AIMS Mathematics

http://www.aimspress.com/journal/Math

AIMS Mathematics, 6(2): 1497–1514.
DOI: 10.3934/math.2021091
Received: 24 September 2020
Accepted: 15 November 2020
Published: 20 November 2020

Research article

New grey forecasting model with its application and computer code

Halis Bilgil*

Department of Mathematics, Aksaray University, Aksaray 68100, Turkey

* **Correspondence:** Email: halis@aksaray.edu.tr; Tel: +903822882155; Fax: +903822882299.

Abstract: Grey theory is an approach that can be used to construct a model with limited samples to provide better forecasting advantage for short-term problems. In some cases, a grey forecasting model may yield unacceptable forecasting errors. In this work, a new exponential grey prediction model, which is called as EXGM (1,1), is proposed. By using this model, new cases, deaths and recovered cases of COVID-19 in Turkey is forecast. Numerical results show that EXGM (1,1) is a model that performs more accurately than the comparison models.

Keywords: grey systems; COVID-19 predictions; grey forecasting model; least squares method
Mathematics Subject Classification: 60G25, 34B60, 68U01

1. Introduction

The grey prediction models play an important role in the grey system theory, which was proposed by Deng in 1982 [1]. These models are often named as grey models (GM) as they are developed based on the grey system theory. The grey models have been useful in solving uncertain problems with small samples and inadequate information.

The grey prediction models have been widely and successfully applied to various fields, such as industry, science and technology, economy, energy consumption and other fields [2–9]. In recent years, some extended and modified the grey prediction models have been developed based on GM (1,1) because of practicality and the prediction accuracy. Ma and Lui [10] proposed a time-delayed polynomial grey prediction model called TDPGM (1, 1) model, Cui et al. [11] developed a parameter optimization method to improve the ONGM (1, 1, k) model, Ma et al. [12] developed a novel nonlinear multivariate forecasting grey model based on the Bernoulli equation named NGBMC (1, n), Wang et al. [13] introduced a seasonal grey model called SGM (1, 1), Wu et al. [14] proposed a new grey model called BNGM (1, 1, t^2) model, Liu and Wu [15] proposed ANDGM model, Ye et al. [16] proposed a novel accumulative time-delay multivariate grey prediction model called ATGM (1, N), Wu et al. [17] developed a novel grey Riccati model (GRM).

Generally all models in the literature are suitable when the given data sequence satisfies exponential growth. However, the abnormal decreasing in the given data will have negatively effects on the prediction accuracy. The main point of this article is to improve predictive accuracy by adding a decreasing term, (e^{-t}), in the whitenization differential equation. Therefore, the monotone decreasing term (e^{-t}) will suppress the growth of the prediction error. The whitening equation is taken as a linear equation of time in the most of studies on the grey modelling mechanism. However, these models generally neglects the second-order Taylor expansion in the right side of the equation. Thence the errors produced by the model will increase with time, and the structural relations of the variables will not be accurately represented. The standart GM (1, 1) model is improved by Kedong et al. [18] for these reasons and they propose a new EOGM (1, 1) model. In this paper, we propose to further improve the EOGM (1, 1) grey forecasting model and obtain a better prediction of the forecasting results. The right side of EOGM (1, 1) is modified by adding a paramater to balance of abnormal changing in the given data. In addition, the parameters are estimated by using the linear least squares estimation to the a parameter is taken as $a = 1$ in EOGM (1, 1).

The new Coronavirus (COVID-19) is an emerging disease responsible for infecting millions of people since the first notification until nowadays. Today (Jan 9, 2020), there have been reported approximately 280 thousand confirmed COVID-19 cases and 6.2 thousand deaths in TURKEY since Jan 3, 2020. Developing efficient short-term forecasting models allow forecasting the number of future cases. The findings of this research may help government and other agencies to reshape their strategies according to the forecasted situation. As the data generating process is identified in terms of time series models, then it can be updated with the arrival of new data and provide forecasted scenario in future.

The novelty of this paper is essentially shown in two viewpoints. Firstly, this paper introduces an exponential optimization grey model termed EXGM (1, 1) model to improve the prediction accuracy of grey forecasting model. Secondly, it is to predict the future output value of short-term total COVID-19 cases in Turkey.

This paper is outlined as follows: Section 1 includes relevant literature. Section 2 introduces the original GM (1, 1) model. The definition and theorem of EXGM (1, 1) is introduced in Section 3. In Section 4, we present a series of samples to validate EXGM (1, 1). The prediction concerning short-term COVID-19 cases in Turkey will be conducted in Section 4. Section 5 includes computer code. Finally, the conclusions of this study are given in Sections 6.

2. Standard GM (1, 1) model

The grey model GM (1, 1) is one of the most commonly used grey forecasting models and requires at least four observations. Firstly, an accumulating generation operator (AGO) is applied to the data and then the governing differential equation of the model is solved to obtain the predicted value of the system. Finally, the predicted value of the original data is obtained by using the inverse accumulating generation operator (IAGO). The traditional GM (1, 1) modeling process is as follows:

$$X^{(0)} = \left\{x^{(0)}(1), x^{(0)}(2), ..., x^{(0)}(n)\right\} \tag{2.1}$$

is a non-negative sequence of raw data and it's the accumulating generation (AGO) sequence $X^{(1)}$ is

$$X^{(1)} = \left\{x^{(1)}(1), x^{(1)}(2), ..., x^{(1)}(n)\right\}, \tag{2.2}$$

where $x^{(1)}(1) = x^{(0)}(1)$ and,

$$x^{(1)}(k) = \sum_{i=1}^{k} x^{(0)}(k), \qquad k = 2, 3, .., n \tag{2.3}$$

and the sequence mean generated of consecutive neighbors of $X^{(1)}$ is,

$$Z^{(1)} = \left\{ z^{(1)}(1), z^{(1)}(2), ..., z^{(1)}(n) \right\} \tag{2.4}$$

where,

$$z^{(1)}(k) = \frac{x^{(1)}(k) + x^{(1)}(k-1)}{2}, \quad k = 2, ..., n. \tag{2.5}$$

