Mathematics

http://www.aimspress.com/journal/Math

AIMS Mathematics, 5(6): 6841–6855.
DOI:10.3934/math.2020439
Received: 16 July 2020
Accepted: 31 August 2020
Published: 04 September 2020

Research article

Dynamic behaviors for reaction-diffusion neural networks with mixed delays

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Abstract: A class of reaction-diffusion neural networks with mixed delays is studied. We will discuss some important properties of the periodic mild solutions including existence and globally exponential stability by using exponential dissipation property of semigroup of operators and some analysis techniques. Finally, an example for the above neural networks is given to show the effectiveness of the results in this paper.

Keywords: reaction-diffusion; neural networks; mixed delays
Mathematics Subject Classification: 35C07, 35K57, 92D30

1. Introduction

The research of neural networks has attracted wide attention due to its successful applications in many areas, such as static image processing, target tracking, associative memory, and optimization problems [1–5]. Hence, a large number of results on dynamic properties of neural networks including stability, periodicity and attractivity, have been obtained. Liu etc. [6] studied the stability of neural networks with time-delay and variable-time impulses by using a valid approach on calculating the upper bound and lower bound of dwell time. Hu and Wang [7] investigated global exponential stability recurrent neural networks with asymmetric connection weight matrices and globally Lipschitz continuous and monotone nondecreasing activation functions. For more results for neural networks, see e.g., [8–12].

Reaction-diffusion neural networks have wide application in control systems, so many results have been obtained for the reaction-diffusion neural networks. Wang, Teng and Jiang [13] discussed the adaptive synchronization in an array of linearly coupled neural networks with reaction-diffusion terms and time delays. In [14], an anti-synchronization problem was considered for an array of linearly coupled reaction-diffusion neural networks with cooperative-competitive interactions and time-varying coupling delays. Wang and Wu [15] studied a coupled reaction-diffusion neural networks with hybrid coupling, which is composed of spatial diffusion coupling and state coupling. By using the Lyapunov functional method combined with the inequality techniques, a sufficient condition was given to ensure that the proposed network model is synchronized. Duan etc. [16] considered a class of reaction-diffusion high-order Hopfield neural networks with time-varying delays subject to the Dirichlet boundary condition in a bounded domain and discussed the existence of periodic mild solutions, and the global exponential stability of the periodic mild solutions by using the exponential dissipation property of semigroup of operators.

On the other hand, the delay is an inherent feature of signal transmission between neurons, which has a significant influence on the dynamical properties of the system. Lisena [17] obtained a new criteria for the global exponential stability of a class of cellular neural networks, with delay and periodic coefficients and inputs. In [18], a secondary delay partitioning method is proposed to study the stability problem for a class of recurrent neural networks with time-varying delay. In [20–22], the dynamic properties have been dealt with for neural networks with discrete and distributed time-delays by using different approaches.

Motivated by the above discussions, in this paper we will study the existence and global exponential stability of periodic mild solution for a class reaction-diffusion neural networks with mixed delays subject to Dirichlet boundary conditions, as well as positive effects of diffusion terms on existence and exponential stability of periodic mild solution. By using Lyapunov functional method, some new stability criteria are obtained which also guarantee the network will be exponentially convergent to the periodic solution. The theoretical methods developed in this paper have universal significance and can be easily extended to investigate many other types of neural networks with mixed delays. Our main contributions are summarized as follows.

(i) Time delay has an important influence on the dynamical properties of the system. The system in this paper includes discrete time delays and distributed time delays which is more general than the existing systems which will be more important for the applications.

(ii) Due to influence of mixed delays, constructing Lyapunov functional is more difficult. By using a new method, we construct a proper functional which overcomes the above difficulty.

(iii) It is nontrivial to establish a unified framework to handle reaction-diffusion terms, discrete time delays and distributed time delays. Our method provides a useful reference for studying more complex systems.

The rest of this paper is organized as follows. In Section 2, the considered model of the reaction-diffusion neural networks with mixed delays is presented. Some preliminaries are also given in this section. In Section 3, the existence and global exponential stability of periodic mild solutions for the considered model are studied. An example is presented to illustrate our theoretical results in Section 4. Finally, conclusions are drawn in Section 5.

2. Preliminaries

Set

$$W^{1,p} = \{u|u, \frac{\partial u}{\partial x_k} \in L^p(\Omega), k = 1, 2, \cdots, m, p > 2\}$$

and

$$H^1 = \{u|u, \frac{\partial u}{\partial x_k} \in L^2(\Omega), k = 1, 2, \cdots, m\}.$$

$L^p(\Omega)(p \geq 2)$ is the space of real functions on Ω which are L^p for the Lebesgue measure.

$(L^p(\Omega))^n(p \geq 2)$ is the space of real functions $u = (u_1, u_2, \cdots, u_n)$ on Ω, where $u_i \in L^p(\Omega)$. Obviously, $L^p(\Omega)(p \geq 2)$ is a Banach space equipped with the norm

$$\|u\|_p = \max_{i=1,2,\cdots,n} \|u_i\|_p,$$

where $\|u_i\|_p = \left(\int_\Omega |u_i(x)|^p dx \right)^{\frac{1}{p}}$. Denote

$$\underline{d}_i = \min_{1\leq k\leq m} d_{ik}, \ \ \underline{c}_i = \inf_{t\in\mathbb{R}} c_i(t), \ \ \overline{c}_i = \sup_{t\in\mathbb{R}} c_i(t),$$

$$\overline{a}_{ij} = \sup_{t\in\mathbb{R}} |a_{ij}(t)|, \ \ \overline{b}_{ij} = \sup_{t\in\mathbb{R}} |b_{ij}(t)|, \ \ \overline{I}_i = \sup_{t\in\mathbb{R}} |I_i(t)|, \ \ \overline{I} = \max_{1\leq i\leq n} \overline{I}_i,$$

$$\overline{\tau} = \{\tau, \ \ \max_{1\leq i,j\leq n} \sup_{t\in\mathbb{R}} \tau_{ij}(t)\}.$$

