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Research article

The stationary distribution of a stochastic rumor spreading model

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Abstract: In this paper, we develop a rumor spreading model by introducing white noise into the model. We establish sufficient conditions for the existence and uniqueness of an ergodic stationary distribution of the positive solutions to the stochastic model by constructing a suitable stochastic Lyapunov function, which provides us a good description of persistence. Finally, we provide some numerical simulations to illustrate the analytical results.

Keywords: rumor spreading; stationary distribution; threshold
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1. Introduction

Rumor spreading as a social contagion process is very similar to the epidemic diffusion, so most rumor spreading models have evolved from epidemic models, such as SI, SIR and SIS. A classic rumor model was DK model proposed by Daley and Kendall in 1964 [2], in which the population was divided into three classes: people who did not know the rumor, people who spread the rumor and people who know but will never spread the rumor. Maki and Thomson modified the DK model into the MK model [8]. Since then, a number of scholars have proposed various rumor spreading models by improving the traditional epidemic models [1, 3, 14–16]. In 2019, Tian and Ding [11] formulated an ordinary differential equation (ODE) compartmental model for rumor, where the population was divided into five disjoint classes, namely the ignorants, the latents, the rumor-spreaders, the debunkers and the stiflers. At time t, the numbers in each of these classes are denoted by $I(t), L(t), R(t), D(t)$ and

$S(t)$, respectively. The rumor spreading model considered by Tian and Ding [11] can be described by a system of ODEs

$$\begin{cases} \dfrac{dI(t)}{dt} = \Lambda - \mu IR - kID - \rho I, \\ \dfrac{dL(t)}{dt} = \mu IR + kID - (\alpha + \beta + \gamma + \rho)L, \\ \dfrac{dR(t)}{dt} = \alpha L - (\delta + \xi + \rho)R, \\ \dfrac{dD(t)}{dt} = \beta L + \xi R - (\theta + \rho)D, \\ \dfrac{dS(t)}{dt} = \delta R + \theta D + \gamma L - \rho S, \end{cases} \tag{1.1}$$

where Λ is the constant immigration rate of the population, μ is the rumor-contacting rate, k is the debunker-contacting rate, ρ is the rate at which all existing users exit from the five classes (i.e. emigration rate), α is the rumor-spreading rate, β is the debunking rate, γ is the silent rate, δ is the rumor-stifling, ξ is the reversal rate and θ represents the debunking-stifling rate. All parameters are assumed to be independent of time t and positive.

The model used in the above study to describe rumor propagation behavior is deterministic model, whereas the random models used to study rumor propagation are few [3]. But in the real world, rumor models are often affected by environmental noise. Especially in emergency events, when rumors are widely spread, the propagation process is affected by many uncertain factors, which increase the volatility of the propagation process. Therefore, it would be necessary and interesting to reveal how the environmental noise affects the rumor spreading model. Following the idea of Jia et al. [3], in this paper, we assume that the stochastic perturbations are of white noise type which are proportional to the system variable, respectively. Then we obtain a stochastic analogue of the deterministic model (1.1) as follows

$$\begin{cases} dI = [\Lambda - \mu IR - kID - \rho I]\, dt + \sigma_1 I dB_1(t), \\ dL = [\mu IR + kID - (\alpha + \beta + \gamma + \rho)L]\, dt + \sigma_2 L dB_2(t), \\ dR = [\alpha L - (\delta + \xi + \rho)R]\, dt + \sigma_3 R dB_3(t), \\ dD = [\beta L + \xi R - (\theta + \rho)D]\, dt + \sigma_4 D dB_4(t), \\ dS = [\delta R + \theta D + \gamma L - \rho S]\, dt + \sigma_5 S\, dB_5(t), \end{cases} \tag{1.2}$$

where $B_i(t)(i = 1, 2, 3, 4, 5)$ are independent Brownian motions and $\sigma_i > 0(i = 1, 2, 3, 4, 5)$ are their intensities. All the other parameters in system (1.2) have the same meaning as in system (1.1). Throughout this paper, unless otherwise specified, let $(\Omega, \mathscr{F}, \{\mathscr{F}_t\}_{t\geq 0}, \mathbb{P})$ be a complete probability space with a filtration $\{\mathscr{F}_t\}_{t\geq 0}$ satisfying the usual conditions, that is, it is rightly continuous and increasing while $\mathscr{F}_0$ contains all $\mathbb{P}-$null sets, and let $B_i(t)(i = 1, 2, 3, 4, 5)$ be scalar Brownian motions defined on the probability space. For the sake of simplicity, we introduce the following notations: $\mathbb{R}^5_+ = \{(x_1, x_2, x_3, x_4, x_5) \in \mathbb{R}^5 : x_i > 0, i = 1, 2, 3, 4, 5\}, a \vee b = \max\{a, b\}, a \wedge b = \min\{a, b\}, \sigma = \sigma_1 \vee \sigma_2 \vee \sigma_3 \vee \sigma_4 \vee \sigma_5$.

With the help of the Lyapunov function methods and the inequality techniques, the existence and uniqueness of an ergodic stationary distribution of the positive solutions to system (1.2) are presented. The main difficulties lies in the construction of Lyapunov function and the construction of a bounded closed domain. The main contribution of this paper are highlighted as follows: (i) a stochastic rumor spreading model is proposed and investigated; (ii) some sufficient conditions for the existence of an ergodic stationary distribution; and (iii) the stationary distribution implies that the rumor can be persistent in the mean. The subsequent part of this paper is as follows: In Section 2, we prove the existence and uniqueness of a global positive solution to system (1.2) with any positive initial value. In Section 3, by constructing a suitable stochastic Lyapunov function, we establish sufficient conditions for the existence and uniqueness of an ergodic stationary distribution of the positive solutions to model (1.2). In Section 4, some numerical simulations are provided to illustrate our theoretical results. Finally, some concluding remarks are presented to end this paper.

2. Existence and uniqueness of the global positive solution

To analyze the dynamical behavior of a rumor spreading model, the first concerning thing is whether the solution is global and positive. In this section, motivated by the methods in [9] and we show that there is a unique global positive solution of system (1.2). The key is to construct a Lyapunov function.

Theorem 2.1. For any given initial value $(I(0), L(0), R(0), D(0), S(0)) \in \mathbb{R}^5_+$, system (1.2) admits a unique solution $(I(t), L(t), R(t), D(t), S(t)) \in \mathbb{R}^5_+$ on $t \geq 0$ and the solution will remain in $\mathbb{R}^5_+$ with probability one, namely $(I(t), L(t), R(t), D(t), S(t)) \in \mathbb{R}^5_+$ for all $t \geq 0$ almost surely (a.s.).