The equation,

$$x^{(0)}(k) + az^{(1)}(k) = b \tag{2.6}$$

is called the basic form of the GM (1, 1) model and the whitenization equation is established as,

$$\frac{dx^{(1)}}{dt} + ax^{(1)} = b. \tag{2.7}$$

The whitenization equation is solved and the prediction value of $X^{(1)}$ can be calculated as following

$$\widehat{x}^{(1)}(k) = \left(x^{(0)}(1) - \frac{b}{a} \right) e^{-a(k-1)} + \frac{b}{a}, \quad k = 2, 3, ... \tag{2.8}$$

Therefore the prediction values can be generated by,

$$\begin{gathered} \widehat{x}^{(0)}(1) = x^{(0)}(1) \\ \widehat{x}^{(0)}(k) = \widehat{x}^{(1)}(k) - \widehat{x}^{(1)}(k-1), \quad k = 2, 3, ..., n. \end{gathered} \tag{2.9}$$

In Eq (2.6), k is a time point, a is a the development and b is called driving coefficients [19]. The parameters a, b in Eq (2.6) can be estimated using the least squares method with the difference Eq (2.6) as,

$$[a, b]^T = \left[B^T B \right]^{-1} B^T Y, \tag{2.10}$$

where

$$B = \begin{bmatrix} -z^{(1)}(2) & 1 \\ -z^{(1)}(3) & 1 \\ . & . \\ . & . \\ -z^{(1)}(n) & 1 \end{bmatrix}, \qquad Y = \begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ . \\ . \\ x^{(0)}(n) \end{bmatrix}. \tag{2.11}$$

The readers may consults the reference [11] for the proof of the traditional GM (1, 1) model, so its details were omitted here.

3. The proposed exponential grey model

In this section, we will introduce a novel exponential grey prediction model, called as the EXGM (1, 1) model. The exponential change of the raw data is an important property of the grey forecasting model. If the exponential variation of the raw data sequence is separated, it can be seen that the grey action quantity is time dependent and this change is exponential with time. The standard GM (1, 1) treats the grey action quantity as a constant and its effect is inadequate for the prediction accuracy. Hence, the estimated error produced by the model will increase with time. Therefore, this study considers the grey action quantity as an exponential function of time and a constant.

Definition 1. *The linear differential equation*

$$\frac{dx^{(1)}(t)}{dt} + ax^{(1)}(t) = b + ce^{-t} \tag{3.1}$$

is called the whitening equation of the EXGM (1, 1) model. One of the reasons for choosing a linear equation is that the sequence Eq (2.1) is monotonically increased and the solution of the linear equations include increased exponential functions. Another reason is that the first order derivative in the linear equation can be written as a difference equation form.

The grey derivative for the first-order grey differential equation with AGO data as the intermediate information is represented as,

$$\frac{dx^{(1)}(t)}{dt} = \lim_{\Delta t \to 0} \frac{x^{(1)}(t + \Delta t) - x^{(1)}(t)}{\Delta t} \tag{3.2}$$

where Δt represents the increment of the parameter t which can be time, position or other usable parameter, and considered to be constant [20]. Hence, we can make it as the unit amount, while $x^{(1)}(t + \Delta t) - x^{(1)}(t)$ is data difference between the consecutive points in the data sequence, therefore

$$\frac{dx^{(1)}(t)}{dt} \approx x^{(1)}(k + 1) - x^{(1)}(k) = x^{(0)}(k). \tag{3.3}$$

Theorem 1.

$$x^{(0)}(k) + az^{(1)}(k) = b + c(e - 1)e^{-k} \tag{3.4}$$

is called the basic difference equation of the EXGM (1, 1) model, where $z^{(1)}(k)$ is given by Eq (2.5).

Proof. Integrating of both sides of the whitenization differential Eq (3.1) in the interval $[k - 1, k]$ following as,

$$\int_{k-1}^{k} \frac{dx^{(1)}(t)}{dt} dt + a \int_{k-1}^{k} x^{(1)}(t)dt = \int_{k-1}^{k} \left(b + ce^{-t}\right) dt. \tag{3.5}$$

Then,

$$x^{(1)}(k) - x^{(1)}(k - 1) + a \int_{k-1}^{k} x^{(1)}(t)dt = b + c(e - 1)e^{-k} \tag{3.6}$$

By using the condition $\int_{k-1}^{k} x^{(1)}(t)dt = 0.5(x^{(1)}(k) + x^{(1)}(k-1)) = z^{(1)}(k)$ and the Eq (3.3), the Eq (3.6) can be written as,

$$x^{(0)}(k) + az^{(1)}(k) = b + c(e-1)e^{-k}.$$

□

The solution of linear Eq (3.1) can be easily obtained as follows,

$$x^{(1)}(k) = \frac{b}{a} + \frac{c}{a-1}e^{-t} + de^{-at} \tag{3.7}$$

where d is integral constant. By using the initial condition $x^{(1)}(1) = x^{(0)}(1),$ the constant d can be found as,

$$d = \left(x^{(0)}(1) - \frac{b}{a} - \frac{c}{a-1}e^{-1}\right)e^{a}.$$

Therefore the grey prediction model Eq (3.7) can be obtained as following

$$\widehat{x}^{(1)}(k) = \left(x^{(0)}(1) - \frac{b}{a} - \frac{c}{a-1}e^{-1}\right)e^{a(1-t)} + \frac{b}{a} + \frac{c}{a-1}e^{-t}\ . \tag{3.8}$$

The response Eq (3.8) is used to compute the values of the series $\widehat{x}^{(1)}(k)$, and the predicted values of the original series $\widehat{x}^{(0)}(k)$ can be obtained as

$$\widehat{x}^{(0)}(k) = \widehat{x}^{(1)}(k) - \widehat{x}^{(1)}(k-1), \qquad k = 2, 3, ..., n. \tag{3.9}$$

