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Research article

Medical waste treatment scheme selection based on single-valued neutrosophic numbers

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Abstract: With the rapid increase in the number of infected people in COVID-19, medical supplies have been increasing significantly. Medical waste treatment scheme selection may have long-term impacts on the economy, society, and environment. Determining the best treatment option is a considerable challenge. To solve this problem, in this paper, we proposed a multi-criteria group decision making (MCGDM) method based on single-valued neutrosophic numbers and partitioned Maclaurin symmetric mean (PMSM) operator. Because of the complexity of the medical waste treatment scheme selection problem, the single-valued neutrosophic numbers are applied to express the uncertain evaluation information. For the medical waste treatment scheme selection problem, the factors or criteria (these two terms can be interchanged.) in the same clusters are closely related, and the criteria in different clusters have no relationships. The partitioned Maclaurin symmetric mean function can handle these complicated criterion relationships. Therefore, we extend the PMSM operator to process the single-valued neutrosophic numbers and propose the single-valued neutrosophic partitioned Maclaurin symmetric mean (SVNPMSM) operator and its weighted form (SVNWPMSM). Then, we analyze their properties and give typical examples of the proposed operators. An MCGDM model based on the SVNWPMSM aggregation operator is developed and applied to solve the medical waste treatment scheme selection problem. Finally, the validity and superiority of the developed model are verified by comparing it with the previous methods.

Keywords: single-valued neutrosophic numbers; medical waste treatment scheme; aggregation operators; the PMSM operator

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1. Introduction

The medical wastes are caused by doctors and nurses during various medical services. They may be the disposable syringe, disposable examination glove, and surgical mask. They have been polluted by bacteria and viruses. They may infect a large number of people when they are used again. Therefore, the medical wastes should be safely treated [1,2]. In the past years, the total amount of medical wastes initially increases every year. Since the horrible COVID-19 outbreaks in 2019, many persons from all over the world have been infected, which makes the COVID-19 pandemic [3]. Treating these patients that have been infected by the COVID-19 has produced massive medical wastes. As COVID-19 spreads rapidly to many countries, the total amount of medical waste per day has increased several times. To cope with this global crisis, a number of new medical waste treatment plants have been built. Some new medical waste treatment schemes also have been developed to improve treatment efficiency. Because of these alternative means, the medical waste treatment efficiency has been dramatically increased. For example, the medical waste treatment efficiency in Wuhan city of China has been increased by more than five times at the beginning of 2020.

There are several types of medical waste treatment schemes that can be evaluated and selected. Different medical waste treatment schemes show different features. The selection problem of medical waste treatment schemes requires consideration of the influences of multiple factors. Determining the best medical waste treatment scheme is a big challenge since each medical waste treatment scheme has different advantages and disadvantages [4,5]. Moreover, the process of determining the best medical waste treatment scheme needs a group of experts to participate. This case makes the process complex. Therefore, the selection problem of medical waste treatment schemes can be formulated to be an MCGDM (multi-criteria group decision making) problem. For the MCGDM process, the evaluation criteria should be determined for alternatives. Then a group of experts is invited to evaluate alternatives with respect to the evaluation criteria. Finally, all the evaluation information is fused to rank the alternatives. Due to the complexity of the selection problem of medical waste treatment schemes, in this paper, we introduce the tool of single-valued neutrosophic sets (SVNSs) to express the evaluation information of medical waste treatment schemes. SVNSs [6,7] are the generalization of fuzzy sets, which can represent the evaluation information in a more accurate way.

The selection process of medical waste treatment schemes should consider multiple criteria, which belong to economic, social, and environmental aspects. These criteria may have interrelationships or be independent of each other. To process these complex interrelationships among criteria, a novel MCGDM method based on single-valued neutrosophic numbers (SVNNs) and partitioned Maclaurin symmetric mean (PMSM) operator is proposed in this paper. Our main contributions are listed as follows:

(1) The PMSM operator is used to fuse the evaluation information in terms of SVNNs, and then a novel single-value neutrosophic PMSM (SVNPMSM) operator and its weighted form are proposed. Their features are also discussed.

(2) A novel MCGDM model based on the single-value neutrosophic weighted PMSM (SVNWPMSM) operator is proposed to deal with the selection problem of medical waste treatment schemes.

(3) An illustrative example is provided to illustrate the implementation process of the proposed MCGDM model. Then, the influences of the parameters on the selection results are discussed, and comparative analyses are also provided.

The rest content of this study is arranged as follows: Section 2 reviews the studies about SVNNs. Section 3 shows some basic information of SVNNs, including the definition, operation rules, and comparison method, and also gives the definition and characteristics of MSM (Maclaurin symmetric mean) and PMSM operators. Section 4 presents the definitions of the proposed SVNPMSM and SVNWPMSM operators and analyzes the relevant properties. In Section 5, a novel selection model based on the SVNWPMSM operator is proposed to solve the selection problem of medical waste treatment schemes. Section 6 uses the proposed selection model to solve a practical selection problem of medical waste treatment schemes. Section 7 verifies the superiority of the proposed selection model. Some valuable conclusions are summarized in the last section.

2. Literature review

The selection problem of medical waste treatment schemes is a complex decision-making problem. It is difficult for experts to use crisp values to evaluate the medical waste treatment schemes [8]. For the complex decision-making problem, the concept of fuzzy sets (FSs) [9,10] is an alternative for modeling uncertain information. The fuzzy sets use the degree of membership to measure the uncertain information. This concept was extended by Atanassov [11] using the degree of non-membership, and the concept of intuitionistic fuzzy sets (IFSs) was designed. When expressing fuzzy and uncertain information, IFSs give a means that is more intuitive and effective than FSs. However, both IFSs and FSs do not have the ability to express inconsistent evaluation information. In this case, Smarandache designed the neutrosophic sets that consist of the truth degree, indeterminacy degree, and falsity degree [12]. The values of the truth degree, indeterminacy degree, falsity degree, and their sum are the non-standard interval subsets of $]0^-,[1^+$. Since it was proposed, its decision-making theories and methods have been studied by researchers [13–15]. However, it is difficult to use the neutrosophic sets in real engineering applications. To promote the real application of neutrosophic sets, an extended concept of single-valued neutrosophic sets (SVNSs) was proposed by Wang et al. [16] by restricting the values of the truth degree, indeterminacy degree, and falsity degree to be in the interval [0,1]. Since its appearance, SVNSs have been used in various fields such as medical diagnosis [17], assessment of consumers' motivations [18], and typhoon disaster assessment [19].

For the complex decision-making problem, how to aggregate or fuse the evaluation information is a big challenge. Information aggregation operators are an effective but simple fusion means [20,21]. Various aggregation operators have been proposed to fuse the evaluation information, such as OWA (ordered weighted averaging) operator [22], IOWLAD (induced ordered weighted logarithmic averaging distance) operator [23], WA (weighted averaging) operator [24], PA (power averaging) operator [25], and Hamacher aggregation operator [26]. However, these aggregation operators do not consider the relationship between the evaluation information [27]. For the complex decision-making problem, there exists a correlation relationship between the evaluation information. To consider this correlation relation, the Bonferroni mean function [28] and Heronian mean function [29] are extended for fusing various fuzzy evaluation information. For example, Ates et al. [30] improved the Bonferroni mean to fuse picture fuzzy information. Lin et al. [31] extended the Heronian mean to fuse linguistic q-rung orthopair fuzzy information.

Both Bonferroni mean and Heronian mean functions only consider the correlation relationship between two input values. The Maclaurin symmetric mean (MSM) [32] is an excellent mapping function that can capture the interrelation among evaluation information. Hence, it is more generic than Bonferroni mean and Heronian mean [33]. It has been extended by scholars and researchers to aggregate complex q-rung orthopair fuzzy sets [34], linguistic intuitionistic fuzzy numbers [26], and intuitionistic fuzzy soft sets [35]. Nevertheless, these criteria are not always correlated with each other. There may exist cluster relationships among these criteria. The criteria in the same clusters are closely related, but the criteria in the different clusters have no relationship. In order to cope with this complex interrelation, Liu et al. [32] proposed the partitioned MSM (PMSM) operator, which can not only capture the correlation relation among the evaluation information of criteria in the same clusters, but also consider the independence relation between clusters. The PMSM operator has been used to process 2-dimensional linguistic information [36], linguistic neutrosophic information [37], q-rung orthopair uncertain linguistic information [38]. However, the information structure of single-valued neutrosophic sets is very different from 2-dimensional linguistic information, linguistic neutrosophic information, and q-rung orthopair uncertain linguistic information. Their research results cannot be simply and directly applied to the single-valued neutrosophic sets.

3. Preliminaries

3.1. Single-valued neutrosophic sets

Definition 1. [12] Suppose X is the collection of discourse, the neutrosophic set p can be denoted as:

$$p=\left\{\left\langle x,u_p(x),\tau_p(x),v_p(x)\right\rangle : x\in X\right\}, \tag{1}$$

where $u_p(x)$, $\tau_p(x)$, and $v_p(x)$ denote truth, indeterminacy, and falsity membership function, respectively. The values satisfy the conditions $u_p(x),\tau_p(x),v_p(x)\in]0^-,[1^+$ and $0^-\le \sup u_p(x)+\sup \tau_p(x)+\sup v_p(x)\le 3^+$.

Definition 2. [12,39] Suppose X is the collection of discourse, the single-valued neutrosophic set (SVNS) p is given by:

$$p=\left\{\left\langle x,u_p(x),\tau_p(x),v_p(x)\right\rangle : x\in X\right\}, \tag{2}$$

where $u_p(x),\tau_p(x),v_p(x)\in[0,1]$. The constraint $0\le u_p(x)+\tau_p(x)+v_p(x)\le 3$ is satisfied. For convenience, we call $p=\left\langle u_p,\tau_p,v_p\right\rangle$ a single-valued neutrosophic number (SVNN).

