



Research article

Stability analysis for time delay systems via a generalized double integral inequality

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Abstract: This paper proposes a new stability condition for a class of time delay systems. Firstly, a generalized double integral inequality is obtained. Then, a less conservative stability criterion is proposed by using the double integral inequality and choosing some new Lyapunov-Krasovskii functionals. Finally, two numerical examples are proposed to show the effectiveness of our method.

Keywords: time delay systems; linear matrix inequality (LMI); double integral inequality; Lyapunov-Krasovskii functional (LKF)

Mathematics Subject Classification: 34D20, 34K20, 34K25

1. Introduction

A time delay widely exists in many fields such as chemistry, biology, industry, and so on. Since a time delay arising in a system may cause instability, the stability analysis of time-delay systems has been widely studied in the past few decades [1,2]. The main purpose is paid to determine the admissible delay, for which the systems remain stable.

It is well known that the LKF method has been widely used to obtain stability conditions for time-delay systems [3, 4]. The main purpose of the LKF method is to estimate the integral term arising in the time derivative of double integral term in the LKF. Therefore, to get less conservative stability criteria, many integral inequality methods are derived. Those inequality methods include Jensen inequality [5, 6], Wirtinger inequality [7–9], double integral inequality [10–12], various improved integral inequalities [13–26]. The Jensen inequality expressed as $V_{ab}(\dot{y}) = \int_a^b \dot{y}^T(t) R \dot{y}(t) dt \geq \frac{1}{b-a} \Omega_0^T R \Omega_0 = V_{Jensen}$, where $a < b$, $R = R^T > 0$, $\Omega_0 = y(b) - y(a)$. The Wirtinger-based inequality expressed as $V_{ab}(\dot{y}) \geq V_{Jensen} + \frac{3}{b-a} \Omega_1^T R \Omega_1 = V_{Seuret}$, where

$\Omega_1 = y(b) + y(a) - \frac{2}{b-a} \int_a^b y(t)dt$. The further improved inequality expressed as $V_{ab}(\dot{y}) \geq V_{Seuret} + \frac{5}{b-a} \Omega_2^T R \Omega_2$, $V_{ab}(\dot{y}) \geq V_{Seuret} + \frac{5}{b-a} \Omega_2^T R \Omega_2 + \frac{7}{b-a} \Omega_3^T R \Omega_3$, $V_{ab}(\dot{y}) \geq V_{Seuret} + \frac{5}{b-a} \Omega_2^T R \Omega_2 + \frac{7}{b-a} \Omega_3^T R \Omega_3 + \frac{9}{b-a} \Omega_4^T R \Omega_4$ in [13–15], respectively, where $\Omega_2, \Omega_3, \Omega_4$ are defined in Lemma 4 [15]. However, these results only estimate the integral term arising in the time derivative of double integral term in the LKF. This paper presents a generalized double integral inequality which includes those in [10–12] as special cases. A new stability criterion is proposed by choosing a new LKF and using the generalized double integral inequality. Both the generalized integral inequality and the new LKF include fourth integrals, which may yield less conservative results. Two examples are introduced to show the effectiveness of the proposed criterion. The contributions of our paper are as follows:

- The integral $-\int_a^b \int_u^b \dot{x}^T(s) P \dot{x}(s) ds du$ is estimated as $-\int_a^b \int_u^b \dot{x}^T(s) P \dot{x}(s) ds du \leq \zeta^T \omega \zeta$, where ω and ζ are defined in Lemma 3. The above inequality includes those in [10–12] as special cases.
- Both the new double integral inequality and the new LKF include fourth integrals, which may obtain more general results.

Notation: See Table 1.

Table 1. Nomenclature.

S_+^n	$\rightarrow$ the set of $n \times n$ symmetric positive definite matrices
$\mathbb{R}^m$	$\rightarrow$ m-dimensional Euclidean space
A^T	$\rightarrow$ The transpose of the A matrix
$Sym(P)$	$\rightarrow P + P^T$, For any square matrix P
0	$\rightarrow$ A zero matrix with appropriate dimensions

2. Preliminary

Consider the time delay systems as

$$\dot{y}(t) = Ay(t) + By(t-h) + C \int_{t-h}^t y(s)ds \quad (2.1)$$

$$y(t) = \phi(t), \quad t \in [-h, 0] \quad (2.2)$$

where $y(t) \in \mathbb{R}^n$ is the state vector, $h > 0$ is constant time-delay and the initial condition $\phi(t)$ is a continuous function.

