Mathematics

http://www.aimspress.com/journal/Math

AIMS Mathematics, 2(1): 102-110
DOI:10.3934/Math.2017.1.102
Received: 5 December 2016
Accepted: 15 January 2017
Published: 14 February 2017

Research article

Large time behavior framework for the time-increasing weak solutions of bipolar hydrodynamic model of semiconductors

Mengmeng Shang *

Department of Mathematics, Shandong Normal University, Jinan, 250014, China

* **Correspondence:** Email : 2523659632@qq.com

Abstract: In this paper, we consider an isentropic Euler-Poisson equations for the bipolar hydrodynamic model of semiconductor devices, which has a non-flat doping profile and insulating boundary conditions. Using a technical energy method and an entropy dissipation estimate, we present a framework for the large time behavior of time-increasing weak entropy solutions. It is shown that the weak solutions converge to the stationary solutions in L^2 norm with exponential decay rate. No regularity and smallness conditions are assumed.

Keywords: Euler-Poisson system; bipolar semiconductor device; entropy solution; stationary solution; large time behavior

---

1. Introduction

In this paper, isentropic Euler-Poisson equations for the bipolar hydrodynamic model of semiconductor devices are considered. This model is as follows

$$\begin{cases} n_{1t} + J_{1x} = 0, \\ J_{1t} + (\dfrac{J_1^2}{n_1} + n_1)_x = n_1 E - J_1, \\ n_{2t} + J_{2x} = 0, \\ J_{2t} + (\dfrac{J_2^2}{n_2} + n_2)_x = -n_2 E - J_2, \\ E_x = n_1 - n_2 - D(x), \end{cases} \tag{1.1}$$

here n_1, n_2, J_1, J_2 and E are the unknown functions of the space variable $x \in [0, 1]$ and time variable $t \geq 0$, representing the electron density, the hole density, the electron current density, the hole current density and the electric field respectively. The function $D(x) > 0$, usually called the doping profile, stands for the density of impurities in semiconductor devices. In this paper, we assume the doping profile $D(x)$ satisfies

$$D^* = \sup_x D(x) \geq \inf_x D(x) = D_*. \tag{1.2}$$

For the system (1.1), the initial-boundary conditions are described by

$$\begin{aligned} &n_i(x,0) = n_{i0}(x) \geq 0, J_i(x,0) = J_{i0}(x), \\ &J_i(0,t) = J_i(1,t) = 0, E(0,t) = E(1,t) = 0, \quad i = 1,2 \end{aligned} \tag{1.3}$$

with the compatibility condition

$$J_{i0}(0) = J_{i0}(1) = 0, \ \ i = 1,2. \tag{1.4}$$

When $n_2 = J_2 = 0$ in (1.1), the bipolar model turns into the unipolar one, that is

$$\begin{cases} n_t + J_x = 0, \\ J_t + (\dfrac{J^2}{n} + n)_x = nE - J, \\ E_x = n - D(x). \end{cases} \tag{1.5}$$

Recently, many efforts are made on the systems (1.1) and (1.5) to considering the large time behavior of their weak entropy solutions. With the smallness assumption on the amplitude of background electron current, [1] first proved the uniformly bounded density weak entropy solutions of the unipolar hydrodynamic model (1.5), decay exponentially to the stationary solutions. [3] considered a similar problem on the bipolar model with a non-flat doping profile. However, the uniform bounded condition

$$0 \leq n_i(x,t) \leq C_0 \tag{1.6}$$

in [1] ([3])is stiff and still be open although it seems natural from physical point of view. For example, the L^∞ bounds obtained in [2, 4, 7] grow with time. In this paper, instead of proving the hard bone (1.6), we will give a large time behavior framework for density time-increasing entropy solutions to the bipolar hydrodynamic model (1.1) − (1.3). The related work on unipolar model, we can see the reference [6]. We make some preparation work before to introduce the primary result.

