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Research article

A new approach to persistence and periodicity of logistic systems with jumps

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Abstract: This paper considers a class of logistic type differential system with jumps. Based on discontinuous control theory, a new approach is developed to guarantee the persistence and existence of a unique globally attractive positive periodic solution. The development results of this paper emphasize the effects of jumps on system, which are different from the existing ones in the literature. Two examples and their simulations are given to illustrate the effectiveness of the proposed results.

Keywords: logistic differential system; persistence; periodicity; globally attractive; jumps
Mathematics Subject Classification: 34A37, 34D20

1. Introduction

For modelling the dynamics of some biological populations, various logistic type differential systems have been proposed and studied in the past several years, see ([1–11]). The classical nonautonomous logistic differential system can be described by

$$N'(t) = N(t)r(t)\Big[1 - \frac{N(t)}{K(t)}\Big], \tag{1.1}$$

where N is the density of population at time t; r and K admit positive upper and lower bounds which models the growth rate and the environmental carrying capacity or saturation level at time t, respectively. Since most populations are affected by the outside environment, it is necessary to study the models of the population dynamics with harvesting, and the following system and its various generalized forms are considered

$$N'(t) = N(t)r(t)\Big[1 - \frac{N(t)}{K(t)}\Big] - E(t, N(t)), \tag{1.2}$$

where function E is a harvesting strategy for the population. In 1959, Holling ([12]) proposed three basic types of the harvesting term:

- Type I (linear): $E(t, N(t)) = \alpha(t)N + \beta(t)$,
- Type II (cyrtoid): $E(t, N(t)) = \dfrac{\alpha(t)N}{N + \beta(t)}$,
- Type III (sigmoid): $E(t, N(t)) = \dfrac{\alpha(t)N^2}{\gamma^2(t) + \beta(t)N^2}$,

where α,β and γ are some scalar functions with positive upper and lower bounds. We refer to the monographs ([13–15]) for the discussions of three types harvesting. In general, population models (1.1) or models (1.2) with the above three basic types harvesting can be uniformly described by the following differential system

$$N'(t) = N(t)f(t, N(t)), \tag{1.3}$$

where f is a scalar function of t and N.

Recently, many research works have paid much attention to the population models with jumps which includes impulsive harvesting and planting, since the discontinuous models governed by impulsive differential systems are more feasible and beneficial than the continuous ones at times, see ([16–29]). For instance, it is often the case that planting and harvesting of the species are intermittent or occur irregularly. Besides, continuous changes such as temperature or rainfall in environment parameters can create some discontinuous outbreaks in biological populations. For fishery management it is unreasonable to assume that fisherman to fish the whole day, and in fact they only fish for some time, and moreover, the seasons and weather variations will also affect the fishing. It has been shown ([17, 29]) that the continuous harvesting policy is superior to the impulsive harvesting policy, however, the latter is more beneficial in realistic operation. Hence, it is significant to consider jumps in the investigation of population models.

Motivated by the above discussions, the main objective of this paper is to study the logistic system (1.3) with jumps

$$\begin{cases} N'(t) = N(t)f(t, N(t)), \quad t \neq t_k, \\ \Delta N|_{t=t_k} = N(t_k) - N(t_k^-) = I_k(N(t_k^-)), \quad k \in \mathbb{Z}_+. \end{cases} \tag{1.4}$$

It shows that there exist jumps when human activities are considered continuously on population model. Especially, it reflects the combination of continuous harvesting and impulsive harvesting. Based on impulsive control theory, we shall investigate the effects of impulsive harvesting and stocking, and establish conditions for the persistence and existence of a unique globally attractive positive periodic solution of system (1.4). The development results are different from the existing ones in the literature and the relation between dynamics of population models and jumps will be emphasized in this paper.

The remainder of this paper is organized as follows. In Section 2, some necessary definitions and preliminary results are presented. In Section 3, some new criteria for persistence and periodicity are presented. In Section 4, simulations are given to illustrate the effectiveness of the main results. Finally, conclusions are drawn in Section 5.

2. Preliminaries

Notations. Let $\mathbb{R}$ denote the set of real numbers, $\mathbb{R}_+$ the set of nonnegative real numbers, $\mathbb{R}_-$ the set of nonpositive real numbers and $\mathbb{Z}_+$ the set of positive integers. For any interval $J \subseteq \mathbb{R}$, set $S \subseteq \mathbb{R}^k (1 \leq k \leq N), C(J,S) = \{\varphi : J \to S$ is continuous$\}$ and $PC(J,S) = \{\varphi : J \to S$ is continuous everywhere except at finite number of points t, at which $\varphi(t^+)$, $\varphi(t^-)$ exist and $\varphi(t^+) = \varphi(t)\}$. $\Sigma_+ = \{c(t) : c \in PC(\mathbb{R}_+, \mathbb{R}_+)$ and for any interval $(\alpha,\beta) \subseteq \mathbb{R}_+, c(t) \not\equiv 0.\ \}$. $\Sigma_- = \{c(t) : c \in PC(\mathbb{R}_+, \mathbb{R}_-)$ and for any interval $(\alpha,\beta) \subseteq \mathbb{R}_+, c(t) \not\equiv 0.\ \}$. Given a continuous function f which is defined on $\Lambda \subseteq \mathbb{R}$, we set $f^I \doteq \inf_{s\in\Lambda} f(s), f^S \doteq \sup_{s\in\Lambda} f(s)$. The jump times $t_k, k \in \mathbb{Z}_+$, satisfy $0 \leq t_0 < t_1 < \ldots < t_k \to +\infty$ as $k \to +\infty$.

