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Research article

Two-uniqueness of rational ghost soliton solution and well-posedness of perturbed Einstein-Yang-Mills equations

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Abstract: In this paper, we discuss the uniqueness and existence of local solutions for the perturbed static, spherically symmetric Einstein-Yang-Mills (EYM) equations with gauge group $SU(2)$. Moreover, we show that the rational expression solutions to the equations only happened in traditional Schwarzschild solutions and Reissner-Nordstrom solutions. From these results, we can infer that there is no rational ghost soliton solution for the EYM equations.

Keywords: perturbed Einstein-Yang-Mills equations; rational expression solutions; $C^{2+\alpha}$ solutions; well-posedness; ghost soliton
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1. Introduction

We recall that for the spherically symmetric EYM equations, the Einstein metric is of the form

$$ds^2 = -AC^2dt^2 + A^{-1}dr^2 + r^2(d\theta^2 + sin^2\theta d\phi^2), \tag{1.1}$$

and the $SU(2)$ Yang-Mills curvature 2-form is

$$F = w'\tau_1 dr \wedge d\theta + w'\tau_2 \wedge (sin\theta d\phi) - (1 - w^2)\tau_3 d\theta \wedge (sin\theta d\phi). \tag{1.2}$$

Here A, C and w are functions of r, and (τ_1, τ_2, τ_3) form a basis for the Lie algebra $SU(2)$. Using (1.1) and (1.2), the spherically symmetric $SU(2)$ EYM equations are

$$rA' + (1 + 2w'^2)A = 1 - \frac{(1 - w^2)^2}{r^2}, \tag{1.3}$$

$$r^2Aw'' + [r(1-A) - \frac{(1-w^2)^2}{r}]w' + w(1-w^2) = 0, \tag{1.4}$$

and

$$\frac{C'}{C} = \frac{2w'^2}{r}. \tag{1.5}$$

We should point out that (1.3) and (1.4) do not involve C.

Neither the vacuum Einstein equations nor the pure Yang-Mills equations have nontrivial static globally regular solutions, so it is natural to conjecture that the coupled EYM equations also have no nontrivial globally regular solutions. In 1988, Bartnik and McKinnon [1] presented numerical evidence for the existence of a discrete family of globally regular solutions of the static EYM equations: the gravitational attraction can balance the Yang-Mills (YM) repulsive force. This prompts the mathematical theory of this conjecture came into being. However, until now, the behavior of the solution in the case of spherical symmetry is still unclear. For example, in 2018, Baxter [2] considered the case of spherical symmetry.

We note that Gross-Pitaevskii equation has been successfully used to deal with Bose-Einstein condensation equation by applying Hirota's method, Lie algebra structure and Backlund transformations, bilinear transform, etc (refer to [3–5] and their references), but we have not seen an example of applying these methods to Einstein-Yang-Mills equation.

In this paper, we mainly prove the following surprising properties of static spherically symmetric solutions of the $SU(2)$ Einstein-Yang-Mills equations.

Theorem 1.1. *There are no any rational expression solutions for static spherically symmetric solutions to the perturbed $SU(2)$ Einstein-Yang-Mills equations*

$$rA' + (1+2w'^2)A = 1 - \frac{(1-w^2)^2}{r^2}, \tag{1.6}$$

$$r^2Aw'' + [r(1-A) - \frac{(1-w^2)^2}{r}]w' + \lambda w(1-w^2) = 0, \tag{1.7}$$

except classical Schwarzschild solutions and Reissner-Nordstrom solutions. Here we add a perturbed coefficient into (1.2)*, and we assume that λ is waving near* 1.

In 1993, Smoller, Wasserman and Yau proved the existence of black hole solutions for the Einstein-Yang-Mills equations (see [7–9]). In 1995, Smoller and Wasserman [6] provided a rigorous proof of the existence and uniqueness of the solutions to the Einstein-Yang-Mills equations with gauge group $SU(2)$. After adding the perturbed term λ to (1.4), we discover that the perturbed equations have the same properties as the original equations in existence and uniqueness, see [9].

Theorem 1.2. *Let $\bar{r} > 0$ be given. Assume that $A(\bar{r}) = 0$, and $(\bar{w}, \beta)$ satisfies*

$$\Phi(\bar{r})\beta + \bar{w}(1-\bar{w}^2) = 0,$$

where $\Phi(\bar{r}) = \bar{r} - \frac{(1-\bar{w}^2)^2}{\bar{r}} \neq 0$. Then there exists a unique $C^{2,\alpha}$ solution $(A(r,\bar{w}), w(r,\bar{w}), w'(r,\bar{w}))$ of (1.6), (1.7) with the initial conditions $(A(\bar{r},\bar{w}), w(\bar{r},\bar{w}), w'(\bar{r},\bar{w})) = (0, \bar{w}, \beta)$, defined on some interval $\bar{r} < r < \bar{r} + s(\bar{w})$. The solution is analytic on $|r - \bar{r}| < s(\bar{w})$, and the one-parameter family $(A(r,\bar{w}), w(r,\bar{w}), w'(r,\bar{w}))$ is continuous about r and $\bar{w}$.

Theorem 1.3. *There are no any rational analytic solutions for EYM equations except classical Schwarzschild solutions or Reissner-Nordstrom solutions.*

2. Proof of Theorem 1.1

In this section, we prove Theorem 1.1. Set

$$A = \frac{a_n r^n + a_{n-1} r^{n-1} + \cdots + a_1 r + a_0}{b_m r^m + b_{m-1} r^{m-1} + \cdots + b_1 r + b_0}, \tag{2.1}$$

$$w = \frac{c_s r^s + c_{s-1} r^{s-1} + \cdots + c_1 r + c_0}{d_t r^t + d_{t-1} r^{t-1} + \cdots + d_1 r + d_0}, \tag{2.2}$$

where a_i, b_i, c_i, d_i $(i \in [1, n], n \geq 1)$ are constants.