The vector function (n_1, n_2, J_1, J_2, E) is a weak solution of problem (1.1) − (1.4), if it satisfies the equation (1.1) in the distributional sense, verifies the restriction (1.3) and (1.4). Furthermore, a weak solution of system (1.1) − (1.4) is called an entropy solution if it satisfies the entropy inequality

$$\eta_{et} + q_{ex} + \frac{J_1^2}{n_1} + \frac{J_2^2}{n_2} - J_1E + J_2E \leq 0 \tag{1.7}$$

in the sense of distribution. And (η_e, q_e) are mechanical entropy-entropy flux pair satisfying

$$\begin{cases} \eta_e(n_1,n_2,J_1,J_2) = \dfrac{J_1^2}{2n_1} + n_1^2 + \dfrac{J_2^2}{2n_2} + n_2^2, \\ \\ q_e(n_1,n_2,J_1,J_2) = \dfrac{J_1^3}{2n_1^2} + 2n_1J_1 + \dfrac{J_2^3}{2n_2^2} + 2n_2J_2. \end{cases} \tag{1.8}$$

The corresponding stationary system of problem (1.1) − (1.4) is

$$\begin{cases} N_{1x} = N_1\mathcal{E}, \\ N_{2x} = -N_2\mathcal{E}, \\ \mathcal{E}_x = N_1 - N_2 - D(x) \end{cases} \tag{1.9}$$

with the boundary condition

$$\mathcal{E}(0) = \mathcal{E}(1) = 0. \tag{1.10}$$

In reference [5], the author give the following existence and uniqueness Theorem, that is
theorem A *Problem* (1.9) − (1.10) *has an unique stationary solution* $(N_1, N_2, \mathcal{E})$ *satisfying*
1) $D_* \le N_1 - N_2 \le D^*$ *and there exist positive constant* N^* *and* N_* *such that* $0 < N_* \le N_1, N_2 \le N^*$*;*
2) $D_* - D^* \le \mathcal{E}, \mathcal{E}_x \le D^* - D_*$.

2. results

This following Theorem is main result of this paper.
Theorem 1 (Large time behavior framework). *Suppose* $(n_1, n_2, J_1, J_2, E)(x, t)$ *be any* L^∞ *weak entropy solution to problem* (1.1) − (1.4) *satisfying*

$$0 \le n_i(x, t) \le Mt^\alpha, \quad M \ge 0, \quad 0 \le \alpha \le 2, \tag{2.1}$$

$(N_1, N_2, \mathcal{E})(x, t)$ *be the unique stationary smooth solution. If*

$$(E - \mathcal{E})(x, 0) \in L^2(R), \quad \sum_{i=1}^{2}(\frac{J_i^2}{2n_i} + (n_i - N_i)^2)(x, 0) \in L^1(R), \tag{2.2}$$

$$\|n_1 - n_2 - N_1 + N_2 - D(x)\|_{L^\infty} < \sqrt{\|8(N_1 + N_2)(x)\|_{L^\infty}}, \tag{2.3}$$

then there exist positive constants $T(\alpha)$*,* C*, and* $\widetilde{C}$ *such that*

$$\begin{aligned} &\int_0^1 [(E - \mathcal{E})^2(x, t) + \sum_{i=1}^{2}(\frac{J_i^2}{2n_i} + (n_i - N_i)^2)(x, t)]dx \\ &\le Ce^{-\widetilde{C}t^{\frac{2-\alpha}{2}}} \int_0^1 [(E - \mathcal{E})^2(x, 0) + \sum_{i=1}^{2}(\frac{J_i^2}{2n_i} + (n_i - N_i)^2)(x, 0)]dx \end{aligned} \tag{2.4}$$

for any $t > T(\alpha)$.

With less regularity of the L^∞ entropy solutions, we can only obtain zero-order estimates. To get the exponential time decay estimate between the entropy solution and the corresponding stationary solution, we need explore the entropy dissipation.

