AIMS Mathematics

http://www.aimspress.com/journal/Math

AIMS Mathematics, 6(11): 12051–12064.
DOI: 10.3934/math.2021698
Received: 25 June 2021
Accepted: 12 August 2021
Published: 18 August 2021

Research article

Adaptive constraint control for nonlinear multi-agent systems with undirected graphs

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Abstract: This paper investigates the problem of adaptive distributed consensus control for stochastic multi-agent systems (MASs) with full state constraints. By utilizing adaptive backstepping control technique and barrier Lyapunov function (BLF), an adaptive distributed consensus constraint control method is proposed. The developed control method can ensure that all signals of the controlled system are semi-globally uniformly ultimately bounded (SGUUB) in probability, and outputs of the follower agents keep consensus with the output of leader. In addition, system states are not transgressed their constrained sets. Finally, simulation results are provided to illustrate the feasibility of the developed control algorithm and theorem.

Keywords: multi-agent systems; undirected graphs; backstepping control; full state constraint; adaptive consensus control
Mathematics Subject Classification: 93B52, 93C42

1. Introduction

Consensus control is one of the basic problems in the coordinated control of multi-agent systems (MASs) and has wide application prospect in the fields of aerospace, robot and autonomous ocean vehicle, which means that the state or output of the follower agents of MASs tend to contain consensus with the state or output of the leaders through local communication. In recent years, the consensus control of MASs has been become the hot research direction in the fields of control engineering, and some critical results have been received, see [1–8]. The authors in [1] first researched the observer-based distributed consensus control for multi-agent networks. The works [2,3] studied the consensus control problems for second-order multi-agent systems, and the work [4] proposed the consensus control for linear MASs with directed topology. The authors in [5,6] studied the adaptive consensus tracking control problems for high-order nonlinear MASs, and [7] investigated the distributed adaptive asymptotically consensus tracking control problem. The authors in [8] developed the event-based robust adaptive consensus tracking control problem for nonlinear MASs.

However, the above proposed control methods all do not consider the constraint control problems. Generally, most physical systems have various forms of constraints, for example, safety specifications, saturation and physical stoppages. If these constraints are violated in control design, which will lead to system performance degradation, and even lead to serious security problems. Hence, the researches of constraints control have great significance for the physical systems, the state or output constraints control problems should be considered in control design. The authors in [9] studied the output constraint control problem by adopting barrier Lyapunov function (BLF) technique. Then, based on the proposed control approach in [9], the work [10] investigated the adaptive NN full state constraint control problem, and the works [11,12] studied the adaptive constraint tracking control problems for nonlinear system with stochastic disturbance and full state constraint. The authors in [13] studied the observer-based adaptive optimal full state constraint control problem for nonlinear systems. Since the finite-time control has the features of better robustness, fast transient performances, the authors in [14] proposed the finite-time adaptive fuzzy full state constraint control method for nonlinear system, and [15] developed the finite-time adaptive full state constraint control method for stochastic nonlinear ones.

Most notably, the above presented constraint control approaches are all for single-agent systems, which cannot be directly applied to solve the control problem for nonlinear MASs. In recently, some adaptive constraint control schemes have been developed for nonlinear MASs, see [16–23]. The authors in [16,17] developed adaptive fixed-time consensus control approaches for MASs with full state constraints. Based on [15], the works [18] studied the adaptive optimal consensus control for stochastic nonlinear MASs. Considering the unmeasured states, the authors in [19,20] studied the observer-based adaptive output constraint control problems for nonlinear MASs. By using the unified transformation functions, the authors in [21] proposed an adaptive containment constraint control approach, and [22] developed distributed adaptive consensus constraint control approach for MASs. The work [23] developed the distributed consensus for nonlinear MASs with output constraints. It should be pointed out that there are no available works for robust adaptive consensus full state constraint control design for nonlinear MASs with stochastic disturbances. Thus, it is necessary to study the adaptive distributed consensus constraint control of stochastic nonlinear MASs.

This paper investigates the problem of adaptive robust consensus constraint control design for nonlinear MASs with stochastic disturbances and full state constraint. BLFs are adopted in control design to deal with the problem of full state constraint control. Compared with the existing works, the major contributions of this paper can be highlighted as:

1. This paper first investigates the adaptive full state constraint consensus control design problem for nonlinear MASs under undirected graphs with stochastic disturbances. Based on adaptive backstepping technique, this paper proposed robust adaptive constraint consensus control approach, which can ensure all signals in the considered system are semi-globally uniformly ultimately bounded (SGUUB), and all states do not exceed their constrain bound.
2. Noted that [16,17,22] are also studied the adaptive full state constraint consensus control problems, but the stochastic disturbances are not be considered in control design, the proposed control approaches cannot be applied to solve the control problem in this paper.

2. Preliminaries and problem description

2.1. Graph theory

Algebraic graph $\mathcal{G}(\Lambda, \Xi, \Upsilon)$, which is an undirected graph, can be used to depict the topology of the interactions between leader and followers, where $\Lambda = \{\varrho_1, \cdots, \varrho_n\}$ denotes the set of all nodes, $\Xi = \{(n_j, n_k) \subseteq \Lambda \times \Lambda\}$ denotes the set of edges, if agent j can get the information from agent i, then $(\varrho_i, \varrho_j) \in \Xi$. $\Upsilon = [a_{ij}]$ is the adjacency matrix of leader, the element a_{ij} is the communication weight, which can be depicted as $a_{ij} = a_{ji} = 1 \Leftrightarrow (\varrho_i, \varrho_j) \in \Xi$, and $a_{ij} = a_{ji} = 0 \Leftrightarrow (\varrho_i, \varrho_j) \notin \Xi$, Υ is symmetric and the diagonal elements $a_{ii} = 0$. Let $\delta_{ij} = (\varrho_i, \varrho_j)$ is the edge connecting between agents i and j and the set of neighbors of node j is defined as $\mathcal{N} = \{j|(\varrho_i, \varrho_j) \in \Xi\}$. The Laplacian matrix $L = [l_{ij}]_{N\times N}$ of the digraph $\mathcal{G}$ can be defined as $L = \mathcal{D} - \Upsilon$, where diagonal matrix $\mathcal{D} = \text{diag}\{d_1, \cdots, d_N\}$ denotes the degree matrix of digraph $\mathcal{G}$ with $d_i = \sum_{j\in\mathcal{N}_i} a_{ij}$.

2.2. Stochastic nonlinear systems

Consider the nonlinear stochastic systems as

$$d\chi(t) = f(\chi(t))dt + h(\chi(t))dw(t), \quad \chi(0) = \chi_0, \tag{2.1}$$

where w denotes r-dimensional independent standard wiener process and the incremental covariance $\mathbb{E}\{dw \cdot dw^T\} = \sigma(t)\sigma^T(t)dt$, $\chi \in \mathbb{R}^n$ denotes state. $f : \mathbb{R}^n \to \mathbb{R}^n$ and $h : \mathbb{R}^n \to \mathbb{R}^{n\times r}$, and satisfy $f(0) = 0$ and $h(0) = 0$.

