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Research article

A logarithmically improved regularity criterion for the 3D MHD equations in Morrey-Campanato space

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Abstract: In this paper, we will establish a sufficient condition for the regularity criterion to the 3D MHD equation in terms of the derivative of the pressure in one direction. It is shown that if the partial derivative of the pressure $\partial_3\pi$ satisfies the logarithmical Serrin type condition $\partial_3\pi$ satisfies the logarithmical Serrin type condition

$$\int_0^T \frac{\|\partial_3\pi(s)\|_{\dot{\mathcal{M}}_{2,\frac{3}{r}}}^{\frac{2}{2-r}}}{1+\ln(1+\|b(s)\|_{L^4})} ds < \infty \quad \text{for } 0 < r < 1,$$

then the solution (u, b) remains smooth on $[0, T]$. Compared to the Navier-Stokes result, there is a logarithmic correction involving b in the denominator.

Keywords: MHD equations; regularity criteria

1. Introduction

The MHD equation plays a significant role of mathematical model in fluid dynamics, which can be stated as follows :

$$\begin{cases} \partial_t u - \Delta u + u \cdot \nabla u + \nabla\pi - b \cdot \nabla b = 0, \\ \partial_t b - \Delta b + u \cdot \nabla b - b \cdot \nabla u = 0, \\ \nabla \cdot u = \nabla \cdot b = 0, \\ u(x, 0) = u_0(x), \qquad b(x, 0) = b_0(x). \end{cases} \tag{1.1}$$

Here $u = u(x, t) \in \mathbb{R}^3$ is the velocity field, $\pi = \pi(x, t) \in \mathbb{R}$, $b = b(x, t) \in \mathbb{R}^3$ denote the velocity vector, scalar pressure and the magnetic field of the fluid, respectively, while $u_0(x)$ and $b_0(x)$ are given initial velocity and initial magnetic fields with $\nabla \cdot u_0 = \nabla \cdot b_0 = 0$ in the sense of distribution.

In their work, Sermange and Temam [19] (see also Duvaut and Lions [6]) proved that the MHD equations admit at least one global weak solution for any divergence-free initial data $(u_0, b_0) \in L^2(\mathbb{R}^3)$ and it has a (unique) local strong solution, if additionally, (u_0, b_0) belongs to some Sobolev space $H^s(\mathbb{R}^3)$ with $s \geq 3$. However, whether a local strong solution can exist globally, or equivalently, whether global weak solutions are smooth is an open and challenge problem.

There are many known mathematical results on the three-dimentional MHD equations (see [4, 5, 10, 14, 20, 21, 22, 25, 26] and the references therein). Realizing the dominant role played by the velocity field, He and Xin [10] were able to derive criteria in terms of the velocity field u alone. In particular, a scaling invariant regularity criterion in terms of u was established (also by Zhou [25] independently) which shows that a weak solution (u, b) is smooth on a time interval $(0, T]$ if

$$\nabla u \in L^\alpha(0, T; L^\gamma(\mathbb{R}^3)) \quad \text{with} \quad 1 \leq \alpha < \infty, \;\; 3/2 < \gamma \leq \infty \;\; \text{and} \;\; \frac{2}{\alpha} + \frac{3}{\gamma} = 2.$$

Moreover, the problem of so-called "regularity criteria via partial components" was shown in [3, 9, 11, 12, 13, 15, 17, 23, 24, 27].

Recently, Cao and Wu in [3] presented the regularity criteria on the derivatives of the pressure in one direction. More precisely, they proved that if

$$\frac{\partial \pi}{\partial x_3}(x, t) \in L^\alpha\left(0, T; L^\gamma(\mathbb{R}^3)\right) \quad \text{with} \;\; \frac{2}{\alpha} + \frac{3}{\gamma} \leq \frac{7}{4} \;\text{and}\; \frac{12}{7} \leq \gamma \leq \infty, \tag{1.2}$$

then (u, b) is smooth on $\mathbb{R}^3 \times [0, T]$. Later, [13] and [24] improve condition (1.2) as:

$$\frac{\partial \pi}{\partial x_3}(x, t) \in L^\alpha\left(0, T; L^\gamma(\mathbb{R}^3)\right) \quad \text{with} \;\; \frac{2}{\alpha} + \frac{3}{\gamma} \leq 2 \;\text{and}\; \frac{3}{2} \leq \gamma \leq \infty. \tag{1.3}$$