Definition 3. [39] Suppose $p=\left\langle u_p,\tau_p,v_p\right\rangle$ and $q=\left\langle u_q,\tau_q,v_q\right\rangle$ are any SVNNs, then the operation rules are defined as:

$$p\oplus q=\left\langle u_p+u_q-u_pu_q,\tau_p\tau_q,v_pv_q\right\rangle=\left\langle 1-(1-u_p)(1-u_q),\tau_p\tau_q,v_pv_q\right\rangle, \tag{3}$$

$$p\otimes q=\left\langle u_pu_q,\tau_p+\tau_q-\tau_p\tau_q,v_p+v_q-v_pv_q\right\rangle, \tag{4}$$

$$\lambda p=\left\langle 1-\left(1-u_p\right)^{\lambda},\tau_p^{\lambda},v_p^{\lambda}\right\rangle, \tag{5}$$

$$p^{\lambda}=\left\langle u_p^{\lambda},1-\left(1-\tau_p\right)^{\lambda},1-\left(1-v_p\right)^{\lambda}\right\rangle, \tag{6}$$

$$p^{c}=\left\langle v_p,1-\tau_p,u_p\right\rangle. \tag{7}$$

Definition 4. [40] Suppose $p=\left\langle u_p,\tau_p,v_p\right\rangle$ and $q=\left\langle u_q,\tau_q,v_q\right\rangle$ are any SVNNs, the score function $S(p)$ of p is given by:

$$S(p)=\frac{2+u_p(x)-\tau_p(x)-v_p(x)}{3}, \tag{8}$$

and the accuracy function $H(p)$ of p is given by:

$$H(p)=u_p(x)+\tau_p(x)+v_p(x). \tag{9}$$

Then, these two SVNNs can be compared according to the following rules:

(1) if $S(p)>S(q)$, then $p>q$;

(2) if $S(p)=S(q)$, then
 if $H(p)=H(q)$, then $p=q$,
 if $H(p)>H(q)$, then $p>q$.

3.2. The partitioned Maclaurin symmetric mean (PMSM) operator

Definition 5. [41] Suppose $\{p_1,p_2,...,p_n\}$ are non-negative real numbers and k is a parameter ($k=1,2,...,n$), then the mathematical expression of the Maclaurin symmetric mean (MSM) operator is given by:

$$MSM^{(k)}\left(p_1,p_2,\ldots,p_n\right)=\left(\frac{\sum_{1\le i_1<\ldots<i_k\le n}\left(\prod_{j=1}^{k}p_{i_j}\right)}{C_n^k}\right)^{1/k}, \tag{10}$$

where $i_1,i_2,...,i_k$ is the set of k integers extracted from the set $\{1,2,...,n\}$, and $1\le i_1<i_2<\cdots<i_k\le n$. C_n^k denotes the coefficients of the binomial and $C_n^k=\frac{n!}{k!(n-k)!}$.

Definition 6. [32] Suppose $\{p_1,p_2,\ldots,p_n\}$ is a non-negative real number set that is divided into S clusters. k is a parameter, $k=1,2,...,h_g$, h_g represents the number of criteria in the cluster P_g. Then, the formula of the partitioned MSM (PMSM) operator is given by:

$$\mathrm{PMSM}^{(k)}\left(p_1,p_2,\ldots,p_n\right)=\frac{1}{s}\sum_{g=1}^{s}\left(\frac{\sum_{1\le i_1<\cdots<i_k\le h_g}\left(\prod_{j=1}^{k}p_{i_j}\right)}{C_{h_g}^k}\right)^{1/k}, \tag{11}$$

where $i_1, i_2, ..., i_k$ is the set of k integers extracted from the set $\{1, 2, ..., h_g\}$, and $1 \le i_1 < i_2 < \cdots < i_k \le h_g$. $C_{h_g}^k$ denotes the coefficients of the binomial and $C_{h_g}^k = \dfrac{h_g!}{k!(h_g - k)!}$.

4. The proposed SVNPMSM operator and SVNWPMSM operator

4.1. The SVNPMSM operator

Definition 7. Suppose $p_i = \langle u_i, \tau_i, v_i \rangle$ $(i = 1, 2, ..., n)$ is a set of SVNNs, which can be divided into S different clusters. k is the parameter, and $k = 1, 2, ..., h_g$, h_g represents the number of criteria in the cluster P_g. Then the SVNPMSM (single-valued neutrosophic partitioned Maclaurin symmetric mean) operator is given by:

$$SVNPMSM^{(k)}\left(p_1, p_2, \ldots, p_n\right) = \frac{1}{S} \overset{s}{\underset{g=1}{\oplus}} \left(\frac{\underset{1 \le i_1 < \cdots < i_k \le h_g}{\oplus} \left(\overset{k}{\underset{j=1}{\otimes}} p_{i_j} \right)}{C_{h_g}^k} \right)^{1/k}, \tag{12}$$

where $i_1, i_2, ..., i_k$ is the set of k integers extracted from the set $\{1, 2, ..., h_g\}$, and $1 \le i_1 < i_2 < \cdots < i_k \le h_g$. $C_{h_g}^k$ denotes the coefficients of the binomial and $C_{h_g}^k = \dfrac{h_g!}{k!(h_g - k)!}$.

Theorem 1. Suppose $p_i = \langle u_i, \tau_i, v_i \rangle$ $(i = 1, 2, ..., n)$ is a set of SVNNs, which can be divided into S different clusters. k is the parameter, and $k = 1, 2, ..., h_g$, h_g represents the number of criteria in the cluster P_g. Then, the result of the SVNPMSM operator is still an SVNN, which is given by:

$$SVNPMSM^{(k)}\left(p_1, p_2, \ldots, p_n\right) = \left\langle 1 - \left(\prod_{g=1}^{s} \left(1 - \left(1 - \left(\prod_{1 \le i_1 < \cdots < i_k \le h_g} (1 - \prod_{j=1}^{k} u_{i_j}) \right)^{\frac{1}{C_{h_g}^k}} \right)^{\frac{1}{k}} \right) \right)^{\frac{1}{s}}, \right.$$
$$\left. \left(\prod_{g=1}^{s} \left(1 - \left(1 - \left(\prod_{1 \le i_1 < \cdots < i_k \le h_g} \left(1 - \prod_{j=1}^{k} (1 - \tau_{i_j}) \right) \right)^{\frac{1}{C_{h_g}^k}} \right)^{\frac{1}{k}} \right) \right)^{\frac{1}{s}}, \left(\prod_{g=1}^{s} \left(1 - \left(1 - \left(\prod_{1 \le i_1 < \cdots < i_k \le h_g} \left(1 - \prod_{j=1}^{k} (1 - v_{i_j}) \right) \right)^{\frac{1}{C_{h_g}^k}} \right)^{\frac{1}{k}} \right) \right)^{\frac{1}{s}} \right\rangle. \tag{13}$$

Proof. According to Eq (3)–(6), we have

$$\overset{k}{\underset{j=1}{\otimes}} p_{i_j} = \left\langle \prod_{j=1}^{k} u_{i_j}, 1 - \prod_{j=1}^{k} \left(1 - \tau_{i_j}\right), 1 - \prod_{j=1}^{k} \left(1 - v_{i_j}\right) \right\rangle.$$

Then we can get

$$\underset{1\le i_1<\cdots<i_k\le h_g}{\oplus}\left(\overset{k}{\underset{i=1}{\otimes}} p_{i_j}\right)=\left\langle 1-\prod_{1\le i_1<\cdots<i_k\le h_g}\left(1-\prod_{j=1}^{k}u_{i_j}\right),\prod_{1\le i_1<\cdots<i_k\le h_g}\left(1-\prod_{j=1}^{k}(1-\tau_{i_j})\right),\prod_{1\le i_1<\cdots<i_k\le h_g}\left(1-\prod_{j=1}^{k}(1-v_{i_j})\right)\right\rangle,$$

and

$$\frac{1}{C_{h_g}^{k}}\left(\underset{1\le i_1<\cdots<i_k\le h_g}{\oplus}\left(\overset{k}{\underset{j=1}{\otimes}} p_{i_j}\right)\right)=\left\langle 1-\left(\prod_{1\le i_1<\cdots<i_k\le h_g}\left(1-\prod_{j=1}^{k}u_{i_j}\right)\right)^{\frac{1}{C_{h_g}^{k}}},\left(\prod_{1\le i_1<\cdots<i_k\le h_g}\left(1-\prod_{j=1}^{k}(1-\tau_{i_j})\right)\right)^{\frac{1}{C_{h_g}^{k}}},\left(\prod_{1\le i_1<\cdots<i_k\le h_g}\left(1-\prod_{j=1}^{k}(1-v_{i_j})\right)\right)^{\frac{1}{C_{h_g}^{k}}}\right\rangle.$$

Further,

$$\left(\frac{\underset{1\le i_1<\cdots<i_k\le h_g}{\oplus}\left(\overset{k}{\underset{j=1}{\otimes}} p_{i_j}\right)}{C_{h_g}^{k}}\right)^{\frac{1}{k}}=\left\langle\left(1-\left(\prod_{1\le i_1<\cdots<i_k\le h_g}\left(1-\prod_{j=1}^{k}u_{i_j}\right)\right)^{\frac{1}{C_{h_g}^{k}}}\right)^{\frac{1}{k}},1-\left(1-\left(\prod_{1\le i_1<\cdots<i_k\le h_g}\left(1-\prod_{j=1}^{k}(1-\tau_{i_j})\right)\right)^{\frac{1}{C_{h_g}^{k}}}\right)^{\frac{1}{k}},1-\left(1-\left(\prod_{1\le i_1<\cdots<i_k\le h_g}\left(1-\prod_{j=1}^{k}(1-v_{i_j})\right)\right)^{\frac{1}{C_{h_g}^{k}}}\right)^{\frac{1}{k}}\right\rangle.$$

