi(z) = 1$ $q \to 1^-$ and $\psi(z)$ is of the form (3.12) in Theorem 4.1, then we have the following result.

Corollary 4.4. *[21] Let* $0 \le \eta < 1 < \vartheta$ *and let the function* $f \in \mathcal{A}$ *belong to* $\mathcal{S}^*(\eta, \vartheta)$*. Then, for any* μ*,*

$$\left|a_3 - \mu a_2^2\right| \le \frac{\vartheta - \eta}{\pi} \sin\left(\frac{\pi(1-\eta)}{\vartheta - \eta}\right) \max\left\{1; \left|\frac{1}{2} + (1-2\mu)\frac{\vartheta - \eta}{\pi} i + \left(\frac{1}{2} - (1-2\mu)\frac{\vartheta - \eta}{\pi} i\right) e^{2\pi i \frac{1-\eta}{\vartheta - \eta}}\right|\right\}.$$

If we take $\lambda = t = 0, \Gamma_k = 1, \xi = 0, \gamma = 1, \phi(z) = 1$ and $q \to 1^-$ in Theorem 4.1, then we have the following corollary.

Corollary 4.5. *[33] Suppose* $f(z) = z + a_2 z^2 + a_3 z^3 + \cdots \in \mathcal{S}^*(\psi)$ $(z \in \mathbb{U})$*. Then*

$$\left|a_3 - \mu a_2^2\right| \le \frac{A_1}{2} \max\left\{1; \left|A_1 + \frac{A_2}{A_1} - 2\mu A_1\right|\right\} \quad (\mu \in \mathbb{C}).$$

The inequality is sharp for the function given by

$$f(z) = \begin{cases} z \exp \int_0^z \left[\psi(t) - 1\right] \frac{1}{t} dt, & \text{if } \left|A_1 + \frac{A_2}{A_1} - 2\mu A_1\right| \ge 1 \\ z \exp \int_0^z \left[\psi(t^2) - 1\right] \frac{1}{t} dt, & \text{if } \left|A_1 + \frac{A_2}{A_1} - 2\mu A_1\right| \le 1. \end{cases}$$

Conclusion 4.1. *By defining Bazilevič functions of complex order using quasi-subordination and Hadamard product, we were able to unify and extend various classes of analytic function. New extensions were discussed in detail. Further, by replacing the ordinary differentiation with quantum differentiation we have attempted at the discretization of some of the well-known results.*

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Conflict of interest

The authors declare that they have no conflicts of interest.

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