Lemma 1. [15] For a matrix $P \in S_+^n$, and any continuously differentiable function $x : [a, b] \rightarrow \mathbb{R}^n$, then we can obtain

$$\int_a^b \dot{x}^T(s) P \dot{x}(s) ds \geq \frac{1}{b-a} \sum_{i=0}^4 (2i+1) \Omega_i^T P \Omega_i \quad (2.3)$$

where

$$\Omega_0 = x(b) - x(a)$$

$$\Omega_1 = x(b) + x(a) - \frac{2}{b-a} \int_a^b x(t)dt$$

$$\Omega_2 = x(b) - x(a) + \frac{6}{b-a} \int_a^b x(t)dt - \frac{12}{(b-a)^2} \int_a^b \int_u^b x(t)dtdu$$

$$\Omega_3 = x(b) + x(a) - \frac{12}{b-a} \int_a^b x(t)dt + \frac{60}{(b-a)^2} \int_a^b \int_u^b x(t)dtdu - \frac{120}{(b-a)^3} \int_a^b \int_u^b \int_v^b x(t)dtdvdvdu$$

$$\Omega_4 = x(b) - x(a) + \frac{20}{b-a} \int_a^b x(t)dt - \frac{180}{(b-a)^2} \int_a^b \int_u^b x(t)dtdu + \frac{840}{(b-a)^3} \int_a^b \int_u^b \int_v^b x(t)dtdvdvdu \\ - \frac{1680}{(b-a)^4} \int_a^b \int_u^b \int_v^b \int_\beta^b x(t)dtd\beta dvdu$$

Lemma 2. [27] For a positive definite matrix $P \in S_+^n$, a integrable function $\{x(s) | s \in [a, b]\}$, and any auxiliary functions $\{f_i(s) | i \in [0, n], s \in [a, b], f_0(s) = 1\}$ satisfying $\int_a^b \int_u^b f_i(s)f_j(s)dsdu = 0$, ($0 \leq i, j \leq n, i \neq j$) with $f_i(s) \neq 0, i = 1, 2, \dots, n$. Let $\lambda_i \in R^{n \times k}, i = 0, 1, \dots, n$ and a vector $\zeta \in R^k$, such that $\int_a^b \int_u^b f_i(s)x(s)dsdu = \lambda_i \zeta$. Then for any matrices $M_i \in R^{k \times n} (i = 0, 1, \dots, n)$, the following inequality holds

$$- \int_a^b \int_u^b x^T(s)Px(s)dsdu \leq \zeta^T \left\{ \sum_{i=0}^n \int_a^b \int_u^b f_i^2(s)dsdu M_i P^{-1} M_i^T + \text{Sym} \left(\sum_{i=0}^n M_i \lambda_i \right) \right\} \zeta \quad (2.4)$$

Proof. Define $M = \begin{bmatrix} M_0^T & M_1^T & \cdots & M_n^T \end{bmatrix}^T, \xi(s) = \begin{bmatrix} f_0(s)\zeta^T & f_1(s)\zeta^T & \cdots & f_n(s)\zeta^T \end{bmatrix}^T$. It is easy to obtain that

$$-2\xi^T(s)Mx(s) \leq \xi^T(s)MP^{-1}M\xi(s) + x^T(s)Px(s) \quad (2.5)$$

Integrating the inequality (2.5) from $[a, b] \times [u, b]$ yields

$$-2\zeta^T \sum_{i=0}^n M_i \int_a^b \int_u^b f_i(s)x(s)dsdu \\ \leq \zeta^T \left\{ \sum_{i=0}^n \int_a^b \int_u^b f_i^2(s)dsdu M_i P^{-1} M_i^T \right. \\ \left. + 2 \sum_{i=0}^n \sum_{j=i+1}^n \int_a^b \int_u^b f_i(s)f_j(s)dsdu M_i P^{-1} M_j^T \right\} \zeta \\ + \int_a^b \int_u^b x^T(s)Px(s)dsdu \\ = \zeta^T \left\{ \sum_{i=0}^n \int_a^b \int_u^b f_i^2(s)dsdu M_i P^{-1} M_i^T \right\} \zeta \\ + \int_a^b \int_u^b x^T(s)Px(s)dsdu \quad (2.6)$$

This completes the proof.

