Mathematics

http://www.aimspress.com/journal/Math

AIMS Mathematics, 5(6): 5564–5570.
DOI:10.3934/math.2020357
Received: 27 April 2020
Accepted: 23 June 2020
Published: 01 July 2020

Research article

Mixed domination and 2-independence in trees

Chang Wan[1], Zehui Shao[2], Nasrin Dehgardi[3,*], Marzieh Soroudi[4] and Asfand Fahad[5]

[1] Guangdong Polytechnic of Science and Technology, Guangzhou 510640, China
[2] Institute of Computing Science and Technology, Guangzhou University, Guangzhou 510006, China
[3] Department of Mathematics and Computer Science, Sirjan University of Technology, Sirjan, I. R. Iran
[4] Department of Mathematics, Azarbaijan Shahid Madani University, Tabriz, I. R. Iran
[5] Department of Mathematics, COMSATS University Islamabad, Vehari Campus, Vehari 61100, Pakistan

* **Correspondence:** Email: n.dehgardi@sirjantech.ac.ir.

Abstract: We investigate some relationships between two vastly studied parameters of a simple graph G. These parameters include mixed domination number (denoted by $\gamma_m(G)$) and 2-independence number ($\beta_2(G)$). For a tree T, we obtain $\frac{3}{4}\beta_2(T) \geq \gamma_m(T)$ and characterized all those trees which attain the equality.

Keywords: mixed dominating function; mixed domination number; 2-independent set; 2-independence number
Mathematics Subject Classification: 05C05, 05C69

1. Introduction

The problem of covering vertices (edges) by vertices (edges) has been widely studied in recent years. The quests of covering: vertices by vertices, edges by vertices, vertices by edges and edges by edges resulted into several significant problems in the area of graph theory. As a natural extension of these problems, the domination of the vertices or edges by vertices or edges was introduced and studied in [1, 2, 16, 17, 19].

Before proceeding further, we fix the notations used in this paper. For other notations and terminologies, see [18]. We consider the simple and connected graphs G, denote a vertex set by $V = V(G)$, edge set by $E = E(G)$ and the cardinality of V by n. Furthermore, we denote the open (closed) neighborhood of a vertex v by $N(v)$ ($N[v]$), children and descendants by $C(v)$ and $D(v)$, maximal subtree at v by T_v, the set of leaves adjacent to v by L_v, pendant path by P, diameter by diam(G), path of order n by P_n, star of order n by $K_{1,n-1}$ and double star by $DS_{p,q}$. In addition, the notions of degree of v, leaf, rooted tree and support vertex which will used in this paper are well known in the literature.

For a graph $G = (V, E)$, a vertex v mixed dominate its closed neighborhood and all edges incident to it. An edge uv mixed dominate all the edges incident to u or v and the vertices u, v. A set D consisting of vertices and edges of G such that each vertex and edge in G can be mixed dominated by an element of D is called a mixed dominating set (MDS). Such a set with minimum cardinality is defined as γ_m-set of G and its cardinality is called the mixed domination number $\gamma_m(G)$. It is clear that the MDS is a variant of dominating set which has been generalized to many other parameters, such as total domination [21], Roman domination [15], semitotal domination [20, 22]. Several researchers studied the problem of mixed domination (MD) in different directions such as: in [14], NP-completeness in split graphs and a primal-dual algorithm for MD problem were presented; some results related to MD problem contributing toward electric power system have been obtained in [19]. For more literature on mixed domination and related notions, see [1, 2, 13, 16].

On the other hand, the notion of k-independent set (k-IS), generalizing the notion of the independent set, was introduced in [11] as: For a positive integer k, a set $X \subseteq V$ is called k-IS if every vertex v in the subgraph induced by vertices of X has degree at most $k-1$. The k-independence number (denoted by $\beta_k(G)$) is the maximum possible cardinality of k-IS and such a set is called $\beta_k(G)$-set. The results on bounding and improvement of $\beta_k(G)$ may be seen in [5, 6, 12]. Another aspect of $\beta_k(G)$ which has been studied is its relationship with $\gamma_k(G)$, for instance see [10]. In such relationships the case $\beta_2(G)$ got especial attention, see [3, 4, 8, 9, 15]. For more on k-independence we refer the readers [7].