The linear equations system (3.4) can be written as following

$$\begin{aligned} x^{(0)}(2) &= -az^{(1)}(2) + b + c(e-1)e^{-2} \\ x^{(0)}(3) &= -az^{(1)}(3) + b + c(e-1)e^{-3} \\ &. \\ &. \\ &. \\ x^{(0)}(n) &= -az^{(1)}(n) + b + c(e-1)e^{-n} \end{aligned} \tag{3.10}$$

or

$$Y = B\widehat{\alpha} \tag{3.11}$$

where

$$B = \begin{bmatrix} -z^{(1)}(2) & 1 & (e-1)e^{-2} \\ -z^{(1)}(3) & 1 & (e-1)e^{-3} \\ . & . & . \\ . & . & . \\ -z^{(1)}(n) & 1 & (e-1)e^{-n} \end{bmatrix}, \qquad Y = \begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ . \\ . \\ x^{(0)}(n) \end{bmatrix}, \qquad \widehat{\alpha} = \begin{bmatrix} a \\ b \\ c \end{bmatrix} \tag{3.12}$$

in which n is the number of samples used to construct the model.

The parameters (a, b and c) estimation of the EXGM (1, 1) model can be easily obtained using a similar way of the GM (1, 1) model as follows

$$[a, b, c]^T = \left(B^T B\right)^{-1} B^T Y. \tag{3.13}$$

The flow mechanism of the EXGM (1, 1) is given in Figure 1.

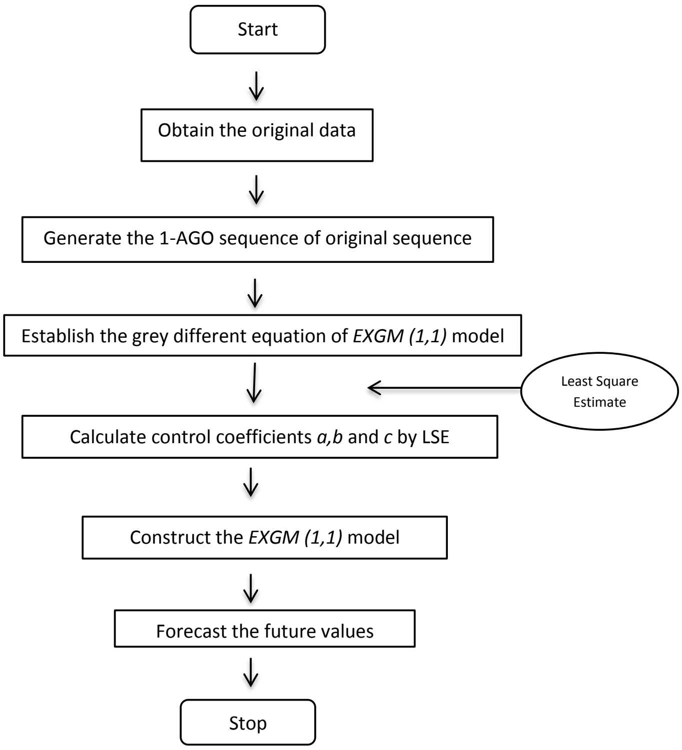

Figure 1. The flowchart of the EXGM (1, 1) model.

Evaluative accuracy of the forecasting model

Relative percentage error (RPE) and mean absolute percentage error (MAPE) are used to evaluate the overall forecast performance of the prediction models. They are defined as follows:

$$RPE(k) = \left| \frac{\widehat{x}^{(0)}(k) - x^{(0)}(k)}{x^{(0)}(k)} \right| \times 100\% \tag{3.14}$$

$$MAPE = \frac{1}{n} \sum_{k=1}^{n} RPE(k) \tag{3.15}$$

where $x^{(0)}(k)$ is the original series, and $\widehat{x}^{(0)}(k)$ is the predicted series. The accuracy evaluation is given in Table 1 [10].

Table 1. Adequacy levels for performance measures.

MAPE (%)	Forecasting power
≤ 10	Excellent
$10 - 20$	Good
$20 - 50$	Reasonable
$50 \leq$	Incorrect

4. An application: Prediction of total COVID-19 cases, deaths and recovers in Turkey by employing the EXGM (1, 1) model

Turkey's Ministry of Health reports complete, correct and official data about COVID-19 situation report of Turkey. The data includes the numbers of total COVID-19 cases, deaths, recovered patients. This section presents a systematic predicting methodology which includes data collection, parameter estimation, result analysis and prediction of near future COVID-19 situation for Turkey.

The modelling values and predicted values by EXGM (1, 1) are tabulated in Tables 2–7 and as shown in Figures 2–7. The forecast values obtained by EXGM (1, 1), GM (1, 1), ONGM (1, 1), TDPGM (1, 1) and the real value are shown in Tables 2–7, it shows that proposed EXGM (1, 1) forecasting model is better for forecasting of COVID-19 situation of Turkey, the precision of EXGM (1, 1) model is much better than the comparison models. The MAPE for the proposed predictions demonstrate the suitability of the proposed model for prediction.

Table 2. Numerical results for total number of COVID-19 cases obtained by the $EXGM\ (1,1)$, $GM\ (1,1)$, $ONGM\ (1,1)$ and $TDPGM\ (1,1)$.

Year	Actual Value	EXGM (1, 1)	Error (Rpe %)	GM(1, 1)	Error (Rpe %)	ONGM (1, 1)	Error (Rpe %)	TDPGM(1, 1)	Error (Rpe %)
22–28 Jun 2020	198284	198284	0.00	198284	0.00	198284	0.00	198284	0.00
29 Jun–05 July 2020	206847	207395	0.27	206432	0.20	173403	16.17	205609	0.60
06–12 July 2020	214029	213612	0.19	213225	0.38	215708	0.78	210665	1.57
13–19 July 2020	220658	220122	0.24	220241	0.19	233470	5.81	215078	2.53
20–26 July 2020	227107	227261	0.07	227489	0.17	240927	6.09	218956	3.59
27 July–02 Aug 2020	233860	234791	0.40	234975	0.48	244058	4.36	222411	4.90
03–09 Aug 2020	241808	242629	0.34	242707	0.37	245373	1.47	225559	6.72
10–16 Aug 2020	250313	250751	0.17	250694	0.15	245925	1.75	228518	8.71
17–23 Aug 2020	259253	259152	0.04	258943	0.12	246157	5.05	237444	8.41
24–30 Aug 2020	268546	267837	0.26	267464	0.40	246254	8.30	239389	10.86
MAPE			0.22		0.27		5.53		5.32

