



Research article

The onset of Marangoni bio-thermal convection in a layer of fluid containing gyrotactic microorganisms

Latifa I. Khayyat¹ and Abdullah A. Abdullah^{2,*}

¹ Department of Biology, College of Applied Sciences, Umm Al-Qura University, Makkah, Saudi Arabia

² Department of Mathematical Sciences, College of Applied Sciences, Umm Al-Qura University, Makkah, Saudi Arabia

* **Correspondence:** Email: aamohammad@uqu.edu.sa; Tel: +966503559660.

Abstract: The problem of the onset of Marangoni bio-thermal convection is investigated for a horizontal layer of fluid containing motile gyrotactic microorganisms. The fluid layer is assumed to rest on a rigid surface with fixed temperature and the top boundary of the layer is assumed to be a free non deformable surface. The resulting equations of the problem constitute an eigenvalue problem which is solved using the Chebyshev tau numerical method. The critical values of the thermal Marangoni number are calculated for several values of the bioconvection Péclet number, bioconvection Marangoni number, bioconvection Lewis number and gyrotaxis number. The results of this study showed that the existence of gyrotactic microorganisms increases the critical thermal Marangoni numbers. Moreover, the critical eigenvalues obtained were real-valued indicating that the mode of instability is via a stationary mode, however oscillatory mode is possible for some ranges of the parameters values.

Keywords: linear stability; bio-thermal convection; Marangoni; gyrotactic microorganisms; Chebyshev tau method

Mathematics Subject Classification: 76E06

1. Introduction

Bioconvection patterns usually appear due to the upswimming of microorganisms which are more dense than water in suspensions. When these species gather on the top boundary of the horizontal layer, it becomes more dense and unstable and the microorganisms fall down causing what is called bioconvection. The term was firstly introduced by Platt [1]. It refers to a phenomenon that happens when instability is driven by the motion of upswimming microorganisms which are heavier than water.

Bioconvection has applications in several areas such as environmental systems, mass transport, fuel cells, biotechnology, and nanofluids stability. Childress et al. [2] were the first who proposed the self-consistent theory for bioconvection containing gravitactic microorganisms and developed the mathematical model for gravitactic bioconvection. The bioconvection in a suspension of gyrotactic microorganisms has been investigated by several workers. Hill et al. [3] discussed the thermal instability in a layer of fluid containing a suspension of gyrotactic microorganisms. Ghorai and Hill [4] extended this problem further and obtained numerical solutions for the development and the stability of gyrotactic plumes in a chamber with stress free sidewalls. Pedley and Kessler [5] investigated the hydrodynamic phenomena of the in suspensions of swimming micro-organisms. The bioconvection in a fluid layer containing suspension of gyrotactic microorganisms and small particles was studied by Kuznetsov and Avramenko [6] and Geng and Kuznetsov [7]. Kuznetsov [8] introduced the theory of bio-thermal convection and studied it in a fluid layer with suspension of gyrotactic microorganisms and proved that they have a destabilizing effect on the system. Nield and Kuznetsov [9] extended this problem by studying the case of oscillatory convection.

The combined effect of bioconvection and thermal instability in a layer of fluid with temperature gradient being inclined to the vertical is studied by Avramenko and Kuznetsov [10]. The thermal instability of a layer of nanofluid containing both nanoparticles and gyrotactic microorganisms was first studied by Kuznetsov [11, 12] for oscillatory and non-oscillatory cases. He showed that the existence of gyrotactic microorganisms has a destabilizing effect on the system. Kuznetsov [13] investigated the instability of bio-thermal convection when two different species of microorganisms are presented. The bio-thermal instability of a layer of fluid containing gyrotactic and oxytactic microorganisms is studied by Kuznetsov [14]. Saini and Sharma [15] used the energy method to investigate the bio-thermal instability of a fluid layer containing gravitactic microorganisms. The onset of Darcy-Brinkman bio-thermal convection in a porous layer heated from below and containing the suspension of gyrotactic microorganisms is investigated by Zhao et al. [16]. They showed that as the Darcy number increases, the critical bioconvection Rayleigh number increases. This indicates that porosity has a stabilizing effect on the system.