Furthermore,

$$\overset{s}{\underset{g=1}{\oplus}}\left(\frac{\underset{1\le i_1<\cdots<i_k\le h_g}{\oplus}\left(\overset{k}{\underset{j=1}{\otimes}} p_{i_j}\right)}{C_{h_g}^{k}}\right)^{\frac{1}{k}}=\left\langle 1-\prod_{g=1}^{s}\left(\left(1-\left(\prod_{1\le i_1<\cdots<i_k\le h_g}\left(1-\prod_{j=1}^{k}u_{i_j}\right)\right)^{\frac{1}{C_{h_g}^{k}}}\right)^{\frac{1}{k}}\right),\prod_{g=1}^{s}\left(1-\left(1-\left(\prod_{1\le i_1<\cdots<i_k\le h_g}\left(1-\prod_{j=1}^{k}(1-\tau_{i_j})\right)\right)^{\frac{1}{C_{h_g}^{k}}}\right)^{\frac{1}{k}}\right),\prod_{g=1}^{s}\left(1-\left(1-\left(\prod_{1\le i_1<\cdots<i_k\le h_g}\left(1-\prod_{j=1}^{k}(1-v_{i_j})\right)\right)^{\frac{1}{C_{h_g}^{k}}}\right)^{\frac{1}{k}}\right)\right\rangle.$$

Thus, we can get

$$\frac{1}{S}\overset{s}{\underset{g=1}{\oplus}}\left(\frac{\underset{1\le i_1<\cdots<i_k\le h_g}{\oplus}\left(\overset{k}{\underset{j=1}{\otimes}} p_{i_j}\right)}{C_{h_g}^{k}}\right)^{\frac{1}{k}}=\left\langle 1-\left(\prod_{g=1}^{s}\left(1-\left(1-\left(\prod_{1\le i_1<\cdots<i_k\le h_g}\left(1-\prod_{j=1}^{k}u_{i_j}\right)\right)^{\frac{1}{C_{h_g}^{k}}}\right)^{\frac{1}{k}}\right)\right)^{\frac{1}{s}},\right.$$

$$\left.\left(\prod_{g=1}^{s}\left(1-\left(1-\left(\prod_{1\le i_1<\cdots<i_k\le h_g}\left(1-\prod_{j=1}^{k}(1-\tau_{i_j})\right)\right)^{\frac{1}{C_{h_g}^{k}}}\right)^{\frac{1}{k}}\right)\right)^{\frac{1}{s}},\left(\prod_{g=1}^{s}\left(1-\left(1-\left(\prod_{1\le i_1<\cdots<i_k\le h_g}\left(1-\prod_{j=1}^{k}(1-v_{i_j})\right)\right)^{\frac{1}{C_{h_g}^{k}}}\right)^{\frac{1}{k}}\right)\right)^{\frac{1}{s}}\right\rangle$$

.

According to Definition 2, it is easy to know that

$$0\le 1-\left(\prod_{g=1}^{s}\left(1-\left(1-\left(\prod_{1\le i_1<\cdots<i_k\le h_g}\left(1-\prod_{j=1}^{k}u_{i_j}\right)\right)^{\frac{1}{C_{h_g}^{k}}}\right)^{\frac{1}{k}}\right)\right)^{\frac{1}{s}}\le 1,$$

$$0\le\left(\prod_{g=1}^{s}\left(1-\left(1-\left(\prod_{1\le i_1<\cdots<i_k\le h_g}\left(1-\prod_{j=1}^{k}(1-\tau_{i_j})\right)\right)^{\frac{1}{C_{h_g}^{k}}}\right)^{\frac{1}{k}}\right)\right)^{\frac{1}{s}}\le 1,$$

$$0\le\left(\prod_{g=1}^{s}\left(1-\left(1-\left(\prod_{1\le i_1<\cdots<i_k\le h_g}\left(1-\prod_{j=1}^{k}(1-v_{i_j})\right)\right)^{\frac{1}{C_{h_g}^{k}}}\right)^{\frac{1}{k}}\right)\right)^{\frac{1}{s}}\le 1.$$

Therefore, we can get

$$0\le 1-\left(\prod_{g=1}^{s}\left(1-\left(1-\left(\prod_{1\le i_1<\cdots<i_k\le h_g}\left(1-\prod_{j=1}^{k}u_{i_j}\right)\right)^{\frac{1}{C_{h_g}^{k}}}\right)^{\frac{1}{k}}\right)\right)^{\frac{1}{s}}+\left(\prod_{g=1}^{s}\left(1-\left(1-\left(\prod_{1\le i_1<\cdots<i_k\le h_g}\left(1-\prod_{j=1}^{k}(1-\tau_{i_j})\right)\right)^{\frac{1}{C_{h_g}^{k}}}\right)^{\frac{1}{k}}\right)\right)^{\frac{1}{s}}+$$

$$\left(\prod_{g=1}^{s}\left(1-\left(1-\left(\prod_{1\le i_1<\cdots<i_k\le h_g}\left(1-\prod_{j=1}^{k}(1-v_{i_j})\right)\right)^{\frac{1}{C_{h_g}^{k}}}\right)^{\frac{1}{k}}\right)\right)^{\frac{1}{s}}\le 3$$

.

According to the above proof process, the proof of Theorem 1 is completed. Then, we will discuss some characteristics of the SVNPMSM operator.

Theorem 2. (Idempotency). Suppose there exist n identical real numbers $p=\langle u,\tau,v\rangle$. Then

$$SVNPMSM^{(k)}(p,p,\ldots,p)=p.$$

Proof. Due to $p=\langle u,\tau,v\rangle$, we have

$$SVNPMSM^{(k)}(p,p,\ldots,p)=\left\langle 1-\left(\prod_{g=1}^{s}\left(1-\left(1-\left(\prod_{1\le i_1<\cdots<i_k\le h_g}\left(1-\prod_{j=1}^{k}u\right)\right)^{\frac{1}{C_{h_g}^k}}\right)^{\frac{1}{k}}\right)\right)^{\frac{1}{s}},\right.$$

$$\left.\left(\prod_{g=1}^{s}\left(1-\left(1-\left(\prod_{1\le i_1<\cdots<i_k\le h_g}\left(1-\prod_{j=1}^{k}(1-\tau)\right)\right)^{\frac{1}{C_{h_g}^k}}\right)^{\frac{1}{k}}\right)\right)^{\frac{1}{s}},\left(\prod_{g=1}^{s}\left(1-\left(1-\left(\prod_{1\le i_1<\cdots<i_k\le h_g}\left(1-\prod_{j=1}^{k}(1-v)\right)\right)^{\frac{1}{C_{h_g}^k}}\right)^{\frac{1}{k}}\right)\right)^{\frac{1}{s}}\right\rangle,$$

$$=\left\langle 1-\left(\prod_{g=1}^{s}\left(1-\left(1-\left(\prod_{1\le i_1<\cdots<i_k\le h_g}(1-u^k)\right)^{\frac{1}{C_{h_g}^k}}\right)^{\frac{1}{k}}\right)\right)^{\frac{1}{s}},\left(\prod_{g=1}^{s}\left(1-\left(1-\left(\prod_{1\le i_1<\cdots<i_k\le h_g}\left(1-(1-\tau)^k\right)\right)^{\frac{1}{C_{h_g}^k}}\right)^{\frac{1}{k}}\right)\right)^{\frac{1}{s}},\right.$$

$$\left.\left(\prod_{g=1}^{s}\left(1-\left(1-\left(\prod_{1\le i_1<\cdots<i_k\le h_g}\left(1-(1-v)^k\right)\right)^{\frac{1}{C_{h_g}^k}}\right)^{\frac{1}{k}}\right)\right)^{\frac{1}{s}}\right\rangle,$$

$$=\left\langle 1-\left(\prod_{g=1}^{s}\left(1-\left(1-\left((1-u^k)^{C_{h_g}^k}\right)^{\frac{1}{C_{h_g}^k}}\right)^{\frac{1}{k}}\right)\right)^{\frac{1}{s}},\left(\prod_{g=1}^{s}\left(1-\left(1-\left(\left(1-(1-\tau)^k\right)^{C_{h_g}^k}\right)^{\frac{1}{C_{h_g}^k}}\right)^{\frac{1}{k}}\right)\right)^{\frac{1}{s}},\right.$$

$$\left.\left(\prod_{g=1}^{s}\left(1-\left(1-\left(\left(1-(1-v)^k\right)^{C_{h_g}^k}\right)^{\frac{1}{C_{h_g}^k}}\right)^{\frac{1}{k}}\right)\right)^{\frac{1}{s}}\right\rangle,$$

$$=\left\langle 1-\left(\prod_{g=1}^{s}\left(1-\left(1-(1-u^{k})\right)^{\frac{1}{k}}\right)\right)^{\frac{1}{s}},\left(\prod_{g=1}^{s}\left(1-\left(1-\left(1-(1-\tau)^{k}\right)\right)^{\frac{1}{k}}\right)\right)^{\frac{1}{s}},\left(\prod_{g=1}^{s}\left(1-\left(1-\left(1-(1-v)^{k}\right)\right)^{\frac{1}{k}}\right)\right)^{\frac{1}{s}}\right\rangle,$$

$$=\left\langle 1-\left(\prod_{g=1}^{s}(1-u)\right)^{\frac{1}{s}},\left(\prod_{g=1}^{s}(1-(1-\tau))\right)^{\frac{1}{s}},\left(\prod_{g=1}^{s}(1-(1-v))\right)^{\frac{1}{s}}\right\rangle=\langle u,\tau,v\rangle=p\,.$$

According to the above proof process, we have completed the proof of Idempotency.

Theorem 3. (Monotonicity). Suppose that $p_1, p_2, \ldots, p_n$ and $q_1, q_2, \ldots, q_n$ are two sets of SVNNs. If $p_i=\langle u_i,\tau_i,v_i\rangle$, $q_i=\langle u_i',\tau_i',v_i'\rangle$, and satisfies $u_i\ge u_i',\tau_i\le\tau_i',v_i\le v_i'$ for all $i=1,2,...,n$. Then

$$SVNPMSM^{(k)}\left(p_1,p_2,...,p_n\right)\ge SVNPMSM^{(k)}\left(q_1,q_2,...,q_n\right).$$

Proof. Suppose that $SVNPMSM^{(k)}\left(p_1,p_2,...,p_n\right)=\langle u,\tau,v\rangle$, $SVNPMSM^{(k)}\left(q_1,q_2,...,q_n\right)=\langle u',\tau',v'\rangle$.