The following result can be found in [1].

Theorem 1. *For a complete graph K_n, $\gamma_m(K_n) = \lceil \frac{n}{2} \rceil$.*

Next result shows, there exist families of graphs such that $\gamma_m(G) < \beta_2(G)$ for a graph G in a family and $\gamma_m(H) > \beta_2(H)$ for a graph H in another family.

Proposition 1. *For any $t \in \mathbb{N}$, there exist graphs G_t and H_t such that $\beta_2(G_t) - \gamma_m(G_t) = t$ and $\gamma_m(H_t) - \beta_2(H_t) = t$.*

Proof. Let $G_t = K_{1,t+1}$. Then clearly $\beta_2(G_t) = t + 1$ and $\gamma_m(G_t) = 1$ and so $\beta_2(G_t) - \gamma_m(G_t) = t$. Now assume that $H_t = K_{2t+4}$. We can see that $\beta_2(H_t) = 2$ and by Theorem 1, $\gamma_m(H_t) = t + 2$. So $\gamma_m(H_t) - \beta_2(H_t) = t$. □

Motivated by the above proposition, the current work is devoted to investigations of relationships between $\beta_2(G)$ and $\gamma_m(G)$. Specifically, by keeping in view the importance of tress, we prove $3\beta_2(T) - 4\gamma_m(T) \geq 0$ and characterize all trees T such that $3\beta_2(T) = 4\gamma_m(T)$. Before proceeding further, we include some definitions, notations and a lemma which will be used later. We start with the following Remark.

Remark 1. *For any G and $v \in V(G)$, $\gamma_m(G) \leq \gamma_m(G - v) + 1$ holds.*

Definition 1. *Let $u \in V \cup E$, and $D \subseteq V \cup E$. The element u is said to be mixed dominated by D if u is adjacent to a vertex v of D or is incident to an edge e of D. Let $v \in V \cup E$. A set D is said to be an almost mixed dominating set (AMDS) corresponding to v, if for any $u \in ((V \cup E) - \{v\})$ is mixed dominated by D. Take $\gamma_m(G; v)$ the cardinality of an AMDS corresponding to v with minimum possible elements.*

We note that any mixed dominating set on G is an AMDS corresponding to any element of G. Therefore, for any $v \in V \cup E$ the inequality $\gamma_m(G; v) \leq \gamma_m(G)$ holds.

Definition 2. *For any G, we define W_G^1 as:*

$$W_G^1 = \{v \in V \mid \gamma_m(G - v) \geq \gamma_m(G)\}$$

and

$$W_G^2 = \{v \in V \cup E \mid \gamma_m(G; v) = \gamma_m(G)\}.$$

Lemma 1. *Let T' be a tree and $u \in V(T')$. If T is a tree constructed from T' by adding a path $P_6 = x_6x_5x_4x_3x_2x_1$, and either joining u to x_3 or joining u to x_2, then $\beta_2(T) = \beta_2(T') + 4$ and $\gamma_m(T) \leq \gamma_m(T') + 3$.*

Proof. From any $\beta_2(T')$-set, a 2-IS of T may be obtained by including x_1, x_2, x_5, x_6, and so $\beta_2(T) \geq \beta_2(T') + 4$. Moreover, for any $\beta_2(T)$-set S, the equality $|S \cap \{x_1, x_2, x_3, x_4, x_5, x_6\}| = 4$ holds and $S \cap V(T')$ is a 2-IS of T', therefore $\beta_2(T) \leq \beta_2(T') + 4$. Thus $\beta_2(T) = \beta_2(T') + 4$. Moreover, by adding x_1, x_3, x_5 to any $\gamma_m(T')$-set D we get an MD-set of T. Consequently, we get $\gamma_m(T) \leq \gamma_m(T') + 3$ as desired. □

2. Main result

The current section is devoted to the proofs of the main results which have been briefly described in the previous section. To achieve this objective, we use $\mathcal{T}$ to denote a family of unlabeled trees T which may be constructed by a sequences of trees T_j, for $j = 1, 2, \ldots m$ $(m \geq 1)$ such that $T_1 = P_6$, and T_{i+1} is obtained from T_i recursively, by using the operations:

Operation $\mathcal{O}_1$. If $u \in W_{T_i}^1$, then $\mathcal{O}_1$ adds a path $P_6 = x_6x_5x_4x_3x_2x_1$, and an edge ux_3 to produce T_{i+1}.