The data for COVID-19 cases of Turkey from 22 June 2020 to 31 August 2020 are applied to construct the grey model, while the data up to 17 January 2021 are used for prediction. The values are listed in Tables 3 and 4, indicating that the EXGM (1, 1) model outperforms the other models in this case. Figures 2 and 3 represent time series actual and forecasted data for COVID-19 cases of Turkey using the new proposed EXGM (1, 1) model. It is seen from the Figure 2 that the actual (black line) and forecasted (red line) data are matched to each other. Table 3 shows forecasted results using EXGM (1, 1). It is seen that trends of new COVID-19 cases will continue in the upcoming to 2021.

Table 3. Forecasting values of total COVID-19 cases for Turkey up to Jan 2021.

Week	Date	Forecasting values	Week	Date	Forecasting values
W1	31 Aug–06 Sept 2020	276815	W11	09–15 Nov 2020	384930
W2	07–13 Sept 2020	286094	W12	16–22 Nov 2020	397833
W3	14-20 Sept 2020	295684	W13	23–29 Nov 2020	411169
W4	21–27 Sept 2020	305595	W14	30 Nov-06 Dec 2020	424952
W5	28 Sept–04 Oct 2020	315839	W15	07–13 Dec 2020	439197
W6	05–11 Oct 2020	326426	W16	14–20 Dec 2020	453919
W7	12–18 Oct 2020	337368	W17	21–27 Dec 2020	469135
W8	19–25 Oct 2020	348677	W18	28 Dec 2020–03 Jan 2021	484861
W9	26 Oct–01 Nov 2020	360365	W19	04–10 Jan 2021	501114
W10	02–08 Nov 2020	372445	W20	11–17 Jan 2021	517911

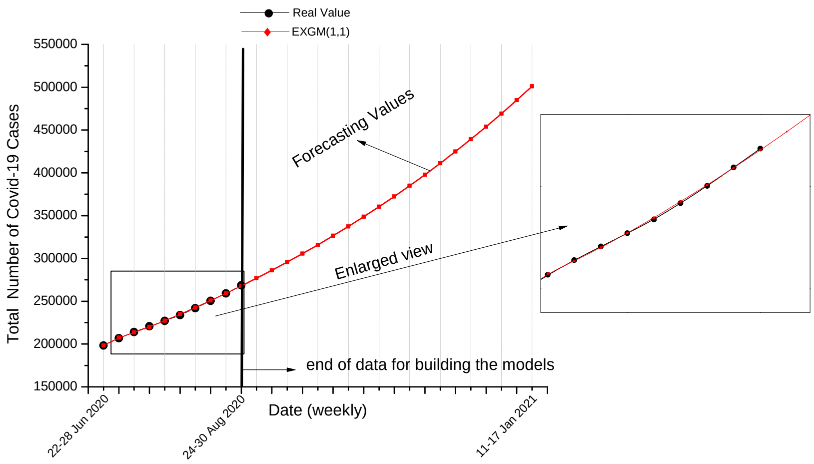

Figure 2. Actual values and forecasting values of total COVID-19 cases in TURKEY.

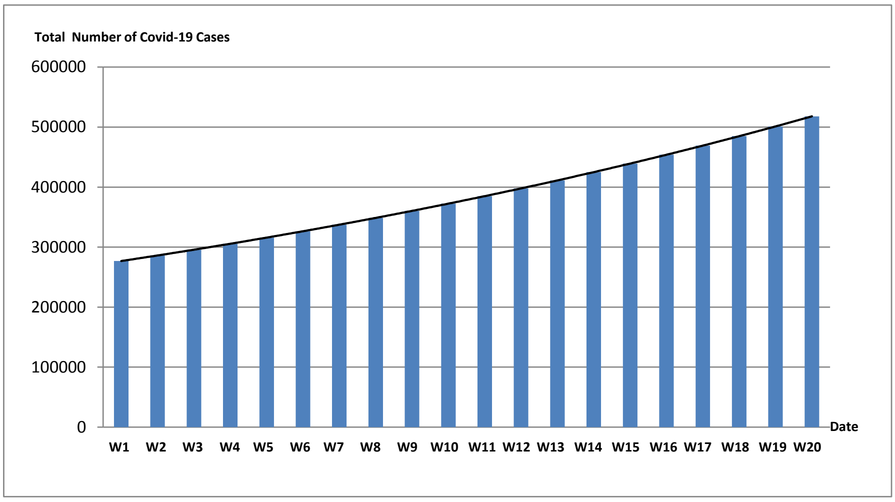

Figure 3. Total weekly COVID-19 cases 31 Aug 2020 to 17 Jan 2021 predicted by EXGM (1, 1) model.

Tables 4 and 5 gives forecasted results using EXGM (1, 1) for deaths numbers. Actual and forecasted data for COVID-19 deaths of Turkey are shown in Figures 4 and 5.

Table 4. Numerical results for total number of COVID-19 death obtained by the $EXGM$ (1, 1), GM (1, 1), $ONGM$ (1, 1) and $TDPGM$ (1, 1).

Date	Actual Value	EXGM (1, 1)	Error (Rpe %)	GM (1, 1)	Error (Rpe %)	ONGM (1,1)	Error (Rpe %)	TDPGM (1, 1)	Error (Rpe %)
22–28 Jun 2020	5097	5097	0.00	5097	0.00	5097	0.00	5097	0.00
29 Jun–05 July 2020	5225	5231	0.11	5230	0.10	4593	12.10	4518	13.53
06–12 July 2020	5363	5352	0.21	5351	0.22	5450	1.62	5066	5.54
13–19 July 2020	5491	5476	0.27	5475	0.29	5738	4.50	5498	0.13
20–26 July 2020	5613	5603	0.18	5602	0.20	5836	3.97	5541	1.28
27 July–02 Aug 2020	5728	5732	0.07	5732	0.07	5868	2.44	5860	2.30
03–09 Aug 2020	5844	5865	0.36	5865	0.36	5879	0.60	5901	0.98
10–16 Aug 2020	5974	6001	0.45	6001	0.45	5883	1.52	5913	1.02
17–23 Aug 2020	6121	6140	0.31	6140	0.31	5889	3.79	5919	3.30
24–30 Aug 2020	6326	6283	0.68	6282	0.70	5901	6.72	6028	4.71
MAPE			0.29		0.30		4.14		3.64

Table 5. Forecasting Values of Total COVID-19 deaths by $EXGM$ (1, 1) model.