System (1.4) may be rewritten as:

$$\begin{cases} x'(t) = xf(t,x), \quad t \geq t_0, \ t \neq t_k, \\ \Delta x|_{t=t_k} = x(t_k) - x(t_k^-) = I_k(x(t_k^-)), \quad k \in \mathbb{Z}_+, \\ x(t_0) = x_0, \end{cases} \tag{2.1}$$

where $x_0 \in \mathbb{R}_+$, $f \in PC([t_0,\infty)\times\mathbb{R}_+,\mathbb{R})$. The numbers $x(t_k^-)$ and $x(t_k)$ denote the population densities of the species before and after jumps at the moments t_k, respectively. $I_k \in C(\mathbb{R}_+,\mathbb{R})$, which characterize the magnitude of the jumps on the species at the moments t_k and satisfy $I_k(s)+s > 0$ for any $s \in \mathbb{R}_+, k \in \mathbb{Z}_+$; In particular, when $I_k > 0$, the perturbation stands for planting of the species, while $I_k < 0$ stands for harvesting. We assume that system (2.1) satisfies some fundamental conditions which guarantee the global existence and uniqueness of the solutions on $[t_0,\infty)$, see ([17, 30]). In the following, denote by $x(t) = x(t,t_0,x_0)$ the solution of system (2.1) with initial value (t_0, x_0).

Definition 2.1. (see [17]) System (2.1) is said to be persistent, if there exist constants $M > 0$ and $m > 0$ such that each positive solution $x(t) = x(t,t_0,x_0)$ of system (2.1) satisfies

$$m \leq \liminf_{t\to+\infty} x(t) \leq \limsup_{t\to+\infty} x(t) \leq M.$$

Definition 2.2. (see [17]) A map $x : \mathbb{R}_+ \to \mathbb{R}_+$ is said to be a ω-periodic solution of system (2.1), if
(i) $x(t)$ is a piecewise continuous map with first-class discontinuity points and satisfies (2.1);
(ii) $x(t)$ satisfies $x(t+\omega) = x(t), t \neq t_k$ and $x(t_k+\omega^+) = x(t_k^+), k \in \mathbb{Z}_+$.

Definition 2.3. (see [17]) Assume that $x^\star(t) = x^\star(t,t_0,x_0^\star)$ be a positive periodic solution of system (2.1). Then $x^\star$ is said to be globally attractive, if for any positive solution $x(t) = x(t,t_0,x_0)$ of system (2.1), it holds that

$$\lim_{t\to+\infty} \|x(t) - x^\star(t)\| = 0.$$

3. Main results

Lemma 3.1. *The set $\mathbb{R}_+$ is the positively invariant set of system (2.1).*

Proof. Note that $I_k(s) + s > 0$ for any $s \in \mathbb{R}_+, k \in \mathbb{Z}_+$. The proof of Lemma 3.1 is obvious.

Theorem 3.1. *Assume that there exist constants $q > 1, \lambda > 1$ and $M > m > 0$ such that*
(i) $\dfrac{1-q}{q} \leq \dfrac{I_k(s)}{s} \leq \lambda - 1, \quad s > 0;$

(ii) $f(t,M) \in \Sigma_-$ *and* $\sup_{k\in\mathbb{Z}_+} \int_{t_{k-1}}^{t_k} f(s,M)ds < -\ln\lambda$;

(iii) $f(t,m) \in \Sigma_+$ *and* $\inf_{k\in\mathbb{Z}_+} \int_{t_{k-1}}^{t_k} f(s,m)ds > \ln q$;

(iv) $[f(t,x) - f(t,y)]sgn(x-y) \le 0, \quad (t,x,y) \in \mathbb{R}_+ \times \mathbb{R}_+ \times \mathbb{R}_+$.

Then the set

$$\Delta = \{x \in \mathbb{R}_+ : \frac{m}{q^3} \le x \le M\lambda^3\}$$

is the ultimately bounded set of system (2.1), that is, system (2.1) is persistent.

Proof. Let $x(t) = x(t,t_0,x_0)$ be the solution of system (2.1) with initial value (t_0,x_0), where $(t_0,x_0) \in \mathbb{R}_+ \times \mathbb{R}_+$. First, we show that there exists $t^* \ge t_0$ such that $x(t^*) \le M$. If $x_0 \in (0,M]$, then the conclusion is obvious. Or else, assume that $x(t) > M$ for all $t \ge t_0$. Then it follows from (iv) and system (2.1) that $x'(t) = xf(t,x) \le xf(t,M), t \in [t_{k-1},t_k), k \in \mathbb{Z}_+$, which, together with (i) and (ii), yields

$$\begin{aligned} x(t_{k+1}) \le \lambda x(t_{k+1}^-) &\le \lambda^{k+1} x(t_0) \exp\Big(\int_{t_0}^{t_{k+1}} f(s,M)ds\Big) \\ &= x(t_0) \exp\Big((k+1)\ln\lambda + \int_{t_0}^{t_{k+1}} f(s,M)ds\Big) \\ &\le x(t_0)\exp\Big((k+1)[\ln\lambda + \mu]\Big) \to 0, \quad as\ k \to \infty, \end{aligned}$$

where $\mu \doteq \sup_{k\in\mathbb{Z}_+} \int_{t_{k-1}}^{t_k} f(s,M)ds$. This is a contradiction with the above assumption that $x(t) > M$ for all $t \ge t_0$ and thus there exists $t^* \ge t_0$ such that $x(t^*) \le M$.

Now we show that $x(t) \le M\lambda^3, t \ge t^*$. Suppose not, then there exists $\bar{t} > t^*$ such that $x(\bar{t}^+) \ge M\lambda^3$ and $x(\bar{t}^-) \le M\lambda^3$. Since $x(t^*) \le M$, there exists $\underline{t} \in [t^*,\bar{t}]$ such that $x(\underline{t}^+) \ge M$ and $x(\underline{t}^-) \le M$. Moreover, $\underline{t} < \bar{t}$. In fact, if $\underline{t} = \bar{t}$, then $\lambda M \ge \lambda x(\underline{t}^-) \ge x(\underline{t}^+) \ge M\lambda^3$, which contradicts $\lambda > 1$. Thus we get $M \le x(t) \le M\lambda^3, t \in [\underline{t},\bar{t}]$. Then there are three cases: (a) If there is no jump on $[\underline{t},\bar{t}]$, then $x'(t) = xf(t,x) \le xf(t,M) \le 0, t \in [\underline{t},\bar{t}]$, which implies that $M\lambda^2 \le x(\bar{t}^-) \le x(\underline{t}^+) \le \lambda M$. This is a contradiction. (b) If there exists a jump on $[\underline{t},\bar{t}]$, assume that $\underline{t} < t_\sigma < \bar{t}$, where $\sigma \in \mathbb{Z}_+$. Then it follows from $f(t,M) \in \Sigma_-$ that