According to Definition 7, we get $h_g\ge1,k\ge1$, and $C_{h_g}^{k}\ge1$. Firstly, let us consider the truth membership function part. Since $u_i\ge u_i'$, then we can get $\prod_{j=1}^{k}u_{i_j}\ge\prod_{j=1}^{k}u_{i_j}'$. Further, we can obtain

$$(1-\prod_{j=1}^{k}u_{i_j})\le(1-\prod_{j=1}^{k}u_{i_j}')\Rightarrow\prod_{1\le i_1<\cdots<i_k\le h_g}(1-\prod_{j=1}^{k}u_{i_j})\le\prod_{1\le i_1<\cdots<i_k\le h_g}(1-\prod_{j=1}^{k}u_{i_j}'),$$

$$\Rightarrow\left(\prod_{1\le i_1<\cdots<i_k\le h_g}(1-\prod_{j=1}^{k}u_{i_j})\right)^{\frac{1}{C_{h_g}^{k}}}\le\left(\prod_{1\le i_1<\cdots<i_k\le h_g}(1-\prod_{j=1}^{k}u_{i_j}')\right)^{\frac{1}{C_{h_g}^{k}}},$$

$$\Rightarrow\left(1-\left(\prod_{1\le i_1<\cdots<i_k\le h_g}(1-\prod_{j=1}^{k}u_{i_j})\right)^{\frac{1}{C_{h_g}^{k}}}\right)^{\frac{1}{k}}\ge\left(1-\left(\prod_{1\le i_1<\cdots<i_k\le h_g}(1-\prod_{j=1}^{k}u_{i_j}')\right)^{\frac{1}{C_{h_g}^{k}}}\right)^{\frac{1}{k}},$$

$$\Rightarrow\prod_{g=1}^{s}\left(1-\left(1-\left(\prod_{1\le i_1<\cdots<i_k\le h_g}(1-\prod_{j=1}^{k}u_{i_j})\right)^{\frac{1}{C_{h_g}^{k}}}\right)^{\frac{1}{k}}\right)\le\prod_{g=1}^{s}\left(1-\left(1-\left(\prod_{1\le i_1<\cdots<i_k\le h_g}(1-\prod_{j=1}^{k}u_{i_j}')\right)^{\frac{1}{C_{h_g}^{k}}}\right)^{\frac{1}{k}}\right),$$

$$\Rightarrow1-\left(\prod_{g=1}^{s}\left(1-\left(1-\left(\prod_{1\le i_1<\cdots<i_k\le h_g}(1-\prod_{j=1}^{k}u_{i_j})\right)^{\frac{1}{C_{h_g}^{k}}}\right)^{\frac{1}{k}}\right)\right)^{\frac{1}{s}}\ge1-\left(\prod_{g=1}^{s}\left(1-\left(1-\left(\prod_{1\le i_1<\cdots<i_k\le h_g}(1-\prod_{j=1}^{k}u_{i_j}')\right)^{\frac{1}{C_{h_g}^{k}}}\right)^{\frac{1}{k}}\right)\right)^{\frac{1}{s}}.$$

Therefore, we can get $u\ge u'$. Similarly, we can obtain the following inequality: $\tau\le\tau',v\le v'$. Through the above analysis, we can obtain that $u\ge u'$ and $\tau\le\tau',v\le v'$. Then, we can obtain

$\langle u,\tau,v\rangle \geq \langle u',\tau',v'\rangle$ and $SVNPMSM^{(k)}(p_1,p_2,...,p_n) \geq SVNPMSM^{(k)}(q_1,q_2,...,q_n)$. According to the detailed proof process, we have completed the proof of monotonicity.

Theorem 4. (Boundedness). Let $p_1,p_2,\ldots,p_n$ be SVNNs, where $p_i=\langle u_i,\tau_i,v_i\rangle$ $(i=1,2,...,n)$, and $p^+=\max_{i=1}^n p_i$, $p^-=\min_{i=1}^n p_i$. Then

$$p^- \leq SVNPMSM^{(k)}(p_1,p_2,...,p_n) \leq p^+ .$$

Proof. Monotonicity and Idempotency of the SVNPMSM operator have been proved, so we can obtain

$$SVNPMSM^{(k)}(p_1,p_2,...,p_n) \leq SVNPMSM^{(k)}(p^+,p^+,...,p^+) = p^+ .$$

Similarly, we can obtain

$$SVNPMSM^{(k)}(p_1,p_2,...,p_n) \geq SVNPMSM^{(k)}(p^-,p^-,...,p^-) = p^- .$$

Therefore, we have

$$p^- \leq SVNPMSM^{(k)}(p_1,p_2,...,p_n) \leq p^+ .$$

According to the above proof process, we have completed the proof of boundedness. Then, we introduce a lemma that will be used in the proof of Theorem 5.

Lemma 1. Let $\alpha_i>0,\beta_i>0$ and $\sum_{i=1}^n \alpha_i=1$, where $i=1,2,...,n$. Then, $\prod_{i=1}^n \alpha_i^{\beta_i} \leq \sum_{i=1}^n \beta_i^{\alpha_i}$.

Theorem 5. Let $p_i=\langle u_i,\tau_i,v_i\rangle$ $(i=1,2,...,n)$ be SVNNs, and $k=1,2,...,\min_g h_g$. The SVNPMSM operator decreases monotonically as the parameter k increases.

Proof. From the definition of the SVNPMSM operator, we have $h_g \geq 1$ and $k\geq 1$, which leads to $C_{h_g}^k \geq 1$.

$$SVNPMSM^{(k)}(p_1,p_2,\ldots,p_n)=\left\langle 1-\left(\prod_{g=1}^{s}\left(1-\left(1-\left(\prod_{1\leq i_1<\cdots<i_k\leq h_g}(1-\prod_{j=1}^{k}u_{i_j})\right)^{\frac{1}{C_{h_g}^k}}\right)^{\frac{1}{k}}\right)\right)^{\frac{1}{s}},\right.$$

$$\left.\left(\prod_{g=1}^{s}\left(1-\left(1-\left(\prod_{1\leq i_1<\cdots<i_k\leq h_g}\left(1-\prod_{j=1}^{k}(1-\tau_{i_j})\right)\right)^{\frac{1}{C_{h_g}^k}}\right)^{\frac{1}{k}}\right)\right)^{\frac{1}{s}},\left(\prod_{g=1}^{s}\left(1-\left(1-\left(\prod_{1\leq i_1<\cdots<i_k\leq h_g}\left(1-\prod_{j=1}^{k}(1-v_{i_j})\right)\right)^{\frac{1}{C_{h_g}^k}}\right)^{\frac{1}{k}}\right)\right)^{\frac{1}{s}}\right\rangle$$

.

Suppose $X(k)=1-\left(\prod_{g=1}^{s}\left(1-\left(1-\left(\prod_{1\leq i_1<\cdots<i_k\leq h_g}(1-\prod_{j=1}^{k}u_{i_j})\right)^{\frac{1}{C_{h_g}^k}}\right)^{\frac{1}{k}}\right)\right)^{\frac{1}{s}}$,

$$Y(k)=\left(\prod_{g=1}^{s}\left(1-\left(1-\left(\prod_{1\le i_1<\cdots<i_k\le h_g}\left(1-\prod_{j=1}^{k}(1-\tau_{i_j})\right)\right)^{\frac{1}{C_{h_g}^{k}}}\right)^{\frac{1}{k}}\right)\right)^{\frac{1}{s}}, \text{ and}$$

$$Z(k)=\left(\prod_{g=1}^{s}\left(1-\left(1-\left(\prod_{1\le i_1<\cdots<i_k\le h_g}\left(1-\prod_{j=1}^{k}(1-\nu_{i_j})\right)\right)^{\frac{1}{C_{h_g}^{k}}}\right)^{\frac{1}{k}}\right)\right)^{\frac{1}{s}}.$$

Next, we analyze the properties of the function $X(k)$. According to Lemma 1, we have

$$\left(\prod_{1\le i_1<\cdots<i_k\le h_g}\left(1-\prod_{j=1}^{k}u_{i_j}\right)\right)^{\frac{1}{C_{h_g}^{k}}}\le \sum_{1\le i_1<\cdots<i_k\le h_g}\frac{1-\prod_{j=1}^{k}u_{i_j}}{C_{h_g}^{k}},$$

$$\Rightarrow 1-\left(\prod_{1\le i_1<\cdots<i_k\le h_g}\left(1-\prod_{j=1}^{k}u_{i_j}\right)\right)^{\frac{1}{C_{h_g}^{k}}}\ge 1-\sum_{1\le i_1<\cdots<i_k\le h_g}\frac{1-\prod_{j=1}^{k}u_{i_j}}{C_{h_g}^{k}}=\sum_{1\le i_1<\cdots<i_k\le h_g}\frac{\prod_{j=1}^{k}u_{i_j}}{C_{h_g}^{k}},$$

$$\Rightarrow \ln\left(1-\left(\prod_{1\le i_1<\cdots<i_k\le h_g}\left(1-\prod_{j=1}^{k}u_{i_j}\right)\right)^{\frac{1}{C_{h_g}^{k}}}\right)\ge \ln\left(\sum_{1\le i_1<\cdots<i_k\le h_g}\frac{\prod_{j=1}^{k}u_{i_j}}{C_{h_g}^{k}}\right),$$

$$\Rightarrow \frac{1}{k}\ln\left(1-\left(\prod_{1\le i_1<\cdots<i_k\le h_g}\left(1-\prod_{j=1}^{k}u_{i_j}\right)\right)^{\frac{1}{C_{h_g}^{k}}}\right)\ge \frac{1}{k}\ln\left(\sum_{1\le i_1<\cdots<i_k\le h_g}\frac{\prod_{j=1}^{k}u_{i_j}}{C_{h_g}^{k}}\right),$$

$$\Rightarrow \left(1-\left(\prod_{1\le i_1<\cdots<i_k\le h_g}\left(1-\prod_{j=1}^{k}u_{i_j}\right)\right)^{\frac{1}{C_{h_g}^{k}}}\right)^{\frac{1}{k}}\ge \left(\sum_{1\le i_1<\cdots<i_k\le h_g}\frac{\prod_{j=1}^{k}u_{i_j}}{C_{h_g}^{k}}\right)^{\frac{1}{k}},$$

$$\Rightarrow \prod_{g=1}^{s}\left(1-\left(1-\left(\prod_{1\le i_1<\cdots<i_k\le h_g}\left(1-\prod_{j=1}^{k}u_{i_j}\right)\right)^{\frac{1}{C_{h_g}^{k}}}\right)^{\frac{1}{k}}\right)\le \prod_{g=1}^{s}\left(1-\left(\sum_{1\le i_1<\cdots<i_k\le h_g}\frac{\prod_{j=1}^{k}u_{i_j}}{C_{h_g}^{k}}\right)^{\frac{1}{k}}\right).$$