Week	Date	Forecasting values	Week	Date	Forecasting values
W1	31 Aug–06 Sept 2020	6428	W1	09–15 Nov 2020	8083
W2	07–13 Sept 2020	6577	W2	16–22 Nov 2020	8271
W3	14–20 Sept 2020	6729	W3	23–29 Nov 2020	8462
W4	21–27 Sept 2020	6885	W4	30 Nov–06 Dec 2020	8658
W5	28 Sept–04 Oct 2020	7045	W5	07–13 Dec 2020	8859
W6	05–11 Oct 2020	7208	W6	14–20 Dec 2020	9065
W7	12–18 Oct 2020	7375	W7	21–27 Dec 2020	9275
W8	19–25 Oct 2020	7546	W8	28 Dec 2020–03 Jan 2021	9490
W9	26 Oct–01 Nov 2020	7721	W9	04–10 Jan 2021	9710
W10	02–08 Nov 2020	7900	W10	11–17 Jan 2021	9935

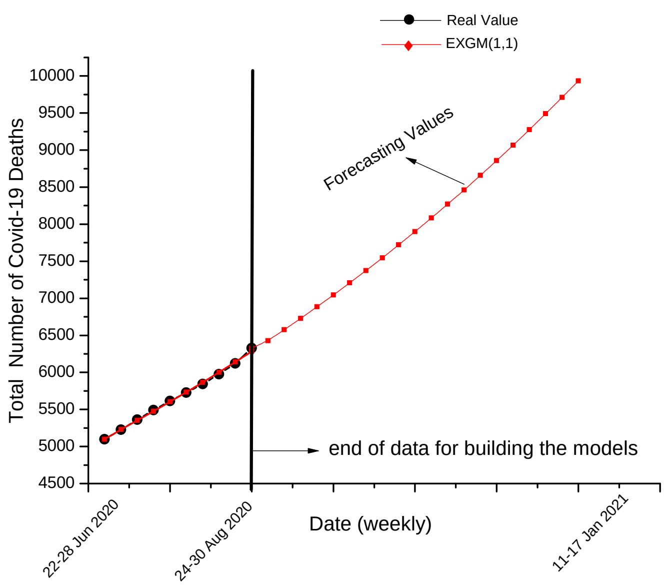

Figure 4. Actual values and forecasting values of total COVID-19 deaths in TURKEY.

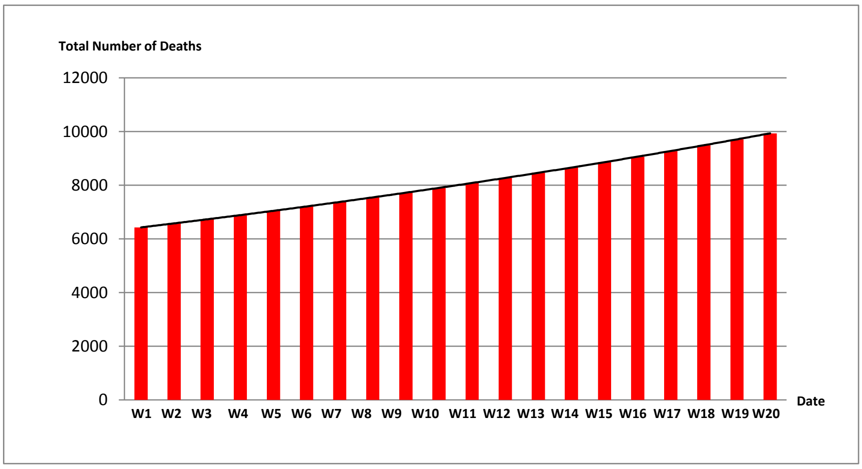

Figure 5. Total weekly COVID-19 deaths 31 Aug 2020 to 17 Jan 2021 predicted by EXGM (1, 1) model.

The real and forecasted results for total number of COVID-19 recovered are given in Tables 6 and 7. Actual and forecasted data for COVID-19 recovered of Turkey are shown in Figures 6 and 7.

Table 6. Numerical results for total number of COVID-19 recovered obtained by the $EXGM$ (1, 1), GM (1, 1), $ONGM$ (1, 1) and $TDPGM$ (1, 1).

Date	Actual Value	EXGM (1, 1)	Error (Rpe %)	GM (1, 1)	Error (Rpe %)	ONGM (1, 1)	Error (Rpe %)	TDPGM (1, 1)	Error (Rpe %)
22–28 Jun 2020	170595	170595	0.00	170595	0.00	170595	0.00	170595	0.00
29 Jun–05 July 2020	180680	172009	0.11	186719	3.34	173341	4.06	176719	2.19
06–12 July 2020	194515	190440	0.21	193322	0.61	193390	0.58	187622	3.54
13–19 July 2020	202010	201084	0.27	200158	0.92	211869	4.88	205567	1.76
20–26 July 2020	209487	208979	0.18	207236	1.07	220393	5.21	211873	1.14
27 July–02 Aug 2020	216494	215985	0.07	214564	0.89	224325	3.62	219508	1.39
03–09 Aug 2020	223759	222788	0.36	222152	0.72	226139	1.06	227665	1.75
10–16 Aug 2020	230969	229645	0.45	230007	0.42	226975	1.73	229010	0.85
17–23 Aug 2020	237165	236654	0.31	238141	0.41	227361	4.13	233109	1.71
24–30 Aug 2020	243839	243855	0.68	246562	1.12	227539	6.68	236903	2.84
MAPE			1.01		1.06		3.55		1.91

Table 7. Forecasting Values of Total COVID-19 recovered by $EXGM$ (1, 1) model.