This paper is devoted to investigating the following non-autonomous dissipative reaction-diffusion system with mixed delays:

$$\begin{aligned} \frac{\partial u_i(t,x)}{\partial t} &= \sum_{k=1}^m d_{ik} \frac{\partial^2 u_i(t,x)}{\partial x_k^2} - c_i(t)u_i(t,x) + \sum_{j=1}^n a_{ij}(t) f_j(u_j(t-\tau_{ij}(t),x)) \\ &+ \sum_{j=1}^n b_{ij}(t) \int_{t-\tau}^t h_j(u_j(s,x))ds + I_i(t), \end{aligned} \tag{2.1}$$

where $d_{ik} > 0$ is diffusion rate, n is the number of the neurons in the neural network, $a_{ij}(t)$ are the discretely delayed connection weights, $b_{ij}(t)$ is the distributively delayed connection weight, of the jth neuron on the i neuron, $c_i(t)$ denotes the rate with which the ith neuron will reset its potential to the resting state in isolation when disconnected from the network and external inputs, $u_i(t)$ denotes the state of the ith neural neuron at time t, $f_j(\cdot)$ and $h_j(\cdot)$ are the activation functions of jth neuron at time t, $I_i(t)$ is the external bias on the ith neuron. The Dirichlet boundary condition and initial condition for system (2.1) are given in the following forms:

$$u_i(t,x) = 0, \ \ (t,x) \in [-\overline{\tau}, +\infty) \times \partial\Omega, \ i = 1, 2, \cdots n, \tag{2.2}$$

and

$$u_i(t,x) = \phi_i(t,x), \ \ (t,x) \in [-\overline{\tau}, 0] \times \Omega, \ i = 1, 2, \cdots n, \tag{2.3}$$

where $\phi(t,x) = (\phi_1(t,x), \phi_2(t,x), \cdots, \phi_n(t,x))^\top \in C([-\overline{\tau}, 0] \times \Omega, (H_0^1(\Omega))^n)$.

Remark 2.1. If $d_{ik} > 0, \ i = 1, 2, \cdots n, \ k = 1, 2, \cdots m$, in view of [19], the operator $A_i = \sum_{k=1}^m d_{ik} \frac{\partial^2 u_i(t,x)}{\partial x_k^2}$ is generator of a semigroup $\{T_i(t)\}_{t\geq 0}$ which is uniformly exponentially stable, that is, there exist positive constants a_i and b_i such that

$$\|T_i(t)\| \leq a_i e^{b_i t}, \ \ t \geq 0.$$

Throughout this paper, we make the following assumptions:
(H_1) $c_i(t), a_{ij}(t), b_{ij}(t), \tau_{ij}(t)$ and $I_i(t)$ are all $\omega-$periodic functions.
(H_2) For any $u, v \in$ R, there exist nonnegative constants L_i such that

$$|f_i(u) - f_i(v)| \leq L_i|u - v|,\ f_i(0) = 0,\ i = 1, 2, \cdots n.$$

(H_3) For any $u, v \in$ R, there exist function $H_i(u_i)$ such that $H_i'(u_i) = h_i(u_i)$, and nonnegative constants M_i such that

$$|H_i(u) - H_i(v)| \leq M_i|u - v|,\ H_i(0) = 0,\ i = 1, 2, \cdots n.$$

Remark 2.2. From Assumptions (H_2) and (H_3), there exist positive constants L_i and M_i such that

$$|f_i(u)| \leq L_i,\quad |H_i(u)| \leq M_i \text{ for any } u \in \mathrm{R},\ i = 1, 2, \cdots n.$$

From Assumptions (H_2),(H_3), and the Faedo Galerkin approximation (see e.g., [23, 24]), the initial boundary value problem (2.1)–(2.3) possesses a unique solution $u_i \in L^2((0, K), H^2(\Omega)) \cup C([-\bar{\tau}, K], H_0^2(\Omega))$ and $u_i' \in L^2((0, K), L^2(\Omega))$ for any $K > 0, i = 1, 2, \cdots, n$.

Definition 2.1. The function $u(t, x) \in C(\mathrm{R}, (L^2(\Omega))^n)$ is $\omega-$periodic on t in the sense of L^2 norm if

$$\sup_{t\in \mathrm{R}} \|u(t + \omega, \cdot) - u(t, \cdot)\|_2 = 0.$$

Definition 2.2. A function $u(t, x) = (u_1(t, x), u_2(t, x), \cdots, u_n(t, x))^\top \in C([-\bar{\tau}, +\infty), (L^2(\Omega))^n)$ said to be a mild solution of system (2.1) if

$$\begin{aligned} u_i(t, x) &= T_i(t - t_0)u_i(t_0, x) + \int_{t_0}^{t} T_i(t - s)\Big[- c_i(s)u_i(s, x) + \textstyle\sum_{j=1}^{n} a_{ij}(s) f_j(u_j(s - \tau_{ij}(s), x)) \\ &\quad + \textstyle\sum_{j=1}^{n} b_{ij}(s) \int_{s-\tau}^{s} h_j(u_j(v, x))dv + I_i(s)\Big]ds. \end{aligned}$$

Lemma 2.1. (Poincar$\acute{e}$'s Inequality [25]) Assume that $u(x) \in H_0^1(\Omega)$, then

$$\int_\Omega u^2(x)dx \leq C_0 \int_\Omega \sum_{i=1}^{m} (\frac{\partial u}{\partial x_k})^2 dx,$$

where C_0 is a constant independent of u.

