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*Research article*

# Research on the new energy transformation strategy of the automobile industry based on differential game

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**Abstract:** In this study, we examined new-energy vehicle (NEV) transformation strategies in low-NEV-penetration automobile markets. We developed a tripartite differential game model involving traditional internal-combustion-engine vehicle (ICE vehicle) manufacturers, new-energy vehicle (NEV) enterprises, and local governments. The model compared Nash non-cooperative, government-led Stackelberg, and cooperative decision-making structures to derive equilibrium effort levels, subsidy policies, stakeholder utilities, and transformation trajectories. The results showed that government cost-sharing subsidies increased the transformation efforts of ICE vehicle manufacturers and NEV enterprises, while the Stackelberg structure improved outcomes relative to the Nash benchmark through government-led cost sharing. Cooperative decision-making generated the highest stakeholder utilities, aggregate ecosystem utility, and new-energy transformation level index. Numerical simulation, sensitivity analysis of revenue-sharing coefficients, and the Changan Automobile case were consistent with the theoretical results. This study provides a dynamic framework for designing cost-sharing subsidy, revenue-sharing, and coordination mechanisms during the low-penetration stage of automotive new-energy transformation.

**Keywords:** industrial transformation; government subsidies; differential games; cooperation; Hamilton-Jacobi-Bellman equation
**Mathematics Subject Classification:** 91A05, 91A10, 91A25

## 1. Introduction

As a major source of carbon emissions, the transportation sector accounts for a substantial share of global energy-related CO2 emissions and has become central to climate governance. Although many countries have adopted carbon-neutrality targets, new-energy vehicle (NEV) adoption remains uneven across markets. By Q2 2025, NEV penetration rates in India, Brazil, and Mexico were below 9%, compared with a global average of 19.2% [1]. This gap reflects an uneven green transition in the automotive industry. In low-penetration markets, internal combustion engine (ICE) vehicle manufacturers face technological obsolescence and market contraction, whereas NEV enterprises face high research and development (R&D) costs and uncertain market acceptance. Local governments therefore need effective policy instruments, especially financial subsidies, to coordinate stakeholder incentives and accelerate industrial upgrading [2].

Studies provide an important basis for understanding industrial transformation and policy intervention. Chandler [3] defined industrial consolidation as a mechanism for improving operational efficiency through structural reorganization. Minford and Meenagh [4] showed that R&D subsidies can have lasting effects on innovation and long-term economic growth. Dynamic models have also been used to study low-carbon transition paths. Cheng [5] applied differential game theory to low-carbon cooperation, while Zhang [6] and Liu [7] examined the regulatory effects of subsidy coefficients. However, most researchers focus on general supply-chain interactions or bilateral government-enterprise games. They do not fully capture the dynamic tripartite interaction among ICE vehicle manufacturers, NEV enterprises, and local governments in low-NEV-penetration markets, where market risks, transition costs, and technology spillovers are especially pronounced [8,9]. More importantly, studies on automotive electrification show that incumbent automakers and emerging NEV-oriented firms face different transition constraints [10,11]. Incumbent ICE vehicle manufacturers are constrained by production assets, conventional supplier networks, and organizational inertia, whereas NEV enterprises are more closely associated with new technology development and market expansion. Therefore, treating the enterprise side as a single homogeneous actor may obscure the different incentives, costs, and strategic responses of these two groups. This limitation is not only a matter of model structure, but also of temporal logic. Many studies on tripartite strategic interaction rely on static analysis and therefore overlook the long-term accumulation process of industrial transition. As a result, they cannot fully explain how enterprise efforts, government subsidies, and coordination behavior jointly affect the accumulated transformation level over time [12].

To address these structural and temporal limitations, we develop a tripartite differential game model involving ICE vehicle manufacturers, NEV enterprises, and local governments [13−15]. As illustrated in Figure 1, the new-energy transition is modeled as a dynamic feedback process in which ICE vehicle manufacturers provide incumbent production capacity and transformation resources, NEV enterprises contribute emerging technology and market expansion capacity, and local governments coordinate the process through policy support and financial incentives. Treating the two enterprise groups as distinct decision-making agents enables the model to capture incumbent transformation pressure and NEV expansion incentives [16]. Because stakeholder efforts and subsidy policies evolve over time, the model uses HJB equations to derive feedback equilibrium strategies under Nash non-cooperative, government-led Stackelberg, and cooperative game structures [17−19]. The framework examines three concerns: How the three stakeholders determine their optimal effort levels during the transformation process; how equilibrium strategies differ across the three governance structures; and how subsidies and cooperative decision-making affect the trajectory of the new-energy transformation level index and stakeholder utilities.