Week	Date	Forecasting values	Week	Date	Forecasting values
W1	31 Aug–06 Sept 2020	251268	W1	09–15 Nov 2020	338928
W2	07–13 Sept 2020	258902	W2	16–22 Nov 2020	349224
W3	14–20 Sept 2020	266768	W3	23–29 Nov 2020	359883
W4	21–27 Sept 2020	274872	W4	30 Nov–06 Dec 2020	370764
W5	28 Sept–04 Oct 2020	283222	W5	07–13 Dec 2020	382027
W6	05–11 Oct 2020	291826	W6	14–20 Dec 2020	393632
W7	12–18 Oct 2020	300691	W7	21–27 Dec 2020	405590
W8	19–25 Oct 2020	309826	W8	28 Dec 2020–03 Jan 2021	417911
W9	26 Oct–01 Nov 2020	319238	W9	04–10 Jan 2021	430606
W10	02–08 Nov 2020	328935	W10	11–17 Jan 2021	443687

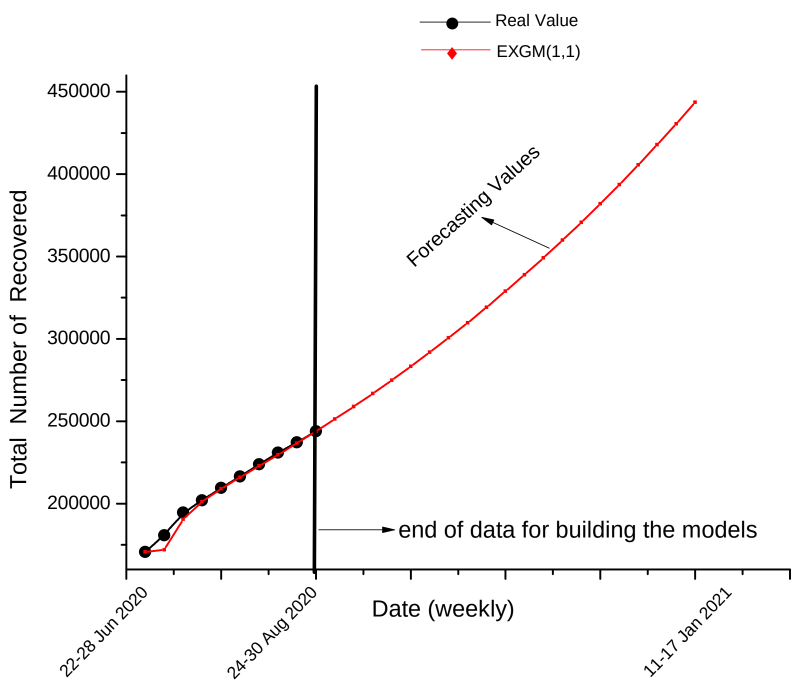

Figure 6. Actual values and forecasting values of total COVID-19 recovered in Turkey.

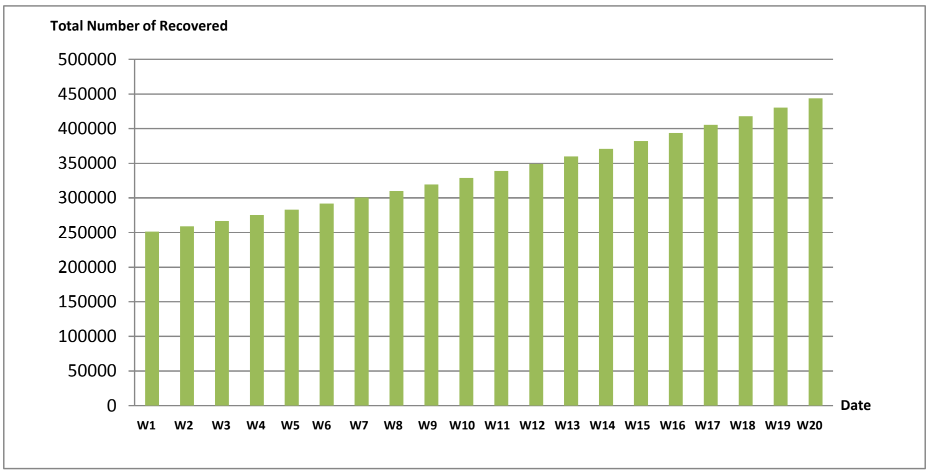

Figure 7. Total weekly COVID-19 recovered 31 Aug 2020 to 17 Jan 2021 predicted by EXGM (1, 1) model.

5. The computer code

In this section a computer code for EXGM(1, 1) model is prepared by using Fortran. Certainly, this code can be modified to C++, Python or another compiler languages. Further predictions can be made with this solution mechanism for COVID-19. In addition, the solution mechanism in this code can be used the other grey models. The working mechanism of the program is as follows:

Steps	Destination	Named as
1	enter the raw data	$x0$
2	build AGO sequence	$x1$
3	build the mean generated sequence,	$z1$
4	create B matrix	BB
5	create Y matrix	YY
6	generate the transpose of B matrix	BT
7	multiply B and B^T	$BC = B^T B$
8	build inverse of BC	BCT
9	multiply BT and Y	BTY
10	multiply BCT and BTY	$SON = \left(B^T B\right)^{-1} B^T Y$
11	find the coefficients a, b, c, d	$SON(k, 1)$
12	write solution of whitezation diff. Eq. $\widehat{x}^{(1)}(k)$	$X1S$
13	calculate the prediction values $\widehat{x}^{(0)}(k)$	$X0S$

The computer code is given in appendix.