Very recently, Benbernou et al. [2] extend (1.3) to the homogeneous Morrey-Campanato space $\dot{\mathcal{M}}_{2,\frac{3}{r}}(\mathbb{R}^3)$. to obtain the regularity of weak solutions. This space has been used successfully in the study of the uniqueness of weak solutions for the Navier-Stokes equations in [16] where it is pointed out that

$$L^{\frac{3}{r}}\left(\mathbb{R}^3\right) \subset L^{\frac{3}{r},\infty}\left(\mathbb{R}^3\right) \subset \dot{\mathcal{M}}_{2,\frac{3}{r}}\left(\mathbb{R}^3\right).$$

The purpose of this manuscript is to establish a logarithmically improved regularity criterion in terms of the derivatives of the pressure in one direction of the systems (1.1). Our result can be stated as follows.

Theorem 1.1. *(regularity criterion) Let* $(u_0, b_0) \in L^2(\mathbb{R}^3) \cap L^4(\mathbb{R}^3)$ *with* $\nabla \cdot u_0 = \nabla \cdot b_0 = 0$. *Suppose that* (u, b) *is a weak solution to the MHD equations (1.1) in the time interval* $[0, T)$ *for some* $0 < T < \infty$. *If the pressure* $\pi(x, t)$ *satisfies the condition :*

$$\int_0^T \frac{\|\partial_3 \pi(s)\|_{\dot{\mathcal{M}}_{2,\frac{3}{r}}}^{\frac{2}{2-r}}}{1 + \ln(1 + \|b\|_{L^4})} ds < \infty \quad \text{for } 0 < r < 1,$$

then (u, b) *is a regular solution on* $\mathbb{R}^3 \times [0, T]$.

Theorem 1.1 is also true for the 3-D incompressible Navier-Stokes equations, so it gives extensions for previous results in [2, 1, 3, 12, 17, 23]. Definitions and basic properties of the Morrey-Campanato spaces can be find in [28] and the references therein. For concision, we omit them here.

Now we are in the position to prove Theorem 1.1.

2. Proof of Theorem 1.1

Throughout this paper, C denotes a generic positive constant (generally large), it may be different from line to line. In order to prove regularity, we need to establish the L^4 bound of (u, b) and the desired regularity then follows from the standard Serrin-type criteria on the 3D MHD equations.

Instead of considering the equations in the form (1.1), we rewrite it in the following form as that in [7, 8]:

$$\begin{cases} \partial_t w^+ + w^- \cdot \nabla w^+ = \Delta w^+ - \nabla\pi, \\ \partial_t w^- + w^+ \cdot \nabla w^- = \Delta w^- - \nabla\pi, \\ \nabla \cdot w^+ = \nabla \cdot w^- = 0, \\ w^+(x,0) = u_0 + b_0, \quad w^-(x,0) = u_0 - b_0, \end{cases} \tag{2.1}$$

with $w^\pm := u \pm b$.

First, taking the inner product of $(2.1)_1$ with $(0, 0, w_3^+ \left|w_3^+\right|^2)$, we have

$$\begin{aligned} &\frac{1}{4}\frac{d}{dt}\int_{\mathbb{R}^3} \left|w_3^+\right|^4 dx + \int_{\mathbb{R}^3} (w^- \cdot \nabla) w_3^+ . \left|w_3^+\right|^2 dx \\ = &\int_{\mathbb{R}^3} \Delta w_3^+ \left|w_3^+\right|^2 dx - \int_{\mathbb{R}^3} \frac{\partial\pi}{\partial x_3} w_3^+ \left|w_3^+\right|^2 dx. \end{aligned}$$

Integrating by parts over $\mathbb{R}^3$ and using the divergence free property $\nabla \cdot w^+ = 0$ into account, we get

$$\int_{\mathbb{R}^3} (w^- \cdot \nabla) w_3^+ \cdot \left|w_3^+\right|^2 dx = 0.$$

