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Research article

Adaptive fuzzy control for nonlinear systems with sampled data and time-varying input delay

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Abstract: In this paper, an adaptive fuzzy backstepping control strategy is studied for nonlinear non-strict feedback systems with sampled data and time-varying input delay. Considering the practical application of the proposed control strategy, a time-varying signal transmission delay is investigated. By using fuzzy logic systems to approximate the unknown nonlinear functions, a fuzzy estimator (FE) model is proposed to estimate the states of the nonlinear plant, which is mainly utilized to support information of estimation states for the adaptive fuzzy controller. In the proposed strategy, the constraint between the signal transmission delay and the time-varying input delay is given to ensure the stability of the closed-loop system, and the state vectors are transformed to address the problem of time-varying input delay. By using the backstepping control technique and the information of the FE model, an adaptive fuzzy backstepping controller is designed. The proposed control strategy can guarantee that all signals of the closed-loop system are semi-globally uniformly ultimately bounded. Ultimately, a numerical simulation example is provided to verify the effectiveness of the proposed control method and theory.

Keywords: adaptive fuzzy control; backstepping; non-strict feedback systems; time-varying input delay; sampled-data
Mathematics Subject Classification: 93B52, 93C42

1. Introduction

For a long time, the adaptive fuzzy control strategy has been attracted a great attention and a variety of remarkable contributions have been proposed [1–6], in which fuzzy logic systems (FLSs) are utilized to approximate the unknown nonlinear functions with arbitrary modeling accuracy. And recently, many adaptive fuzzy control approaches have been reported by backstepping technique for nonlinear systems [7–11]. These approaches don't need the nonlinear systems to satisfy matching conditions, or to satisfy that the unknown parameters of the known linear functions should be linear. Therefore, adaptive fuzzy control scheme with the backstepping method has become one of the most prevalent control strategies.

It should be mentioned that aforementioned methods belong to the strict feedback problem, that is, they need to satisfy that the unknown functions of sub-systems only have the current state vector. If the unknown functions possess the variables of the whole states in the sub-systems, for this case, the design method of non-strict feedback control will be required. In non-strict feedback system, the unknown nonlinear functions in the sub-systems have full states, which give rise to the more complexity of calculation and augment the difficulty of control design. To solve these questions, based on the property of monotonically increasing about the unknown functions, [12–16] have proposed the variable separation schemes. But recently, a state feedback scheme and an output feedback control algorithm were proposed in [17], which have combined the backstepping technique and the property of FLS to solve the non-strict feedback problem, meanwhile, by using the property of FLSs, the unknown nonlinear functions do not need to satisfy the property in [12–16].

In practical engineering application, the delay often occurs and is a critical reason of degrading the system performance. Thus, it is necessary to take it into account in the system analysis process. The authors in [18] investigated the issue of adaptive fuzzy control scheme for strict feedback systems with delay. Furthermore, considering the problem of input delay, the authors in [19–21] addressed the problem of adaptive fuzzy control for nonlinear systems with input delay. The control scheme in [22] can achieve better stable effect. However, [19–21] consider the input delays are constant input delays but not the time-varying condition. Therefore, time-varying input delay is still a significant problem to investigate. In this approach, the negative effect of time-varying input delay has been compensated by sampling information and the transformation of state vectors.

On other research front, because of the development of information computing, sampled-data control method is attracting increasing scholars. However, the delay problems often occur in the process of downloading and uploading data, which cause the sampled-data control often accompany with the issue of the delay, this situation motivates us to solve these issues. In addition, in most of the works on the controller design processes of sampled-data control method, both continuous-time and discrete-time controllers were constructed by sampling information [23–29]. Recently, the sampled-data control scheme has received considerable contributions, however, there are few works on both the adaptive fuzzy control technique and sampled-data control approach simultaneously, only a few ways have been considered in [30–32]. In [30], a restrain of the average dwell time is given to guarantee stability of the closed-loop system. Furthermore, in [31], a state observer with the unknown disturbance is investigated to construct a sampled-data fuzzy output feedback controller. Thus, considering simultaneously the problems about the input-delay and sampled-data under the framework of adaptive fuzzy backstepping control is still a significant challenge.

This paper first investigates the control problem of non-strict feedback nonlinear systems with sampled data and time-varying input delay. Based on the Lyapunov theory, it is proved that all signals in the closed-loop system are semi-globally uniformly ultimately bounded and fuzzy approximation errors are eventually cover to a compact set. Compared with the existing works, the main contributions of this paper are two aspects.

(1) This is the first work on time varying input delay and signal transmission delay of the nonlinear systems with sampled data in non-strict form.

(2) The adaptive fuzzy controller does not need the information of the original plant, which consists of sampled-data and positive design parameters, so that the controller implements much easier than the conventional adaptive fuzzy controllers.

The rest of this paper is consisted by the following. In Section 2.2, a constrain between the signal transmission delay and time varying input delay has been given to ensure the stability of the proposed method. In Section 2.3, an adaptive fuzzy controller is designed in the framework of backstepping. In Section 2.4, a numerical example has been given to verify the effectiveness of the proposed control strategy. In Section 3, we show the results of this paper. In Section 4, we take a discuss of this paper. In Section 5, we conclude this paper.

2. Materials and method

2.1. System descriptions

Consider the following SISO nonlinear system with time-varying input delay

$$\begin{aligned} \dot{x}_1 &= x_2 + f_1(x), \\ \dot{x}_2 &= x_3 + f_2(x), \\ &\vdots \\ \dot{x}_{n-1} &= x_n + f_{n-1}(x), \\ \dot{x}_n &= u(t-\tau(t)) + f_n(x), \\ y &= x_1, \end{aligned} \tag{2.1}$$

where $x_i, i = 1, 2, \ldots n$ and $x = (x_1, x_2, \cdots x_n)^T \in R^n$ are the system state vectors, $u(t - \tau(t)) \in R$ is the input with time-varying delay, which satisfies $0 \leq \tau(t) \leq \tau$, τ is known constant. $y \in R$ is the output of the system, $f_i(x), i = 1, 2, \cdots, n$ are unknown smooth nonlinear functions.

To investigate the stability of the closed-loop system, we consider the sampling strategy, here we take $t_k (k = 1, 2, \ldots)$ as the sampling times, $h = t_{k+1} - t_k$ denotes the sampling period, which is a constant, and the zero-order hold (ZOH) receiving the signals experiences a time-varying signal transmission delays h_k , and the input arrives at nonlinear plant after a time-varying transmission delay $\tau(t)$. The structure of the proposed control scheme is shown in Figure 1.

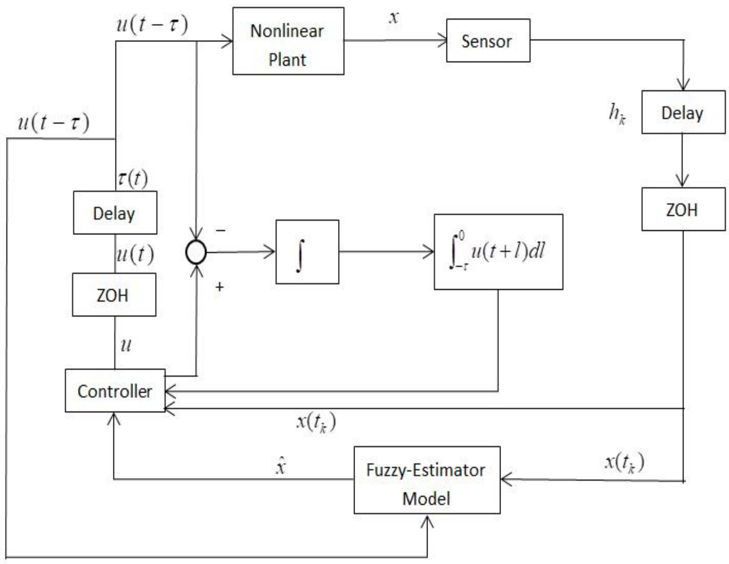

Figure 1. The schematic diagram of the proposed control scheme.

In order to investigate the stability of closed-loop system, we need to give the following assumption.