Lemma 2.2. Assume that $a, b \geq 0$ and $p, q > 0$ with $\frac{1}{p} + \frac{1}{q} = 1$. Then

$$ab \leq \frac{1}{p}a^p + \frac{1}{q}b^q.$$

3. Main results

Theorem 3.1. Suppose that

$$\gamma = \min_{1\leq i\leq n} \Big\{ \frac{\underline{d}_i}{C_0} + \underline{c}_i - \sum_{j=1}^{n} \overline{a}_{ij} L_j - 2\sum_{j=1}^{n} \overline{b}_{ij} M_j - \frac{1}{2}\overline{I}_i \Big\} > 0 \tag{3.1}$$

and $u(t, x)$ is a solution of (2.1) with initial condition

$$u_i(s, x) = \phi_i(s, x),\ \|\phi_i(s, x)\|_2^2 \leq \frac{\overline{I}|\Omega|}{\gamma},\ s \in [-\overline{\tau}, 0],\ i = 1, 2, \cdots, n. \tag{3.2}$$

Then we have

$$\|u_i(t, \cdot)\|_2^2 < \frac{\overline{I}|\Omega|}{\gamma},\ t \in [0, +\infty),\ i = 1, 2, \cdots, n. \tag{3.3}$$

Furthermore, suppose that $p > 2$ is even integer,

$$\eta = \min_{1\leq i\leq n} \left\{p\underline{c}_i - p\sum_{j=1}^{n} \overline{a}_{ij}L_j - 2p\sum_{j=1}^{n} \overline{b}_{ij}M_j - (p-1)\overline{I}_i\right\} > 0 \tag{3.4}$$

and $u(t, x)$ is a solution of (2.1) with initial condition

$$u_i(s, x) = \phi_i(s, x),\ \|\phi_i(s, x)\|_p^p \leq 2\frac{\overline{I}|\Omega|}{\eta},\ s \in [-\overline{\tau}, 0],\ i = 1, 2, \cdots, n. \tag{3.5}$$

Then we have

$$\|u_i(t, \cdot)\|_p^p < \frac{(\overline{I})^p|\Omega|}{\eta},\ t \in [0, +\infty),\ i = 1, 2, \cdots, n. \tag{3.6}$$

Proof. The first section of the proof in Theorem 3.1 is similar to the corresponding one in [16]. In order to completeness of the present paper, we give the detail proof. Suppose (3.3) is not true, then there exist $t_0 > 0$ and some $i \in \{1, 2, \cdots, n\}$ such that

$$\|u_i(t_0, \cdot)\|_2^2 = \frac{\overline{I}|\Omega|}{\gamma},\ \ \|u_i(t, \cdot)\|_2^2 \leq \frac{\overline{I}|\Omega|}{\gamma},\ t \in [-\overline{\tau}, t_0],$$

$$\|u_j(t, \cdot)\|_2^2 \leq \frac{\overline{I}|\Omega|}{\gamma},\ t \in [-\overline{\tau}, t_0], j \neq i,$$

$$\frac{d}{dt}\|u_i(t_0, \cdot)\|_2^2 \geq 0.$$

Along the solutions of system (2.1) with initial condition (3.2), we can calculate the value of derivative of $\|u_i(t, \cdot)\|_2^2$ at $t = t_0$, and in view of (H_2),(H_3) and Lemma 2.1, we have

$$
\begin{aligned}
0 &\le \tfrac{d}{dt}\|u_i(t_0,\cdot)\|_2^2 \\
&= 2\int_\Omega u_i(t_0,x)\Big[\textstyle\sum_{k=1}^m d_{ik}\frac{\partial^2 u_i(t_0,x)}{\partial x_k^2} - c_i(t_0)u_i(t_0,x) + \sum_{j=1}^n a_{ij}(t_0)f_j(u_j(t_0-\tau_{ij}(t_0),x)) \\
&+ \textstyle\sum_{j=1}^n b_{ij}(t_0)\int_{t_0-\tau}^{t_0} h_j(u_j(s,x))ds + I_i(t_0)\Big]dx \\
&\le -2\int_\Omega \textstyle\sum_{k=1}^m d_{ik}(\frac{\partial u_i(t_0,x)}{\partial x_k})^2 dx - 2\int_\Omega \underline{c}_i u_i^2(t_0,x)dx \\
&+2\textstyle\sum_{j=1}^n \overline{a}_{ij}\int_\Omega |u_i(t_0,x)||f_j(u_j(t_0-\tau_{ij}(t_0),x))|dx \\
&+2\textstyle\sum_{j=1}^n \overline{b}_{ij}\int_\Omega |u_i(t_0,x)||H_j(u_j(t_0,x))|dx \\
&+2\textstyle\sum_{j=1}^n \overline{b}_{ij}\int_\Omega |u_i(t_0,x)||H_j(u_j(t_0-\tau,x))|dx + 2\int_\Omega |u_i(t_0,x)|\overline{I}_i dx \\
&\le -2\int_\Omega \textstyle\sum_{k=1}^m d_{ik}(\frac{\partial u_i(t_0,x)}{\partial x_k})^2 dx - 2\int_\Omega \underline{c}_i u_i^2(t_0,x)dx \\
&+2\textstyle\sum_{j=1}^n \overline{a}_{ij}L_j\int_\Omega |u_i(t_0,x)||u_j(t_0-\tau_{ij}(t_0),x))|dx \\
&+2\textstyle\sum_{j=1}^n \overline{b}_{ij}M_j\int_\Omega |u_i(t_0,x)|^2 dx \\
&+2\textstyle\sum_{j=1}^n \overline{b}_{ij}M_j\int_\Omega |u_i(t_0,x)||u_j(t_0-\tau,x))|dx + 2\int_\Omega |u_i(t_0,x)|\overline{I}_i dx \\
&\le -2\tfrac{d_i}{C_0}\int_\Omega u_i^2(t_0,x)dx - 2\underline{c}_i\int_\Omega u_i^2(t_0,x)dx \\
&+ \textstyle\sum_{j=1}^n \overline{a}_{ij}L_j\int_\Omega u_i^2(t_0,x)dx + \sum_{j=1}^n \overline{a}_{ij}L_j\int_\Omega u_j^2(t_0-\tau_{ij}(t_0),x))dx \\
&+2\textstyle\sum_{j=1}^n \overline{b}_{ij}M_j\int_\Omega u_i^2(t_0,x)dx \\
&+ \textstyle\sum_{j=1}^n \overline{b}_{ij}M_j\int_\Omega u_i^2(t_0,x)dx + \sum_{j=1}^n \overline{b}_{ij}M_j\int_\Omega u_i^2(t_0-\tau,x)dx \\
&+\overline{I}_i\int_\Omega u_i^2(t_0,x)dx + \overline{I}_i|\Omega| \\
&\le -2(\tfrac{d_i}{C_0} + \underline{c}_i - \textstyle\sum_{j=1}^n \overline{a}_{ij}L_j - 2\sum_{j=1}^n \overline{b}_{ij}M_j - \frac{1}{2}\overline{I}_i)\|u_i(t_0,x)\|^2 + \overline{I}_i|\Omega| \\
&\le -2\gamma\tfrac{\overline{I}|\Omega|}{\gamma} + \overline{I}_i|\Omega| \\
&< 0
\end{aligned}
$$