To obtain a simple form, we suppose

$$A = \frac{q}{p}, \tag{2.3}$$

$$w = \frac{k}{h}, \tag{2.4}$$

where p, q, h, k represent the corresponding polynomial of denominator and numerator. A and w are simplified, i.e., q, p, h and k are irreducible. Without loss of generality, assuming $\lambda \in (0.5, 2)$ and $\lambda \neq 1$, then (1.6) and (1.7) can be simplified as

$$h^4 r^3 (q' p - p' q) + p h^4 r^2 q + 2pqr^2 (k' h - h' k)^2 = p^2 h^4 r^2 - p^2 (h^2 - k^2)^2, \tag{2.5}$$

$$r^3 h^2 q (k'' h^3 - h'' k h^2 - 2k' h^2 h' + 2hkh'^2) + (k' h - kh')[pr^2 h^4 - qr^2 h^4 - p(h^2 - k^2)^2] = \lambda prkh^3 (k^2 - h^2). \tag{2.6}$$

In the remainder of this paper, we use $\tilde{p}$, $\tilde{q}$, $\tilde{h}$, $\tilde{k}$ to donate the corresponding highest degree of p, q, h, k. Using $L_1 = h^4 r^3 (q' p - p' q)$, $L_2 = p h^4 r^2 q$, $L_3 = 2pqr^2 (k' h - h' k)^2$, $R_1 = p^2 h^4 r^2$, $R_2 = -p^2 (h^2 - k^2)^2$ to facilitate the writing of (2.5), and using $\tilde{L_1}$, $\tilde{L_2}$, $\tilde{L_3}$, $\tilde{R_1}$, $\tilde{R_2}$ to donate the corresponding highest degree of L_1, L_2, L_3, R_1, R_2. Similarly, using $L_{21} = r^3 h^2 q (k'' h^3 - h'' k h^2 - 2k' h^2 h' + 2hkh'^2)$, $L_{22} = (k' h - kh')[pr^2 h^4 - qr^2 h^4 - p(h^2 - k^2)^2]$, $R_{21} = \lambda prkh^3 (k^2 - h^2)$ to facilitate the writing of (2.6), and using $\tilde{L_{21}}$, $\tilde{L_{22}}$, $\tilde{R_{21}}$ to donate the corresponding highest degree of L_{21}, L_{22}, R_{21}.

Lemma 2.1. *If the solution of the perturbed EYM equations can be written as* (2.1) *and* (2.2). *Then*

$$\tilde{p} = \tilde{q}, \tilde{h} \geq \tilde{k} \tag{2.7}$$

or

$$\tilde{p} < \tilde{q}, \tilde{h} < \tilde{k} \tag{2.8}$$

holds. Moreover, if (2.7) *holds, then* $a_n = b_m$. *If* (2.8) *holds, then* $a_n b_m < 0$, $\tilde{q} - \tilde{p} = 2(\tilde{k} - \tilde{h}) \geq 2$. *Here* A *is the form of* (2.1).

Proof. It is obvious that p and h can not be zero. To prove this lemma, we start with the simple case and move on to the general case.

When q=0, i.e., $A \equiv 0$, then, from (1.6) and (1.7), $w \equiv 0$, $r = 1$ or $w^2 \equiv 1$, $r = 0$.

When $k = 0$, i.e., $w \equiv 0$, the solution is Reissner-Nordstrom Solution.

When q and p are constants, i.e., A is a constant, this case are included in the later discussion, where we discuss the situation that $\tilde{p} = \tilde{q}$, i.e., $p = q$, then $A \equiv 1$ and $w^2 \equiv 1$.

When h and k are constants, i.e., w is a constant, then $w^2 \equiv 1$ from (1.6) and (1.7), which is corresponding to Schwarzschild Solution.

Next, we discuss the case $k \neq 0$, $q \neq 0$, and A, w may not be constants. Before observing the highest degree of (2.5) and (2.6) to each terms, we pick out some special cases as which have been discussed above: If $q'p - p'q \equiv 0$, then $A' \equiv 0$; If $k'h - h'k \equiv 0$, then $w' \equiv 0$; If $h^2 = k^2$, then $w^2 \equiv 1$. Then in the following discussion, each terms in (2.5) can not be constant except R_2. Noted that

$$L_1 : 4\tilde{h} + \tilde{p} + \tilde{q} + 2 \sim 4\tilde{h} + 3; \qquad L_2 : 4\tilde{h} + \tilde{p} + \tilde{q} + 2;$$
$$L_3 : 2\tilde{h} + 2\tilde{k} + \tilde{p} + \tilde{q} \sim \tilde{p} + \tilde{q} + 2; \quad R_1 : 4\tilde{h} + 2\tilde{p} + 2; \quad R_2 : \max(4\tilde{h}, \tilde{k}) + 2\tilde{p} \sim 2\tilde{p},$$

where the part on the right of colon means the highest degree.

Hint: If $\tilde{p} = \tilde{q}$, then $\tilde{L_1} < 4\tilde{h} + \tilde{p} + \tilde{q} + 2$. If $\tilde{p} \neq \tilde{q}$, then $\tilde{L_1} = 4\tilde{h} + \tilde{p} + \tilde{q} + 2$. If $\tilde{h} = \tilde{k}$, then $\tilde{L_3} < 2\tilde{h} + 2\tilde{k} + \tilde{p} + \tilde{q}$. If $\tilde{h} \neq \tilde{k}$, then $\tilde{L_3} = 2\tilde{h} + 2\tilde{k} + \tilde{p} + \tilde{q}$ and $\tilde{R_2} = \max(4\tilde{h},\tilde{k})+2\tilde{p}$.

When $\tilde{p} = \tilde{q}$, $\tilde{h} < \tilde{k}$, then $\tilde{R_2} > \max(\tilde{L_1}, \tilde{L_2}, \tilde{L_3}, \tilde{R_1})$, which leads to a contradiction; When $\tilde{p} = \tilde{q}$, $\tilde{h} \geq \tilde{k}$, then $\tilde{L_2}=\tilde{R_1} > \max(\tilde{L_1}, \tilde{L_3}, \tilde{R_2})$ and $a_n = b_n$, i.e., $A = 1$, as $r \to \infty$.

When $\tilde{p} \neq \tilde{q}$, $\tilde{h} = \tilde{k}$, then $\tilde{L_3} \leq 4\tilde{h}+\tilde{p}+\tilde{q}-2 < \tilde{L_1}=\tilde{L_2}=4\tilde{h}+\tilde{p}+\tilde{q}+2, \tilde{R_2} \leq 4\tilde{h}+2\tilde{p} < \tilde{R_1}=4\tilde{h}+2\tilde{p}+2$. If $\tilde{p} > \tilde{q}$, then $\tilde{R_1} > \max(\tilde{L_1}, \tilde{L_2}, \tilde{L_3}, \tilde{R_2})$, which leads to a contradiction. If $\tilde{p} < \tilde{q}$, A and w as (2.1) and (2.2), here $n - m > 0$ ($a_n \neq 0$, $b_m \neq 0$, $c_s \neq 0$, $d_t \neq 0$), then the coefficient of highest degree in $(L_1 + L_2)$ is

$$d_t^4 a_n b_m (n - m + 1) \neq 0,$$

hence $L_1 \tilde{+} L_2 = \tilde{R_1}$ and $\tilde{q} = \tilde{p}$, which leads to a contradiction.