Assumption 1 [1]. Both the sensor and the controller are all time driven, they all activate under the same time period h, the sensor activates at every sampling time, and the controller activates latter time τ than the sensor.

Remark 1. In order to maintain generality of the proposed strategy, we consider the transmission delay and time-varying input delay as follows:

case(1). Considering the signal transmission delay satisfies $0 < h_k < h, k = 1, 2, \ldots$, the time relationship is shown in Figure 2.

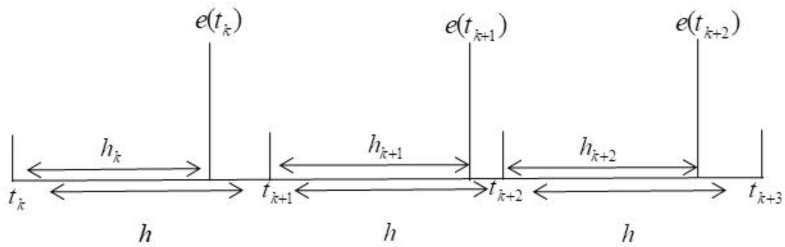

Figure 2. the time relationship in case (1).

If a sampling activity takes place at time t_k, then the sampled-data will arrive at the FE system at time $t_k + h_k$. In the entire control process, we need to guarantee that the control signal sends to the nonlinear plant before the current sampled-data is covered by the sampling information at next sampling moment. Therefore, the input delay should satisfy $t_k + h_k \leq t_{k+1} - \tau(t) \leq t_{k+1}$ and $t_{k+1} \leq t_{k+2} - \tau(t) < t_{k+1} + h_{k+1}$, thus, we have $0 \leq \tau(t) \leq h - h_k$ and $h - h_{k+1} < \tau(t) \leq h$.

case (2). If the signal transmission delay holds $h \le h_k < 2h, k = 1, 2, \ldots$, then the time relationship is shown in Figure 3.

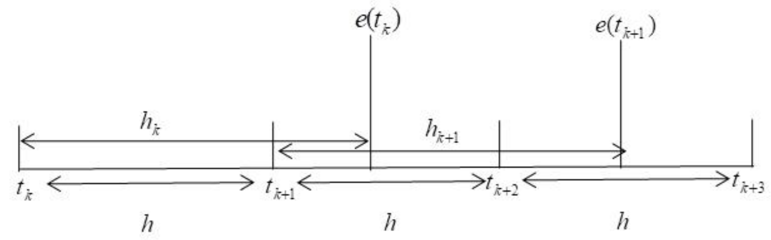

Figure 3. the time relationship in case (2).

Based on the case (1), the signal transmission delay and time-varying input delay should satisfy $t_k + h_k \le t_{k+2} - \tau(t) \le t_{k+2}$ and $t_{k+2} \le t_{k+3} - \tau(t) < t_{k+1} + h_{k+1}$, one has $0 \le \tau(t) \le 2h - h_k$ and $2h - h_{k+1} < \tau(t) \le h$.

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case (n). Considering the signal transmission delay holds $(n-1)h \le h_k < nh, k = 1, 2 \ldots$, the time relationship is shown in Figure 4.

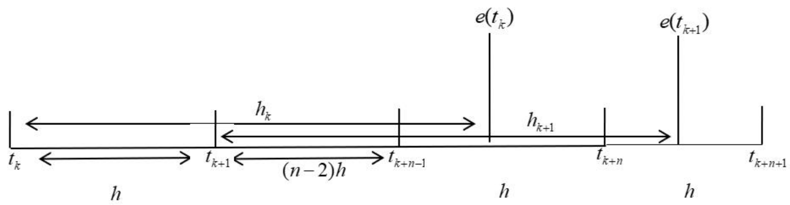

Figure 4. the time relationship in case (n).

Based on the case (1) and case (2), two delays should hold $t_k + h_k \le t_{k+n} - \tau(t)$ and $t_{k+n} \le t_{k+n+1} - \tau(t) < t_{k+1} + h_{k+1}$. Therefore, we have $0 \le \tau(t) \le nh - h_k$ and $nh - h_{k+1} < \tau(t) \le h$.

Through repeating these processes, we design signal transmission delay to satisfy $(a-1)h \le h_k < ah, a = 1, 2, \ldots$ and naturally the design parameter a is bounded. To guarantee the nonlinear plant getting the effective control, we design the signal transmission h_k and time-varying input delay $\tau(t)$ to satisfy $0 \le \tau(t) \le ah - h_k$ and $ah - h_{k+1} < \tau(t) \le h$. Moreover, it should be noticed that $u(t - \tau) = 0$, while $t - \tau \le 0$.

Control Objective-Considering the time varying input delay and the signal transmission delay, and by using the positive design parameters and information of FE model, an adaptive fuzzy controller in the framework of backstepping and adaptive fuzzy control method is proposed, which can make the original system and FE model reach the stable condition.

Remark 2. It should be mentioned that signal transmission delay has also been investigated in strict feedback form of Ref. [1]. However, they have not taken the time-varying input delay into account, which means their controller cannot be directly applied to this paper. In this paper, by taking advantage of sampling strategy, a constraint between them has been investigated.

Remark 3. In Refs. [19]- [21], they have solved the problem of input delay by introducing an integral term. But in this paper, based on the works of Refs. [19]- [21], we solve the problem of the time-varying input delay and time-varying transmission delay in the sampling control strategy.

2.2. Fuzzy logical systems

Lemma 1 [2]. For any continues function $f(x)$, there always exist a compact set Ω and a positive constant ε , by using FLS, there is an ideal parameter θ^* satisfies

$$\sup_{x\in\Omega}\left|f(x)-\theta^{*T}\phi(x)\right|\le\varepsilon. \tag{2.2}$$

There are unknown functions $f_i(\bar{x})$, which can be approximated by using the universal approximation property of the FLS,

$$f_i(\bar{x}_i|\,\theta_i)=\theta_i^T\phi_i(\bar{x}_i),\,1\le i\le n, \tag{2.3}$$

where $\bar{x}_i=(x_1,\ldots,x_i)$, $1\le i\le n$, and θ_i are the estimation of the optimal parameter vector θ_i^* .

Take the optimal parameter vector θ_i^* as

$$\theta_i^*=\arg\min_{\theta_i\in\Omega_i}[\sup_{\bar{x}_i\in U_i}\left|\hat{f}_i\,(\bar{x}_i|\,\theta_i)-f_i(\bar{x}_i)\right|],\,1\le i\le n, \tag{2.4}$$

where Ω_i and U_i are bounded compact sets for θ_i and $\bar{x}_i$, respectively. The corresponding fuzzy minimum approximated error ε_i is defined by

$$f_i(\bar{x}_i)=f_i\,(\bar{x}_i|\,\theta_i^*)+\varepsilon_i, \tag{2.5}$$

where ε_i satisfies $|\varepsilon_i|\le\varepsilon_i^*$ and ε_i^* is a constant.

2.3. Adaptive fuzzy control design

According to the form of system (2.1), we design the FE model in the following form

$$\begin{aligned}\dot{\hat{x}}_1&=\hat{x}_2+\theta_1^T\phi_1(\hat{x}),\\ \dot{\hat{x}}_2&=\hat{x}_3+\theta_2^T\phi_2(\hat{x}),\\ &\vdots\\ \dot{\hat{x}}_n&=u(t-\tau(t))+\theta_n^T\phi_n(\hat{x}),\\ \hat{y}&=\hat{x}_1,\end{aligned} \tag{2.6}$$

where $\hat{x}=[\hat{x}_1,\hat{x}_2,\ldots\hat{x}_n]^T$ and θ_i is the estimation of the ideal parameter vector θ_i^* .