Definition 1: ([6,24]) For any function $V(\chi)$ and stochastic nonlinear system (2.1), defining the differential operator ℓ as

$$\ell V(\chi) = \frac{\partial V}{\partial \chi} f(\chi) + \frac{1}{2} Tr\{\sigma^T h^T(\chi) \frac{\partial^2 V}{\partial \chi^2} h(\chi)\sigma\}, \tag{2.2}$$

where $Tr\{A\}$ is the trace of matrix A.

2.3. System description

Consider the ith agent of nonlinear MASs as

$$\begin{cases} d\chi_{i,j} = [\chi_{i,j+1} + \varphi_{i,j}^T(\bar{\chi}_{i,j})\theta_{i,j}]dt + h_{i,j}^T(\bar{y})dw \\ d\chi_{i,n} = [u_i + \varphi_{i,n}^T(\bar{\chi}_{i,n})\theta_{i,n}]dt + h_{i,n}^T(\bar{y})dw \\ y_i = \chi_{i,1}, \ 1 \le i \le N, \quad 1 \le j \le n-1 \end{cases} \tag{2.3}$$

where $\bar{\chi}_{i,j} = [\chi_{i,1}, \cdots, \chi_{i,j}]^T (\bar{\chi}_{i,n} = [\chi_{i,1}, \cdots, \chi_{i,n}]^T)$ is the state vector, u_i is control input and y_i is systems output, $\bar{y} = [y_1, \cdots, y_N]^T$. $\theta_{i,j}$ is an unknown constant vector, and $\varphi_{i,j}(\cdot)$ is continuous nonlinear function. $h_{i,j}^T(\cdot)$ is an unknown nonlinear function, w is an r-dimensional independent standard Wiener process, and the disturbance covariance is bounded and assumes $h_{i,j}^T h_{i,j} = \beta$ with unknown constant β. All stat variables are limited to a compact set, that is, $|\chi_{i,k}| < K_{c_{i,k}}$ with constant $K_{c_{i,k}} > 0$.

Assumption 1: [7] The undirected graph $\mathcal{G}$ is connected. Assume that at least one follower agent can obtain the information from leader.

Assumption 2: [12] The reference signal function $y_d(t)$ is bounded and satisfies $|y_d(t)| \le M_0$ with constant $M_0 > 0$. In addition, its $i-$ order derivatives are also bounded and satisfy $\left|y_d^{(i)}\right| \le M_i$ with constant $M_i > 0$.

In this paper, we will develop an adaptive consensus full-state constraint control approach for MASs (2.3), such that:

1. All signals of the controlled system are SGUUB in probability;
2. All states in controlled system do not beyond the constrained sets;
3. The each agent outputs y_i can keep consensus the leader's output y_d.

3. Adaptive consensus tracking control design

In this section, an adaptive distributed consensus constraint controller will be designed based on the backstepping control technique. First, define the local synchronization error $s_{i,1}$ of the follower agent i as

$$s_{i,1}(t) = \sum_{k\in\mathcal{N}_i} a_{i,k}(y_i - y_k) + \psi_i(y_i - y_d), \tag{3.1}$$

where $\psi_i \ge 0$ is the edge weight from agent i to leader. if follower agent i can receive the information from the leader, then $\psi_i > 0$, or else $\psi_i = 0$. y_d is the output of leader.

Let $s_1 = [s_{1,1}, s_{2,1}, \cdots, s_{N,1}]^T$, we have

$$s_1 = (\mathcal{L} + \mathcal{B})(y - \mathbf{1}y_d), \tag{3.2}$$

where $y = [y_1, y_2, \cdots, y_N]^T$, $\mathbf{1} = [1, 1, \cdots, 1]^T$, define $y - \mathbf{1}y_d$ indicates the actual tracking error between each subsystem's output and leader's output y_d.

Define the coordinate transformation as follows:

$$\begin{aligned} e_{i,1} &= s_{i,1} \\ e_{i,k} &= \chi_{i,k} - \alpha_{i,k-1}, \end{aligned} \tag{3.3}$$

where $\alpha_{i,k-1}$ is the local optimal virtual control and $s_{i,1}$ is the synchronization error and defined in (3.2).

Step 1: According to definition of $e_{i,1}$, $de_{i,1}$ is

$$\begin{aligned} de_{i,1} &= (\mathcal{L} + \mathcal{B}) \begin{bmatrix} \chi_{1,1} + \varphi_{1,1}^T\theta_{1,1} - \dot{y}_d \\ \vdots \\ \chi_{N,1} + \varphi_{N,1}^T\theta_{N,1} - \dot{y}_d \end{bmatrix} dt + (\mathcal{L} + \mathcal{B}) \begin{bmatrix} h_{1,1}^T \\ \vdots \\ h_{N,1}^T \end{bmatrix} dw \\ &= (\mathcal{L} + \mathcal{B}) \left(\begin{bmatrix} \alpha_{1,1} + e_{1,2} + \varphi_{1,1}^T\theta_{1,1} - \dot{y}_d \\ \vdots \\ \alpha_{N,1} + e_{N,2} + \varphi_{N,1}^T\theta_{N,1} - \dot{y}_d \end{bmatrix} dt + \begin{bmatrix} h_{1,1}^T \\ \vdots \\ h_{N,1}^T \end{bmatrix} dw \right). \end{aligned} \tag{3.4}$$

Choose the Barrier Lyapunov function as

$$V_1 = \sum_{i=1}^{N} \{ \tfrac{1}{4} \log \tfrac{K_{b_{i,1}}^4}{K_{b_{i,1}}^4 - e_{i,1}^4} + \tfrac{1}{2}\tilde{\theta}_{i,m1}^T \Gamma_{i,1}^{-1} \tilde{\theta}_{i,m1} \}, \tag{3.5}$$

where $\Gamma_{i,1}=\Gamma_{i,1}^T>0$ is a gain matrix. $K_{b_{i,1}}=K_{c_{i,1}}-M_0$ and the consensus tracking error $e_{i,1}$ satisfies $\Omega_{i,1}=\{e_{i,1}||e_{i,1}|<K_{b_{i,1}}\}$. $\hat{\theta}_{i,m1}$ is the estimation of $\theta_{i,m1}$ and $\tilde{\theta}_{i,m1}=\theta_{i,m1}-\hat{\theta}_{i,m1}$ is the estimation error. $\theta_{i,m1}=\theta_{i,1}$.