When $\tilde{p} \neq \tilde{q}$ and $\tilde{h} \neq \tilde{k}$, we have $\tilde{L_1} = 4\tilde{h} + \tilde{p} + \tilde{q} + 2, \tilde{L_2} = 4\tilde{h} + \tilde{p} + \tilde{q} + 2, \tilde{L_3} = 2\tilde{h} + 2\tilde{k} + \tilde{p} + \tilde{q}, \tilde{R_1} = 4\tilde{h} + 2\tilde{p} + 2, \tilde{R_2} = 4\max(\tilde{h}, \tilde{k}) + 2\tilde{p}$.

When $\tilde{k} > \tilde{h}$, $\tilde{p} > \tilde{q}$, then $\tilde{R_2} > \max(\tilde{L_1}, \tilde{L_2}, \tilde{L_3}, \tilde{R_1})$, which leads to a contradiction.

When $\tilde{k} < \tilde{h}$, $\tilde{p} > \tilde{q}$, then $\tilde{R_1} > \max(\tilde{L_1},\tilde{L_2},\tilde{L_3},\tilde{R_2})$, which leads to a contradiction.

When $\tilde{k} < \tilde{h}$, $\tilde{p} < \tilde{q}$, then $\tilde{L_3} < \tilde{L_1}=\tilde{L_2}=\tilde{L_1} + \tilde{L_2}=4\tilde{h} + \tilde{p} + \tilde{q} + 2$ and $\tilde{L_1} + \tilde{L_2} > \tilde{R_1}=4\tilde{h} + 2\tilde{p} + 2 > \tilde{R_2}$. It means that the highest degree of $(L_1{+}L_2)$ can not be $4\tilde{h} + \tilde{p} + \tilde{q} + 2$ from (2.1) and (2.2), here $n > m$. Calculating the coefficient of highest degree term in L_1 and L_2, one gets $n - m = -1$, which leads to a contradiction.

When $\tilde{k} > \tilde{h}$, $\tilde{p} < \tilde{q}$, we consider $\tilde{k} = \tilde{h}+1$ firstly. In this case, we have $\tilde{L_1} = \tilde{L_2} = \tilde{L_3} = 4\tilde{h}+\tilde{p}+\tilde{q}+2$, $\tilde{R_2} = 4\tilde{h} + 2\tilde{p} + 4 > \tilde{R_1}$. Letting

$$A = \frac{a_n r^n + a_{n-1} r^{n-1} + \cdots + a_1 r + a_0}{b_m r^m + b_{m-1} r^{m-1} + \cdots + b_1 r + b_0},$$

$$w = \frac{c_{t+1} r^{t+1} + c_t r^t + \cdots + c_1 r + c_0}{d_t r^t + d_{t-1} r^{t-1} + \cdots + d_1 r + d_0},$$

and calculating the coefficient of highest degree term in left, one sees that the result of coefficient can not be 0, i.e., $\tilde{L_1} = \tilde{L_2} = \tilde{L_3} = \tilde{R_2}$, thus $\tilde{q} - \tilde{p} = 2$. Solving the both sides of coefficient of highest degree term, one gets $a_n b_m < 0$. Next, if $\tilde{k} > \tilde{h} + 1$, one has $\tilde{L_3}=2\tilde{h} + 2\tilde{k} + \tilde{p} + \tilde{q}=\tilde{R_2}=4\tilde{k} + 2\tilde{p}$, hence $\tilde{q} - \tilde{p} = 2(\tilde{k} - \tilde{h}) > 2$. What's more, since $R_2 < 0$, the coefficient of highest degree term to pq in L_2 and L_3 should be negative, i.e., $a_n b_m < 0$.

Thus Lemma 2.1 holds. □

Lemma 2.2. *If $\tilde{k} > \tilde{h}$, $\tilde{p} < \tilde{q}$, then $\tilde{k} = \tilde{h} + 1$, $\tilde{q} = \tilde{p} + 2$.*

Proof. In order to prove this lemma, we observe (2.6). Firstly, we consider some simple cases.

When $\tilde{h} = 0$, $\tilde{k} = 1$ and $w = ar + b$, $a \neq 0$, by using coefficient of variation method to (1.6), one can see the expression of A as follows.

If $a^2 \neq \frac{1}{2}$, then

$$A = \frac{1 + 4ab - 2a^2(b^2 - 1)}{2a^2 + 1} - \frac{a^4}{2a^2 + 3}r^2 - \frac{4a^3 b}{2a^2 + 2}r - \frac{4ab(b^2 - 1)}{2a^2 r} - \frac{(b^2 - 1)^2}{(2a^2 - 1)r^2} + \frac{c}{r^{2a^2+1}},$$

where c is a constant. Then we generate the results into (1.7) and discover the fact that $(\frac{a^5}{2a^2+3} - a^5 - \lambda a^3)r^3 + (\frac{4a^4 b}{2a^2+2} - 4a^4 b - 3\lambda a^2 b)r^2 + a(\frac{2a^2b^2-4ab}{2a^2+1} - 4ab - 2a^2b^2 + 2a^2 + \lambda - 3\lambda b^2)r + a[\frac{(b^2-1)^2}{2a^2-1} - (b^2 - 1)^2]r^{-1} - \frac{ac}{r^{2a^2}} + \lambda(4a^2 - 1)(b - b^3) = 0$. For $(\frac{a^5}{2a^2+3} - a^5 - \lambda a^3) = 0$ has solutions, one has $\lambda \leq 2 - \sqrt{3}$ or $\lambda \geq 2 + \sqrt{3}$, which leads to a contradiction.

If $a^2 = \frac{1}{2}$, then

$$A = \frac{2 + 4ab - b^2}{2} - \frac{1}{16}r^2 - \frac{2}{3}abr - \frac{4ab(b^2 - 1)}{r} - \frac{(b^2 - 1)^2 lnr}{r^2} + \frac{c}{r^2},$$

where c is a constant. Then we generate the results into (1.7) and discover the fact that $-\frac{a(3+8\lambda)}{16}r^3 - \frac{b(4+9\lambda)}{6}r^2 + [(-\frac{1}{2} - 3\lambda)ab^2 - 3b + (1 + \lambda)a]r + a(b^2 - 1)^2(\ln r - 1)r^{-1} - \frac{ac}{r} + \lambda(b - b^3) = 0$. Let the coefficient of r^3 be 0, one gets $\lambda = -\frac{3}{8}$, which leads to a contradiction.