Proof. Since the coefficients of system (1.2) satisfy the local Lipschitz condition, we know that, for any initial value $(I(0), L(0), R(0), D(0), S(0)) \in \mathbb{R}^5_+$, there is a unique local solution $(I(t), L(t), R(t), D(t), S(t)) \in \mathbb{R}^5_+$ on $t \in [0, \tau_e]$, where τ_e is the explosion time [10]. Now we prove the solution is global, i.e. to prove $\tau_e = \infty$ a.s. To this end, let $m_0 > 0$ be sufficiently large such that each component of $(I(0), L(0), R(0), D(0), S(0))$ all lies in the interval $[\frac{1}{m_0}, m_0]$. For each integer $m \geq m_0$, define the following stopping time

$$\tau_m = \inf\left\{t \in [0, \tau_e) : \min\{I(t), L(t), R(t), D(t), S(t)\} \leq \frac{1}{m} \text{ or } \max\{I(t), L(t), R(t), D(t), S(t)\} \geq m\right\}.$$

Throughout this paper, we set $\inf \emptyset = \infty$ (as usual $\emptyset$ denotes the empty set). Obviously, τ_m is an increasing function as $m \to \infty$. Set $\tau_\infty = \lim_{m\to\infty} \tau_m$. Then $\tau_\infty \leq \tau_e$ a.s. If $\tau_\infty = \infty$ a.s. is true, then $\tau_e = \infty$ a.s. and $(I(t), L(t), R(t), D(t), S(t)) \in \mathbb{R}^5_+$ a.s. for all $t \geq 0$. In other words, in order to show this assertion, we only need to prove $\tau_\infty = \infty$ a.s. If the assertion is false, then there is a pair of constants $T > 0$ and $\bar{\epsilon} \in (0, 1)$ such that $\mathbb{P}\{\tau_m \leq T\} \geq \bar{\epsilon}$ for each integer $m \geq m_0$. Define a C^2-function $V : \mathbb{R}^5_+ \to \mathbb{R}_+ \cup \{0\}$ by

$$V(I, L, R, D, S) = (I - a - a \ln \frac{I}{a}) + (L - 1 - \ln L) + (R - 1 - \ln R) + (D - 1 - \ln D) + (S - 1 - \ln S),$$

where a is a positive constant to be determined later. The nonnegativity of this function can be seen from $u - 1 - \ln u \geq 0, \forall u > 0$. According to the general Itô formula (see, for example, Theorem 4.2.1 of [10]), we have

$$\begin{aligned} dV(I, L, R, D, S) &= \mathcal{L}V(I, L, R, D, S)dt + \sigma_1(I - a)dB_1(t) + \sigma_2(L - 1)dB_2(t) + \sigma_3(R - 1)dB_3(t) \\ &+ \sigma_4(D - 1)dB_4(t) + \sigma_5(S - 1)dB_5(t), \end{aligned}$$

where $\mathcal{L}V : \mathbb{R}^5_+ \to \mathbb{R}$ is defined by

$$\begin{aligned}
\mathcal{L}V(I, L, R, D, S) &= \left(1 - \frac{a}{I}\right)(\Lambda - \mu IR - kID - \rho I) + \left(1 - \frac{1}{L}\right)(\mu IR + kID - (\alpha + \beta + \gamma + \rho)L) \\
&\quad + \left(1 - \frac{1}{R}\right)(\alpha L - (\delta + \xi + \rho)R) + \left(1 - \frac{1}{D}\right)(\beta L + \xi R - (\theta + \rho)D) \\
&\quad + \left(1 - \frac{1}{S}\right)(\delta R + \theta D + \gamma L - \rho S) + \frac{a\sigma_1^2 + \sigma_2^2 + \sigma_3^2 + \sigma_4^2 + \sigma_5^2}{2} \\
&= \Lambda - \rho I - \rho L - \rho R - \rho D - \rho S - a\frac{\Lambda}{I} + a\mu R + akD + a\rho - \frac{\mu IR}{L} - \frac{kID}{L} - \frac{\alpha L}{R} \\
&\quad + (\alpha + \beta + \gamma + \rho) + (\delta + \xi + \rho) - \frac{\beta L}{D} - \frac{\xi R}{D} + (\theta + \rho) - \frac{\delta R}{S} - \frac{\theta D}{S} - \frac{\gamma L}{S} + \rho \\
&\quad + \frac{a\sigma_1^2 + \sigma_2^2 + \sigma_3^2 + \sigma_4^2 + \sigma_5^2}{2} \\
&\leq \Lambda + (a\mu - \rho)R + (ak - \rho)D + (a + 4)\rho + \alpha + \beta + \gamma + \delta + \xi + \theta \\
&\quad + \frac{a\sigma_1^2 + \sigma_2^2 + \sigma_3^2 + \sigma_4^2 + \sigma_5^2}{2}.
\end{aligned}$$

Choose $a = min\{\frac{\rho}{\mu}, \frac{\rho}{k}\}$, then we obtain

$$\mathcal{L}V(I, L, R, D, S) \leq \Lambda + (a + 4)\rho + \alpha + \beta + \gamma + \delta + \xi + \theta + \frac{a\sigma_1^2 + \sigma_2^2 + \sigma_3^2 + \sigma_4^2 + \sigma_5^2}{2} := K.$$

and K is a positive constant. Thus,

$$\begin{aligned}
dV(I, L, R, D, S) &\leq Kdt + \sigma_1(I - a)dB_1(t) + \sigma_2(L - 1)dB_2(t) + \sigma_3(R - 1)dB_3(t) \\
&\quad + \sigma_4(D - 1)dB_4(t) + \sigma_5(S - 1)dB_5(t). \qquad (2.1)
\end{aligned}$$

For any $m \geq m_0$, integrating (2.1) on both sides from 0 to $\tau_m \wedge T$ and then taking expectation yield

$$\mathbb{E}V(I(\tau_m \wedge T), L(\tau_m \wedge T), R(\tau_m \wedge T), D(\tau_m \wedge T), S(\tau_m \wedge T)) \leq V(I(0), L(0), R(0), D(0), S(0)) + KT.$$

Let $\Omega_m = \{\omega \in \Omega : \tau_m = \tau_m(\omega) \leq T\}$ for $m \geq m_0$. Then we have $\mathbb{P}(\Omega_m) \geq \bar{\epsilon}$. Note that, for every $\omega \in \Omega_m$, there exists $I(\tau_m, \omega)$ or $L(\tau_m, \omega)$ or $R(\tau_m, \omega)$ or $D(\tau_m, \omega)$ or $S(\tau_m, \omega)$ equaling either m or $\frac{1}{m}$. Thus $V(I(\tau_m, \omega), L(\tau_m, \omega)), R(\tau_m, \omega), D(\tau_m, \omega), S(\tau_m, \omega)$ is no less than either