Therefore, we have

$$X(k)=1-\left(\prod_{g=1}^{s}\left(1-\left(1-\left(\prod_{1\le i_1<\cdots<i_k\le h_g}\left(1-\prod_{j=1}^{k}u_{i_j}\right)\right)^{\frac{1}{C_{h_g}^{k}}}\right)^{\frac{1}{k}}\right)\right)^{\frac{1}{s}}\ge 1-\left(\prod_{g=1}^{s}\left(1-\left(\sum_{1\le i_1<\cdots<i_k\le h_g}\frac{\prod_{j=1}^{k}u_{i_j}}{C_{h_g}^{k}}\right)^{\frac{1}{k}}\right)\right)^{\frac{1}{s}}.$$

Then, we prove the monotonicity of $X(k)$ using the contradiction method. Suppose that the function $X(k)$ decreases monotonously with the increase of the variable k. When $k=1$, we can get

$$X(1)\ge 1-\left(\prod_{g=1}^{s}\left(1-\left(\sum_{1\le i_1<\cdots<i_k\le h_g}\frac{\prod_{j=1}^{1}u_{i_j}}{C_{h_g}^{1}}\right)^{\frac{1}{1}}\right)\right)^{\frac{1}{s}}=1-\left(\prod_{g=1}^{s}\left(1-\left(\frac{\sum_{i_j=1}^{h_g}u_{i_j}}{h_g}\right)\right)\right)^{\frac{1}{s}}.$$

Further, assume that the number of criteria is equal in each cluster, i. e., $h_g=h\ (g=1,2,...,s)$. Since $\min_g h_g=h$, we can obtain

$$X\left(\min_g h_g\right)=X(h)=1-\left(\prod_{g=1}^{s}\left(1-\left(1-\left(\prod_{1\le i_1<\cdots<i_k\le h_g}\left(1-\prod_{j=1}^{h}u_{i_j}\right)\right)^{\frac{1}{C_h^h}}\right)^{\frac{1}{h}}\right)\right)^{\frac{1}{s}}=1-\left(\prod_{g=1}^{s}\left(1-\left(\prod_{j=1}^{h}u_{i_j}\right)^{\frac{1}{h}}\right)\right)^{\frac{1}{s}}.$$

Then, according to the assumption $X(h)>X(1)$, we can obtain

$$X(h)=1-\left(\prod_{g=1}^{s}\left(1-\left(\prod_{j=1}^{h}u_{i_j}\right)^{\frac{1}{h}}\right)\right)^{\frac{1}{s}}>X(1)\ge 1-\left(\prod_{g=1}^{s}\left(1-\left(\frac{\sum_{i_j=1}^{h_g}u_{i_j}}{h_g}\right)\right)\right)^{\frac{1}{s}}=1-\left(\prod_{g=1}^{s}\left(1-\left(\frac{\sum_{i=1}^{h}u_{i_j}}{h}\right)\right)\right)^{\frac{1}{s}},$$

$$\Rightarrow\left(\prod_{i_j=1}^{h}u_{i_j}\right)^{\frac{1}{h}}>\left(\frac{\sum_{i_j=1}^{h}u_{i_j}}{h}\right).$$

However, from the above Lemma 1, we know that if $\alpha_i=u_{i_j}$, $\beta_i=\frac{1}{h}$, then we have $\prod_{i_j=1}^{h}u_{i_j}^{\frac{1}{h}}\le\left(\frac{1}{h}\sum_{i_j=1}^{h}u_{i_j}\right)$, which is the opposite of Lemma 1. Therefore, the function $X(k)$ decreases monotonically as the variable k increases. Similarly, as the variable k increases, the functions $Y(k)$ and $Z(k)$ monotonically increase. Based on the above analysis, we can obtain

$X(k) > X(k+1), Y(k) < Y(k+1), Z(k) < Z(k+1)$. Then, we can get $SVNPMSM^{(k)}(p_1, p_2, ..., p_n) > SVNPMSM^{(k+1)}(p_1, p_2, ..., p_n)$. Therefore, it is proved that the SVNPMSM operator will decrease monotonically as the variable k increases.

Then, several special examples of the SVNPMSM operator are briefly described.

(1) When the number of clusters $S = 1$, it means that no cluster is required among the criteria. The SVNPMSM operator will become the single-valued neutrosophic Maclaurin symmetric mean (SVNMSM) operator:

$$SVNPMSM_{s=1}^{(k)}(p_1, p_2, \ldots, p_n) = \left(\frac{\bigoplus_{1 \le i_1 < \cdots < i_k \le h_1} \left(\bigotimes_{j=1}^{k} p_{i_j} \right)}{C_{h_1}^k} \right)^{1/k} = \left(\frac{\bigoplus_{1 \le i_1 < \cdots < i_k \le n} \left(\bigotimes_{j=1}^{k} p_{i_j} \right)}{C_n^k} \right)^{1/k}.$$

(2) When the parameter $k = 1$, the SVNPMSM operator will become the single-valued neutrosophic partitioned mean (SVNPM) operator:

$$SVNPMSM^{(1)}(p_1, p_2, \ldots, p_n) = \frac{1}{S} \bigoplus_{g=1}^{s} \left(\frac{\bigoplus_{1 \le i_1 < \cdots < i_k \le h_g} \left(\bigotimes_{j=1}^{1} p_{i_j} \right)}{C_{h_g}^1} \right)^{1/1} = \frac{1}{S} \bigoplus_{g=1}^{s} \left(\frac{\bigoplus_{i_j=1}^{h_g} p_{i_j}}{h_g} \right).$$

(3) When the parameter $k = 2$, the SVNPMSM operator will become the single-valued neutrosophic PBM (SVNPBM) operator ($p = q = 1$):

$$SVNPMSM^{(2)}(p_1, p_2, \ldots, p_n) = \frac{1}{S} \bigoplus_{g=1}^{s} \left(\frac{\bigoplus_{1 \le i_1 < \cdots < i_k \le h_g} \left(\bigotimes_{j=1}^{2} p_{i_j} \right)}{C_{h_g}^2} \right)^{1/2} = \frac{1}{S} \bigoplus_{g=1}^{s} \left(\frac{\bigoplus_{\substack{i_j, i_l = 1 \\ i_j \ne i_l}}^{h_g} (p_{i_j} \otimes p_{i_l})}{h_g (h_g - 1)} \right)^{1/2}.$$

4.2. The SVNWPMSM operator

Definition 8. Suppose $p_i = \langle u_i, \tau_i, v_i \rangle$ $(i = 1, 2, ..., n)$ is a set of SVNNs, which can be divided into S clusters. k is a parameter, and $k = 1, 2, ..., h_g$, h_g represents the number of criteria in the cluster P_g. ω_i denotes the weight of p_i that satisfies the condition $0 \le \omega_i \le 1$ and $\sum_{i=1}^{n} \omega_i = 1$. Then the SVNWPMSM (single-valued neutrosophic weighted partitioned Maclaurin symmetric mean) operator is given by:

$$SVNWPMSM^{(k)}(p_1, p_2, \ldots, p_n) = \frac{1}{S} \bigoplus_{g=1}^{s} \left(\frac{\bigoplus_{1 \le i_1 < \cdots < i_k \le h_g} \left(\bigotimes_{j=1}^{k} \omega_{i_j} p_{i_j} \right)}{C_{h_g}^k} \right)^{1/k}, \tag{14}$$

where $i_1, i_2, ..., i_k$ is the set of k integers extracted from the set $\{1, 2, ..., h_g\}$, and

$1 \le i_1 < i_2 < \cdots < i_k \le h_g$. $C_{h_g}^k$ denotes the coefficients of the binomial and $C_{h_g}^k = \frac{h_g!}{k!(h_g-k)!}$.

Theorem 6. Suppose $p_i = \langle u_i, \tau_i, v_i \rangle$ $(i = 1, 2, ..., n)$ is a set of SVNNs, which can be divided into S different clusters. k is the parameter, and $k = 1, 2, ..., h_g$, h_g represents the number of criteria in the cluster P_g. ω_i is the weight of p_i satisfying the following constraints $0 \le \omega_i \le 1$ and $\sum_{i=1}^{n} \omega_i = 1$. Then, the result of the SVNWPMSM operator is still an SVNN that is given by:

$$SVNWPMSM^{(k)}(p_1, p_2, \ldots, p_n) = \left\langle 1 - \left(\prod_{g=1}^{s} \left(1 - \left(1 - \left(\prod_{1 \le i_1 < \cdots < i_k \le h_g} \left(1 - \prod_{j=1}^{k} \left(1 - \left(1 - u_{i_j} \right)^{\omega_{i_j}} \right) \right) \right)^{\frac{1}{C_{h_g}^k}} \right)^{\frac{1}{k}} \right) \right)^{\frac{1}{s}}, \right.$$

$$\left. \left(\prod_{g=1}^{s} \left(1 - \left(1 - \left(\prod_{1 \le i_1 < \cdots < i_k \le h_g} \left(1 - \prod_{j=1}^{k} \left(1 - \tau_{i_j}^{\omega_{i_j}} \right) \right) \right)^{\frac{1}{C_{h_g}^k}} \right)^{\frac{1}{k}} \right) \right)^{\frac{1}{s}}, \left(\prod_{g=1}^{s} \left(1 - \left(1 - \left(\prod_{1 \le i_1 < \cdots < i_k \le h_g} \left(1 - \prod_{j=1}^{k} \left(1 - v_{i_j}^{\omega_{i_j}} \right) \right) \right)^{\frac{1}{C_{h_g}^k}} \right)^{\frac{1}{k}} \right) \right)^{\frac{1}{s}} \right\rangle. \tag{15}$$

Theorem 7. Let $p_1, p_2, \ldots, p_n$ be a set of SVNNs, and $k = 1, 2, ..., \min_g h_g$. As the parameter k decreases, the SVNWPMSM operator monotonically increases.

