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Research article

Computation of reverse degree-based topological indices of hex-derived networks

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Abstract: Network theory gives an approach to show huge and complex frameworks through a complete arrangement of logical devices. A network is made is made of vertices and edges, where the degree of a vertex refers to the number of joined edges. The degree appropriation of a network represents the likelihood of every vertex having a particular degree and shows significant worldwide network properties. Network theory has applications in many disciplines like basic sciences, computer science, engineering, medical, business, public health and sociology. There are some important networks like logistical networks, gene regulatory networks, metabolic networks, social networks, derived networks. Topological index is a numerical number assigned to the molecular structure/netwrok which is used for correlation analysis in physical, theoretical and environmental chemistry. The hex-derived networks are created by hexagonal networks of dimension t, these networks have an assortment of valuable applications in computer science, medical science and engineering. In this paper we discuss the reverse degree-based topological for third type of hex-derived networks.

Keywords: topological indices; reverse degree; hex-derived networks
Mathematics Subject Classification: 05C90

1. Introduction

Graph theory has provided the researcher with various useful tools, such as graph labeling, locating numbers and topological indices. Graph theory subject has many applications and implementations in different research subjects like chemistry, medicine and engineering. A graph can be recognized by a numeric value, a polynomial, a sequence of numbers or a matrix. The representation of the chemical compound in terms of diagram, known as its molecular graph, in which its atoms and the chemical bonding between them represent the nodes and edges, respectively. Recently, a new subject caught attention of the researchers was introduced, which is the combination of chemistry, information science and mathematics is called Cheminformatics, which studies QSAR/QSPR relationship, bioactivity and characterization of chemical compounds [1].

The topological index is a numeric value related with chemical compositions maintaining the correlation of chemical structures with many physico-chemical properties, chemical reactivity or biological activity. Topological indices are prepared on the grounds of the transformation of a chemical network into a number that describes the topology of the chemical network. Some of the main types of topological indices of graphs are distance-based topological indices, degree-based topological indices, and counting-related topological indices. Recently, numerous researchers have found topological indices for the study of fundamental properties of molecular graph or network. These networks have very motivating topological properties which have been considered in different characteristics in [2–9].

Let $G = (V, E)$ be a simple connected graph, with V be the vertex set and E be the edge set of graph G, with order $|V| = p$, size $|E| = q$. The number of edges incident with a vertex ω is known as the degree of ω, denoted by $\zeta(\omega)$. The reverse vertex degree ($\Re(\omega)$) was introduced by Kulli [10] defined as: $\Re(\omega) = 1 - \zeta(\omega) + \Delta$, where Δ denoted the maximum degree of the given graph. Let $\mathbf{E}_{\Re(\omega),\Re(\mu)}$ represents the edge partition of the given graph based on reverse degree of end vertices of an edge $\omega\mu \in E$ and $|\mathbf{E}_{\Re(\omega),\Re(\mu)}|$ represents its cardinality. There are detailed variations of topological indices mainly distance-based and degree-based indices, see [11–16]. Milan Randic [30] was the first who defined the degree-based indices and its reverse Randic index is defined as:

$$\Re\mathbb{R}_{\alpha}(G) = \sum_{\omega\mu\in E(G)} \Big(\Re(\omega) \times \Re(\mu)\Big)^{\alpha}, \quad \alpha = \frac{1}{2}, -\frac{1}{2}, 1, -1. \tag{1.1}$$

Estrada *et al.* presented the atom bond connectivity ($\mathbb{ABC}$) index in [18] and the reverse atom bond connectivity ($\Re\mathbb{ABC}$) is defined as:

$$\Re\mathbb{ABC}(G) = \sum_{\omega\mu\in E(G)} \sqrt{\frac{\Re(\omega) + \Re(\mu) - 2}{\Re(\omega) \times \Re(\mu)}} \tag{1.2}$$

Vukicevic and Furtula defined the geometric arithmetic ($\mathbb{GA}$) index in [19] and the reverse geometric arithmetic ($\Re\mathbb{GA}$) is presented as:

$$\Re\mathbb{GA}(G) = \sum_{\omega\mu\in E(G)} \frac{2\sqrt{\Re(\omega) \times \Re(\mu)}}{\Re(\omega) + \Re(\mu)} \tag{1.3}$$

Gutman *et al.* [20, 21] defined the first and second Zagreb and its reverse indices as:

$$\Re\mathbb{M}_1(G) = \sum_{\omega\mu\in E(G)} \Big(\Re(\omega) + \Re(\mu)\Big) \tag{1.4}$$

$$\Re\mathbb{M}_2(G) = \sum_{\omega\mu\in E(G)} \Big(\Re(\omega) \times \Re(\mu)\Big) \tag{1.5}$$

Shirdel *et al.* [22] introduced hyper Zagreb index. We defined the reverse hyper Zagreb index as:

$$\Re\mathbb{HM}(G) = \sum_{\omega\mu\in E(G)} \Big(\Re(\omega) + \Re(\mu)\Big)^2 \tag{1.6}$$

Furtula and Gutman [23] accomplished the forgotten index and its reverse forgotten index as:

$$\Re\mathbb{F}(G) = \sum_{\omega\mu\in E(G)} \Big((\Re(\omega))^2 + (\Re(\mu))^2 \Big) \tag{1.7}$$

Augmented Zagreb index was introduced by Furtula *et al.* [24] and the reverse augmented Zagreb index as:

$$\Re\mathbb{AZI}(G) = \sum_{\omega\mu\in E(G)} \left(\frac{\Re(\omega) \times \Re(\mu)}{\Re(\omega) + \Re(\mu) - 2} \right)^3 \tag{1.8}$$

Ranjini *et al.* [25] introduced the first redefined, second redefined and third redefined Zagreb indices. The reverse first redefined, second redefined and third redefined Zagreb indices are defined as:

$$\Re\mathbb{RZ}_1(G) = \sum_{\omega\mu\in E(G)} \frac{\Re(\omega) + \Re(\mu)}{\Re(\omega) \times \Re(\mu)} \tag{1.9}$$

$$\Re\mathbb{RZ}_2(G) = \sum_{\omega\mu\in E(G)} \frac{\Re(\omega) \times \Re(\mu)}{\Re(\omega) + \Re(\mu)} \tag{1.10}$$

$$\Re\mathbb{RZ}_3(G) = \sum_{\omega\mu\in E(G)} \Big(\Re(\omega) + \Re(\mu)\Big)\Big(\Re(\omega) \times \Re(\mu)\Big) \tag{1.11}$$

For latest results on topological indices see [26–35]. In this paper, we compute the exact results for all the above reverse indices.

2. Structure of third type hex-derived networks

With the help of complete graphs of order 3 (K_3), Chen *et al.* [36] assembled a hexagonal mesh. In terms of chemistry, these K_3 graphs are also called oxide graphs. The Figure 1 is obtained by joining these K_3 graphs. Two dimensional mesh graph HX(2) (see Figure 1 (a)), is obtained by joining six K_3 graphs and three dimensional mesh graph HX(3) (see Figure 1 (b)) is obtained by putting K_3 graphs around all side of HX(2). Furthermore, repeating the same process by putting the t K_3 graph around each hexagon, we obtained the t^{th} hexagonal mesh. To be noted that the one dimensional hexagonal mesh graph does not exist.