3. Large time behavior framework of time-increasing entropy solutions

In this part, we will prove the large time behavior framework for the L^∞ entropy solutions, in which the bounds of densities may increase with time, that is Theorem 1. Specifically speaking, for any global entropy solutions of (1.1) – (1.4) with the densities satisfy (2.1), we get an exponential decay rate for the electric field and the relative entropy between the entropy solution and the stationary solution. To this purpose, we introduce new variables

$$y_i(x,t) = -\int_0^x (n_i(s,t) - N_i(s))ds \quad i = 1,2. \tag{3.1}$$

Naturally, $y_i (i = 1,2)$ is absolutely continuous in x for a.e $t > 0$. Moreover, we have

$$\begin{aligned} &y_{ix} = -(n_i - N_i), \qquad y_{it} = J_i, \\ &y_2 - y_1 = E - \mathcal{E}, \qquad y_i(0,t) = y_i(1,t) = 0, \quad i = 1,2. \end{aligned} \tag{3.2}$$

From (1.1) and the corresponding stationary equation, we get y_i $(i = 1,2)$ admits the equations

$$y_{itt} + (\frac{y_{it}^2}{n_i})_x - y_{ixx} + y_{it} = (-1)^{i+1}(n_i E - N_i \mathcal{E}). \tag{3.3}$$

Multiplying y_i with (3.3) integrating over the spatial domain (0,1) and then adding the results together for $i = 1,2$, we have

$$\begin{aligned} &\sum_{i=1}^2 [\frac{d}{dt}\int_0^1 (y_i y_{it} + \frac{1}{2}y_i^2)dx - \int_0^1 (\frac{y_{it}^2}{n_i})y_{ix}dx + \int_0^1 y_{ix}^2 dx - \int_0^1 y_{it}^2 dx] \\ &= \sum_{i=1}^2 (-1)^{i+1} \int_0^1 [N_i(y_2 - y_1)y_i + \frac{E_x}{2}y_i^2]dx. \end{aligned} \tag{3.4}$$

We calculate that

$$\begin{aligned} &\sum_{i=1}^2 (-1)^{i+1} \int_0^1 [N_i(y_2 - y_1)y_i + \frac{E_x}{2}y_i^2]dx \\ &= (-1)^{i+1} \int_0^1 \frac{n_1 - N_1 - n_2 + N_2 - D(x)}{2} y_i^2 dx - \int_0^1 \frac{N_1 + N_2}{2}(y_1 - y_2)^2 dx, \end{aligned} \tag{3.5}$$

then (3.4) turns into

$$\begin{aligned} &\frac{d}{dt}\int_0^1 \sum_{i=1}^2 (y_i y_{it} + \frac{y_i^2}{2})dx + \sum_{i=1}^2 \int_0^1 y_{ix}^2 dx + \int_0^1 \frac{N_1 + N_2}{2}(y_1 - y_2)^2 dx \\ &= \sum_{i=1}^2 \int_0^1 \frac{N_i}{n_i} y_{it}^2 dx + \int_0^1 \frac{n_1 - N_1 - n_2 + N_2 - D(x)}{2}(y_1^2 - y_2^2)dx. \end{aligned} \tag{3.6}$$

Noticing

$$|y_i(x)| = |\int_0^x y_{is}(s)ds| \le x^{\frac{1}{2}}(\int_0^x y_{is}^2 ds)^{\frac{1}{2}} \le x^{\frac{1}{2}}(\int_0^1 y_{is}^2 ds)^{\frac{1}{2}}, \tag{3.7}$$

then we have

$$\|y_i\|_{L^2}^2 = \int_0^1 y_i^2 dx \le \int_0^1 x \int_0^1 y_{is}^2 ds dx \le \|y_{ix}\|_{L^2}^2 \int_0^1 x dx = \frac{1}{2}\|y_{ix}\|_{L^2}^2. \tag{3.8}$$

While if (2.3) satisfies, we have

$$\begin{aligned}
&\int_0^1 |\frac{n_1 - N_1 - n_2 + N_2 - D(x)}{2}(y_1 - y_2)(y_1 + y_2)|dx \\
&\le (1-\delta)\int_0^1 (y_1 + y_2)^2 dx + \frac{1}{(1-\delta)}\int_0^1 (y_1 - y_2)^2 \frac{(n_1 - n_2 - N_1 + N_2 - D(x))^2}{16} dx \qquad (3.9)\\
&< (1-\delta)\int_0^1 (y_1 + y_2)^2 dx + (1-\widetilde{\delta})\int_0^1 \frac{N_1 + N_2}{2}(y_1 - y_2)^2 dx,
\end{aligned}$$

