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Research article

Adaptive NN control based on Butterworth low-pass filter for quarter active suspension systems with actuator failure

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Abstract: This paper focuses on the adaptive neural network (NN) control problem for nonlinear quarter active suspension systems with actuator failure. By using Butterworth low-pass filter (LPF), the second order active suspension system is converted to a fourth order system, which solves the problem of zero dynamics analysis in the second order system. Based on the adaptive backstepping technique, considering the actuator fault of vehicle, the corresponding fault tolerant controller is designed. At the same time, the unknown smooth functions are estimated by the NN. It is proved by stability analysis that all states in active suspension system are bounded. Finally, a simulation example is given to verify the effectiveness of the proposed method in a quarter active suspension system.

Keywords: adaptive control; fault tolerance; vehicle suspension systems; Butterworth low-pass filter; NN control
Mathematics Subject Classification: 93B52, 93C95, 93D05

1. Introduction

In the last twenty years, with the rapid development of industry, automobile has become a necessary tool for people's daily travel. The vehicle can run smoothly on the road, which is achieved by the cooperation of various subsystems of the vehicle. The suspension system, as one of the main subsystems of the vehicle, plays an indispensable role. Vehicle suspension system is mainly divided into passive suspension system, semi-active suspension system and active suspension system (ASS). Today, many vehicles are equipped with only passive or semi-active suspension [1–4]. However, their spring stiffness and damping are not adjustable, according to the random vibration theory, it can only guarantee to achieve results in specific road conditions. Compared with them, the active suspension system has an actuator which can adjust the force, and greatly increases the controllability of the vehicle in the driving process. Therefore, the control problem of the active suspension systems [5] has been widely concerned by lots of scholars. Many scholars have proposed excellent control algorithms

[6–8] for active suspension systems.

According to the established vehicle model, the active suspension control research is divided into three categories: quarter vehicle model [9], half-car model [10] and full vehicle model [11]. There are many excellent control algorithms applied to active suspension systems, but in practice, many uncertain factors are ignored in nonlinear suspension systems. In order to deal with the uncertain factors, NN control methods [12–19] and fuzzy control methods [20–23] have been proposed in many articles. In addition, some intelligent decision-making [24] and nonlinear analysis methods [25] are proposed. Many systems also use NN control such as stochastic systems [26, 27], pure-feedback systems [28], and nonlinear delayed systems [29]. At the same time, NN control is also used in a lot of practical systems. At a result, ASS has become a hot issue in recent years, and many different control strategies have been reported. In [30] sliding mode control is developed for quarter active suspension systems. The finite-time stability is proposed in [31], and the problem of actuator failure in semi vehicle active suspension system has been raised in [32]. As the quarter ASS is most in line with the real situation, it is representative to study some control strategies such as [33, 34]. However, the general quarter ASS [35, 36] is a second-order system, which needs zero dynamics analysis. This may lead to the loss of information. Therefore, it is proposed in [37] by using Butterworth low-pass filter (LPF) to convert the non-strict feedback system to strict feedback system. Based on the Butterworth LPF, it makes the quarter ASS expressed from a second-order system to a fourth-order system.

Generally speaking, due to the influence of structural, environmental and other problems, many practical systems have different constraints. In view of this, many scholars have achieved a lot of works. For example, in [38], the robotic system with full-state constraints is considered. Constraint methods are proposed for switched systems in [39, 40]. In [41], the delayed systems with time-varying full-state constraints is considered. An effective finite-frequency fault detection method is proposed for descriptor system in [42]. An input constrained control method is proposed in [43]. In addition, many excellent constraint control methods have been applied to many practical systems. Although a large number of scholars have carried out a series of studies on ASS, there are also many difficult problems in the research on active suspension control. The problem of actuator failure can be not ignored. Actuator failure is equally important for vehicle stability.

In the actual production and life, the actuator failure of machines cannot be ignored. of course, the actuator failure of active suspension system is quite important as well. This kind of fault may lead to the instability of the control systems and even lead to catastrophic accidents. Therefore, it is necessary to study fault-tolerant control (FTC) strategy [44, 45]. In order to deal with the problem of actuator failure, some control strategies have been proposed [46–48]. FTC is also very important for ASS. In [49–51], some fault tolerant control strategies are proposed for quarter ASSs or half-car ASSs. Meanwhile, the corresponding control strategy is proposed for the vehicle ASS in [52]. Through the intelligent FTC of the active suspension, the control reliability of the suspension system can be effectively improved. Therefore, it is very meaningful to propose an FTC strategy for a quarter ASS.

An adaptive NN control based on Butterworth low-pass filter for quarter active suspension systems with actuator failure is proposed in this paper. The main contributions of the proposed control are as follows.

1) Different from the previous works [35, 36], by using Butterworth LPF, the second-order quarter active suspension system is changed into a fourth-order system. Considering all states in the quarter suspension system is more general.

2) The actuator failure of quarter ASS is considered. An adaptive NN control is put forward in this paper, and the FTC solves the problem of actuator failure in the process of vehicle driving, so that the vehicle can quickly return to normal driving.

The rest of this paper is organized as follows: Section II introduces the quarter active suspension system based on LPF and the fault model used in this paper. An adaptive controller with actuator failure and stability analysis are considered in Section III. In Section IV, a simulation example is given to demonstrate the effectiveness of the proposed method, and the conclusion is given in Section V.