Simonraj *et al.* [37] created the new network which is named as third type of hex-derived networks. The graphically construction algorithm for third type of hexagonal hex-derived network $HHDN_3(t)$ (see Figure 2), triangular hex-derived network $THDN_3(t)$ (see Figure 3) and rectangular hex-derived network $RHDN_3(t)$ (see Figure 4) are defined in [38,39] and they determined some topological indices of these new derived networks. Some networks such as hexagonal, honeycomb, and grid networks, for instance, endure closeness to atomic or molecular lattice configurations. Related research that applies this theory and which could get additional advantages from the visions of the new research is found in [40–46].

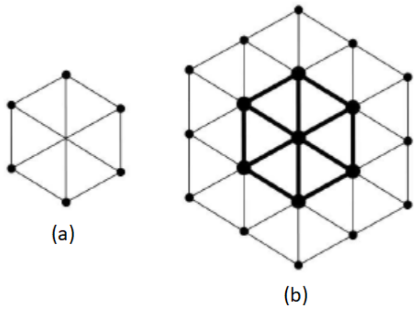

Figure 1. Hexagonal meshes: (a) $HX(2)$ and (b) $HX(3)$.

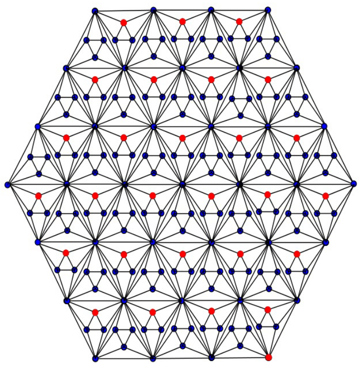

Figure 2. Third type of hexagonal hex-derived network $HHDN_3(t)$ for $t = 4$.

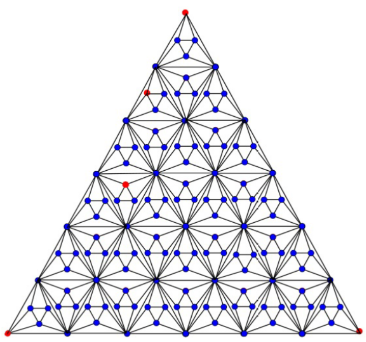

Figure 3. Third type of triangular hex-derived network $THDN_3(t)$ for $t = 7$.

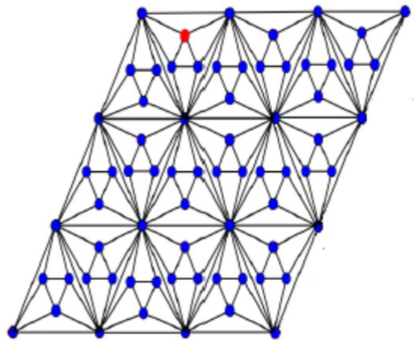

Figure 4. Third type of rectangular hex-derived network $RHDN_3(t)$ for $t = 4$.

3. Hexagonal hex-derived network $HHDN_3(t)$

Let $\Gamma_1 = HHDN_3(t)$ be the third type of hexagonal hex-derived network which is shown in Figure 2, where $t \geq 4$. The graph Γ_1 has $21t^2 - 39t + 19$ vertices from which $18t^2 - 36t + 18$ vertices of reverse degree 15, 4 vertices of reverse degree 12, $6t - 12$ vertices of reverse degree 9 and $3t^2 - 9t + 9$ vertices of reverse degree 1. There are $63t^2 - 123t + 60$ number of edges of Γ_1 is partitioned into nine classes based on their reverse degrees which are given in Eq (3.1).

$$|\mathbf{E}_{\Re(\omega),\Re(\mu)}(\Gamma_1)| = \begin{cases} 9t^2 - 33t + 30, & \text{for; } \Re(\omega) = 1, \Re(\mu) = 1 \\ 12t - 24, & \text{for; } \Re(\omega) = 9, \Re(\mu) = 1 \\ 6t - 18, & \text{for; } \Re(\omega) = 9, \Re(\mu) = 9 \\ 6, & \text{for; } \Re(\omega) = 12, \Re(\mu) = 1 \\ 12, & \text{for; } \Re(\omega) = 12, \Re(\mu) = 9 \\ 36t^2 - 108t + 84, & \text{for; } \Re(\omega) = 15, \Re(\mu) = 1 \\ 36t - 72, & \text{for; } \Re(\omega) = 15, \Re(\mu) = 9 \\ 24, & \text{for; } \Re(\omega) = 15, \Re(\mu) = 12 \\ 18t^2 - 36t + 18, & \text{for; } \Re(\omega) = 15, \Re(\mu) = 15 \end{cases} \tag{3.1}$$

In the next theorem, we will calculate the reverse Randic index, reverse Zagreb indices, reverse forgotten index for Γ_1 graph.

Theorem 3.1. *Let Γ_1 be the third type of hexagonal hex-derived network, then*

- $\Re\mathbb{R}_\alpha(\Gamma_1) = [9+36(15)^\alpha+18(225)^\alpha]t^2+[-33+12(9)^\alpha+6(81)^\alpha-108(15)^\alpha+36(135)^\alpha-36(225)^\alpha]t+30 - 24(9)^\alpha - 18(81)^\alpha + 6(12)^\alpha + 12(108)^\alpha + 84(15)^\alpha - 72(135)^\alpha + 24(180)^\alpha + 18(225)^\alpha$
- $\Re\mathbb{M}_1(\Gamma_1) = 1134\,t^2 - 1782\,t + 630$
- $\Re\mathbb{M}_2(\Gamma_1) = 4599\,t^2 - 4299\,t - 366$
- $\Re\mathbb{HM}(\Gamma_1) = 25452\,t^2 - 36300\,t + 11922$
- $\Re\mathbb{F}(\Gamma_1) = 16254\,t^2 - 27702\,t + 12654$

Proof. Let Γ_1 be the third type of hexagonal hex-derived network which is shown in Figure 2. The order of hexagonal hex derived network Γ_1 is $p = |\Gamma_1| = 21t^2 - 39t + 19$ and size is $q = 63t^2 - 123t + 60$. The edge partitioned of Γ_1 based on their reverse degrees are shown in Eq (3.1). Reverse Randic index

can be calculated by using Eq (3.1). Thus, from Eq (1.1), it follows,
$\Re\mathbb{R}_\alpha(\Gamma_1) = (1)^\alpha|\mathbf{E}_{1,1}(\Gamma_1)| + (9)^\alpha|\mathbf{E}_{9,1}(\Gamma_1)| + (81)^\alpha|\mathbf{E}_{9,9}(\Gamma_1)| + (12)^\alpha|\mathbf{E}_{12,1}(\Gamma_1)|$
$+ (108)^\alpha|\mathbf{E}_{12,9}(\Gamma_1)| + (15)^\alpha|\mathbf{E}_{15,1}(\Gamma_1)| + (135)^\alpha|\mathbf{E}_{15,9}(\Gamma_1)| + (180)^\alpha|\mathbf{E}_{15,12}(\Gamma_1)| + (225)^\alpha|\mathbf{E}_{15,15}(\Gamma_1)|$.
$= (9t^2 - 33t + 30) + (9)^\alpha(12t - 24) + (81)^\alpha(6t - 18) + (12)^\alpha(6) + (108)^\alpha(12) + (15)^\alpha(36t^2 - 108t + 84) + (135)^\alpha(36t - 72) + (180)^\alpha(24) + (225)^\alpha(18t^2 - 36t + 18)$.
$= [9 + 36(15)^\alpha + 18(225)^\alpha]t^2 + [-33 + 12(9)^\alpha + 6(81)^\alpha - 108(15)^\alpha + 36(135)^\alpha - 36(225)^\alpha]t + 30 - 24(9)^\alpha - 18(81)^\alpha + 6(12)^\alpha + 12(108)^\alpha + 84(15)^\alpha - 72(135)^\alpha + 24(180)^\alpha + 18(225)^\alpha$. Put $\alpha = 1$ and after some calculation, we get reverse second Zagreb index $\Re\mathbb{M}_2(\Gamma_1)$ as:

$$\Re\mathbb{M}_2(\Gamma_1) = 4599\,t^2 - 4299\,t - 366. \tag{3.2}$$

Using the Eq (1.4), we can determine the reverse first Zagreb index $\Re\mathbb{M}_1(\Gamma_1)$ as:
$\Re\mathbb{M}_1(\Gamma_1) = 2 \times |\mathbf{E}_{1,1}(\Gamma_1)| + 10 \times |\mathbf{E}_{9,1}(\Gamma_1)| + 18 \times |\mathbf{E}_{9,9}(\Gamma_1)| + 13 \times |\mathbf{E}_{12,1}(\Gamma_1)| + 21 \times |\mathbf{E}_{12,9}(\Gamma_1)| + 16 \times |\mathbf{E}_{15,1}(\Gamma_1)| + 24 \times |\mathbf{E}_{15,9}(\Gamma_1)| + 27 \times |\mathbf{E}_{15,12}(\Gamma_1)| + 30 \times |\mathbf{E}_{15,15}(\Gamma_1)|$.
By putting the values of from equation (3.1) and after simplification, we obtain:

$$\Re\mathbb{M}_1(\Gamma_1) = 1134\,t^2 - 1782\,t + 630. \tag{3.3}$$

Using the Eq (1.6), we can determine the reverse hyper Zagreb index $\Re\mathbb{HM}(\Gamma_1)$ as:
$\Re\mathbb{HM}(\Gamma_1) = 4 \times |\mathbf{E}_{1,1}(\Gamma_1)| + 100 \times |\mathbf{E}_{9,1}(\Gamma_1)| + 324 \times |\mathbf{E}_{9,9}(\Gamma_1)| + 169 \times |\mathbf{E}_{12,1}(\Gamma_1)| + 441 \times |\mathbf{E}_{12,9}(\Gamma_1)| + 256 \times |\mathbf{E}_{15,1}(\Gamma_1)| + 576 \times |\mathbf{E}_{15,9}(\Gamma_1)| + 729 \times |\mathbf{E}_{15,12}(\Gamma_1)| + 900 \times |\mathbf{E}_{15,15}(\Gamma_1)|$.
After simplification, we get

$$\Re\mathbb{HM}(\Gamma_1) = 25452\,t^2 - 36300\,t + 11922$$

Using the Eq (1.7), we can determine the reverse forgotten index $\Re\mathbb{F}(\Gamma_1))$ as:
$\Re\mathbb{F}(\Gamma_1)) = 2 \times |\mathbf{E}_{1,1}(\Gamma_1)| + 82 \times |\mathbf{E}_{9,1}(\Gamma_1)| + 162 \times |\mathbf{E}_{9,9}(\Gamma_1)| + 145 \times |\mathbf{E}_{12,1}(\Gamma_1)| + 225 \times |\mathbf{E}_{12,9}(\Gamma_1)| + 226 \times |\mathbf{E}_{15,1}(\Gamma_1)| + 306 \times |\mathbf{E}_{15,9}(\Gamma_1)| + 369 \times |\mathbf{E}_{15,12}(\Gamma_1)| + 450 \times |\mathbf{E}_{15,15}(\Gamma_1)|$.
After simplification, we get

$$\Re\mathbb{F}(\Gamma_1) = 16254\,t^2 - 27702\,t + 12654$$

□

In the next theorem, we will calculate the reverse atom bond connectivity index, reverse geometric arithmetic index for Γ_1 graph.

Theorem 3.2. *Let Γ_1 be the third type of hexagonal hex-derived network, then*

- $\Re\mathbb{ABC}(\Gamma_1) = \left(\frac{12\sqrt{210}}{5} + \frac{12\sqrt{7}}{5}\right)t^2 + \left(8\sqrt{2} + \frac{8}{3} - \frac{36\sqrt{210}}{5} + \frac{4\sqrt{330}}{5} - \frac{24\sqrt{7}}{5}\right)t - 8 - 16\sqrt{2} + \sqrt{33} + \frac{2\sqrt{57}}{3} + \frac{28\sqrt{210}}{5} - \frac{8\sqrt{330}}{5} + 4\sqrt{5} + \frac{12\sqrt{7}}{5}$
- $\Re\mathbb{GA}(\Gamma_1) = \left(27 + \frac{9\sqrt{15}}{2}\right)t^2 + \left(-\frac{279}{5} - \frac{9\sqrt{15}}{2}\right)t + \frac{78}{5} + \frac{792\sqrt{3}}{91} - \frac{15\sqrt{15}}{2} + \frac{32\sqrt{5}}{3}$.

Proof. The reverse atom bond connectivity ($\Re\mathbb{ABC}(\Gamma_1)$), can be determined by using Eq (1.2) and Eq (3.1), as follows:
$\Re\mathbb{ABC}(\Gamma_1) = 0 \times |\mathbf{E}_{1,1}(\Gamma_1)| + \sqrt{\frac{8}{9}} \times |\mathbf{E}_{9,1}(\Gamma_1)| + \sqrt{\frac{16}{81}} \times |\mathbf{E}_{9,9}(\Gamma_1)| + \sqrt{\frac{11}{12}} \times |\mathbf{E}_{12,1}(\Gamma_1)| + \sqrt{\frac{19}{108}} \times |\mathbf{E}_{12,9}(\Gamma_1)| +$

$\sqrt{\frac{14}{15}} \times |\mathbf{E}_{15,1}(\Gamma_1)| + \sqrt{\frac{22}{135}} \times |\mathbf{E}_{15,9}(\Gamma_1)| + \sqrt{\frac{25}{180}} \times |\mathbf{E}_{15,12}(\Gamma_1)| + \sqrt{\frac{28}{225}} \times |\mathbf{E}_{15,15}(\Gamma_1)|$.
After some simplification, we get
$\Re\mathbb{ABC}(\Gamma_1) = \left(\frac{12\sqrt{210}}{5} + \frac{12\sqrt{7}}{5}\right)t^2 + \left(8\sqrt{2} + \frac{8}{3} - \frac{36\sqrt{210}}{5} + \frac{4\sqrt{330}}{5} - \frac{24\sqrt{7}}{5}\right)t - 8 - 16\sqrt{2} + \sqrt{33} + \frac{2\sqrt{57}}{3} + \frac{28\sqrt{210}}{5} - \frac{8\sqrt{330}}{5} + 4\sqrt{5} + \frac{12\sqrt{7}}{5}$.
The reverse geometric arithmetic ($\Re\mathbb{GA}(\Gamma_1)$), can be determined by using Eq (1.3) and Eq (3.1), as follows:
$\Re\mathbb{GA}(\Gamma_1) = |\mathbf{E}_{1,1}(\Gamma_1)| + \frac{2\sqrt{9}}{10} \times |\mathbf{E}_{9,1}(\Gamma_1)| + \frac{2\sqrt{81}}{18} \times |\mathbf{E}_{9,9}(\Gamma_1)| + \frac{2\sqrt{12}}{13} \times |\mathbf{E}_{12,1}(\Gamma_1)| + \frac{2\sqrt{108}}{21} \times |\mathbf{E}_{12,9}(\Gamma_1)| + \frac{2\sqrt{15}}{16} \times |\mathbf{E}_{15,1}(\Gamma_1)| + \frac{2\sqrt{135}}{24} \times |\mathbf{E}_{15,9}(\Gamma_1)| + \frac{2\sqrt{180}}{27} \times |\mathbf{E}_{15,12}(\Gamma_1)| + \frac{2\sqrt{225}}{30} \times |\mathbf{E}_{15,15}(\Gamma_1)|$.
After some simplification, we get
$\Re\mathbb{GA}(\Gamma_1) = \left(27 + \frac{9\sqrt{15}}{2}\right)t^2 + \left(-\frac{279}{5} - \frac{9\sqrt{15}}{2}\right)t + \frac{78}{5} + \frac{792\sqrt{3}}{91} - \frac{15\sqrt{15}}{2} + \frac{32\sqrt{5}}{3}$. □

In the next theorems, we will calculate the reverse redefined Zagreb indices for Γ_1 graph.