From (3.4) and (3.5), we have

$$\begin{aligned}\ell V_1=&\sum_{i=1}^N\frac{e_{i,1}^3(\mathcal{L}+\mathcal{B})}{K_{b_{i,1}}^4-e_{i,1}^4}(\alpha_{i,1}+e_{i,2}+\varphi_{i,1}^T\theta_{i,1}-\dot{y}_d)\\&+\frac{(\mathcal{L}+\mathcal{B})(3K_{b_{i,1}}^4+e_{i,1}^4)}{2(K_{b_{i,1}}^4-e_{i,1}^4)^2}e_{i,1}^2h_{i,1}^Th_{i,1}-\sum_{i=1}^N\tilde{\theta}_{i,m1}^T\Gamma_{i,1}^{-1}\dot{\hat{\theta}}_{i,m1}.\end{aligned}\tag{3.6}$$

Utilizing Young's inequality $a^Tb\le\frac{\varpi^c}{c}||a||^c+\frac{1}{d\varpi^d}||b||^d$ from, we have

$$\frac{e_{i,1}^3e_{i,2}(\mathcal{L}+\mathcal{B})}{K_{b_{i,1}}^4-e_{i,1}^4}\le\frac{3}{4}(\frac{e_{i,1}^3(\mathcal{L}+\mathcal{B})}{K_{b_{i,1}}^4-e_{i,1}^4})^{\frac{4}{3}}+\frac{1}{4}e_{i,2}^4,\tag{3.7}$$

$$\frac{(\mathcal{L}+\mathcal{B})(3K_{b_{i,1}}^4+e_{i,1}^4)}{2(K_{b_{i,1}}^4-e_{i,1}^4)^2}e_{i,1}^2h_{i,1}^Th_{i,1}\le\frac{e_{i,1}^3(\mathcal{L}+\mathcal{B})}{K_{b_{i,1}}^4-e_{i,1}^4}\frac{e_{i,1}(3K_{b_{i,1}}^4+e_{i,1}^4)^2}{8(K_{b_{i,1}}^4-e_{i,1}^4)^3}+\frac{\mathcal{L}+\mathcal{B}}{2}\beta^2.\tag{3.8}$$

Inserting (3.7), (3.8) into (3.6) gets

$$\begin{aligned}\ell V_1\le&\sum_{i=1}^N\{\frac{e_{i,1}^3(\mathcal{L}+\mathcal{B})}{K_{b_{i,1}}^4-e_{i,1}^4}(\alpha_{i,1}+e_{i,1}(K_{b_{i,1}}^4-e_{i,1}^4)^{-\frac{1}{3}}+\varphi_{i,m1}^T\hat{\theta}_{i,m1}\\&-\dot{y}_d+\frac{e_{i,1}(3K_{b_{i,1}}^4+e_{i,1}^4)^2}{8(K_{b_{i,1}}^4-e_{i,1}^4)^3})+\tilde{\theta}_{i,m1}^T\Gamma_{i,1}^{-1}(\frac{e_{i,1}^3(\mathcal{L}+\mathcal{B})}{K_{b_{i,1}}^4-e_{i,1}^4}\Gamma_{i,1}\varphi_{i,m1}\\&-\dot{\hat{\theta}}_{i,m1})+\frac{1}{4}e_{i,2}^4+\frac{\mathcal{L}+\mathcal{B}}{2}\beta^2\}.\end{aligned}\tag{3.9}$$

Design the virtual control law $\alpha_{i,1}$ and adaptive law $\dot{\hat{\theta}}_{i,m1}$ as

$$\alpha_{i,1}=-(\mathcal{L}+\mathcal{B})^{-1}\{\kappa_{i,1}e_{i,1}+\frac{e_{i,1}(\mathcal{L}+\mathcal{B})^{\frac{4}{3}}}{(K_{b_{i,1}}^4-e_{i,1}^4)^3}+\varphi_{i,m1}^T\hat{\theta}_{i,m1}-\dot{y}_d+\frac{e_{i,1}(3K_{b_{i,1}}^4+e_{i,1}^4)^2}{8(K_{b_{i,1}}^4-e_{i,1}^4)^3}\},\tag{3.10}$$

$$\dot{\hat{\theta}}_{i,m1}=\frac{e_{i,1}^3(\mathcal{L}+\mathcal{B})}{K_{b_{i,1}}^4-e_{i,1}^4}\varphi_{i,m1}-\sigma_{i,1}\hat{\theta}_{i,m1},\tag{3.11}$$

where $\varphi_{i,m1}=\varphi_{i,1}$, $\kappa_{i,1}>0$ and $\sigma_{i,1}>0$ are design parameters.

From (3.9)–(3.11), we have

$$\ell V_1\le\sum_{i=1}^N\{-\frac{\kappa_{i,1}e_{i,1}^4}{K_{b_{i,1}}^4-e_{i,1}^4}+\sigma_{i,1}\tilde{\theta}_{i,m1}^T\hat{\theta}_{i,m1}+\frac{1}{4}e_{i,2}^4+\frac{\mathcal{L}+\mathcal{B}}{2}\beta^2\}.\tag{3.12}$$

Step $j(j=2,\cdots,n-1)$: From (2.3) and (3.4), we have

$$\begin{aligned}de_{i,k}&=[\chi_{i,j+1}+\varphi_{i,j}^T(\bar{\chi}_{i,j})\theta_{i,j}-\ell\alpha_{i,j-1}]dt+[h_{i,j}^T(\bar{y})-\sum_{k=1}^{j-1}\frac{\partial\alpha_{i,j-1}}{\partial\chi_{i,k}}h_{i,k}(\bar{y})]dw\\&=[e_{i,j+1}+\alpha_{i,j}+\varphi_{i,j}^T(\bar{\chi}_{i,j})\theta_i-\ell\varphi_{i,j-1}]dt+[h_{i,j}^T(\bar{y})-\sum_{k=1}^{j-1}\frac{\partial\alpha_{i,j-1}}{\partial\chi_{i,k}}h_{i,k}(\bar{y})]dw\\&=[e_{i,j+1}+\varphi_{i,mj}^T\theta_{i,mj}-\frac{1}{2}\sum_{s=1,l=1}^{j-1}\frac{\partial^2\alpha_{i,j-1}}{\partial\chi_s\partial\chi_l}h_{i,s}^Th_{i,l}+\alpha_{i,j}-H_{i,j}]dt+[h_{i,j}^T-\sum_{k=1}^{j-1}\frac{\partial\alpha_{i,j-1}}{\partial\chi_{i,k}}h_{i,k}]dw,\end{aligned}\tag{3.13}$$

where

$\ell\varphi_{i,j-1} = \sum_{k=1}^{j-1} \frac{\partial\alpha_{i,j-1}}{\partial\chi_{i,k}}(\chi_{i,k+1} + \varphi_{i,k}^T(\bar{\chi}_{i,k})\theta_{i,k}) + \sum_{k=0}^{j-1} \frac{\partial\alpha_{i,j-1}}{\partial y_r^k} y_r^{(k+1)} + \sum_{s=1}^{N} a_{is}\frac{\partial\alpha_{i,j-1}}{\partial\hat{\theta}_{i,s}}\dot{\hat{\theta}}_{i,s} + \frac{\partial\alpha_{i,j-1}}{\partial\hat{\theta}_i}\dot{\hat{\theta}}_i + \frac{1}{2}\sum_{s=1,l=1}^{j-1} \frac{\partial^2\alpha_{i,j-1}}{\partial\chi_s\partial\chi_l} h_{i,s}^T h_{i,l}$. $\varphi_{i,mj} = [\varphi_{i,j}, -\frac{\partial\alpha_{i,j-1}}{\partial\chi_{i,j-1}}\varphi_{i,j-1}^T, \cdots, -\frac{\partial\alpha_{i,j-1}}{\partial\chi_{i,1}}\varphi_{i,1}^T, -\frac{\partial\alpha_{i,j-1}}{\partial\chi_{i,j-1}}\chi_{i,j}, \cdots, -\frac{\partial\alpha_{i,j-1}}{\partial\chi_{i,1}}\chi_{i,2}]^T$, $\theta_{i,mj} = [\theta_{i,j}, \theta_{i,j-1}, \cdots, \theta_{i,1}, 1, \cdots, 1]^T$ and $H_{i,j} = \sum_{k=0}^{j-1} \frac{\partial\alpha_{i,j-1}}{\partial y_r^k} y_r^{(k+1)} + \sum_{s=1}^{N} a_{is}\frac{\partial\alpha_{i,j-1}}{\partial\hat{\theta}_{i,s}}\dot{\hat{\theta}}_{i,s} + \frac{\partial\alpha_{i,j-1}}{\partial\hat{\theta}_i}\dot{\hat{\theta}}_i$.