For the second integral term, applying the integration by parts and the incompressible conditions again yield

$$\int_{\mathbb{R}^3} \Delta w_3^+ \left|w_3^+\right|^2 dx = -\frac{3}{4}\int_{\mathbb{R}^3} \left|\nabla \left|w_3^+\right|^2\right|^2 dx.$$

We easily get

$$\frac{1}{4}\frac{d}{dt}\int_{\mathbb{R}^3} \left|w_3^+\right|^4 dx + \frac{3}{4}\int_{\mathbb{R}^3} \left|\nabla \left|w_3^+\right|^2\right|^2 dx = -\int_{\mathbb{R}^3} \frac{\partial\pi}{\partial x_3} w_3^+ \left|w_3^+\right|^2 dx. \tag{2.2}$$

Similarly, taking the inner product of the second equation of (2.1) with $(0, 0, w_3^- \left|w_3^-\right|^2)$, we obtain

$$\frac{1}{4}\frac{d}{dt}\int_{\mathbb{R}^3} \left|w_3^-\right|^4 dx + \frac{3}{4}\int_{\mathbb{R}^3} \left|\nabla \left|w_3^-\right|^2\right|^2 dx = -\int_{\mathbb{R}^3} \frac{\partial\pi}{\partial x_3} w_3^- \left|w_3^-\right|^2 dx. \tag{2.3}$$

Summing (2.2) and (2.3) together yields

$$\frac{1}{4}\frac{d}{dt}\int_{\mathbb{R}^3} \left(\left|w_3^+\right|^4 + \left|w_3^-\right|^4\right) dx + \frac{3}{4}\int_{\mathbb{R}^3} \left(\left|\nabla \left|w_3^+\right|^2\right|^2 + \left|\nabla \left|w_3^-\right|^2\right|^2\right) dx$$

$$
\begin{aligned}
&= -\int_{\mathbb{R}^3} \frac{\partial \pi}{\partial x_3} w_3^+ \left|w_3^+\right|^2 dx - \int_{\mathbb{R}^3} \frac{\partial \pi}{\partial x_3} w_3^- \left|w_3^-\right|^2 dx \\
&= J_1 + J_2. \qquad (2.4)
\end{aligned}
$$

In what follows, we will deal with each term on the right-hand side of (2.4) separately. We estimate $\left\|\frac{\partial \pi}{\partial x_3} \cdot \left|w_3^+\right|^2\right\|_{L^2}$ as follows :

$$
\begin{aligned}
\left\|\frac{\partial \pi}{\partial x_3} \cdot \left|w_3^+\right|^2\right\|_{L^2} &\leq C \left\|\frac{\partial \pi}{\partial x_3}\right\|_{\dot{\mathcal{M}}_{2,\frac{3}{r}}} \left\|\left|w_3^+\right|^2\right\|_{\dot{B}^r_{2,1}} \\
&\leq C \left\|\frac{\partial \pi}{\partial x_3}\right\|_{\dot{\mathcal{M}}_{2,\frac{3}{r}}} \left\|\nabla \left|w_3^+\right|^2\right\|_{L^2}^r \left\|\left|w_3^+\right|^2\right\|_{L^2}^{1-r}.
\end{aligned}
$$

Here we have used the following inequality due to Machihara and Ozawa [18]

$$
\|f\|_{\dot{B}^r_{2,1}} \leq C \|f\|_{L^2}^{1-r} \|\nabla f\|_{L^2}^r \quad \text{for } 0 < r < 1.
$$

Hence, it follows from the Hölder inequality and Young's inequality that

$$
\begin{aligned}
|J_1| &\leq \int_{\mathbb{R}^3} \left|\frac{\partial \pi}{\partial x_3}\right| |w_3^+|^3 dx \\
&\leq C \left\|\frac{\partial \pi}{\partial x_3} \cdot \left|w_3^+\right|^2\right\|_{L^2} \left\|w_3^+\right\|_{L^2} \\
&\leq C \left(\left\|\frac{\partial \pi}{\partial x_3}\right\|_{\dot{\mathcal{M}}_{2,\frac{3}{r}}}^{\frac{2}{1-r}} \left\|w_3^+\right\|_{L^4}^4\right)^{\frac{1-r}{2}} \left(\left\|\nabla \left|w_3^+\right|^2\right\|_{L^2}^2\right)^{\frac{r}{2}} \left(\left\|w^+\right\|_{L^2}^2\right)^{\frac{1}{2}} \\
&\leq C \left\|\frac{\partial \pi}{\partial x_3}\right\|_{\dot{\mathcal{M}}_{2,\frac{3}{r}}}^{\frac{2}{1-r}} \left\|w_3^+\right\|_{L^4}^4 + \frac{1}{2} \left\|\nabla \left|w_3^+\right|^2\right\|_{L^2}^2 + C \left\|w^+\right\|_{L^2}^2, \qquad (2.5)
\end{aligned}
$$