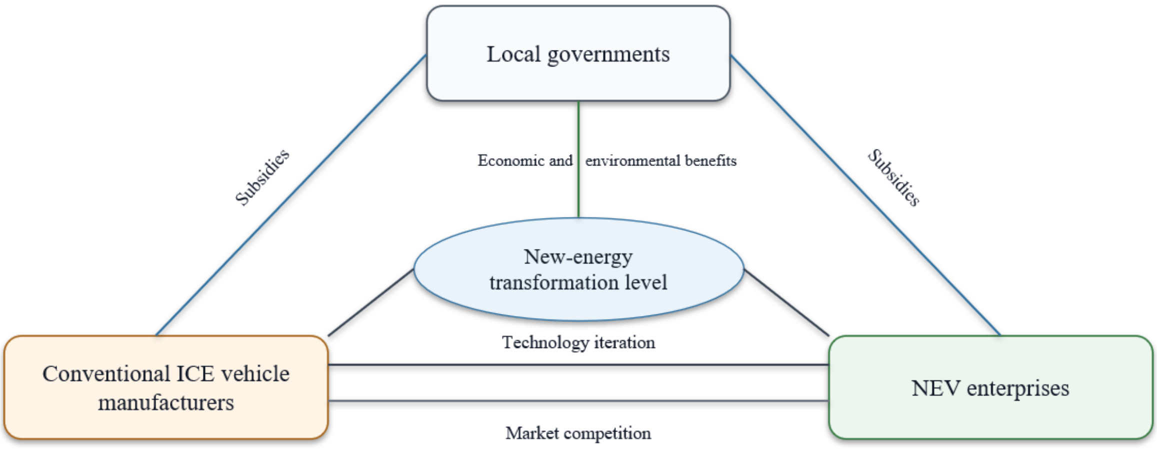



**Figure 1.** Structural framework of the tripartite strategic interaction.

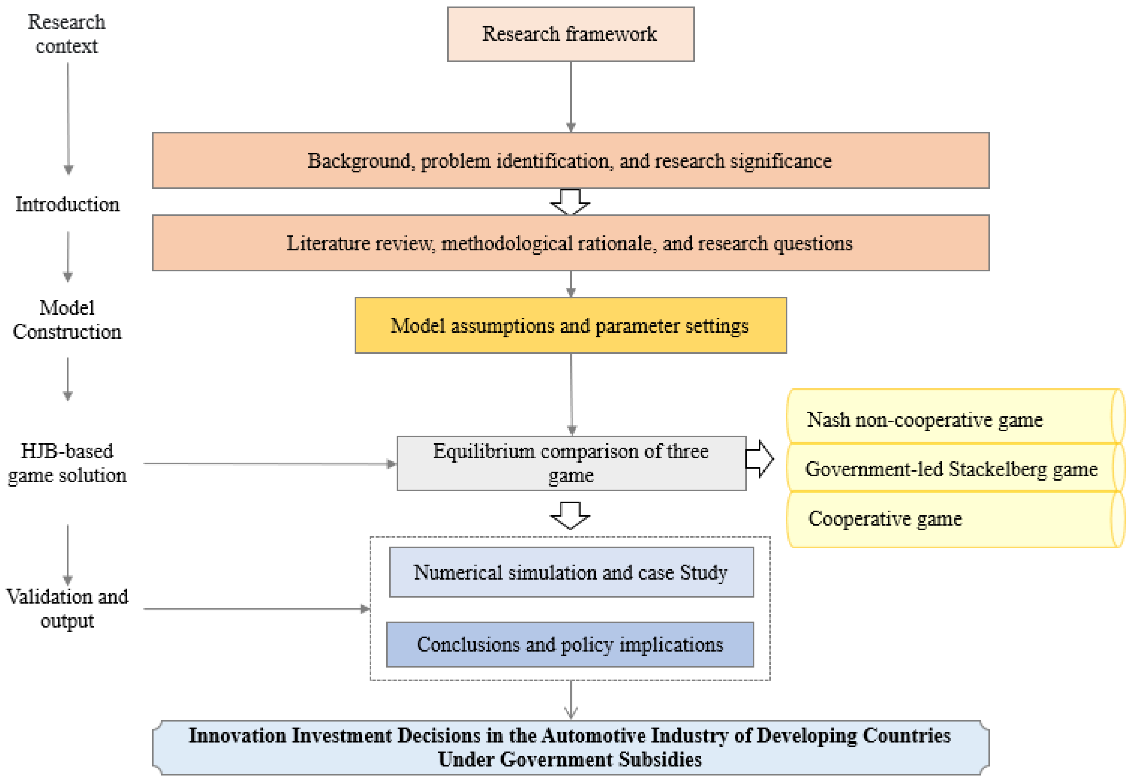



**Figure 2**. Research roadmap.

The study contributes to the literature in three ways. First, we extend traditional bilateral government-enterprise models by distinguishing incumbent ICE vehicle manufacturers from emerging NEV enterprises within a unified tripartite framework. Second, we define the state variable as a new-energy transformation level index, representing accumulated transformation capacity rather than a bounded market penetration rate. Third, we compare Nash, Stackelberg, and cooperative scenarios to reveal how decentralized decision-making, government-led subsidies, and full cooperation shape stakeholder efforts, subsidy policies, transformation trajectories, and system utilities. The analytical results are further examined through numerical simulation, sensitivity analysis of revenue-sharing coefficients, and a Changan Automobile case study, which together support the model's theoretical implications for low-NEV-penetration markets. The overall research design and analytical procedure are summarized in Figure 2.

## 2. Model construction

In this section, we develop the model setting for the new-energy transformation of the automobile industry. For the analysis, we focus on strategic interactions among ICE vehicle manufacturers, NEV enterprises, and local governments under policy incentives. The model is designed for low-NEV-penetration markets, where incumbent manufacturers face transformation pressure, NEV enterprises seek technology expansion, and local governments use subsidies and regulatory instruments to coordinate the transition.

The Nash non-cooperative game, government-led Stackelberg game, and cooperative game are selected to represent three typical governance structures. The Nash game provides a decentralized benchmark in which ICE vehicle manufacturers, NEV enterprises, and local governments independently maximize their own utilities. The Stackelberg game captures a policy-guided structure in which the local government first designs the subsidy mechanism and the two enterprise groups then adjust their effort levels in response. The cooperative game represents a coordinated scenario in which all three stakeholders act as a unified decision-making system to maximize aggregate ecosystem utility.

**Assumption 1.** Let $E_1(t)$ , $E_2(t)$ , and $E_3(t)$ denote the effort levels of ICE vehicle manufacturers, NEV enterprises, and local governments at time t, respectively. $E_1(t)$ and $E_2(t)$ represent enterprise investment efforts in new-energy transformation, while $E_3(t)$ represents the local government's fiscal commitment and policy support for the NEV sector [20]. To reflect diminishing marginal returns, each stakeholder's effort cost is assumed to be convex in its effort level. Accordingly, the cost incurred by stakeholder $i$ at time $t$ is defined as follows:

$$C_i(t)=\frac{l_i}{2}E_i^2(t), \quad (i=1,2,3) \tag{1}$$

Here, $l_1$ , $l_2$ , and $l_3$ denote the effort cost coefficients of ICE vehicle manufacturers, NEV enterprises, and local governments, respectively. $C_1(t), C_2(t)$ , and $C_3(t)$ denote the corresponding effort costs at time t. The quadratic cost specification reflects increasing marginal costs of stakeholder effort during the new-energy transformation process.

**Assumption 2.** Let $x_{(t)}$ denote the new-energy transformation level index of the automobile industry at time t. It is defined as a latent stock variable that represents the accumulated transformation capacity generated by the efforts of ICE vehicle manufacturers, NEV enterprises, and local governments, rather than as a direct measure of market penetration. This index captures the cumulative effects of technological upgrading, green production capacity, policy support, infrastructure improvement, and inter-firm coordination [21,22].

Following the stock-accumulation logic of dynamic goodwill models [23], $x_{(t)}$ increases with stakeholder efforts and depreciates naturally over time. The dynamic evolution of $x_{(t)}$ is specified as follows:

$$\begin{cases} \dot{x} = \sum_{i=1}^{3} \lambda_i E_i(t) - \alpha x(t) \\ x(0) = x_0 \geq 0 \end{cases} \tag{2}$$

where $\lambda_i$ measures the marginal contribution of stakeholder i's effort to the transformation stock, and $\alpha > 0$ denotes the natural depreciation rate of the accumulated transformation level. The initial value $x_0$ represents the initial transformation level of the automobile industry. Since $x_{(t)}$ is a theoretical transformation level index rather than a market-share variable, the numerical trajectories should be interpreted as relative changes in the industry’s accumulated transformation level.

**Assumption 3.** ICE vehicle manufacturers, NEV enterprises, and local governments jointly participate in the automobile industry's new-energy transformation. Their effort levels and coordination determine the aggregate revenue of the tripartite ecosystem. Following the revenue-aggregation logic in product-quality governance models [24], the aggregate revenue at time $t$ is formulated as follows:

$$\pi(t) = \sum_{i=1}^{3} \theta_i E_i(t) + \beta x(t) \tag{3}$$

where $\theta_1, \theta_2$ and $\theta_3$ denote the marginal revenue coefficients associated with the efforts of ICE vehicle manufacturers, NEV enterprises, and local governments, respectively. The parameter $\beta > 0$ measures the effect of the new-energy transformation level index on aggregate ecosystem revenue [25].

