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Research article

A new application of conformable Laplace decomposition method for fractional Newell-Whitehead-Segel equation

Muammer Ayata* and Ozan Özkan

Department of Mathematics, Faculty of Science, Selcuk University, Konya, 42003, Turkey

* Correspondence: Email: muammerayata55@gmail.com.

Abstract: In this study, it is the first time that conformable Laplace decomposition method (CLDM) is applied to fractional Newell-Whitehead-Segel (NWS) equation which is one of the most significant amplitude equations in physics. The method consists of the unification of conformable Laplace transform and Adomian decomposition method (ADM) and it is used for finding approximate analytical solutions of linear-nonlinear fractional PDE's. The results show that this CLDM is quite powerful in solving fractional PDE's.

Keywords: Conformable fractional derivative; Conformable differential equations; Newell-Whitehead-Segel equation; Amplitude equations; Laplace transform; Adomian decomposition method **Mathematics Subject Classification:** 26A33, 34A08, 35R11

1. Introduction

The history of Fractional calculus is based on a question that Leibniz asked L'Hospital on 30 September, 1695. Since that time the developments in fractional derivatives have been done only in pure theoretical mathematics. In recent years, it is observed that fractional analysis allows an elegant modeling of many interdisciplinary applications [1–6]. Until recently, fractional derivative definitions such as Caputo, Riemann-Liouville (RL) have been widely used in the solution methods to find the approximate solutions of differential equations. However, since these derivative definitions contain integral operators, the calculations are very challenging. In addition, analytical solutions usually can not be found in the models using these derivative definitions and to solve these equations researchers sometimes use numerical methods [3,4].

Although the most common fractional derivatives such as Riemann-Liouville and Caputo are used by many mathematicians, there are many studies in the literature that these fractional derivatives have some deficiencies as in the following [7, 8],

- In RL, the derivative of a constant is not equal to zero.
- In RL initial conditions must be given in RL sense.
- For the Caputo derivative, to be differentiable the function must be differentiable in classical sense.
- In both RL and Caputo derivatives; chain rule is not satisfied.
- In both RL and Caputo derivatives; derivatives for integer order does not coincides with the derivatives in classical sense.

All these negativities increased the search for a new derivative. In 2014, R.Khalil et al. presented a new derivative named conformable derivative, which satisfies many of the properties in fractional derivatives that cannot be satisfied by the existing fractional derivatives [7, 8]. This derivative is quite similar to the definition of derivative in the classical limit form and it is quite easy to handle. So it has been accepted so quickly and has been the subject of many studies for researchers [9–12].

The fluctuations in sand, the lines of the seashells and many other striped patterns like these occur in various spatial systems that can be modeled by amplitude equations. NWS equation is one of the most important amplitude equations in applied sciences and it explains how stripe patterns appear in two dimensional systems [13, 14]. NWS equation is of the form

$$\frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial x^2} + cu - du^r$$
(1.1)

where *r* is positive integer, *k*, *d*, *c* are real numbers, k > 0, $t \ge 0$ and $x \in R$ [13]. Here u(x, t) may be taken as the velocity of a fluid or temperature distribution in a thin and infinitely long pipe. Therefore, in the last decades, so many methods were applied to NWS equation to find its analytical and approximate solutions. Some of which are; Variational iteration method [13], ADM and Multiquadric Quasi-Interpolation methods [15], Differential transform method [16] and so.

In this study, CLDM is applied to the time-fractional form of NWS Eq (1.1) given below,

$$\frac{\partial^{\alpha} u}{\partial t^{\alpha}} = k \frac{\partial^{2} u}{\partial x^{2}} + cu - du^{r}$$
(1.2)

2. Preliminaries

Let us give some definitions and theorems needed as follows.

Definition 2.1. *Given a function* $f : [0, \infty) \to \mathbb{R}$, t > 0 and $\alpha \in (0, 1)$ [7],

• Conformable derivative of f with respect to t of order α is defined by

$$D_t^{\alpha}(f)(t) = \lim_{\varepsilon \to 0} \frac{f(t + \varepsilon t^{1-\alpha}) - f(t)}{\varepsilon}$$

• If f is α -differentiable in conformable sense in (0, a) for some a > 0 and if $\lim_{t\to 0} D_t^{\alpha}(f)(t)$ exists, then $D_t^{\alpha}(f)(0) = \lim_{t\to 0} D_t^{\alpha}(f)(t)$