In an arbitrary time interval $[t_k+h_k,t_{k+a})$, design the state $\breve{x}_i(t)=x_i(t_k)$, $i=1,2,\ldots n$, $\breve{x}(t)=x(t_k)$, and $u(t)=u(t_k+h_k)$, $\forall t\in[t_k+h_k,t_{k+a})$. The estimation errors $e_i=e_i(t)=\breve{x}_i(t)-\hat{x}_i(t)$, and the derivative of the estimation errors can be described as follows

$$\begin{aligned}\dot{e}_1&=e_2+\theta_1^{*T}[\phi_1(\breve{x})-\phi_1(\hat{x})]+\tilde{\theta}_1^T\phi_1(\hat{x})+\varepsilon_1,\\ \dot{e}_2&=e_3+\theta_2^{*T}[\phi_2(\breve{x})-\phi_2(\hat{x})]+\tilde{\theta}_2^T\phi_2(\hat{x})+\varepsilon_2,\\ &\vdots\\ \dot{e}_n&=\theta_n^{*T}[\phi_n(\breve{x})-\phi_n(\hat{x})]+\tilde{\theta}_n^T\phi_n(\hat{x})+\varepsilon_n,\end{aligned} \tag{2.7}$$

where $\tilde{\theta}_i = \theta_i^* - \theta_i$. Furthermore, rewrite (2.7) in the vector form

$$\dot{e} = Ae + K(\breve{x}_1 - \hat{x}_1) + \psi + \vartheta + \varepsilon, \tag{2.8}$$

where $e = \begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_n \end{bmatrix}, A = \begin{bmatrix} -k_1 & & & \\ \vdots & & I & \\ -k_n & 0 & \cdots & 0 \end{bmatrix}, K = \begin{bmatrix} k_1 \\ \vdots \\ k_n \end{bmatrix}, \psi = \begin{bmatrix} \theta_1^{*T}[\phi_1(\breve{x}) - \phi_1(\hat{x})] \\ \vdots \\ \theta_n^{*T}[\phi_n(\breve{x}) - \phi_n(\hat{x})] \end{bmatrix}, \vartheta = \begin{bmatrix} \tilde{\theta}_1^T \phi_1(\hat{x}) \\ \vdots \\ \tilde{\theta}_n^T \phi_n(\hat{x}) \end{bmatrix},$

$\varepsilon = \begin{bmatrix} \varepsilon_1 \\ \vdots \\ \varepsilon_n \end{bmatrix}$ and K is designed to make sure that A is a strict Hurwitz matrix. Thereby, for any given matrix $Q = Q^T > 0$, there always exists a positive definite matrix $P = P^T > 0$ such that

$$A^T P + PA = -Q. \tag{2.9}$$

To investigate the stability of the closed-loop system, we define the change of coordinates of the FE model as follows

$$\begin{aligned} \chi_1 &= \hat{x}_1, \\ \chi_i &= \hat{x}_i - \alpha_{i-1}, i = 2 \cdots, n-1, \\ \chi_n &= \hat{x}_n - \alpha_{n-1} + \int_{-\tau}^{0} u(\hat{t} + l)dl, \end{aligned} \tag{2.10}$$

where $\hat{t}$ denotes the arriving time, and $\alpha_i, i = 1, \ldots n-1$ are the virtual controllers.

For the nonlinear plant, we design the change of coordinates as follows

$$\begin{aligned} \xi_1 &= \breve{x}_1, \\ \xi_i &= \breve{x}_i - \alpha_{i-1}, i = 2 \cdots, n-1, \\ \xi_n &= \breve{x}_n - \alpha_{n-1} + \int_{-\tau}^{0} u(\hat{t} + l)dl. \end{aligned} \tag{2.11}$$

Refer to Ref. [33] , by taking the nonlinear system (2.1) and the derivative of (2.11) into account, one has

$$\begin{aligned} \dot{\xi}_1 &= \xi_2 + \alpha_1 + f_1(x), \\ \dot{\xi}_i &= \xi_{i+1} + \alpha_i + f_i(x) - \dot{\alpha}_{i-1}, i = 2 \cdots, n-2, \\ \dot{\xi}_{n-1} &= \xi_n + \alpha_{n-1} + \int_{-\tau}^{0} u(\hat{t} + l)dl + f_{n-1}(x) - \dot{\alpha}_{n-2}, \\ \dot{\xi}_n &= u(\hat{t}) + \alpha_n + f_n(x) - \dot{\alpha}_{n-1}. \end{aligned} \tag{2.12}$$

As a result of the signal transmission delay and time-varying input delay satisfy $0 \leq \tau(t) \leq ah - h_k$ and $ah - h_{k+1} < \tau(t) \leq h$, therefore, the part of controller design is divided into three cases.

Case (1). In case (1), we design $h_k < h_{k+1}$, thus, the time-varying input delay satisfies $0 \leq \tau(t) \leq h$. According to Ref. [21], by using integral mean value theorem and the constraint of the input delay, the integral term can be described as $\left|\int_{-\tau}^{0} u(\hat{t} + l)dl\right| \leq |\tau u(z)|, z \in [t_{k+a-1}, \hat{t}) \leq \left|\bar{\zeta}_1\right|$, where $\bar{\zeta}_1$ is a constant.

Step 1. To investigate the stability of nonlinear system, construct the Lyapunov function candidate as follows

$$V_1 = V_0 + \frac{\xi_1^2}{2} + \frac{\tilde{\theta}_1^T \tilde{\theta}_1}{2\gamma_1} + \frac{\tilde{\Theta}_1^T \tilde{\Theta}_1}{2\bar{\gamma}_1}, \tag{2.13}$$

where $\gamma_1 > 0$ and $\bar{\gamma}_1 > 0$ are the positive design parameters, $V_0 = e^T Pe + \int_{t-ah}^{t} \int_{s}^{t} e^T(v)Pe(v)dvds$, ah is the upper bound of the h_k , and $\tilde{\Theta}$ will be defined later. In order to reduce the complexity of $\dot{V}_1$, we firstly compute $\dot{V}_0$

$$\begin{aligned} \dot{V}_0 = & -\lambda_{\min}(Q)|e|^2 + 2e^T P[K(\breve{x}_1 - \hat{x}_1) + \psi + \vartheta + \varepsilon] \\ & + ah\lambda_{\max}(P)|e|^2 - \int_{t-ah}^{t} e^T(s)Pe(s)ds. \end{aligned} \tag{2.14}$$

On account of Young's inequality, we can obtain the following inequalities

$$\begin{aligned} 2e^T PK(\breve{x}_1 - \hat{x}_1) & \leq \|PK\|^2\|e\|^2 + |e_1|^2 \\ & \leq (\|PK\|^2 + 1)\|e\|^2, \end{aligned} \tag{2.15}$$

$$2e^T P\psi \leq 2\|P\|^2\|e\|^2 + 2\|\theta^*\|^2, \tag{2.16}$$

$$2e^T P\vartheta \leq \|P\|^2\|e\|^2 + \left\|\tilde{\theta}\right\|^2, \tag{2.17}$$

$$2e^T P\varepsilon \leq \|P\|^2\|e\|^2 + \|\varepsilon^*\|^2, \tag{2.18}$$

where $\varepsilon^* = \begin{bmatrix} \varepsilon_1^* \\ \vdots \\ \varepsilon_n^* \end{bmatrix}$, $\theta^* = \begin{bmatrix} \theta_1^* \\ \vdots \\ \theta_n^* \end{bmatrix}$ and $\tilde{\theta} = \begin{bmatrix} \tilde{\theta}_1 \\ \vdots \\ \tilde{\theta}_n \end{bmatrix}$.

By using above inequalities, (2.14) can be described as

$$\begin{aligned} \dot{V}_0 \leq & -\lambda_{\min}(Q)\|e\|^2 + 4\|P\|^2\|e\|^2 + (\|PK\|^2 + 1)\|e\|^2 \\ & + 2\|\theta^*\|^2 + \left\|\tilde{\theta}\right\|^2 + \|\varepsilon^*\|^2 + h\lambda_{\max}(P)|e|^2 - \int_{t-ah}^{t} e^T(s)Pe(s)ds. \end{aligned} \tag{2.19}$$

Hence, the time derivative of V_1 can be expressed as

$$\begin{aligned} \dot{V}_1 \leq & -\lambda_{\min}(Q)\|e\|^2 + 4\|P\|^2\|e\|^2 + (\|PK\|^2 + 1)\|e\|^2 + 2\|\theta^*\|^2 \\ & + \left\|\tilde{\theta}\right\|^2 + \|\varepsilon^*\|^2 + ah\lambda_{\max}(P)|e|^2 - \int_{t-ah}^{t} e^T(s)Pe(s)ds \\ & - \frac{\tilde{\theta}_1^T \dot{\theta}_1}{\gamma_1} - \frac{\tilde{\Theta}_1^T \dot{\Theta}_1}{\bar{\gamma}_1} + \xi_1[\xi_2 + \alpha_1 + \theta_1^{*T}\phi_1(\breve{x}) + \varepsilon_1], \end{aligned} \tag{2.20}$$

where $\tilde{\Theta}_1 = \Theta_1^* - \Theta_1$ and $\Theta_1^* = \theta_1^{*T}\theta_1^*$.