**Assumption 4.** The aggregate ecosystem revenue generated by the new-energy transformation process is allocated among ICE vehicle manufacturers, NEV enterprises, and local governments according to a budget-balanced revenue-sharing rule. Let $\omega_1$ and $\omega_2$ denote the revenue-sharing coefficients of ICE vehicle manufacturers and NEV enterprises, respectively. The local government receives the remaining share $1 - \omega_1 - \omega_2$. To ensure budget-balanced revenue allocation, non-negative equilibrium subsidy coefficients, and the validity of the interior Stackelberg solution, the revenue-sharing coefficients are assumed to satisfy $0 < \omega_1 < 1$, $0 < \omega_2 < 1$, $0 < \omega_1 + \omega_2 < 1$, $3\omega_1 + 2\omega_2 < 2, \quad 2\omega_1 + 3\omega_2 < 2$.

In the government-subsidized Stackelberg scenario, the local government provides cost-sharing subsidies to the two enterprise groups. Let $\sigma_1$ and $\sigma_2$ denote the subsidy coefficients for ICE vehicle manufacturers and NEV enterprises, respectively, where $0 \leq \sigma_1 \leq 1, \quad 0 \leq \sigma_2 \leq 1$. Accordingly, the effort cost borne by ICE vehicle manufacturers is $\frac{(1-\sigma_1) l_1 E_1^2(t)}{2}$, and the effort cost borne by NEV enterprises is $\frac{(1-\sigma_2) l_2 E_2^2(t)}{2}$. The local government bears the corresponding subsidy costs $\frac{\sigma_1 l_1 E_1^2(t)}{2} + \frac{\sigma_2 l_2 E_2^2(t)}{2}$. Given a discount rate $\rho$, the three stakeholders maximize their respective discounted payoffs over an infinite horizon.

## 3. Formulation of the differential game model

Before solving the three game scenarios, we specify the admissibility and verification conditions for the HJB derivations [26]. For each stakeholder $i=1,2,3$, the effort strategy $E_i(t)$ is required to be non-negative and measurable over time. In the Stackelberg scenario, the subsidy coefficients satisfy $0 \le \sigma_1 \le 1$ and $0 \le \sigma_2 \le 1$. Under Assumption 4 and the positivity of the model parameters, the equilibrium effort levels derived below are non-negative, and the equilibrium subsidy coefficients remain within the admissible range under the interior solution. Because the effort costs are quadratic with $l_i > 0$, the instantaneous payoff functions are concave in the corresponding effort variables. Therefore, the first-order conditions characterize the unique maximizers in the HJB equations. The linear value-function form is consistent with the linear state equation and the linear dependence of the payoff functions on the state variable $x(t)$. Moreover, since $\rho > 0$ and $\alpha > 0$, the discounted value functions satisfy the standard transversality condition:

$$\lim_{t\to\infty} e^{-\rho t} U_i(x(t)) = 0, \quad i=1,2,3 \tag{4}$$

For the cooperative game, the corresponding condition is $\lim_{t\to\infty} e^{-\rho t} U^C(x(t)) = 0$ (5)

Thus, once the candidate linear value functions satisfy the corresponding HJB equations, and the derived controls are admissible, the verification theorem for discounted infinite-horizon control problems ensures that the obtained feedback strategies constitute equilibrium strategies for the respective game scenarios.

### *3.1. Nash non-cooperative game*

In the Nash non-cooperative game, each stakeholder maximizes its own discounted payoff without contractual coordination or government cost-sharing subsidies. ICE vehicle manufacturers, NEV enterprises, and local governments act as decentralized decision makers [27,28]. Each stakeholder contributes to the new-energy transformation level but chooses its effort independently. The resulting feedback strategies define the Nash equilibrium. The intertemporal payoff functionals are formulated as follows:

$$\begin{aligned} \Pi_{E_i} &= \int_0^\infty e^{-\rho t}\left[\omega_i \pi(t) - \frac{l_i}{2} E_i^2(t)\right] dt, \quad (i=1,2) \\ \Pi_G &= \int_0^\infty e^{-\rho t}\left[(1-\omega_1-\omega_2)\pi(t) - \frac{l_3}{2} E_3^2(t)\right] dt \end{aligned} \tag{6}$$

**Proposition 1.** Under the Nash non-cooperative game, the optimal feedback effort strategies of ICE vehicle manufacturers, NEV enterprises, and local governments are given by:

$$E_1^N = \frac{\omega_1[\theta_1(\rho+\alpha)+\lambda_1\beta]}{l_1(\rho+\alpha)} \tag{7}$$

$$E_2^N = \frac{\omega_2[\theta_2(\rho+\alpha)+\lambda_2\beta]}{l_2(\rho+\alpha)} \tag{8}$$

$$E_3^N = \frac{(1-\omega_1-\omega_2)[\theta_3(\rho+\alpha)+\lambda_3\beta]}{l_3(\rho+\alpha)} \tag{9}$$

**Proof.** To derive the Markovian feedback Nash equilibrium, let $U_i^N(x)(i\in\{1,2,3\})$ denote the continuously differentiable and finite value functions of ICE vehicle manufacturers, NEV enterprises, and local governments, respectively. For any state $x \geq 0$, the Bellman optimality principle gives the following HJB system:

$$\rho U_1^N = \max_{E_1\geq 0}\left\{\omega_1\left(\sum_{i=1}^{3}\theta_i E_i+\beta x\right)-\frac{l_1}{2}E_1^2+\frac{\partial U_1^N}{\partial x}\left(\sum_{i=1}^{3}\lambda_i E_i-\alpha x\right)\right\} \tag{10}$$

$$\rho U_2^N = \max_{E_2\geq 0}\left\{\omega_2\left(\sum_{i=1}^{3}\theta_i E_i+\beta x\right)-\frac{l_2}{2}E_2^2+\frac{\partial U_2^N}{\partial x}\left(\sum_{i=1}^{3}\lambda_i E_i-\alpha x\right)\right\} \tag{11}$$

$$\rho U_3^N = \max_{E_3\geq 0}\left\{(1-\omega_1-\omega_2)\left(\sum_{i=1}^{3}\theta_i E_i+\beta x\right)-\frac{l_3}{2}E_3^2+\frac{\partial U_3^N}{\partial x}\left(\sum_{i=1}^{3}\lambda_i E_i-\alpha x\right)\right\} \tag{12}$$

Solving the first-order conditions of the HJB system yields the following optimal response functions:

$$\begin{cases} E_1 = \dfrac{\omega_1\theta_1+\lambda_1\dfrac{\partial U_1^N}{\partial x}}{l_1} \\ E_2 = \dfrac{\omega_2\theta_2+\lambda_2\dfrac{\partial U_2^N}{\partial x}}{l_2} \\ E_3 = \dfrac{(1-\omega_1-\omega_2)\theta_3+\lambda_3\dfrac{\partial U_3^N}{\partial x}}{l_3} \end{cases} \tag{13}$$