Choose the Barrier Lyapunov function as

$$V_j = V_{j-1} + \sum_{i=1}^{N}\{\frac{1}{4}\log\frac{K_{b_{i,j}}^4}{K_{b_{i,j}}^4 - e_{i,j}^4} + \frac{1}{2}\tilde{\theta}_{i,mj}^T\Gamma_{i,j}^{-1}\tilde{\theta}_{i,mj}\}, \tag{3.14}$$

where $\hat{\theta}_{i,mj}$ is the estimation of $\theta_{i,mj}$, $\tilde{\theta}_{i,mj} = \theta_{i,mj} - \hat{\theta}_{i,mj}$ is the estimation error, $\Gamma_{i,j} = \Gamma_{i,j}^T > 0$ is gain matrix. $K_{b_{i,j}} = K_{c_{i,j}} - M_{i,j}$ ($|\alpha_{i,j-1}| < M_{i,j}$ with positive constant $M_{i,j}$) and the consensus tracking error $e_{i,j}$ satisfies $\Omega_{i,j} = \{e_{i,j}| \, |e_{i,j}| < K_{b_{i,j}}\}$.

From (3.13) and (3.14), we have

$$\begin{aligned}\ell V_j = \ell V_{i-1} + \sum_{i=1}^{N}\{\frac{e_{i,j}^3}{K_{b_{i,j}}^4 - e_{i,j}^4}[e_{i,j+1} + \varphi_{i,mj}^T\theta_{i,mj} - H_{i,j} - \frac{1}{2}\sum_{s=1,l=1}^{j-1}\frac{\partial^2\alpha_{i,j-1}}{\partial\chi_s\partial\chi_l}h_{i,s}^T h_{i,l} \\ + \alpha_{i,j}] + \frac{3K_{b_{i,j}}^4 + e_{i,j}^4}{2(K_{b_{i,j}}^4 - e_{i,j}^4)}e_{i,j}^2\left\|h_{i,j} - \sum_{k=1}^{j-1}\frac{\partial\alpha_{i,j-1}}{\partial\chi_{i,k}}h_{i,k}\right\|^2 - \tilde{\theta}_{i,mj}^T\Gamma_{i,j}^{-1}\dot{\hat{\theta}}_{i,mj}\}.\end{aligned} \tag{3.15}$$

By using Young's inequality, we have

$$-\frac{1}{2}\frac{e_{i,j}^3}{K_{b_{i,j}}^4 - e_{i,j}^4}\sum_{s=1,l=1}^{j-1}\frac{\partial^2\alpha_{i,j-1}}{\partial\chi_s\partial\chi_l}h_{i,s}^T h_{i,l} \le \frac{e_{i,j}^3}{K_{b_{i,j}}^4 - e_{i,j}^4}\frac{e_{i,j}^3}{4(K_{b_{i,j}}^4 - e_{i,j}^4)}\sum_{s=1,l=1}^{j-1}(\frac{\partial^2\alpha_{i,j-1}}{\partial\chi_s\partial\chi_l})^2 + \frac{1}{4}\beta^2, \tag{3.16}$$

$$\frac{e_{i,j}^3 e_{i,j+1}}{K_{b_{i,j}}^4 - e_{i,j}^4} \le \frac{3}{4}(\frac{e_{i,j}^3}{K_{b_{i,j}}^4 - e_{i,j}^4})^{\frac{4}{3}} + \frac{1}{4}e_{i,j+1}^4, \tag{3.17}$$

$$\begin{aligned}&\frac{3K_{b_{i,j}}^4 + e_{i,j}^4}{2(K_{b_{i,j}}^4 - e_{i,j}^4)}e_{i,j}^2\left\|h_{i,j} - \sum_{k=1}^{j-1}\frac{\partial\alpha_{i,j-1}}{\partial\chi_{i,k}}h_{i,k}\right\|^2 \\ &\le (j-1)^2\frac{e_{i,j}^3}{K_{b_{i,j}}^4 - e_{i,j}^4}\frac{e_{i,j}(3K_{b_{i,j}}^4 + e_{i,j}^4)^2}{8(K_{b_{i,j}}^4 - e_{i,j}^4)^3}\sum_{k=1}^{j-1}(\frac{\partial\alpha_{i,j-1}}{\partial\chi_{i,k}})^4 \\ &\quad + \frac{e_{i,j}^3}{K_{b_{i,j}}^4 - e_{i,j}^4}\frac{e_{i,j}(3K_{b_{i,j}}^4 + e_{i,j}^4)^2}{4(K_{b_{i,j}}^4 - e_{i,j}^4)^3}\sum_{k=1}^{j-1}(\frac{\partial\alpha_{i,j-1}}{\partial\chi_{i,k}})^2 + 2\beta^2 \\ &\quad + \frac{e_{i,j}^3}{K_{b_{i,j}}^4 - e_{i,j}^4}\frac{e_{i,j}(3K_{b_{i,j}}^4 + e_{i,j}^4)^2}{8(K_{b_{i,j}}^4 - e_{i,j}^4)^3}.\end{aligned} \tag{3.18}$$

Substituting (3.16)–(3.18) into (3.15) yields

$$\begin{aligned}\ell V_j \le &\sum_{i=1}^{N}\{\sum_{k=1}^{j-1}[-\frac{\kappa_{i,k}e_{i,k}^4}{K_{b_{i,k}}^4 - e_{i,k}^4} + \sigma_{i,k}\tilde{\theta}_{i,mk}\hat{\theta}_{i,mk} + (\frac{9(k-1)}{4} + \frac{\mathcal{L}+\mathcal{B}}{2})\beta^2]\} \\ &+ \sum_{i=1}^{N}\{\frac{e_{i,j}^3}{K_{b_{i,j}}^4 - e_{i,j}^4}[\alpha_{i,j} + \frac{1}{4}e_{i,j}(K_{b_{i,j}}^4 - e_{i,j}^4) + \frac{3}{4}(\frac{e_{i,j}^3}{K_{b_{i,j}}^4 - e_{i,j}^4})^{\frac{1}{3}} - \bar{H}_{i,j} \\ &+ \varphi_{i,mj}^T\hat{\theta}_{i,mj}] + \tilde{\theta}_{i,mj}^T\Gamma_{i,j}^{-1}(\frac{\varphi_{i,mj}^T\Gamma_{i,j}e_{i,j}^3}{K_{b_{i,j}}^4 - e_{i,j}^4} - \dot{\hat{\theta}}_{i,mj}) + \frac{1}{4}e_{i,j+1}^4\},\end{aligned} \tag{3.19}$$