Theorem 3.3. *Let Γ_1 be the third type of hexagonal hex-derived network, then*

- $\Re\mathbb{RZ}_1(\Gamma_1) = \frac{294\,t^2}{5} - \frac{2474\,t}{15} + \frac{3629}{30}$
- $\Re\mathbb{RZ}_2(\Gamma_1) = \frac{693\,t^2}{4} - \frac{2949\,t}{20} - \frac{93907}{1820}$
- $\Re\mathbb{RZ}_3(\Gamma_1) = 130158\,t^2 - 142518\,t + 24828$

Proof. Reverse redefined Zagreb indices can be calculated by using Eq (3.1), the $\Re\mathbb{RZ}_1(\Gamma_1)$ by using Eq (1.9) as follows:
$\Re\mathbb{RZ}_1(\Gamma_1) = 2 \times |\mathbf{E}_{1,1}(\Gamma_1)| + \frac{10}{9} \times |\mathbf{E}_{9,1}(\Gamma_1)| + \frac{18}{81} \times |\mathbf{E}_{9,9}(\Gamma_1)| + \frac{13}{12} \times |\mathbf{E}_{12,1}(\Gamma_1)| + \frac{21}{108} \times |\mathbf{E}_{12,9}(\Gamma_1)| + \frac{16}{15} \times |\mathbf{E}_{15,1}(\Gamma_1)| + \frac{24}{135} \times |\mathbf{E}_{15,9}(\Gamma_1)| + \frac{27}{108} \times |\mathbf{E}_{15,12}(\Gamma_1)| + \frac{30}{225} \times |\mathbf{E}_{15,15}(\Gamma_1)|$.
After some simplification, we get

$$\Re\mathbb{RZ}_1(\Gamma_1) = \frac{294\,t^2}{5} - \frac{2474\,t}{15} + \frac{3629}{30}.$$

The $\Re\mathbb{RZ}_2(\Gamma_1)$ can be determined by using Eq (1.10) as follows:
$\Re\mathbb{RZ}_2(\Gamma_1) = \frac{1}{2} \times |\mathbf{E}_{1,1}(\Gamma_1)| + \frac{9}{10} \times |\mathbf{E}_{9,1}(\Gamma_1)| + \frac{81}{18} \times |\mathbf{E}_{9,9}(\Gamma_1)| + \frac{12}{13} \times |\mathbf{E}_{12,1}(\Gamma_1)| + \frac{108}{21} \times |\mathbf{E}_{12,9}(\Gamma_1)| + \frac{15}{16} \times |\mathbf{E}_{15,1}(\Gamma_1)| + \frac{135}{24} \times |\mathbf{E}_{15,9}(\Gamma_1)| + \frac{108}{27} \times |\mathbf{E}_{15,12}(\Gamma_1)| + \frac{225}{30} \times |\mathbf{E}_{15,15}(\Gamma_1)|$.
After some simplification, we get

$$\Re\mathbb{RZ}_2(\Gamma_1) = \frac{693\,t^2}{4} - \frac{2949\,t}{20} - \frac{93907}{1820}.$$

The $\Re\mathbb{RZ}_3(\Gamma_1)$ can be calculated by using Eq (1.11) as follows:
$\Re\mathbb{RZ}_3(\Gamma_1) = 2 \times |\mathbf{E}_{1,1}(\Gamma_1)| + 90 \times |\mathbf{E}_{9,1}(\Gamma_1)| + 1458 \times |\mathbf{E}_{9,9}(\Gamma_1)| + 156 \times |\mathbf{E}_{12,1}(\Gamma_1)| + 2268 \times |\mathbf{E}_{12,9}(\Gamma_1)| + 240 \times |\mathbf{E}_{15,1}(\Gamma_1)| + 3240 \times |\mathbf{E}_{15,9}(\Gamma_1)| + 2916 \times |\mathbf{E}_{15,12}(\Gamma_1)| + 6750 \times |\mathbf{E}_{15,15}(\Gamma_1)|$.
After some simplification, we get

$$\Re\mathbb{RZ}_3(\Gamma_1) = 130158\,t^2 - 142518\,t + 24828.$$

□

4. Triangular hex-derived network $THDN3(p)$

Let $\Gamma_2 = THDN_3(t)$ be the third type of triangular hex-derived network which is shown in Figure 3, where $t \geq 4$. The graph Γ_2 has $\frac{7t^2-11t+6}{2}$ vertices. There are $\frac{21t^2-39t+18}{2}$ number of edges of Γ_2 is partitioned into six classes based on their reverse degrees which are given in Eq (4.1). Now we calculated reverse degree based indices such as: reverse Randic index $\Re\mathbb{R}_\alpha$, reverse atom bond connectivity index $\Re\mathbb{ABC}$, reverse geometric arithmetic index $\Re\mathbb{GA}$, first reverse Zagreb index $\Re\mathbb{M}_1$, second reverse Zagreb index $\Re\mathbb{M}_2$, reverse forgotten index $\Re\mathbb{F}$, reverse hyper Zagreb index $\Re\mathbb{HM}$ and reverse redefined Zagreb indices for Γ_2 graph.

$$|\mathbf{E}_{\Re(\omega),\Re(\mu)}(\Gamma_2)| = \begin{cases} \frac{3t^2}{2} - \frac{21t}{2} + 18, & \text{for; } \Re(\omega) = 1, \Re(\mu) = 1 \\ 6t - 18, & \text{for; } \Re(\omega) = 9, \Re(\mu) = 1 \\ 3t - 6, & \text{for; } \Re(\omega) = 9, \Re(\mu) = 9 \\ 6t^2 - 30t + 36, & \text{for; } \Re(\omega) = 15, \Re(\mu) = 1 \\ 18t - 30, & \text{for; } \Re(\omega) = 15, \Re(\mu) = 9 \\ 3t^2 - 6t + 9, & \text{for; } \Re(\omega) = 15, \Re(\mu) = 15 \end{cases} \tag{4.1}$$

In the next theorem, we will calculate the reverse Randic index, reverse Zagreb indices, reverse forgotten index for Γ_2 graph.