which is a contradiction and hence (3.3) holds. Now, we proof that (3.6) also holds. Suppose that (3.6) is not true, then there exist $t_0 > 0$ and some $i \in \{1, 2, \cdots, n\}$ such that

$$\|u_i(t_0,\cdot)\|_p^p = \frac{(\overline{I})^p|\Omega|}{\eta},\ \ \|u_i(t,\cdot)\|_p^p \le \frac{(\overline{I})^p|\Omega|}{\eta},\ t \in [-\overline{\tau}, t_0],$$

$$\|u_j(t,\cdot)\|_p^p \le \frac{(\overline{I})^p|\Omega|}{\eta},\ t \in [-\overline{\tau}, t_0],\ j \ne i,$$

$$\frac{d}{dt}\|u_i(t_0,\cdot)\|_p^p \ge 0.$$

In view of (H_2), (H_3), Lemma 2.2 and Hölder inequality, we have

$$
\begin{aligned}
&0 \le \tfrac{d}{dt}\|u_i(t_0,\cdot)\|_p^p \\
&= p\int_\Omega [u_i(t_0,x)]^{p-1}\Big[\textstyle\sum_{k=1}^m d_{ik}\frac{\partial^2 u_i(t_0,x)}{\partial x_k^2} - c_i(t_0)u_i(t_0,x) + \sum_{j=1}^n a_{ij}(t_0)f_j(u_j(t_0-\tau_{ij}(t_0),x)) \\
&+ \textstyle\sum_{j=1}^n b_{ij}(t_0)\int_{t_0-\tau}^{t_0} h_j(u_j(s,x))ds + I_i(t_0)\Big]dx \\
&\le -p\int_\Omega \textstyle\sum_{k=1}^m d_{ik}[u_i(t_0,x)]^{p-2}(\frac{\partial u_i(t_0,x)}{\partial x_k})^2 dx - p\int_\Omega \underline{c}_i u_i^p(t_0,x)dx \\
&+p\textstyle\sum_{j=1}^n \overline{a}_{ij}\int_\Omega |u_i(t_0,x)|^{p-1}|f_j(u_j(t_0-\tau_{ij}(t_0),x))|dx \\
&+p\textstyle\sum_{j=1}^n \overline{b}_{ij}\int_\Omega |u_i(t_0,x)|^{p-1}|H_j(u_j(t_0,x))|dx \\
&+p\textstyle\sum_{j=1}^n \overline{b}_{ij}\int_\Omega |u_i(t_0,x)|^{p-1}|H_j(u_j(t_0-\tau,x))|dx + p\int_\Omega |u_i(t_0,x)|^{p-1}\overline{I}_i dx \\
&\le -p\int_\Omega \underline{c}_i u_i^p(t_0,x)dx \\
&+p\textstyle\sum_{j=1}^n \overline{a}_{ij}L_j\int_\Omega |u_i(t_0,x)|^{p-1}|u_j(t_0-\tau_{ij}(t_0),x))|dx \\
&+p\textstyle\sum_{j=1}^n \overline{b}_{ij}M_j\int_\Omega |u_i(t_0,x)|^{p-1}|u_j(t_0,x)|dx \\
&+p\textstyle\sum_{j=1}^n \overline{b}_{ij}M_j\int_\Omega |u_i(t_0,x)|^{p-1}|u_j(t_0-\tau,x))|dx + p\int_\Omega |u_i(t_0,x)|^{p-1}\overline{I}_i dx \\
&\le -p\underline{c}_i\int_\Omega u_i^p(t_0,x)dx \\
&+p\textstyle\sum_{j=1}^n \overline{a}_{ij}L_j\Big(\int_\Omega |u_i(t_0,x)|^p dx\Big)^{\frac{p-1}{p}}\Big(\int_\Omega |u_j(t_0-\tau_{ij}(t_0),x)|^p dx\Big)^{\frac{1}{p}} \\
&+p\textstyle\sum_{j=1}^n \overline{b}_{ij}M_j\Big(\int_\Omega |u_i(t_0,x)|^p dx\Big)^{\frac{p-1}{p}}\Big(\int_\Omega |u_j(t_0,x)|^p dx\Big)^{\frac{1}{p}} \\
&+p\textstyle\sum_{j=1}^n \overline{b}_{ij}M_j\Big(\int_\Omega |u_i(t_0,x)|^p dx\Big)^{\frac{p-1}{p}}\Big(\int_\Omega |u_j(t_0-\tau,x)|^p dx\Big)^{\frac{1}{p}} \\
&+p\Big(\int_\Omega |u_i(t_0,x)|^p dx\Big)^{\frac{p-1}{p}}\overline{I}_i|\Omega|^{\frac{1}{p}} \\
&\le -p\underline{c}_i\int_\Omega |u_i(t_0,x)|^p dx \\
&+(p-1)\textstyle\sum_{j=1}^n \overline{a}_{ij}L_j\int_\Omega |u_i(t_0,x)|^p dx + \sum_{j=1}^n \overline{a}_{ij}L_j\int_\Omega |u_j(t_0-\tau_{ij}(t_0),x)|^p dx \\
&+(p-1)\textstyle\sum_{j=1}^n \overline{b}_{ij}M_j\int_\Omega |u_i(t_0,x)|^p dx + \sum_{j=1}^n \overline{b}_{ij}M_j\int_\Omega |u_j(t_0,x)|^p dx \\
&+(p-1)\textstyle\sum_{j=1}^n \overline{b}_{ij}M_j\int_\Omega |u_i(t_0,x)|^p dx + \sum_{j=1}^n \overline{b}_{ij}M_j\int_\Omega |u_j(t_0-\tau,x)|^p dx \\
&+(p-1)\overline{I}_i\int_\Omega |u_i(t_0,x)|^p dx + (\overline{I}_i)^p|\Omega| \\
&\le -(p\underline{c}_i - p\textstyle\sum_{j=1}^n \overline{a}_{ij}L_j - 2p\sum_{j=1}^n \overline{b}_{ij}M_j - (p-1)\overline{I}_i)\|u_i(t_0,\cdot)\|_p^p + (\overline{I}_i)^p|\Omega| \\
&\le -2\eta\tfrac{(\overline{I})^p|\Omega|}{\eta} + (\overline{I}_i)^p|\Omega| \\
&< 0
\end{aligned}
$$

which is a contradiction and hence (3.6) holds.