When $\tilde{h} = 0$, $\tilde{k} \geq 2$, then $\tilde{L_{21}} = \tilde{k} + \tilde{q} + 1$, $\tilde{L_{22}} = 5\tilde{k} + \tilde{p} - 1$, $\tilde{R_{21}} = 3\tilde{k} + \tilde{p} + 1$. Through a comparative analysis, one sees $\tilde{L_{22}} > \max(\tilde{L_{21}}, \tilde{R_{21}})$, which leads to a contradiction.

When $\tilde{h} = 1$, then $\tilde{L_{21}} \leq 6 + \tilde{k} + \tilde{q}$. If $\tilde{q} - \tilde{p} \geq 4$, i.e., $\tilde{k} - \tilde{h} \geq 2$, then $\tilde{L_{22}}=5\tilde{k} + \tilde{p}$, $\tilde{R_{21}}=3\tilde{k} + \tilde{p} + 4$. Through a comparative analysis, one gets $\tilde{L_{22}} > \max(\tilde{L_{21}},\tilde{R_{21}})$, which leads to a contradiction.

When $\tilde{h} \geq 2$, if $\tilde{q}-\tilde{p} \geq 4$, i.e., $\tilde{k}-\tilde{h} \geq 2$, then $\tilde{L_{21}} \leq 5\tilde{h}+1+\tilde{k}+\tilde{q}$, $\tilde{L_{22}}=5\tilde{k}+\tilde{h}+\tilde{p}-1$,$\tilde{R_{21}}=3\tilde{k}+3\tilde{h}+\tilde{p}+1$. By analysis, one has $\tilde{L_{22}} > \max(\tilde{L_{21}},\tilde{R_{21}})$, which leads to a contradiction.

Thus, Lemma 2.2 holds. □

Lemma 2.3. *Let $w = \frac{k}{h} = \frac{a_n r^n + a_{n-1} r^{n-1} + \cdots + a_1 r + a_0}{b_n r^n + b_{n-1} r^{n-1} + \cdots + b_1 r + b_0}$. If $\tilde{p} = \tilde{q}$, $\tilde{h} \geq \tilde{k}$, then $\tilde{p} = \tilde{q}$, $\tilde{h} = \tilde{k}$, and $a_n^2 = b_n^2$.*

Proof. When $\tilde{k} = 0$, $\tilde{h} = 1$, then $w = \frac{k}{h} = \frac{c}{r+b}$, so $\tilde{L_{22}} \leq 5 + \tilde{p} < \tilde{L_{21}}=6 + \tilde{p}=\tilde{R_{21}}$. Solving both sides of the coefficient of highest degree term, one gets that $\lambda = -2$, which leads to a contradiction.

When $\tilde{k} = 0$, $\tilde{h} \geq 2$, then $\tilde{L_{22}} \leq 5\tilde{h} + \tilde{p}$, $\tilde{L_{21}} \leq 5\tilde{h} + \tilde{p} + 1=\tilde{R_{21}}$. Let $k = 1$, $h = d_n r^n + d_{n-1} r^{n-1} + \cdots + d_1 r + d_0 (n \geq 2)$, $A = \frac{q}{p} = \frac{r^s + a_{s-1} r^{s-1} + \ldots + a_1 r + a_0}{r^s + b_{s-1} r^{s-1} + \cdots + b_1 r + b_0}$. Then the coefficient of highest degree term in L_{21} is

$$n(n + 1)d_n^5, \tag{2.9}$$

and the coefficient of highest degree term in R_{21} is

$$-\lambda d_n^5. \tag{2.10}$$

Solving the both sides of the coefficient of highest degree term, one has $\lambda = -n(n + 1) < 0$, which leads to a contradiction.

When $\tilde{k} = 1$, $\tilde{h} = 1$, then $\tilde{L_{21}} \leq 6 + \tilde{p}$, $\tilde{L_{22}} \leq 5 + \tilde{p}$. Let $w = \frac{br+c}{r+a}$, if $b^2 \neq 1$, then $\tilde{R_{21}}=7 + \tilde{p}$, which leads to a contradiction. So $b^2 = 1$, i.e., $w^2 = 1$ $(r \to \infty)$.

When $\tilde{k} = 1, \tilde{h} \geq 2$, then $\tilde{L_{22}} \leq 5\tilde{h} + \tilde{p} + 1$, $\tilde{L_{21}} \leq 5\tilde{h} + \tilde{p} + 2 = \tilde{R_{21}}$. Let $k = ar + c$, $h = d_n r^n + d_{n-1} r^{n-1} + \cdots + d_1 r + d_0$ $(n \geq 2)$, then the coefficient of highest degree term in L_{21} is

$$n(n-1)ad_n^5, \tag{2.11}$$

and the coefficient of highest degree term in R_{21} is

$$-\lambda a d_n^5. \tag{2.12}$$

Solving both sides of the coefficient of highest degree term, one gets $\lambda = -n(n-1)$, which leads to a contradiction.

When $\tilde{k} \geq 2$, and $w = \frac{k}{h} = \frac{a_n r^n + a_{n-1} r^{n-1} + \cdots + a_1 r + a_0}{b_m r^m + b_{m-1} r^{m-1} + \cdots + b_1 r + b_0}$ $(m \geq n \geq 2)$, considering $\tilde{h} > \tilde{k}$ and $m > n$ firstly, then $\tilde{L_{22}} \leq 5\tilde{h} + \tilde{p} + \tilde{k}$, $\tilde{L_{21}} \leq 5\tilde{h} + \tilde{p} + \tilde{k} + 1 = \tilde{R_{21}}$. The coefficient of highest degree term in L_{21} is

$$a_n b_m^5 [2m^2 + n(n-1) - 2nm - m(m-1)], \tag{2.13}$$

and the coefficient of highest degree term in R_{21} is

$$-\lambda a_n b_m^5. \tag{2.14}$$

Solving both sides of the coefficient of highest degree term, one gets

$$\lambda = -[2m^2 + n(n-1) - 2nm - m(m-1)]. \tag{2.15}$$

In fact, by (2.13), one has

$$2m^2 + n(n-1) - 2nm - m(m-1) = (m-n)(m-n+1) > 0. \tag{2.16}$$

So $\lambda < 0$, which leads to a contradiction. If $\tilde{h} = \tilde{k}$, $a_n^2 \neq b_n^2$, then $\tilde{L_{21}} \leq 6\tilde{h} + \tilde{p}$, $\tilde{L_{22}} \leq 6\tilde{h} + \tilde{p} - 1$, $\tilde{R_{21}} = 6\tilde{h} + \tilde{p} + 1$, which lead to a contradiction, i.e., $w^2 = 1$ $(r \to \infty)$.