for some small positive constant δ and $\widetilde{\delta} = \dfrac{\delta}{1-\delta} < 1$. To see this, let $\varepsilon = 2\delta > 0$, we have

$$\begin{aligned}
&\frac{1}{(1-\delta)}\int_0^1 (y_1 - y_2)^2 \frac{(n_1 - n_2 - N_1 + N_2 - D(x))^2}{16} dx \\
&< \frac{1}{(1-\delta)}\int_0^1 \frac{N_1 + N_2}{2}(y_1 - y_2)^2 dx - \frac{\varepsilon}{(1-\delta)}\int_0^1 \frac{N_1 + N_2}{2}(y_1 - y_2)^2 dx \qquad (3.10)\\
&= \frac{1-2\delta}{(1-\delta)}\int_0^1 \frac{N_1 + N_2}{2}(y_1 - y_2)^2 dx.
\end{aligned}$$

Thus (3.6) turns into

$$\begin{aligned}
&\frac{d}{dt}\int_0^1 \sum_{i=1}^2 (y_i y_{it} + \frac{y_i^2}{2})dx + \frac{\delta}{2}\int_0^1 \sum_{i=1}^2 (y_{ix}^2 + y_i^2)dx + \widetilde{\delta}\int_0^1 \frac{N_1 + N_2}{2}(y_1 - y_2)^2 dx \\
&\le \sum_{i=1}^2 \int_0^1 \frac{N_i}{n_i} y_{it}^2 dx.
\end{aligned} \tag{3.11}$$

Now we explore the entropy dissipation estimate. To this end, we introduce the relative entropy-entropy flux pair to make full use of the entropy inequality.

The relative entropy-entropy flux are:

$$\begin{aligned}
\eta^*(x,t) &= \sum_{i=1}^2 \Big(\frac{J_i^2}{2n_i} + n_i^2 - N_i^2 - 2N_i(n_i - N_i)\Big)(x,t) \\
&= \Big(\eta_e - \sum_{i=1}^2 Q_i\Big)(x,t) \ge 0,
\end{aligned} \tag{3.12}$$

$$
\begin{aligned}
q^*(x,t) &= \sum_{i=1}^{2}\Big(\frac{J_i^3}{2n_i^2}+2n_iJ_i-2N_iJ_i\Big)(x,t) \\
&= \Big(q_e-\sum_{i=1}^{2}P_i\Big)(x,t),
\end{aligned}
\tag{3.13}
$$

where

$$
Q_i = N_i^2+2N_i(n_i-N_i), \quad P_i = 2N_iJ_i,
$$

η_e and q_e are the entropy-entropy flux pair defined in (1.8).

Using the entropy inequality, we have the following estimates on the relative entropy-entropy flux pair (η^*, q^*):

$$
\begin{aligned}
0 &\geq \eta_{et}+q_{ex}+\frac{J_1^2}{n_1}+\frac{J_2^2}{n_2}-J_1E+J_2E \\
&= \eta_t^*+q_x^*+\frac{J_1^2}{n_1}+\frac{J_2^2}{n_2}-J_1E+J_2E+J_1\mathcal{E}-J_2\mathcal{E} \\
&= \eta_t^*+q_x^*+\frac{y_{1t}^2}{n_1}+\frac{y_{2t}^2}{n_2}+\frac{1}{2}(y_2-y_1)_t^2,
\end{aligned}
\tag{3.14}
$$

that is

$$
\frac{d}{dt}\int_0^1(\eta^*+\frac{1}{2}(y_2-y_1)^2)dx+\int_0^1(\frac{y_{1t}^2}{n_1}+\frac{y_{2t}^2}{n_2})dx \leq 0. \tag{3.15}
$$

The estimates (3.11) and (3.15) are elemental. Any L^∞ weak entropy solutions satisfying (2.3) have these two estimates. Let $\lambda(t) = Mt^{\frac{\alpha}{2}}+N^*+1$, where M and N^* are the constants in (2.1) and theorem A.