Theorem 4.1. *Let Γ_2 be the third type of triangular hex-derived network, then*

- $\Re\mathbb{R}_\alpha(\Gamma_2) = \left(\frac{3}{2} + 6\,(15)^\alpha + 3\,(225)^\alpha\right) t^2 + \left(\frac{-21}{2} + 6\,(9)^\alpha + 3\,(81)^\alpha - 30\,(15)^\alpha + 18\,(135)^\alpha - 6\,(225)^\alpha\right) t + 18 - 18\,(9)^\alpha - 6\,(81)^\alpha + 36\,(15)^\alpha - 30\,(135)^\alpha + 9\,(225)^\alpha.$
- $\Re\mathbb{M}_1(\Gamma_2) = 189\,t^2 - 135\,t - 126$
- $\Re\mathbb{M}_2(\Gamma_2) = \frac{1533\,t^2}{2} + \frac{1833\,t}{2} - 2115$
- $\Re\mathbb{HM}(\Gamma_2) = 4242\,t^2 - 1182\,t - 3636$
- $\Re\mathbb{F}(\Gamma_2) = 2709\,t^2 - 3015\,t + 594$

Proof. Let Γ_2 be the third type of triangular hex-derived network which is shown in Figure 3. The order of triangular hex derived network Γ_2 is $p = |\Gamma_2| = \frac{7t^2-11t+6}{2}$ and size is $q = \frac{21t^2-39t+18}{2}$. The edge partitioned of Γ_2 based on their reverse degrees are shown in Eq (4.1). Reverse Randic index can be calculated by using Eq (4.1). Thus, from Eq (1.1), it follows,
$\Re\mathbb{R}_\alpha(\Gamma_2) = (1)^\alpha|\mathbf{E}_{1,1}(\Gamma_2)| + (9)^\alpha|\mathbf{E}_{9,1}(\Gamma_2)| + (81)^\alpha|\mathbf{E}_{9,9}(\Gamma_2)| + (15)^\alpha|\mathbf{E}_{15,1}(\Gamma_2)| + (135)^\alpha|\mathbf{E}_{15,9}(\Gamma_2)| + (225)^\alpha|\mathbf{E}_{15,15}(\Gamma_2)|.$
After simplification, we get
$\Re\mathbb{R}_\alpha(\Gamma_2) = \left(\frac{3}{2} + 6\,(15)^\alpha + 3\,(225)^\alpha\right) t^2 + \left(\frac{-21}{2} + 6\,(9)^\alpha + 3\,(81)^\alpha - 30\,(15)^\alpha + 18\,(135)^\alpha - 6\,(225)^\alpha\right) t + 18 - 18\,(9)^\alpha - 6\,(81)^\alpha + 36\,(15)^\alpha - 30\,(135)^\alpha + 9\,(225)^\alpha.$
Put $\alpha = 1$ and after some calculation, we get reverse second Zagreb index $\Re\mathbb{M}_2(\Gamma_2)$ as:

$$\Re\mathbb{M}_2(\Gamma_2) = \frac{1533\,t^2}{2} + \frac{1833\,t}{2} - 2115 \tag{4.2}$$

Using the Eq (1.4), we can determine the reverse first Zagreb index $\Re\mathbb{M}_1(\Gamma_2)$ as:
$\Re\mathbb{M}_1(\Gamma_2) = 2\times|\mathbf{E}_{1,1}(\Gamma_1)| + 10\times|\mathbf{E}_{9,1}(\Gamma_1)| + 18\times|\mathbf{E}_{9,9}(\Gamma_1)| + 16\times|\mathbf{E}_{15,1}(\Gamma_1)| + 24\times|\mathbf{E}_{15,9}(\Gamma_1)| + 30\times|\mathbf{E}_{15,15}(\Gamma_1)|.$
By putting the values of from Eq (4.1) and after simplification, we obtain:

$$\Re\mathbb{M}_1(\Gamma_2) = 189\,t^2 - 135\,t - 126 \tag{4.3}$$

Using the Eq (1.6), we can determine the reverse hyper Zagreb index $\mathfrak{R}\mathbb{HM}(\Gamma_2)$ as:
$\mathfrak{R}\mathbb{HM}(\Gamma_2) = 4 \times |\mathbf{E}_{1,1}(\Gamma_2)| + 100 \times |\mathbf{E}_{9,1}(\Gamma_2)| + 324 \times |\mathbf{E}_{9,9}(\Gamma_2)| + 256 \times |\mathbf{E}_{15,1}(\Gamma_2)| + 576 \times |\mathbf{E}_{15,9}(\Gamma_2)| + 900 \times |\mathbf{E}_{15,15}(\Gamma_2)|$.
After simplification, we get

$$\mathfrak{R}\mathbb{HM}(\Gamma_2) = 4242\,t^2 - 1182\,t - 3636.$$

Using the Eq (1.7), we can determine the reverse forgotten index $\mathfrak{R}\mathbb{F}(\Gamma_2))$ as:
$\mathfrak{R}\mathbb{F}(\Gamma_2)) = 2 \times |\mathbf{E}_{1,1}(\Gamma_2)| + 82 \times |\mathbf{E}_{9,1}(\Gamma_2)| + 162 \times |\mathbf{E}_{9,9}(\Gamma_2)| + 226 \times |\mathbf{E}_{15,1}(\Gamma_2)| + 306 \times |\mathbf{E}_{15,9}(\Gamma_2)| + 450 \times |\mathbf{E}_{15,15}(\Gamma_2)|$.
After simplification, we get

$$\mathfrak{R}\mathbb{F}(\Gamma_2) = 2709\,t^2 - 3015\,t + 594.$$

□

In the next theorem, we will calculate the reverse atom bond connectivity index, reverse geometric arithmetic index for Γ_2 graph.

Theorem 4.2. *Let Γ_2 be the third type of triangular hex-derived network, then*

- $\mathfrak{R}\mathbb{ABC}(\Gamma_2) = \left(\frac{2\sqrt{210}}{5} + \frac{2\sqrt{7}}{5}\right)t^2 + \left(4\sqrt{2} + \frac{4}{3} - 2\sqrt{210} + \frac{2\sqrt{330}}{5} - \frac{4\sqrt{7}}{5}\right)t - \frac{8}{3} - 12\sqrt{2} + \frac{12\sqrt{210}}{5} - \frac{2\sqrt{330}}{3} + \frac{6\sqrt{7}}{5}$
- $\mathfrak{R}\mathbb{GA}(\Gamma_2) = \left(\frac{9}{2} + \frac{3\sqrt{15}}{4}\right)t^2 + \left(-\frac{99}{10} + \frac{3\sqrt{15}}{4}\right)t + \frac{51}{5} - 3\sqrt{15}$.

Proof. The reverse atom bond connectivity ($\mathfrak{R}\mathbb{ABC}(\Gamma_2)$), can be determined by using Eq (1.2) and Eq (4.1), as follows:
$\mathfrak{R}\mathbb{ABC}(\Gamma_2) = 0 \times |\mathbf{E}_{1,1}(\Gamma_2)| + \sqrt{\frac{8}{9}} \times |\mathbf{E}_{9,1}(\Gamma_2)| + \sqrt{\frac{16}{81}} \times |\mathbf{E}_{9,9}(\Gamma_2)| + \sqrt{\frac{14}{15}} \times |\mathbf{E}_{15,1}(\Gamma_2)| + \sqrt{\frac{22}{135}} \times |\mathbf{E}_{15,9}(\Gamma_2)| + \sqrt{\frac{28}{225}} \times |\mathbf{E}_{15,15}(\Gamma_2)|$.
After some simplification, we get
$\mathfrak{R}\mathbb{ABC}(\Gamma_2) = \left(\frac{2\sqrt{210}}{5} + \frac{2\sqrt{7}}{5}\right)t^2 + \left(4\sqrt{2} + \frac{4}{3} - 2\sqrt{210} + \frac{2\sqrt{330}}{5} - \frac{4\sqrt{7}}{5}\right)t - \frac{8}{3} - 12\sqrt{2} + \frac{12\sqrt{210}}{5} - \frac{2\sqrt{330}}{3} + \frac{6\sqrt{7}}{5}$.
The reverse geometric arithmetic ($\mathfrak{R}\mathbb{GA}(\Gamma_2)$), can be determined by using Eq (1.3) and Eq (4.1), as follows:
$\mathfrak{R}\mathbb{GA}(\Gamma_2) = |\mathbf{E}_{1,1}(\Gamma_2)| + \frac{2\sqrt{9}}{10} \times |\mathbf{E}_{9,1}(\Gamma_2)| + \frac{2\sqrt{81}}{18} \times |\mathbf{E}_{9,9}(\Gamma_2)| + \frac{2\sqrt{15}}{16} \times |\mathbf{E}_{15,1}(\Gamma_2)| + \frac{2\sqrt{135}}{24} \times |\mathbf{E}_{15,9}(\Gamma_2)| + \frac{2\sqrt{225}}{30} \times |\mathbf{E}_{15,15}(\Gamma_2)|$.
After some simplification, we get