The onset of Marangoni convection in a base fluid is induced by the dependence of surface tension on the temperature. Pearson [17] studied the stability of Marangoni convection of when a layer of base fluid is heated from below and showed that the instability is caused by surface tension and not by the buoyancy forces. Nield [18] demonstrated that the effect of buoyancy forces can be neglected if the fluid layer depth is less than 1 mm. The results of Pearson [17] and Nield [18] have been expanded and refined by many researchers (e.g., Takashima [19], Benguria and Depassier [20], Wilson [21], Shivakumara et al. [22], Hashim and Arifin [23], Shivakumara et al. [24], Abdullah et al. [25], Abdullah et al. [26]).

The literature revealed that the problem of the onset of Marangoni bio-thermal convection in a fluid layer containing gyrotactic microorganisms has not been studied before, so the object of the present study is to investigate how the Marangoni bio-thermal convection is affected by the existence of gyrotactic microorganisms.

The work is set out as follows. Section 2 formulates the problem, introduces the field equations, describes the boundary conditions of the model and constructs the steady state solution. Section 3 constructs the linearized and non-dimensional equations. Section 4 formulates the normal modes analysis. Section 5 describes the numerical procedure used to treat the eigenvalue problem. Section 6

presents results and section 7 concludes.

2. Problem Formulation

Consider an infinite horizontal layer of an incompressible viscous fluid with gyrotactic microorganisms confined between the planes $x_3^* = 0$ and $x_3^* = d$ of a rectangular Cartesian system of coordinates with position vector $\mathbf{x}^* = x_1^* \mathbf{e}_1 + x_2^* \mathbf{e}_2 + x_3^* \mathbf{e}_3$. In this, \mathbf{e}_3 is directed vertically upwards. Let $T^*(t^*, \mathbf{x}^*)$ and $n^*(t^*, \mathbf{x}^*)$ denote the Kelvin temperature and concentration of microorganisms in the fluid at time t^* and position \mathbf{x}^* respectively. On $x_3^* = 0$, the fluid is assumed to rest on a rigid boundary which is maintained at a constant Kelvin temperature T_0 , whereas on $x_3^* = d$, the motion of the fluid is driven by the thermocapillary effect of the surface tension, say $\gamma(T^*, n^*)$. Otherwise, the surface is non-deformable and loses heat by convection to an environment at constant temperature T_{atmos} . The present problem is relevant to certain species of thermophilic microorganisms that live in hot springs.

2.1. Field equations

The field equations for a layer of fluid with gyrotactic microorganisms has the form

$$\frac{\partial V_j^*}{\partial x_j^*} = 0, \quad (2.1)$$

$$\rho_0 \left(\frac{\partial V_j^*}{\partial t^*} + V_k^* \frac{\partial V_j^*}{\partial x_k^*} \right) = -\frac{\partial P^*}{\partial x_j^*} + \mu \frac{\partial^2 V_j^*}{\partial x_k^{*2}}, \quad (2.2)$$

$$\rho c \left(\frac{\partial T^*}{\partial t^*} + V_k^* \frac{\partial T^*}{\partial x_k^*} \right) = \kappa \nabla^2 T^*, \quad (2.3)$$

$$\frac{\partial n^*}{\partial t^*} = -\frac{\partial}{\partial x_k^*} \left(n^* V_k^* + n^* W_c \tilde{\mathbf{P}}^* \delta_{k3} - D_m \frac{\partial n^*}{\partial x_k^*} \right), \quad (2.4)$$

where \mathbf{V}^* is the fluid velocity, P^* is the hydrostatic pressure, $W_c \tilde{\mathbf{P}}^*$ is the microorganisms average swimming velocity vector relative to the fluid, D_m is the diffusivity of microorganisms, ρ_0 is the fluid density at reference temperature and μ , ρ , c and κ are respectively the viscosity, density, specific heat and thermal conductivity of the fluid.