2. Problem analysis

The quarter active suspension system is a representative model in vehicle system, which is displayed in Figure 1. It is often used in the design and analysis of automotive auxiliary system. According to the dynamic characteristics of the active suspension system, the force analysis is carried out as

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = \dfrac{1}{m_s}(-F_s - F_d + u) \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = \dfrac{1}{m_u}(-F_s - F_d + F_t + F_b - u) \end{cases} \tag{2.1}$$

where the control input of the quarter active suspension system is u, the sprung mass and the unsprung mass are represented by m_s, m_u. z_s and z_w represent the displacement of the sprung and unsprung mass, z_r is the disturbance of the road surface, k_a, k_t, l_a and l_t stand for the stiffness coefficients and the damping coefficients.

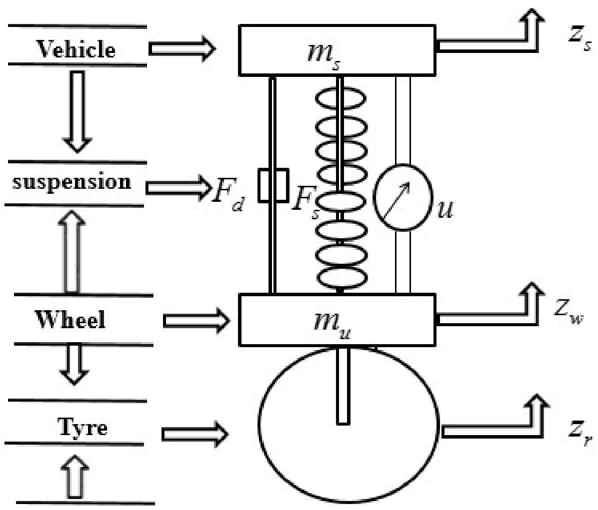

Figure 1. Quarter-car model.

According to the stress analysis, these forces are expressed as

$$\begin{aligned} F_s &= k_a (z_s - z_w), F_d = l_a (\dot{z}_s - \dot{z}_w) \\ F_t &= k_t (z_w - z_r), F_b = l_t (\dot{z}_w - \dot{z}_r) \end{aligned} \tag{2.2}$$

Define the following state variables $x_1 = z_s$, $x_2 = \dot{z}_s$, $x_3 = z_w$, $x_4 = \dot{z}_w$. It easy to know that the state space form of quarter car active suspension systems is

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = \dfrac{1}{m_s}(-F_s - F_d + u) \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = \dfrac{1}{m_u}(-F_s - F_d + F_t + F_b - u) \end{cases} \tag{2.3}$$

Remark 1: Previous works [37, 38] considered the equation of states with respect to x_1 and x_2, which may result in the information loss of the state variables x_3 and x_4. In this work, by using Butterworth low-pass filter, all states are considered when designing the controller.

Assumption 1: The desired tracking trajectory y_d and its time derivative $\dot{y}_d$ are assumed to be bounded with $|y_d| \leq A$.

Assumption 2: There is only one failure of the quarter active suspension system at the same time.

Assumption 3: Because the load of the vehicle is limited, the car-body mass is limited by $m_{s\min} \leq m_s \leq m_{s\max}$, where $m_{s\min}$ and $m_{s\max}$ are constants.

Assumption 4: [37] The control signal can be expressed as $u = u_f + \Delta u$, where Δu is an error and u_f is a filtered signal defined as:

$$u_f = H_L(s)\, u \approx u \tag{2.4}$$

where $H_L(s)$ is a Butterworth low-pass filter.

The corresponding filter parameters of Butterworth filters with the cutoff frequency are shown in Table 1, in which it holds that ω_c=1rad/s. for different values of n.

Table 1. Parameters of the Butterworth LPF.
$\left[H_L(s) = a_0/\left(s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0\right)\right]$

Filter order	a_4	a_3	a_2	a_1	a_0
2				1.141	1.000
3			2.000	2.000	1.000
4		2.613	3.141	2.613	1.000
5	3.236	5.236	5.236	3.236	1.000

In this way, it will not only lead to algebraic cycles, but also is unable to implement directly. In order to avoid the above problems, the filtered signal is used to avoid the algebraic loop problem effectively. According to the approach proposed in [37], it is worth noting that, due to the low pass characteristics of most actuators, the following replacement is reasonable.

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = \dfrac{1}{m_s}\left(-f_2 + u_f + \Delta u + x_3\right) \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = \dfrac{1}{m_u}\left(f_4 - u\right) \end{cases} \tag{2.5}$$

where $f_2 = F_s + F_d$ and $f_4 = -F_s - F_d + F_t + F_b$ are unknown functions.

In this paper, the actuator faults include deviation faults and ineffective faults, which are given as [53]

$$u = \rho u_c + \zeta, t \in [t_v, t_e] \tag{2.6}$$

where $0 \leq \rho \leq 1$ is the effectiveness factor of the fault model, ζ is an unknown constant and $\zeta_{\max} = v_m$. Let t_v and t_e be the time instants when the actuator takes place and ends. The actuator fault model includes the following situations:

1) when ρ=1 and ζ=0, it means that there is not any actuator fault.

2) when $0 < \underline{\rho} \leq \rho < \bar{\rho}$ and ζ=0, it shows partial actuator failure, where $0 < \underline{\rho} \leq 1$, and $0 < \bar{\rho} \leq 1$ are constant.