$m - a - a\ln\frac{m}{a}$ or $\frac{1}{m} - a - a\ln\frac{1}{ma} = \frac{1}{m} - a + a\ln(ma)$ or $m - 1 - \ln m$ or $\frac{1}{m} - 1 - \ln\frac{1}{m} = \frac{1}{m} - 1 + \ln m$. So we have

$$\begin{aligned}
V(I(\tau_m, \omega), L(\tau_m, \omega), R(\tau_m, \omega), D(\tau_m, \omega), S(\tau_m, \omega)) &\geq \left(m - a - a\ln\frac{m}{a}\right) \wedge \left(\frac{1}{m} - a + a\ln(ma)\right) \\
&\quad \wedge (m - 1 - \ln m) \wedge \left(\frac{1}{m} - 1 + \ln m\right).
\end{aligned}$$

Consequently,

$$V(I(0), L(0), R(0), D(0), S(0)) + KT \geq \mathbb{E}[1_{\Omega_m(\omega)} V(I(\tau_m, \omega), L(\tau_m, \omega), R(\tau_m, \omega), D(\tau_m, \omega), S(\tau_m, \omega))]$$

$$\geq \quad \bar{\epsilon}\left(m - a - a\ln\frac{m}{a}\right) \wedge \left(\frac{1}{m} - a + a\ln(ma)\right) \wedge (m - 1 - \ln m)$$
$$\wedge \quad \left(\frac{1}{m} - 1 + \ln m\right),$$

where 1_{Ω_m} denotes the indicator function of Ω_m. Letting $m \to \infty$ leads to the contradiction

$$\infty > V(I(0), L(0), R(0), D(0), S(0)) + KT = \infty.$$

This completes the proof.

3. Existence of ergodic stationary distribution

When considering rumor propagation model, we are also interested to know when the rumor will persist and prevail in a population. In the deterministic models, it can be solved by proving the rumor-epidemic equilibrium of the corresponding model is a global attractor or globally asymptotically stable. But there is no rumor-epidemic equilibrium in system (1.2). In this section, based on the theory of Khasminskii([7]), we prove that there is a stationary distribution which reveals that the rumor will persist in the mean. Here we present some theory about the stationary distribution which is introduced in ([7]).

Definition 3.1. ([7]) The transition probability function $P(s, x, t, A)$ is said to be time-homogeneous (and the corresponding Markov process is called time-homogeneous) if the function $P(s, x, t + s, A)$ is independent of s, where $0 \leq s \leq t$, $x \in \mathbb{R}^l$ and $A \in \mathcal{B}$ and $\mathcal{B}$ denotes the $\sigma-$ algebra of Borel sets in $\mathbb{R}^l$.

Definition 3.2. ([7]) Let $X(t)$ be a regular time-homogeneous Markov process in $\mathbb{R}^l$ described by the stochastic differential equation:

$$dX(t) = f(X(t))dt + \sum_{r=1}^{k} g_r(X(t))dB_r(t).$$

The diffusion matrix of the process $X(t)$ is defined as follows:

$$A(x) = (a_{ij}(x)), \quad a_{ij}(x) = \sum_{r=1}^{k} g_r^i(x)g_r^j(x).$$

Lemma 3.1. ([7]) The Markov process $X(t)$ has a unique ergodic stationary distribution $\pi(\cdot)$ if there exists a bounded domain $D \subset \mathbb{R}^n$ with regular boundary Γ and

A1: there is a positive number M such that

$$\sum_{i,j=1}^{d} a_{ij}(x)\xi_i\xi_j \geq M|\xi|^2, \quad \forall x \in D, \ \forall \xi \in \mathbb{R}^n.$$

A2: there exists a nonnegative C^2-function V such that $\mathcal{L}V$ is negative for any $\mathbb{R}^n \setminus D$, where $\mathcal{L}$ denotes the differential operator defined by

$$\mathcal{L} = \sum_{i=1}^{n} b_i(x)\frac{\partial}{\partial x_i} + \frac{1}{2}\sum_{i,j=1}^{n} a_{ij}(x)\frac{\partial^2}{\partial x_i \partial x_j}.$$

Then

$$\mathbb{P}_x\left\{\lim_{T\to\infty}\frac{1}{T}\int_0^T f(X(t))dt=\int_{\mathbb{R}^n} f(x)\pi(dx)\right\}=1$$

for all $x\in\mathbb{R}^n$, where $f(\cdot)$ is a function integrable with respect to the measure π.

Theorem 3.1. Assume

$$R_0^s:=\frac{\Lambda\mu\alpha}{4\left(\rho+\frac{\sigma_1^2}{2}\right)\left(\alpha+\beta+\gamma+\rho+\frac{\sigma_2^2}{2}\right)\left(\delta+\xi+\rho+\frac{\sigma_3^2}{2}\right)}>1.$$

Then, for any initial value $(I(0),L(0),R(0),D(0),S(0))\in\mathbb{R}_+^5$, system (1.2) has a unique stationary distribution $\pi(\cdot)$ and the ergodicity holds.

Proof. Theorem 2.1 tells us that for any initial value $(I(0),L(0),R(0),D(0),S(0))\in\mathbb{R}_+^5$, there is a unique global solution $(I(t),L(t),R(t),D(t),S(t))\in\mathbb{R}_+^5$. In order to prove this Theorem, it suffices to validate $A1$ and $A2$ in Lemma 3.1. First, we verify $A1$. The diffusion matrix of system (1.2) is given by

$$A=\begin{pmatrix}\sigma_1^2I^2&0&0&0&0\\0&\sigma_2^2L^2&0&0&0\\0&0&\sigma_3^2R^2&0&0\\0&0&0&\sigma_4^2D^2&0\\0&0&0&0&\sigma_5^2S^2\end{pmatrix}.$$

It is easy to see that the matrix A is positive definite for any compact subset of $\mathbb{R}_+^5$, so condition $A1$ in Lemma 3.1 is satisfied. Now we prove condition $A2$. Define a C^2-function

$$\begin{aligned}\tilde{Q}(I,L,R,D,S)&=M(-n_1\ln I-n_2\ln L-n_3\ln R-n_4\ln S-n_5D)\\&\quad+\frac{1}{\varrho+1}(I+L+R+D+S)^{(\varrho+1)}-\ln I-\ln L-\ln R-\ln S\\&:=MQ_1+Q_2-\ln I-\ln L-\ln R-\ln S,\end{aligned}$$

where $n_1,n_2,n_3,n_4,n_5,\varrho$ and M are positive constants, which will be determined later. It is easy to check that

$$\liminf_{k\to\infty,(I,L,R,D,S)\in\mathbb{R}_+^5\setminus U_k}\tilde{Q}(I,L,R,D,S)=+\infty,$$

where $U_k=\Pi_{i=1}^5\left(\frac{1}{k},k\right)$. In addition, $\tilde{Q}(I,L,R,D,S)$ is a continuous function. Hence, $\tilde{Q}(I,L,R,D,S)$ must have a minimum point $(I_0,L_0,R_0,D_0,S_0)\in\mathbb{R}_+^5$. Therefore, we define a nonnegative C^2-function $Q:\mathbb{R}_+^5\to\mathbb{R}_+$