2.2. Boundary conditions

The thermodynamic, mechanical and microorganisms conditions must be imposed on the boundaries $x_3^* = 0$ and $x_3^* = d$. The thermodynamic boundary conditions are embodied in the equations given below

$$T^*(t^*; x_\lambda^*, 0) = T_0, \quad (2.5)$$

$$-\kappa \frac{\partial T^*(t^*; x_\lambda^*, 0)}{\partial x_j^*} = R(T^*(t^*; x_\lambda^*, 0) - T_{\text{atmos}}). \quad (2.6)$$

Here, λ takes the values 1 and 2 and R is the Robin parameter. The mechanical conditions require the fluid velocity to be zero on the boundary $x_3^* = 0$ whereas the boundary $x_3^* = d$ is assumed to be flat

and free of shear stress after taking account of the influence of surface tension. These conditions are embodied in the equations:

$$V_k^*(t^*; x_\lambda^*, 0) = 0, \quad k = (1, 2, 3) \quad (2.7)$$

$$V_3^*(t^*; x_\lambda^*, d) = 0, \quad (2.8)$$

$$\sigma_{3\lambda}(t^*; x_\lambda^*, d) = \frac{\partial \gamma}{\partial x_\lambda^*}. \quad (2.9)$$

Here, σ_{31} and σ_{32} denote the shear components of fluid stress in the plane $x_3^* = d$. The condition $V_3^* = 0$ on $x_3^* = d$ simplifies the surface tension boundary condition (2.9) to give the following:

$$\mu \frac{\partial V_\lambda^*(t^*; x_\lambda^*, d)}{\partial x_3^*} = \frac{\partial \gamma}{\partial T^*} \frac{\partial T^*(t^*; x_\lambda^*, d)}{\partial x_\lambda^*} + \frac{\partial \gamma}{\partial n^*} \frac{\partial n^*(t^*; x_\lambda^*, d)}{\partial x_\lambda^*}, \quad (2.10)$$

in which n^* and T^* are evaluated on $x_3^* = d$. Finally, the normal fluxes of microorganisms are zero in both boundaries, i.e.,

$$J_3^* = 0, \quad \text{on } x_3 = 0, d, \quad (2.11)$$

where $J_k^* = n^* V_k^* + n^* W_c \tilde{\mathbf{P}}^* \delta_{k3} - D_m \frac{\partial n^*}{\partial x_k^*}$.

2.3. Steady state solution

Equations (2.1–2.4) together with the boundary conditions (2.5–2.11) have a steady state solution in which the fluid is at rest and all other variables are functions of x_3^* alone. Let $T^* = T_b(x_3^*)$ and $n^* = n_b(x_3^*)$ denote the solution of the steady state problem, then it follows that $T_b(x_3^*)$ and $n_b(x_3^*)$ satisfy:

$$T_b(x_3^*) = T_0 + (T_{\text{atmos}} - T_0) \frac{x_3^*}{d}, \quad n_b(x_3^*) = \Omega \exp\left(\frac{W_c x_3^*}{D_m}\right), \quad (2.12)$$

where

$$\Omega = \frac{\bar{n} Q}{\exp(Q) - 1}, \quad (2.13)$$

is the integration constant which represents the value of the basic number density of microorganisms at the bottom of the layer and

$$\bar{n} = \frac{1}{d} \int_0^d n_b(x_3^*) dx_3^*, \quad (2.14)$$

is the average concentration of microorganisms and $Q = \frac{W_c d}{D_m}$ is the biconvection Péclet number.

3. The linearized and non-dimensional equations

Following standard procedures Eqs (2.1–2.4) and their associated boundary conditions are linearized about the steady state solution by introducing the following perturbations

$$V_k^* = \mathbf{0} + \epsilon \hat{v}_k, \quad P^* = P_b + \epsilon \hat{p}, \quad T^* = T_b + \epsilon \hat{T}, \quad n^* = n_b + \epsilon \hat{n}, \quad \tilde{\mathbf{P}}^* = \mathbf{k} + \epsilon \hat{\mathbf{P}}, \quad (3.1)$$

where \mathbf{k} is the vertically upward unit vector. Thus, the linearized field equations have the following form

$$\frac{\partial \hat{v}_j}{\partial x_j^*} = 0, \quad (3.2)$$

$$\rho_0 \frac{\partial \hat{v}_j}{\partial t^*} = -\frac{\partial \hat{p}}{\partial x_j^*} + \mu \frac{\partial^2 \hat{v}_j}{\partial x_k^* \partial x_k^*}, \quad (3.3)$$

$$(\rho c) \left(\frac{\partial \hat{T}}{\partial t^*} + \hat{v}_k \frac{dT_b}{dx_3^*} \delta_{k3} \right) = \kappa \nabla^2 \hat{T}, \quad (3.4)$$

$$\frac{\partial \hat{n}}{\partial t^*} = -\frac{\partial}{\partial x_k^*} \left[n_b (\hat{v}_k + W_c \hat{\mathbf{P}} \delta_{k3}) + \hat{\mathbf{k}} W_c \hat{n} - D_m \frac{\partial \hat{n}}{\partial x_k^*} \right]. \quad (3.5)$$