Operation $\mathcal{O}_2$. If $u \in W_{T_i}^2$, then $\mathcal{O}_2$ adds a path $P_6 = x_6x_5x_4x_3x_2x_1$, and an edge ux_2 to produce T_{i+1}.

Lemma 2. *If T_i is a tree with $\gamma_m(T_i) = \frac{3}{4}\beta_2(T_i)$ and T_{i+1} is a tree obtained from T_i by Operation $\mathcal{O}_1$, then $\gamma_m(T_{i+1}) = \frac{3}{4}\beta_2(T_{i+1})$.*

Proof. By Lemma 1, $\beta_2(T_{i+1}) = \beta_2(T_i) + 4$ and $\gamma_m(T_{i+1}) \leq \gamma_m(T_i) + 3$. Let D be a $\gamma_m(T_{i+1})$-set. We have

$$k = |D \cap \{x_1, x_2, x_3, x_4, x_5, x_6, x_1x_2, x_2x_3, x_3x_4, x_4x_5, x_5x_6, ux_3\}| \geq 3.$$

If $u \in D$ or $wu \in D$ for some $w \in N(u) - \{x_3\}$, then the set D, restricted to T_i is an MDS of T_i and this implies that $\gamma_m(T_{i+1}) \geq \gamma_m(T_i) + 3$. Now let $u \notin D$ and $wu \notin D$ for every $w \in N(u) - \{x_3\}$. Then the set D, restricted to $T_i - u$ is an MDS of $T_i - u$. We deduce from $u \in W_{T_i}^1$ that $\gamma_m(T_i - u) \geq \gamma_m(T_i)$ and this implies that $\gamma_m(T_{i+1}) \geq \gamma_m(T_i) + 3$. Thus $\gamma_m(T_{i+1}) = \gamma_m(T_i) + 3$.

By the assumption $\gamma_m(T_i) = \frac{3}{4}\beta_2(T_i)$, we obtain $\frac{3}{4}\beta_2(T_{i+1}) = \frac{3}{4}\beta_2(T_i) + 3 = \gamma_m(T_i) + 3 = \gamma_m(T_{i+1})$. □

Lemma 3. If T_i is a tree with $\gamma_m(T_i) = \frac{3}{4}\beta_2(T_i)$ and T_{i+1} is a tree obtained from T_i by Operation $\mathcal{O}_2$, then $\gamma_m(T_{i+1}) = \frac{3}{4}\beta_2(T_{i+1})$.

Proof. By Lemma 1, $\beta_2(T_{i+1}) = \beta_2(T_i) + 4$ and $\gamma_m(T_{i+1}) \leq \gamma_m(T_i) + 3$. Let D be a $\gamma_m(T_{i+1})$-set. We have

$$k = |D \cap \{x_1, x_2, x_3, x_4, x_5, x_6, x_1x_2, x_2x_3, x_3x_4, x_4x_5, x_5x_6, ux_2\}| \geq 3.$$

If $u \in D$ or $wu \in D$ for some $w \in N(u) - \{x_2\}$, then the set D, restricted to T_i is an MDS of T_i and this implies that $\gamma_m(T_{i+1}) \geq \gamma_m(T_i) + 3$. Now let $u \notin D$ and $wu \notin D$ for every $w \in N(u) - \{x_3\}$. If $ux_3 \in D$, then we can see that $k = 4$ and the result follows as above. If $ux_3 \notin D$, then the set D, restricted to T_i is an MDS of T_i corresponding to u and the assumption $u \in W^2_{T_i}$ implies $\gamma_m(T_{i+1}) + 3 \geq \gamma_m(T_i; u) = \gamma_m(T_i)$. Thus $\gamma_m(T_{i+1}) = \gamma_m(T_i) + 3$. By the assumption $\gamma_m(T_i) = \frac{3}{4}\beta_2(T_i)$, we obtain $\frac{3}{4}\beta_2(T_{i+1}) = \frac{3}{4}\beta_2(T_i) + 3 = \gamma_m(T_i) + 3 = \gamma_m(T_{i+1})$. □