Note that the weak solution $(u, b) \in L^\infty(0, T; L^2(\mathbb{R}^3))$, this leads to

$$
(w^+, w^-) \in L^\infty(0, T; L^2(\mathbb{R}^3)).
$$

Similarly, one can prove that

$$
|J_2| \leq C \left\|\frac{\partial \pi}{\partial x_3}\right\|_{\dot{\mathcal{M}}_{2,\frac{3}{r}}}^{\frac{2}{1-r}} \left\|w_3^-\right\|_{L^4}^4 + \frac{1}{2} \left\|\nabla \left|w_3^-\right|^2\right\|_{L^2}^2 + C \left\|w^-\right\|_{L^2}^2. \qquad (2.6)
$$

Substituting (2.5) and (2.6) into (2.4), we obtain

$$
\begin{aligned}
&\frac{1}{4}\frac{d}{dt} \int_{\mathbb{R}^3} \left(\left|w_3^+\right|^4 + \left|w_3^-\right|^4\right) dx + \frac{1}{4} \int_{\mathbb{R}^3} \left(\left|\nabla \left|w_3^+\right|^2\right|^2 + \left|\nabla \left|w_3^-\right|^2\right|^2\right) dx \\
\leq\ & C \left(\left\|\frac{\partial \pi}{\partial x_3}\right\|_{\dot{\mathcal{M}}_{2,\frac{3}{r}}}^{\frac{2}{1-r}} + 1\right) \left(\left\|w_3^+\right\|_{L^4}^4 + \left\|w_3^-\right\|_{L^4}^4\right), \qquad (2.7)
\end{aligned}
$$

for all $0 \le t < T$. Setting

$$J = \left\| \frac{\partial \pi}{\partial x_3} \right\|_{\dot{\mathcal{M}}_{2,\frac{3}{r}}}^{\frac{2}{1-r}} \left(e + \left\| w_3^+ \right\|_{L^4}^4 + \left\| w_3^- \right\|_{L^4}^4 \right).$$

On the other hand, we see that

$$\begin{aligned} 1 + \ln\left(1 + \|b\|_{L^4}\right) &\le 1 + \ln\left(1 + \|b\|_{L^4}^4 + \frac{9}{8}\right) \\ &\le 1 + \ln\left(e + \|b\|_{L^4}^4\right), \end{aligned}$$

where we have used the following inequality

$$x \le x^4 + \frac{9}{8} \text{ for all } x \ge 0.$$

Consequently, J can be estimated as follows:

$$\begin{aligned} J &= \frac{\left\| \frac{\partial \pi}{\partial x_3} \right\|_{\dot{\mathcal{M}}_{2,\frac{3}{r}}}^{\frac{2}{1-r}}}{1 + \ln(1 + \|b\|_{L^4})} (e + \left\| w_3^+ \right\|_{L^4}^4 + \left\| w_3^- \right\|_{L^4}^4) \left[1 + \ln(1 + \|b\|_{L^4})\right] \\ &\le \frac{\left\| \frac{\partial \pi}{\partial x_3} \right\|_{\dot{\mathcal{M}}_{2,\frac{3}{r}}}^{\frac{2}{1-r}}}{1 + \ln(1 + \|b\|_{L^4})} (e + \left\| w_3^+ \right\|_{L^4}^4 + \left\| w_3^- \right\|_{L^4}^4) \left[1 + \ln(e + \|b\|_{L^4}^4)\right] \\ &\le \frac{\left\| \frac{\partial \pi}{\partial x_3} \right\|_{\dot{\mathcal{M}}_{2,\frac{3}{r}}}^{\frac{2}{1-r}}}{1 + \ln(1 + \|b\|_{L^4})} (e + \left\| w_3^+ \right\|_{L^4}^4 + \left\| w_3^- \right\|_{L^4}^4) \left[1 + \ln(e + \left\| w_3^+ \right\|_{L^4}^4 + \left\| w_3^- \right\|_{L^4}^4)\right]. \end{aligned} \tag{2.8}$$