The perturbation of the swimming direction of a gyrotactic microorganism is related to a perturbation in convection fluid velocity by the following (see Pedley and Kessler [27])

$$\hat{\mathbf{P}} = B(\eta, -\xi, 0), \quad (3.6)$$

where

$$\xi = (1 - \alpha_0) \frac{\partial \hat{v}_3}{\partial x_2^*} - (1 + \alpha_0) \frac{\partial \hat{v}_2}{\partial x_3^*}, \quad (3.7)$$

$$\eta = -(1 - \alpha_0) \frac{\partial \hat{v}_3}{\partial x_1^*} + (1 + \alpha_0) \frac{\partial \hat{v}_1}{\partial x_3^*}, \quad (3.8)$$

here α_0 is a measure of the cell eccentricity and B is the gyrotactic orientation parameter. Thus Eq 3.5 becomes:

$$\frac{\partial \hat{n}}{\partial t^*} = -\hat{v}_3 \frac{\partial n_b}{\partial x_3^*} - W_c \frac{\partial \hat{n}}{\partial x_3^*} + D_m \frac{\partial^2 \hat{n}}{\partial x_k^{*2}} + W_c B n_b \left[(1 - \alpha_0) \left(\frac{\partial^2 \hat{v}_3}{\partial x_1^{*2}} + \frac{\partial^2 \hat{v}_3}{\partial x_2^{*2}} \right) + (1 + \alpha_0) \frac{\partial^2 \hat{v}_3}{\partial x_3^{*2}} \right]. \quad (3.9)$$

The associated linearized boundary conditions on $x_3^* = 0$ are:

$$\hat{v}_3 = \frac{\partial \hat{v}_3}{\partial x_3^*} = \hat{T} = \hat{n} W_c - D_m \frac{\partial \hat{n}}{\partial x_3^*} = 0. \quad (3.10)$$

The associated linearized boundary conditions on $x_3^* = d$ are:

$$\hat{v}_3 = \kappa \frac{\partial \hat{T}}{\partial x_3^*} + R \hat{T} = \hat{n} W_c - D_m \frac{\partial \hat{n}}{\partial x_3^*} = 0, \quad \frac{\partial \hat{v}_\lambda}{\partial x_3} = \frac{1}{\mu} \left(\frac{\partial \gamma}{\partial T} \frac{\partial \hat{T}}{\partial x_\lambda^*} + \frac{\partial \gamma}{\partial n_b} \frac{\partial \hat{n}}{\partial x_\lambda^*} \right). \quad (3.11)$$

Equations (3.2–3.4) and (3.9) and their corresponding boundary conditions are non-dimensionalized by introducing the following non-dimensional variables

$$v_k = \frac{d}{\alpha} \hat{v}_k, \quad p = \frac{d^2}{\rho_0 \nu \alpha} \hat{p}, \quad \theta = \frac{\hat{T}}{\beta d}, \quad t = \frac{\alpha}{d^2} t^*, \quad n = \frac{\hat{n}}{\bar{n}}, \quad x_k = \frac{x_k^*}{d}, \quad (3.12)$$

where α is the diffusivity and β is the adverse temperature gradient. Thus the non-dimensional equations have the final form as following:

$$\frac{\partial v_j}{\partial x_j} = 0, \quad (3.13)$$

$$\frac{1}{\text{Pr}} \frac{\partial v_j}{\partial t} = -\frac{\partial p}{\partial x_j} + \nabla^2 v_j, \quad (3.14)$$

$$\frac{\partial \theta}{\partial t} - v_3 = \nabla^2 \theta, \quad (3.15)$$

$$\frac{\partial n}{\partial t} = -\frac{v_3}{\bar{n}} \frac{\partial n_b}{\partial x_3} - \frac{Q}{\text{Lb}} \frac{\partial n}{\partial x_3} + \frac{1}{\text{Lb}} \frac{\partial^2 n}{\partial x_k^2} + \frac{Q G n_b}{\bar{n}} \left[(1 - \alpha_0) \left(\frac{\partial^2 v_3}{\partial x_1^2} + \frac{\partial^2 v_3}{\partial x_2^2} \right) + (1 + \alpha_0) \frac{\partial^2 v_3}{\partial x_3^2} \right]. \quad (3.16)$$