Lemma 3. For a differential function $x : [a, b] \rightarrow R^n$, a matrix $P \in R_+^n$, a vector $\zeta \in R^k$, and any matrices $M_i \in R^{k \times n}$ ($i = 1, 2, 3, 4$), then the following inequality holds:

$$-\int_a^b \int_u^b \dot{x}^T(s) P \dot{x}(s) ds du \leq \zeta^T \omega \zeta \quad (2.7)$$

where

$$\begin{aligned} \omega = & \frac{(b-a)^2}{2} \left(M_1 P^{-1} M_1^T + \frac{1}{2} M_2 P^{-1} M_2^T + \frac{1}{3} M_3 P^{-1} M_3^T + \frac{1}{4} M_4 P^{-1} M_4^T \right) \\ & + (b-a) \text{Sym}(M_1 \lambda_1 + M_2 \lambda_2 + M_3 \lambda_3 + M_4 \lambda_4) \end{aligned}$$

$$\lambda_1 \zeta = x(b) - \frac{1}{b-a} \int_a^b x(s) ds$$

$$\lambda_2 \zeta = x(b) + \frac{2}{b-a} \int_a^b x(s) ds - \frac{6}{(b-a)^2} \int_a^b \int_u^b x(s) ds du$$

$$\begin{aligned} \lambda_3 \zeta = & x(b) - \frac{3}{b-a} \int_a^b x(s) ds + \frac{24}{(b-a)^2} \int_a^b \int_u^b x(s) ds du \\ & - \frac{60}{(b-a)^3} \int_a^b \int_u^b \int_v^b x(s) ds dv du \end{aligned}$$

$$\begin{aligned} \lambda_4 \zeta = & x(b) + \frac{4}{b-a} \int_a^b x(s) ds - \frac{60}{(b-a)^2} \int_a^b \int_u^b x(s) ds du \\ & + \frac{360}{(b-a)^3} \int_a^b \int_u^b \int_v^b x(s) ds dv du \\ & - \frac{840}{(b-a)^4} \int_a^b \int_u^b \int_v^b \int_\beta^b x(s) ds d\beta dv du \end{aligned}$$

Proof. The result can be easily obtained by choosing $n = 3$, $f_1(s) = \frac{3}{b-a} \left(s - \frac{2b+a}{3} \right)$, $f_2(s) = \frac{10}{(b-a)^2} \left[\left(s - \frac{3b+2a}{5} \right)^2 - \frac{3(b-a)^2}{50} \right]$, $f_3(s) = -4 + 30 \frac{s-a}{b-a} - 60 \left(\frac{s-a}{b-a} \right)^2 + 35 \left(\frac{s-a}{b-a} \right)^3$ in (2.4). So the details of proof is omitted.

Remark 1. The inequality (25) of Lemma 5.1 in [10] is a special case of Lemma 3 by setting $M_1 = -\frac{2}{b-a} \lambda_1^T R$, $M_2 = -\frac{4}{b-a} \lambda_2^T R$, $M_3 = 0$, and $M_4 = 0$. The inequality (4) of Lemma 2.3 in [11] is a special case of Lemma 3 by setting $M_1 = -\frac{2}{b-a} \lambda_1^T R$, $M_2 = -\frac{4}{b-a} \lambda_2^T R$, $M_3 = -\frac{6}{b-a} \lambda_3^T R$, and $M_4 = 0$. In addition, the inequality (12) of Lemma 5 in [12] is a special case of Lemma 3 by setting $M_1 = -\frac{2}{b-a} \lambda_1^T R$, $M_2 = -\frac{4}{b-a} \lambda_2^T R$, $M_3 = -\frac{6}{b-a} \lambda_3^T R$, and $M_4 = -\frac{8}{b-a} \lambda_4^T R$.