Multiplying (3.15) by $\lambda(t)$ and adding the result to (3.11), we obtain

$$
\begin{aligned}
&\frac{d}{dt}\int_0^1[\lambda\eta^*+\frac{\lambda}{2}(y_2-y_1)^2)+\sum_{i=1}^{2}(y_iy_{it}+\frac{y_i^2}{2})]dx-\frac{\alpha M}{2}t^{\frac{\alpha}{2}-1}\int_0^1(\eta^*+\frac{(y_2-y_1)^2}{2})dx \\
&+\frac{\delta}{2}\int_0^1\sum_{i=1}^{2}(y_{ix}^2+y_i^2)dx+\widetilde{\delta}\int_0^1\frac{N_1+N_2}{2}(y_1-y_2)^2dx \\
&+\int_0^1[(\lambda-N_1)\frac{y_{1t}^2}{n_1}+(\lambda-N_2)\frac{y_{2t}^2}{n_2}]dx \leq 0.
\end{aligned}
\tag{3.16}
$$

Since $\eta^* \sim \sum_{i=1}^{2}(\frac{y_{it}^2}{n_i}+y_{ix}^2)$ and $\alpha<2$, we get

$$
\begin{aligned}
&\sum_{i=1}^{2}\frac{\delta}{4}\int_0^1 y_{ix}^2dx+\frac{\widetilde{\delta}}{2}\int_0^1\frac{N_1+N_2}{2}(y_1-y_2)^2dx+\int_0^1 Mt^{\frac{\alpha}{2}}(\frac{y_{1t}^2}{n_1}+\frac{y_{2t}^2}{n_2})dx \\
&> \frac{\alpha M}{2}t^{\frac{\alpha}{2}-1}\int_0^1(\eta^*+\frac{(y_2-y_1)^2}{2})dx
\end{aligned}
\tag{3.17}
$$

for big enough $t > t_*$. * Then (3.16) turns into

$$\begin{aligned}&\frac{d}{dt}\int_0^1[\lambda\eta^* + \frac{\lambda}{2}(y_2-y_1)^2 + \sum_{i=1}^{2}(y_iy_{it}+\frac{y_i^2}{2})]dx + C_1[\int_0^1\sum_{i=1}^{2}(y_{ix}^2+y_i^2)dx\\&+\int_0^1\frac{N_1+N_2}{2}(y_1-y_2)^2dx+\int_0^1(\frac{y_{1t}^2}{n_1}+\frac{y_{2t}^2}{n_2})dx]\le 0,\end{aligned}\tag{3.18}$$

where $C_1 = \min\{\frac{\delta}{4},\frac{\widetilde{\delta}}{2},1\}$.

Since

$$\begin{aligned}&\lambda\eta^* + \frac{\lambda}{2}(y_2-y_1)^2 + \sum_{i=1}^{2}(y_iy_{it}+\frac{y_i^2}{2})\\&\le\sum_{i=1}^{2}(\frac{\sqrt{n_i}y_i^2}{2}+\frac{y_{it}^2}{2\sqrt{n_i}}+\frac{y_i^2}{2}+\frac{\lambda y_{it}^2}{2n_i}+\lambda O(1)\sum_{i=1}^{2}y_{ix}^2)+\frac{\lambda}{2}(y_2-y_1)^2\\&\le O(1)\lambda C_1[\sum_{i=1}^{2}(y_{ix}^2+y_i^2)+\frac{N_1+N_2}{2}(y_1-y_2)^2+(\frac{y_{1t}^2}{n_1}+\frac{y_{2t}^2}{n_2})],\end{aligned}\tag{3.19}$$

then there exists positive constant C_2 such that (3.18) turns into

$$\begin{aligned}&\frac{d}{dt}\int_0^1[\lambda\eta^* + \frac{\lambda}{2}(y_2-y_1)^2 + \sum_{i=1}^{2}(y_iy_{it}+\frac{y_i^2}{2})]dx\\&+\frac{1}{C_2}t^{-\frac{\alpha}{2}}\int_0^1[(\lambda\eta^* + \frac{\lambda}{2}(y_2-y_1)^2 + \sum_{i=1}^{2}(y_iy_{it}+\frac{y_i^2}{2})]dx\le 0.\end{aligned}\tag{3.20}$$

Let $F(x,t) = \int_0^1[\lambda\eta^* + \frac{\lambda}{2}(y_2-y_1)^2 + \sum_{i=1}^{2}(y_iy_{it}+\frac{y_i^2}{2})]dx$, then Gronwal inequality denotes

$$F(x,t)\le e^{-C_3t^{\frac{2-\alpha}{2}}}F(x,0)\tag{3.21}$$

for some positive constant $C_3 > 0$.