Substituting these response functions into Equations (10)−(12) and simplifying gives:

$$\rho U_1^N(x) = \left[\omega_1\beta-\alpha\frac{\partial U_1^N}{\partial x}\right]x+\frac{\left[\omega_1\theta_1+\lambda_1\dfrac{\partial U_1^N}{\partial x}\right]^2}{2l_1} \\ +\frac{\left[\omega_1\theta_2+\lambda_2\dfrac{\partial U_1^N}{\partial x}\right]\left[\omega_2\theta_2+\lambda_2\dfrac{\partial U_2^N}{\partial x}\right]}{l_2}+\frac{\left[\omega_1\theta_3+\lambda_3\dfrac{\partial U_1^N}{\partial x}\right]\left[(1-\omega_1-\omega_2)\theta_3+\lambda_3\dfrac{\partial U_3^N}{\partial x}\right]}{l_3} \tag{14}$$

$$\rho U_2^N(x)=\left[\omega_2\beta-\alpha\frac{\partial U_2^N}{\partial x}\right]x+\frac{\left[\omega_2\theta_2+\lambda_2\frac{\partial U_2^N}{\partial x}\right]^2}{2l_2}$$
$$+\frac{\left[\omega_2\theta_1+\lambda_1\frac{\partial U_2^N}{\partial x}\right]\left[\omega_1\theta_1+\lambda_1\frac{\partial U_1^N}{\partial x}\right]}{l_2}+\frac{\left[\omega_2\theta_3+\lambda_3\frac{\partial U_2^N}{\partial x}\right]\left[(1-\omega_1-\omega_2)\theta_3+\lambda_3\frac{\partial U_3^N}{\partial x}\right]}{l_3} \tag{15}$$

$$\rho U_3^N(x)=\left[(1-\omega_1-\omega_2)\beta-\alpha\frac{\partial U_3^N}{\partial x}\right]x+\frac{\left[(1-\omega_1-\omega_2)\theta_3+\lambda_3\frac{\partial U_3^N}{\partial x}\right]^2}{2l_3}$$
$$+\frac{\left[(1-\omega_1-\omega_2)\theta_1+\lambda_1\frac{\partial U_3^N}{\partial x}\right]\left[\omega_1\theta_1+\lambda_1\frac{\partial U_1^N}{\partial x}\right]}{l_1}+\frac{\left[(1-\omega_1-\omega_2)\theta_2+\lambda_2\frac{\partial U_3^N}{\partial x}\right]\left[\omega_2\theta_2+\lambda_2\frac{\partial U_2^N}{\partial x}\right]}{l_2} \tag{16}$$

Because the state equation and payoff functions are linear in $x(t)$, we use linear candidate value functions of the following form:

$$U_i^N(x)=k_i^N x+B_i^N,\quad (i=1,2,3) \tag{17}$$

where $k_i^N$ and $B_i^N$ are constants to be determined for stakeholder i = 1, 2, 3. Substituting the linear value functions into the HJB system gives the following coefficient equations [29]:

$$\rho(k_1^N x+B_1^N)=\left[\omega_1\beta-\alpha k_1^N\right]x+\frac{\left[\omega_1\theta_1+\lambda_1k_1^N\right]^2}{2l_1}+\frac{\left[\omega_1\theta_2+\lambda_2k_1^N\right]\left[\omega_2\theta_2+\lambda_2k_2^N\right]}{l_2}$$
$$+\frac{\left[\omega_1\theta_3+\lambda_3k_1^N\right]\left[(1-\omega_1-\omega_2)\theta_3+\lambda_3k_3^N\right]}{l_3} \tag{18}$$

$$\rho(k_2^N x+B_2^N)=\left[\omega_2\beta-\alpha k_2^N\right]x+\frac{\left[\omega_2\theta_2+\lambda_2k_2^N\right]^2}{2l_2}+\frac{\left[\omega_2\theta_1+\lambda_1k_2^N\right]\left[\omega_1\theta_1+\lambda_1k_1^N\right]}{l_1}$$
$$+\frac{\left[\omega_2\theta_3+\lambda_3k_2^N\right]\left[(1-\omega_1-\omega_2)\theta_3+\lambda_3k_3^N\right]}{l_3} \tag{19}$$

$$\rho(k_3^N x+B_3^N)=\left[(1-\omega_1-\omega_2)\beta-\alpha k_3^N\right]x+\frac{\left[(1-\omega_1-\omega_2)\theta_3+\lambda_3k_3^N\right]^2}{2l_3}+\frac{\left[(1-\omega_1-\omega_2)\theta_1+\lambda_1k_3^N\right]\left[\omega_1\theta_1+\lambda_1k_1^N\right]}{l_1}$$
$$+\frac{\left[(1-\omega_1-\omega_2)\theta_2+\lambda_2k_3^N\right]\left[\omega_2\theta_2+\lambda_2k_2^N\right]}{l_2} \tag{20}$$

Equating the coefficients of $x$ and the constant terms yields the following values of $k_i^N$ and $B_i^N$:

$$\begin{cases} k_1^N = \dfrac{\omega_1\beta}{\rho+\alpha}, \quad k_2^N = \dfrac{\omega_2\beta}{\rho+\alpha}, \quad k_3^N = \dfrac{(1-\omega_1-\omega_2)\beta}{\rho+\alpha} \\ B_1^N = \dfrac{\omega_1{}^2\left[\theta_1(\rho+\alpha)+\lambda_1\beta\right]^2}{2\rho l_1(\rho+\alpha)^2} + \dfrac{\omega_1\omega_2\left[\theta_2(\rho+\alpha)+\lambda_2\beta\right]^2}{\rho l_2(\rho+\alpha)^2} \\ +\dfrac{\omega_1(1-\omega_1-\omega_2)\left[\theta_3(\rho+\alpha)+\lambda_3\beta\right]^2}{\rho l_3(\rho+\alpha)^2} \\ B_2^N = \dfrac{\omega_2{}^2\left[\theta_2(\rho+\alpha)+\lambda_2\beta\right]^2}{2\rho l_2(\rho+\alpha)^2} + \dfrac{\omega_1\omega_2\left[\theta_1(\rho+\alpha)+\lambda_1\beta\right]^2}{\rho l_1(\rho+\alpha)^2} \\ +\dfrac{\omega_2(1-\omega_1-\omega_2)\left[\theta_3(\rho+\alpha)+\lambda_3\beta\right]^2}{\rho l_3(\rho+\alpha)^2} \\ B_3^N = \dfrac{(1-\omega_1-\omega_2)^2\left[\theta_3(\rho+\alpha)+\lambda_3\beta\right]^2}{2\rho l_3(\rho+\alpha)^2} + \dfrac{\omega_1(1-\omega_1-\omega_2)^2\left[\theta_1(\rho+\alpha)+\lambda_1\beta\right]^2}{\rho l_1(\rho+\alpha)^2} \\ +\dfrac{\omega_2(1-\omega_1-\omega_2)^2\left[\theta_2(\rho+\alpha)+\lambda_2\beta\right]^2}{\rho l_2(\rho+\alpha)^2} \end{cases} \tag{21}$$