Theorem 2. *If $T \in \mathcal{T}$, then $\gamma_m(T) = \frac{3}{4}\beta_2(T)$.*

Proof. Let $T \in \mathcal{T}$, then by the definition of $\mathcal{T}$, T can be constructed by using the operations $\mathcal{O}_1$ and $\mathcal{O}_2$. Suppose T is obtained recursively by the sequence $(T_k)_{k=1}^{j}$. Now, we proof it by induction on j. If $j = 1$, $T = P_6 \in \mathcal{T}$ and the equality is true. Assume that the equality hold for T constructed from $j - 1$ operations. Now, let T be a tree constructed from j operations, take $T' = T_{j-1}$. Then T is constructed from T' by either $\mathcal{O}_1$ or $\mathcal{O}_2$. Consequently, by applying the Lemmas 2- 3, we get $\gamma_m(T) = \frac{3}{4}\beta_2(T)$. □

Up to now, we have developed the sufficient results to prove the following main theorem of this paper.

Theorem 3. *For any tree T of order n, the inequality $\gamma_m(T) \leq \frac{3}{4}\beta_2(T)$ holds. Moreover, $\gamma_m(T) = \frac{3}{4}\beta_2(T)$ if and only if $T \in \mathcal{T}$.*

Proof. We prove it by the induction on $|T| = n$.
If $n \leq 5$, it can be observed that $\gamma_m(T) < \frac{3}{4}\beta_2(T)$.
If $n = 6$, then with a simple verification, we see that $\gamma_m(T) \leq \frac{3}{4}\beta_2(T)$ and the only tree with $\gamma_m(T) = \frac{3}{4}\beta_2(T)$ is $P_6 \in \mathcal{T}$.

Consider the case when $n \geq 7$ and the statement is true for $n - 1$ or less. Then
If $\mathrm{diam}(T) = 2$, then $T = K_{1,n-1}$ and we get $\gamma_m(T) = 1 < \frac{3(n-1)}{4} = \frac{3}{4}\beta_2(T)$.
If $\mathrm{diam}(T) = 3$, then $T = DS_{p,q}$ $(q \geq p \geq 1)$ and it can be verified that $\gamma_m(T) = 2 < \frac{3}{4}\beta_2(T)$ as $n \geq 7$.

Assume now, that $\mathrm{diam}(T) \geq 4$ and let $v_1 v_2 \ldots v_{d+1}$ denote a diametral path with the property that d_{v_2} is maximum (possible). Root T at v_{d+1}, then:
Case 1. If $k = d_{v_2} \geq 3$. Take $T' = T - T_{v_2}$, then any $\beta_2(T')$-set and $\gamma_m(T')$-set may be extended to a 2-IS (MDS) of T by adding $k - 1$ leaves adjacent to v_2 (v_2). Therefore $\beta_2(T) \geq \beta_2(T') + (k - 1)$ and $\gamma_m(T) \leq \gamma_m(T') + 1$. Consequently, the induction hypothesis yield,

$$\begin{aligned}
\tfrac{3}{4}\beta_2(T) &\geq \tfrac{3}{4}\beta_2(T') + \tfrac{3}{4}k - \tfrac{3}{4} \\
&\geq \gamma_m(T') + \tfrac{3}{4}k - \tfrac{3}{4} \\
&\geq \gamma_m(T) - 1 + \tfrac{3}{4}k - \tfrac{3}{4} \\
&\geq \gamma_m(T) + \tfrac{3}{4}k - \tfrac{7}{4} \\
&> \gamma_m(T).
\end{aligned}$$