The non-dimensional boundary conditions on the lower boundary become

$$v_3 = \frac{\partial v_3}{\partial x_3} = \theta = Q n - \frac{\partial n}{\partial x_3} = 0, \quad (3.17)$$

and on the upper boundary $x_3 = 1$ the conditions to be satisfied are

$$v_3 = \frac{\partial \theta}{\partial x_3} + Nu \theta = Q n - \frac{\partial n}{\partial x_3} = 0, \quad \frac{\partial^2 v_3}{\partial x_3^2} - M_T \Delta_2 \theta - M_n \Delta_2 n = 0. \quad (3.18)$$

In Eqs (3.13–3.18), the Prandtl number Pr , the biconvection Péclet number Q , the biconvection Lewis number Lb , the gyrotaxis number G , the Nusselt number Nu , and the temperature and bioconvection Marangoni numbers M_T and M_n have definitions

$$\text{Pr} = \frac{\mu c}{\kappa}, \quad Q = \frac{W_c d}{D_m}, \quad \text{Lb} = \frac{\alpha}{D_m}, \quad G = \frac{B D_m}{d^2}, \quad Nu = \frac{R d}{\kappa}, \quad M_T = -\frac{\beta d^2}{\alpha \mu} \frac{\partial \gamma}{\partial T}, \quad M_n = -\frac{\bar{n} d}{\alpha \mu} \frac{\partial \gamma}{\partial n_b}. \quad (3.19)$$

Following standard procedures of applying the curl operator twice to Eq (3.14) and taking the third component of the resulting equation and of Eqs (3.15) and (3.16), the final equations become

$$\frac{1}{\text{Pr}} \frac{\partial}{\partial t} (\nabla^2 v_3) = \nabla^4 v_3, \quad (3.20)$$

$$\frac{\partial \theta}{\partial t} - v_3 = \nabla^2 \theta, \quad (3.21)$$

$$\frac{\partial n}{\partial t} = -\frac{Q}{\text{Lb}} \frac{\partial n}{\partial x_3} + \frac{1}{\text{Lb}} \frac{\partial^2 n}{\partial x_3^2} - \frac{Q^2 e^{Q x_3}}{e^Q - 1} \left[v_3 - G(1 - \alpha_0) \left(\frac{\partial^2 v_3}{\partial x_1^2} + \frac{\partial^2 v_3}{\partial x_2^2} \right) - G(1 + \alpha_0) \frac{\partial^2 v_3}{\partial x_3^2} \right]. \quad (3.22)$$

4. Normal modes analysis

The stability of perturbations to the steady state solution is investigated using a normal mode analysis. Solutions of Eqs (3.20–3.22) satisfying the boundary conditions (3.17) and (3.18) are sought in the form

$$(w, \theta, n) = (w(x_3), \theta(x_3), n(x_3)) e^{\sigma t + im_1 x_1 + im_2 x_2}, \quad (4.1)$$

where w is the third component of velocity, m_1, m_2 are the wave numbers along the \mathbf{e}_1 and \mathbf{e}_2 directions respectively and σ is a growth rate. The eigenvalue problem for σ is constructed following standard procedures and becomes

$$\frac{\sigma}{\text{Pr}}L(w) = L^2(w), \quad (4.2)$$

$$\sigma\theta = L(\theta) + w, \quad (4.3)$$

$$\sigma n = -\frac{Q}{\text{Lb}} \frac{dn}{dx_3} + \frac{1}{\text{Lb}}L(n) - \frac{Q^2 e^{Qx_3}}{e^Q - 1} [w - GL(w) - G\alpha_0(D^2 + a^2)w], \quad (4.4)$$

where $L(\Phi) = (D^2 - a^2)(\Phi)$ and $a^2 = m_1^2 + m_2^2$. These equations are solved with the conditions

$$v_3 = \frac{\partial v_3}{\partial x_3} = \theta = Qn - \frac{\partial n}{\partial x_3} = 0, \quad (4.5)$$

on the lower boundary and the conditions

$$v_3 = D\theta + Nu\theta = Qn - \frac{\partial n}{\partial x_3} = 0, \quad \frac{\partial^2 v_3}{\partial x_3^2} + a^2 M_T \theta + a^2 M_n n = 0, \quad (4.6)$$

on the upper boundary.