$$\begin{aligned} M\lambda^2 \le x(\bar{t}^-) &\le \lambda x(\underline{t}^+) \exp\Big(\int_{\underline{t}}^{\bar{t}} f(s,M)ds\Big) \\ &\le \lambda^2 M \exp\Big(\int_{\underline{t}}^{\bar{t}} f(s,M)ds\Big) < \lambda^2 M, \end{aligned}$$

which is also a contradiction. (c) If there exist some jumps on $[\underline{t},\bar{t}]$, assume that $\underline{t} < t_\sigma < t_{\sigma+1} < \cdots < t_{\sigma+l} < \bar{t}$, where $\sigma, l \in \mathbb{Z}_+$. Then it can be deduced that

$$x(t_{\sigma+l}^-) \le \lambda^l x(\underline{t}^+) \exp\Big(\int_{\underline{t}}^{t_{\sigma+l}} f(s,M)ds\Big),$$

which lead to

$$M\lambda^2 \le x(\bar{t}^-) \le x(t_{\sigma+l}^+) \exp\Big(\int_{t_{\sigma+l}}^{\bar{t}} f(s,M)ds\Big)$$

$$
\begin{aligned}
&\le \lambda^{l+2} M \exp\Big(\int_{\underline{t}}^{\bar{t}} f(s,M)ds\Big)\\
&\le \lambda^{2} M \exp\Big(l\ln\lambda + \int_{t_\sigma}^{t_{\sigma+l}} f(s,M)ds\Big)\\
&\le \lambda^{2} M \exp\Big(l[\ln\lambda+\mu]\Big) < \lambda^2 M.
\end{aligned}
$$

Obviously, this is also a contradiction and thus all cases (a-c) are impossible. Hence, it holds that $x(t) \le M\lambda^3, t \ge t^*$.

Next we show that there exists $t^\star \ge t^*$ such that $x(t^\star) > m$. Suppose not, then $x(t) \le m, t \ge t^*$. It follows from (iv) and (2.1) that $x'(t) = xf(t,x) \ge xf(t,m), t \in [t_{k-1}, t_k) \cap [t^*, \infty), k \in \mathbb{Z}_+$. Assume that $t^* \in [t_{l-1,} t_l)$, for some $l \in \mathbb{Z}_+$, then it can be deduced from (i) and (iii) that

$$
\begin{aligned}
x(t_{l+k}) \ge \frac{1}{q}x(t_{l+k}^-) &\ge \frac{1}{q^{k+1}}x(t^*)\exp\Big(\int_{t^*}^{t_{l+k}} f(s,m)ds\Big)\\
&\ge \frac{1}{q}x(t^*)\exp\Big(-k\ln q + \int_{t_l}^{t_{l+k}} f(s,m)ds\Big)\\
&\ge \frac{1}{q}x(t^*)\exp\Big(k[\eta - \ln q]\Big) \to \infty, \quad as\ k\to\infty,
\end{aligned}
$$

where $\eta \doteq \inf_{k\in\mathbb{Z}_+} \int_{t_{k-1}}^{t_k} f(s,m)ds$. This is a contradiction with $x(t) \le m, t \ge t^*$ and thus there exists $t^\star \ge t^*$ such that $x(t^\star) > m$.

Furthermore, we show that $x(t) \ge \frac{m}{q^3}, t \ge t^\star$. Suppose not, then there exists $\hat{t} \ge t^\star$ such that $x(\hat{t}^+) \le \frac{m}{q^3}$ and $x(\hat{t}^-) \ge \frac{m}{q^3}$. Note that $x(t^\star) > m$ and $q > 1$, there exists $\tilde{t} \in [t^\star, \hat{t})$ such that $x(\tilde{t}^+) \le m$ and $x(\tilde{t}^-) \ge m$. Thus we get that $\frac{m}{q^3} \le x(t) \le m, t \in [\tilde{t}, \hat{t}]$. Then there are also three cases: (d) If there is no jump on $[\tilde{t}, \hat{t}]$, then $x'(t) = xf(t,x) \ge xf(t,m) \ge 0, t \in [\tilde{t}, \hat{t}]$, which implies that $\frac{m}{q} \le x(\tilde{t}^+) \le x(\hat{t}^-) \le \frac{m}{q^2}$. This is a contradiction. (e) If there exists a jump on $[\tilde{t}, \hat{t}]$, assume that $\tilde{t} < t_\rho < \hat{t}$, where $\rho \in \mathbb{Z}_+$. Then it follows from $f(t,m) \in \Sigma_+$ that

$$
\begin{aligned}
\frac{m}{q^2} \ge x(\hat{t}^-) &\ge \frac{1}{q}x(\tilde{t}^+)\exp\Big(\int_{\tilde{t}}^{\hat{t}} f(s,m)ds\Big)\\
&\ge \frac{m}{q^2}\exp\Big(\int_{\tilde{t}}^{\hat{t}} f(s,m)ds\Big) > \frac{m}{q^2},
\end{aligned}
$$

which is a contradiction. (f) If there exist some jumps on $[\tilde{t}, \hat{t}]$, assume that $\tilde{t} < t_\rho < t_{\rho+1} < \cdots < t_{\rho+s} < \hat{t}$, where $\rho, s \in \mathbb{Z}_+$. Then it can be deduced that

$$
x(t_{\rho+s}^-) \ge \frac{1}{q^s}x(\tilde{t}^+)\exp\Big(\int_{\tilde{t}}^{t_{\rho+s}} f(s,m)ds\Big),
$$

which lead to

$$
\frac{m}{q^2} \ge x(\hat{t}^-) \ge x(t_{\rho+s}^+)\exp\Big(\int_{t_{\rho+s}}^{\hat{t}} f(s,m)ds\Big)
$$

$$
\begin{aligned}
&\geq \frac{1}{q^{s+1}} x(\tilde{t}^{+}) \exp\Big(\int_{\tilde{t}}^{\hat{t}} f(s,m)ds \Big) \\
&\geq \frac{1}{q} x(\tilde{t}^{+}) \exp\Big(-s\ln q + \int_{\tilde{t}}^{\hat{t}} f(s,m)ds \Big) \\
&\geq \frac{m}{q^2} \exp\Big(s[\eta - \ln q] \Big) > \frac{m}{q^2}.
\end{aligned}
$$

Obviously, it is also a contradiction and thus all cases (d-f) are impossible. The proof of Theorem 3.1 is there completed.