Obviously, thanks to $\int_{t-ah}^{t} e^T(s)Pe(s)ds > 0$, so the integral term $-\int_{t-ah}^{t} e^T(s)Pe(s)ds < 0$ is missed.

According to the property of fuzzy basis function $0 < \phi_i^T(\cdot)\phi_i(\cdot) \leq 1$ and by using Young's inequality, we have

$$\begin{aligned} \dot{V}_1 \leq & -p_0\|e\|^2 + 2\|\theta^*\|^2 + \left\|\tilde{\theta}\right\|^2 + \xi_1(\xi_2 + \alpha_1 + \frac{w_1\Theta_1^*}{4} + \tilde{\theta}_1^T\phi_1(\hat{x}_1) - \tilde{\theta}_1^T\phi_1(\hat{x}_1)) \\ & + \xi_1^2 - \frac{\tilde{\theta}_1^T \dot{\theta}_1}{\gamma_1} - \frac{\tilde{\Theta}_1^T \dot{\Theta}_1}{\bar{\gamma}_1} + \frac{1}{2w_1^2} + \frac{\varepsilon_1^{*2}}{2} + T_0, \end{aligned} \tag{2.21}$$

where $p_0 = \lambda_{\min}(Q) - ah\lambda_{\max}(P) - (\|PK\|^2 + 1) - 4\|P\|^2$, $T_0 = \|\varepsilon^*\|^2$, and w_1 is a design positive parameter.

Design the virtual controller and parameter adaptive laws as follows

$$\alpha_1 = -(c_1 + \frac{3}{2})\chi_1 - \frac{w_1}{4}\Theta_1 + \theta_1^T\phi_1(\hat{x}_1), \tag{2.22}$$

$$\dot{\theta}_1 = \gamma_1(-\chi_1\phi_1(\hat{x}_1) - \sigma_1\theta_1), \tag{2.23}$$

$$\dot{\Theta}_1 = \bar{\gamma}_1(\frac{\chi_1 w_1}{4} - \bar{\sigma}_1\Theta_1), \tag{2.24}$$

where c_1 , σ_1 and $\bar{\sigma}_1$ are positive design constants.

Substituting (2.22-2.24) into (2.21), it follows that

$$\begin{aligned}\dot{V}_1 \leq & -(c_1 - 1)\xi_1^2 + \frac{\xi_2^2}{2} + \frac{1}{2}\tilde{\Theta}_1^T\tilde{\Theta}_1 + \frac{1}{2}\tilde{\theta}_1^T\tilde{\theta}_1 + \bar{\sigma}_1\tilde{\Theta}_1^T\Theta_1 \\ & + \frac{1}{2}\theta_1^{*T}\theta_1^* + \sigma_1\tilde{\theta}_1^T\theta_1 - p_1\|e\|^2 + T_1 + 2\|\theta^*\|^2 + \left\|\tilde{\theta}\right\|^2,\end{aligned} \tag{2.25}$$

where $p_1 = p_0 - [\frac{1}{2}(c_1 + \frac{3}{2})^2 + \frac{w_1^2}{32} + \frac{1}{2}]$ and $T_1 = T_0 + \frac{\varepsilon_1^{*2}}{2} + \frac{1}{2w_1^2}$.

Step $i, i = 2, ..., n-2$. Choose the following Lyapunov function as

$$V_i = V_{i-1} + \frac{\xi_i^2}{2} + \frac{\tilde{\theta}_i^T\tilde{\theta}_i}{2\gamma_i} + \frac{\tilde{\Theta}_i^T\tilde{\Theta}_i}{2\bar{\gamma}_i}. \tag{2.26}$$

where $\tilde{\Theta}_i$ is the approximation error of the ideal parameter Θ_i^* , and $\Theta_i^* = \theta_i^{*T}\theta_i^*$.

Consider the Young's inequality with the time derivative of (2.26), one yields

$$\begin{aligned}\dot{V}_i = & \dot{V}_{i-1} + \xi_i(\xi_{i+1} + \alpha_i + \frac{w_i\Theta_i^*}{4} - \dot{\alpha}_{i-1} + \tilde{\theta}_i^T\phi_i(\hat{x}_i) - \tilde{\theta}_i^{\mathrm{T}}\phi_i(\hat{x}_i)) + \xi_i^2 \\ & + \frac{\varepsilon_i^{*2}}{2} + \frac{1}{2w_i^2} - \frac{\tilde{\theta}_i^T\dot{\theta}_i}{\gamma_i} - \frac{\tilde{\Theta}_i^T\dot{\Theta}_i}{\bar{\gamma}_i}.\end{aligned} \tag{2.27}$$

where w_i is a positive design parameter.

Design i-th virtual controller and parameter adaptive laws in the following

$$\alpha_i = -(c_i + 2)\chi_i - \frac{w_i\Theta_i}{4} + \theta_i^T\phi_i(\hat{x}_i) + \dot{\alpha}_{i-1}, \tag{2.28}$$

$$\dot{\theta}_i = \gamma_i(-\chi_i\phi_i(\hat{x}_i) - \sigma_i\theta_i), \tag{2.29}$$

$$\dot{\Theta}_i = \bar{\gamma}_i(\frac{\chi_i w_i}{4} - \bar{\sigma}_i\Theta_i), i = 2, \cdots, n-2, \tag{2.30}$$

where $c_i, \gamma_i, \bar{\gamma}_i, \sigma_i$ and $\bar{\sigma}_i, i = 2, \ldots, n-2$ are positive design constants.

From virtual controller (2.28) and parameter adaptive laws (2.29-2.30), (2.27) can be rewritten in the following

$$\begin{aligned}V_i \leq & -\sum_{m=1}^{i-1}(c_m - 1)\xi_m^2 - p_i\|e\|^2 + \sum_{m=1}^{i}\sigma_m\tilde{\theta}_m^T\theta_m + \sum_{m=1}^{i}\bar{\sigma}_m\tilde{\Theta}_m^T\Theta_m + T_i + 2\|\theta^*\|^2 \\ & + \left\|\tilde{\theta}\right\|^2 - \xi_i(c_i + 2)\chi_i + \frac{5}{2}\xi_{i+1}^2 + \frac{1}{2}\xi_{i+1}^2 + \sum_{m=1}^{i}\frac{\tilde{\Theta}_m^T\tilde{\Theta}_m}{2} + \sum_{m=1}^{i}\frac{\tilde{\theta}_m^T\tilde{\theta}_m}{2},\end{aligned} \tag{2.31}$$

where $p_i = p_{i-1} - [\frac{1}{2}(c_i+2)^2 + \frac{w_i^2}{32} + \frac{1}{2}]$ and $T_i = T_{i-1} + \frac{\varepsilon_i^{*2}}{2} + \frac{1}{2w_i^2}$.