Similarly, the SVNWPMSM operator also has the characteristics of idempotency, boundedness, and monotonicity. In the following part, we will present some typical examples of the SVNWPMSM operator.

(1) When the number of clusters $S = 1$, it indicates that there are no clusters among criteria. The proposed SVNWPMSM operator will change to the single-valued neutrosophic weighted Maclaurin symmetric mean (SVNWMSM) operator:

$$SVNWPMSM_{s=1}^{(k)}(p_1, p_2, \ldots, p_n) = \left(\frac{\bigoplus_{1 \le i_1 < \cdots < i_k \le h_1} \left(\bigotimes_{j=1}^{k} \omega_{i_j} p_{i_j} \right)}{C_{h_1}^k} \right)^{1/k} = \left(\frac{\bigoplus_{1 \le i_1 < \cdots < i_k \le n} \left(\bigotimes_{j=1}^{k} \omega_{i_j} p_{i_j} \right)}{C_n^k} \right)^{1/k}.$$

(2) When the parameter $k = 1$, the SVNWPMSM operator will become the single-valued neutrosophic weighted partitioned mean (SVNWPM) operator:

$$SVNWPMSM^{(1)}(p_1, p_2, \ldots, p_n) = \frac{1}{S} \bigoplus_{g=1}^{s} \left(\frac{\bigoplus_{1 \le i_1 < \cdots < i_k \le h_g} \left(\bigotimes_{j=1}^{1} \omega_{i_j} p_{i_j} \right)}{C_{h_g}^1} \right)^{1/1} = \frac{1}{S} \bigoplus_{g=1}^{s} \left(\frac{\bigoplus_{i_j=1}^{h_g} \omega_{i_j} p_{i_j}}{h_g} \right).$$

(3) When the parameter $k = 2$, the SVNWPMSM operator will become the single-valued neutrosophic weighted PBM (SVNWPBM) operator:

$$SVNWPMSM^{(2)}\left(p_1, p_2, \ldots, p_n\right) = \frac{1}{S}\overset{s}{\underset{g=1}{\oplus}}\left(\frac{\underset{1\le i_1<\cdots<i_k\le h_g}{\oplus}\left(\overset{2}{\underset{j=1}{\otimes}}\omega_{i_j} p_{i_j}\right)}{C_{h_g}^2}\right)^{1/2} = \frac{1}{S}\overset{s}{\underset{g=1}{\oplus}}\left(\frac{\overset{h_g}{\underset{\substack{i_j,i_l=1\\ i_j\ne i_l}}{\oplus}}(\omega_{i_j} p_{i_j} \otimes \omega_{i_l} p_{i_l})}{h_g(h_g-1)}\right)^{1/2}.$$

5. A novel MCGDM model using the proposed SVNWPMSM operator

The proposed SVNWPMSM operator can efficiently handle the complex relationship among criteria, so we apply it to solve the MCGDM problem. Let us suppose that $\delta = \{\delta_1, \delta_2, ..., \delta_m\}$ is a set of alternatives and $\psi = \{\psi_1, \psi_2, ..., \psi_n\}$ is a group of criteria. $\omega = \left[\omega_1, \omega_2, ..., \omega_n\right]^T$ denotes the weight vector of the criteria, which satisfies the following constraints $0 \le \omega_j \le 1\ (j = 1, 2, ..., n)$ and $\sum_{j=1}^{n}\omega_j = 1$. The evaluation of alternatives is performed by a group of decision-makers $D = \{D_1, D_2, ..., D_e\}$. The weight vector of each decision-maker $D_l\,(l = 1, 2, ..., e)$ is $\gamma = \left[\gamma_1, \gamma_2, ..., \gamma_e\right]^T$, which satisfies the constraint $0 \le \gamma_l \le 1\ (l = 1, 2, ..., e)$ and $\sum_{l=1}^{e}\gamma_l = 1$. The decision-maker D_l evaluates the alternative δ_i according to the criteria ψ_j and generates the evaluation matrix X^l of SVNNs, where $X^l = [\alpha_{ij}^l]_{m\times n} = \left\langle u_{ij}^l, \tau_{ij}^l, v_{ij}^l \right\rangle_{m\times n}$. The criteria ψ_j can be divided into S clusters $P_1, P_2, ..., P_s$ according to certain rules, and it satisfies the constraints $P_r \cap P_g = \varnothing$ and $\cup_{r=1}^{s} P_r = \psi$.

We developed a novel MCGDM model using the SVNWPMSM operator to fuse the evaluation matrices. The detailed steps of the model are shown in the following part.

Step 1: Standardization of the evaluation information.

In the MCGDM problem, the criteria may be either benefit type or cost type. For subsequent processing, the data of cost type needs to be converted to be benefit type. Assume that $Y^l = [\beta_{ij}^l]_{m\times n} = \left\langle T_{ij}^l, I_{ij}^l, F_{ij}^l \right\rangle_{m\times n}$ is a standardized evaluation matrix. The evaluation matrix is normalized in the following way.

$$\beta_{ij}^l = \left\langle T_{ij}^l, I_{ij}^l, F_{ij}^l \right\rangle = \begin{cases} \left\langle u_{ij}^l, \tau_{ij}^l, v_{ij}^l \right\rangle & \textit{for} \text{ the benefit criterion of } \psi_j \\ \left\langle v_{ij}^l, 1-\tau_{ij}^l, u_{ij}^l \right\rangle & \textit{for} \text{ the cost criterion of } \psi_j \end{cases}, \tag{16}$$

where $1 \le i \le m, 1 \le j \le n, 1 \le l \le e$.

Step 2: Calculate the aggregation values of criteria of alternatives.

According to Theorem 6, the aggregated values are obtained by calculating the evaluation vector β_i^l for each alternative δ_i, where $\beta_i^l = \left\langle T_i^l, I_i^l, F_i^l \right\rangle = SVNWPMSM^{(k)}\left(\beta_{i1}^l, \beta_{i2}^l, \ldots, \beta_{in}^l\right)$.

$$SVNWPMSM^{(k)}\left(\beta_{i1}^{l},\beta_{i2}^{l},\ldots,\beta_{in}^{l}\right)=\left\langle 1-\left(\prod_{g=1}^{s}\left(1-\left(1-\left(\prod_{1\le i_1<\cdots<i_r\le h_g}\left(1-\prod_{\phi=1}^{k}\left(1-\left(1-T_{i_{j_\phi}}^{l}\right)^{\omega_{j_\phi}}\right)\right)\right)^{\frac{1}{C_{h_g}^{k}}}\right)^{\frac{1}{k}}\right)\right)^{\frac{1}{s}},\right.$$

$$\left.\left(\prod_{g=1}^{s}\left(1-\left(1-\left(\prod_{1\le i_1<\cdots<i_r\le h_g}\left(1-\prod_{\phi=1}^{k}\left(1-(I_{i_{j_\phi}}^{l})^{\omega_{j_\phi}}\right)\right)\right)^{\frac{1}{C_{h_g}^{k}}}\right)^{\frac{1}{k}}\right)\right)^{\frac{1}{s}},\left(\prod_{g=1}^{s}\left(1-\left(1-\left(\prod_{1\le i_1<\cdots<i_r\le h_g}\left(1-\prod_{\phi=1}^{k}\left(1-(F_{i_{j_\phi}}^{l})^{\omega_{j_\phi}}\right)\right)\right)^{\frac{1}{C_{h_g}^{k}}}\right)^{\frac{1}{k}}\right)\right)^{\frac{1}{s}}\right\rangle. \tag{17}$$

where S denotes the number of clusters, h_g is the number of criteria in the cluster P_g. k is a parameter, ω_{q_ϕ} is the weight of the criterion $\psi_{j_\phi}\,(j_\phi=1,2,...,h_g)$, and $C_{h_g}^{k}=\dfrac{h_g!}{k!\left(h_g-k\right)!}$ is the binomial coefficient.

Step 3: Calculate the aggregation value of the evaluation information of decision-makers.

According to Theorem 6, the aggregated values are obtained by calculating the overall evaluation value β_i for alternative δ_i, where $\beta_i=\langle T_i,I_i,F_i\rangle=SVNWPMSM^{(k)}\left(\beta_i^1,\beta_i^2,\ldots,\beta_i^e\right)$:

$$SVNWPMSM^{(k)}\left(\beta_{i}^{1},\beta_{i}^{2},\ldots,\beta_{i}^{e}\right)=\left\langle 1-\left(\prod_{g=1}^{d}\left(1-\left(1-\left(\prod_{1\le i_1<\cdots<i_r\le h_g}\left(1-\prod_{\varphi=1}^{k}\left(1-\left(1-T_{i}^{l_\varphi}\right)^{\gamma_{l_\varphi}}\right)\right)\right)^{\frac{1}{C_{h_g}^{k}}}\right)^{\frac{1}{k}}\right)\right)^{\frac{1}{d}},\right.$$

$$\left.\left(\prod_{g=1}^{d}\left(1-\left(1-\left(\prod_{1\le i_1<\cdots<i_r\le h_g}\left(1-\prod_{\varphi=1}^{k}\left(1-(I_{i}^{l_\varphi})^{\gamma_{l_\varphi}}\right)\right)\right)^{\frac{1}{C_{h_g}^{k}}}\right)^{\frac{1}{k}}\right)\right)^{\frac{1}{d}},\left(\prod_{g=1}^{s}\left(1-\left(1-\left(\prod_{1\le i_1<\cdots<i_r\le h_g}\left(1-\prod_{\varphi=1}^{k}\left(1-(F_{i}^{l_\varphi})^{\gamma_{l_\varphi}}\right)\right)\right)^{\frac{1}{C_{h_g}^{k}}}\right)^{\frac{1}{k}}\right)\right)^{\frac{1}{d}}\right\rangle. \tag{18}$$

where d represents the number of clusters, and decision-makers usually do not need to be partitioned, i.e., $d=1$. h_g denotes the number of decision-makers in the cluster P_g. k is a parameter, and γ_{l_φ} denotes the weight of D_{l_φ}.