Definition 2.2. Let $f : [t_0, \infty) \to R$ be a real valued function with $t_0 \in R$ and $0 < \beta \le 1$. Then conformable laplace transform of the function f of order β is defined by [9]

$$\mathscr{L}_{\beta}^{t_0}[f(t)](s) = \int_{t_0}^{\infty} e^{-s\frac{(t-t_0)^{\beta}}{\beta}} f(t)d_{\beta}(t,t_0) = \int_{t_0}^{\infty} e^{-s\frac{(t-t_0)^{\beta}}{\beta}} f(t)(t-t_0)^{\beta-1}dt$$
(2.1)

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Theorem 2.1. Let $a \in R$, $0 < \beta \le 1$ and $f : (a, \infty) \to R$ be a differentiable function. Then [9] $\mathscr{L}_{\beta}[D_t^{\beta}(f(t))] = s\mathscr{L}_{\beta}[f(t)] - f(a).$

There are so many scientific researches to test the usefulness and accuracy of this derivative some of which are as follows: Abdeljawad [9] expressed Laplace transform, chain rule, integration by parts and power series in conformable sense. Tayyan and Sakka [17] used Lie symmetry analysis to investigate the invariance properties of some conformable time and space fractional PDEs. Kurt et al. [18] found new solutions to conformable fractional Nizhnik-Novikov-Veselov system using homotopy analysis method and G'/G expansion method. Islam et al. [19] found new general trawelling wave solutions for some conformable fractional dispersive long wave equations, modified regularized long-wave equation and mKdV-ZK equation. Eslami and Rezazadeh [20] extracted analytical solutions of conformable time fractional Wu-Zhang system by the aid of first integral method. Hammad and Khalil [21] proved the existence of Abel's formula for the conformable fractional differential equations. Zhao and Luo [22] generalized conformable fractional derivative (GCFD) concept and give the physical and geometrical interpretations of GCFD. Qi and Wang [23] analysed the asymptotical stability of conformable fractional systems. Ladrani and Cherif [24] made oscillation tests for damping conformable fractional differential equations. Therefore, there are many open problems in this new area that need to be considered.

For a better accuracy and convergence, there are so many modifications and hybrid forms of ADM in the literature. Some of which are double Laplace ADM [25–27], Fourier Transform ADM [28], spectral ADM [29], Legendre polynomials combined with ADM [30], combination of reproducing kernel method and ADM [31]. CLDM is one of those hybrid forms of ADM. In CLDM, Laplace transform is combined with ADM in conformable sense and it is applied to linear-nonlinear fractional PDE's. Now let us give CLDM algorithm.

3. Conformable Laplace decomposition method (CLDM)

To show the basic idea of CLDM, we consider the following fractional PDE's in general operator form

$$D_t^{\alpha}u(x,t) + D_x^n u(x,t) + R(u(x,t)) + N(u(x,t)) = g(x,t) \quad t > 0, \quad x > 0, \quad 0 < \alpha \le 1$$
(3.1)

$$u(x,0) = h(x)$$
 (3.2)

where D_t^{α} is the linear derivative operator in conformable sense of order α in t, D_x^n is the highest order linear classical derivative operator in x, R is the other linear terms with lower derivatives, N is the nonlinear term and g(x, t) is the nonhomogenous part.

If the conformable Laplace transform \mathscr{L}_{α} with respect to *t* is applied to both sides of the Eq (3.1), it becomes,

$$\mathscr{L}_{\alpha}[D_{t}^{\alpha}u] + \mathscr{L}_{\alpha}[D_{x}^{n}u] + \mathscr{L}_{\alpha}[R(u) + N(u)] = \mathscr{L}_{\alpha}[g(x,t)]$$
(3.3)

From the differential property of the conformable Laplace transform [9], Eq (3.3) equation turns into

$$s\mathscr{L}_{\alpha}[u] - u(x,0) + \mathscr{L}_{\alpha}[D_{x}^{n}u] + \mathscr{L}_{\alpha}[R(u) + N(u)] = \mathscr{L}_{\alpha}[g(x,t)]$$
(3.4)

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If the Eq (3.4) is simplified

$$\mathscr{L}_{\alpha}[u] = \frac{1}{s}[u(x,0) + \mathscr{L}_{\alpha}[g(x,t)]] - \frac{1}{s}\mathscr{L}_{\alpha}[D_{x}^{n}u] - \frac{1}{s}\mathscr{L}_{\alpha}[R(u) + N(u)]$$
(3.5)