Substituting these coefficients into Equation (13) gives the optimal feedback strategies in Proposition 1. Substituting the corresponding effort levels into the state equation yields the following transformation trajectory:

$$\begin{cases} x^N(t) = \dfrac{A^N}{\alpha} + \left(x_0 - \dfrac{A^N}{\alpha}\right)e^{-\alpha t} \\ x(0) = x_0 \end{cases} \tag{22}$$

where $A^N$ denotes the effort-induced accumulation term under the Nash non-cooperative game:

$$A^N = \lambda_1 E_1^N + \lambda_2 E_2^N + \lambda_3 E_3^N = \frac{\lambda_1\omega_1[\theta_1(\rho+\alpha)+\lambda_1\beta]}{l_1(\rho+\alpha)} + \frac{\lambda_2\omega_2[\theta_2(\rho+\alpha)+\lambda_2\beta]}{l_2(\rho+\alpha)} + \frac{\lambda_3(1-\omega_1-\omega_2)[\theta_3(\rho+\alpha)+\lambda_3\beta]}{l_3(\rho+\alpha)}$$

The corresponding value functions of ICE vehicle manufacturers, NEV enterprises, and local governments are:

$$\begin{aligned} U_1^N(x) = {} & \frac{\omega_1\beta}{\rho+\alpha}x^N + \frac{\omega_1{}^2\left[\theta_1(\rho+\alpha)+\lambda_1\beta\right]^2}{2\rho l_1(\rho+\alpha)^2} + \frac{\omega_1\omega_2\left[\theta_2(\rho+\alpha)+\lambda_2\beta\right]^2}{\rho l_2(\rho+\alpha)^2} \\ & + \frac{\omega_1(1-\omega_1-\omega_2)\left[\theta_3(\rho+\alpha)+\lambda_3\beta\right]^2}{\rho l_3(\rho+\alpha)^2} \end{aligned} \tag{23}$$

$$\begin{aligned} U_2^N(x) = {} & \frac{\omega_2\beta}{\rho+\alpha}x^N + \frac{\omega_2{}^2\left[\theta_2(\rho+\alpha)+\lambda_2\beta\right]^2}{2\rho l_2(\rho+\alpha)^2} + \frac{\omega_1\omega_2\left[\theta_1(\rho+\alpha)+\lambda_1\beta\right]^2}{\rho l_1(\rho+\alpha)^2} \\ & + \frac{\omega_2(1-\omega_1-\omega_2)\left[\theta_3(\rho+\alpha)+\lambda_3\beta\right]^2}{\rho l_3(\rho+\alpha)^2} \end{aligned} \tag{24}$$

$$U_3^N(x)=\frac{(1-\omega_1-\omega_2)\beta}{\rho+\alpha}x^N+\frac{(1-\omega_1-\omega_2)^2\left[\theta_3(\rho+\alpha)+\lambda_3\beta\right]^2}{2\rho l_3(\rho+\alpha)^2}+\frac{\omega_1(1-\omega_1-\omega_2)\left[\theta_1(\rho+\alpha)+\lambda_1\beta\right]^2}{\rho l_1(\rho+\alpha)^2}$$
$$+\frac{\omega_2(1-\omega_1-\omega_2)\left[\theta_2(\rho+\alpha)+\lambda_2\beta\right]^2}{\rho l_2(\rho+\alpha)^2} \tag{25}$$

The aggregate value function under the Nash non-cooperative game is defined as the sum of the three individual value functions:

$$U^N(x)=U_1^N(x)+U_2^N(x)+U_3^N(x)$$
$$=\frac{\beta}{\rho+\alpha}x^N+\frac{(2\omega_1-\omega_1^2)\left[\theta_1(\rho+\alpha)+\lambda_1\beta\right]^2}{2\rho l_1(\rho+\alpha)^2}+\frac{(2\omega_2-\omega_2^2)\left[\theta_2(\rho+\alpha)+\lambda_2\beta\right]^2}{2\rho l_2(\rho+\alpha)^2}$$
$$+\frac{\left[2(1-\omega_1-\omega_2)-(1-\omega_1-\omega_2)^2\right]\left[\theta_3(\rho+\alpha)+\lambda_3\beta\right]^2}{2\rho l_3(\rho+\alpha)^2} \tag{26}$$

### *3.2. Stackelberg game*

In the Stackelberg game, local governments act as leaders in the new-energy transformation, while ICE vehicle manufacturers and NEV enterprises act as followers. Local governments provide cost-sharing subsidies to support the effort costs of the two enterprise groups. The local government first determines the subsidy coefficients $\sigma_1,\sigma_2$ and anticipates the firms' response strategies. The enterprises then choose their effort levels to maximize their own payoffs [30,31]. The intertemporal payoff functionals under this leader-follower structure are formulated as follows:

$$\prod_{E1}=\int_0^\infty e^{-\rho t}\left[\omega_1(\theta_1E_1(t)+\theta_2E_2(t)+\theta_3E_3(t)+\beta x(t))-\frac{l_1}{2}E_1^2(t)+\sigma_1\frac{l_1}{2}E_1^2(t)\right]dt \tag{27}$$

$$\prod_{E2}=\int_0^\infty e^{-\rho t}\left[\omega_2(\theta_1E_1(t)+\theta_2E_2(t)+\theta_3E_3(t)+\beta x(t))-\frac{l_2}{2}E_2^2(t)+\sigma_2\frac{l_2}{2}E_2^2(t)\right]dt \tag{28}$$

$$\prod_{G}=\int_0^\infty e^{-\rho t}\left[(1-\omega_1-\omega_2)(\theta_1E_1(t)+\theta_2E_2(t)+\theta_3E_3(t)+\beta x(t))-\frac{l_3}{2}E_3^2(t)-\sigma_1\frac{l_1}{2}E_1^2(t)-\sigma_2\frac{l_2}{2}E_2^2(t)\right]dt \tag{29}$$

**Proposition 2.** Under the government-led Stackelberg game, the subgame-perfect feedback strategies of ICE vehicle manufacturers, NEV enterprises, and local governments are given by:

$$E_1^S=\frac{(2-\omega_1-2\omega_2)\left[\theta_1(\rho+\alpha)+\lambda_1\beta\right]}{2l_1(\rho+\alpha)} \tag{30}$$

$$E_2^s=\frac{(2-2\omega_1-\omega_2)\left[\theta_2(\rho+\alpha)+\lambda_2\beta\right]}{2l_2(\rho+\alpha)} \tag{31}$$

$$E_3^S=\frac{(1-\omega_1-\omega_2)\left[\theta_3(\rho+\alpha)+\lambda_3\beta\right]}{l_3(\rho+\alpha)} \tag{32}$$