Theorem 3.2. Suppose that

$$\xi_1 = \min_{1\le i\le n}\left\{\frac{d_i}{C_0} + \underline{c}_i - \sum_{j=1}^n \overline{a}_{ij}L_j - 2\sum_{j=1}^n \overline{b}_{ij}M_j\right\} > 0 \tag{3.7}$$

Let $u(t,x), u^0(t,x)$ be any two solutions of system (2.1) with initial function $\phi(t,x)$ and $\phi^0(x)$ satisfying (3.2), then there exists a positive constant $\varepsilon > 0$ such that

$$\max_{1\le i\le n}\{\|u_i(t,\cdot) - u_i^0(t,\cdot)\|_2^2\} \le \sup_{s\in[-\overline{\tau},0]}\max_{1\le i\le n}\{\|\phi_i(s,\cdot) - \phi_i^0(s,\cdot)\|_2^2\}e^{-\varepsilon t}, \quad t > 0 \tag{3.8}$$

Furthermore, suppose that

$$\xi_2 = \min_{1\le i\le n}\Big\{p\underline{c}_i - p\sum_{j=1}^{n}\overline{a}_{ij}L_j - 2p\sum_{j=1}^{n}\overline{b}_{ij}M_j\Big\} > 0, \tag{3.9}$$

where $p > 2$ is even number. Let $u(t,x), u^0(t,x)$ be any two solutions of system (2.1) with initial function $\phi(t,x)$ and $\phi^0(x)$ satisfying (3.5), then there exists a positive constant $\varepsilon > 0$ such that

$$\max_{1\le i\le n}\{\|u_i(t,\cdot) - u_i^0(t,\cdot)\|_p^p\} \le \sup_{s\in[-\overline{\tau},0]}\max_{1\le i\le n}\{\|\phi_i(s,\cdot) - \phi_i^0(s,\cdot)\|_p^p\}e^{-\varepsilon t}, \quad t > 0. \tag{3.10}$$

Proof. The first section of the proof in Theorem 3.2 is similar to the corresponding one in [16]. In order to completeness of the present paper, we give the detail proof. Define the following continuous functions:

$$\Xi_{1,i}(\mu) = \mu - 2\frac{\underline{d}_i}{C_0} - 2\underline{c}_i + (\sum_{j=1}^{n}\overline{a}_{ij}L_j + 2\sum_{j=1}^{n}\overline{b}_{ij}M_j)(1 + e^{\mu\overline{\tau}}),\ \mu \ge 0,\ i = 1,2,\cdots,n.$$

By (3.7), we have

$$\Xi_{1,i}(0) = -2\frac{\underline{d}_i}{C_0} - 2\underline{c}_i + 2(\sum_{j=1}^{n}\overline{a}_{ij}L_j + 2\sum_{j=1}^{n}\overline{b}_{ij}M_j) < 0,\ i = 1,2,\cdots,n.$$

In view of the continuity of $\Xi_i(\mu)$, there exists a constant $\varepsilon > 0$ such that

$$\Xi_{1,i}(\varepsilon) = \varepsilon - 2\frac{\underline{d}_i}{C_0} - 2\underline{c}_i + (\sum_{j=1}^{n}\overline{a}_{ij}L_j + 2\sum_{j=1}^{n}\overline{b}_{ij}M_j)(1 + e^{\varepsilon\overline{\tau}}) < 0,\ i = 1,2,\cdots,n. \tag{3.11}$$

Set $y_i(t,x) = u_i(t,x) - u_i^0(t,x)$. Then

$$\begin{cases} \frac{\partial y_i(t,x)}{\partial t} = \sum_{k=1}^{m} d_{ik}\frac{\partial^2 y_i(t,x)}{\partial x_k^2} - c_i(t)y_i(t,x) \\ \quad + \sum_{j=1}^{n} a_{ij}(t)[f_j(u_j(t-\tau_{ij}(t),x)) - f_j(u_j^0(t-\tau_{ij}(t),x))] \\ \quad + \sum_{j=1}^{n} b_{ij}(t)[\int_{t-\tau}^{t} h_j(u_j(s,x)) - h_j^0(u_j(s,x))]ds,\ (t,x) \in (0,\infty)\times\Omega, \\ y_i(t,x) = 0,\ (t,x) \in [0,\infty)\times\partial\Omega, \\ y_i(t,x) = \phi_i(t,x) - \phi_i^0(t,x)\ (t,x) \in [-\overline{\tau},0]\times\Omega. \end{cases} \tag{3.12}$$

Let

$$V_1(t) = \max_{1\le i\le n}\{e^{\varepsilon t}\|y_i(t,\cdot)\|_2^2\},\quad M_1(t) = \sup_{-\overline{\tau}\le s\le t} V_1(s).$$

Then

$$V_1(t) \le M_1(t) \text{ for all } t \ge 0.$$

We claim

$$M_1(t) = M_1(0) \text{ for all } t \ge 0.$$

In fact, for any $t_0 \ge 0$, there exist two cases as follows:
Case 1. $V_1(t_0) < M_1(t_0)$. Obviously, there exists a $\varepsilon_1 > 0$ such that

$$V_1(t) \le M_1(t_0) \text{ for } t \in [t_0, t_0 + \varepsilon_1).$$

Hence

$$M_1(t) = M_1(t_0) \text{ for } t \in [t_0, t_0 + \varepsilon_1).$$

Case 2. $V_1(t_0) = M_1(t_0)$. Then $V_1(t) \leq V_1(t_0),\ t \in [-\overline{\tau}, t_0]$. Hence, there exists an $i \in \{1, 2, \cdots, n\}$ such that

$$e^{\varepsilon t_0} \|y_i(t_0, \cdot)\|_2^2 = V_1(t_0),\ \ e^{\varepsilon t} \|y_j(t, \cdot)\|_2^2 \leq V_1(t_0),\ j \neq i,\ t \in [-\overline{\tau}, t_0].$$