Step 4: Choose the optimal alternative.

In order to obtain the best alternative, the score function $S(\beta_i)$ of β_i $(1\le i\le m)$ needs to be calculated. Then, the score functions $S(\beta_1)$, $S(\beta_2)$, …, $S(\beta_m)$ are compared according to Definition 4, and the optimal alternative is selected.

6. Case study

In this section, the developed MCGDM model is applied to solve the medical waste treatment schemes selection problem.

6.1. Illustrate example

Medical waste treatment schemes selection is a critical problem in the field of environmental protection. This problem is a crisis looming, especially after the outbreak of the COVID-19. To validate the effectiveness of the proposed MCGDM model, we illustrate an example from Fuzhou city, China. Fuzhou is located in the eastern part of China, with a population of more than 8 million. About 2 tons of medical wastes were produced in Fuzhou per day. Currently, medical wastes are mainly treated by incineration. Incineration has caused many problems for people living near the medical waste treatment stations. Therefore, the authorities are considering new solutions for the treatment of medical wastes.

This study provides the authorities with four available schemes: Depositing in landfills (δ_1), Gasification (δ_2), Autoclaving (δ_3), and Microwaving (δ_4). To evaluate the above medical waste treatment schemes, we define seven main sustainability criteria: emission (ψ_1), investment cost (ψ_2), energy recovery (ψ_3), operation cost (ψ_4), efficiency in waste reduction (ψ_5), technology accessibility (ψ_6), and employment potential (ψ_7). Then, we consulted with the MWMB (Medical Waste Management Board) and determined the weight vector of the above criteria as $\omega = (0.1, 0.2, 0.2, 0.1, 0.1, 0.2, 0.1)^T$. The criteria emission, energy recovery, efficiency in waste reduction, and technology accessibility describe the technical aspects of the medical waste treatment schemes. The criteria investment cost, operation cost, and employment potential refer to the economic aspects of the medical waste treatment schemes. Thus, we divided the criteria into two clusters: $P_1 = \{\psi_1, \psi_3, \psi_5, \psi_6\}$ and $P_2 = \{\psi_2, \psi_4, \psi_7\}$. To ensure the rationality of the medical waste treatment schemes selection result, we invited three experts from different fields. The invited experts have rich experience in medical waste treatment. According to the experts from different fields, the weight vector of the expert $D_l (l = 1, 2, 3)$ is determined as $\gamma = (0.4, 0.3, 0.3)^T$. Each expert D_l evaluated the medical waste treatment schemes and gave the evaluation matrix $\chi^l = \left[\alpha_{ij}^l\right]_{4\times 7} (l = 1, 2, 3)$, as shown in Tables 1–3.

Table 1. The evaluation matrix χ^1 supplied by D_1.

	ψ_1	ψ_2	ψ_3	ψ_4	ψ_5	ψ_6	ψ_7
δ_1	<0.9,0.2,0.2>	<0.6,0.3,0.2>	<0.5,0.4,0.1>	<0.6,0.4,0.1>	<0.5,0.2,0.2>	<0.9,0.2,0.1>	<0.6,0.4,0.2>
δ_2	<0.8,0.3,0.2>	<0.7,0.3,0.2>	<0.7,0.2,0.2>	<0.5,0.4,0.1>	<0.6,0.4,0.3>	<0.5,0.3,0.2>	<0.8,0.3,0.2>
δ_3	<0.6,0.4,0.1>	<0.8,0.4,0.2>	<0.6,0.3,0.1>	<0.7,0.3,0.2>	<0.7,0.2,0.2>	<0.6,0.4,0.2>	<0.6,0.4,0.3>
δ_4	<0.7,0.3,0.3>	<0.9,0.1,0.1>	<0.4,0.1,0.2>	<0.8,0.2,0.1>	<0.5,0.3,0.2>	<0.7,0.4,0.1>	<0.5,0.2,0.2>

Table 2. The evaluation matrix χ^2 supplied by D_2.

	ψ_1	ψ_2	ψ_3	ψ_4	ψ_5	ψ_6	ψ_7
δ_1	<0.8,0.2,0.2>	<0.6,0.3,0.2>	<0.5,0.3,0.1>	<0.7,0.2,0.3>	<0.4,0.5,0.2>	<0.7,0.3,0.2>	<0.8,0.2,0.3>
δ_2	<0.7,0.4,0.3>	<0.7,0.2,0.1>	<0.6,0.4,0.1>	<0.8,0.3,0.1>	<0.5,0.3,0.3>	<0.6,0.4,0.3>	<0.7,0.2,0.2>
δ_3	<0.5,0.3,0.3>	<0.6,0.5,0.3>	<0.7,0.4,0.1>	<0.4,0.6,0.1>	<0.8,0.4,0.1>	<0.7,0.4,0.2>	<0.5,0.4,0.3>
δ_4	<0.6,0.4,0.1>	<0.9,0.2,0.2>	<0.8,0.3,0.1>	<0.7,0.3,0.1>	<0.9,0.2,0.2>	<0.5,0.5,0.1>	<0.6,0.3,0.1>

Table 3. The evaluation matrix χ^3 supplied by D_3.

	ψ_1	ψ_2	ψ_3	ψ_4	ψ_5	ψ_6	ψ_7
δ_1	<0.7,0.3,0.3>	<0.6,0.2,0.1>	<0.5,0.4,0.3>	<0.6,0.1,0.1>	<0.5,0.5,0.3>	<0.3,0.6,0.3>	<0.9,0.2,0.1>
δ_2	<0.6,0.2,0.2>	<0.7,0.3,0.1>	<0.6,0.2,0.1>	<0.6,0.2,0.3>	<0.5,0.2,0.1>	<0.8,0.3,0.2>	<0.6,0.3,0.2>
δ_3	<0.4,0.5,0.3>	<0.5,0.4,0.2>	<0.6,0.3,0.2>	<0.4,0.3,0.2>	<0.6,0.4,0.1>	<0.6,0.5,0.1>	<0.5,0.5,0.3>
δ_4	<0.8,0.3,0.2>	<0.6,0.3,0.2>	<0.5,0.5,0.2>	<0.7,0.2,0.3>	<0.8,0.2,0.2>	<0.5,0.2,0.2>	<0.5,0.3,0.1>

6.2. Steps to address the selection problem of medical waste treatment schemes

Step 1: Standardization of the evaluation information.

According to the description of the criteria $\psi_j (j = 1, 2, ..., 7)$, all of them are benefit types. Thus, there is no need to normalize the evaluation matrix. Then, we can get that

$$\beta_{ij}^l = \left\langle T_{ij}^l, I_{ij}^l, F_{ij}^l \right\rangle = \alpha_{ij}^l = \left\langle u_{ij}^l, \tau_{ij}^l, v_{ij}^l \right\rangle.$$

Step 2: Calculate the aggregation values of the criteria of medical waste treatment schemes.

We can use Eq (17) to obtain the aggregation values β_i^l of the criteria of medical waste treatment scheme δ_i, where $1 \le i \le 4,\ 1 \le l \le 3$. Since the criteria are divided into two clusters, the number of clusters $S = 2$. Assume that the parameter $k = 2$, then the aggregation values of the criteria of medical waste treatment schemes are obtained as follows:

$$\beta_1^1 = (0.1474, 0.8432, 0.7720), \beta_1^2 = (0.1368, 0.8388, 0.8091), \beta_1^3 = (0.1220, 0.8402, 0.7907);$$
$$\beta_2^1 = (0.1347, 0.8550, 0.7978), \beta_2^2 = (0.1411, 0.8474, 0.7909), \beta_2^3 = (0.1294, 0.8233, 0.7824);$$
$$\beta_3^1 = (0.1407, 0.8606, 0.7932), \beta_3^2 = (0.1231, 0.8917, 0.7919), \beta_3^3 = (0.0956, 0.8842, 0.7921);$$
$$\beta_4^1 = (0.1478, 0.8096, 0.7740), \beta_4^2 = (0.1686, 0.8312, 0.7530), \beta_4^3 = (0.1217, 0.8331, 0.8012).$$

Step 3: Calculate the aggregation value of the evaluation information of decision-makers.

We apply Eq (18) to calculate the aggregation values β_i of the evaluation information of decision-makers. The number of clusters $d=1$ since the decision-makers usually do not need to be clustered. Assume the parameter $k=2$, the aggregation values of the evaluation information of decision-makers are obtained as follows:

$$\beta_1=(0.0472,0.9442,0.9249),\beta_2=(0.0470,0.9448,0.9251),$$
$$\beta_3=(0.0415,0.9579,0.9258),\beta_4=(0.0521,0.9379,0.9195).$$

Step 4: Choose the optimal medical waste treatment scheme.

The score values $S(\beta_i)$ of the aggregation values β_i are obtained as follows:

$$S(\beta_1)=0.0594,S(\beta_2)=0.0590,S(\beta_3)=0.0526,S(\beta_4)=0.0649\,.$$

Then, we compare the above score values $S(\beta_i)$, where $1\le i\le 4$. Since $S(\beta_4)>S(\beta_1)>S(\beta_2)>S(\beta_3)$, the medical waste treatment schemes can be ranked as $\delta_4\succ\delta_1\succ\delta_2\succ\delta_3$, where the symbol "$\succ$" indicates "preferred to". Therefore, the optimal medical waste treatment scheme is Microwaving (δ_4).

6.3. *The effects of variable S and parameter K on the ranking results*

According to the above example, we will discuss the influence of the cluster number S and the parameter k on the ranking results. Different values of the cluster number S may lead to changes in the morphology of the proposed operator. For example, when $S=1$, the proposed SVNWPMSM operator will change to the SVNWMSM operator. Moreover, it is known from Theorem 7 that the SVNWPMSM operator monotonically increases as the parameter k decreases. We assign different values to the cluster number S and parameter k in the above example. The effects of S and k on the ranking results of medical waste treatment schemes are discussed. Table 4 shows the ranking results obtained by the proposed SVNWPMSM operator when the cluster number S and the parameter k take different values.