If the inverse Laplace transform in conformable sense is applied to Eq (3.5), we get

$$u(x,t) = \mathscr{L}_{\alpha}^{-1} \left[\frac{1}{s} [u(x,0) + \mathscr{L}_{\alpha}[g(x,t)]] - \mathscr{L}_{\alpha}^{-1} \left[\frac{1}{s} \mathscr{L}_{\alpha}[D_{x}^{n}u] \right] - \mathscr{L}_{\alpha}^{-1} \left[\frac{1}{s} \mathscr{L}_{\alpha}[R(u) + N(u)] \right]$$
(3.6)

According to the ADM, the solution u(x, t) with its convergency, the nonlinear term N(u(x, t)) of the Eq (3.1) and the Adomian polynomials A_k which depend on $u_0, u_1, u_2, ..., u_k$ are respectively given as follows [32–35],

$$u(x,t) = \sum_{k=0}^{\infty} u_k(x,t)$$
(3.7)

$$N(u(x,t)) = \sum_{k=0}^{\infty} A_k$$
(3.8)

$$A_{k} = \frac{1}{k!} \frac{d^{k}}{d\lambda^{k}} [N(\sum_{i=0}^{k} \lambda^{i} u_{i})]|_{\lambda=0}, \quad k = 0, 1, 2, 3, \dots$$
(3.9)

If the Eqs (3.7)–(3.9) are substituted in Eq (3.6),

$$\sum_{k=0}^{\infty} u_k = \mathcal{L}_{\alpha}^{-1} [\frac{1}{s} [u(x,0) + \mathcal{L}_{\alpha}[g(x,t)]]] - \mathcal{L}_{\alpha}^{-1} [\frac{1}{s} \mathcal{L}_{\alpha}[D_x^n \sum_{k=0}^{\infty} u_k]] - \mathcal{L}_{\alpha}^{-1} [\frac{1}{s} \mathcal{L}_{\alpha}[R(\sum_{k=0}^{\infty} u_k) + \sum_{k=0}^{\infty} A_k]]$$
(3.10)

If both sides of the Eq (3.10) is matched, the following iterative algorithm is obtained

$$u_0 = \mathscr{L}_{\alpha}^{-1} \left[\frac{1}{s} [u(x,0) + \mathscr{L}_{\alpha}[g(x,t)]] \right]$$
(3.11)

$$u_{k+1} = \mathscr{L}_{\alpha}^{-1} \left[\frac{1}{s} \mathscr{L}_{\alpha} [D_x^n u_k] \right] - \mathscr{L}_{\alpha}^{-1} \left[\frac{1}{s} \mathscr{L}_{\alpha} [R(u_k) + A_k] \right], \quad k = 0, 1, 2, 3, \dots$$
(3.12)

Hence by calculating as many u_k components as needed, the solution u(x, t) can be obtained from Eq (3.7).

To show the effectiveness of this method, let us give some examples.

4. Applications

Example 4.1. Regard the linear conformable fractional NWS equation below [13],

$$D_t^{\alpha} u(x,t) = D_x^2 u(x,t) - 2u(x,t), \quad 0 < \alpha \le 1$$
(4.1)

with initial condition

$$u(x,0) = e^x \tag{4.2}$$

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When the conformable Laplace transform with respect to t is applied to both sides of the Eq (4.1), it becomes

$$s\mathscr{L}_{\alpha}[u(x,t)] - u(x,0) = \mathscr{L}_{\alpha}[D_x^2u(x,t)] - 2\mathscr{L}_{\alpha}[u(x,t)].$$
(4.3)

Using initial conditions given in Eq (4.2) and simplifying (4.3), we get

$$\mathscr{L}_{\alpha}[u] = \frac{1}{s+2}e^{x} + \frac{1}{s+2}\mathscr{L}_{\alpha}[D_{x}^{2}u]$$
(4.4)

If the inverse Laplace transform in conformable sense is applied to Eq (4.4), we get

$$u(x,t) = e^{x}e^{-\frac{2t^{\alpha}}{\alpha}} + \mathscr{L}_{\alpha}^{-1}\left[\frac{1}{s+2}\mathscr{L}_{\alpha}[D_{x}^{2}u]\right]$$
(4.5)