Case 2. If $d_{v_3} \geq 3$. Let v_3 have $t \geq 1$ children with depth 1, and $\ell \geq 0$ children with depth 0. If $t \geq 2$, then let $T' = T - T_{v_2}$. Suppose that xyv_3 be a path in T with $d_y = 2$ and $d_x = 1$. Assume that S' be a $\beta_2(T')$-set. If $v_3 \in S'$, then $|S \cap \{x, y\}| = 1$ and $S = (S' - \{v_3\}) \cup \{v_1, v_2, x, y\}$ is a 2-IS of T, and if $v_3 \notin S'$, then $S = S' \cup \{v_1, v_2\}$ is a 2-IS of T, so $\beta_2(T') + 2 \leq \beta_2(T)$. On the other hand, by including v_2 in any $\gamma_m(T')$-set, we may get a MDS of T. Therefore, $\gamma_m(T) \leq \gamma_m(T') + 1$. Therefore, the induction hypothesis yield $\frac{3}{4}\beta_2(T) > \gamma_m(T)$. Now let $t = 1$ and $T' = T - T_{v_3}$. Since $\deg(v_3) \geq 3$, we have $\ell \geq 1$.

Clearly, a 2-IS of T can be obtained from any $\beta_2(T')$-set by including all children and descendants of v_3. So, $\beta_2(T') + 2 + \ell \le \beta_2(T)$. On the other hand, by including v_2, v_3 in any $\gamma_m(T')$-set, we may get a MDS of T. Therefore, we have $\gamma_m(T) \le \gamma_m(T') + 2$ and the induction hypothesis yield:

$$\begin{array}{rcl} \frac{3}{4}\beta_2(T) & \ge & \frac{3}{4}\beta_2(T') + \frac{3}{2} + \frac{3}{4}\ell \\ & \ge & \gamma_m(T') + \frac{3}{2} + \frac{3}{4}\ell \\ & \ge & \gamma_m(T) - 2 + \frac{3}{2} + \frac{3}{4}\ell \\ & \ge & \gamma_m(T) - \frac{1}{2} + \frac{3}{4}\ell \\ & > & \gamma_m(T) \end{array}$$

Case 3. If $d_{v_3} = 2$. Considering Case 1 and the choice of diametrical path, we may assume that any child of v_4 with depth 2, is of degree 2. We divide this case as:

Subcase 3.1 If $d_{v_4} \ge 3$. Let $x_1^1, x_1^2, \dots, x_1^k (k \ge 1)$ be the children of v_4 with depth 2, and $v_4 x_1^i x_2^i x_3^i$ be a path in T for $1 \le i \le k$. Assume $y_1^1, y_1^2, \dots, y_1^t$ be the children of v_4 with depth 1, and $v_4 y_1^j y_2^j$ be a path in T for $1 \le j \le t$. Assume that v_4 has ℓ children with depth 0. Let $T' = T - T_{v_4}$. Clearly, by including all children and descendants of v_3 except the children with depth 2 in any $\beta_2(T')$-set, we may get a 2-IS of T. Therefore, $\beta_2(T') + 2k + 2t + \ell \le \beta_2(T)$. On the other hand, by including v_4, x_2^i, y_1^j for $1 \le i \le k, 1 \le j \le t$ to any $\gamma_m(T')$-set, we get a MDS of T. Therefore, $\gamma_m(T) \le \gamma_m(T') + k + t + 1$ and the induction hypothesis yields,

$$\begin{array}{rcl} \frac{3}{4}\beta_2(T) & \ge & \frac{3}{4}\beta_2(T') + \frac{3}{2}k + \frac{3}{2}t + \frac{3}{4}\ell \\ & \ge & \gamma_m(T') + \frac{3}{2}k + \frac{3}{4}t\frac{3}{4}\ell \\ & \ge & \gamma_m(T) - k - t - 1 + \frac{3}{2}k + \frac{3}{2}t + \frac{3}{4}\ell \\ & = & \gamma_m(T) + \frac{k}{2} + \frac{t}{2} + \frac{3}{4}\ell - 1 \end{array}.$$

If the equality holds, then we must have $\frac{3}{4}\beta_2(T') = \gamma_m(T')$, and $\frac{k}{2} + \frac{t}{2} + \frac{3}{4}\ell = 1$. Therefore $k = t = 1$ and $\ell = 0$. Now we show that $v_5 \in W^1_{T'}$. If $v_5 \notin W^1_{T'}$, then $\gamma_m(T) \le \gamma_m(T' - v_5) + 3 < \gamma_m(T') + 3$ which is a contradiction. Hence $v_5 \in W^1_{T'}$ and T' with O_1 produces T, thus $T \in \mathcal{T}$.