By using Young's inequality, we can obtain following inequality

$$\begin{aligned} -\xi_i(c_i+2)\chi_i &= -\xi_i(c_i+2)(\xi_i - e_i) \\ &\le -(c_i+2)\xi_i^2 + \frac{\xi_i^2}{2} + \frac{1}{2}(c_i+2)^2 e_i^2. \end{aligned} \tag{2.32}$$

Substituting (2.32) into (2.31), results in

$$\begin{aligned} \dot{V}_i \le& -\sum_{m=1}^{i}(c_m-1)\xi_m^2 - p_i|e|^2 + \sum_{m=1}^{i}\sigma_m\tilde{\theta}_m^T\theta_m + \sum_{m=1}^{i}\bar{\sigma}_m\tilde{\Theta}_m^T\Theta_m \\ &+\frac{1}{2}\xi_{i+1}^2 + T_i + \frac{1}{2}\sum_{m=1}^{i}\tilde{\theta}_m^T\tilde{\theta}_m + \sum_{m=1}^{i}\frac{\tilde{\Theta}_m^T\tilde{\Theta}_m}{2} + 2\|\theta^*\|^2 + \left\|\tilde{\theta}\right\|^2. \end{aligned} \tag{2.33}$$

Step $n-1$**.** Consider following Lyapunov candidate function

$$V_{n-1} = V_{n-2} + \frac{\xi_{n-1}^2}{2} + \frac{\tilde{\theta}_{n-1}^T\tilde{\theta}_{n-1}}{2\gamma_{n-1}} + \frac{\tilde{\Theta}_{n-1}^T\tilde{\Theta}_{n-1}}{2\bar{\gamma}_{n-1}}, \tag{2.34}$$

where $\tilde{\Theta}_{n-1}$ is the approximation error of the ideal parameter Θ_{n-1}^* , and $\Theta_{n-1}^* = \theta_{n-1}^{*T}\theta_{n-1}^*$.

Considering Young's inequality, (2.34) can be transformed into

$$\begin{aligned} \dot{V}_{n-1} =& \dot{V}_{n-2} + 2\xi_{n-1}^2 + \frac{\xi_n^2}{2} + \frac{1}{2}(\bar{\zeta}_1)^2 + \frac{{\varepsilon_{n-1}^*}^2}{2} + \frac{1}{2w_{n-1}^2} - \frac{\tilde{\Theta}_{n-1}^T\dot{\Theta}_{n-1}}{\bar{\gamma}_{n-1}} - \frac{\tilde{\theta}_{n-1}^T\dot{\theta}_{n-1}}{\gamma_{n-1}} \\ &+\xi_{n-1}(\alpha_{n-1} + \frac{w_{n-1}\Theta_{n-1}^*}{4} - \dot{\alpha}_{n-2} + \theta_{n-1}^{*T}\phi_{n-1}(\hat{x}_{n-1}) - \theta_{n-1}^{*T}\phi_{n-1}(\hat{x}_{n-1})). \end{aligned} \tag{2.35}$$

where w_{n-1} is a positive design parameter.

Design the virtual controller and parameter adaptive laws as follows

$$\alpha_{n-1} = -(c_{n-1} + \frac{5}{2})\chi_{n-1} - \frac{w_{n-1}\Theta_{n-1}}{4} + \theta_{n-1}^T\phi_{n-1}(\hat{x}_{n-1}) + \dot{\alpha}_{n-2}, \tag{2.36}$$

$$\dot{\theta}_{n-1} = \gamma_{n-1}(-\chi_{n-1}\phi_{n-1}(\hat{x}_{n-1}) - \sigma_{n-1}\theta_{n-1}), \tag{2.37}$$

$$\dot{\Theta}_{n-1} = \bar{\gamma}_{n-1}(\frac{\chi_{n-1}w_{n-1}}{4} - \bar{\sigma}_{n-1}\Theta_{n-1}), \tag{2.38}$$

where $c_{n-1}, \gamma_{n-1}, \bar{\gamma}_{n-1}, \sigma_{n-1}$ and $\bar{\sigma}_{n-1}$ are positive design parameters.

Substituting (2.36)-(2.38) into (2.35), one has

$$\begin{aligned} \dot{V}_{n-1} \le& \dot{V}_{n-2} + \xi_{n-1}(-c_{n-1} - \frac{5}{2})\chi_{n-1} + 2\xi_{n-1}^2 + \frac{\xi_n^2}{2} + \tilde{\theta}_{n-1}^T\tilde{\theta}_{n-1} + \frac{1}{2}\theta_{n-1}^{*T}\theta_{n-1}^* \\ &+\tilde{\Theta}_{n-1}^T[w_{n-1}(\frac{\xi_{n-1} - \chi_{n-1}}{4}) + \bar{\sigma}_{n-1}\Theta_{n-1}] + \frac{{\varepsilon_{n-1}^*}^2}{2} + \frac{1}{2w_{n-1}^2} \\ &+\tilde{\theta}_{n-1}^T[\varpi_{n-1}(\xi_{n-1} - \chi_{n-1}) + \sigma_{n-1}\theta_{n-1}] + \frac{1}{2}(\bar{\zeta}_1)^2. \end{aligned} \tag{2.39}$$

Consider the time derivative of V_{n-2} and by employing Young's inequality, we have

$$\begin{aligned}\dot{V}_{n-1} \leq &-\sum_{m=1}^{n-1}(c_m-1)\xi_m^2 - p_{n-1}|e|^2 + \sum_{m=1}^{n-1}\sigma_m\tilde{\theta}_m^T\theta_m + \sum_{m=1}^{n-1}\bar{\sigma}_m\tilde{\Theta}_m^T\Theta_m \\ &+\frac{\xi_n^2}{2} + T_{n-1} + \frac{1}{2}\sum_{m=1}^{n-1}\tilde{\theta}_m^T\tilde{\theta}_m + \frac{1}{2}\sum_{m=1}^{n-1}{\theta_m^*}^T\theta_m^* + 2\|\theta^*\|^2 + \left\|\tilde{\theta}\right\|^2 \\ &+\frac{1}{2}(\bar{\zeta}_1)^2 + \frac{1}{2}\sum_{m=1}^{n-1}\tilde{\Theta}_m^T\tilde{\Theta}_m.\end{aligned} \tag{2.40}$$

where $p_{n-1} = p_{n-2} - [\frac{1}{2}(c_{n-1}+\frac{5}{2})^2 + \frac{w_{n-1}^2}{32} + \frac{1}{2}]$ and $T_{n-1} = T_{n-2} + \frac{{\varepsilon_{n-1}^*}^2}{2} + \frac{1}{2w_{n-1}^2}$.

Step n. Finally, consider the n-th Lyapunov function in the following

$$V_n = V_{n-1} + \frac{\xi_n^2}{2} + \frac{\tilde{\theta}_n^T\tilde{\theta}_n}{2\gamma_n} + \frac{\tilde{\Theta}_n^T\tilde{\Theta}_n}{2\bar{\gamma}_n}. \tag{2.41}$$

The time-derivative of (2.41) is

$$\begin{aligned}\dot{V}_n = \dot{V}_{n-1} + \xi_n[u(\hat{t}) + \tilde{\theta}_n^T\phi_n(\hat{x}) - \tilde{\theta}_n^T\phi_n(\hat{x}) + \varepsilon_n - \dot{\alpha}_{n-1} + u(t_k+h_k) - u(\hat{t})] \\ -\frac{1}{\gamma_n}\tilde{\theta}_n^T\dot{\theta}_n - \frac{1}{\bar{\gamma}_n}\tilde{\Theta}_n^T\dot{\Theta}_n.\end{aligned} \tag{2.42}$$

Design the actual controller and parameter adaptive laws as follows

$$u = -(c_n+\frac{5}{2})\chi_n - \frac{w_n\Theta_n}{4} + \theta_n^T\phi_n(\hat{x}) + \dot{\alpha}_{n-1}, \tag{2.43}$$

$$\dot{\theta}_n = \gamma_n(-\chi_n\phi_n(\hat{x}_n) - \sigma_n\theta_n), \tag{2.44}$$

$$\dot{\Theta}_n = \bar{\gamma}_n(\frac{\chi_n w_n}{4} - \bar{\sigma}_n\Theta_n), \tag{2.45}$$

where $c_n, \gamma_n, \bar{\gamma}_n, \sigma_n$ and $\bar{\sigma}_n$ are positive design constants.