Base on the serial solution formula in Eq (3.7), (4.5) turns into

$$\sum_{k=0}^{\infty} u_k = e^x e^{-\frac{2t^{\alpha}}{\alpha}} + \mathscr{L}_{\alpha}^{-1} [\frac{1}{s+2} \mathscr{L}_{\alpha} [D_x^2 \sum_{k=0}^{\infty} u_k]]$$
(4.6)

If the iterative algorithm in Eq (3.11) and Eq (3.12) are used, we obtain

$$u_0 = e^{x - \frac{2t^{\alpha}}{\alpha}} \tag{4.7}$$

$$u_1 = \frac{\Gamma^{\alpha}}{\alpha} e^{x - \frac{2\Gamma^{\alpha}}{\alpha}}$$
(4.8)

$$u_2 = \frac{t^{2\alpha}}{\alpha^2 2!} e^{x - \frac{2t^{\alpha}}{\alpha}}$$
(4.9)

$$u_{3} = \frac{t^{3\alpha}}{\alpha^{3}3!} e^{x - \frac{2t^{\alpha}}{\alpha}}$$
(4.10)

- (4.11)
 - (4.12)

So the series solution can be found as

$$u(x,t) = u_0 + u_1 + u_2 + u_3 + \dots$$

= $e^{x - \frac{2t^{\alpha}}{\alpha}} [1 + \frac{t^{\alpha}}{\alpha} + \frac{t^{2\alpha}}{\alpha^2 2!} + \frac{t^{3\alpha}}{\alpha^3 3!} + \dots]$
= $e^{x - \frac{2t^{\alpha}}{\alpha}} e^{\frac{t^{\alpha}}{\alpha}}$
= $e^{x - \frac{t^{\alpha}}{\alpha}}$ (4.13)

In Figure 1, to show how the change in derivative orders affects the physical behavior of the solution, 5-step approximate solution graphs of u(x, t) are given for different α values. Also, for $\alpha = 1$, semi-analytical solution graph of u(x, t) obtained by CLDM is given. It is seen that, for $\alpha = 1$ our CLDM solution is overlapped with the exact solution in [13].

:



Figure 1. The first four graphics are the approximate solutions of u(x, t) in five steps for different α values, the fifth one is the analytical solution of u(x, t) we found for $\alpha = 1$ and the last one is the error between approximate solution in five step and exact solution for $\alpha = 1$.

Example 4.2. Consider the conformable nonlinear fractional NWS equation below [14],

$$D_t^{\alpha}u(x,t) = 5D_x^2u(x,t) + 2u(x,t) + u^2(x,t), \quad 0 < \alpha \le 1$$
(4.14)

with initial condition

$$u(x,0) = \rho \tag{4.15}$$

where ρ is a constant.

If the conformable Laplace transform with respect to t is applied to both sides of the Eq (4.14), it becomes

$$s\mathscr{L}_{\alpha}[u(x,t)] - u(x,0) = 5\mathscr{L}_{\alpha}[D_{x}^{2}u(x,t)] + 2\mathscr{L}_{\alpha}[u(x,t)] + \mathscr{L}_{\alpha}[u^{2}(x,t)].$$
(4.16)

Using the initial conditions given in Eq (4.15) and simplifying Eq (4.16), we get

$$\mathscr{L}_{\alpha}[u] = \frac{\rho}{s-2} + \frac{5}{s-2}\mathscr{L}_{\alpha}[D_x^2 u] + \frac{1}{s-2}\mathscr{L}_{\alpha}[u^2(x,t)]$$

$$(4.17)$$

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If the inverse Laplace transform in conformable sense is applied to Eq (4.17), we get

$$u = \rho e^{\frac{2t^{\alpha}}{\alpha}} + \mathscr{L}_{\alpha}^{-1} [\frac{5}{s-2} \mathscr{L}_{\alpha}[D_x^2 u]] + \mathscr{L}_{\alpha}^{-1} [\frac{1}{s-2} \mathscr{L}_{\alpha}[u^2]].$$
(4.18)

Putting the infinite serial solution formula in Eq (3.7) and the Adomian polynomials for nonlinear term in Eq (3.8), Eq (4.18) turns into

$$\sum_{k=0}^{\infty} u_k = \rho e^{\frac{2t^{\alpha}}{\alpha}} + \mathcal{L}_{\alpha}^{-1} \left[\frac{5}{s-2} \mathcal{L}_{\alpha} \left[D_x^2 \sum_{k=0}^{\infty} u_k \right] \right] + \mathcal{L}_{\alpha}^{-1} \left[\frac{1}{s-2} \mathcal{L}_{\alpha} \sum_{k=0}^{\infty} A_k \right]$$
(4.19)