$$Q(I,L,R,D,S)=\tilde{Q}(I,L,R,D,S)-\tilde{Q}(I_0,L_0,R_0,D_0,S_0).$$

Applying the general Itô formula [10] to Q_1, one obtains the differential operator $\mathcal{L}$ of Q_1 as follows:

$$\begin{aligned}\mathcal{L}Q_1&=-\frac{n_1}{I}[\Lambda-\mu IR-kID-\rho I]+\frac{n_1\sigma_1^2}{2}-\frac{n_2}{L}[\mu IR+kID-(\alpha+\beta+\gamma+\rho)L]+\frac{n_2\sigma_2^2}{2}\\&\quad-\frac{n_3}{R}[\alpha L-(\delta+\xi+\rho)R]+\frac{n_3\sigma_3^2}{2}-\frac{n_4}{S}[\delta R+\theta D+\gamma L-\rho S]+\frac{n_4\sigma_5^2}{2}-n_5[\beta L+\xi R-(\theta+\rho)D]\end{aligned}$$

$$
\begin{aligned}
&\leq -\left(\frac{n_1\Lambda}{I}+\frac{n_2\mu IR}{L}+\frac{n_3\alpha L}{R}\right)+(n_1\mu-n_5\xi)R+(n_1k+n_5(\theta+\rho))D+n_1\rho+n_2(\alpha+\beta+\gamma+\rho)\\
&\quad+n_3(\delta+\xi+\rho)+n_4\rho+\frac{n_1\sigma_1^2}{2}+\frac{n_2\sigma_2^2}{2}+\frac{n_3\sigma_3^2}{2}+\frac{n_4\sigma_5^2}{2}\\
&\leq -3\sqrt[3]{n_1n_2n_3\Lambda\mu\alpha}+(n_1\mu-n_5\xi)R+(n_1k+n_5(\theta+\rho))D+n_1\rho+n_2(\alpha+\beta+\gamma+\rho)\\
&\quad+n_3(\delta+\xi+\rho)+n_4\rho+\frac{n_1\sigma_1^2}{2}+\frac{n_2\sigma_2^2}{2}+\frac{n_3\sigma_3^2}{2}+\frac{n_4\sigma_5^2}{2}\\
&= -3\sqrt[3]{n_1n_2n_3\Lambda\mu\alpha}+(n_1\mu-n_5\xi)R+(n_1k+n_5(\theta+\rho))D+n_1\left(\rho+\frac{\sigma_1^2}{2}\right)\\
&\quad+n_2\left(\alpha+\beta+\gamma+\rho+\frac{\sigma_2^2}{2}\right)+n_3\left(\delta+\xi+\rho+\frac{\sigma_3^2}{2}\right)+n_4\left(\rho+\frac{\sigma_5^2}{2}\right).
\end{aligned}
$$

Let

$$
n_1=1,\ n_2=\frac{\rho+\frac{\sigma_1^2}{2}}{2\left(\alpha+\beta+\gamma+\rho+\frac{\sigma_2^2}{2}\right)},\ n_3=\frac{\rho+\frac{\sigma_1^2}{2}}{2\left(\delta+\xi+\rho+\frac{\sigma_3^2}{2}\right)},\ n_4=\frac{\rho+\frac{\sigma_1^2}{2}}{\rho+\frac{\sigma_5^2}{2}},\ n_5=\left(1+\frac{1}{M}\right)\frac{\mu}{\xi}.
$$

Then it follows that

$$
\begin{aligned}
\mathcal{L}Q_1 &\leq -3\left[\frac{\Lambda\mu\alpha\left(\rho+\frac{\sigma_1^2}{2}\right)^2}{4\left(\alpha+\beta+\gamma+\rho+\frac{\sigma_2^2}{2}\right)\left(\delta+\xi+\rho+\frac{\sigma_3^2}{2}\right)}\right]^{\frac{1}{3}}-\frac{\mu}{M}R\\
&\quad+\left[k+\left(1+\frac{1}{M}\right)\frac{\mu(\theta+\rho)}{\xi}\right]D+3\left(\rho+\frac{\sigma_1^2}{2}\right)\\
&= -3\left(\rho+\frac{\sigma_1^2}{2}\right)\left(R_0^{s\frac{1}{3}}-1\right)-\frac{\mu}{M}R+\left[k+\left(1+\frac{1}{M}\right)\frac{\mu(\theta+\rho)}{\xi}\right]D. \qquad (3.1)
\end{aligned}
$$

Similarly, one has

$$
\begin{aligned}
\mathcal{L}Q_2 &= (I+L+R+D+S)^{\varrho}[\Lambda-\rho(I+L+R+D+S)]\\
&\quad+\frac{\varrho}{2}(I+L+R+D+S)^{\varrho-1}\left(\sigma_1^2I^2+\sigma_2^2L^2+\sigma_3^2R^2+\sigma_4^2D^2+\sigma_5^2S^2\right)\\
&\leq \Lambda(I+L+R+D+S)^{\varrho}-\rho(I+L+R+D+S)^{\varrho+1}\\
&\quad+\frac{\varrho}{2}\left(\sigma_1^2\vee\sigma_2^2\vee\sigma_3^2\vee\sigma_4^2\vee\sigma_5^2\right)(I+L+R+D+S)^{\varrho+1}\\
&= \Lambda(I+L+R+D+S)^{\varrho}-\phi(I+L+R+D+S)^{\varrho+1}\\
&\leq C_0-\frac{1}{2}\phi(I+L+R+D+S)^{\varrho+1}\\
&\leq C_0-\frac{1}{2}\phi(I^{\varrho+1}+L^{\varrho+1}+R^{\varrho+1}+D^{\varrho+1}+S^{\varrho+1}), \qquad (3.2)
\end{aligned}
$$

where we choose ϱ sufficiently small such that $\phi=\rho-\frac{\varrho\sigma^2}{2}>0$ and

$$
C_0 = \sup_{(I,L,R,D,S)\in\mathbb{R}_+^5}\left\{\Lambda(I+L+R+D+S)^{\varrho}-\frac{\phi}{2}(I+L+R+D+S)^{\varrho+1}\right\}<\infty.
$$

Moreover, one has

$$\begin{cases} \mathcal{L}(-\ln I) = -\dfrac{\Lambda}{I} + \mu R + kD + \rho + \dfrac{\sigma_1^2}{2}, \\ \mathcal{L}(-\ln L) = -\dfrac{\mu IR}{L} - \dfrac{kID}{L} + (\alpha + \beta + \gamma + \rho) + \dfrac{\sigma_2^2}{2}, \\ \mathcal{L}(-\ln R) = -\dfrac{\alpha L}{R} + (\delta + \xi + \rho) + \dfrac{\sigma_3^2}{2}, \\ \mathcal{L}(-\ln S) = -\dfrac{\delta R}{S} - \dfrac{\theta D}{S} - \dfrac{\gamma L}{S} + \rho + \dfrac{\sigma_5^2}{2}. \end{cases} \tag{3.3}$$