$$\sigma_1^S=\frac{2-3\omega_1-2\omega_2}{2-\omega_1-2\omega_2} \tag{33}$$

$$\sigma_2^S = \frac{2-2\omega_1-3\omega_2}{2-2\omega_1-\omega_2} \tag{34}$$

**Proof.** To derive the Stackelberg equilibrium, we use backward induction. Let $U_i^S(x)$, $i=1,2,3$ denote the continuously differentiable and finite value functions of ICE vehicle manufacturers, NEV enterprises, and local governments, respectively. For any state $x \geq 0$, we first derive the response functions of the two enterprise groups. The HJB equations for ICE vehicle manufacturers and NEV enterprises are:

$$\rho U_1^S(x) = \max_{E_1\geq 0}\left\{\omega_1(\theta_1 E_1+\theta_2 E_2+\theta_3 E_3+\beta x)-\frac{l_1}{2}E_1^2(1-\sigma_1)+\frac{\partial U_1^S}{\partial x}(\lambda_1 E_1+\lambda_2 E_2+\lambda_3 E_3-\alpha x)\right\} \tag{35}$$

$$\rho U_2^S(x) = \max_{E_2\geq 0}\left\{\omega_2(\theta_1 E_1+\theta_2 E_2+\theta_3 E_3+\beta x)-\frac{l_2}{2}E_2^2(1-\sigma_2)+\frac{\partial U_2^S}{\partial x}(\lambda_1 E_1+\lambda_2 E_2+\lambda_3 E_3-\alpha x)\right\} \tag{36}$$

Given the concavity of the objective functions in Equations (35) and (36) with respect to the enterprise effort variables, the first-order conditions characterize the optimal response functions. Differentiating the enterprise objective functions with respect to $E_1$ and $E_2$ and setting the derivatives equal to zero gives:

$$E_1 = \frac{\theta_1\omega_1+\lambda_1\dfrac{\partial U_1^s}{\partial x}}{l_1(1-\sigma_1)} \tag{37}$$

$$E_2 = \frac{\theta_2\omega_2+\lambda_2\dfrac{\partial U_2^s}{\partial x}}{l_2(1-\sigma_2)} \tag{38}$$

Given the enterprise response functions, the local government then chooses its effort level and subsidy coefficients under perfect information. Substituting the enterprise response functions into the government objective gives the following HJB equation for the local government:

$$\rho U_3^S(x) = \max_{E_3\geq 0}\left\{(1-\omega_1-\omega_2)(\theta_1 E_1+\theta_2 E_2+\theta_3 E_3+\beta x)-\frac{l_3}{2}E_3^2-\sigma_1\frac{l_1}{2}E_1^2-\sigma_2\frac{l_2}{2}E_2^2+\frac{\partial U_3^S}{\partial x}(\lambda_1 E_1+\lambda_2 E_2+\lambda_3 E_3-\alpha x)\right\} \tag{39}$$

Solving the first-order conditions of the government HJB equation with respect to $E_3$, $\sigma_1$, and $\sigma_2$ yields:

$$E_3 = \frac{(1-\omega_1-\omega_2)\theta_3+\lambda_3\dfrac{\partial U_3^S}{\partial x}}{l_3} \tag{40}$$

$$\sigma_1 = \frac{\theta_1(2-3\omega_1-2\omega_2)-\lambda_1\dfrac{\partial U_1^S}{\partial x}+2\lambda_1\dfrac{\partial U_3^S}{\partial x}}{\theta_1(2-\omega_1-2\omega_2)+\lambda_1\dfrac{\partial U_1^S}{\partial x}+2\lambda_1\dfrac{\partial U_3^S}{\partial x}} \tag{41}$$

$$\sigma_2 = \frac{\theta_2(2-3\omega_2-2\omega_1) - \lambda_2 \frac{\partial U_2^S}{\partial x} + 2\lambda_2 \frac{\partial U_3^S}{\partial x}}{\theta_2(2-\omega_2-2\omega_1) + \lambda_2 \frac{\partial U_2^S}{\partial x} + 2\lambda_2 \frac{\partial U_3^S}{\partial x}} \tag{42}$$

Substituting the response functions and government policies into Equations (35), (36), and (39) and simplifying gives:

$$\begin{aligned}\rho U_1^S(x) = \left[\omega_1\beta - \alpha\frac{\partial U_1^S}{\partial x}\right]x + \frac{\left[\omega_1\theta_1 + \lambda_1\frac{\partial U_1^S}{\partial x}\right]\left[\theta_1(2-\omega_1-2\omega_2) + \lambda_1\left(\frac{\partial U_1^S}{\partial x} + 2\frac{\partial U_3^S}{\partial x}\right)\right]}{4l_1}\\ + \frac{\left[\omega_1\theta_2 + \lambda_2\frac{\partial U_2^S}{\partial x}\right]\left[\theta_2(2-\omega_2-2\omega_1) + \lambda_2\left(\frac{\partial U_2^S}{\partial x} + 2\frac{\partial U_3^S}{\partial x}\right)\right]}{2l_2} + \frac{\left[\omega_1\theta_3 + \lambda_3\frac{\partial U_1^S}{\partial x}\right]\left[(1-\omega_1-\omega_2)\theta_3 + \lambda_3\frac{\partial U_3^S}{\partial x}\right]}{l_3}\end{aligned} \tag{43}$$

$$\begin{aligned}\rho U_2^S(x) = \left[\omega_2\beta - \alpha\frac{\partial U_2^S}{\partial x}\right]x + \frac{\left[\omega_2\theta_2 + \lambda_2\frac{\partial U_2^S}{\partial x}\right]\left[\theta_2(2-\omega_2-2\omega_1) + \lambda_2\left(\frac{\partial V_2^S}{\partial x} + 2\frac{\partial V_3^S}{\partial x}\right)\right]}{4\mu_2}\\ + \frac{\left[\omega_2\theta_1 + \lambda_1\frac{\partial U_2^S}{\partial x}\right]\left[\theta_1(2-\omega_1-2\omega_2) + \lambda_1\left(\frac{\partial U_1^S}{\partial x} + 2\frac{\partial U_3^S}{\partial x}\right)\right]}{2l_1} + \frac{\left[\omega_2\theta_3 + \lambda_3\frac{\partial U_2^S}{\partial x}\right]\left[(1-\omega_1-\omega_2)\theta_3 + \lambda_3\frac{\partial U_3^S}{\partial x}\right]}{l_3}\end{aligned} \tag{44}$$

$$\begin{aligned}\rho U_3^S(x) = \left[(1-\omega_1-\omega_2)\beta - \alpha\frac{\partial U_3^S}{\partial x}\right]x + \frac{\left[(1-\omega_1-\omega_2)\theta_3 + \lambda_3\frac{\partial U_3^S}{\partial x}\right]^2}{2l_3}\\ + \frac{\left[\theta_1(2-\omega_1-2\omega_2) + \lambda_1\left(\frac{\partial U_1^S}{\partial x} + 2\frac{\partial U_3^S}{\partial x}\right)\right]^2}{8l_1} + \frac{\left[\theta_2(2-\omega_2-2\omega_1) + \lambda_2\left(\frac{\partial U_2^S}{\partial x} + 2\frac{\partial U_3^S}{\partial x}\right)\right]^2}{8l_2}\end{aligned} \tag{45}$$