where $\bar{H}_{i,j} = H_{i,j} - \frac{e_{i,j}^3}{4(K_{b_{i,j}}^4 - e_{i,j}^4)} \sum_{s=1,l=1}^{j-1} (\frac{\partial^2 \alpha_{i,j-1}}{\partial \chi_s \partial \chi_l})^2 - (j-1)^2 \frac{e_{i,j}(3K_{b_{i,j}}^4 + e_{i,j}^4)^2}{8(K_{b_{i,j}}^4 - e_{i,j}^4)^3} \sum_{k=1}^{j-1} (\frac{\partial \alpha_{i,j-1}}{\partial \chi_{i,k}})^4 - \frac{e_{i,j}(3K_{b_{i,j}}^4 + e_{i,j}^4)^2}{4(K_{b_{i,j}}^4 - e_{i,j}^4)^3} \sum_{k=1}^{j-1} (\frac{\partial \alpha_{i,j-1}}{\partial \chi_{i,k}})^2 - \frac{e_{i,j}(3K_{b_{i,j}}^4 + e_{i,j}^4)^2}{8(K_{b_{i,j}}^4 - e_{i,j}^4)^3}$.

Design the intermediate control law $\alpha_{i,j}$ and adaptive law $\dot{\hat{\theta}}_{i,mj}$ as

$$\alpha_{i,j} = -\kappa_{i,j} e_{i,j} - \frac{1}{4} e_{i,j}(K_{b_{i,j}}^4 - e_{i,j}^4) - \frac{3}{4}(\frac{e_{i,j}^3}{K_{b_{i,j}}^4 - e_{i,j}^4})^{\frac{1}{3}} + \bar{H}_{i,j} - \varphi_{i,mj}^T \hat{\theta}_{i,mj} \tag{3.20}$$

$$\dot{\hat{\theta}}_{i,mj} = \Gamma_{i,j} \frac{\varphi_{i,mj}^T e_{i,j}^3}{K_{b_{i,j}}^4 - e_{i,j}^4} - \sigma_{i,j} \hat{\theta}_{i,mj} \tag{3.21}$$

where $\kappa_{i,j} > 0$ and $\sigma_{i,j} > 0$ are design parameters.

By invoking (3.19)–(3.21), we have

$$\ell V_j \leq \sum_{i=1}^{N} \{ \sum_{k=1}^{j} [-\frac{\kappa_{i,k} e_{i,k}^4}{K_{b_{i,k}}^4 - e_{i,k}^4} + \sigma_{i,k} \tilde{\theta}_{i,mk} \hat{\theta}_{i,mk} + (\frac{9(k-1)}{4} + \frac{\mathcal{L}+\mathcal{B}}{2})\beta^2] + \frac{1}{4} e_{i,j+1}^4 \}. \tag{3.22}$$

Step n: From (2.3) and (3.4), we have

$$\begin{aligned} de_{i,n} &= [u_i + \varphi_{i,n}^T \theta_{i,n} - \ell \alpha_{i,n-1}]dt + [h_{i,n}^T(\bar{y}) - \sum_{k=1}^{n-1} \frac{\partial \alpha_{i,n-1}}{\partial \chi_{i,k}} h_{i,k}(\bar{y})]dw \\ &= [u_i + \varphi_{i,mn}^T \theta_{i,mn} - \frac{1}{2} \sum_{s=1,l=1}^{n-1} \frac{\partial^2 \alpha_{i,n-1}}{\partial \chi_s \partial \chi_l} h_{i,s}^T h_{i,l} - H_{i,n}]dt + [h_{i,n}^T - \sum_{k=1}^{n-1} \frac{\partial \alpha_{i,n-1}}{\partial \chi_{i,k}} h_{i,k}]dw, \end{aligned} \tag{3.23}$$

where $\ell \varphi_{i,n-1} = \sum_{k=1}^{n-1} \frac{\partial \alpha_{i,n-1}}{\partial \chi_{i,k}} (\chi_{i,k+1} + \varphi_{i,k}^T(\bar{\chi}_{i,k}) \theta_{i,k}) + \sum_{k=0}^{n-1} \frac{\partial \alpha_{i,n-1}}{\partial y_r^k} y_r^{(k+1)} + \sum_{s=1}^{N} a_{is} \frac{\partial \alpha_{i,n-1}}{\partial \hat{\theta}_{i,s}} \dot{\hat{\theta}}_{i,s} + \frac{1}{2} \sum_{s=1,l=1}^{n-1} \frac{\partial^2 \alpha_{i,n-1}}{\partial \chi_s \partial \chi_l} h_{i,s}^T h_{i,l} + \frac{\partial \alpha_{i,n-1}}{\partial \hat{\theta}_i} \dot{\hat{\theta}}_i$. $\varphi_{i,mn} = [\varphi_{i,n}, -\frac{\partial \alpha_{i,n-1}}{\partial \chi_{i,n-1}} \varphi_{i,n-1}^T, \cdots, -\frac{\partial \alpha_{i,n-1}}{\partial \chi_{i,1}} \varphi_{i,1}^T, -\frac{\partial \alpha_{i,n-1}}{\partial \chi_{i,n-1}} \chi_{i,n}, \cdots, -\frac{\partial \alpha_{i,n-1}}{\partial \chi_{i,1}} \chi_{i,2}]^T$, $\theta_{i,mn} = [\theta_{i,n}, \theta_{i,n-1}, \cdots, \theta_{i,1}, 1, \cdots, 1]^T$ and $H_{i,n} = \sum_{k=0}^{n-1} \frac{\partial \alpha_{i,n-1}}{\partial y_r^k} y_r^{(k+1)} + \sum_{s=1}^{N} a_{is} \frac{\partial \alpha_{i,n-1}}{\partial \hat{\theta}_{i,s}} \dot{\hat{\theta}}_{i,s} + \frac{\partial \alpha_{i,n-1}}{\partial \hat{\theta}_i} \dot{\hat{\theta}}_i$.

Choose the Barrier Lyapunov function as

$$V_n = V_{n-1} + \sum_{i=1}^{N} \{ \frac{1}{4} \log \frac{K_{b_{i,n}}^4}{K_{b_{i,n}}^4 - e_{i,n}^4} + \frac{1}{2} \tilde{\theta}_{i,mn}^T \Gamma_{i,n}^{-1} \tilde{\theta}_{i,mn} \}, \tag{3.24}$$

where $\hat{\theta}_{i,mn}$ is the estimation of $\theta_{i,mn}$, $\tilde{\theta}_{i,mn} = \theta_{i,mn} - \hat{\theta}_{i,mn}$ is the estimation error, $\Gamma_{i,n} = \Gamma_{i,n}^T > 0$ is gain matrix. $K_{b_{i,n}} = K_{c_{i,n}} - M_{i,n}$ ($|\alpha_{i,n-1}| < M_{i,n}$ with positive constant $M_{i,n}$) and error $e_{i,n}$ satisfies $\Omega_{i,n} = \{e_{i,n} | |e_{i,n}| < K_{b_{i,n}}\}$.