Along the trajectories of system (3.12), in view of (H_2), (H_3), Lemma 2.1 and (3.11), calculating the value of derivative $V_{1,i}(t)$ at $t = t_0$ leads to

$$\begin{aligned}
\frac{d}{dt} V_{1,i}(t_0) = {} & \varepsilon e^{\varepsilon t_0} \|y_i(t_0, x)\|_2^2 + 2e^{\varepsilon t_0} \int_\Omega y_i(t_0, x) \Big[\sum_{k=1}^m d_{ik} \frac{\partial^2 y_i(t_0, x)}{\partial x_k^2} - c_i(t_0) y_i(t_0, x) \\
& + \sum_{j=1}^n a_{ij}(t_0)(f_j(u_j(t_0 - \tau_{ij}(t_0), x)) - f_j(u_j^0(t_0 - \tau_{ij}(t_0), x))) \\
& + \sum_{j=1}^n b_{ij}(t_0) \int_{t_0 - \tau}^{t_0} (h_j(u_j(s, x)) - h_j(u_j^0(s, x))) ds \Big] dx \\
& < \varepsilon e^{\varepsilon t_0} \|y_i(t_0, x)\|_2^2 + e^{\varepsilon t_0} \Big[-2 \int_\Omega \sum_{k=1}^m d_{ik} (\frac{\partial y_i(t_0, x)}{\partial x_k})^2 dx - 2 \int_\Omega \underline{c}_i y_i^2(t_0, x) dx \\
& + 2 \sum_{j=1}^n \overline{a}_{ij} L_j \int_\Omega |y_i(t_0, x)||y_j(t_0 - \tau_{ij}(t_0), x)| dx \\
& + 2 \sum_{j=1}^n \overline{b}_{ij} M_j \int_\Omega |y_i(t_0, x)||y_j(t_0, x)| dx \\
& + 2 \sum_{j=1}^n \overline{b}_{ij} M_j \int_\Omega |y_i(t_0, x)||y_j(t_0 - \tau, x)| dx \Big] \\
& \leq \varepsilon e^{\varepsilon t_0} \|y_i(t_0, x)\|_2^2 + e^{\varepsilon t_0} \Big[-2 \frac{\underline{d}_i}{C_0} \|y_i(t_0, x)\|_2^2 - 2 \underline{c}_i \|y_i(t_0, x)\|_2^2 \\
& + \sum_{j=1}^n \overline{a}_{ij} L_j (\|y_i(t_0, x)\|_2^2 + \|y_j(t_0 - \tau_{ij}(t_0), x)\|_2^2) \\
& + \sum_{j=1}^n \overline{b}_{ij} M_j (\|y_i(t_0, x)\|_2^2 + \|y_j(t_0, x)\|_2^2) \\
& + \sum_{j=1}^n \overline{b}_{ij} M_j (\|y_i(t_0, x)\|_2^2 + \|y_j(t_0 - \tau, x)\|_2^2) \Big] \\
& \leq \Big[\varepsilon - 2 \frac{\underline{d}_i}{C_0} - 2 \underline{c}_i + (\sum_{j=1}^n \overline{a}_{ij} L_j + 2 \sum_{j=1}^n \overline{b}_{ij} M_j)(1 + e^{\varepsilon \overline{\tau}}) \Big] V(t_0) \\
& < 0.
\end{aligned}$$

From the above inequality, there exists a $\varepsilon_2 > 0$ such that

$$V_1(t) < V_1(t_0) \text{ for } t \in [t_0, t_0 + \varepsilon_2).$$

Thus, $M_1(t) = M_1(0)$ for $t \geq 0$ and (3.8) holds. Furthermore, define the following continuous functions:

$$\Xi_{2,i}(\mu) = \mu - p\underline{c}_i + (p-1)\sum_{j=1}^{n}\overline{a}_{ij}L_j + 2(p-1)\sum_{j=1}^{n}\overline{b}_{ij}M_j + (\sum_{j=1}^{n}\overline{a}_{ij}L_j + 2\sum_{j=1}^{n}\overline{b}_{ij}M_j)e^{\mu\overline{\tau}},\ \mu \geq 0,\ i = 1, 2, \cdots, n.$$

By (3.7), we have

$$\Xi_{2,i}(0) = -p\underline{c}_i + p\sum_{j=1}^{n}\overline{a}_{ij}L_j + 2p\sum_{j=1}^{n}\overline{b}_{ij}M_j < 0,\ i = 1, 2, \cdots, n.$$

In view of the continuity of $\Xi_{2,i}(\mu)$, there exists a constant $\varepsilon > 0$ such that

$$\Xi_{2,i}(\varepsilon) = \varepsilon - p\underline{c}_i + (p-1)\sum_{j=1}^{n}\overline{a}_{ij}L_j + 2(p-1)\sum_{j=1}^{n}\overline{b}_{ij}M_j + (\sum_{j=1}^{n}\overline{a}_{ij}L_j + 2\sum_{j=1}^{n}\overline{b}_{ij}M_j)e^{\varepsilon\overline{\tau}} < 0,\ i = 1, 2, \cdots, n. \tag{3.13}$$

Let

$$V_2(t) = \max_{1\leq i\leq n}\{e^{\varepsilon t}\|y_i(t,\cdot)\|_p^p\},\quad M_2(t) = \sup_{-\overline{\tau}\leq s\leq t} V_2(s).$$