3) when ρ=0 and ζ=1, it implies that the signal u_c can no longer be influenced by the control inputs u.

Then, (2.5) becomes

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = \dfrac{1}{m_s}\left(-f_2 + \rho u_c + \Delta u + x_3 + \zeta\right) \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = \dfrac{1}{m_u}\left(f_4 - \rho u_c + \zeta\right) \end{cases} \tag{2.7}$$

In the next section, the actuator failure problem will be solved for the ASS.

3. Adaptive FTC controller design

3.1. Adaptive FTC controller design

The controller design is implemented by backstepping technologies. Define the errors variable as follows:

$$z_1 = x_1 - y_d$$
$$z_i = x_i - \alpha_{i-1}\,(i = 2, 3, 4)$$

where y_d is the expected trajectory of x_1 and α_{i-1} are the virtual controls which will be given in the later.

Step 1: From above definition, $\dot{z}_1$ is given as

$$\dot{z}_1 = \dot{x}_1 - \dot{y}_d \tag{3.1}$$

The Lyapunov function V_1 is selected as

$$V_1 = \frac{1}{2}z_1^2 \tag{3.2}$$

The time derivative of V_1 is became

$$\dot{V}_1 = z_1 (z_2 + \alpha_1 - \dot{y}_d) \tag{3.3}$$

The virtual control input $\alpha_1$is selected as

$$\alpha_1 = -c_1 z_1 + \dot{y}_d \tag{3.4}$$

where $c_1 > 0$ is a positive constant. The time derivative of V_1 is written as

$$\dot{V}_1 = -c_1 z_1^2 + z_1 z_2 \tag{3.5}$$

Step 2: The time derivative of z_2 as defined in above is

$$\begin{aligned} \dot{z}_2 &= \dot{x}_2 - \dot{\alpha}_1 \\ &= \frac{1}{m_s} (-f_2 + \Delta u + \rho u_c + \zeta + x_3) - \dot{\alpha}_1 \end{aligned} \tag{3.6}$$

It is easy to get

$$\begin{aligned} m_s z_2 \dot{z}_2 = &- z_2 f_2 + z_2 \Delta u + z_2 (\rho u_c + \zeta) \\ &+ z_2 z_3 + z_2 \alpha_2 - m_s z_2 \dot{\alpha}_1 \end{aligned} \tag{3.7}$$

In order to deal with the influence of unknown functions, the neural network is introduced to approximate it as

$$H(X) = w^T \phi(X) + \kappa \tag{3.8}$$

where $w = [w_1, w_2, \cdots w_n]^T \in R$ is the weight vector of NN, $\phi(X)$ is the basis functions, $X = [x_1, x_2, x_3, x_4]^T$ is the input of the basis functions, κ represents the approximation error and $\bar{\kappa}$ is a constant with $\kappa \leq \bar{\kappa}$.

In addition, define $H_2(X_2) = w_2^T \phi_2(X_2) + \kappa_2$, where $H_2(X_2)$ denotes an unknown function, given as

$$\begin{aligned} H_2(X_2) &= (\rho u_c + \zeta) - m_s z_2 \dot{\alpha}_1 \\ &= w_2^T \phi_2(X_2) + \kappa_2 \end{aligned} \tag{3.9}$$

The error weight vector is defined as $\tilde{w}_2 = w_2 - \hat{w}_2$, $\hat{w}_2$ represents the estimate of w_2.

The Lyapunov function V_2 is selected as

$$V_2 = V_1 + \frac{m_s}{2} z_2^2 + \frac{1}{2} \tilde{w}_2^T \Lambda_2^{-1} \tilde{w}_2 \tag{3.10}$$

where $\Lambda_i\,(i = 2, 3, 4)$ is a positive definite symmetric matrix. The time derivative of V_2 is regarded as

$$\dot{V}_2 = \dot{V}_1 + m_s z_2 \dot{z}_2 - \tilde{w}_2^T \Lambda_2^{-1} \dot{\hat{w}} \tag{3.11}$$

Substituting (3.7) and (3.9) into (3.11), one obtains

$$\dot{V}_2 = m_s z_2 \dot{z}_2 - \tilde{w}_2^T \Lambda_2^{-1} \dot{\hat{w}}_2 - c_1 z_1^2 + z_1 z_2$$

$$
\begin{aligned}
&= z_2 \left(\mathrm{w}_2^T \phi_2 (X_2) + \kappa_2 + \Delta u + z_3 + \alpha_2 - f_2 \right) \\
&- \tilde{w}_2^T \Lambda_2^{-1} \dot{\hat{w}}_2 - c_1 z_1^2 + z_1 z_2
\end{aligned} \tag{3.12}
$$

Lemma 1: [54, 55] For any $x, y \in R^n$, there are following inequalities where $a > 0, p > 1, q > 1$ and $(p-1)(q-1) = 1$.

$$
x^T y \leq \frac{a^p}{p} \|x\|^p + \frac{1}{q a^q} \|y\|^q \tag{3.13}
$$

Then, the time derivative of V_2 is

$$
\begin{aligned}
\dot{V}_2 \leq & z_2 \left(\mathrm{w}_2^T \phi_2 (X_2) + z_3 + \alpha_2 - f_2 \right) - \tilde{w}_2^T \Lambda_2^{-1} \dot{\hat{w}}_2 \\
& - c_1 z_1^2 + z_1 z_2 + \frac{1}{2} \bar{\kappa}_2^2 + \frac{1}{2} (\Delta u)^2 + z_2^2
\end{aligned} \tag{3.14}
$$