From (3.23) and (3.24), we have

$$\begin{aligned} \ell V_n \leq & \sum_{i=1}^{N} \{ \sum_{k=1}^{n-1} [-\frac{\kappa_{i,k} e_{i,k}^4}{K_{b_{i,k}}^4 - e_{i,k}^4} + \sigma_{i,k} \tilde{\theta}_{i,mk} \hat{\theta}_{i,mk} + (\frac{9(k-1)}{4} + \frac{\mathcal{L}+\mathcal{B}}{2})\beta^2] \} \\ & + \sum_{i=1}^{N} \{ \frac{e_{i,n}^3}{K_{b_{i,n}}^4 - e_{i,n}^4} [u_i + \frac{1}{4} e_{i,n}(K_{b_{i,n}}^4 - e_{i,n}^4) - \bar{H}_{i,n} \\ & + \varphi_{i,mn}^T \hat{\theta}_{i,mn}] + \tilde{\theta}_{i,mn}^T \Gamma_{i,n}^{-1} (\frac{\varphi_{i,mn}^T \Gamma_{i,n} e_{i,n}^3}{K_{b_{i,n}}^4 - e_{i,n}^4} - \dot{\hat{\theta}}_{i,mn}) \}, \end{aligned} \tag{3.25}$$

where $\bar{H}_{i,n} = H_{i,n} - \frac{e_{i,n}(3K_{b_{i,n}}^4 + e_{i,n}^4)^2}{8(K_{b_{i,n}}^4 - e_{i,n}^4)^3} - \frac{e_{i,n}^3}{4(K_{b_{i,n}}^4 - e_{i,n}^4)} \sum_{s=1,l=1}^{n-1} (\frac{\partial^2 \alpha_{i,n-1}}{\partial \chi_s \partial \chi_l})^2 - (n-1)^2 \frac{e_{i,n}(3K_{b_{i,n}}^4 + e_{i,n}^4)^2}{8(K_{b_{i,n}}^4 - e_{i,n}^4)^3} \sum_{k=1}^{n-1} (\frac{\partial \alpha_{i,n-1}}{\partial \chi_{i,k}})^4 - \frac{e_{i,n}(3K_{b_{i,n}}^4 + e_{i,n}^4)^2}{4(K_{b_{i,n}}^4 - e_{i,n}^4)^3} \times \sum_{k=1}^{n-1} (\frac{\partial \alpha_{i,n-1}}{\partial \chi_{i,k}})^2$.

Design the control law u_i and adaptive law $\dot{\hat{\theta}}_{i,mn}$ as

$$u_i = -\kappa_{i,n} e_{i,n} - \frac{1}{4} e_{i,n}(K_{b_{i,n}}^4 - e_{i,n}^4) + \bar{H}_{i,n} - \varphi_{i,mn}^T \hat{\theta}_{i,mn} \tag{3.26}$$

$$\dot{\hat{\theta}}_{i,mn} = \Gamma_{i,n} \frac{\varphi_{i,mn}^T e_{i,n}^3}{K_{b_{i,n}}^4 - e_{i,n}^4} - \sigma_{i,n} \hat{\theta}_{i,mn} \tag{3.27}$$

where $\kappa_{i,n} > 0$ and $\sigma_{i,n} > 0$ are design parameters.

Substituting (3.26), (3.27) into (3.25) yields

$$\ell V_n \leq \sum_{i=1}^{N} \{ \sum_{k=1}^{n} [-\frac{\kappa_{i,k} e_{i,k}^4}{K_{b_{i,k}}^4 - e_{i,k}^4} + \sigma_{i,k} \tilde{\theta}_{i,mk} \hat{\theta}_{i,mk} + (\frac{9(k-1)}{4} + \frac{\mathcal{L}+\mathcal{B}}{2}) \beta^2] \}. \tag{3.28}$$

Stability analysis

Based on m-step backstepping design, the properties of the proposed adaptive distributed consensus control method can be described as the following Theorem.

Theorem 1: Consider the nonlinear stochastic MASs (2.3), the designed adaptive consensus controller (3.26), virtual controllers (3.10), (3.20), adaptive laws (3.11), (3.21) and (3.27), cannot only ensure all signals of the controlled system are bounded in probability, and the outputs y_i of follower agents can keep consensus with leader's output y_d. Finally, all states do not beyond their constrained sets.

Proof: By adopting Young's inequality, we have

$$\sigma_{i,k} \tilde{\theta}_{i,mk} \hat{\theta}_{i,mk} \leq -\frac{\sigma_{i,k}}{2} \tilde{\theta}_{i,mk}^T \tilde{\theta}_{i,mk} + \frac{\sigma_{i,k}}{2} \theta_{i,mk}^T \theta_{i,mk}. \tag{3.29}$$

Thus, (3.29) can be rewritten as

$$\ell V_n \leq \sum_{i=1}^{N} \{ \sum_{k=1}^{n} [-\frac{\kappa_{i,k} e_{i,k}^4}{K_{b_{i,k}}^4 - e_{i,k}^4} - \frac{\sigma_{i,k}}{2} \tilde{\theta}_{i,mk}^T \tilde{\theta}_{i,mk} + D_{i,n}] \}, \tag{3.30}$$

where $D_{i,n} = \frac{\sigma_{i,k}}{2} \theta_{i,mk}^T \theta_{i,mk} + (\frac{9(k-1)}{4} + \frac{\mathcal{L}+\mathcal{B}}{2}) \beta^2$.

According to [11], we have, $\log \frac{K_{b_{i,k}}^4}{K_{b_{i,k}}^4 - e_{i,k}^4} \leq \frac{e_{i,k}^4}{K_{b_{i,k}}^4 - e_{i,k}^4}$ holds for $|e_{i,k}| \leq K_{b_{i,k}}$, thus, (3.30) can be further rewritten as

$$\ell V_n \leq -C V_n + D, \tag{3.31}$$

where $C = \min\{ \sum_{i=1}^{N} \sum_{k=1}^{n} 4\kappa_{i,k}, \sigma_{i,k} \lambda_{\max}\{\Gamma_{i,k}^{-1}\} \}$ and $D = \sum_{i=1}^{N} \sum_{k=1}^{n} D_{i,k}$.

From Definition 1, we have

$$E(V_n(t)) \leq V_n(0) \mathrm{e}^{-Ct} + \frac{D}{C}. \tag{3.32}$$

Thus, form (3.32), it impled that all signals of controlled system are SGUUB.

In addition, since $\left|e_{i,1}\right| \leq K_{b_{i,1}}$ and $e_{*1} = [e_{1,1}, \cdots, e_{N,1}]^T = \mathcal{H}(y - \mathbf{1}y_d)$, we have

$$\|y - \mathbf{1}y_d\| \leq \frac{1}{\lambda_{\min}(\mathcal{H})} \|e_{*1}\|, \tag{3.33}$$

where $\mathcal{H} = \mathcal{L} + \mathcal{B}$ and $\lambda_{\min}(\mathcal{H})$ is the minimum singular value of $\mathcal{H}$.