Then

$$V_2(t) \leq M_2(t) \text{ for all } t \geq 0.$$

We claim

$$M_2(t) = M_2(0) \text{ for all } t \geq 0.$$

In fact, for any $t_0 \geq 0$, there exist two cases as follows:
Case 1. $V_2(t_0) < M_2(t_0)$. Obviously, there exists a $\varepsilon_1 > 0$ such that

$$V_2(t) \leq M_2(t_0) \text{ for } t \in [t_0, t_0 + \varepsilon_1).$$

Hence

$$M_2(t) = M_2(t_0) \text{ for } t \in [t_0, t_0 + \varepsilon_1).$$

Case 2. $V_2(t_0) = M_2(t_0)$. Then $V_2(t) \leq V_2(t_0),\ t \in [-\overline{\tau}, t_0]$. Hence, there exists an $i \in \{1, 2, \cdots, n\}$ such that

$$e^{\varepsilon t_0}\|y_i(t_0,\cdot)\|_p^p = V_2(t_0),\quad e^{\varepsilon t}\|y_j(t,\cdot)\|_p^p \leq V_2(t_0),\ j \neq i,\ t \in [-\overline{\tau}, t_0].$$

Along the trajectories of system (3.12), in view of (H_2), (H_3), Lemma 2.2, Hölder inequality and (3.13), calculating the value of derivative $V_{2,i}(t)$ at $t = t_0$ leads to

$$
\begin{aligned}
\frac{d}{dt}V_{2,i}(t_0) &= \varepsilon e^{\varepsilon t_0}\|y_i(t_0,x)\|_p^p + pe^{\varepsilon t_0}\int_\Omega (y_i(t_0,x))^{p-1}\Big[\sum_{k=1}^m d_{ik}\frac{\partial^2 y_i(t_0,x)}{\partial x_k^2} - c_i(t_0)y_i(t_0,x) \\
&+ \sum_{j=1}^n a_{ij}(t_0)(f_j(u_j(t_0-\tau_{ij}(t_0),x)) - f_j(u_j^0(t_0-\tau_{ij}(t_0),x))) \\
&+ \sum_{j=1}^n b_{ij}(t_0)\int_{t_0-\tau}^{t_0}(h_j(u_j(s,x)) - h_j(u_j^0(s,x)))ds\Big]dx \\
&\le \varepsilon e^{\varepsilon t_0}\|y_i(t_0,x)\|_p^p - p\underline{c}_i e^{\varepsilon t_0}\int_\Omega (y_i(t_0,x))^p \\
&+ pe^{\varepsilon t_0}\sum_{j=1}^n \overline{a}_{ij}L_j\int_\Omega |y_i(t_0,x)|^{p-1}|y_j(t_0-\tau_{ij}(t_0),x)|dx \\
&+ pe^{\varepsilon t_0}\sum_{j=1}^n \overline{b}_{ij}M_j\int_\Omega |y_i(t_0,x)|^{p-1}|y_j(t_0,x)|dx \\
&+ pe^{\varepsilon t_0}\sum_{j=1}^n \overline{b}_{ij}M_j\int_\Omega |y_i(t_0,x)|^{p-1}|y_j(t_0-\tau,x)|dx \\
&\le \varepsilon e^{\varepsilon t_0}\|y_i(t_0,x)\|_p^p - pe^{\varepsilon t_0}\underline{c}_i\int_\Omega |y_i|^p(t_0,x)dx \\
&+ pe^{\varepsilon t_0}\sum_{j=1}^n \overline{a}_{ij}L_j\Big(\int_\Omega |y_i(t_0,x)|^p dx\Big)^{\frac{p-1}{p}}\Big(\int_\Omega |y_j(t_0-\tau_{ij}(t_0),x)|^p dx\Big)^{\frac{1}{p}} \\
&+ pe^{\varepsilon t_0}\sum_{j=1}^n \overline{b}_{ij}M_j\Big(\int_\Omega |y_i(t_0,x)|^p dx\Big)^{\frac{p-1}{p}}\Big(\int_\Omega |y_j(t_0,x)|^p dx\Big)^{\frac{1}{p}} \\
&+ pe^{\varepsilon t_0}\sum_{j=1}^n \overline{b}_{ij}M_j\Big(\int_\Omega |y_i(t_0,x)|^p dx\Big)^{\frac{p-1}{p}}\Big(\int_\Omega |y_j(t_0-\tau,x)|^p dx\Big)^{\frac{1}{p}} \\
&\le \varepsilon e^{\varepsilon t_0}\|y_i(t_0,x)\|_p^p - pe^{\varepsilon t_0}\underline{c}_i\int_\Omega |y_i|^p(t_0,x)dx \\
&+ (p-1)e^{\varepsilon t_0}\sum_{j=1}^n \overline{a}_{ij}L_j\int_\Omega |y_i(t_0,x)|^p dx + e^{\varepsilon t_0}\sum_{j=1}^n \overline{a}_{ij}L_j\int_\Omega |y_j(t_0-\tau_{ij}(t_0),x)|^p dx \\
&+ (p-1)e^{\varepsilon t_0}\sum_{j=1}^n \overline{b}_{ij}M_j\int_\Omega |y_i(t_0,x)|^p dx + e^{\varepsilon t_0}\sum_{j=1}^n \overline{b}_{ij}M_j\int_\Omega |y_j(t_0,x)|^p dx \\
&+ (p-1)e^{\varepsilon t_0}\sum_{j=1}^n \overline{b}_{ij}M_j\int_\Omega |y_i(t_0,x)|^p dx + e^{\varepsilon t_0}\sum_{j=1}^n \overline{b}_{ij}M_j\int_\Omega |y_j(t_0-\tau,x)|^p dx \\
&\le \Big[\varepsilon - p\underline{c}_i + (p-1)\sum_{j=1}^n \overline{a}_{ij}L_j + 2(p-1)\sum_{j=1}^n \overline{b}_{ij}M_j + (\sum_{j=1}^n \overline{a}_{ij}L_j + 2\sum_{j=1}^n \overline{b}_{ij}M_j)e^{\varepsilon\overline{\tau}}\Big]V_{2,i}(t_0) \\
&< 0.
\end{aligned}
$$

From the above inequality, there exists a $\varepsilon_2 > 0$ such that

$$V_2(t) < V_2(t_0) \text{ for } t \in [t_0, t_0 + \varepsilon_2).$$

Thus, $M_2(t) = M_2(0)$ for $t \geq 0$ and (3.10) holds.

Remark 3.1. From Theorem 3.2, the periodic solution of system (2.1) is globally exponentially stable.