3. Results

Based on Lemma 1 and Lemma 3, a new stability condition can be obtained.

Theorem 1. System (1) is asymptotically stable if there exist matrices $P \in S_+^{5n}$, $R_1, R_2, R_3 \in S_+^n$, and any matrices $M_1, M_2, M_3, M_4 \in R^{6n \times n}$ such that

$$\begin{aligned} \Psi = & Sym(\Pi_1^T P \Pi_2) + \delta_1^T R_1 \delta_1 - \delta_2^T R_1 \delta_2 + h^2 \delta_0^T R_2 \delta_0 + \frac{h^2}{2} \delta_0^T R_3 \delta_0 - \Pi_3^T R_2 \Pi_3 - 3\Pi_4^T R_2 \Pi_4 - 5\Pi_5^T R_2 \Pi_5 \\ & - 7\Pi_6^T R_2 \Pi_6 - 9\Pi_7^T R_2 \Pi_7 + \frac{h^2}{2} \left(M_1 R_3^{-1} M_1^T + \frac{1}{2} M_2 R_3^{-1} M_2^T + \frac{1}{3} M_3 R_3^{-1} M_3^T + \frac{1}{4} M_4 R_4^{-1} M_4^T \right) \\ & + h Sym(M_1 \Pi_8 + M_2 \Pi_9 + M_3 \Pi_{10} + M_4 \Pi_{11}) < 0 \end{aligned} \quad (3.1)$$

where

$$\begin{aligned} \Pi_1 &= \begin{bmatrix} \delta_1^T & \delta_3^T & \delta_4^T & \delta_5^T & \delta_6^T \end{bmatrix}^T, \\ \Pi_2 &= \begin{bmatrix} \delta_0^T & \delta_1^T - \delta_2^T & h\delta_1^T - \delta_3^T & \frac{h^2}{2}\delta_1^T - \delta_4^T & \frac{h^3}{6}\delta_1^T - \delta_5^T \end{bmatrix}, \\ \Pi_3 &= \delta_1 - \delta_2, \\ \Pi_4 &= \delta_1 + \delta_2 - \frac{2}{h}\delta_3, \\ \Pi_5 &= \delta_1 - \delta_2 + \frac{6}{h}\delta_3 - \frac{12}{h^2}\delta_4, \\ \Pi_6 &= \delta_1 + \delta_2 - \frac{12}{h}\delta_3 + \frac{60}{h^2}\delta_4 - \frac{120}{h^3}\delta_5, \\ \Pi_7 &= \delta_1 - \delta_2 + \frac{20}{h}\delta_3 - \frac{180}{h^2}\delta_4 + \frac{840}{h^3}\delta_5 - \frac{1680}{h^4}\delta_6, \\ \Pi_8 &= \delta_1 - \frac{1}{h}\delta_3, \\ \Pi_9 &= \delta_1 + \frac{2}{h}\delta_3 - \frac{6}{h^2}\delta_4, \\ \Pi_{10} &= \delta_1 - \frac{3}{h}\delta_3 + \frac{24}{h^2}\delta_4 - \frac{60}{h^3}\delta_5, \\ \Pi_{11} &= \delta_1 + \frac{4}{h}\delta_3 - \frac{60}{h^2}\delta_4 + \frac{360}{h^3}\delta_5 - \frac{840}{h^4}\delta_6, \\ \delta_0 &= A\delta_1 + B\delta_2 + C\delta_3, \\ \delta_i &= \begin{bmatrix} 0_{n \times (i-1)n} & I_n & 0_{n \times (7-i)n} \end{bmatrix}, i = 1, 2, \dots, 6. \end{aligned}$$