5. Numerical procedure

The governing Eqs (4.2–4.4), together with the boundary conditions (4.5) and (4.6), are solved numerically using the Chebyshev spectral Tau method when the fluid layer is heated from below. This method has high accuracy and allows stationary and overstable modes to be treated simultaneously, which is important whenever the critical eigenvalue flits between stationary and overstable modes in response to changing parameter values. We first map the interval $x_3 \in [0, 1]$ into the interval $[-1, 1]$ by transformation $z = 2x_3 - 1$. Then the variables of the problem are assigned the Chebyshev spectral expansions

$$y_r(z) = \sum_{k=0}^N a_{kr} T_k(z), \quad 1 \leq r \leq 8, \quad (5.1)$$

where N is a user-specified even integer (40 in this problem), $T_k(z)$ are the Chebyshev polynomials of the first kind, a_{kr} are its coefficients and y_1, \dots, y_8 have definitions

$$\begin{aligned} y_1 &= w(z), & y_2 &= \frac{dw}{dz}, & y_3 &= \frac{d^2w}{dz^2} - a^2w, & y_4 &= \frac{d}{dz} \left(\frac{d^2w}{dz^2} - a^2w \right), \\ y_5 &= \theta(z), & y_6 &= \frac{d\theta}{dz}, & y_7 &= n(z), & y_8 &= \frac{dn}{dz}. \end{aligned} \quad (5.2)$$

Applying the definitions (5.2) to the governing Eqs (4.2–4.4) and boundary conditions (4.5) and (4.6), we obtain a system of equations which leads to the generalized eigenvalue problem $AY = \sigma BY$. The eigenvalues, σ , and corresponding eigenvectors are calculated using a specialized routine.

The critical stability boundaries are obtained for which the finite eigenvalue with largest real part has real part with the value zero. The stationary instability arises whenever this eigenvalue is real-valued,

otherwise instability occurs via an oscillatory state or over-stable mode. The numerical results obtained showed that all critical eigenvalues were real-valued indicating that the mechanism of instability in this problem is by a stationary mode, however an oscillatory mode can be obtained for some ranges of the parameters values.

6. Results

To study the stability of this problem, the critical temperature Marangoni numbers M_{Tcrit} were obtained numerically for different values of the bioconvection parameters. Figure 1 plots the bioconvection Péclet number Q , against the critical temperature Marangoni number M_{Tcrit} , for different values of the bioconvection Lewis number Lb , when the gyrotaxis number $G = 0.03$ and $\alpha_0 = 0.2$. The figure shows that an increase in Q (slowly, intermediate and faster swimmers) led to an increase of the critical temperature Marangoni number which indicates that the bioconvection Péclet number had a stabilizing effect on the system and do not improve the bioconvection process. In fact, we also concluded from the figure that the suspension of faster cells is more stable than the suspension of slower and intermediate cells. Moreover, it is clear from the figure that the effect of the bioconvection Lewis number is to increase the values of the critical temperature Marangoni number. This indicates that Lb has a stabilizing effect on the system.

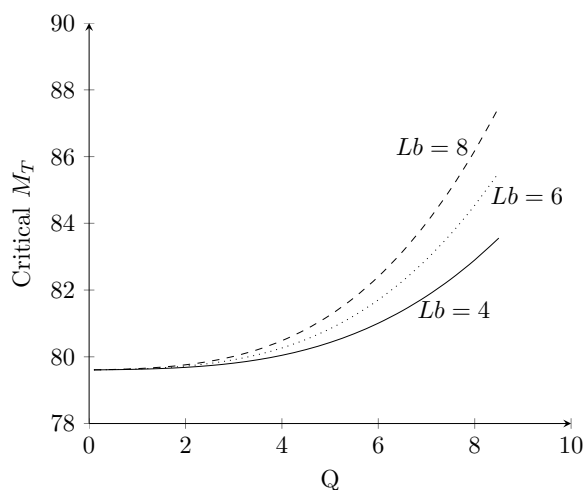


Figure 1. The variation of the critical temperature Marangoni number with the bioconvection Péclet number for different values of the bioconvection Lewis number.