6. Conclusions

Since the traditional GM (1, 1) is one of the basic and most important grey prediction model, there are many scholars proposing new methods to improve the precision of the traditional GM (1, 1). Hence, an optimization for the traditional GM (1, 1) model has been developed in this study. The result of the numerical example indicated that the proposed improved grey prediction model aims to achieve effective performance for medium and short term predictions. The structural parameters (a, b and c) of the model can be dynamically adjusted according to the actual system. In addition, the traditional GM (1, 1) model can be considered as the special case of EXGM (1, 1) model (for $c = 0$). The proposed EXGM (1, 1) model is suitable for predicting the data sequence with the characteristics of non-homogeneous exponential law. The EXGM (1, 1) model has achieved higher accuracy than the comparison models such as GM (1, 1), ONGM (1, 1) ,TDPGM (1, 1). However, they can all be employed for estimations.

Scientific simulation and accurate forecasting of future COVID-19 data is a crucial issue for environmental sciences, health sciences and government policy of a country. In this paper, a new exponential grey prediction model, namely EXGM (1, 1) model, has been proposed and applied in order to forecast future values of Turkey's COVID-19 situation by applying the grey modelling technique. Furthermore, Turkey's short-term COVID-19 situation has been predicted by the EXGM (1, 1). It is clear that further predictions (long-term) can be made with this solution mechanism for COVID-19 or another applications.These results provide the growth trend of the future COVID-19 cases of Turkey (if no vaccine for the virus is found or mutation is not seen), and also offer a guideline for policymaking and project planning. It is clear that EXGM (1, 1) can be practically used, with much better accuracy for COVID-19 data forecasting with the smallest MAPE than other models in this comparison.

Short-term daily or weekly estimates are important for making strategic decisions for the future days. Short-term forecasting can provide information to decision makers to schedule to prevent the spreading of COVID-19. Figure 2 and Table 3 show that if adequate precautions are not taken, the confirmed cases of COVID-19 in Turkey will continue to grow. The public health officials and government should take hard decisions to control the rapid increase of the COVID-19. Out sides of officials, the general public should keep social distancing, mask, hygiene and the other precautions to ensure their safety and prevent the spread of disease.

Finally, a new grey forecasting model is introduced and the proposed model was applied to forecast the number of confirmed COVID-19 cases, deaths and recovers. The results indicate that the introduced approach acts well in forecasting the future confirmed COVID-19 indicators. It is clear that, these forecastings can be further expanded for the future months and it can also be applied for estimating COVID-19 data of other countries or the other applications.

The proposed EXGM (1, 1) model play an important role in enriching the theoretical system of grey forecasting theory. However, the proposed EXGM (1, 1) model is only one variable grey prediction model. It is suitable for medium and short term prediction especially. For multi-variable and long term prediction, further research is needed in future. Some modifications such as optimization of the grey derivative, optimization of the background value will be focused mainly onto improve predictive accuracy of EXGM (1, 1) model in future work.

7. Appendix

```
  parameter (n=12,m=3)
  doubleprecision x0,x1,z1,t,a,b,c,a1,a2,a3,a4,a5,c1,c2,c3,xxx,
                   BB,yy,bt,bc,bct,bty,son,X1S,top,ff1,ff2,fonk,X0S

  dimension x0(n),x1(n),z1(n),BB(n-1,m), YY(n-1,1),BT(m,n-1),X1S(2*n),
              Bc(m,m),bct(m,m),bty(m,1),son(m,1),ff1(n),ff2(n),
              top(n),fonk(n),X0S(2*n)

   open(2,file='exgm.txt')
!  The raw data is entered
   do 44 I=1,n
44  read*, x0(I)
! The AGO is arranged
   do 1 k=1,n
   do 2 I=1,k
   T=T+x0(I)
2   continue
   x1(k)=T
   T=0
```

```
1   continue
! The sequence Z1 is generated
   do 4 k=2,n
4   z1(k)=(0.5d0)*(x1(k)+x1(k-1))

! The B matrix is created.
   do 5 k=1,n-1
   BB(k,1)=-z1(k+1)
5   continue

   do 55 k=1,n-1
   BB(k,2)=1.d0
55  continue

   do 56 k=1,n-1
   BB(k,3)=BB(k,3)+dexp(-0.5*k-0.5d0)
56  continue

! The Y matrix is created.
   do 6 k=1,n-1
   YY(k,1)=x0(k+1)
6   continue

! The transpose of the B matrix is created (BT).
   do 8 I=1,m
   do 9 J=1,n-1
    BT(I,J)=BB(J,I)
9   continue
8   continue
! The matrix multiply of B and it's transpose is generated (BC).
   do 10 i=1,m
   do 11 j=1,m
    BC(i,j)=0
   do 12 k=1,n-1
       BC(i,j)=BC(i,j)+BT(i,k)*BB(k,j)
12      continue
11   continue
10 continue

! The inverse of the BC matrix is calculated.
   call inverse(bc,bct)
! The multiply of BT and Y matrix is calculated (BTY).
   do 13 i=1,m
```

```
   do 14 j=1,1
    BTY(i,j)=0
   do 15 k=1,n-1
       BTY(i,j)=BTY(i,j)+BT(i,k)*YY(k,j)
15      continue
14     continue
13  continue

! The matrix multiply of BCT and BTY is calculated (named SON)
  to find of the coefficients a,b,c,d.
   do 16 i=1,m
   do 17 j=1,1
   son(i,j)=0
   do 18 k=1,m
    SON(i,j)=SON(i,j)+BCT(i,k)*BTY(k,j)
18       continue
17      continue
16     continue

! The coefficients a,b,c,d are writing.
   do 19 k=1,m
19    write(2,*) SON(k,1)

! The solution of the whitenization
  differential equations is, $x^1(k)$, calculated (X1S).

   do 24 k=1,14
    X1S(k)=(x0(1)-SON(2,1)/SON(1,1)-
       SON(m,1)*dexp(-0.5d0)/(SON(1,1)-0.5d0))*dexp(-SON(1,1)*(k-1))+
       (SON(m,1)/(SON(1,1)-0.5d0))*dexp(-k*0.5d0)+SON(2,1)/SON(1,1)
24  continue

! X1S(1)=x0(1)
! The prediction values is genereted and printed.

   do 26 k=2,13
   x0S(k)=X1S(k)-X1S(k-1)
   write(2,*) ,k,X0S(k)
26  continue
   end

! print the inverse matrix $C = A^{-1} $
```

```
    subroutine inverse(BC,BCT)
   parameter (m=3)
    double precision BC(m,m), BCT(m,m)
   double precision L7(m,m), U7(m,m), b7(m), d7(m), x7(m)
   double precision coeff
   integer i, j, k

! step 0: initialization for matrices L and U and b

   L7=0.0
   U7=0.0
   b7=0.0

! step 1: forward elimination
   do 1 k=1, m-1
    do 2 i=k+1,m
         coeff=BC(i,k)/BC(k,k)
        L7(i,k) = coeff
        do 3 j=k+1,m
        BC(i,j) = BC(i,j)-coeff*BC(k,j)
  3          continue
  2      continue
 1    continue

! Step 2: prepare L and U matrices
! L matrix is a matrix of the elimination coefficient
! + the diagonal elements are 1.0
   do 5 i=1,m
   L7(i,i) = 1.0
5   continue

! U matrix is the upper triangular part of A
   do 7 j=1,m
   do 6 i=1,j
      U7(i,j) = BC(i,j)
6   continue
7 continue

! Step 3: compute columns of the inverse matrix C
   do 8 k=1,m
   b7(k)=1.0
   d7(1) = b7(1)
! Step 3a: Solve Ld=b using the forward substitution
```

```
   do 9 i=2,m
   d7(i)=b7(i)
   do 10 j=1,i-1
       d7(i) = d7(i) - L7(i,j)*d7(j)
10       continue
9   continue
! Step 3b: Solve Ux=d using the back substitution
   x7(m)=d7(m)/U7(m,m)
   do 11 i = m-1,1,-1
    x7(i) = d7(i)
   do 12 j=m,i+1,-1
       x7(i)=x7(i)-U7(i,j)*x7(j)
12  continue
   x7(i) = x7(i)/u7(i,i)
11  continue
! Step 3c: fill the solutions x(n) into column k of C
   do 13 i=1,m
     BCT(i,k) = x7(i)
13  continue
    b7(k)=0.0
8   continue
   return
end
```