Based on the $1 \sim n-1$ process, we have

$$\begin{aligned}\dot{V}_n \leq &-\sum_{i=1}^{n}(c_i-1)\xi_i^2 - p_n\|e\|^2 + \sum_{i=1}^{n}\sigma_i\tilde{\theta}_i^T\theta_i + \sum_{i=1}^{n}\bar{\sigma}_i\tilde{\Theta}_i^T\Theta_i + T_n \\ &+\sum_{i=1}^{n}\frac{\tilde{\Theta}_i^T\tilde{\Theta}_i}{2} + \frac{1}{2}\sum_{i=1}^{n}\tilde{\theta}_i^T\tilde{\theta}_i + \frac{1}{2}\sum_{i=1}^{n}\theta_i^{*T}\theta_i^* + 2\|\theta^*\|^2 + \left\|\tilde{\theta}\right\|^2 + \frac{3}{2}(\bar{\zeta}_1)^2,\end{aligned} \tag{2.46}$$

where $p_n = p_{n-1} - \frac{1}{2}[\frac{1}{2}(c_n+\frac{5}{2})^2 + (\frac{1}{32w_n})^2 + \frac{1}{2}]$ and $T_n = T_{n-1} + \frac{\varepsilon_n^{*2}}{2} + \frac{1}{2w_n^2}$.

On account of Young's inequality, results in

$$\sigma_i\tilde{\theta}_i^T\theta_i \leq -\frac{1}{2}\sigma_i\tilde{\theta}_i^T\tilde{\theta}_i + \frac{1}{2}\sigma_i\theta_i^{*T}\theta_i^*, \tag{2.47}$$

$$\bar{\sigma}_i\tilde{\Theta}_i^T\Theta_i \leq -\frac{1}{2}\bar{\sigma}_i\tilde{\Theta}_i^T\tilde{\Theta}_i + \frac{1}{2}\bar{\sigma}_i\Theta_i^{*T}\Theta_i^*. \tag{2.48}$$

From (2.46), (2.47)and (2.48), one yields

$$\begin{aligned}\dot{V}_n \leq & -\sum_{i=1}^{n}(c_i-1)\xi_i^2 - p_n\|e\|^2 - \frac{1}{2}\sum_{i=1}^{n}(1-\bar{\sigma}_i)\tilde{\Theta}_i^T\tilde{\Theta}_i + \frac{1}{2}\sum_{i=1}^{n}\bar{\sigma}_i\Theta_i^{*T}\Theta_i^* \\ & -\frac{1}{2}\sum_{i=1}^{n}(\sigma_i-3)\tilde{\theta}_i^T\tilde{\theta}_i + \frac{1}{2}\sum_{i=1}^{n}(\sigma_i+5)\theta_i^{*T}\theta_i^* + T_n + \frac{3}{2}(\bar{\zeta}_1)^2.\end{aligned} \tag{2.49}$$

Let $C = \min\{\frac{p_n}{\lambda\min(P)}, 2(c_i-1), (\sigma_i-3)\gamma_i, (1-\bar{\sigma}_i)\bar{\gamma}_i\}$, and $\kappa_1 = \frac{1}{2}\sum\limits_{i=1}^{n}(\sigma_i+5)\theta_i^{*T}\theta_i^* + T_n + \frac{1}{2}\sum\limits_{i=1}^{n}\bar{\sigma}_i\Theta_i^{*T}\Theta_i^* + \frac{3}{2}(\bar{\zeta}_1)^2$.

Finally, (2.49) can be expressed as

$$\dot{V} \leq -CV + \kappa_1. \tag{2.50}$$

1) If $h_k > h_k + 1$, then we differentiate it into two parts

Case 2. In this case, we design that input delay holds $0 \leq \tau(t) \leq ah - h_k$, and the integral term satisfies $\left|\int_{-\tau}^{0} u(\hat{t}+l)dl\right| \leq |\tau u(z)|, z \in [t_k + h_k, \hat{t}) \leq \left|\bar{\zeta}_2\right|$, where $\bar{\zeta}_2$ is a constant.

Consider the following Lyapunov function

$$V = e^TPe + \int_{t-ah}^{t}\int_{s}^{t} e^T(v)Pe(v)dvds + \sum_{i=1}^{n}\frac{\xi_i^2}{2} + \sum_{i=1}^{n}\frac{\tilde{\theta}_i^T\tilde{\theta}_i}{2\gamma_i} + \sum_{i=1}^{n}\frac{\tilde{\Theta}_i^T\tilde{\Theta}_i}{2\bar{\gamma}_i}. \tag{2.51}$$

Take the same controller design approach, we have the result

$$\dot{V} \leq -CV + \kappa_2, \tag{2.52}$$

where $\kappa_2 = \frac{1}{2}\sum\limits_{i=1}^{n}(\sigma_i+5)\theta_i^{*T}\theta_i^* + T_n + \frac{1}{2}\sum\limits_{i=1}^{n}\bar{\sigma}_i\Theta_i^{*T}\Theta_i^* + \frac{3}{2}(\bar{\zeta}_2)^2$.

Case 3. Under this part, input delay satisfies $ah - h_{k+1} < \tau(t) \leq h$, and based on above analysis, we have $\left|\int_{-\tau}^{0} u(\hat{t}+l)dl\right| \leq |\tau u(z)|, z \in [t_k + h_{k+1}, \hat{t}] \leq \left|\bar{\zeta}_3\right|$, where $\bar{\zeta}_3$ is a constant. Through the same process of controller design, the final form of the time derivative of the Lyapunov function is

$$\dot{V} \leq -CV + \kappa_3, \tag{2.53}$$

where $\kappa_3 = \frac{1}{2}\sum\limits_{i=1}^{n}(\sigma_i+5)\theta_i^{*T}\theta_i^* + T_n + \frac{1}{2}\sum\limits_{i=1}^{n}\bar{\sigma}_i\Theta_i^{*T}\Theta_i^* + \frac{3}{2}(\bar{\zeta}_3)^2$.

Theorem 1. Under the actual controller (2.43) with the virtual controllers (2.22), (2.28) and (2.36), adaptive parameter laws (2.23-2.24), (2.29-2.30), (2.37-2.38) and (2.44-2.45), there exist sufficiently large impact sets $\Omega_i \in R_i$, $i = 1, 2, \ldots n$, which satisfy $\Omega_i \in R_i$, $i = 1, 2, \ldots n$ for interval $[t_k + h_k, t_{k+1} + h_{k+1})$. Then we proved that all signals in the closed-loop system are bounded, and the states x , and fuzzy estimates $\theta_1^T, \ldots, \theta_n^T$ and $\Theta_1^T, \ldots, \Theta_n^T$ are all ultimately coverage to the compact set $\Omega_{s1} \overset{\Delta}{=} \{x, \theta_1^T, \ldots, \theta_n^T, \Theta_1^T, \ldots, \Theta_n^T \big| V < \frac{\kappa}{C}\}$, where $\kappa = \max(\kappa_1, \kappa_2, \kappa_3)$.

Proof. Multiply e^{ct} and integrate both side of $\dot{V} \leq -CV + \kappa$ for time $[t_k + h_k, t_{k+1} + h_{k+1})$, we have

$$V(t_{k+1} + h_{k+1}) \leq e^{(h_k - h_{k+1} - h)C}V(t_k + h_k) + \frac{\kappa}{C}e^{C(t_{k+1}+h_{k+1})} - \frac{\kappa}{C}e^{C(t_k+h_k)}. \tag{2.54}$$

Take $k = -1$ as the first sampling time. Take time interval $[t_0, t_0+h_0)$ into (2.54), we have $V(t_0+h_0) \leq e^{(-h_0-h)C}V(t_0) + \frac{\kappa}{C}(e^{C(t_0+h_0)} - e^{C(t_0)})$. Due to the initial $V(t_0)$ is finite, and according to Remark 1, we have

$0 \le h_0 \le ah$, thus, $V(t_0 + h_0)$ is bounded. Repeating this produce, we can know $V(t_k + h_k)$ is bounded. So, all signals in the closed-loop system are SGUUB for time $[t_0, \infty)$. To provide the attractiveness of (2.54) for a region, we distinct two conditions:

(1)If $(x(t_k + h_k), \theta_1^T(t_k + h_k), \dots, \theta_n^T(t_k + h_k), \Theta_1^T(t_k + h_k), \dots, \Theta_n^T(t_k + h_k)) \in \Omega_{01} \in \Omega_{s1}$. Refer to Theorem 2.14 in Ref. [32], all the states x and the fuzzy estimates $\theta_1^T, \dots, \theta_n^T$ and $\Theta_1^T, \dots, \Theta_n^T$ remain in Ω_{s1} for $\forall t \in [t_k + h_k, t_{k+1} + h_{k+1})$.