Making use of (3.1)–(3.3), we then derive that

$$\begin{aligned} \mathcal{L}Q \le\ & -3M\left(\rho + \frac{\sigma_1^2}{2}\right)\left(R_0^{s^{\frac{1}{3}}} - 1\right) + (M+1)\left(k + \frac{\mu(\theta+\rho)}{\xi}\right)D + C_0 - \frac{\Lambda}{I} + \rho \\ & - \frac{1}{2}\phi(I^{\varrho+1} + L^{\varrho+1} + R^{\varrho+1} + D^{\varrho+1} + S^{\varrho+1}) + \frac{\sigma_1^2}{2} - \frac{\mu IR}{L} - \frac{kID}{L} + (\alpha+\beta+\gamma+\rho) \\ & + \frac{\sigma_2^2}{2} - \frac{\alpha L}{R} + (\delta+\xi+\rho) + \frac{\sigma_3^2}{2} - \frac{\delta R}{S} - \frac{\theta D}{S} - \frac{\gamma L}{S} + \rho + \frac{\sigma_5^2}{2} \\ \le\ & -3M\left(\rho + \frac{\sigma_1^2}{2}\right)\left(R_0^{s^{\frac{1}{3}}} - 1\right) + (M+1)\left(k + \frac{\mu(\theta+\rho)}{\xi}\right)D - \frac{\Lambda}{I} - \frac{kID}{L} - \frac{\alpha L}{R} - \frac{\theta D}{S} + C_0 + 4\rho \\ & + \alpha + \beta + \gamma + \delta + \xi + \frac{\sigma_1^2 + \sigma_2^2 + \sigma_3^2 + \sigma_5^2}{2} - \frac{\phi}{2}\left(I^{\varrho+1} + L^{\varrho+1} + R^{\varrho+1} + D^{\varrho+1} + S^{\varrho+1}\right) \end{aligned} \tag{3.4}$$

For the convenience, we define

$$H_1 = \sup_{D\in\mathbb{R}_+}\left\{-3M\left(\rho + \frac{\sigma_1^2}{2}\right)\left(R_0^{s^{\frac{1}{3}}} - 1\right) + (M+1)\left(k + \frac{\mu(\theta+\rho)}{\xi}\right)D - \frac{1}{4}\phi D^{\varrho+1}\right\} < \infty$$

and

$$H_2 = C_0 + 4\rho + \alpha + \beta + \gamma + \delta + \xi + \frac{1}{2}(\sigma_1^2 + \sigma_2^2 + \sigma_3^2 + \sigma_5^2).$$

Now we are in the position to construct a bounded closed domain U_ϵ such that the condition $A2$ in Lemma 3.1 holds. To this end, we define a compact set as follows

$$U_\epsilon = \left\{(I, L, R, D, S) \in \mathbb{R}_+^5 : \epsilon \le I \le \frac{1}{\epsilon}, \epsilon^3 \le L \le \frac{1}{\epsilon^3}, \epsilon^4 \le R \le \frac{1}{\epsilon^4}, \epsilon \le D \le \frac{1}{\epsilon}, \epsilon^2 \le S \le \frac{1}{\epsilon^2}\right\},$$

where $\epsilon > 0$ is a sufficiently small constant such that

$$-3M\left(\rho + \frac{\sigma_1^2}{2}\right)\left(R_0^{s^{\frac{1}{3}}} - 1\right) + (M+1)\left(k + \frac{\mu(\theta+\rho)}{\xi}\right)\epsilon + H_2 < -1 \tag{3.5}$$

and

$$-\left(\frac{\Lambda}{\epsilon} \wedge \frac{\theta}{\epsilon} \wedge \frac{k}{\epsilon} \wedge \frac{\alpha}{\epsilon} \wedge \frac{\phi}{2\epsilon^{4(\varrho+1)}} \wedge \frac{\phi}{4\epsilon^{\varrho+1}}\right) + H_1 + H_2 < -1, \tag{3.6}$$

For convenience, we can divide $\mathbb{R}^5_+ \setminus U_\epsilon$ into ten domains, where

$$U_1 = \left\{(I,L,R,D,S) \in \mathbb{R}^5_+ | 0 < I < \epsilon\right\}, \quad U_2 = \left\{(I,L,R,D,S) \in \mathbb{R}^5_+ | 0 < D < \epsilon\right\},$$

$$U_3 = \left\{(I,L,R,D,S) \in \mathbb{R}^5_+ | \epsilon \le D, 0 < S < \epsilon^2\right\}, \quad U_4 = \left\{(I,L,R,D,S) \in \mathbb{R}^5_+ | \epsilon \le I, \epsilon \le D, 0 < L < \epsilon^3\right\},$$

$$U_5 = \left\{(I,L,R,D,S) \in \mathbb{R}^5_+ | \epsilon^3 \le L, 0 < R < \epsilon^4\right\}, \quad U_6 = \left\{(I,L,R,D,S) \in \mathbb{R}^5_+ | I > \frac{1}{\epsilon}\right\},$$

$$U_7 = \left\{(I,L,R,D,S) \in \mathbb{R}^5_+ | L > \frac{1}{\epsilon^3}\right\}, \quad U_8 = \left\{(I,L,R,D,S) \in \mathbb{R}^5_+ | R > \frac{1}{\epsilon^4}\right\},$$

$$U_9 = \left\{(I,L,R,D,S) \in \mathbb{R}^5_+ | D > \frac{1}{\epsilon}\right\}, \quad U_{10} = \left\{(I,L,R,D,S) \in \mathbb{R}^5_+ | S > \frac{1}{\epsilon^2}\right\}.$$

Obviously, $\mathbb{R}^5_+ \setminus U_\epsilon = \bigcup_{i=1}^{10} U_i$. Next, we will prove that $\mathcal{L}Q(I,L,R,D,S) \le -1$ for any $(I,L,R,D,S) \in \mathbb{R}^5_+ \setminus U_\epsilon$, which is equivalent to proving it on the above ten domains, respectively.
Case 1. For any $(I,L,R,D,S) \in U_1$, then (3.4) implies that

$$\begin{aligned}
\mathcal{L}Q &\le -3M\left(\rho + \frac{\sigma_1^2}{2}\right)\left(R_0^{s\frac{1}{3}} - 1\right) + (M+1)\left(k + \frac{\mu(\theta+\rho)}{\xi}\right)D - \frac{1}{2}\phi D^{\varrho+1} \\
&\quad - \frac{\Lambda}{I} + C_0 + 4\rho + \alpha + \beta + \gamma + \delta + \xi + \frac{1}{2}(\sigma_1^2 + \sigma_2^2 + \sigma_3^2 + \sigma_5^2) \\
&\le -\frac{\Lambda}{I} + H_1 + H_2 \le -\frac{\Lambda}{\epsilon} + H_1 + H_2. \qquad (3.7)
\end{aligned}$$