Remark 3.2. Theorems 3.1 and 3.2 generalize the corresponding results of [16]. In fact, in [16], the authors obtained that the dynamic properties of solution to system (2.1) based on the sense of L_2 norm. However, in this paper we generalize the sense of L_2 norm to the sense of L_p norm, where $p > 2$ is even number. When $p > 2$ is odd number, some classic inequalities are not available for system (2.1), we can not obtain relevant results for system (2.1). When $p > 2$ is odd number, we wish that some further results can be obtained in the future.

Similar to the proof of Theorem 3.3 and 3.4 in [16], we have the following theorems for the existence and globally exponential stability of periodic mild solution to system (2.1).

Theorem 3.3. Under the assumptions of Theorem 3.1, and $d_{ik} > 0, i = 1, 2, \cdots, n, k = 1, 2, \cdots, m$, system (2.1) admits a $\omega-$periodic mild solution.

Theorem 3.4. Under the assumptions of Theorem 3.3, and assume that

$$\max_{1\leq i\leq n}\{K_i[\overline{c}_i + \sum_{j=1}^{n}\overline{a}_{ij}L_j + 2\sum_{j=1}^{n}\overline{b}_{ij}M_j]\} < \alpha,$$

where K_i and α_i are positive constants, $\alpha = \min_{1\leq i\leq n}\{\alpha_i\}$. Then system (2.1) admits a $\omega-$periodic mild solution $u^*(t, x)$ which is globally exponentially stable in the sense of L_2 norm. That is, for any $\varepsilon > 0$ and any solution $u(t, x)$ of system (2.1),

$$e^{\varepsilon t}\max_{1\leq i\leq n}\|u_i(t, x) - u_i^*(t, x)\|_2 = 0 \text{ as } t \to \infty.$$

4. Example

In this section, we present an example so as to illustrate the usefulness of our main results.

$$\begin{cases} \frac{\partial u_1(t,x)}{\partial t} = \frac{\partial^2 u_1(t,x)}{\partial x^2} - (11 + \sin t)u_1(t, x) + \frac{1}{2}\cos t f_1(u_1(t - 0.5, x)) + \frac{1}{4}\cos t f_2(u_2(t - 0.5, x)) \\ \quad + \sin t\int_{t-0.2}^{t} h_1(u_1(s, x))ds + \frac{1}{4}\cos t\int_{t-0.2}^{t} h_2(u_2(s, x))ds + 0.4\cos t \\ \frac{\partial u_2(t,x)}{\partial t} = \frac{\partial^2 u_2(t,x)}{\partial x^2} - (12 + 2\sin t)u_2(t, x) + \frac{1}{3}\cos t f_1(u_1(t - 0.5, x)) + \frac{1}{5}\cos t f_2(u_2(t - 0.5, x)) \\ \quad + \sin t\int_{t-0.2}^{t} h_1(u_1(s, x))ds + \frac{1}{2}\cos t\int_{t-0.2}^{t} h_2(u_2(s, x))ds + 0.4\cos t \\ u_i(t, 0) = u_i(t, 1) = 0, \quad t \in [0, \infty), \\ u_i(t, x) = \phi_i(t, x), \quad (t, x) \in [-0.5, 0] \times (0, 1), i = 1, 2, \end{cases} \tag{4.1}$$

where $f_i(v) = |v + 1| - |v - 1|$, $i = 1, 2$, then $f_i(v)$ satisfies (H_2) with $L_i = 2$, $i = 1, 2$; $h_i(v) = 2$, then $H_i(v) = 2v$ satisfies (H_3) with $M_i = 2$, $i = 1, 2$. From simple calculation, we have

$$\underline{c}_1 = \underline{c}_2 = 10, \ \overline{a}_{11} = \frac{1}{2}, \ \overline{a}_{12} = \frac{1}{4}, \ \overline{b}_{11} = 1, \ \overline{b}_{12} = \frac{1}{4},$$

$$\overline{a}_{21} = \frac{1}{3},\ \overline{a}_{22} = \frac{1}{5},\ \overline{b}_{21} = 1,\ \overline{b}_{22} = \frac{1}{2},\ \overline{I}_1 = \overline{I}_2 = 0.4,$$

$$\underline{c}_1 - \sum_{j=1}^{2} \overline{a}_{1j} L_j - 2\sum_{j=1}^{2} \overline{b}_{1j} M_j - \frac{1}{2}\overline{I}_1 = 4.3 > 0,$$

$$\underline{c}_2 - \sum_{j=1}^{2} \overline{a}_{2j} L_j - 2\sum_{j=1}^{2} \overline{b}_{2j} M_j - \frac{1}{2}\overline{I}_2 = \frac{56}{15} > 0.$$

Furthermore, let $p = 4$, we have

$$p\underline{c}_1 - p\sum_{j=1}^{2} \overline{a}_{1j} L_j - 2p\sum_{j=1}^{2} \overline{b}_{1j} M_j - (p-1)\overline{I}_1 = 12.8 > 0$$

$$p\underline{c}_2 - p\sum_{j=1}^{2} \overline{a}_{2j} L_j - 2p\sum_{j=1}^{2} \overline{b}_{2j} M_j - (p-1)\overline{I}_2 = \frac{158}{15} > 0.$$

Thus, system (4.1) satisfies all the conditions in Theorem 3.1. Hence, system (4.1) has one 2π-periodic mild solution which is bounded and globally exponentially stable.

5. Conclusion

In this article, we study a new reaction-diffusion neural networks with mixed delays. In [16], the properties of periodic mild solution of L_2 norm sense were obtained. In the present paper, we generalize the above results to the L_p norm sense, where $p > 2$ is even number.

Both by theoretical analysis and numerical simulations, we show how the parameters of system affect dynamic characteristic. It is noted that system (2.1) is a non-autonomous partial differential equation, we use some new techniques to deal with this system. Furthermore, an example verify the correctness of theoretical analyses. However, many important questions about reaction-diffusion neural networks remain to be studied, such as clustering for complex networks, optimal control, LMI-based stability criteria, and bifurcation problems.

Acknowledgments

The authors would like to express the sincere appreciation to the editor and reviewers for their helpful comments in improving the presentation and quality of the paper.

Conflict of interest

The authors confirm that they have no conflict of interest.

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