$$\mathfrak{R}\mathbb{GA}(\Gamma_2) = \left(\frac{9}{2} + \frac{3\sqrt{15}}{4}\right)t^2 + \left(-\frac{99}{10} + \frac{3\sqrt{15}}{4}\right)t + \frac{51}{5} - 3\sqrt{15}.$$

□

In the next theorems, we will calculate the reverse redefined Zagreb indices for Γ_2 graph.

Theorem 4.3. *Let Γ_2 be the third type of triangular hex-derived network, then*

- $\mathfrak{R}\mathbb{RZ}_1(\Gamma_2) = \frac{49\,t^2}{5} - \frac{649\,t}{15} + \frac{734}{15}$
- $\mathfrak{R}\mathbb{RZ}_2(\Gamma_2) = \frac{231\,t^2}{8} + \frac{1671\,t}{40} - \frac{1017}{10}$

- $\Re\mathbb{RZ}_3(\Gamma_2) = 21693\,t^2 + 15513\,t - 38142.$

Proof. Reverse redefined Zagreb indices can be calculated by using Eq (4.1), the $\Re\mathbb{RZ}_1(\Gamma_2)$ by using Eq (1.9) as follows:
$\Re\mathbb{RZ}_1(\Gamma_2) = 2\times|\mathbf{E}_{1,1}(\Gamma_2)|+\frac{10}{9}\times|\mathbf{E}_{9,1}(\Gamma_2)|+\frac{18}{81}\times|\mathbf{E}_{9,9}(\Gamma_2)|+\frac{16}{15}\times|\mathbf{E}_{15,1}(\Gamma_2)|+\frac{24}{135}\times|\mathbf{E}_{15,9}(\Gamma_2)|+\frac{30}{225}\times|\mathbf{E}_{15,15}(\Gamma_2)|$.
After some simplification, we get

$$\Re\mathbb{RZ}_1(\Gamma_2) = \frac{49\,t^2}{5} - \frac{649\,t}{15} + \frac{734}{15}.$$

The $\Re\mathbb{RZ}_2(\Gamma_2)$ can be determined by using Eq (1.10) as follows:
$\Re\mathbb{RZ}_2(\Gamma_2) = \frac{1}{2}\times|\mathbf{E}_{1,1}(\Gamma_2)|+\frac{9}{10}\times|\mathbf{E}_{9,1}(\Gamma_2)|+\frac{81}{18}\times|\mathbf{E}_{9,9}(\Gamma_2)|+\frac{15}{16}\times|\mathbf{E}_{15,1}(\Gamma_2)|+\frac{135}{24}\times|\mathbf{E}_{15,9}(\Gamma_2)|+\frac{225}{30}\times|\mathbf{E}_{15,15}(\Gamma_2)|$.
After some simplification, we get

$$\Re\mathbb{RZ}_2(\Gamma_2) = \frac{231\,t^2}{8} + \frac{1671\,t}{40} - \frac{1017}{10}.$$

The $\Re\mathbb{RZ}_3(\Gamma_2)$ can be calculated by using Eq (1.11) as follows:
$\Re\mathbb{RZ}_3(\Gamma_2) = 2\times|\mathbf{E}_{1,1}(\Gamma_2)| + 90\times|\mathbf{E}_{9,1}(\Gamma_2)| + 1458\times|\mathbf{E}_{9,9}(\Gamma_2)| + 240\times|\mathbf{E}_{15,1}(\Gamma_2)| + 3240\times|\mathbf{E}_{15,9}(\Gamma_2)| + 6750\times|\mathbf{E}_{15,15}(\Gamma_2)|$.
After some simplification, we get

$$\Re\mathbb{RZ}_3(\Gamma_2) = 21693\,t^2 + 15513\,t - 38142.$$

□

5. Rectangular hex-derived network $RHDN3(p)$

In this section, we calculate certain reverse degree based topological indices of the third type of rectangular hex-derived network, $RHDN_3(t,w)$ of dimension $t = w$. Now we calculated reverse degree based indices such as: reverse Randic index $\Re\mathbb{R}_\alpha$, reverse atom bond connectivity index $\Re\mathbb{ABC}$, reverse geometric arithmetic index $\Re\mathbb{GA}$, first reverse Zagreb index $\Re\mathbb{M}_1$, second reverse Zagreb index $\Re\mathbb{M}_2$, reverse forgotten index $\Re\mathbb{F}$, reverse augmented Zagreb index $\Re\mathbb{AZI}$, reverse hyper Zagreb index $\Re\mathbb{HM}$ and reverse redefined Zagreb indices for Γ_3 graph.

$$|\mathbf{E}_{\Re(\omega),\Re(\mu)}(\Gamma_3)| = \begin{cases} 3t^2-16t+21, & \text{for; } \Re(\omega)=1, \Re(\mu)=1 \\ 8t-20, & \text{for; } \Re(\omega)=9, \Re(\mu)=1 \\ 4t-10, & \text{for; } \Re(\omega)=9, \Re(\mu)=9 \\ 2, & \text{for; } \Re(\omega)=12, \Re(\mu)=1 \\ 4, & \text{for; } \Re(\omega)=12, \Re(\mu)=9 \\ 12t^2-48t+48, & \text{for; } \Re(\omega)=15, \Re(\mu)=1 \\ 24t-44, & \text{for; } \Re(\omega)=15, \Re(\mu)=9 \\ 8, & \text{for; } \Re(\omega)=15, \Re(\mu)=12 \\ 6t^2-12t+10, & \text{for; } \Re(\omega)=15, \Re(\mu)=15 \end{cases} \tag{5.1}$$

In the next theorem, we will calculate the reverse Randic index, reverse Zagreb indices, reverse forgotten index for Γ_3 graph.