(2)If $(x(t_k + h_k), \theta_1^T(t_k + h_k), \dots, \theta_n^T(t_k + h_k), \Theta_1^T(t_k + h_k), \dots, \Theta_n^T(t_k + h_k)) \in \Omega_{02} \in \Omega_{s1}^c$, where Ω_{s1}^c is the complementary of Ω_{s1}. Due to $\dot{V}$ remains negative definite until the states x and fuzzy estimates $\theta_1^T, \dots, \theta_n^T$ and $\Theta_1^T, \dots, \Theta_n^T$ will eventually enter and stay in Ω_{s1} for $\forall t \in [t_k + h_k, t_{k+1} + h_{k+1})$.

3. Results

In this section, a numerical simulation example is given to illustrate the effectiveness of the proposed control strategy.

Consider the following second-order system with time-varying input delay

$$\begin{aligned} \dot{x}_1 &= x_2 + f_1(x), \\ \dot{x}_2 &= u(t - \tau(t)) + f_2(x), \\ y &= x_1, \end{aligned} \tag{3.1}$$

where $f_1(x) = 5x_1x_2\sin(x_1)$, $f_2(x) = 2x_1x_2\cos(x_2^2)$.

To approximate the unknown nonlinear functions, select fuzzy basic functions in the following$\mu_{F_j}^l(\hat{x}_1, \hat{x}_2) = \exp([-0.5(\hat{x}_1 - 2 + 0.5j)^2/4] + [0.5(\hat{x}_2 - 2 + 0.5j)^2/4])$, $\mu_{F_j}^l(\hat{x}_1, \hat{x}_2) = \exp([-0.5(\hat{x}_1 - 1.5 + 0.5j)^2/4] + [0.5(\hat{x}_2 - 1.5 + 0.5j)^2/4])$, $j = 1, ..., 7$, $l = 1, 2$.

Define fuzzy membership function as $\phi_j^l(\hat{x}) = \mu_{F_j}(\hat{x}_1, \hat{x}_2)/\sum_{l=1}^7 \mu_{F_l}(\hat{x}_1, \hat{x}_2)$, $j = 1, \dots, 7, l = 1, 2$. where $\theta_1^T = [\theta_{11}, \theta_{12}, \theta_{13}, \theta_{14}, \theta_{15}, \theta_{16}, \theta_{17}]^T$, $\theta_2^T = [\theta_{21}, \theta_{22}, \theta_{23}, \theta_{24}, \theta_{25}, \theta_{26}, \theta_{27}]^T$, $\phi_1(\hat{x}) = [\phi_1^1(\hat{x}_1, \hat{x}_2), \phi_2^1(\hat{x}_1, \hat{x}_2), \phi_3^1(\hat{x}_1, \hat{x}_2), \phi_4^1(\hat{x}_1, \hat{x}_2), \phi_5^1(\hat{x}_1, \hat{x}_2), \phi_6^1(\hat{x}_1, \hat{x}_2), \phi_7^1(\hat{x}_1, \hat{x}_2)]$, $\phi_2(\hat{x}) = [\phi_1^2(\hat{x}_1, \hat{x}_2), \phi_2^2(\hat{x}_1, \hat{x}_2), \phi_3^2(\hat{x}_1, \hat{x}_2), \phi_4^2(\hat{x}_1, \hat{x}_2), \phi_5^2(\hat{x}_1, \hat{x}_2), \phi_6^2(\hat{x}_1, \hat{x}_2), \phi_7^2(\hat{x}_1, \hat{x}_2)]$.

Based on FLSs, design FE model in the following

$$\begin{aligned} \dot{\hat{x}}_1 &= \hat{x}_2 + \theta_1^T \phi_1(\hat{x}), \\ \dot{\hat{x}}_2 &= u(t - \tau(t)) + \theta_2^T \phi_2(\hat{x}). \end{aligned} \tag{3.2}$$

Choose $Q = diag[5, 5]$, $k_1 = 10$, $k_2 = 10$ and according to (2.9), we can get the positive matrix $P = \begin{bmatrix} 0.275 & 0.25 \\ 0.25 & 5.25 \end{bmatrix}$.

The positive design parameters are selected as $c_1 = 1$, $c_2 = 1$, $\gamma_1 = 0.01$, $\bar{\gamma}_1 = 5$, $\gamma_2 = 0.01$, $\bar{\gamma}_2 = 1$, $\sigma_1 = 0.5$, $\bar{\sigma}_1 = 0.5$, $\sigma_2 = 0.5$, $\bar{\sigma}_1 = 1.5$, $w_1 = 20$ and $w_2 = 20$.

The initial conditions are set as $x_1(0) = 0$, $x_2(0) = 0$, $\hat{x}_1(0) = 0$, $\hat{x}_2(0) = 0$, $\theta_1(0) = \theta_2(0) = [0, 0, 0, 0, 0, 0, 0]^T$, $\Theta_1(0) = \Theta_2(0) = 0$. While the sampling period h is chosen as 0.01, take $a = 50$, $h_k \in (00.5], k = 0, 1, 2, \dots$, $\tau(t) \in (0, 0.01]$. And while the sampling period h is chosen as 0.005, take $a = 50$, $h_k \in (00.25], k = 0, 1, 2, \dots$, $\tau(t) \in (0, 0.005]$. Form the simulation results in Figures 5–10, when the sampling period is 0.01, Figure 5 shows the response of state x_1 and estimation $\hat{x}_1$, Figure 6 shows the response of statex_2 and estimation $\hat{x}_2$, Figure 7 shows the response of input u of the closed-loop system. When the sampling period is 0.005, Figure 8 shows the response of state x_1 and estimation $\hat{x}_1$, Figure 9 shows the response of state x_2 and estimation $\hat{x}_2$, Figure 10 shows the response of input u of the closed-loop system.

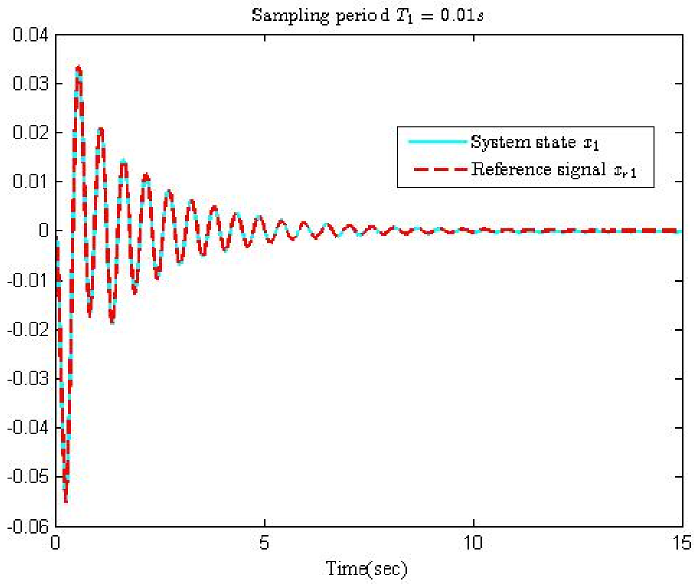

Figure 5. The response of state x_1 (red line)and estimation $\hat{x}_1$ (blue line)of the closed-loop system when the sampling period is 0.01.

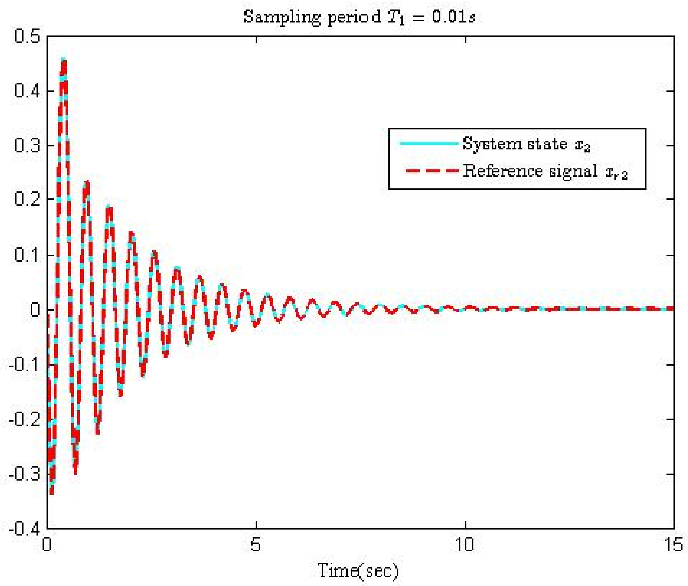

Figure 6. The response of state x_2 (red line)and estimation $\hat{x}_2$ (blue line)of the closed-loop system when the sampling period is 0.01.