Figures 2 and 3 illustrates similar plots of Figure 1. Figure 2 plots the bioconvection Péclet number Q , against the critical temperature Marangoni number M_{Tcrit} , for different values of the gyrotaxis number G , when the bioconvection Lewis number $Lb = 4$ and $\alpha_0 = 0.2$. The figure shows that an increase in G leads to an increase of the critical temperature Marangoni number which indicate that the gyrotaxis number has a stabilizing effect on the system. This means that the suspension containing more gyrotactic cells is more stable than that containing less gyrotactic cells. Figure 3 plots the bioconvection Péclet number Q , against the critical temperature Marangoni number M_{Tcrit} , for different values of the bioconvection Marangoni number M_n , when the bioconvection Lewis

number $Lb = 4$, the gyrotaxis number $G = 0.03$ and $\alpha_0 = 0.2$. The figure shows that the bioconvection Marangoni number has an insignificant stabilizing effect for slow and intermediate swimmers, however for faster swimmers significant stability exists.

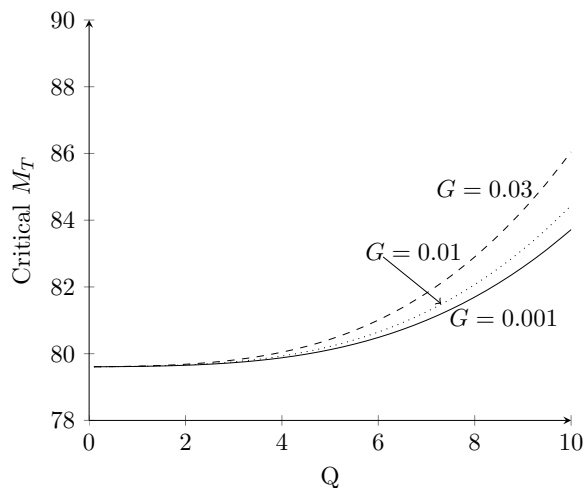


Figure 2. The variation of the critical temperature Marangoni number with the bioconvection Péclet number for different values of the gyrotaxis number.

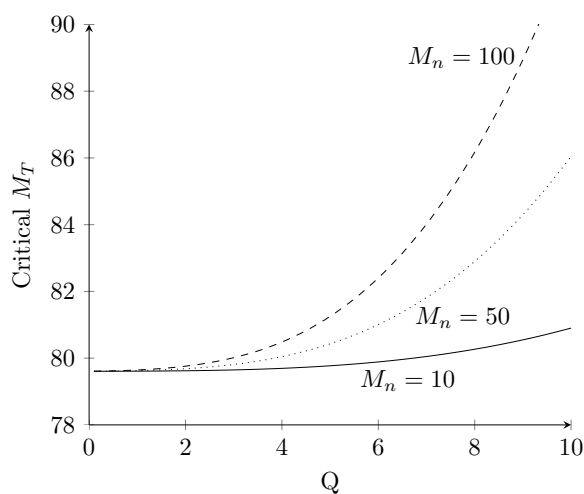


Figure 3. The variation of the critical temperature Marangoni number with the bioconvection Péclet number for different values of the bioconvection Marangoni number.

The effect of bioconvection Péclet number Q , on the critical wave number for different values of the bioconvection Lewis number Lb , when the gyrotaxis number $G = 0.03$ and $\alpha_0 = 0.2$ is displayed in Figure 4. The figure shows that an increase in Q (slowly, intermediate and faster swimmers) leads to an increase of the critical wave number which indicates that the bioconvection Péclet number reduces the size of the cells. In fact, for slow and intermediate swimmers, the effect is less as compared to the effect for faster swimmers. This means that for slow and intermediate swimmers, the size of the

convection cell is elongated. However, as the swimming speed of microorganism's increases, the cell size becomes narrower. Moreover, the figure shows that the effect of the bioconvection Lewis number is to increase the values of the critical wave number. Similar effects are obtained for the gyrotaxis number and the bioconvection Marangoni number.