From $|y_d| \leq M_0$, we have $\left|e_{i,1}\right| + |y_d| \leq K_{b_{i,1}} + M_0$, since $K_{c_{i,1}} = K_{b_{i,1}} + M_0$, thus, $\left|\chi_{i,1}\right| < K_{c_{i,1}}$. From (3.32) and the definition of $\tilde{\theta}_{i,m1}$, we have $\tilde{\theta}_{i,m1}$ is bounded, since $\theta_{i,m1}$ is bounded and $\hat{\theta}_{i,m1} = \theta_{i,m1} - \tilde{\theta}_{i,m1}$, thus, $\hat{\theta}_{i,m1}$ is needed to be bounded. Thus, we have $\alpha_{i,1}$ is bounded because of $e_{i,1}$, $\hat{\theta}_{i,m1}$ and $\dot{y}_d$ are bounded. Similarly, it can be seen $\left|\chi_{i,k}\right| < K_{c_{i,k}}$. Obviously, it impled that $e_{i,1}, \cdots, e_{i,n}, \chi_{i,1}, \cdots, \chi_{i,n}, \hat{\theta}_{i,m1}, \cdots, \hat{\theta}_{i,mn}$ and u_i are all bounded.

4. Simulation example

In this section, the numerical example is provided to illustrate the feasibility of the proposed control method.

Example: The following stochastic nonlinear MASs are considered in this paper, which contain five follower agents and one leader, and can be depicted as

$$\begin{cases} d\chi_{i,1} = [\chi_{i,2} + \varphi_{i,1}^T(\chi_{i,1})\theta_{i,1}]dt + h_{i,1}(\bar{y})dw \\ d\chi_{i,2} = [u_i + \varphi_{i,2}^T(\bar{\chi}_{i,2})\theta_{i,2}]dt + h_{i,2}(\bar{y})dw \\ y_i = \chi_{i,1} \end{cases} \tag{4.1}$$

where $\bar{\chi}_{i,2} = [\chi_{i,1}, \chi_{i,2}]^T (i = 1, \cdots, 5)$, $\bar{y} = [y_1, \cdots, y_5]^T$, $\varphi_{i,1} = x_{i,1}^2$, $\varphi_{i,2} = [x_{i,2}\cos(x_{1,1}), x_{i,1}x_{i,2}]^T$, $\theta_{i,1} = 0.2$, $\theta_{i,2} = [0.1, 0.6]^T$ $h_{1,1} = \sin(0.5y_1y_4 + y_2y_5 + y_3y_4)$, $h_{1,2} = y_1y_2y_3y_4y_5$, $h_{2,1} = 0.5\sin(0.2 * y_1y_5) + \cos(y_2 + y_3y_4) - 1$, $h_{2,2} = 0.5\cos(y_1y_2y_4 + y_3y_5)$, $h_{3,1} = 0.5(y_1y_4 + 2y_2 + y_3y_5)$, $h_{3,2} = 0.5(y_1 + y_1y_2y_3 + y_4 + y_5)$, $h_{4,1} = 0.2y_1\cos(y_1y_2) + y_3 + y_4 + y_3y_5$, $h_{4,2} = \sin y_1y_2y_3 + y_4y_5$, $h_{5,1} = 0.5y_1y_2y_3y_4y_5$, $h_{5,2} = y_1^2y_2 + y_3 + y_4y_5$.

The dynamic of the leader is depicted as $y_d = 0.5(\sin(t) + \sin(0.5t))$. The communication graph is as Figure 1.

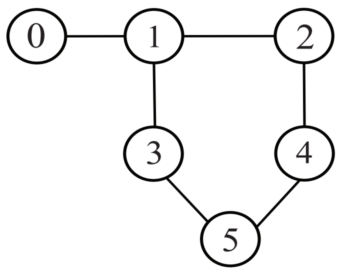

Figure 1. Communication topology.

The adjacency matrix $\mathcal{N}$ and Laplacian matrix $\mathcal{L}$ as

$$\mathcal{N} = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix} \text{ and } \mathcal{L} = \begin{bmatrix} 2 & -1 & -1 & 0 & 0 \\ -1 & 2 & 0 & -1 & 0 \\ -1 & 0 & 2 & 0 & -1 \\ 0 & -1 & 0 & 2 & -1 \\ 0 & 0 & -1 & -1 & 2 \end{bmatrix}.$$

The design parameters in virtual control, controller and adaptive laws are selected as $K_{c_{i,k}} = 1.7$, $K_{b_{i,k}} = 0.7$, $\Gamma_{i,1} = 0.16 I_{5\times5}$, $\Gamma_{i,2} = 0.18 I_{5\times5}$, $\sigma_{i,k} = 0.1$, $c_{1,1} = 12$, $c_{1,2} = 15$, $c_{2,1} = 18$, $c_{2,2} = 12$, $c_{3,1} = 15$, $c_{4,1} = 12$, $c_{4,2} = 15$, $c_{5,1} = 12$, $c_{5,2} = 15$.

The initial values are chosen as $\chi_{i,k} = 0$, $\hat{\theta}_{1,m1} = 0.03$, $\hat{\theta}_{1,m2} = [0.2, 0.1, 0.2, 0.3]$, $\hat{\theta}_{2,m1} = 0.01$, $\hat{\theta}_{1,m2} = [0.1, 0, 0.2, 0.2]$, $\hat{\theta}_{3,m1} = 0.02$, $\hat{\theta}_{3,m2} = [0.2, 0.1, 0.21, 0.2]$, $\hat{\theta}_{4,m1} = 0.04$, $\hat{\theta}_{4,m2} = [0.2, 0.3, 0.1, 0.2]$, $\hat{\theta}_{5,m1} = 0.05$, $\hat{\theta}_{5,m2} = [0.1, 0, 0.2, 0.1]$. Thus, the simulation results are displayed by Figures 2–7. Figure 2 is the curves of y_i, y_d and constraint bound; Figure 3 is the curves of tracking errors $e_{i,1}$ between followers and leader; Figure 4 is the curves of states $\chi_{i,2} (i = 1, \cdots, 5)$ and constraint bound; Figure 5 is the trajectories of control laws u_i; Figure 6 is the curves of adaptive parameters $\hat{\theta}_{i,m1}$, and Figure 7 is the curves of $\left\|\hat{\theta}_{i,m2}\right\|$.

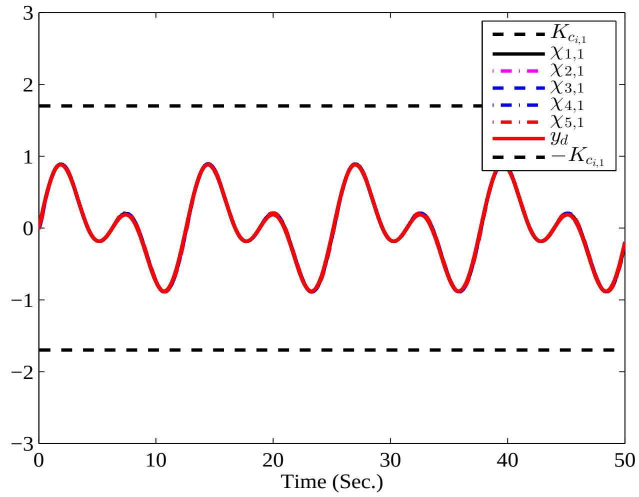

Figure 2. Curves of states $\chi_{i,1}$, y_d and constraint bound.