Thus, Lemma 2.3 holds. □

Lemma 2.4. *The case $\tilde{p} = \tilde{q}$, $\tilde{h} = \tilde{k}$ can not happen.*

Proof. When $\tilde{h} = \tilde{k} = 1$, $w = \frac{r+b}{r+a}$, then $\tilde{L_{22}} < \tilde{L_{21}} = 6 + \tilde{p} = \tilde{R_{21}}$, thus the coefficient of highest degree term in L_{21} is

$$2(b-a), \tag{2.17}$$

and the coefficient of highest degree term in R_{21} is

$$2\lambda(b-a). \tag{2.18}$$

Solving the both sides of the coefficient of highest degree term, we discover $\lambda = 1$, which leads to a contradiction. If let $w = \frac{-r+b}{r+a}$, also leads to a contradiction in a same way.

For $\tilde{h} = \tilde{k} \geq 2$, let $A = \frac{r^m + c_{m-1} r^{m-1} + \cdots + c_1 r + c_0}{r^m + d_{m-1} r^{m-1} + \cdots + d_1 r + d_0}$, $w = \frac{r^n + a_{n-1} r^{n-1} + \cdots + a_1 r + a_0}{r^n + b_{n-1} r^{n-1} + \cdots + b_1 r + b_0}$ $\left(w = \frac{-r^n + a_{n-1} r^{n-1} + \cdots + a_1 r + a_0}{r^n + b_{n-1} r^{n-1} + \cdots + b_1 r + b_0}\right.$ also leads to the same result). If $a_{n-1} \neq b_{n-1}$, then $\tilde{L_{22}} \leq 6\tilde{h} + \tilde{p} - 1 < \tilde{L_{21}} = \tilde{R_{21}} = 6\tilde{h} + \tilde{p}$. Solving both sides of the coefficient of highest degree term, one gets $\lambda = 1$, which leads to a contradiction. So, $a_{n-1} = b_{n-1}$. If $a_{n-2} \neq b_{n-2}$, then $\tilde{L_{22}} \leq 6\tilde{h} + \tilde{p} - 2 < \tilde{L_{21}} = \tilde{R_{21}} = 6\tilde{h} + \tilde{p} - 1$. Solving both sides of the coefficient of highest degree term, one has $\lambda = 3$, which leads to a contradiction. So, $a_{n-2} = b_{n-2}$. This reminds us that perhaps $a_i = b_i (i \leq n-2)$. In the following, we use mathematical induction to prove $a_i = b_i$. With the reduction to absurdity, assume $\exists\, a_i \neq b_i$ and $a_{i+j} = b_{i+j}$, $j \geq 1$, $0 \leq i < n-2$. So $w = \frac{k}{h} = \frac{r^n + a_{n-1}r^{n-1} + a_{n-2}r^{n-2} + \cdots + a_{i+1}r^{i+1} + a_i r^i + \cdots}{r^n + a_{n-1}r^{n-1} + a_{n-2}r^{n-2} + \cdots + a_{i+1}r^{i+1} + b_i r^i + \cdots}$. By (2.6), one has

$$L_{21} = (a_i - b_i)(i^2 - i + n^2 + n - 2ni)r^{5n+m+i+1} + \cdots, \tag{2.19}$$

$$L_{22} = (i-n)(a_i - b_i)(d_{m-1} - c_{m-1})r^{5n+m+i-2} + \cdots, \tag{2.20}$$

$$R_{21} = 2\lambda(a_i - b_i)r^{5n+m+i+1} + \cdots. \tag{2.21}$$

Solving both sides of the coefficient of highest degree term, one gets

$$n^2 + n = 2ni + i - i^2 + 2\lambda \leq 2n(n-2) + n - 2 - (n-2)^2 + 2\lambda = n^2 + n + 2\lambda - 6. \tag{2.22}$$

Thus $\lambda \geq 3$, which leads to a contradiction. Then $w \equiv 1$, and contradicts the hypothesis. Here, if we assume $w = \frac{k}{h} = \frac{-r^n + a_{n-1}r^{n-1} + \cdots + a_1 r + a_0}{r^n + b_{n-1}r^{n-1} + \cdots + b_1 r + b_0}$ in the beginning, we will get the result that $w \equiv -1$, which is also a contradiction.

Thus, Lemma 2.4 holds. □

Lemma 2.5. *The case $\tilde{q} = \tilde{p} + 2$, $\tilde{k} = \tilde{h} + 1$ can not happen.*

Proof. We consider $\tilde{h} \geq 2$ firstly. Let

$$w = \frac{k}{h} = \frac{ar^{n+1} + a_n r^n + \cdots + a_1 r + a_0}{r^n + c_{n-1}r^{n-1} + \cdots + c_1 r + c_0},$$

$$A = \frac{q}{p} = \frac{r^{m+2} + d_{m+1}r^{m+1} + \cdots + d_1 r + d_0}{br^m + b_{m-1}r^{m-1} + \cdots + b_1 r + b_0},$$

and $b < 0$, then $\tilde{L_{21}} \leq 5\tilde{h} + \tilde{k} + \tilde{q} + 1$. By calculating, the corresponding coefficient of highest degree term is 0, thus $\tilde{L_{21}} \leq 5\tilde{h} + \tilde{k} + \tilde{q}$. What's more, $\tilde{L_{22}} \leq 5\tilde{h} + \tilde{k} + \tilde{q} + 1$, and the coefficient of highest degree term in $5\tilde{h} + \tilde{k} + \tilde{q} + 1$ is $-(a + a^5 b)$ and $\tilde{R_{21}} = 5\tilde{h} + \tilde{k} + \tilde{q} + 1$. The corresponding coefficient is $\lambda a^3 b$. Solving both sides of coefficient of highest degree term, one gets

$$a^4 b + \lambda a^2 b + 1 = 0. \tag{2.23}$$

From (2.5), one gets $\tilde{L_1} = \tilde{L_2} = \tilde{L_3} = \tilde{R_2} = 4\tilde{h} + 2\tilde{p} + 4 > \tilde{R_1} = 4\tilde{h} + 2\tilde{p} + 2$. Comparing the coefficient of highest degree term in $(L_1 + L_2 + L_3)(3b + 2ba^2)$ and R_2 $(-a^4 b^2)$, one gets

$$a^4 b + 2a^2 + 3 = 0. \tag{2.24}$$

Calculating (2.23) and (2.24), one has

$$\frac{2}{\lambda}a^4 + (\frac{2}{\lambda} + 2)a^2 + 3 = 0. \tag{2.25}$$

If (2.25) has solutions, then $\lambda \leq 2 - \sqrt{3}$ or $\lambda \geq 2 + \sqrt{3}$, which leads to a contradiction.