Case 2. For any $(I,L,R,D,S) \in U_2$, using (3.4) one obtains

$$\begin{aligned}
\mathcal{L}Q &\le -3M\left(\rho + \frac{\sigma_1^2}{2}\right)\left(R_0^{s\frac{1}{3}} - 1\right) + (M+1)\left(k + \frac{\mu(\theta+\rho)}{\xi}\right)D \\
&\quad + C_0 + 4\rho + \alpha + \beta + \gamma + \delta + \xi + \frac{1}{2}\left(\sigma_1^2 + \sigma_2^2 + \sigma_3^2 + \sigma_5^2\right) \\
&\le -3M\left(\rho + \frac{\sigma_1^2}{2}\right)\left(R_0^{s\frac{1}{3}} - 1\right) + (M+1)\left(k + \frac{\mu(\theta+\rho)}{\xi}\right)\epsilon + H_2. \qquad (3.8)
\end{aligned}$$

Case 3. For any $(I,L,R,D,S) \in U_3$, in view of (3.4), we get

$$\begin{aligned}
\mathcal{L}Q &\le -3M\left(\rho + \frac{\sigma_1^2}{2}\right)\left(R_0^{s\frac{1}{3}} - 1\right) + (M+1)\left(k + \frac{\mu(\theta+\rho)}{\xi}\right)D - \frac{1}{2}\phi D^{\varrho+1} \\
&\quad - \frac{\theta D}{S} + C_0 + 4\rho + \alpha + \beta + \gamma + \delta + \xi + \frac{1}{2}\left(\sigma_1^2 + \sigma_2^2 + \sigma_3^2 + \sigma_5^2\right) \\
&\le -\frac{\theta D}{S} + H_1 + H_2 \le -\frac{\theta}{\epsilon} + H_1 + H_2. \qquad (3.9)
\end{aligned}$$

Case 4. For any $(I,L,R,D,S) \in U_4$, it follows from (3.4) that

$$\mathcal{L}Q \le -3M\left(\rho + \frac{\sigma_1^2}{2}\right)\left(R_0^{s\frac{1}{3}} - 1\right) + (M+1)\left(k + \frac{\mu(\theta+\rho)}{\xi}\right)D - \frac{1}{2}\phi D^{\varrho+1}$$

$$
\begin{aligned}
& - \frac{kID}{L} + C_0 + 4\rho + \alpha + \beta + \gamma + \delta + \xi + \frac{1}{2}\left(\sigma_1^2 + \sigma_2^2 + \sigma_3^2 + \sigma_5^2\right) \\
\leq \;& -\frac{k}{\epsilon} + H_1 + H_2.
\end{aligned} \tag{3.10}
$$

Case 5. For any $(I, L, R, D, S) \in U_5$, according to (3.4), we derive

$$
\begin{aligned}
\mathcal{L}Q \leq \;& -3M\left(\rho + \frac{\sigma_1^2}{2}\right)\left(R_0^{s\frac{1}{3}} - 1\right) + (M+1)\left(k + \frac{\mu(\theta+\rho)}{\xi}\right)D - \frac{1}{2}\phi D^{\varrho+1} \\
& - \frac{\alpha L}{R} + C_0 + 4\rho + \alpha + \beta + \gamma + \delta + \xi + \frac{1}{2}\left(\sigma_1^2 + \sigma_2^2 + \sigma_3^2 + \sigma_5^2\right) \\
\leq \;& -\frac{\alpha}{\epsilon} + H_1 + H_2.
\end{aligned} \tag{3.11}
$$

Case 6. For any $(I, L, R, D, S) \in U_6$, by (3.4), we obtain

$$
\begin{aligned}
\mathcal{L}Q \leq \;& -3M\left(\rho + \frac{\sigma_1^2}{2}\right)\left(R_0^{s\frac{1}{3}} - 1\right) + (M+1)\left(k + \frac{\mu(\theta+\rho)}{\xi}\right)D - \frac{1}{2}\phi D^{\varrho+1} \\
& - \frac{1}{2}\phi I^{\varrho+1} + C_0 + 4\rho + \alpha + \beta + \gamma + \delta + \xi + \frac{1}{2}\left(\sigma_1^2 + \sigma_2^2 + \sigma_3^2 + \sigma_5^2\right) \\
\leq \;& -\frac{1}{2}\phi\frac{1}{\epsilon^{\varrho+1}} + H_1 + H_2.
\end{aligned} \tag{3.12}
$$

Case 7. For any $(I, L, R, D, S) \in U_7$, note from (3.4) that

$$
\begin{aligned}
\mathcal{L}Q \leq \;& -3M\left(\rho + \frac{\sigma_1^2}{2}\right)\left(R_0^{s\frac{1}{3}} - 1\right) + (M+1)\left(k + \frac{\mu(\theta+\rho)}{\xi}\right)D - \frac{1}{2}\phi D^{\varrho+1} \\
& - \frac{1}{2}\phi L^{\varrho+1} + C_0 + 4\rho + \alpha + \beta + \gamma + \delta + \xi + \frac{1}{2}\left(\sigma_1^2 + \sigma_2^2 + \sigma_3^2 + \sigma_5^2\right) \\
\leq \;& -\frac{1}{2}\phi\frac{1}{\epsilon^{3(\varrho+1)}} + H_1 + H_2.
\end{aligned} \tag{3.13}
$$

Case 8. For any $(I, L, R, D, S) \in U_8$, making use of (3.4) one obtains that

$$
\begin{aligned}
\mathcal{L}Q \leq \;& -3M\left(\rho + \frac{\sigma_1^2}{2}\right)\left(R_0^{s\frac{1}{3}} - 1\right) + (M+1)\left(k + \frac{\mu(\theta+\rho)}{\xi}\right)D - \frac{1}{2}\phi D^{\varrho+1} \\
& - \frac{1}{2}\phi R^{\varrho+1} + C_0 + 4\rho + \alpha + \beta + \gamma + \delta + \xi + \frac{1}{2}\left(\sigma_1^2 + \sigma_2^2 + \sigma_3^2 + \sigma_5^2\right) \\
\leq \;& -\frac{1}{2}\phi\frac{1}{\epsilon^{4(\varrho+1)}} + H_1 + H_2.
\end{aligned} \tag{3.14}
$$