Because the state equation and payoff functions are linear in $x(t)$ , we use linear candidate value functions of the following form:

$$U_i^S(x) = k_i^S x + B_i^S, \quad (i = 1,2,3) \tag{46}$$

where $k_i^S$ and $B_i^S$ are constants to be determined for stakeholder $i = 1,2,3$ . Substituting these candidate value functions into the HJB equations gives the following coefficient equations:

$$\begin{aligned}\rho(k_1^S x + B_1^S) = \left[\omega_1\beta - \alpha k_1^S\right]x + \frac{\left[\omega_1\theta_1 + \lambda_1 k_1^S\right]\left[\theta_1(2-\omega_1-2\omega_2) + \lambda_1(k_1^S + 2k_3^S)\right]}{4l_1}\\ + \frac{\left[\omega_1\theta_2 + \lambda_2 k_2^S\right]\left[\theta_2(2-2\omega_2+\omega_1+1) + \lambda_2(k_2^S + 2k_3^S)\right]}{2l_2}\\ + \frac{\left[\omega_1\theta_3 + \lambda_3 k_1^S\right]\left[(1-\omega_1-\omega_2)\theta_3 + \lambda_3 k_3^S\right]}{l_3}\end{aligned} \tag{47}$$

$$\rho(k_2^S x+B_2^S)=\left[\omega_2\beta-\alpha k_2^S\right]x+\frac{\left[\omega_2\theta_2+\lambda_2 k_2^S\right]\left[\theta_2(2-\omega_2-2\omega_1)+\lambda_2(k_2^S+2k_3^S)\right]}{4l_2}$$
$$+\frac{\left[\omega_2\theta_1+\lambda_1 k_2^S\right]\left[\theta_1(2-\omega_1-2\omega_2+1)+\lambda_1(k_1^S+2k_3^S)\right]}{2l_1} \tag{48}$$
$$+\frac{\left[\omega_2\theta_3+\lambda_3 k_2^S\right]\left[(1-\omega_1-\omega_2)\theta_3+\lambda_3 k_3^S\right]}{l_3}$$

$$\rho(k_3^S x+B_3^S)=\left[(1-\omega_1-\omega_2)\beta-\alpha k_3^S\right]x+\frac{\left[(1-\omega_1-\omega_2)\theta_3+\lambda_3 k_3^S\right]^2}{2l_3}+\frac{\left[\theta_1(2-\omega_1-2\omega_2)+\lambda_1(k_1^S+2k_3^S)\right]^2}{8l_1}$$
$$+\frac{\left[\theta_2(2-\omega_2-2\omega_1)+\lambda_2(k_2^S+2k_3^S)\right]^2}{8l_2} \tag{49}$$

Equating the coefficients of x and the constant terms yields the following values of $k_i^S$ and $B_i^S$ :

$$\begin{cases} k_1^S=\dfrac{\omega_1\beta}{\rho+\alpha},\quad k_2^S=\dfrac{\omega_2\beta}{\rho+\alpha},\quad k_3^S=\dfrac{(1-\omega_1-\omega_2)\beta}{\rho+\alpha} \\ B_1^S=\dfrac{\omega_1(2-\omega_1-2\omega_2)\left[\theta_1(\rho+\alpha)+\lambda_1\beta\right]^2}{4\rho l_1(\rho+\alpha)^2}+\dfrac{\omega_1(2-2\omega_1-\omega_2)\left[\theta_3(\rho+\alpha)+\lambda_3\beta\right]^2}{\rho l_3(\rho+\alpha)^2} \\ +\dfrac{\omega_1(2-2\omega_1-\omega_2)\left[\theta_2(\rho+\alpha)+\lambda_2\beta\right]^2}{2\rho l_2(\rho+\alpha)^2} \\ B_2^S=\dfrac{\omega_2(2-\omega_2-2\omega_1)\left[\theta_2(\rho+\alpha)+\lambda_2\beta\right]^2}{4\rho l_2(\rho+\alpha)^2}+\dfrac{\omega_2(1-\omega_1-\omega_2)\left[\theta_3(\rho+\alpha)+\lambda_3\beta\right]^2}{\rho l_3(\rho+\alpha)^2} \\ +\dfrac{\omega_2(2-\omega_1-2\omega_2)\left[\theta_1(\rho+\alpha)+\lambda_1\beta\right]^2}{2\rho l_1(\rho+\alpha)^2} \\ B_3^S=\dfrac{(1-\omega_1-\omega_2)^2\left[\theta_3(\rho+\alpha)+\lambda_3\beta\right]^2}{2\rho l_3(\rho+\alpha)^2}+\dfrac{(2-\omega_1-2\omega_2)^2\left[\theta_1(\rho+\alpha)+\lambda_1\beta\right]^2}{8\rho l_1(\rho+\alpha)^2} \\ +\dfrac{(2-\omega_2-2\omega_1)^2\left[\theta_2(\rho+\alpha)+\lambda_2\beta\right]^2}{8\rho l_2(\rho+\alpha)^2} \end{cases} \tag{50}$$

Substituting these coefficients into the response and policy functions gives the Stackelberg equilibrium strategies $E_1^S$, $E_2^S$ , and $E_3^S$, together with the optimal subsidy coefficients $\sigma_1^S$ and $\sigma_2^S$. Substituting the corresponding effort levels into the state equation yields:

$$\begin{cases} \dot{x}=\dfrac{dx}{dt}=A^S-\alpha x(t) \\ x(0)=x_0 \end{cases} \tag{51}$$

where AS denotes the effort-induced accumulation term under the Stackelberg game:

$$A^S=\lambda_1E_1^S+\lambda_2E_2^S+\lambda_3E_3^S=\frac{\lambda_1(2-\omega_1-2\omega_2)\left[\theta_1(\rho+\alpha)+\lambda_1\beta\right]}{2l_1(\rho+\alpha)}+\frac{\lambda_2(2-2\omega_1-\omega_2+1)\left[\theta_2(\rho+\alpha)+\lambda_2\beta\right]}{2l_2(\rho+\alpha)}+\frac{\lambda_3(1-\omega_1-\omega_2)\left[\theta_3(\rho+\alpha)+\lambda_3\beta\right]}{l_3(\rho+\alpha)}$$

The corresponding trajectory of the new-energy transformation level index is:

$$x^S(t) = \frac{A^S}{\alpha} + \left(x_0 - \frac{A^S}{\alpha}\right)e^{-\alpha t} \tag{52}$$