Remark 3.1. Note that condition (*iv*) in Theorem 3.1 can be replaced by the following stronger one: $\frac{\partial f(t,x)}{\partial x} \leq 0,\ (t,x) \in \mathbb{R}_+ \times \mathbb{R}_+$, which can be checked more easily in practical problems. In addition, from Theorem 3.1 one may note that it is possible that $I_k \geq 0$ or $I_k \leq 0$. Thus the development results can be applied to logistic systems with impulsive planting or/and impulsive harvesting. In particular, when there is no jump, we have the following two Corollaries.

Corollary 3.1. *Assume that there exist constants $M > m > 0$ such that*
(i) $f(t,M) \leq 0, t \geq t_0$, and $\int_{t_0}^{+\infty} f(s,M)ds \to -\infty$;
(ii) $f(t,m) \geq 0, t \geq t_0$, and $\int_{t_0}^{+\infty} f(s,m)ds \to +\infty$;
(iii) $[f(t,x) - f(t,y)]sgn(x-y) \leq 0,\ \ (t,x,y) \in \mathbb{R}_+ \times \mathbb{R}_+ \times \mathbb{R}_+$.

Then the set $\Delta = \{x \in \mathbb{R}_+ : m \leq x \leq M\}$ is the ultimately bounded set of system (2.1) without jumps, that is, system (2.1) without jumps is persistent.

Corollary 3.2. *Assume that there exist constants $M > m > 0$ such that $f(t,M) \leq 0$ and $f(t,m) \geq 0, t \geq t_0$, then $\Delta = \{x \in \mathbb{R}_+ : m \leq x \leq M\}$ is the invariant set of system (2.1) without jumps.*

Remark 3.2. Corollaries 3.1 and 3.2 can be easily derived by the proof process of Theorem 3.1.

Consider the following logistic differential system with jumps

$$
\begin{cases}
x'(t) = x(t)[r(t) - a(t)x(t)], & t \geq t_0,\ t \neq t_k, \\
\Delta x|_{t=t_k} = x(t_k) - x(t_k^-) = I_k(x(t_k^-)), & k \in \mathbb{Z}_+, \\
x(t_0) = x_0,
\end{cases}
\tag{3.1}
$$

where $x_0 \in \mathbb{R}_+$, $r, a \in PC(\mathbb{R}_+, \mathbb{R}_+)$ and admit positive upper and lower bounds which are natural for biological meanings.

Corollary 3.3. *Suppose that there exist constants $q > 1, \lambda > 1$ and $M > m > 0$ such that*
(i) $\frac{1-q}{q} \leq \frac{I_k(s)}{s} \leq \lambda - 1,\ \ s > 0$;
(ii) $m < \frac{r^I}{a^S}$ and $M > \frac{r^S}{a^I}$;
(iii) $\inf_{k\in\mathbb{Z}_+}\{t_k - t_{k-1}\} > \max\Big\{\frac{\ln q}{r^I - a^S m}, \frac{\ln \lambda}{a^I M - r^S}\Big\}$.

Then the set

$$
\Delta = \{x \in \mathbb{R}_+ : \frac{m}{q^3} \leq x \leq M\lambda^3\}
$$

is the ultimately bounded set of system (3.1).

Corollary 3.4. *Suppose that there exist constants $q > 1, \lambda > 1$ such that*
(i) $\frac{1-q}{q} \leq \frac{I_k(s)}{s} \leq \lambda - 1,\ \ s > 0$;

(ii) $\inf_{k\in\mathbb{Z}_+}\{t_k - t_{k-1}\} > \frac{\ln q}{r^I}$.

Then system (3.1) is persistent.

Consider the following logistic differential system with jumps and cyrtoid type harvesting

$$\begin{cases} x'(t) = x(t)[r(t) - a(t)x(t)] - \dfrac{\alpha(t)x(t)}{x(t)+\beta(t)}, \quad t \geq t_0, \ t \neq t_k, \\ \Delta x|_{t=t_k} = x(t_k) - x(t_k^-) = I_k(x(t_k^-)), \quad k \in \mathbb{Z}_+, \\ x(t_0) = x_0, \end{cases} \tag{3.2}$$

where $x_0 \in \mathbb{R}_+$, $r, a, \alpha, \beta \in PC(\mathbb{R}_+, \mathbb{R}_+)$ and admit positive upper and lower bounds which are natural for biological meanings.

Corollary 3.5. *Suppose that $r^I\beta^I > \alpha^S$ and $a^I(\beta^I)^2 > \alpha^S$. Moreover, there exist constants $q > 1, \lambda > 1$ and $M > m > 0$ such that*

(i) $\frac{1-q}{q} \leq \frac{I_k(s)}{s} \leq \lambda - 1, \quad s > 0;$

(ii) $r^S - a^I M - \frac{\alpha^I}{M+\beta^S} < 0;$

(iii) $r^I - a^S m - \frac{\alpha^S}{m+\beta^I} > 0;$

(iii) $\inf_{k\in\mathbb{Z}_+}\{t_k - t_{k-1}\} > \max\Big\{\frac{\ln q}{r^I - a^S m - \frac{\alpha^S}{m+\beta^I}}, \ \frac{\ln \lambda}{a^I M + \frac{\alpha^I}{M+\beta^S} - r^S}\Big\}.$

Then the set

$$\Delta = \{x \in \mathbb{R}_+ : \frac{m}{q^3} \leq x \leq M\lambda^3\}$$

is the ultimately bounded set of system (3.2).