Case 9. For any $(I, L, R, D, S) \in U_9$, we know from (3.4) that

$$
\begin{aligned}
\mathcal{L}Q \leq \;& -3M\left(\rho + \frac{\sigma_1^2}{2}\right)\left(R_0^{s\frac{1}{3}} - 1\right) + (M+1)\left(k + \frac{\mu(\theta+\rho)}{\xi}\right)D - \frac{1}{4}\phi D^{\varrho+1} \\
& - \frac{1}{4}\phi D^{\varrho+1} + C_0 + 4\rho + \alpha + \beta + \gamma + \delta + \xi + \frac{1}{2}\left(\sigma_1^2 + \sigma_2^2 + \sigma_3^2 + \sigma_5^2\right) \\
\leq \;& -\frac{1}{4}\phi\frac{1}{\epsilon^{\varrho+1}} + H_1 + H_2.
\end{aligned} \tag{3.15}
$$

Case 10. For any $(I, L, R, D, S) \in U_{10}$, using (3.4), we can show that

$$\begin{aligned}
\mathcal{L}Q &\leq -3M\left(\rho + \frac{\sigma_1^2}{2}\right)\left(R_0^{s\frac{1}{3}} - 1\right) + (M+1)\left(k + \frac{\mu(\theta+\rho)}{\xi}\right)D - \frac{1}{2}\phi D^{\varrho+1} \\
&\quad - \frac{1}{2}\phi S^{\varrho+1} + C_0 + 4\rho + \alpha + \beta + \gamma + \delta + \xi + \frac{1}{2}\left(\sigma_1^2 + \sigma_2^2 + \sigma_3^2 + \sigma_5^2\right) \\
&\leq -\frac{1}{2}\phi\frac{1}{\epsilon^{2(\varrho+1)}} + H_1 + H_2.
\end{aligned} \tag{3.16}$$

It follows from (3.5)–(3.16) that

$$\mathcal{L}Q < -1, \quad \forall (I, L, R, D, S) \in \mathbb{R}_+^5 \setminus U_\epsilon,$$

which proves that condition $A2$ holds. Thus the conditions in Lemma 3.1 are verified, the proof is completed.

Remark 3.1. Comparing with the threshold parameter R_0 in [11], the parameter R_0^s in stochastic system (1.2) is less than R_0, which reveals that the extinction of the rumor is much easier than in the corresponding deterministic model (1.1). Moreover, taking attention to the expression of R_0^s, we can control the rumor propagation by environmental white noise.

4. Numerical simulations

In this section, we give some examples to illustrate the obtained theoretical results. We illustrate our findings by the Milstein's Higher Order Method developed in [5]. According to this method, we can get the following discretization equation of system (1.2):

$$\begin{cases}
I_{j+1} = I_j + [\Lambda - \mu I_j R_j - k I_j D_j - \rho I_j]\Delta t + \sigma_1 I_j\sqrt{\Delta t}\xi_{1,j} + \dfrac{\sigma_1^2}{2} I_j \Delta t(\xi_{1,j}^2 - 1), \\
L_{j+1} = L_j + [\mu I_j R_j + k I_j D_j - (\alpha + \beta + \gamma + \rho)L_j]\Delta t + \sigma_2 L_j\sqrt{\Delta t}\xi_{2,j} + \dfrac{\sigma_2^2}{2} L_j \Delta t(\xi_{2,j}^2 - 1), \\
R_{j+1} = R_j + [\alpha L_j - (\delta + \xi + \rho)R_j]\Delta t + \sigma_3 R_j\sqrt{\Delta t}\xi_{3,j} + \dfrac{\sigma_3^2}{2} R_j \Delta t(\xi_{3,j}^2 - 1), \\
D_{j+1} = D_j + [\beta L_j + \xi R_j - (\theta + \rho)D_j]\Delta t + \sigma_4 D_j\sqrt{\Delta t}\xi_{4,j} + \dfrac{\sigma_4^2}{2} D_j \Delta t(\xi_{4,j}^2 - 1), \\
S_{j+1} = S_j + [\delta R_j + \theta D_j + \gamma L_j - \rho S_j]\Delta t + \sigma_5 S_j\sqrt{\Delta t}\xi_{5,j} + \dfrac{\sigma_5^2}{2} S_j \Delta t(\xi_{5,j}^2 - 1),
\end{cases}$$

where time increment $\Delta t > 0$, and $\xi_{1,k}, \xi_{2,k}, \xi_{3,k}, \xi_{4,k}, \xi_{5,k}$ are independent Gaussian random variables which follows $N(0, 1)$. However, we would like to point out that the values of parameters of system (1.2) and the initial values in the following numerical simulations are chosen for illustration purposes and are not taken from any real life data for any rumors. To this end, we set$(I(0), L(0), R(0), D(0), S(0)) = (0.3, 0.6, 0.2, 0.9, 0.6)$.

Example 1. Choose the parameters: $\Lambda = 5, \mu = 0.045, k = 0.2, \alpha = 0.05, \beta = 0.05, \gamma = 0.05, \rho = 0.01, \delta = 0.01, \xi = 0.01, \theta = 0.2$.

By simple calculation, we have $R_0^s = 1.213 > 1$, which means that the conditions of Theorem 3.1 hold. Therefore, system (1.2) has a unique ergodic stationary distribution. Figures 1 and 2 illustrate this fact.

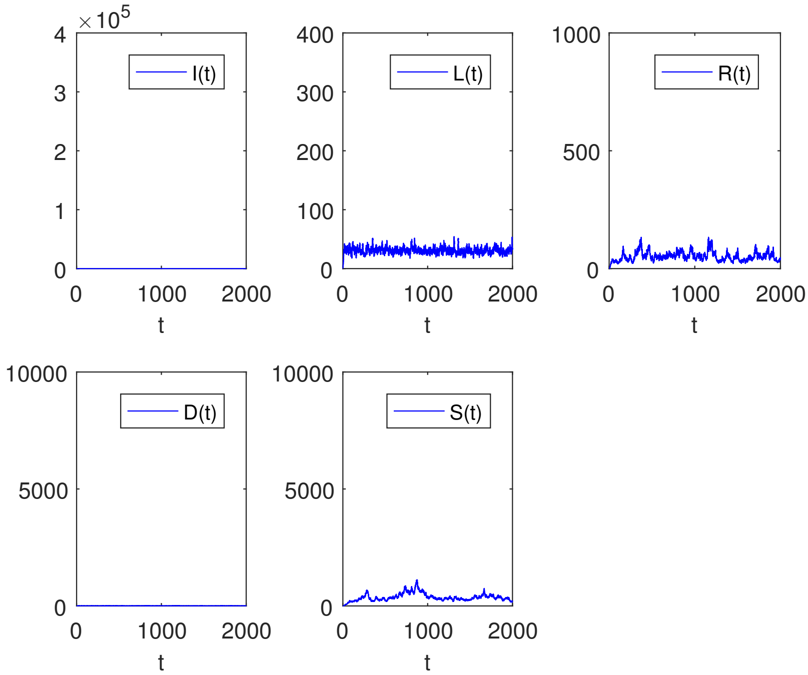

Figure 1. $\sigma_1 = 0.9, \sigma_2 = \sigma_3 = 0.09, \sigma_4 = \sigma_5 = 0.05.$