It is easy to know

$$
\begin{aligned}
\dot{V}_2 \leq & z_2 \left(\hat{\mathrm{w}}_2^T \phi_2 (X_2) + z_3 + \alpha_2 - f_2 \right) - \tilde{w}_2^T \Lambda_2^{-1} \dot{\hat{w}}_2 \\
& - c_1 z_1^2 + z_1 z_2 + z_2^2 + \frac{1}{2} \bar{\kappa}_2^2 + \frac{1}{2} (\Delta u)^2 + z_2 \tilde{w}_2^T \phi_2 (X_2)
\end{aligned} \tag{3.15}
$$

The virtual control input and parameter adaptive law are designed as

$$
\begin{aligned}
\alpha_2 &= -(c_2 + 1) z_2 + f_2 - z_1 - \hat{w}_2^T \phi_2 (X_2) \\
\dot{\hat{w}}_2 &= \Lambda_2 (z_2 \phi_2 (X_2) - \gamma_1 \hat{w}_2)
\end{aligned} \tag{3.16}
$$

Now, substituting (3.16) into (3.15) gives

$$
\begin{aligned}
\dot{V}_2 \leq & z_2 z_3 - c_1 z_1^2 - c_2 z_2^2 - \frac{1}{2} \gamma_1 \|\tilde{w}_2\|^2 \\
& + \frac{1}{2} \gamma_1 \|w_2\|^2 + \frac{1}{2} \bar{\kappa}_2^2 + \frac{1}{2} (\Delta u)^2
\end{aligned} \tag{3.17}
$$

where $c_2 > 0$ and $\gamma_1 > 0$ are designed constants.

Step 3: The Lyapunov function V_3 is defined as

$$
V_3 = V_2 + \tfrac{1}{2} z_3^2 + \tfrac{1}{2} \tilde{w}_3^T \Lambda_3^{-1} \tilde{w}_3 \tag{3.18}
$$

The error weight vector is defined as $\tilde{w}_3 = w_3 - \hat{w}_3$, $\hat{w}_3$ represents the estimate of w_3.

The time derivative of V_3 is regarded as

$$
\begin{aligned}
\dot{V}_3 &= z_3 \dot{z}_3 - \tilde{w}_3^T \Lambda_3^{-1} \dot{\hat{w}}_3 + \dot{V}_2 \\
&= z_3 (z_4 + \alpha_3 - \dot{\alpha}_2) - \tilde{w}_3^T \Lambda_3^{-1} \dot{\hat{w}}_3 + \dot{V}_2
\end{aligned} \tag{3.19}
$$

Define

$$
H_3 (X_3) = -\dot{\alpha}_2 = \mathrm{w}_3^T \phi_3 (X_3) + \kappa_3 \tag{3.20}
$$

Then, the time derivative of V_3 becomes

$$
\dot{V}_3 = z_3 \left(z_4 + \alpha_3 + \mathrm{w}_3^T \phi_3 (X_3) + \kappa_3 \right)
$$

$$-\tilde{w}_3^T\Lambda_3^{-1}\dot{\hat{w}}_3+\dot{V}_2 \tag{3.21}$$

The virtual control input of active suspension system and parameter adaptive law are given as

$$\begin{aligned}\alpha_3 &= -\left(c_3+0.5\right)z_3 - z_2 - \hat{w}_3^T\phi_3\left(X_3\right)\\ \dot{\hat{w}}_3 &= \Lambda_3\left(z_3\phi_3\left(X_3\right)-\gamma_2\hat{w}_3\right)\end{aligned} \tag{3.22}$$

From (3.22) $\dot{V}_3$ becomes

$$\begin{aligned}\dot{V}_3 \leq & z_3z_4 - c_1z_1^2 - c_2z_2^2 - c_3z_3^2\\ & -\frac{1}{2}\gamma_1\|\tilde{w}_2\|^2+\frac{1}{2}\gamma_1\|w_2\|^2+\frac{1}{2}\bar{\kappa}_2^2\\ & +\frac{1}{2}(\Delta u)^2-\frac{1}{2}\gamma_2\|\tilde{w}_3\|^2+\frac{1}{2}\gamma_2\|w_3\|^2+\frac{1}{2}\bar{\kappa}_3^2\end{aligned} \tag{3.23}$$

where $c_3>0$ and $\gamma_2>0$ are designed constants.

Step 4: The time derivative of z_4 is as follows

$$\dot{z}_4=\tfrac{1}{m_u}\left(f_4-\rho u_c-\zeta\right)-\dot{\alpha}_3 \tag{3.24}$$

Then, it gets

$$m_u\rho z_4\dot{z}_4 = z_4\left(\tfrac{f_4}{\rho}-u_c-\tfrac{\zeta}{\rho}\right)-\tfrac{m_u}{\rho}z_4\dot{\alpha}_3 \tag{3.25}$$

Let

$$H_4\left(X_4\right)=\left(\tfrac{f_4}{\rho}-\tfrac{\zeta}{\rho}\right)-\tfrac{m_u}{\rho}\dot{\alpha}_3 = w_4^T\phi_4\left(X_4\right)+\kappa_4 \tag{3.26}$$

The Lyapunov function V_4 is defined as

$$V_4 = V_3+\tfrac{m_u}{2\rho}z_4^2+\tfrac{1}{2}\tilde{w}_4^T\Lambda_4^{-1}\tilde{w}_4 \tag{3.27}$$

The error weight vector is defined as $\tilde{w}_4=w_4-\hat{w}_4$, $\hat{w}_4$ represents the estimate of w_4.