If $\tilde{h} = 0$, then $w = ar + b$. This case has been discussed in the proof of Lemma 2.2. When $\tilde{h} = 1$, $\tilde{k} = 2$, we suppose $w = \frac{k}{h} = \frac{br^2 + b_1 r + b_0}{r + c}$, $A = \frac{ar^{m+2} + a_{m+1}r^{m+1} + \cdots + a_1 + a_0}{r^m + d_{m-1}r^{m-1} + \cdots + d_1 r + d_0}$, $(a \neq 0, b \neq 0)$. Solving the coefficient of highest degree term in L_{21}, L_{22}, R_{21} and observing (2.6), one gets $\tilde{L_{21}} \leq 10 + \tilde{p}$. By calculating, the coefficient of highest degree term is 0, thus $\tilde{L_{21}} \leq 9 + \tilde{p}$. What's more, $\tilde{L_{22}} \leq 10 + \tilde{p}$, the corresponding coefficient of highest degree term is $-b(a + b^4)$, $\tilde{R_{21}} = 10 + \tilde{p}$, the corresponding coefficient of highest degree term is λb^3, then one has

$$b^4 + \lambda b^2 + a = 0. \tag{2.26}$$

Observing (2.5), one gets $\tilde{L_1} = \tilde{L_2} = \tilde{L_3} = \tilde{R_2} = 2m + 8 > \tilde{R_1} = 2m + 6$. Comparing the coefficient of highest degree term in $(L_1 + L_2 + L_3)$: $(3a + 2ab^2)$ and in R_2: $(-b^4)$, one has

$$3a + 2ab^2 + b^4 = 0. \tag{2.27}$$

Calculating (2.26) and (2.27), one can see that

$$2b^4 + 2(1 + \lambda)b^2 + 3\lambda = 0. \tag{2.28}$$

Thus, when (2.28) has solutions, $\lambda \leq 2 - \sqrt{3}$ or $\lambda \geq 2 + \sqrt{3}$ holds, which leads to a contradiction.

Thus, the proof of Lemma 2.5 is completed. □

By Lemma 2.1–2.5, we complete the proof of Theorem 1.1.

Remark 2.6. *If we consider the case that adding the perturbed coefficient into* (1.4) *for the first term as follows:*

$$\zeta r^2 A w'' + [r(1 - A) - \frac{(1 - w^2)^2}{r}]w' + w(1 - w^2) = 0, \tag{2.29}$$

we can also get the same result as the conclusion in this paper. Here $\zeta \in (0.5, 2)$ *and* $\zeta \neq 1$. *The proof of this result is similar to Theorem 1.1 if we add the perturbed coefficient* λ *into* (1.4)*, so we omit here.*

3. Proof of Theorem 1.2

In this section, we prove the existence and uniqueness of the perturbed EYM equations. The reference [9] provides a rigorous proof of the existence and uniqueness of the solutions to the Einstein-Yang-Mills equations with gauge group $SU(2)$. After adding the perturbed term λ to (1.4), we discover that the perturbed equations have the same properties as the original equations in existence and uniqueness.

For the definitions of $C^{2+\alpha}(\bar{r}, \bar{r}+\varepsilon)$, $D_1, D_2, D_3, X, d(\cdot,\cdot), T, T_1, T_2$ and T_3, we still use the definitions in [9]. The main work in [9] is to prove that T_1, T_2 and T_3 are contract mappings. By analysis, to prove our results, we just need to prove T_2 is a contract mapping. Recalling u_1, u_2, w_1, w_2 in $C^{2+\alpha}(\bar{r}, \bar{r} + \epsilon)$, $T_2(w, z, A) = w'(\bar{r}) - \int_{\bar{r}}^{r} \frac{\lambda u w + \phi z}{s^2 A} ds$, and denoting $\theta = (w, z, A)$, then

$$\begin{aligned} T_2(\theta_1) - T_2(\theta_2) &= \int_{\bar{r}}^{r} [\frac{\lambda u_2 w_2}{s^2 A_2} - \frac{\lambda u_1 w_1}{s^2 A_1} + \frac{\phi_2 z_2}{s^2 A_2} - \frac{\phi_1 z_1}{s^2 A_1}] ds \\ &= \int_{\bar{r}}^{r} [\frac{\lambda(u_2 w_2 - \bar{u}\bar{w})}{s^2 A_2} - \frac{\lambda(u_1 w_1 - \bar{u}\bar{w})}{s^2 A_1} + \frac{\phi_2 z_2 - \bar{\phi}\bar{z}}{s^2 A_2} \\ &\quad - \frac{\phi_1 z_1 - \bar{\phi}\bar{z}}{s^2 A_1} + \frac{1}{s^2}(\frac{1}{A_2} - \frac{1}{A_1})(\lambda \bar{u}\bar{w} + \bar{\phi}\bar{z})] ds. \end{aligned}$$

For convenience, denoting

$$\Delta f := f_1 - f_2 \quad (f \in C^{1+\alpha}),$$

then

$$\Delta(\frac{\lambda uw - \lambda\bar{u}\bar{w}}{s^2 A}) = \frac{\lambda u_2 w_2 - \lambda\bar{u}\bar{w}}{s^2 A_2} - \frac{\lambda u_1 w_1 - \lambda\bar{u}\bar{w}}{s^2 A_1},$$

and

$$\begin{aligned}\|T_2(\theta_1) - T_2(\theta_2)\|_{1+\alpha} =&\Big\|\Delta(\frac{\lambda uw - \lambda\bar{u}\bar{w}}{r^2 A}) + \Delta(\frac{\phi z - \bar{\phi}\bar{z}}{r^2 A}) - \Delta(\frac{1}{A})\frac{1}{r^2}(\lambda\bar{u}\bar{w} + \bar{\phi}\bar{z})\Big\|_\alpha \\ \leq&\lambda\Big\|\Delta(\frac{uw - \bar{u}\bar{w}}{r^2 A})\Big\|_\alpha + \Big\|\Delta(\frac{\phi z - \bar{\phi}\bar{z}}{r^2 A})\Big\|_\alpha + \Big\|\Delta(\frac{1}{A})\frac{1}{r^2}(\lambda\bar{u}\bar{w} + \bar{\phi}\bar{z})\Big\|_\alpha.\end{aligned}$$

Next, we estimate the above equation term by term.