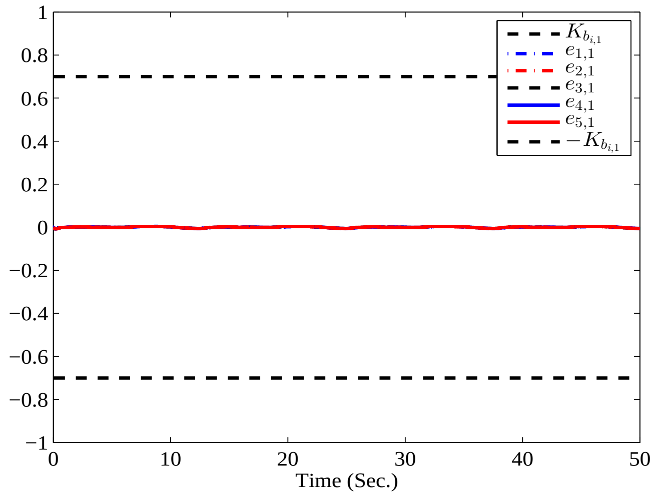

Figure 3. Curves of tracking errors $e_{i,1}$ between follower and leader.

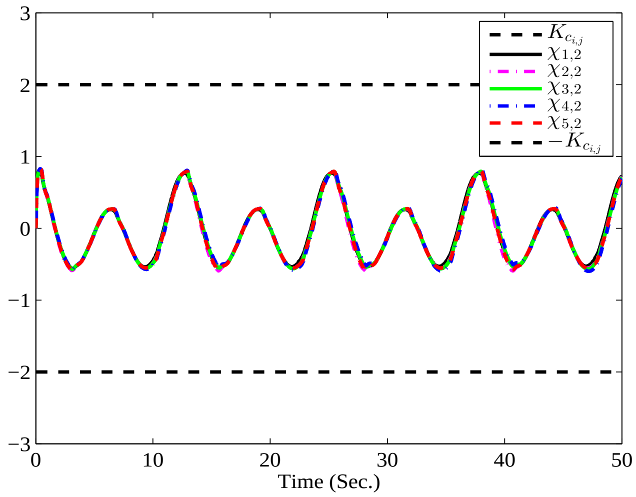

Figure 4. Curves of states $\chi_{i,2}$ and constraint bound.

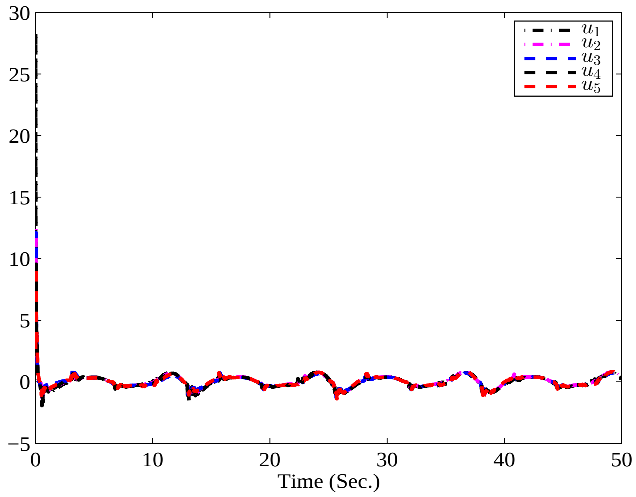

Figure 5. Curves of control laws u_i.

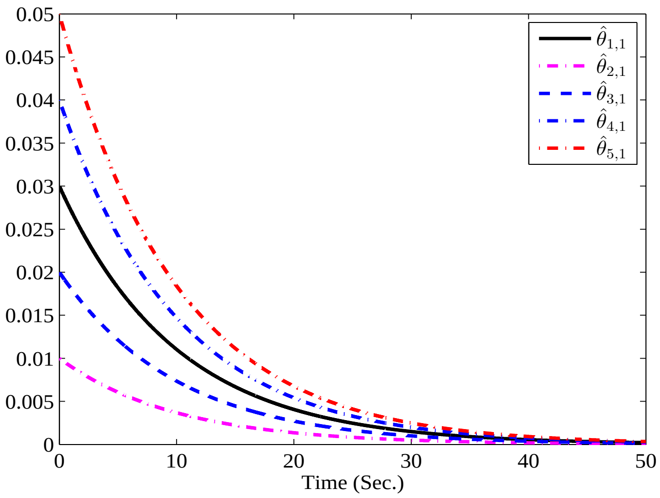

Figure 6. Curves of $\hat{\theta}_{i,m1}(i = 1, \cdots, 5)$.

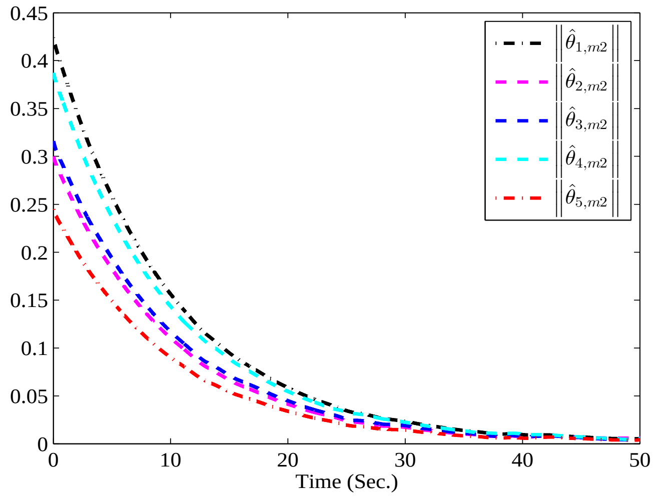

Figure 7. Curves of $\|\hat{\theta}_{i,m2}\|$ $(i = 1, \cdots, 5)$.

Obviously, it can be seen from the simulation results in example that the proposed adaptive consensus control method can ensure all signals of controlled system are bounded in probability, and the states are not beyond their constrained sets.

5. Conclusions

We have investigated the adaptive consensus constraint control problem for stochastic nonlinear MASs under undirected topological graph subjected to the full state constraints. Based on the adaptive backstepping control and BLF theory, an adaptive consensus constraint control method has been proposed. It has proved that all signals of controlled system are all bounded in probability, and the leader-follower consensus has been achieved. In addition, it also has proved the states do not beyond their constrained sets.

Since the convergence time in fixed-time control does not depend on the initial conditions of nonlinear systems, thus, inspires by this point, our future research direction will focus on the fixed-time consensus adaptive robust control for stochastic MASs under directed topological graph or switched systems similar to [25–28].

Acknowledgments

This work was supported in part by the National Natural Science Foundation of China under Grant 61903167.

Conflict of interest

The authors declare no conflict of interest.

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