The time derivative of V_4 is regarded as

$$\begin{aligned}\dot{V}_4 &= \frac{m_uz_4}{\rho}\dot{z}_4-\tilde{w}_4^T\Lambda_4^{-1}\dot{\hat{w}}_4+\dot{V}_3\\ &= z_4\left(w_4^T\phi_4\left(X_4\right)+\kappa_4-u_c\right)-\tilde{w}_4^T\Lambda_4^{-1}\dot{\hat{w}}_4+\dot{V}_3\end{aligned} \tag{3.28}$$

The controller of active suspension system is designed as

$$u_c = \left(c_4+0.5\right)z_4+z_3+\hat{w}_4^T\phi_4\left(X_4\right) \tag{3.29}$$

The parameter adaptive law is given as

$$\dot{\hat{w}}_4 = \Lambda_4\left(z_4\phi_4\left(X_4\right)-\gamma_3\hat{w}_4\right) \tag{3.30}$$

Then, from (3.29) and (3.30), it leads to

$$\begin{aligned}\dot{V}_4 \leq & -\sum_{i=1}^{n=4}c_iz_i^2+\sum_{i=2}^{4}\frac{1}{2}\bar{\kappa}_i^2+\frac{1}{2}(\Delta u)^2\\ & +\sum_{i=1}^{3}\left(-\frac{1}{2}\gamma_i\|\tilde{w}_{i+1}\|^2+\frac{1}{2}\gamma_i\|w_{i+1}\|^2\right)\end{aligned} \tag{3.31}$$

where $c_4>0$ and $\gamma_3>0$ are designed constants.

3.2. Stability analysis

Theorem 1: The quarter active suspension systems with actuator faults based on Butterworth low-pass filter is considered (2.7). Through the design of virtual controllers α_1, α_2 and α_3, actual controller u_c and adaptive laws $\hat{w}_2$, $\hat{w}_3$ and $\hat{w}_4$, all the signals in the closed-loop systems are bounded.
Proof: Based on (3.28), (3.29), and (3.30), (3.31) is rewritten as

$$\dot{V}_4 \leq -MV + N \tag{3.32}$$

where $N = \sum_{i=2}^{4} \frac{1}{2}\bar{\kappa}_i^2 + \frac{1}{2}(\Delta u)^2 + \sum_{i=1}^{3} \frac{1}{2}\gamma_i \|w_{i+1}\|^2$, $M = \min\{2c_1, 2a_2/m_{s\max},\ 2a_3, 2a_4/m_{b\max},\ \gamma_i\Lambda_{i+1}\ i = 2, 3, 4\}$, $a_2 = c_2 + 1$, $a_3 = c_3 + 0.5$, and $a_4 = c_4 - 0.5$.

Then, (3.32) can be further written as

$$V_4(t) \leq \left(V_4(0) - \frac{N}{M}\right) e^{-Mt} + \frac{N}{M} \tag{3.33}$$

From (3.33), it is clear that z_i, x_i, u are bounded.

Meanwhile, the errors satisfy

$$|z_i| \leq \sqrt{2\left(\left(V_4(0) - \frac{N}{M}\right) e^{-Mt} + \frac{N}{M}\right)} = B, i = 1, 2, 3, 4 \tag{3.34}$$

From (3.34), it clear that the errors of quarter active suspension system are within bounds. z_1 is bounded as $|z_1| \leq B$ and the desired trajectory $|y_d| \leq A$ is also bounded. At the same time, $|x_1| = |z_1| + |y_d| \leq A + B$. We can get x_2, x_3, $x_4$are bound similar. It implies that all the signals of the quarter active suspension closed-loop system with actuator failure are bounded.

4. Simulation example

In this section, an example of quarter active suspension system is provided to prove the effectiveness of proposed method. The quarter-car parameters are given.

The initial state values are selected as: $x_1(0) = 0.03$, $x_2(0) = 0.03$, $x_3(0)$=0.03, $x_4(0) = 0.03$, $x_5(0) = 0.03$, $x_6(0) = 0.03$ and $x_7(0) = 0.03$. The desired tracking trajectories y_d is given as $y_d = 0$, then $\dot{y}_d = 0$. The road excitation is represented as: z_r=0.002 sin(0.001πt). In addition, the designed parameters are as follows $c_1 3$, $c_2 = 3.2$, $c_3 = 1$, $c_4 = 64.5$ γ_1=100, γ_2=100, and γ_3=100. The problem of actuator failure is considered in this paper. So, the actuator faults deviation coefficient ρ and unknown constant ζ are designed as ρ=0.8, ζ=0.5. The parameters of the vehicle is described by Table 2.

Table 2. The quarter-car model parameters.