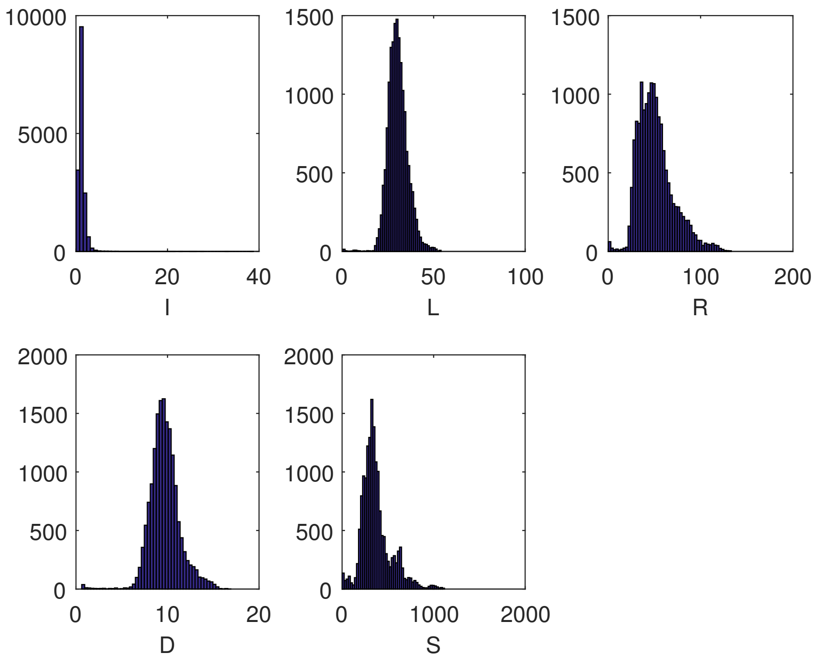

Figure 2. Relative frequency density.

Example 2. Choose the parameters: $\Lambda = 0.6, \mu = 0.04, k = 0.8, \alpha = 0.5, \beta = 0.05, \gamma = 0.5, \rho = 0.01, \delta = 0.1, \xi = 0.2, \theta = 0.2$.

In what follows, we start comparing the stochastic system and deterministic system. In Figure 3, the rumor-spreaders in stochastic system are shown in red lines, compared with the rumor-spreaders in deterministic system which are shown in blue lines. It reveals that the environmental disturbance may help to curb the outbreak of rumors. Further, if we increase the environmental noise, the simulation results in Figure 4 suggests that the extinction of rumor-spreaders in stochastic system is much more easier than that in the corresponding deterministic system.

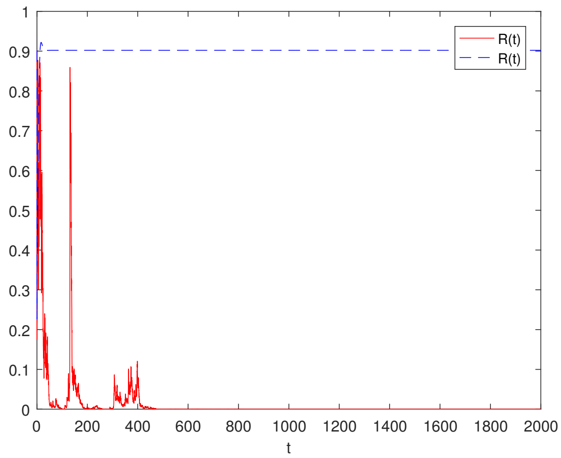

Figure 3. Red line: Rumor-spreaders in stochastic system. Blue line: Rumor-spreaders in deterministic system. The intensities of the white noise are $\sigma_1 = 2.3, \sigma_2 = 1, \sigma_3 = 0.25, \sigma_4 = 0.3, \sigma_5 = 0.5$, and other parameter values are presented in Example 2.

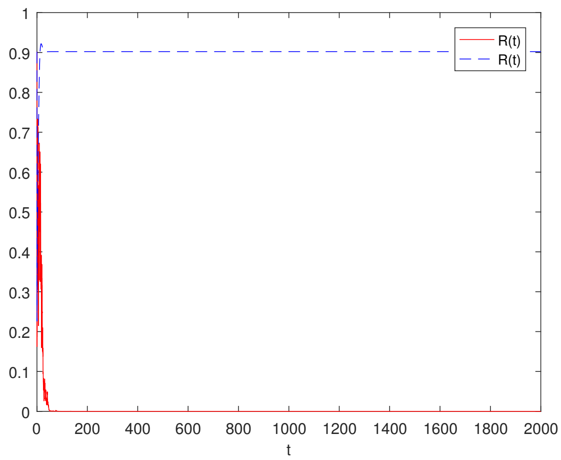

Figure 4. Red line: Rumor-spreaders in stochastic system. Blue line: Rumor-spreaders in deterministic system. The intensities of the white noise are $\sigma_1 = 2.7, \sigma_2 = 1.5, \sigma_3 = 0.3, \sigma_4 = 0.4, \sigma_5 = 0.6$, and other parameter values are presented in Example 2.

5. Conclusions

In the current paper, we have studied a stochastic rumor spreading model. We have established sufficient conditions for the existence and uniqueness of an ergodic stationary distribution of the positive solutions to system (1.2) by using the stochastic Lyapunov function method. The ergodic property can help us better understand cycling phenomena of a rumor spreading model, and so describe the persistence of a rumor spreading model in practice. More precisely, we have obtained the following result:

• Assume

$$R_0^s := \frac{\Lambda\mu\alpha}{4\left(\rho + \frac{\sigma_1^2}{2}\right)\left(\alpha + \beta + \gamma + \rho + \frac{\sigma_2^2}{2}\right)\left(\delta + \xi + \rho + \frac{\sigma_3^2}{2}\right)} > 1.$$

Then, for any initial value $(I(0), L(0), R(0), D(0), S(0)) \in \mathbb{R}_+^5$, system (1.2) has a unique stationary distribution $\pi(\cdot)$ and the ergodicity holds. The stationary distribution indicates that the rumor can become persistent in vivo. The theoretic work extended the results of the corresponding deterministic system. The results show that the rumors will maintain its persistence if the environmental noise is sufficiently small, while large stochastic noise can suppress the spread of rumors.

Some interesting topics deserve further consideration. As we all know, time-delay occurs frequently in many practical engineering systems, which is usually the source of oscillation, instability and poor performance of the systems. Now time-delay has been considered into many stochastic models (see, for example, [4, 6, 12, 13]). We leave time-delay case for our future work.

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Conflict of interest

The authors declare that there are no conflicts of interest.

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