*since we consider the large time behavior, without loss of generality, we always assume $t > t_*$.

On the other hand, noticing

$$\begin{aligned}
&\int_0^1 [\lambda\eta^* + \frac{\lambda}{2}(y_2 - y_1)^2 + \sum_{i=1}^{2}(y_i y_{it} + \frac{y_i^2}{2})]dx \\
&\geq \int_0^1 [\sum_{i=1}^{2}(-\frac{\sqrt{n_i}y_i^2}{2} - \frac{y_{it}^2}{2\sqrt{n_i}} + \frac{\lambda y_{it}^2}{2n_i} + \frac{y_i^2}{2} + \lambda O(1)y_{ix}^2) + \frac{\lambda(y_2 - y_1)^2}{2}]dx \\
&\geq C_4 \int_0^1 [\eta^* + \sum_{i=1}^{2} y_i^2 + (y_2 - y_1)^2]dx
\end{aligned} \tag{3.22}$$

for some constant $C_4 > 0$, we have

$$\begin{aligned}
&\int_0^1 [\eta^* + \sum_{i=1}^{2} y_i^2 + (y_2 - y_1)^2]dx \\
&\leq C_5 e^{-C_3 t^{\frac{2-\alpha}{2}}} \int_0^1 [\eta^* + \sum_{i=1}^{2} y_i^2 + (y_2 - y_1)^2](x, 0)dx
\end{aligned} \tag{3.23}$$

for some constant $C_5 > 0$. Thus, we prove Theorem 1.

4. Remark on the assumption (2.3)

The assumption (2.3) is important to get relation (3.11). However, if we suppose

$$\max_{i=1,2} |N_i - n_i + (-1)^{i+1} D(x)| < 4, \tag{4.1}$$

(3.11) can be obtained too. To see this, we calculate

$$\begin{aligned}
&\int_0^1 \frac{n_1 - N_1 - n_2 + N_2 - D(x)}{2}(y_1^2 - y_2^2)dx \\
&= \frac{1}{2}\int_0^1 (y_{2x} - y_{1x} - D(x))(y_1^2 - y_2^2)dx \\
&= \frac{1}{2}[\int_0^1 (y_{2x} - D(x))y_1^2 dx + \int_0^1 (y_{1x} + D(x))y_2^2 dx] \\
&< 2\int_0^1 (y_1^2 + y_2^2)dx.
\end{aligned} \tag{4.2}$$

It is worthy to point out that (4.1) indicates that n_i is bounded with respect to x and t. While the assumption (2.3) permits the bounds of n_i ($i = 1, 2$) grow with time.

Conflict of Interest

The author declares no conflicts of interest in this paper.

References

1. F. Huang, R. Pan, H. Yu, *Large time behavior of Euler-Possion system for semiconductor*. Science in Chian Series A., **51** (2008), 965-972.
2. L. Hsiao, K.J. Zhang, *The relaxation of the hydrodynamic model for semiconducts to the drift-diffusion equations*, J. Differential Equations., **165** (2000), 315-354.
3. J. Li, H. Yu, *Large time behavior of solutions to a bipolar hydrodynamic model with big data and vacuum*, Nonlinear Analysis: Real world applications, **34** (2017), 446-458.
4. P. Marcati, R. Natalini, *Weak solutions to a hydrodynamic model for semiconductors and relaxation to the drift-diffusion equation*, Arch. Ration. Mech., **129** (1995), 129-145.
5. H. Yu, *On the stationary solutions of multi-dimensional bipolar hydrodynamic model of semiconductors*, Appl. Math. Lett., **64** (2007), 108-112.
6. H. Yu, *Large time behavior of entropy solution to a unipolar hydropynamic model of semiconductors*, Commun. Math. Sci., **14** (2016), 69-82.
7. B. Zhang, *Convergence of Godunov scheme for a simplified one-dimensional hydrodynamic model for semiconductor devices*, Comm. Math. Phys., **157** (1993), 1-22.