Proof. Introduce a LKF as

$$\begin{aligned} V(y_t) = & \zeta^T(t) P \zeta(t) + \int_{t-h}^t y^T(s) R_1 y(s) ds + h \int_{t-h}^t \int_u^t \dot{y}^T(s) R_2 \dot{y}(s) ds du \\ & + \int_{t-h}^t \int_u^t \int_v^t \dot{y}^T(s) R_3 \dot{y}(s) ds dv du \end{aligned} \quad (3.2)$$

where

$$\begin{aligned} \zeta(t) &= \begin{bmatrix} y^T(t) & \int_{t-h}^t y^T(s) ds & v_1^T(t) & v_2^T(t) & v_3^T(t) \end{bmatrix}^T \\ v_1^T(t) &= \int_{t-h}^t \int_{u_1}^t y^T(s) ds du_1 \\ v_2^T(t) &= \int_{t-h}^t \int_{u_1}^t \int_{u_2}^t y^T(s) ds du_2 du_1 \\ v_3^T(t) &= \int_{t-h}^t \int_{u_1}^t \int_{u_2}^t \int_{u_3}^t y^T(s) ds du_3 du_2 du_1 \end{aligned}$$

Then, the time derivative of $V(y_t)$ along the trajectories of system (1) as follows

$$\begin{aligned} \dot{V}(y_t) = & 2\zeta^T(t)P\dot{\zeta}(t) + y^T(t)R_1y(t) - y^T(t-h)R_1y(t-h) + h^2\dot{y}^T(t)R_2\dot{y}(t) + \frac{h^2}{2}\dot{y}^T(t)R_3\dot{y}(t) \\ & - h \int_{t-h}^t \dot{y}^T(s)R_2\dot{y}(s)ds - \int_{t-h}^t \int_u^t \dot{y}^T(s)R_3\dot{y}(s)dsdu \\ = & \eta^T(t) \left\{ Sym(\Pi_1^T P \Pi_2) + \delta_0^T Q \delta_0 - \delta_7^T Q \delta_7 + \delta_1^T R_1 \delta_1 - \delta_2^T R_1 \delta_2 + h^2 \delta_0^T R_2 \delta_0 + \frac{h^2}{2} \delta_0^T R_3 \delta_0 \right\} \eta(t) \\ & - h \int_{t-h}^t \dot{y}^T(s)R_2\dot{y}(s)ds - \int_{t-h}^t \int_u^t \dot{y}^T(s)R_3\dot{y}(s)dsdu \end{aligned} \quad (3.3)$$

where

$$\begin{aligned} \eta(t) = & \begin{bmatrix} y^T(t) & y^T(t-h) & \int_{t-h}^t y^T(s)ds & v_1^T(t) & v_2^T(t) & v_3^T(t) \end{bmatrix}^T \\ \text{By Lemma 1, we have} \\ & - h \int_{t-h}^t \dot{y}^T(s)R_2\dot{y}(s)ds \\ \leq & \eta^T(t) \left(-\Pi_3^T R_2 \Pi_3 - 3\Pi_4^T R_2 \Pi_4 - 5\Pi_5^T R_2 \Pi_5 - 7\Pi_6^T R_2 \Pi_6 - 9\Pi_7^T R_2 \Pi_7 \right) \eta(t) \end{aligned} \quad (3.4)$$

By Lemma 3, we have

$$\begin{aligned} & - \int_{t-h}^t \int_u^t \dot{y}^T(s)R_3\dot{y}(s)dsdu \\ \leq & \eta^T(t) \left[\frac{h^2}{2} \sum_{i=1}^4 \frac{1}{i} M_i R_3^{-1} M_i^T + h Sym \left(\sum_{i=1}^4 M_i \Pi_{i+7} \right) \right] \eta(t) \end{aligned} \quad (3.5)$$

Thus, according to (3.2)–(3.5), we have $\dot{V}(y_t) \leq \eta^T(t)\Psi\eta(t)$. Thus, if (3.1) holds, then, for a sufficient small scalar $\varepsilon > 0$, $\dot{V}(y_t) \leq -\varepsilon \|y(t)\|^2$ holds, which ensures system (1) is asymptotically stable. The proof is completed.

Remark 2. Both the double integral inequality and the new LKF include fourth integrals, which may yield novel stability results. Furthermore, in order to fully consider relevant information of the double integral inequality in Lemma 3, the $\int_{t-h}^t \int_{u_1}^t \int_{u_2}^t \int_{u_3}^t y^T(s)dsdu_3du_2du_1$ is added as a state vector.