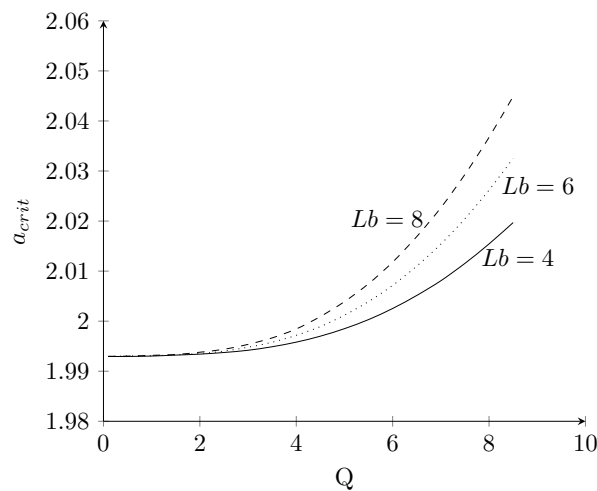


Figure 4. The variation of the critical wave number with the bioconvection Péclet number for different values of the bioconvection Lewis number.

The numerical results investigated in this problem indicate that the marginal state is of stationary type. Interestingly, oscillatory convection is possible for higher values of the bioconvection Marangoni number M_n , provided that the bioconvection Péclet number Q , is greater than a certain value, i.e., for faster swimmers only. When this happened, convection occurs as an oscillatory motion. Figure 5 illustrates the variations of the critical temperature Marangoni number M_{Tcrit} , with the bioconvection Marangoni number M_n , for different values of the bioconvection Péclet number. The figure shows that for intermediate swimmers, $Q = 4$, convection occurs as stationary motion, however for faster swimmers, $Q = 7$ and $Q = 10$ convection occurs as oscillatory motion. When $Q = 7$, oscillatory convection occurs if $M_n > 400$ and when $Q = 10$, oscillatory convection occurs if $M_n > 150$.

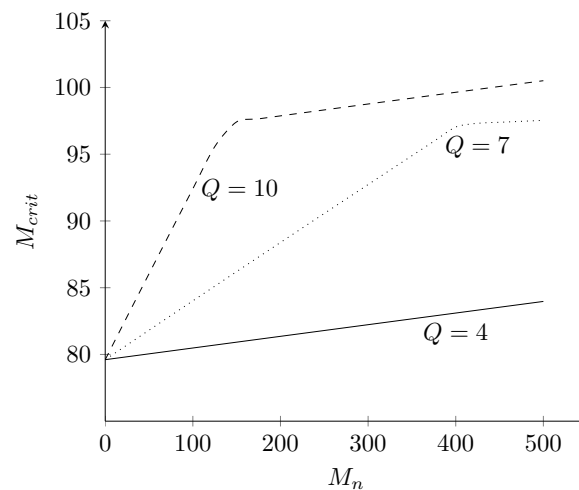


Figure 5. Variation of the critical temperature Marangoni number with the bioconvection Marangoni number for different values of the bioconvection Péclet number.

7. Conclusions

This study had investigated the linear stability of Marangoni bio-thermal convection in a layer of fluid containing gyrotactic microorganisms. All previous studies used the buoyancy effect (Rayleigh effect) to study the stability of a fluid layer containing microorganisms. However, this study uses the effect of surface tension (Marangoni effect) to study the stability of this problem. Findings of this study indicate the followings:

- The most interesting outcome of this analysis is that the marginal stability boundary in this problem depends on two Marangoni numbers: the thermal Marangoni number which characterizes the effect of surface tension and the bioconvection Marangoni number which characterize the effect of upswimming of microorganisms.
- The critical thermal Marangoni number is evaluated for a range of bioconvection Péclet number corresponding to slower, intermediate, and faster swimmers.
- The presence of gravitactic microorganisms increases the values of the critical Marangoni number which indicates that gravitactic microorganisms have a stabilizing effect in this case.
- The bioconvection Péclet number, the bioconvection Lewis number, the gyrotaxis number and the bioconvection Marangoni number have a stabilizing effect in this problem.
- The critical wave number increases with an increase in bioconvection Péclet number and bioconvection Lewis number.
- All critical eigenvalues were real valued indicating that the mechanism of instability is by a stationary mode. However, oscillatory mode is possible only for faster swimmers when the value of the bioconvection Marangoni number exceeds a certain value.

Using surface tension effect, this work opens a wide range of research. For example, this work can be extended by adding the effect of magnetic field, rotation, porosity or solute. Moreover, further research should address the combined effect of gyrotactic and oxytactic microorganisms on the onset of Marangoni bio-thermal convection in a fluid layer.

Conflict of interest

The authors declare that they have no conflicts of interest to this work.

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