Theorem 5.1. *Let Γ_3 be the third type of rectangular hex-derived network, then*

- $\Re\mathbb{R}_\alpha(\Gamma_3) = (3 + 12\,(15)^\alpha + 6\,(225)^\alpha)\,t^2 + (-16 + 8\,(9)^\alpha + 4\,(81)^\alpha - 48\,(15)^\alpha + 24\,(135)^\alpha - 12\,(225)^\alpha)\,t + 21 - 20\,(9)^\alpha - 10\,(81)^\alpha + 2\,(12)^\alpha + 4\,(108)^\alpha + 48\,(15)^\alpha - 44\,(135)^\alpha + 8\,(180)^\alpha + 10\,(225)^\alpha$
- $\Re\mathbb{M}_1(\Gamma_3) = 378\,t^2 - 432\,t$
- $\Re\mathbb{M}_2(\Gamma_3) = 1533\,t^2 + 200\,t - 2043$
- $\Re\mathbb{HM}(\Gamma_3) = 8484\,t^2 - 7232\,t - 1278$
- $\Re\mathbb{F}(\Gamma_3) = 5418\,t^2 - 7632\,t + 2808.$

Proof. Let Γ_3 be the third type of rectangular hex-derived network which is shown in Figure 4. The order of hexagonal hex derived network Γ_3 is $p = |\Gamma_1| = 7t^2 - 12t + 6$ and size is $q = 21t^2 - 40t + 19$. The edge partitioned of Γ_3 based on their reverse degrees are shown in Eq (5.1). Reverse Randic index can be calculated by using Eq (5.1). Thus, from Eq (1.1), it follows,
$\Re\mathbb{R}_\alpha(\Gamma_3) = (1)^\alpha|\mathbf{E}_{1,1}(\Gamma_3)| + (9)^\alpha|\mathbf{E}_{9,1}(\Gamma_3)| + (81)^\alpha|\mathbf{E}_{9,9}(\Gamma_3)| + (12)^\alpha|\mathbf{E}_{12,1}(\Gamma_3)|$
$+ (108)^\alpha|\mathbf{E}_{12,9}(\Gamma_3)| + (15)^\alpha|\mathbf{E}_{15,1}(\Gamma_3)| + (135)^\alpha|\mathbf{E}_{15,9}(\Gamma_3)| + (180)^\alpha|\mathbf{E}_{15,12}(\Gamma_3)| + (225)^\alpha|\mathbf{E}_{15,15}(\Gamma_3)|.$
After Simplification, we get
$\Re\mathbb{R}_\alpha(\Gamma_3) = (3 + 12\,(15)^\alpha + 6\,(225)^\alpha)\,t^2 + (-16 + 8\,(9)^\alpha + 4\,(81)^\alpha - 48\,(15)^\alpha + 24\,(135)^\alpha - 12\,(225)^\alpha)\,t +$
$21 - 20\,(9)^\alpha - 10\,(81)^\alpha + 2\,(12)^\alpha + 4\,(108)^\alpha + 48\,(15)^\alpha - 44\,(135)^\alpha + 8\,(180)^\alpha + 10\,(225)^\alpha.$
Put $\alpha = 1$ and after some calculation, we get reverse second Zagreb index $\Re\mathbb{M}_2(\Gamma_3)$ as:

$$\Re\mathbb{M}_2(\Gamma_3) = 1533\,t^2 + 200\,t - 2043. \tag{5.2}$$

Using the Eq (1.4), we can determine the reverse first Zagreb index $\Re\mathbb{M}_1(\Gamma_3)$ as:
$\Re\mathbb{M}_1(\Gamma_3) = 2 \times |\mathbf{E}_{1,1}(\Gamma_3)| + 10 \times |\mathbf{E}_{9,1}(\Gamma_3)| + 18 \times |\mathbf{E}_{9,9}(\Gamma_3)| + 13 \times |\mathbf{E}_{12,1}(\Gamma_3)| + 21 \times |\mathbf{E}_{12,9}(\Gamma_3)| + 16 \times |\mathbf{E}_{15,1}(\Gamma_3)| + 24 \times |\mathbf{E}_{15,9}(\Gamma_3)| + 27 \times |\mathbf{E}_{15,12}(\Gamma_3)| + 30 \times |\mathbf{E}_{15,15}(\Gamma_3)|.$
By putting the values of from Eq (5.1) and after simplification, we obtain:

$$\Re\mathbb{M}_1(\Gamma_3) = 378\,t^2 - 432\,t. \tag{5.3}$$

Using the Eq (1.6), we can determine the reverse hyper Zagreb index $\Re\mathbb{HM}(\Gamma_3)$ as:
$\Re\mathbb{HM}(\Gamma_3) = 4 \times |\mathbf{E}_{1,1}(\Gamma_3)| + 100 \times |\mathbf{E}_{9,1}(\Gamma_3)| + 324 \times |\mathbf{E}_{9,9}(\Gamma_3)| + 169 \times |\mathbf{E}_{12,1}(\Gamma_3)| + 441 \times |\mathbf{E}_{12,9}(\Gamma_3)| + 256 \times |\mathbf{E}_{15,1}(\Gamma_3)| + 576 \times |\mathbf{E}_{15,9}(\Gamma_3)| + 729 \times |\mathbf{E}_{15,12}(\Gamma_3)| + 900 \times |\mathbf{E}_{15,15}(\Gamma_3)|.$
After simplification, we get

$$\Re\mathbb{HM}(\Gamma_3) = 8484\,t^2 - 7232\,t - 1278.$$

Using the Eq (1.7), we can determine the reverse forgotten index $\Re\mathbb{F}(\Gamma_3))$ as:
$\Re\mathbb{F}(\Gamma_3)) = 2 \times |\mathbf{E}_{1,1}(\Gamma_3)| + 82 \times |\mathbf{E}_{9,1}(\Gamma_3)| + 162 \times |\mathbf{E}_{9,9}(\Gamma_3)| + 145 \times |\mathbf{E}_{12,1}(\Gamma_3)| + 225 \times |\mathbf{E}_{12,9}(\Gamma_3)| + 226 \times |\mathbf{E}_{15,1}(\Gamma_3)| + 306 \times |\mathbf{E}_{15,9}(\Gamma_3)| + 369 \times |\mathbf{E}_{15,12}(\Gamma_3)| + 450 \times |\mathbf{E}_{15,15}(\Gamma_3)|.$
After simplification, we get

$$\Re\mathbb{F}(\Gamma_3) = 5418\,t^2 - 7632\,t + 2808.$$

□

In the next theorem, we will calculate the reverse atom bond connectivity index, reverse geometric arithmetic index for Γ_3 graph.

Theorem 5.2. *Let Γ_3 be the third type of rectangular hex-derived network, then*

- $\Re\mathbb{ABC}(\Gamma_3) = \left(\frac{4\sqrt{210}}{5} + \frac{4\sqrt{7}}{5}\right)t^2 + \left(\frac{16\sqrt{2}}{3} + \frac{16}{9} - \frac{16\sqrt{210}}{5} + \frac{8\sqrt{330}}{15} - \frac{8\sqrt{7}}{5}\right)t - \frac{40}{9} - \frac{40\sqrt{2}}{3} + \frac{\sqrt{33}}{3} + \frac{2\sqrt{57}}{9} + \frac{16\sqrt{210}}{5} - \frac{44\sqrt{330}}{45} + \frac{4\sqrt{5}}{3} + \frac{4\sqrt{7}}{3}$.
- $\Re\mathbb{GA}(\Gamma_3) = \left(9 + \frac{3\sqrt{15}}{2}\right)t^2 - \frac{96t}{5} + 9 + \frac{264\sqrt{3}}{91} - 5\sqrt{15} + \frac{32\sqrt{5}}{9}$.