For $\lambda\|\Delta(\frac{uw-\bar{u}\bar{w}}{r^2 A})\|_\alpha$, denoting $\varrho_2 = r_2 - \bar{r}$ and by lemma 5.3 in [9], one can get

$$\begin{aligned}\lambda\Big\|\Delta(\frac{uw - \bar{u}\bar{w}}{r^2 A})\Big\|_\alpha =&\frac{\lambda}{r^2}\Big\|\frac{u_2 w_2}{A_2} - \frac{u_1 w_1}{A_2} + \frac{u_1 w_1}{A_2} - \frac{u_1 w_1}{A_1} + \frac{\bar{u}\bar{w}}{A_1} - \frac{\bar{u}\bar{w}}{A_2}\Big\|_\alpha \\ \leq&\frac{\lambda}{c\bar{r}^2}[\Big\|\frac{1}{B_2}\Big\|_\alpha\Big\|\frac{u_2 w_2 - u_1 w_1}{\varrho_2}\Big\|_\infty + \Big\|\frac{1}{B_2}\Big\|_\infty\Big\|\frac{u_2 w_2 - u_1 w_1}{\varrho_2}\Big\|_\alpha] \\ &+ \frac{\lambda}{\bar{r}^2}[\|u_1 w_1\|_\alpha\Big\|\frac{1}{A_2} - \frac{1}{A_1}\Big\|_\infty + \|u_1 w_1\|_\infty\Big\|\frac{1}{A_2} - \frac{1}{A_1}\Big\|_\alpha + \bar{u}\bar{w}\Big\|\frac{1}{A_1} - \frac{1}{A_2}\Big\|_\alpha].\end{aligned}$$

Due to u_1, u_2, w_1, w_2 in $C^{2+\alpha}(\bar{r}, \bar{r} + \epsilon)$, $(u_1 w_1)(\bar{r}) = (u_2 w_2)(\bar{r})$, $(u_1 w_1)'(\bar{r}) = (u_2 w_2)'(\bar{r})$, $(u_1 w_1)''(\bar{r}) = (u_2 w_2)''(\bar{r})$, then $\|\frac{\Delta(uw)}{\varrho_2}\|_\alpha \to 0$, $\|\frac{\Delta(uw)}{\varrho_2}\|_\infty \to 0$. Hence,

$$\Big\|\frac{1}{B_2}\Big\|_\alpha\Big\|\frac{u_2 w_2 - u_1 w_1}{\varrho_2}\Big\|_\infty \to 0, \qquad \Big\|\frac{1}{B_2}\Big\|_\infty\Big\|\frac{u_2 w_2 - u_1 w_1}{\varrho_2}\Big\|_\alpha \to 0, \tag{3.1}$$

as $\varepsilon \to 0$. By $(\frac{1}{A_2} - \frac{1}{A_1})(\bar{r}) = 0, (\frac{1}{A_2} - \frac{1}{A_1})'(\bar{r}) = 0$, $\|u_1 w_1\|_\infty \to 0$ and $\|\bar{u}\bar{w}\|_\infty \to 0$, one has

$$\|u_1 w_1\|_\infty\Big\|(\frac{1}{A_2} - \frac{1}{A_1})\Big\|_\alpha \to 0, \qquad \bar{u}\bar{w}\Big\|\frac{1}{A_1} - \frac{1}{A_2}\Big\|_\alpha \to 0. \tag{3.2}$$

as $\varepsilon \to 0$. Combining (3.1) and (3.2) to get

$$\lambda\Big\|\Delta(\frac{uw - \bar{u}\bar{w}}{r^2 A})\Big\|_\alpha \to 0 \quad (\varepsilon \to 0). \tag{3.3}$$

For $\|\Delta(\frac{\phi z-\bar{\phi}\bar{z}}{r^2 A})\|_\alpha$, one has

$$\Big\|\Delta(\frac{\phi z - \bar{\phi}\bar{z}}{r^2 A})\Big\|_\alpha \leq\Big\|\Delta(\frac{\phi z - \bar{\phi}z}{r^2 A})\Big\|_\alpha + \Big\|(\frac{\bar{\phi}z - \bar{\phi}\bar{z}}{r^2 A})\Big\|_\alpha.$$

It is easy to see $\|\Delta(\frac{\phi z-\bar{\phi}z}{r^2 A})\|_\alpha \to 0$, as $\varepsilon \to 0$. For convenience, defining

$$n(r) = \frac{\bar{\phi}z - d\varrho\bar{\phi} - \bar{\phi}\bar{z}}{cr^2},$$

one has $n_i(r) = \frac{\bar{\phi}z_i - d\varrho\bar{\phi} - \bar{\phi}\bar{z}_i}{cr^2}$, and

$$\|\Delta(\frac{\bar{\phi}z - \bar{\phi}\bar{z}}{r^2 A})\|_\alpha = \|\frac{n_2}{\varrho B_2} - \frac{n_1}{\varrho B_1}\|_\alpha \leq \|\frac{n_2}{\varrho B_2} - \frac{n_2}{\varrho B_1}\|_\alpha + \|\frac{n_2}{\varrho B_1} - \frac{n_1}{\varrho B_1}\|_\alpha. \tag{3.4}$$

Because $\frac{n_2}{\varrho B_2} - \frac{n_2}{\varrho B_1} = \frac{n_2}{B_1 B_2}(\frac{B_1 - B_2}{\varrho})$, one gets

$$\|\frac{n_2}{\varrho B_2} - \frac{n_2}{\varrho B_1}\|_\alpha \leq \|\frac{n_2}{B_1 B_2}\|_\infty \|\frac{B_1 - B_2}{\varrho}\|_\alpha + \|\frac{n_2}{B_1 B_2}\|_\alpha \|\frac{B_1 - B_2}{\varrho}\|_\infty.$$

Due to $\frac{n_2}{B_1 B_2} \in C^{1+\alpha}$, $n_2 = \frac{\bar{\phi}}{cr^2}(z_2 - d\varrho - \bar{z})$, then

$$n_2(\bar{r}) = \frac{\bar{\phi}}{c\bar{r}^2}(\bar{z} - \bar{z} - d\varrho).$$

Because of $\varrho = \bar{r} - \bar{r} = 0$, $n_2(\bar{r}) = 0$,

$$n_2'(\bar{r}) = \frac{\bar{\phi}}{c}[\frac{1}{r^2}(z_2' - d) - \frac{2}{r^3}(z_2 - d\varrho - \bar{z})](\bar{r}) = \frac{\bar{\phi}}{c}[\frac{1}{\bar{r}^2}(d - d) - \frac{2}{\bar{r}^3}(\beta - \beta - d\varrho(\bar{r}))] = 0.$$