Conflict of interest

The author declares no conflict of interest in this paper.

References

1. J. L. Deng, Control problems of grey systems, *Syst. Control Lett.*, **1** (1982), 288–294.
2. L. F. Wu, S. F. Liu, D. Chen, L. Yao, W. Cui, Using gray model with fractional order accumulation to predict gas emission, *Nat. Hazards*, **71** (2014), 2231–2236.
3. Y. G. Zhang, Y. Xu, Z. P. Wang, GM (1, 1) grey prediction of lorenz chaotic system, *Chaos Solitons Fract.*, **42** (2009), 1003–1009.
4. W. Zhou, J. M. He, Generalized GM (1, 1) model and its application in forecasting of fuel production, *Appl. Math. Modell.*, **37** (2013), 6234–6243.
5. S. A. Javed, S. Liu, Predicting the research output/growth of selected countries: application of even GM (1, 1) and ndgm models, *Scientometrics*, **115** (2018), 395–413.
6. S. L. Ou, Forecasting agricultural output with an improved grey forecasting model based on the genetic algorithml, *Comput. Electron. Agric.*, **85** (2012), 33–39.

7. B. Zeng, Y. Tan , H. Xu, J. Quan, L. Wang, X. Zhou, Forecasting the electricity consumption of commercial sector in hong kong using a novel grey dynamic prediction model, *J. Grey Syst.*, **30** (2018), 157–172.

8. S. Ene, N. Ozturk, Grey modelling based forecasting system for return flow of end-of-life vehicles, *Technol. Forecast. Soc. Change*, **117** (2018), 155–166.

9. U. Şahin, T. Şahin, Forecasting the cumulative number of confirmed cases of COVID-19 in italy, uk and usa using fractional nonlinear grey bernoulli model, *Chaos Solitons Fract.*, **138** (2020), 109948.

10. X. Ma, Z. B. Liu, Application of a novel time-delayed polynomial grey model to predict the natural gas consumption in china, *J. Comput. Appl. Math.*, **324** (2017), 17–24.

11. J. Cui, S. Liu, B. Zeng, N. Xie, A novel grey forecasting model and its optimization, *Appl. Math. Model.*, **37** (2013), 4399–4406.

12. X. Ma, Z. Liu, Y. Wang, Application of a novel nonlinear multivariate grey bernoulli model to predict the tourist income of china, *J. Comput. Appl. Math.*, **347** (2019), 84–94.

13. Z. X. Wang, Q. Li, L. L. Pei, A seasonal GM (1, 1) model for forecasting the electricity consumption of the primary economic sectors, *Energy*, **154** (2018), 522–534.

14. L. Z. Wu, S. H. Li, R. Q. Huang, Q. Xi, A new grey prediction model and its application to predicting landslide displacement, *Appl. Soft Comput.*, **95** (2020), 106543.

15. L. Liu, L. Wu, Forecasting the renewable energy consumption of the european countries by an adjacent non-homogeneous grey model, *Appl. Math. Modell.*, **89** (2021), 1932–1948.

16. L. Ye, N. Xie, A. Hu, A novel time-delay multivariate grey model for impact analysis of CO_2 emissions from chinas transportation sectors, *Appl. Math. Modell.*, **91** (2021), 493–507.

17. W. Wu, X. Ma, Y. Wang, W. Cai, B. Zeng, Predicting chinas energy consumption using a novel grey riccati model, *Appl. Soft Comput.*, **95** (2020), 106555.

18. K. D. Yin, Y. Geng, X. M. Li, Improved grey prediction model based on exponential grey action quantity, *J. Syst. Eng. Electron.*, **29** (2018), 560–570.

19. D. Akay, M. Atak, Grey prediction with rolling mechanism for electricity demand forecasting of Turkey, *Energy*, **32** (2007), 1670–1675.

20. J. Y. Chiang, C. K. Chen, Application of grey prediction to inverse nonlinear heat conduction problem, *Int. J. Heat Mass Transfer*, **51** (2008), 576–585.