Corollary 3.6. *Suppose that $r^I\beta^I > \alpha^S$ and $a^I(\beta^I)^2 > \alpha^S$. Moreover, there exist constants $q > 1, \lambda > 1$ such that*

(i) $\frac{1-q}{q} \leq \frac{I_k(s)}{s} \leq \lambda - 1, \quad s > 0;$

(ii) $\inf_{k\in\mathbb{Z}_+}\{t_k - t_{k-1}\} > \frac{\ln q}{r^I - \frac{\alpha^S}{\beta^I}}$.

Then system (3.2) is persistent.

Remark 3.3. Corollaries 3.3 and 3.5 can be directly derived by Theorem 3.1. For Corollaries 3.4 and 3.6, one only needs choose small enough $m > 0$ and large enough $M > 0$ such that (*ii*) and (*iii*) in the corresponding Corollary hold.

In the following, we shall investigate the periodic solution problem of system (2.1).

Theorem 3.2. *Assume that there exist constants $q > 1, m > 0, \omega > 0$ and $\theta \in \mathbb{Z}_+$ such that*

(i) $I_k(s) = (\rho_k - 1)s, s \in \mathbb{R}_+$, *where* $\rho_k \geq \frac{1}{q}, \ k \in \mathbb{Z}_+;$

(ii) $f(t, m) \in \Sigma_+$ *and* $\inf_{k\in\mathbb{Z}_+} \int_{t_{k-1}}^{t_k} f(s, m)ds > \ln q;$

(iii) $f(t+\omega, \bullet) = f(t, \bullet)$, $t_k + \omega = t_{k+\theta}$ *and* $\rho_{k+\theta} = \rho_k, k \in \mathbb{Z}_+;$

(iv) $\frac{f(t,x) - f(t,y)}{x-y} \leq -p(t)$, *where* $p \in PC(\mathbb{R}_+, \mathbb{R}_+)$ *and satisfies* $\int_{t_0}^{\infty} p(s)ds = +\infty.$

Then system (2.1) has a unique positive ω-periodic solution, which is globally attractive.

Proof. Let $y(t) = \ln x(t)$, then system (2.1) may be rewritten as

$$\begin{cases} y'(t) = f(t, e^{y(t)}), \quad t \geq t_0,\ t \neq t_k, \\ y(t_k) - y(t_k^-) = \ln \rho_k, \quad k \in \mathbb{Z}_+, \\ y(t_0) = \ln x_0 \in \mathbb{R}. \end{cases} \tag{3.3}$$

Obviously, the investigation of the positive periodic solution problem for (2.1) is equal to investigate the periodic solution problem for system (3.3).

First, by condition (*iii*) it is easy to show that system (3.3) has an ω-periodic solution if there exists a $y_0 \in \mathbb{R}$ such that $y(t_0 + \omega, t_0, y_0) = y_0$, where $y(t, t_0, y_0)$ is the solution of system (3.3) through (t_0, y_0). In fact, one may establish a solution as follows:

$$\widetilde{y}(t) = \begin{cases} y(t), & t \in [t_0, t_0 + \omega], \\ y(t - n\omega), & t \in [t_0 + n\omega, t_0 + (n+1)\omega]. \end{cases}$$

Obviously, $\widetilde{y}$ is ω-periodic. Next we show that $\widetilde{y}$ is a solution of system (3.3). For any $t \in [t_0 + n\omega, t_0 + (n+1)\omega]$, if $t \neq t_k, k \in \mathbb{Z}_+$, then

$$\widetilde{y}'(t) = y'(t - n\omega) = f(t - n\omega, e^{y(t-n\omega)}) = f(t, e^{y(t-n\omega)}) = f(t, e^{\widetilde{y}(t)});$$

if $t = t_m$ for some $m \in \mathbb{Z}_+$, then it follows from (*iii*) that $t_m - n\omega = t_{m-n\theta}$, which yields that

$$\widetilde{y}(t_m) = y(t_{m-n\theta}) = y(t_{m-n\theta}^-) + \ln \rho_{m-n\theta} = y(t_m^- - n\omega) + \ln \rho_m = \widetilde{y}(t_m^-) + \ln \rho_m.$$

By the above discussion, $\widetilde{y}$ is a solution of system (3.3). Moreover, by the existence-uniqueness theorem, $\widetilde{y} \equiv y, t \geq t_0$. That is, y is an ω-periodic solution of system (3.3).

Let $z(t) = z(t, t_0, z_0)$ and $h(t) = h(t, t_0, h_0)$ be two solutions of system (3.3) through (t_0, z_0) and (t_0, h_0), respectively, where $z_0 \neq h_0$. Define $\Gamma(t) = |h(t) - z(t)|, t \geq t_0$. Then

$$\Gamma(t_k) = |h(t_k) - z(t_k)| = |h(t_k^-) - z(t_k^-)| = \Gamma(t_k^-),\ k \in \mathbb{Z}_+. \tag{3.4}$$

In addition, it can be deduced that

$$\begin{aligned} \Gamma'(t) &= \left[f(t, e^{h(t)}) - f(t, e^{z(t)})\right] sgn(h - z) \\ &= \frac{f(t, e^{h(t)}) - f(t, e^{z(t)})}{e^{h(t)} - e^{z(t)}} |e^{h(t)} - e^{z(t)}| \\ &\leq -p(t) e^{\xi(t)} \Gamma(t), t \geq t_0, \end{aligned}$$

where $\xi(t)$ is a real value between $h(t)$ and $z(t)$. By conditions (*i*), (*ii*) and Theorem 3.1, we know that there exists $T_0 \geq t_0$ such that $e^{\xi(t)} \geq \frac{m}{q^3}, t \geq T_0$, which yields that

$$\Gamma'(t) \leq -p(t) \frac{m}{q^3} \Gamma(t), t \geq T_0. \tag{3.5}$$

For given $T_0 \geq t_0$, define

$$G(T_0) \doteq \frac{|h(T_0) - z(T_0)|}{|h(t_0) - z(t_0)|}. \tag{3.6}$$

Considering (3.4), (3.5) and (3.6), we have

$$\Gamma(t) = |h(t) - z(t)| \leq G(T_0)|h(t_0) - z(t_0)| \exp\Big(- \frac{m}{q^3} \int_{T_0}^{t} p(s)ds\Big), t \geq T_0, \tag{3.7}$$