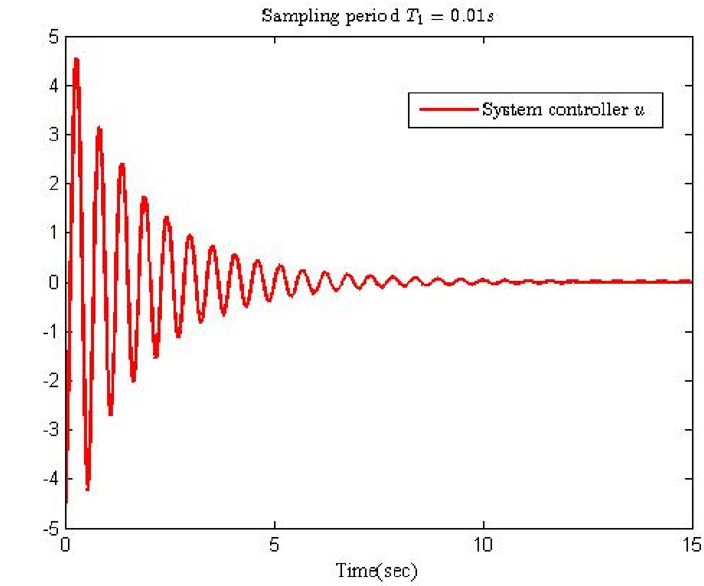

Figure 7. The response of input u of the closed-loop system when the sampling period is 0.01.

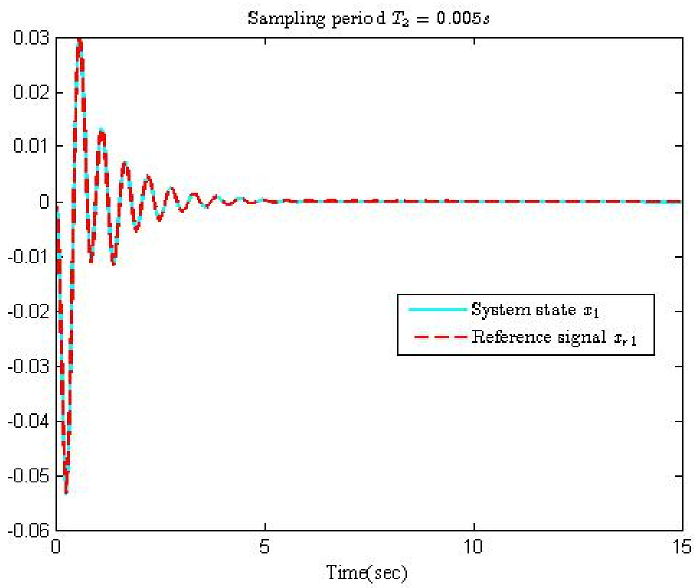

Figure 8. The response of state x_1 (red line)and estimation $\hat{x}_1$ (blue line)of the closed-loop system when the sampling period is 0.005.

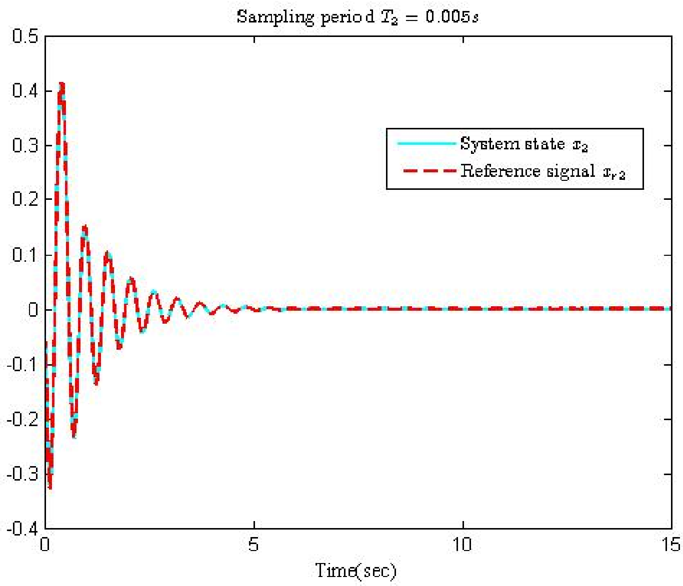

Figure 9. The response of state x_2 (red line)and estimation $\hat{x}_2$ (blue line)of the closed-loop system when the sampling period is 0.005.

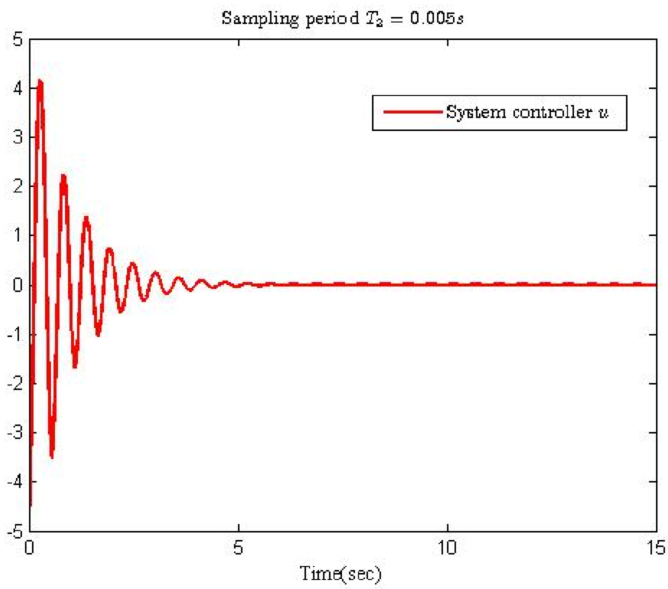

Figure 10. The response of input u of the closed-loop system when the sampling period is 0.005.

Ultimately, it is easy to see that all signals of the closed-loop system are bounded, which proved the effectiveness of the proposed control strategy.

4. Discussion

Now while the development of computer network is rapidly expending, the problems of delay are attracting increasing attention. To deal with the input delay, [19–21] have proposed a integral term to solve the problem of input delay. Based on their works, under the sampled-data strategy, this paper have addressed the problem of time-varying input delay via employing the integral term. Subsequently, a time-varying transmission delay has been considered during the state signal of controlled plant transmitting to the FE model. To stabilize the controlled plant, the restricted condition of them have been proposed. Under the restricted condition, the proposed control strategy can stabilize the non-strict feedback system, which accompany with the time-varying input delay and time-varying transmission delay. However, one limitation should be noticed, which is the input signal have to be bounded during the control process. Although [19–21] have been involved the limitation also. In next work, we will focus on removing this limitation in the control process, and extend this conclusion to the nonlinear switched systems or stochastic systems.

5. Conclusion

This paper investigates the control design and the property of stability for a class of nonlinear systems with sampled data and time-varying input delay. Fuzzy logical systems have been utilized to approximate unknown nonlinear functions, and a fuzzy estimator (FE) model is introduced to estimate state vector of the original plant, which mainly provides states information to the controller. In the proposed strategy, the constraint between transmission delay and input delay are given and the state vectors are transformed to compensate the effect of time-varying input delay. Moreover, the proposed adaptive fuzzy controller and adaptive parameter laws are able to make all signals of the closed-loop system are SGUUB by choosing the appropriate design parameters. Simulation results also prove the effectiveness of the proposed strategy. The next work will focus on the sampled-data control for the nonlinear switched systems or stochastic systems [34–37].

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Conflict of interest

The authors declare no conflict of interest.

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