A few components of Adomian polynomials above are as follows

$$A_{0} = u_{0}^{2}$$

$$A_{1} = 2u_{0}u_{1}$$

$$A_{2} = 2u_{0}u_{2} + u_{1}^{2}$$

$$A_{3} = 2u_{0}u_{3} + 2u_{1}u_{2}$$
:
$$(4.20)$$

Putting these Adomian polynomials into Eq (4.19) and using the iterative algorithm in Eq (3.11) and Eq (3.12), we obtain

$$u_{0} = \rho e^{\frac{2t^{\alpha}}{\alpha}}$$

$$u_{1} = \frac{1}{2} \rho^{2} e^{\frac{2t^{\alpha}}{\alpha}} (e^{\frac{2t^{\alpha}}{\alpha}} - 1)$$

$$u_{2} = (\frac{1}{2})^{2} \rho^{3} e^{\frac{2t^{\alpha}}{\alpha}} (e^{\frac{2t^{\alpha}}{\alpha}} - 1)^{2}$$

$$u_{3} = (\frac{1}{2})^{3} \rho^{4} e^{\frac{2t^{\alpha}}{\alpha}} (e^{\frac{2t^{\alpha}}{\alpha}} - 1)^{3}$$

$$\vdots$$

$$(4.21)$$

So the series solution can be obtained as

$$\begin{split} u(x,t) &= u_0 + u_1 + u_2 + u_3 + \dots \\ &= \rho e^{\frac{2t^{\alpha}}{\alpha}} + \frac{1}{2} \rho^2 e^{\frac{2t^{\alpha}}{\alpha}} (e^{\frac{2t^{\alpha}}{\alpha}} - 1) + (\frac{1}{2})^2 \rho^3 e^{\frac{2t^{\alpha}}{\alpha}} (e^{\frac{2t^{\alpha}}{\alpha}} - 1)^2 + (\frac{1}{2})^3 \rho^4 e^{\frac{2t^{\alpha}}{\alpha}} (e^{\frac{2t^{\alpha}}{\alpha}} - 1)^3 + \dots \\ &= \rho e^{\frac{2t^{\alpha}}{\alpha}} [1 + \frac{\rho (e^{\frac{2t^{\alpha}}{\alpha}} - 1)}{2} + (\frac{\rho (e^{\frac{2t^{\alpha}}{\alpha}} - 1)}{2})^2 + (\frac{\rho (e^{\frac{2t^{\alpha}}{\alpha}} - 1)}{2})^3 + \dots] \\ &= \rho e^{\frac{2t^{\alpha}}{\alpha}} \left(\frac{1}{1 - \frac{\rho (e^{\frac{2t^{\alpha}}{\alpha}} - 1)}{2}} \right) \\ &= \frac{2\rho e^{\frac{2t^{\alpha}}{\alpha}}}{2 + \rho (1 - e^{\frac{2t^{\alpha}}{\alpha}})} \end{split}$$

$$(4.22)$$

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In Figure 2, to show how the change in derivative orders affects the physical behavior of the solution, 3-step approximate solution graphs of u(x, t) are given for different α values and $\rho = 1$. Also, for $\rho = 1$ and $\alpha = 1$, semi-analytical solution graph of u(x, t) obtained by CLDM is given. It is seen that, for $\alpha = 1$ our CLDM solution is overlapped with the exact solution in [14].



Figure 2. The first four graphics are the approximate solutions of u(x, t) in three steps for different α values, the fifth one is the analytical solution of u(x, t) we found for $\alpha = 1$ and the last one is the error between approximate solution in three step and exact solution for $\alpha = 1$.

5. Conclusion

In this study, CLDM is applied to fractional Newell-Whitehead-Segel equation for the first time. In the applications it is seen that even the three step approximate solutions of the nonlinear problems give us a very accurate solutions. Morover if infinitely many terms are taken, this method gives us the approximate analytical solutions. Therefore, this shows that CLDM is an effective and easy mathematical tool for obtaining the approximate analytical solutions of the linear-nonlinear fractional PDE's of the given type. Also it can be said that CLDM is a promising method in solving other

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nonlinear fractional PDE's and it will guide the researchers who study on the approximate analytical solutions of fractional PDE's.

Conflict of interest

The authors declare that no competing interests exist.

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