which implies that there exists $T_1 \geq T_0$ such that

$$|h(t) - z(t)| \leq \frac{1}{2}|h(t_0) - z(t_0)|, t \geq T_1. \tag{3.8}$$

Define an operator

$$\mathscr{F} : r_0 \to r(t_0 + \omega, t_0, r_0),$$

where $r(t, t_0, r_0)$ is the solution of system (3.3) through (t_0, r_0). Obviously, operator $\mathscr{F}$ maps the set $\mathbb{R}$ into itself. Moreover,

$$\mathscr{F}^k(r_0) = r(t_0 + k\omega, t_0, r_0), k \in \mathbb{Z}_+.$$

Let k large enough such that $t_0 + k\omega \geq T_1$, then it follows from (3.8) that

$$|\mathscr{F}^k(r_0) - \mathscr{F}^k(s_0)| \leq \frac{1}{2}|r_0 - s_0|,$$

where $s_0 \in \mathbb{R}$. Thus, operator $\mathscr{F}$ is a contraction mapping in Banach space. Using Banach fixed point theorem, there exists a unique $r_0^* \in \mathbb{R}$ such that $\mathscr{F}(r_0^*) = r_0^*$. Hence, system (3.3) has a ω-periodic solution $r^*(t) = r^*(t, t_0, r_0^*)$. That is, system (2.1) has a positive ω-periodic solution $x^*(t) = e^{r^*(t)}$.

Next we show that $r^*(t) = r^*(t, t_0, r_0^*)$ is the unique ω-periodic solution of system (3.3) and all other solutions converge to it. Suppose that $r(t) = r(t, t_0, r_0)$ is any another solution of system (3.3) through (t_0, r_0), then it follows from (3.7) that

$$|r^*(t) - r(t)| \leq G(T_0)|r_0^* - r_0| \exp\Big(- \frac{m}{q^3} \int_{T_0}^{t} p(s)ds\Big), t \geq T_0,$$

where

$$G(T_0) \doteq \frac{|r^*(T_0) - r(T_0)|}{|r_0^* - r_0|}.$$

It is obvious that $|r^*(t) - r(t)| \to 0$, as $t \to \infty$, which implies that system (2.1) has a unique positive ω-periodic solution, which is globally attractive. The proof of Theorem 3.1 is completed.

Remark 3.4. One may observe from Theorem 3.2 that, to investigate the periodic problem of system (2.1), there is no any restriction on the upper bound of jump constant ρ_k, that is, ρ_k may be large enough if it satisfies the periodic condition $\rho_{k+\theta} = \rho_k$. Moreover, condition $\rho_k \geq \frac{1}{q}$ in Theorem 3.2 can be replaced by $\min_{1\leq i\leq\theta} \rho_i \geq \frac{1}{q}$.

For system (3.1) and (3.2), we have

Corollary 3.7. *Suppose that there exist constants $q > 1, \omega > 0$ and $\theta \in \mathbb{Z}_+$ such that*

(i) $I_k(s) = (\rho_k - 1)s, s \in \mathbb{R}_+$, where $\min\limits_{1\leq i\leq\theta} \rho_i \geq \dfrac{1}{q}$;

(ii) $\min_{1\le i\le\theta}\{t_i - t_{i-1}\} > \dfrac{\ln q}{r^I}$;
(iii) $r(t+\omega) = r(t), a(t+\omega) = a(t), t_k + \omega = t_{k+\theta}$ and $\rho_{k+\theta} = \rho_k, k \in \mathbb{Z}_+$.

Then system (3.1) has a unique positive ω-periodic solution, which is globally attractive.

Corollary 3.8. *Suppose that* $r^I\beta^I > \alpha^S$ *and* $a^I(\beta^I)^2 > \alpha^S$. *Moreover, there exist constants* $q > 1, \omega > 0$ *and* $\theta \in \mathbb{Z}_+$ *such that*
(i) $I_k(s) = (\rho_k - 1)s, s \in \mathbb{R}_+$, *where* $\min_{1\le i\le\theta}\rho_i \ge \dfrac{1}{q}$;
(ii) $\min_{1\le i\le\theta}\{t_i - t_{i-1}\} > \dfrac{\ln q}{r^I - \frac{\alpha^S}{\beta^I}}$;
(iii) $r(t+\omega) = r(t), a(t+\omega) = a(t), t_k + \omega = t_{k+\theta}$ and $\rho_{k+\theta} = \rho_k, k \in \mathbb{Z}_+$.

Then system (3.2) has a unique positive ω-periodic solution, which is globally attractive.

Remark 3.5. The research thought in the paper is completely new and can be extended to the investigation of jump for delay logistic differential system.

4. Applications

In this section, two examples and their simulations are presented to show the effectiveness of our obtained results.

Example 4.1. Consider the logistic type differential equations with jumps:

$$\begin{cases} x'(t) = x\Big[\ln\Big(7 + \dfrac{2t}{1+t^2}\Big) - \dfrac{1+t^2}{1+t+t^2}\ln(1+x^2)\Big], & t \ge 0, t \ne t_k, \\ x(t_k^+) = \gamma x(t_k), & k \in \mathbb{Z}_+, \\ x(0) = x_0 \in \mathbb{R}_+, \end{cases} \tag{4.1}$$

where $\gamma > 0$ is a given constant. For system (4.1), we have

Property 4.1. *Case* $\gamma < 1$. *System (4.1) is persistent, if* $\inf_{k\in\mathbb{Z}_+}\{t_k - t_{k-1}\} > -\log_7^\gamma$.

Proof. Let

$$f(t,x) = \ln\Big(7 + \frac{2t}{1+t^2}\Big) - \frac{1+t^2}{1+t+t^2}\ln(1+x^2), (t,x) \in \mathbb{R}_+ \times \mathbb{R}_+.$$

Then it is easy to check that

$$\ln 7 - \ln(1+x^2) \le f(t,x) \le \ln 9 - \frac{2}{3}\ln(1+x^2).$$

Since $\inf_{k\in\mathbb{Z}_+}\{t_k - t_{k-1}\} > -\log_7^\gamma$, there exists $m > 0$ small enough such that

$$\inf_{k\in\mathbb{Z}_+}\{t_k - t_{k-1}\} > -\frac{\ln\gamma}{\ln\frac{7}{1+m^2}} > 0. \tag{4.2}$$

For given m, let $\lambda = 2$, then there exists $M > m$ large enough such that

$$0 < \frac{\ln 2}{\frac{2}{3}\ln(1+M^2) - \ln 9} < -\frac{\ln\gamma}{\ln\frac{7}{1+m^2}}. \tag{4.3}$$

Choose $q = \frac{1}{\gamma}$ and considering (4.2), (4.3), it is easy to check that all conditions in Theorem 3.1 hold and thus system (4.1) is persistent for the case $\gamma < 1$.