Subcase 3.2 If $d_{v_4} = 2$. Let v_5 have ℓ children with depth 0. Also assume that $z_1^1, z_1^2, \dots, z_1^t$ be the children of v_5 with depth 1, and $v_5 z_1^l z_2^l$ be a path in T for $1 \le l \le t$, and let $y_1^1, y_1^2, \dots, y_1^k$ be the children of v_5 with depth 2, and $v_5 y_1^j y_2^j y_3^j$ be a path in T for $1 \le j \le k$. Let $x_1^1, x_1^2, \dots, x_1^s$ be the children of v_5 with depth 3, and $v_5 x_1^i x_2^i x_3^i x_4^i$ be a path in T for $1 \le i \le s$. Considering above cases and subcases, we may assume that all above paths are pendant paths. Take $T' = T - T_{v_5}$, then by including all children of v_5 with depth 0, x_1^i, x_3^i, x_4^i for $1 \le i \le s$, y_2^j, y_3^j for $1 \le j \le k$, and z_1^l, z_2^l for $1 \le l \le t$ in any $\beta_2(T')$-set we may get a 2-IS of T. Hence, $\beta_2(T) \ge \beta_2(T') + 3s + 2k + 2t + \ell$. On the other hand, let D a $\gamma_m(T')$-set. If $\ell = 0$, then by including $x_3^i, x_1^i v_5, y_2^j, z_1^l$ for $1 \le i \le s, 1 \le j \le k, 1 \le l \le t$ in D, we get an MDS of T. Hence $\gamma_m(T) \le \gamma_m(T') + 2s + k + t$ and the induction hypothesis yield

$$\begin{array}{rcl} \frac{3}{4}\beta_2(T) & \ge & \frac{3}{4}\beta_2(T') + \frac{9}{4}s + \frac{3}{2}k + \frac{3}{2}t \\ & \ge & \gamma_m(T') + \frac{9}{4}s + \frac{3}{2}k + \frac{3}{4}t \\ & \ge & \gamma_m(T) - 2s - k - t + \frac{9}{4}s + \frac{3}{2}k + \frac{3}{2}t \\ & \ge & \gamma_m(T) + \frac{1}{4}s + \frac{1}{2}k + \frac{1}{2}t \\ & > & \gamma_m(T) \end{array}.$$

If $\ell \geq 1$, then by including $v_5, x_3^i, x_1^i v_5, y_2^j, z_1^l$ for $1 \leq i \leq s, 1 \leq j \leq k, 1 \leq l \leq t$ in D, we get an MDS of T. Hence $\gamma_m(T) \leq \gamma_m(T') + 2s + k + t + 1$ the induction hypothesis yield:

$$\begin{array}{rcl} \frac{3}{4}\beta_2(T) & \geq & \frac{3}{4}\beta_2(T') + \frac{9}{4}s + \frac{3}{2}k + \frac{3}{2}t + \frac{3}{4}\ell \\ & \geq & \gamma_m(T') + \frac{9}{4}s + \frac{3}{2}k + \frac{3}{4}t + \frac{3}{4}\ell \\ & \geq & \gamma_m(T) - 2s - k - t - 1 + \frac{9}{4}s + \frac{3}{2}k + \frac{3}{2}t + \frac{3}{4}\ell \\ & \geq & \gamma_m(T) + \frac{1}{4}s + \frac{1}{2}k + \frac{1}{2}t + \frac{3}{4}\ell - 1 \\ & \geq & \gamma_m(T) \qquad\qquad\qquad\qquad\qquad . \end{array}$$

If the equality holds, then we must have $\frac{3}{4}\beta_2(T') = \gamma_m(T')$, and $\frac{s}{4} + \frac{k}{2} + \frac{t}{2} + \frac{3}{4}\ell = 1$. Therefore $k = t = 0, \ell = s = 1$. Finally, we show that $v_6 \in W^2_{T'}$. If $v_6 \notin W^2_{T'}$, then $\gamma_m(T) \leq \gamma_m(T'; v_6) + 3 < \gamma_m(T') + 3$ which is a contradiction. Hence $v_6 \in W^2_{T'}$ and T' along with O_2 produces T. Thus $T \in \mathcal{T}$ and the proof is completed. □

Acknowledgments

This work is supported by the 2019 teaching reform project of Guangdong Polytechnic of science and technology, JG201926, Research on the course construction of exhibition data analysis under the background of digital economy, and Guangdong Provincial Education Department, 2017GGXJK020, "Internet plus" technology service and management platform research and design.