Parameter	Value	Parameter	Value
k_a	$18000N/m$	m_u	$59kg$
l_a	$2400Ns/m$	m_s	$590kg$
k_t	$15000N/m$	$m_{s\max}$	$700kg$
l_t	$1200Ns/m$	$m_{s\min}$	$520kg$

Figures 2 and 4 show that the vertical displacement of the car-body converges to a small neighborhood of zero. Figures 3 and 5 show that the vertical speed converges to the neighborhood of zero. At the same time, z_1, z_2, z_3 and z_4 are given in Figures 6 and 7. Figure 8 shows that the suspension space is small enough around zero. The adaptive laws are plotted in Figure 9. It is clear that all the adaptive laws converge to a small neighborhood of zero. In Figure 10, we can see that the control input u is stable.

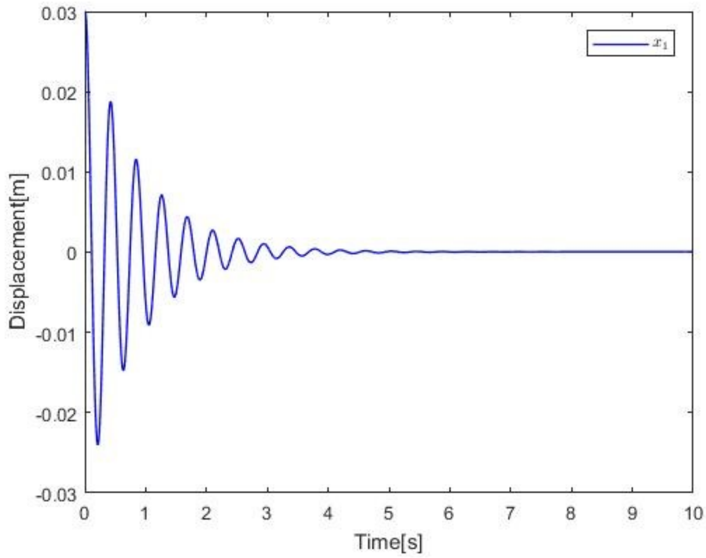

Figure 2. The vertical displacement of vehicle x_1.

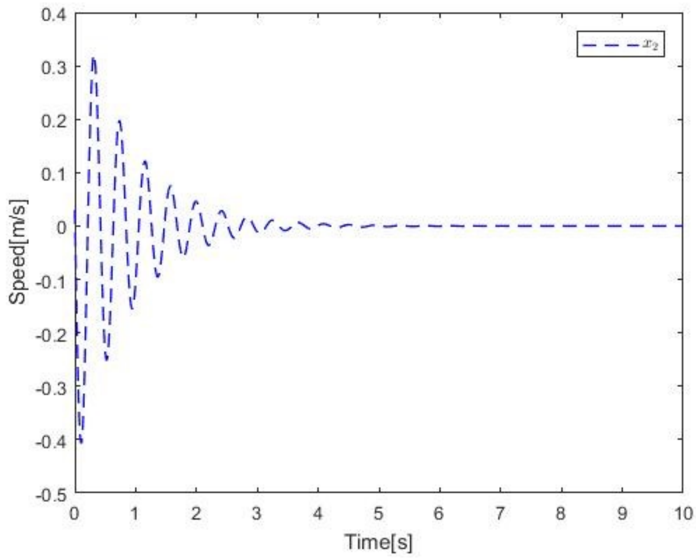

Figure 3. The vertical speed of the car x_2.

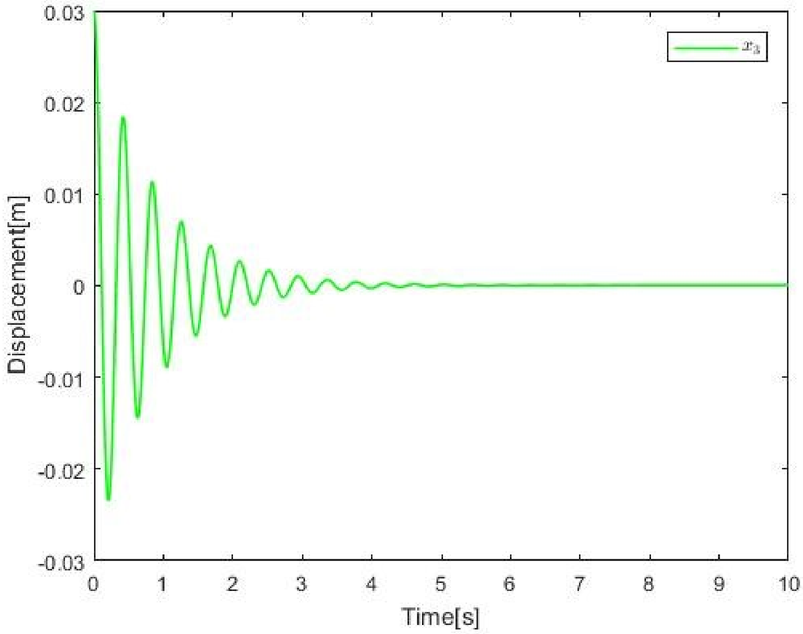

Figure 4. The vertical displacement of vehicle x_3.