Property 4.2. *Case* $\gamma > 1$. *System (4.1) is persistent, if* $\inf\limits_{k\in\mathbb{Z}_+}\{t_k - t_{k-1}\} > 0$.

Proof. Choose $q \in (1, \gamma]$ and $m > 0$ small enough such that

$$0 < \frac{\ln q}{\ln \frac{7}{1+m^2}} < \eta, \tag{4.4}$$

where $\eta \doteq \inf\limits_{k\in\mathbb{Z}_+}\{t_k - t_{k-1}\} > 0$. Let $\lambda = \gamma > 1$ and then choose $M > m$ large enough such that

$$\frac{\ln \lambda}{\frac{2}{3}\ln(1 + M^2) - \ln 9} < \eta. \tag{4.5}$$

By (4.4) and (4.5), it is easy to check that all conditions in Theorem 3.1 hold and thus system (4.1) is persistent for the case $\gamma > 1$.

Property 4.3. *Case* $\gamma = 1$. *System (4.1) is persistent.*

Proof. In view of Corollary 3.1 and the analysis of Properties 4.1 and 4.2, the above property is obvious.

Remark 4.1. In particular, if let $\gamma = 0.5$, then by Property 4.1 system (4.1) is persistent, if $\inf_{k\in\mathbb{Z}_+}\{t_k - t_{k-1}\} > 0.36$. For example, when $t_k = 0.4k$, Figure 1(a) shows that system (4.1) is persistent. However, when $t_k = 0.3k$, Property 4.1 is invalid. In this case, it is interesting to see that system (4.1) will become extinct, which is shown in Figure 1(b). This partially reflects the advantage of our development results. In addition, if let $\gamma = 2$ and 8, then by Property 4.2 system (4.1) is persistent, if $\inf_{k\in\mathbb{Z}_+}\{t_k - t_{k-1}\} > 0$. Figure 1(c) shows the case that $\gamma = 2, t_k = 0.2k$, Figure 1(d) shows the case that $\gamma = 8, t_k = k$.

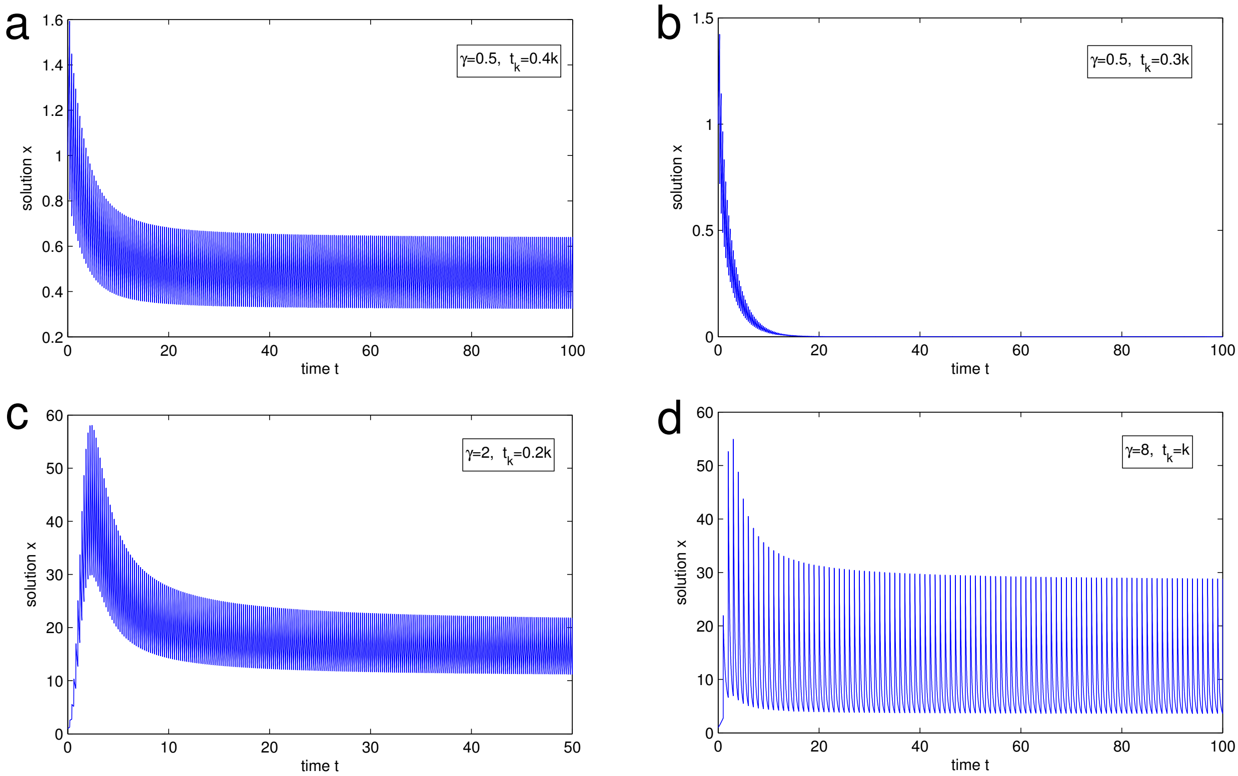

Figure 1. (a) State trajectory of system (4.1) with $\gamma = 0.5$ and $t_k = 0.4k$; (b) State trajectory of system (4.1) with $\gamma = 0.5$ and $t_k = 0.3k$; (c) State trajectory of system (4.1) with $\gamma = 2$ and $t_k = 0.2k$; (d) State trajectory of system (4.1) with $\gamma = 8$ and $t_k = k$.