4. Numerical examples

In this section, we demonstrate the advantages of our proposed criterion by two numerical examples.

Example 1. Consider system(1) with:

$$A = \begin{bmatrix} 0.2 & 0 \\ 0.2 & 0.1 \end{bmatrix}, B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, C = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix}.$$

Table 2 lists the allowable upper bounds of h by different methods. Table 2 shows that the maximum delay bounds of h obtained by our method are much larger than those in [4, 6, 7, 9, 11].

Example 2. Consider system(1) with:

$$A = \begin{bmatrix} 0 & 1 \\ -100 & -1 \end{bmatrix}, B = \begin{bmatrix} 0.0 & 0.1 \\ 0.1 & 0.2 \end{bmatrix}, C = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Table 2. Maximal bound h for Example 1.

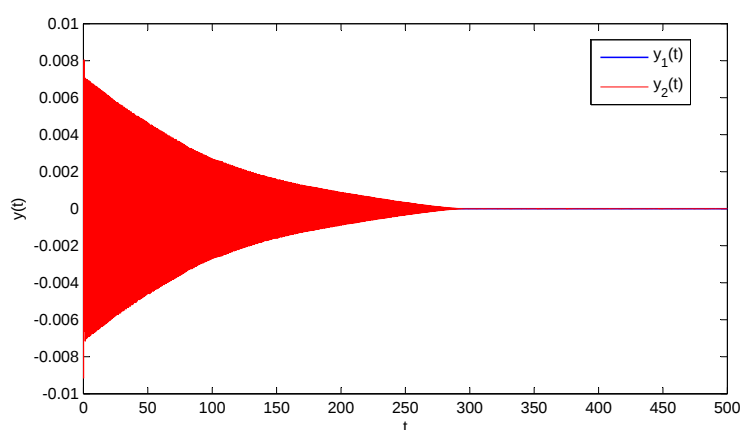
methods	Maximal h	NoDv
[7]	1.877	16
[9]	1.9504	59
[4]	2.0395	75
[6]	2.0395	27
[11]	2.0402	45
Theorem 1	2.0412	64

Table 3 lists the allowable upper bounds of h by different methods. Table 3 shows that the maximum delay bounds of h obtained by our method are much larger than those in [4, 7, 9–11, 14]. For $h = 0.750$, $y(0) = (0.001, -0.001)^T$, the state trajectories of the system(1) is given in Figure 1.

Remark 3. According to Example 1 and Example 2, although our method can reduce the conservatism of the system effectively, it increases the computational burden.

Table 3. Maximal bound h for Example 2.

methods	Maximal h	NoDv
[7]	0.126	16
[9]	0.126	59
[4]	0.577	75
[10]	0.577	96
[11]	0.675	45
[14]	0.728	45
Theorem 1	0.750	64

**Figure 1.** the state trajectories of the system(1) of example 2.

5. Conclusion

This paper focus on a new stability condition for a class of time delay systems. By using two generalized integral inequalities and a new augmented LKF, a new stability criterion is obtained. Both the double integral inequality and the new LKF include fourth integrals, which may yield more general results. Two numerical examples are proposed to show the effectiveness of the proposed criterion.

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Conflict of interest

The authors declare that there are no conflicts of interest.