Conflict of interest

The authors wish to confirm that there are no known conflicts of interest associated with this paper and there has been no significant financial support for this work that could have influenced its outcome.

References

1. Y. Alavi, M. Behzad, L. Lesniak, et al. *Total matchings and total coverings of graphs*, J. Graph Theor., **1** (1977), 135–140.
2. Y. Alavi, J. Q. Liu, J. F. Wang, et al. *On total covers of graphs*, Discrete Math., **100** (1992), 229–233.
3. J. Amjadi, S. M. Sheikholeslami, M. Valinavaz, et al. *Independent Roman domination and 2-independence in trees*, Discrete Mathematics, Algorithms and Applications, **10** (2018), 1850052.
4. N. Dehgardi, S. M. Sheikholeslami, M. Valinavaz, et al. *Domination number, independent domination number and 2-independence number in trees*, Discuss. Math. Graph T., 2018, 1–11.
5. Y. Caro, A. Hansberg, *New approach to the k-independence number of a graph*, Electron. J. Comb., **20** (2013), 1–17.
6. Y. Caro, Z. Tuza, *Improved lower bounds on k-independence*, J. Graph Theor., **15** (1991), 99–107.
7. M. Chellali, O. Favaron, A. Hansberg, et al. *k-domination and k-independence in graphs*: A survey, Graph. Combinator., **28** (2012), 1–55.
8. M. Chellali, N. Meddah, *Trees with equal 2-domination and 2-independence numbers*, Discuss. Math. Graph T., **32** (2012), 263–270.

9. N. Dehgardi, *Mixed Roman domination and 2-independence in trees*, Communications in Combinatorics and Optimization, **3** (2018), 79–91.
10. O. Favaron, *On a conjecture of Fink and Jacobson concerning k-domination and k-dependence*, J. Combin. Theory, B, **39** (1985), 101–102.
11. J. F. Fink, M. S. Jacobson, *On n-domination, n-dependence and forbidden subgraphs*, In: *Graph Theory with Applications to Algorithms and Computer Science*, John Wiley & Sons, Inc., 1985, 301–311.
12. S. Kogan, *New results on k-independence of graphs*, Electron. J. Comb., **24** (2017), 2–15.
13. P. Hatami, *An approximation algorithm for the total covering problem*, Discuss. Math. Graph T., **27** (2007), 553–558.
14. J. K. Lan, G. J. Chang, *On the mixed domination problem in graphs*, Theor. Comput. Sci., **467** (2013), 84–93.
15. N. Meddah, M. Chellali, *Roman domination and 2-independence in trees*, Discrete Mathematics, Algorithms and Applications, **9** (2017), 1–6.
16. D. F. Manlove, *On the algorithmic complexity of twelve covering and independence parameters of graphs*, Discrete Appl. Math., **91** (1999), 155–175.
17. E. Nordhaus, *Generalizations of graphical parameters*, in: *Theory and Applications of Graphs*, 1978.
18. D. B. West, *Introduction to Graph Theory (Second Edition)*, Prentice Hall, USA, 2001.
19. Y. Zhao, L. Kang, M. Y. Sohn, *The algorithmic complexity of mixed domination in graphs*, Theor. Comput. Sci., **412** (2011), 2387–2392.
20. E. Zhu, C. Liu, *On the semitotal domination number of line graphs*, Discrete Appl. Math., **254** (2019), 295–298.
21. E. Zhu, C. Liu, F. Deng, et al. *On upper total domination versus upper domination in graphs*, Graph. Combinator., **35** (2019), 767–778.
22. E. Zhu, Z. Shao, J. Xu, *Semitotal domination in claw-free cubic graphs*, Graph. Combinator., **33** (2017), 1119–1130.