Inserting (2.8) into (2.7) and setting

$$F(t) = e + \left\| w_3^+(t) \right\|_{L^4}^4 + \left\| w_3^-(t) \right\|_{L^4}^4,$$

we obtain

$$\frac{dF}{dt} \le C \frac{\left\| \frac{\partial \pi}{\partial x_3} \right\|_{\dot{\mathcal{M}}_{2,\frac{3}{r}}}^{\frac{2}{1-r}}}{1 + \ln(1 + \|b\|_{L^4})} (1 + \ln F) F + CF.$$

for all $t \in [0, T]$. Thank's to Gronwall inequality, we get

$$F(t) \le F(0) \exp\left(C \int_0^t \frac{\left\| \frac{\partial \pi}{\partial x_3}(s) \right\|_{\dot{\mathcal{M}}_{2,\frac{3}{r}}}^{\frac{2}{1-r}}}{1 + \ln(1 + \|b(s)\|_{L^4})} (1 + \ln F(s)) ds \right) \exp(CT),$$

which implies

$$1 + \ln F(t) \le CT + \ln F(0) + C \int_0^t \frac{\left\| \frac{\partial \pi}{\partial x_3}(s) \right\|_{\dot{\mathcal{M}}_{2,\frac{3}{r}}}^{\frac{2}{1-r}}}{1 + \ln(1 + \|b(s)\|_{L^4})} (1 + \ln F(s)) ds.$$

Applying Gronwall's inequality again, one has

$$\ln F(t) \leq c(u_0, b_0, T)\exp\left(C\int_0^T \frac{\left\|\frac{\partial\pi}{\partial x_3}(s)\right\|_{\dot{\mathcal{M}}_{2,\frac{3}{r}}}^{\frac{2}{1-r}}}{1+\ln(1+\|b(s)\|_{L^4})} ds \right),$$

which implies that

$$\sup_{0\leq t\leq T}(\left\|w^+(.,t)\right\|_{L^4} + \left\|w^-(.,t)\right\|_{L^4}) < \infty \tag{2.9}$$

Hence, it follows from the triangle inequality and (2.9) that

$$\begin{aligned}\sup_{0\leq t\leq T}\|u(.,t)\|_{L^4} &= \frac{1}{2}\sup_{0\leq t\leq T}\|(u+b)(.,t)+(u-b)(.,t)\|_{L^4}\\ &\leq \frac{1}{2}\sup_{0\leq t\leq T}(\|(u+b)(.,t)\|_{L^4}+\|(u-b)(.,t)\|_{L4})\\ &\leq \frac{1}{2}\sup_{0\leq t\leq T}(\left\|w^+(.,t)\right\|_{L^4}+\left\|w^-(.,t)\right\|_{L^4}) < \infty\end{aligned}$$

and

$$\begin{aligned}\sup_{0\leq t\leq T}\|b(.,t)\|_{L^4} &= \frac{1}{2}\sup_{0\leq t\leq T}\|(u+b)(.,t)-(u-b)(.,t)\|_{L^4}\\ &\leq \frac{1}{2}\sup_{0\leq t\leq T}(\|(u+b)(.,t)\|_{L^4}+\|(u-b)(.,t)\|_{L^4})\\ &\leq \frac{1}{2}\sup_{0\leq t\leq T}(\left\|w^+(.,t)\right\|_{L^4}+\left\|w^-(.,t)\right\|_{L^4}) < \infty.\end{aligned}$$

Thus,

$$\sup_{0\leq t\leq T}(\|u(.,t)\|_{L^4}+\|b(.,t)\|_{L^4}) < \infty. \tag{2.10}$$

This completes the proof of Theorem 1.1.

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Conflict of Interest

All authors declare no conflicts of interest in this paper.

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