References

1. H. B. Zeng, Y. He, M. Mu, et al. *Free-matrix-based integral inequality for stability analysis of systems with time-varying delay*, IEEE T. Automat. Contr., **60** (2015), 2768–2772.
2. Y. He, Q. G. Wang, L. Xie, et al. *Further improvement of free-weighting matrices technique for systems with time-varying delay*, IEEE T. Automat. Contr., **52** (2007), 293–299.
3. W. H. Chen, W. X. Zheng, *Delay-dependent robust stabilization for uncertain neutral systems with distributed delays*, Automatica, **43** (2007), 95–104 .
4. H. B. Zeng, Y. He, M. Mu, et al. *New results on stability ananlysis for systems with discrete distributed delay*, Automatica, **60** (2015), 189–192.
5. K. Gu, J. Chen, V. Kharitonov, *Stability of Time-Delay Systems*, Springer Science & Business Media, 2003.
6. L. V. Hien, H. Trinh, *Refined Jensen-based ineuqality approach to stability analysis of time-delay systems*, IET Control Theory A., **9** (2015), 2188–2194.
7. A. Seuret, F. Gouaisbaut, *Wirtinger-based integral inequality: Application to time-delay systems*, Automatica, **49** (2013), 2860–2866.
8. O. M. Kwon, M. J. Park, J. H. Park, et al. *Improved results on stability of linear systems with time-varying delays via Wirtinger-based integral inequality*, J. Franklin I., **351** (2015), 5386–5398.
9. M. J. Park, O. M. Kwon, J. H. Park, et al. *Stability of time-delay systems via Wirtinger-based double integral inequality*, Automatica, **55** (2015), 204–208.
10. P. G. Park, W. I. Lee, S. Y. Lee, *Auxiliary function-based integral inequalities for quadratic functions and their applications to time-delay systems*, J. Franklin I., **352** (2015), 1378–1396.

11. N. Zhao, C. Lin, B. Chen, et al. *A new double integral inequality and application to stability test for time-delay systems*, Appl. Math. Lett., **65** (2017), 26–31.
12. J. Chen, S. Xu, W. Chen, et al. *Two general integral inequalities and their applications to stability analysis for systems with time-varying delay*, Int. J. Robust Nonlin., **201** (2016), 4088–4103.
13. J. H. Kim, *Further improvement of Jensen inequality and application to stability of time-delayed systems*, Automatica, **64** (2016), 121–125.
14. J. Tian, Z. Ren, S. Zhong, *A new integral inequality and application to stability of time-delay systems*, Appl. Math. Lett., **101** (2010), 106058.
15. J. Tian, Z. Ren, *Stability analysis of systems with time-varying delays via an improved integral inequality*, IEEE access, **8** (2020), 90889–90894.
16. H. B. Zeng, Y. He, M. Wu, et al. *New results on stability analysis for systems with discrete distributed delay*, Automatica, **60** (2015), 189–192.
17. K. Liu, A. Seuret, Y. Xia, *Stability analysis of systems with time-varying delays via the second-order Bessel-Legendre inequality*, Automatica, **76** (2017), 138–142.
18. A. Seuret, F. Gouaisbaut, *Hierarchy of LMI conditions for the stability analysis of time-delay systems*, Syst. Control Lett., **81** (2015), 1–7.
19. A. Seuret, K. Liu, F. Gouaisbaut, *Generalized reciprocally convex combination lemmas and its application to time-delay systems*, Automatica, **95** (2018), 488–493.
20. J. Chen, J. H. Park, S. Xu, *Stability analysis of systems with time varying delay: A quadratic-partitioning method*, IET control Theory A., **13** (2019), 3184–3189.
21. C. K. Zhang, Y. He, L. Jiang, et al. *Delay-dependent stability analysis of neural networks with time-varying delay: A generalized free-weighting-matrix approach*, Appl. Math. Comput., **294** (2017), 102–120.
22. H. B. Zeng, X. G. Liu, W. Wang, *A generalized free-matrix-based integral inequality for stability analysis of time-varying delay systems*, Appl. Math. Comput., **354** (2019), 1–8.
23. H. B. Zeng, X. G. Liu, W. Wang, et al. *New results on stability analysis of systems with time-varying delays using a generalized free-matrix-based inequality*, J. Franklin I., **356** (2019), 7312–7321.
24. Z. Zhang, Z. Quan, *Global exponential stability via inequality technique for inertial BAM neural networks with time delays*, Neurocomputing, **151** (2015), 1316–1326.
25. Z. Zhang, W. Liu, D. Zhou, *Global asymptotic stability to a generalized Cohen-Grossberg BAM neural networks of neutral type delays*, Neural Networks, **25** (2012), 94–105.
26. F. X. Wang, X. G. Liu, M. L. Tang, et al. *Improved integral inequalities for stability analysis of delayed neural networks*, Neurocomputing, **273** (2018), 178–189.
27. J. J. Wei, M. L. Tang, *Stability analysis of several kinds of neural networks with time-varying delays* (master's dissertation), Changsha, Central South University, 2018.