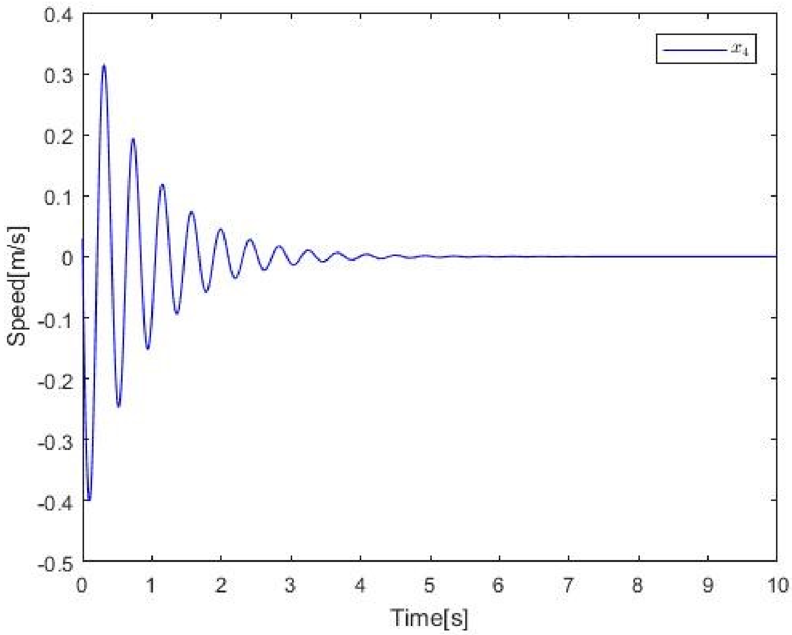

Figure 5. The speed of the vehicle x_4.

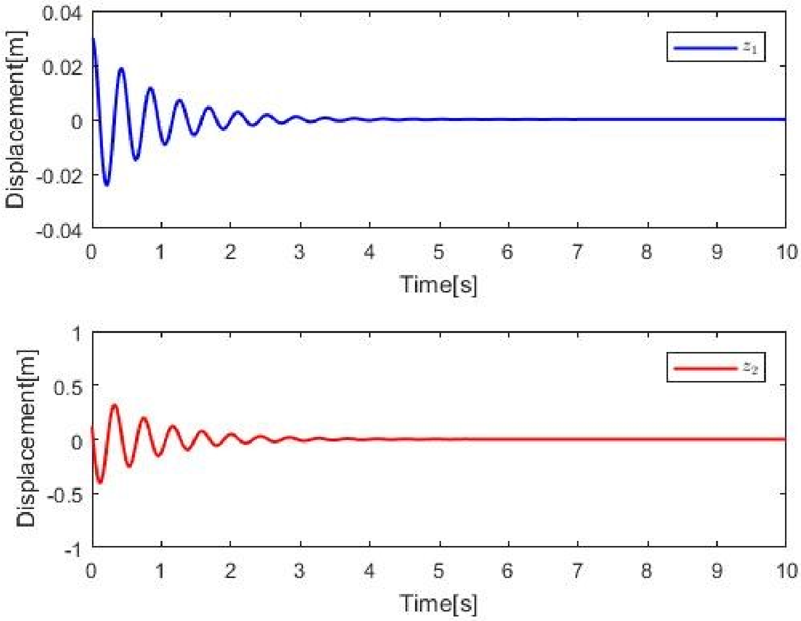

Figure 6. The errors z_1 and z_2.

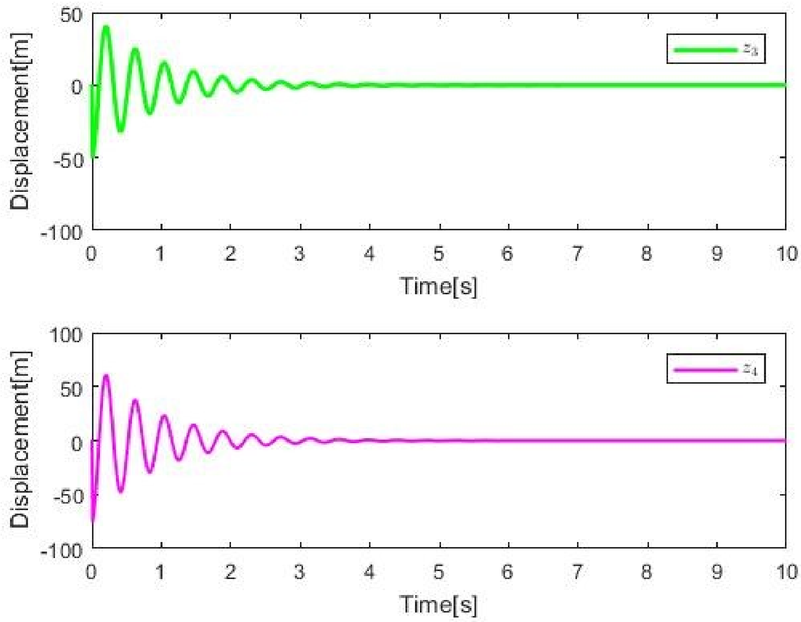

Figure 7. The errors z_3 and z_4.