Hence $(\frac{n_2}{B_1 B_2})(\bar{r}) = 0, (\frac{n_2}{B_1 B_2})'(\bar{r}) = 0$, furthermore

$$\|\frac{n_2}{B_1 B_2}\|_\infty \to 0, \qquad \|\frac{n_2}{B_1 B_2}\|_\alpha \to 0,$$

and

$$\|\frac{n_2}{B_1 B_2}\|_\infty \|\frac{B_1 - B_2}{\varrho}\|_\alpha \to 0,$$

as $\varepsilon \to 0$. Due to $(\frac{B_2 - B_1}{\varrho})(\bar{r}) = 0$, one has

$$\|\frac{B_2 - B_1}{\varrho}\|_\infty \leq \varepsilon^\alpha \|(B_2 - B_1)/\varrho\|_\alpha \leq \frac{\epsilon^\alpha}{1+\alpha}\|\frac{B_1 - B_2}{\varrho}\|_{1+\alpha},$$

and then $\varepsilon \to 0$,

$$\|\frac{n_2}{B_1 B_2}\|_\alpha \|\frac{B_1 - B_2}{\varrho}\|_\infty \to 0, \qquad \|\frac{n_2}{\varrho B_2} - \frac{n_2}{\varrho B_1}\|_\alpha \to 0. \tag{3.5}$$

For the last term in (3.4), denoting $h(r) = \frac{\bar{\phi}}{B_1 r^2 c}$ for convenience, then

$$\|\frac{n_2}{\varrho B_1} - \frac{n_1}{\varrho B_1}\|_\alpha = \|\frac{\bar{\phi}}{B_1 r^2 c}(\frac{z_2 - d\varrho - \bar{z} - z_1 + d\varrho + \bar{z}}{\varrho})\|_\alpha = \|h\frac{z_2 - z_1}{\varrho}\|_\alpha.$$

Because $h \in C^{1+\alpha}, h(\bar{r}) = \frac{\bar{\phi}}{B(\bar{r})\bar{r}^2 c}$, one has

$$h(\bar{r}) = \frac{c}{B(\bar{r})c} = \frac{1}{1 + \frac{e}{2c}\varrho(\bar{r})} = 1,$$

and $(h-1) \in C^{1+\alpha}$, $(h-1)(\bar{r}) = h(\bar{r}) - 1 = 0$, $\|h-1\|_\infty \to 0$. Moreover, $0 < \delta < \alpha, \|h\|_\infty \leq 1 + \delta$, then

$$\|\frac{n_2}{\varrho B_1} - \frac{n_1}{\varrho B_1}\|_\alpha \leq \|h\|_\infty \|\frac{z_2 - z_1}{\varrho}\|_\alpha + \|h\|_\alpha \|\frac{z_2 - z_1}{\varrho}\|_\infty.$$

Due to the fact that $(z_2 - z_1)(\bar{r}) = 0$ and $\|\frac{z_2-z_1}{\varrho}\| \leq \frac{1}{1+\alpha}\|z_2 - z_1\|_{1+\alpha}$, then

$$\Big\|\frac{n_2}{\varrho B_1} - \frac{n_1}{\varrho B_1}\Big\|_\alpha \leq (1+\delta)\frac{1}{1+\alpha}\|z_2 - z_1\|_{1+\alpha} + \|h\|_\alpha\Big\|\frac{z_2 - z_1}{\varrho}\Big\|_\infty, \tag{3.6}$$

and

$$\Big\|\frac{z_2 - z_1}{\varrho}\Big\|_\infty \to 0, \qquad \|h\|_\alpha\Big\|\frac{z_2 - z_1}{\varrho}\Big\|_\infty \to 0,$$

as $\varepsilon \to 0$. Let $0 < k = \frac{1+\delta}{1+\alpha} < 1$, then

$$\Big\|\frac{n_2}{\varrho B_1} - \frac{n_1}{\varrho B_1}\Big\|_\alpha \leq k\|z_2 - z_1\|_{1+\alpha} + O(\varepsilon). \tag{3.7}$$

Furthermore, $\lambda\bar{u}\bar{w} + \bar{\phi}\bar{z} = \bar{r}^2\bar{A}\bar{w}'' = 0$, then

$$\|T_2(\theta_1) - T_2(\theta_2)\|_{1+\alpha} \leq k\|z_2 - z_1\|_{1+\alpha} + O(\varepsilon). \tag{3.8}$$

So T_2 is a contract mapping. Thus, by Banach fixed-point theorem, in the interval $\bar{r} \leq r \leq \bar{r} + \varepsilon$, T has a unique fixed point $(A(r, \bar{w}), w(r, \bar{w}), w'(r, \bar{w})) \in X$. This completes the proof of Theorem 1.2.

4. Conclusions

By Theorems 1.1 and 1.2, we can see that the local solution is unique. Moreover, this solution is not rational solution, but $C^{2+\alpha}$ solution. This means the results in Theorem 1.3 holds. Since the current methods of finding ghost solitons are all based on the rational fraction solution, these results show that there is no rational ghost soliton solution for the EYM equation.

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Conflict of interest

All authors declare no conflicts of interest in this paper.

References

1. R. Bartnik, M. John, Particlelike solutions of the Einstein-Yang-Mills equations, *Phys. Rev. Lett.*, **61** (1988), 141–144.
2. J. E. Baxter, On the existence of topological dyons and dyonic black holes in anti-de Sitter Einstein-Yang-Mills theories with compact semisimple gauge groups, *J. Math. Phys.*, **59** (2018), 052502.
3. F. Bethuel, P. Gravejat, J. C. Saut, D. Smets, Orbital stability of the black soliton for the Gross-Pitaevskii equation, *Indiana U. Math. J.*, **57** (2008), 2611–2642.

4. A. M. Kamchatnov, A. Gammal, F. K. Abdullaev, R. A. Kraenkel, Formation of soliton trains in Bose-Einstein condensates as a nonlinear Fresnel diffraction of matter waves, *Phys. Lett. A*, **319** (2003), 406–412.
5. R. Radha, V. R. Kumar, M. Wadati, Line-soliton dynamics and stability of Bose-Einstein condensates in (2+1) Gross-Pitaevskii equation, *J. Math. Phys.*, **51** (2010), 043507.
6. J. A. Smoller, A. G. Wasserman, Regular solutions of the Einstein-Yang-Mills equations, *J. Math. Phys.*, **36** (1995), 4301–4323.
7. J. Smoller, A. Wasserman, Reissner-Nordstrom-like solutions of the $SU(2)$ Einstein-Yang-Mills equations, *J. Math. Phys.*, **38** (1997), 6522–6559.
8. J. A. Smoller, A. G. Wasserman, Extendability of solutions of the Einstein-Yang-Mills equations, *Commun. Math. Phys.*, **194** (1998), 707–732.
9. J. A. Smoller, A. G. Wasserman, S. T. Yau, Existence of black hole solutions for the Einstein-Yang-Mills equations, *Commun. Math. Phys.*, **154** (1993), 377–401.