Example 4.2. Consider the logistic type differential equations with jumps:

$$\begin{cases} x'(t) = x\Big[r + 0.1\sin\dfrac{2\pi}{\omega}t - \Big(1 + 0.2\cos\dfrac{2\pi}{\omega}t\Big)x\Big], & t \geq 0, t \neq t_k, \\ x(t_k^+) = \gamma_k x(t_k), & k \in \mathbb{Z}_+, \\ x(0) = x_0 \in \mathbb{R}_+, & \end{cases} \tag{4.6}$$

where $t_k = \dfrac{k\omega}{\theta}$; $r > 0.1, \omega > 0$ and $\theta \in \mathbb{Z}_+$ are some given constants; $\gamma_k > 0$ satisfying $\gamma_{k+\theta} = \gamma_k, k \in \mathbb{Z}_+$. For system (4.6), we have

Property 4.4. *System (4.6) has a unique positive ω-periodic solution, which is globally attractive, if*

$$\min_{1\leq i\leq\theta}\gamma_i < 1 \ \ \textit{and} \ \ \frac{1}{\min_{1\leq i\leq\theta}\gamma_i} < \exp\Big[\frac{\omega}{\theta}(r - 0.1)\Big].$$

Property 4.5. *System (4.6) has a unique positive ω-periodic solution, which is globally attractive, if* $\min\limits_{1\leq i\leq\theta}\gamma_i > 1$.

Proof. By Corollary 3.6, then above two Properties can be easily derived.

Remark 4.2. In particular, if let $\gamma_k = 0.8, r = 0.4$ and $\omega = \theta$, then by Property 4.4 system (4.6) has a unique globally attractive positive ω-periodic solution. When $\omega = 2$ and 5, the corresponding simulations are given in Figure 2(a) and 2(b), respectively. Under the same conditions, if let $\gamma_k = 2$, then by Property 4.5 system (4.6) has a unique globally attractive positive ω-periodic solution, which are shown in Figure 2(c) and 2(d) for $\omega = 1$ and 10, respectively.

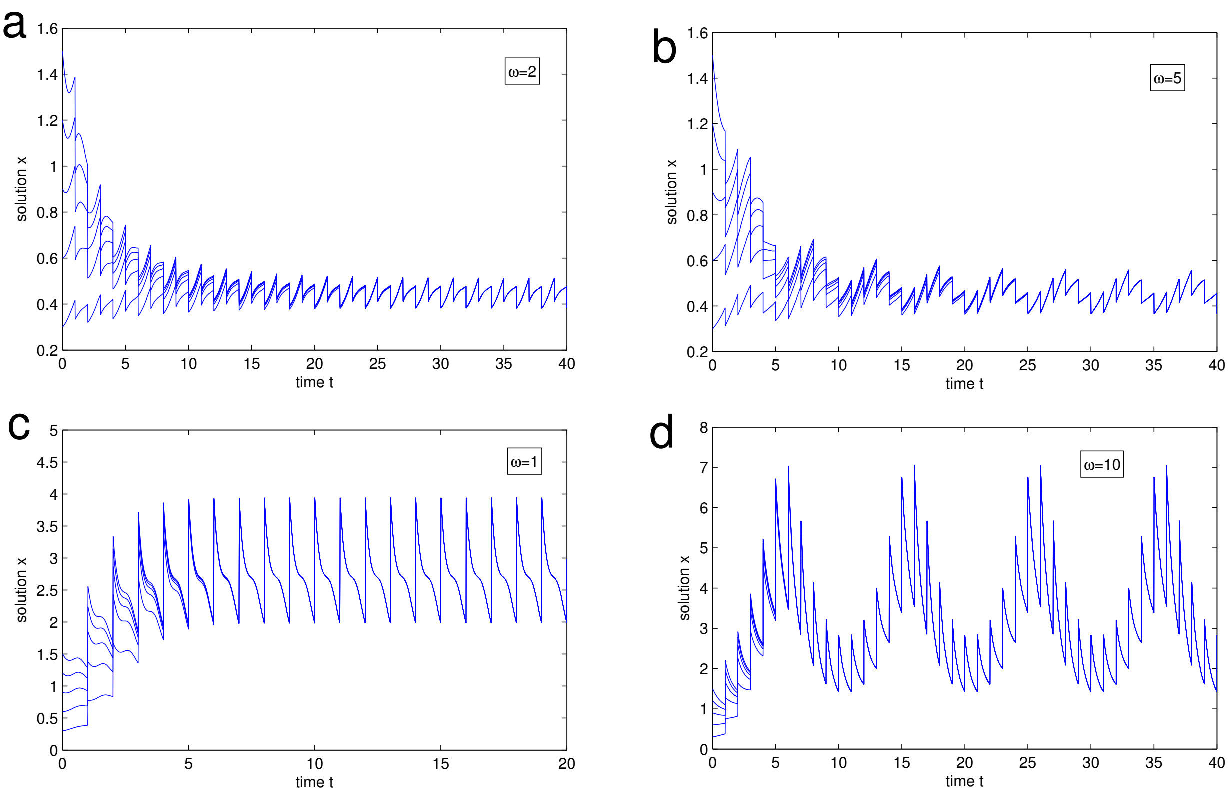

Figure 2. (a) State trajectories of system (4.6) with $\gamma_k = 0.8$ and $\omega = 2$; (b) State trajectories of system (4.6) with $\gamma_k = 0.8$ and $\omega = 5$; (c) State trajectories of system (4.6) with $\gamma_k = 2$ and $\omega = 1$; (d) State trajectories of system (4.6) with $\gamma_k = 2$ and $\omega = 10$.

5. Conclusions

In this paper, we investigated a class of logistic type differential system with jumps. Based on impulsive control theory, some new sufficient condition ensuring the permanence and existence of a unique globally attractive positive periodic solution were derived. The developed method is different from the usual methods in other literatures. Two numerical examples were given to illustrate the effectiveness and advantages of the results. The research thought in the paper can be extended to the investigation of jump for impulsive logistic differential system with time delays. In the near future, we shall do some further research on this topic.

Conflict of interest

The author declares no conflict of interest.

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