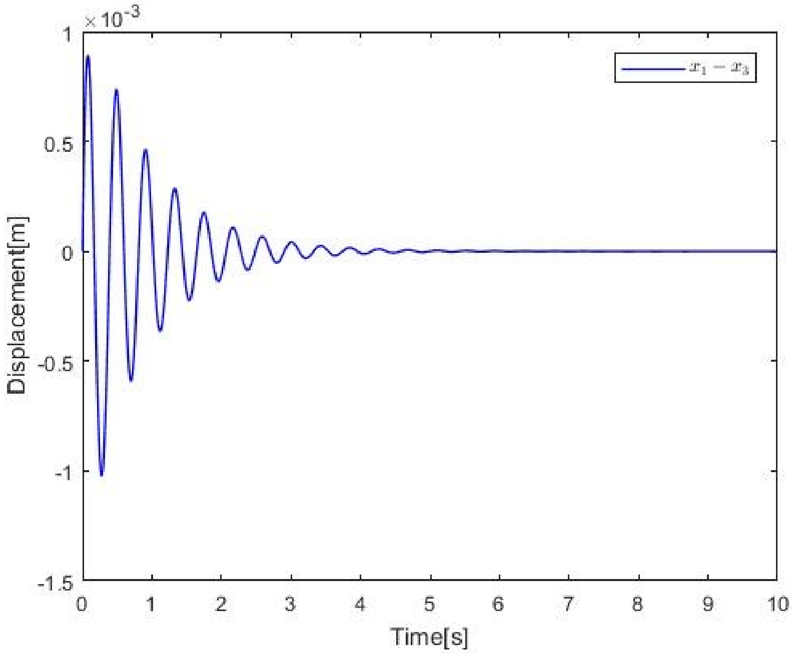

Figure 8. Suspension space $x_1 - x_3$.

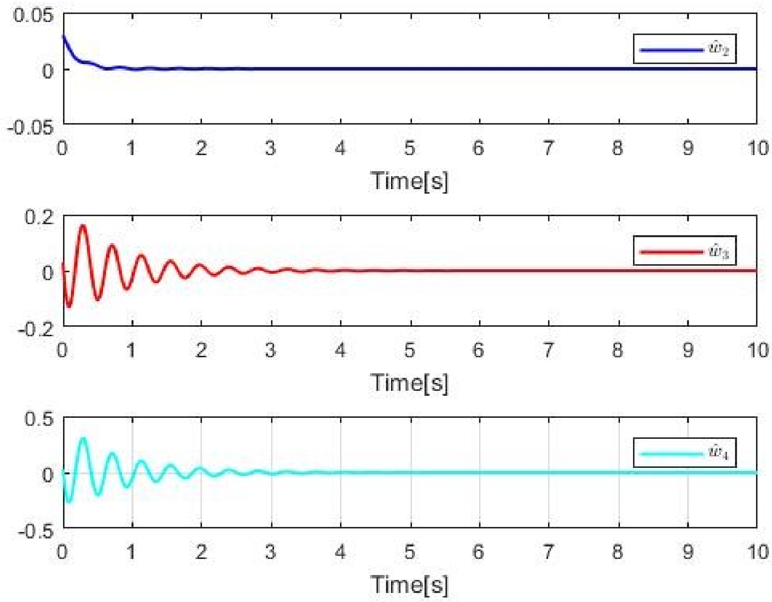

Figure 9. The adaptive laws $\hat{w}_2,\ \hat{w}_3$ and $\hat{w}_4$.

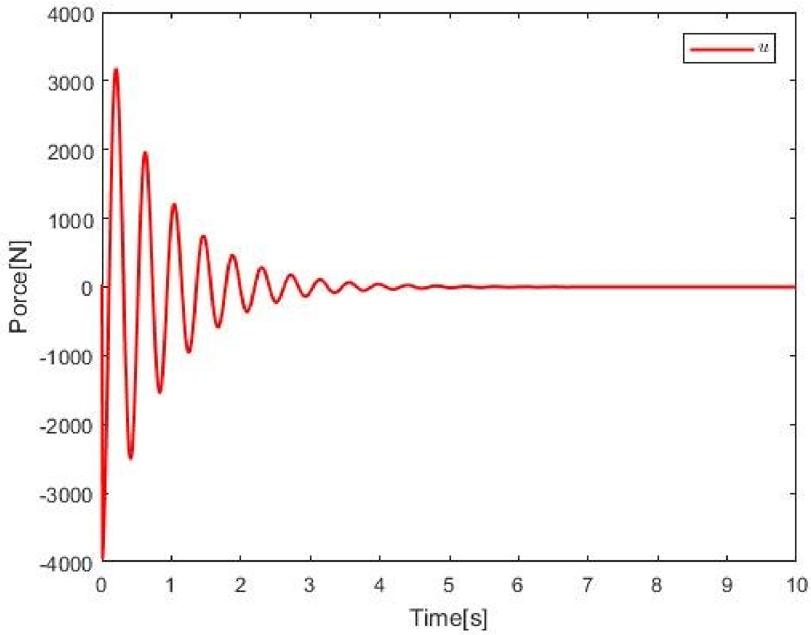

Figure 10. Control input u.

5. Conclusion

An adaptive fault tolerant controller is put forward for a quarter ASS in this paper. By using Butterworth low-pass filter, the second order ASS is converted to a fourth order system, which includes all the vehicle variables in the quarter suspension system. At the same time, the fault of vehicle actuator is also considered. NN control is used to approximate unknown functions in quarter suspension system. On account of backstepping method, the corresponding adaptive controller is designed. Through stability analysis, it can be concluded that all signals in the quarter active suspension system are bounded. Finally, the simulation results show that the method is effective. In the future, more complex situations can be considered in the controller design of the fault-tolerant control, such as the performance constraints, saturated or dead-zone, etc. The control studies will be further enriched in active suspension systems.

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Conflict of interest

All authors declare no conflicts of interest.

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