Proof. The reverse atom bond connectivity ($\Re\mathbb{ABC}(\Gamma_3)$), can be determined by using Eq (1.2) and Eq (5.1), as follows:
$\Re\mathbb{ABC}(\Gamma_3) = 0\times|\mathbf{E}_{1,1}(\Gamma_3)| + \sqrt{\frac{8}{9}}\times|\mathbf{E}_{9,1}(\Gamma_3)| + \sqrt{\frac{16}{81}}\times|\mathbf{E}_{9,9}(\Gamma_3)| + \sqrt{\frac{11}{12}}\times|\mathbf{E}_{12,1}(\Gamma_3)| + \sqrt{\frac{19}{108}}\times|\mathbf{E}_{12,9}(\Gamma_3)| + \sqrt{\frac{14}{15}}\times|\mathbf{E}_{15,1}(\Gamma_3)| + \sqrt{\frac{22}{135}}\times|\mathbf{E}_{15,9}(\Gamma_3)| + \sqrt{\frac{25}{180}}\times|\mathbf{E}_{15,12}(\Gamma_3)| + \sqrt{\frac{28}{225}}\times|\mathbf{E}_{15,15}(\Gamma_3)|$.
After some simplification, we get
$\Re\mathbb{ABC}(\Gamma_3) = \left(\frac{4\sqrt{210}}{5} + \frac{4\sqrt{7}}{5}\right)t^2 + \left(\frac{16\sqrt{2}}{3} + \frac{16}{9} - \frac{16\sqrt{210}}{5} + \frac{8\sqrt{330}}{15} - \frac{8\sqrt{7}}{5}\right)t - \frac{40}{9} - \frac{40\sqrt{2}}{3} + \frac{\sqrt{33}}{3} + \frac{2\sqrt{57}}{9} + \frac{16\sqrt{210}}{5} - \frac{44\sqrt{330}}{45} + \frac{4\sqrt{5}}{3} + \frac{4\sqrt{7}}{3}$.
The reverse geometric arithmetic ($\Re\mathbb{GA}(\Gamma_3)$), can be determined by using Eq (1.3) and Eq (5.1), as follows:
$\Re\mathbb{GA}(\Gamma_3) = |\mathbf{E}_{1,1}(\Gamma_3)| + \frac{2\sqrt{9}}{10}\times|\mathbf{E}_{9,1}(\Gamma_3)| + \frac{2\sqrt{81}}{18}\times|\mathbf{E}_{9,9}(\Gamma_3)| + \frac{2\sqrt{12}}{13}\times|\mathbf{E}_{12,1}(\Gamma_3)| + \frac{2\sqrt{108}}{21}\times|\mathbf{E}_{12,9}(\Gamma_3)| + \frac{2\sqrt{15}}{16}\times|\mathbf{E}_{15,1}(\Gamma_3)| + \frac{2\sqrt{135}}{24}\times|\mathbf{E}_{15,9}(\Gamma_3)| + \frac{2\sqrt{180}}{27}\times|\mathbf{E}_{15,12}(\Gamma_3)| + \frac{2\sqrt{225}}{30}\times|\mathbf{E}_{15,15}(\Gamma_3)|$.
After some simplification, we get
$\Re\mathbb{GA}(\Gamma_3) = \left(9 + \frac{3\sqrt{15}}{2}\right)t^2 - \frac{96t}{5} + 9 + \frac{264\sqrt{3}}{91} - 5\sqrt{15} + \frac{32\sqrt{5}}{9}$.

□

In the next theorems, we will calculate the reverse redefined Zagreb indices for Γ_3 graph.

Theorem 5.3. *Let Γ_3 be the third type of rectangular hex-derived network, then*

- $\Re\mathbb{Z}_1(\Gamma_3) = \frac{98t^2}{5} - \frac{3184t}{45} + \frac{5977}{90}$
- $\Re\mathbb{Z}_2(\Gamma_3) = \frac{231t^2}{4} + \frac{86t}{5} - \frac{28460}{273}$
- $\Re\mathbb{Z}_3(\Gamma_3) = 43386t^2 - 8240t - 31614$.

Proof. Reverse redefined Zagreb indices can be calculated by using Eq (5.1), the $\Re\mathbb{Z}_1(\Gamma_3)$ by using Eq (1.9) as follows:
$\Re\mathbb{Z}_1(\Gamma_3) = 2\times|\mathbf{E}_{1,1}(\Gamma_3)| + \frac{10}{9}\times|\mathbf{E}_{9,1}(\Gamma_3)| + \frac{18}{81}\times|\mathbf{E}_{9,9}(\Gamma_3)| + \frac{13}{12}\times|\mathbf{E}_{12,1}(\Gamma_3)| + \frac{21}{108}\times|\mathbf{E}_{12,9}(\Gamma_3)| + \frac{16}{15}\times|\mathbf{E}_{15,1}(\Gamma_3)| + \frac{24}{135}\times|\mathbf{E}_{15,9}(\Gamma_3)| + \frac{27}{108}\times|\mathbf{E}_{15,12}(\Gamma_3)| + \frac{30}{225}\times|\mathbf{E}_{15,15}(\Gamma_3)|$.
After some simplification, we get

$$\Re\mathbb{Z}_1(\Gamma_3) = \frac{98t^2}{5} - \frac{3184t}{45} + \frac{5977}{90}.$$

The $\Re\mathbb{Z}_2(\Gamma_1)$ can be determined by using equation (1.10) as follows:
$\Re\mathbb{Z}_2(\Gamma_3) = \frac{1}{2}\times|\mathbf{E}_{1,1}(\Gamma_3)| + \frac{9}{10}\times|\mathbf{E}_{9,1}(\Gamma_3)| + \frac{81}{18}\times|\mathbf{E}_{9,9}(\Gamma_3)| + \frac{12}{13}\times|\mathbf{E}_{12,1}(\Gamma_3)| + \frac{108}{21}\times|\mathbf{E}_{12,9}(\Gamma_3)| + \frac{15}{16}\times|\mathbf{E}_{15,1}(\Gamma_3)| + \frac{135}{24}\times|\mathbf{E}_{15,9}(\Gamma_3)| + \frac{108}{27}\times|\mathbf{E}_{15,12}(\Gamma_3)| + \frac{225}{30}\times|\mathbf{E}_{15,15}(\Gamma_3)|$.
After some simplification, we get

$$\Re\mathbb{Z}_2(\Gamma_3) = \frac{231t^2}{4} + \frac{86t}{5} - \frac{28460}{273}.$$

The $\Re\mathbb{Z}_3(\Gamma_3)$ can be calculated by using Eq (1.11) as follows:
$\Re\mathbb{Z}_3(\Gamma_3) = 2\times|\mathbf{E}_{1,1}(\Gamma_3)| + 90\times|\mathbf{E}_{9,1}(\Gamma_3)| + 1458\times|\mathbf{E}_{9,9}(\Gamma_3)| + 156\times|\mathbf{E}_{12,1}(\Gamma_3)| + 2268\times|\mathbf{E}_{12,9}(\Gamma_3)| +$

$240 \times |\mathbf{E}_{15,1}(\Gamma_3)| + 3240 \times |\mathbf{E}_{15,9}(\Gamma_3)| + 2916 \times |\mathbf{E}_{15,12}(\Gamma_3)| + 6750 \times |\mathbf{E}_{15,15}(\Gamma_3)|$.
After some simplification, we get

$$\Re\mathbb{RZ}_3(\Gamma_3) = 43386\,t^2 - 8240\,t - 31614.$$

□

6. Conclusions

In this article, we have calculated the exact solutions of reverse degree-based topological descriptors for hex-derived networks of third type. Hex-derived network has a variety of useful applications in pharmacy, electronics, and networking. We obtained the reverse degree-based indices such as reverse Randic index, reverse atom bond connectivity index, reverse geometric arithmetic index, reverse Zagreb indices, reverse redefined Zagreb indices for hex derived networks. These results may be helpful for people working in computer science and chemistry who encounter hex-derived networks.

Conflict of interest

The authors declare that there is no conflict of financial interests regarding the publication of this paper.

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