Table 4. The ranking results for different S and k.

S and k	Score values	Ranking results
$S=1,k=1$	$S(\beta_1)=0.0630,S(\beta_2)=0.0614,S(\beta_3)=0.0573,S(\beta_4)=0.0692$	$\delta_4\succ\delta_1\succ\delta_2\succ\delta_3$
$S=1,k=2$	$S(\beta_1)=0.0599,S(\beta_2)=0.0598,S(\beta_3)=0.0546,S(\beta_4)=0.0663$	$\delta_4\succ\delta_1\succ\delta_2\succ\delta_3$
$S=2,k=1$	$S(\beta_1)=0.0630,S(\beta_2)=0.0615,S(\beta_3)=0.0562,S(\beta_4)=0.0692$	$\delta_4\succ\delta_1\succ\delta_2\succ\delta_3$
$S=2,k=2$	$S(\beta_1)=0.0594,S(\beta_2)=0.0590,S(\beta_3)=0.0526,S(\beta_4)=0.0649$	$\delta_4\succ\delta_1\succ\delta_2\succ\delta_3$
$S=2,k=3$	$S(\beta_1)=0.0573,S(\beta_2)=0.0576,S(\beta_3)=0.0503,S(\beta_4)=0.0620$	$\delta_4\succ\delta_2\succ\delta_1\succ\delta_3$

From Table 4, we can get the following information:

(1) When the cluster number $S=1$ or $S=2$, the score value $S(\beta_i)$ decreases monotonically as the value of the parameter k increases.

(2) When the parameter $k=1$ or $k=2$, the ranking result of the medical waste treatment schemes is $\delta_4 \succ \delta_1 \succ \delta_2 \succ \delta_3$ and the optimal scheme is δ_4.

(3) When the parameter $k=3$, the ranking result is $\delta_4 \succ \delta_2 \succ \delta_1 \succ \delta_3$ and the optimal scheme is δ_4.

Obviously, when the cluster number S or the parameter k is changed, the ranking result of the medical waste treatment schemes may change, but the optimal scheme keeps unchanged, i.e., δ_4. As shown in Table 4, the change of the value of the parameter k may lead to the change of the ranking results of medical waste treatment schemes. The parameter k is a reflection of the expert's risk preference. The reference value of k is $\eta = 0.5\left[\min_g h_g\right]$, where the symbol [] indicates rounding off the data in the symbol and h_g is the number of criteria in the cluster P_g. If k is greater than η, it means that the decision-maker is aggressive. If k is less than η, it means that the decision-maker is conservative. If k is equal to η, it means that the decision-maker is neutral.

7. Verification and comparative analysis

In this section, the validity, reliability, and other advantages of the proposed MCGDM model based on the SVNWPMSM operator are verified. For comparison, we apply the previous methods, such as Peng et al.'s [40] method and Wang et al.'s [33] method, to solve the medical waste scheme selection problem in Fuzhou. Table 5 shows the ranking results of the medical waste treatment schemes. We compare the ranking results obtained by the previous methods with the ranking results obtained by the proposed MCGDM model.

Table 5. Ranking results obtained by different methods

Operators	Score values	Sorting results
SVNWA [40]	$S(\beta_1)=0.7343, S(\beta_2)=0.7255, S(\beta_3)=0.6898, S(\beta_4)=0.7686$	$\delta_4 \succ \delta_1 \succ \delta_2 \succ \delta_3$
SVNWG [40]	$S(\beta_1)=0.6924, S(\beta_2)=0.7064, S(\beta_3)=0.6712, S(\beta_4)=0.7395$	$\delta_4 \succ \delta_2 \succ \delta_1 \succ \delta_3$
LNWBM [33]	$S(\beta_1)=0.8047, S(\beta_2)=0.8042, S(\beta_3)=0.7620, S(\beta_4)=0.8423$	$\delta_4 \succ \delta_1 \succ \delta_2 \succ \delta_3$
LNWPBM [33]	$S(\beta_1)=0.8096, S(\beta_2)=0.8089, S(\beta_3)=0.7552, S(\beta_4)=0.8453$	$\delta_4 \succ \delta_1 \succ \delta_2 \succ \delta_3$
SVNWPMSM (S=1 and k=1)	$S(\beta_1)=0.0630, S(\beta_2)=0.0614, S(\beta_3)=0.0573, S(\beta_4)=0.0692$	$\delta_4 \succ \delta_1 \succ \delta_2 \succ \delta_3$
SVNWPMSM (S=1 and k=2)	$S(\beta_1)=0.0599, S(\beta_2)=0.0598, S(\beta_3)=0.0546, S(\beta_4)=0.0663$	$\delta_4 \succ \delta_1 \succ \delta_2 \succ \delta_3$
SVNWPMSM (S=1 and k=3)	$S(\beta_1)=0.0582, S(\beta_2)=0.0589, S(\beta_3)=0.0530, S(\beta_4)=0.0647$	$\delta_4 \succ \delta_2 \succ \delta_1 \succ \delta_3$
SVNWPMSM (S=2 and k=1)	$S(\beta_1)=0.0630, S(\beta_2)=0.0615, S(\beta_3)=0.0562, S(\beta_4)=0.0692$	$\delta_4 \succ \delta_1 \succ \delta_2 \succ \delta_3$
SVNWPMSM (S=2 and k=2)	$S(\beta_1)=0.0594, S(\beta_2)=0.0590, S(\beta_3)=0.0526, S(\beta_4)=0.0649$	$\delta_4 \succ \delta_1 \succ \delta_2 \succ \delta_3$
SVNWPMSM (S=2 and k=3)	$S(\beta_1)=0.0573, S(\beta_2)=0.0576, S(\beta_3)=0.0503, S(\beta_4)=0.0620$	$\delta_4 \succ \delta_2 \succ \delta_1 \succ \delta_3$

(1) Comparative analysis of the proposed MCGDM model with Peng et al.'s [40] method

Peng et al.'s method [40] uses the single-valued neutrosophic weighted averaging (SVNWA) operator and the single-valued neutrosophic weighted geometric (SVNWG) operator to aggregate the evaluation information. The ranking results obtained by Peng et al.'s method [40] and the proposed MCGDM model are shown in Table 5. The ranking results of SVNWA tend to the medical waste treatment scheme that owns the largest evaluation information value. In contrast, the ranking results of SVNWG tend to the medical waste treatment scheme with the smallest weight value of the evaluation information. Therefore, their ranking results are different. The ranking results from Peng et al.'s method using the SVNWA operator are the same as those from the proposed MCGDM model (when $k=1$ or $k=2$), i.e., $\delta_4 \succ \delta_1 \succ \delta_2 \succ \delta_3$. The ranking results of Peng et al.'s method using the SVNWG operator are the same as those of the proposed MCGDM model (when $k=3$), i.e., $\delta_4 \succ \delta_2 \succ \delta_1 \succ \delta_3$. The best and worst medical waste treatment schemes of Peng et al.'s method [40] are the same as those of the proposed MCGDM model. This proves the practicality and effectiveness of the proposed model.

(2) Comparative analysis of the proposed MCGDM model with Wang et al.'s method [33]

Wang et al.'s method [33] uses the linguistic neutrosophic weighted Bonferroni mean (LNWBM) operator and linguistic neutrosophic weighted partitioned Bonferroni mean (LNWPBM) operator to process the evaluation information. From Table 5, we can see that the ranking results of Wang et al.'s method [33] using LNWBM operator and LNWPBM operators are the same as those of the proposed MCGDM model (when $k=1$ or $k=2$), i.e., $\delta_4 \succ \delta_1 \succ \delta_2 \succ \delta_3$. While the ranking results of Wang et al.'s method using LNWBM and LNWPBM operators are different from those of the proposed MCGDM model (when $k=3$). Then, we analyze the above situation in detail. Both the LNWBM operator and LNWPBM operator can capture the interrelationship between two criteria. The LNWPBM operator can handle the situation where the criteria need to be divided into different clusters, while the LNWBM operator cannot handle the above situation. The proposed MCGDM method can deal with the case where the criteria in the same cluster are closely related and the criteria in different clusters have no relationships. When $k=1$ or $k=2$, the ranking results obtained by Wang et al.'s method and the proposed MCGDM model are the same. It proves the practicality of the proposed MCGDM model. In addition, when $k=3$, the ranking results obtained by Wang et al.'s approach [33] and the proposed model are different. The reason is that Wang et al.'s method can only capture the correlation relation between two criteria, but cannot handle the case where there are interrelationships among criteria in the same cluster. For the above illustrate example, four criteria $\{\psi_1, \psi_3, \psi_5, \psi_6\}$ are closely related and the other three criteria $\{\psi_2, \psi_4, \psi_7\}$ are closely related. Wang et al.'s method [33] can only consider the correlation relation between any two criteria in these two clusters, while the proposed MCGDM model can capture the interrelationships among all the criteria in each cluster. Therefore, the proposed MCGDM model shows better performance in terms of ranking results.

8. Conclusions

During the outbreak of COVID-19, it is a big challenge to dispose of a large number of medical wastes. This paper proposes a novel MCGDM model to solve the medical waste treatment schemes selection problem to improve medical waste treatment efficiency. The proposed MCGDM model is composed of three main phases. In the first phase, the SVNNs are used to represent the evaluation information provided by the experts. In the second phase, we extend the PMSM operator to process

SVNNs, and propose the SVNWPMSM operator. In the third phase, we propose a novel MCGDM model using the SVNWPMSM operator. Then we apply the proposed MCGDM model to solve the medical waste treatment schemes selection problem in Fuzhou city, China. The reliability and superiority of the proposed MCGDM model are verified by comparing it with the previous methods.

The proposed MCGDM model can instruct the government management department to choose the best treatment scheme. In addition, the proposed MCGDM model can also be applied to solve the decision-making problems in other fields, such as site selection, energy management, and production evaluation.

Data availability

The data used to support the findings of this study are included within the article.

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Conflict of interest

The authors declare that there is no conflict of interest regarding the publication of this paper.

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