Using the solved coefficients, the value functions of ICE vehicle manufacturers, NEV enterprises, and local governments under the Stackelberg game are:

$$\begin{aligned} U_1^S(x) = {} & \frac{\omega_1\beta}{\rho+\alpha}x^S + \frac{\omega_1(2-\omega_1-2\omega_2)\left[\theta_1(\rho+\alpha)+\lambda_1\beta\right]^2}{4\rho l_1(\rho+\alpha)^2} + \frac{\omega_1(1-\omega_1-\omega_2)\left[\theta_3(\rho+\alpha)+\lambda_3\beta\right]^2}{\rho l_3(\rho+\alpha)^2} \\ & + \frac{\omega_1(2-2\omega_1-\omega_2)\left[\theta_2(\rho+\alpha)+\lambda_2\beta\right]^2}{2\rho l_2(\rho+\alpha)^2} \end{aligned} \tag{53}$$

$$\begin{aligned} U_2^S(x) = {} & \frac{\omega_2\beta}{\rho+\alpha}x^S + \frac{\omega_2(2-2\omega_1-\omega_2)\left[\theta_2(\rho+\alpha)+\lambda_2\beta\right]^2}{4\rho l_2(\rho+\alpha)^2} + \frac{\omega_2(1-\omega_1-\omega_2)\left[\theta_3(\rho+\alpha)+\lambda_3\beta\right]^2}{\rho l_3(\rho+\alpha)^2} \\ & + \frac{\omega_2(2-\omega_1-2\omega_2)\left[\theta_1(\rho+\alpha)+\lambda_1\beta\right]^2}{2\rho l_1(\rho+\alpha)^2} \end{aligned} \tag{54}$$

$$\begin{aligned} U_3^S(x) = {} & \frac{(1-\omega_1-\omega_2)\beta}{\rho+\alpha}x^S + \frac{(1-\omega_1-\omega_2)^2\left[\theta_3(\rho+\alpha)+\lambda_3\beta\right]^2}{2\rho l_3(\rho+\alpha)^2} + \frac{(2-\omega_1-2\omega_2)^2\left[\theta_1(\rho+\alpha)+\lambda_1\beta\right]^2}{8\rho l_1(\rho+\alpha)^2} \\ & + \frac{(2-2\omega_1-\omega_2)^2\left[\theta_2(\rho+\alpha)+\lambda_2\beta\right]^2}{8\rho l_2(\rho+\alpha)^2} \end{aligned} \tag{55}$$

The aggregate value function under the Stackelberg game is defined as the sum of the three individual value functions:

$$\begin{aligned} U^S(x) = {} & U_1^S(x) + U_2^S(x) + U_3^S(x) = \frac{\beta}{\rho+\alpha}x^S + \frac{(1-\omega_1-\omega_2)(1+\omega_1+\omega_2)\left[\theta_3(\rho+\alpha)+\lambda_3\beta\right]^2}{2\rho l_3(\rho+\alpha)^2} \\ & + \frac{(2-2\omega_1-\omega_2)(2+2\omega_1+\omega_2)\left[\theta_1(\rho+\alpha)+\lambda_1\beta\right]^2}{8\rho l_2(\rho+\alpha)^2} \\ & + \frac{(1-\omega_1-\omega_2)(1+\omega_2+\omega_1)\left[\theta_3(\rho+\alpha)+\lambda_3\beta\right]^2}{2\rho l_3(\rho+\alpha)^2} \end{aligned} \tag{56}$$

### *3.3. Cooperative game*

In the cooperative game, ICE vehicle manufacturers, NEV enterprises, and local governments are treated as a unified decision-making system. The three stakeholders jointly choose their effort levels to maximize aggregate ecosystem utility. The cooperative benchmark represents the coordinated outcome against which the Nash non-cooperative and Stackelberg scenarios are compared. The cooperative payoff functional is formulated as follows:

$$\Pi = \sum_{i=1}^{3}\Pi_{Ei} = \int_0^\infty e^{-\rho t}\left[\pi(t) - \sum_{i=1}^{3}\frac{l_i}{2}E_i^2(t)\right]dt \tag{57}$$

**Proposition 3.** Under the cooperative game, the jointly optimal feedback effort strategies of ICE vehicle manufacturers, NEV enterprises, and local governments are given by:

$$E_1^C = \frac{\left[\theta_1(\rho+\alpha)+\lambda_1\beta\right]}{l_1(\rho+\alpha)} \tag{58}$$

$$E_2^C = \frac{[\theta_2(\rho+\alpha)+\lambda_2\beta]}{l_2(\rho+\alpha)} \tag{59}$$

$$E_3^C = \frac{[\theta_3(\rho+\alpha)+\lambda_3\beta]}{l_3(\rho+\alpha)} \tag{60}$$

Proof. Let $U^C(x)$ denote the continuously differentiable and finite aggregate value function under the cooperative game. For any state $x \geq 0$, the HJB equation for the unified decision-making system is:

$$\rho U^C(x) = \max_{\substack{E_1\geq 0\\ E_2\geq 0\\ E_3\geq 0}} \left\{ (\theta_1 E_1 + \theta_2 E_2 + \theta_3 E_3 + \beta x) - \frac{l_1}{2}E_1^2 - \frac{l_2}{2}E_2^2 - \frac{l_3}{2}E_3^2 + \frac{\partial U^C}{\partial x}(\lambda_1 E_1 + \lambda_2 E_2 + \lambda_3 E_3 - \alpha x) \right\} \tag{61}$$

Because the objective function in Equation (61) is concave in the effort variables, the first-order conditions characterize the jointly optimal effort levels. Differentiating the right-hand side of Equation (61) with respect to $E_1$, $E_2$, and $E_3$ and setting the derivatives equal to zero gives:

$$E_i = \frac{\theta_i + \lambda_i \dfrac{\partial U^C}{\partial x}}{l_i}, \quad (i = 1,2,3) \tag{62}$$

Substituting the jointly optimal effort levels into Equation (61) and simplifying gives:

$$\rho U^c(x) = \left(\beta - \alpha\frac{\partial U^c}{\partial x}\right)x + \frac{\left(\theta_1 + \lambda_1\dfrac{\partial U^c}{\partial x}\right)^2}{2l_1} + \frac{\left(\theta_2 + \lambda_2\dfrac{\partial U^c}{\partial x}\right)^2}{2l_2} + \frac{\left(\theta_3 + \lambda_3\dfrac{\partial U^c}{\partial x}\right)^2}{2l_3} \tag{63}$$

Because the state equation and cooperative payoff function are linear in $x(t)$, we use a linear candidate value function:

$$U^C(x) = k^C x + B^C \tag{64}$$

where $k^C$ and $B^C$ are constants to be determined. Substituting this linear form into the cooperative HJB equation gives:

$$\rho(k^c x + B^c) = (\beta - \alpha k^c)x + \frac{(\theta_1 + \lambda_1 k^c)^2}{2l_1} + \frac{(\theta_2 + \lambda_2 k^c)^2}{2l_2} + \frac{(\theta_3 + \lambda_3 k^c)^2}{2l_3} \tag{65}$$

Equating the coefficients of $x$ and the constant terms yields:

$$\begin{cases} k^C = \dfrac{\beta}{\rho+\alpha} \\ B^C = \dfrac{[\theta_1(\rho+\alpha)+\lambda_1\beta]^2}{\rho l_1(\rho+\alpha)^2} + \dfrac{[\theta_2(\rho+\alpha)+\lambda_2\beta]^2}{\rho l_2(\rho+\alpha)^2} + \dfrac{[\theta_3(\rho+\alpha)+\lambda_3\beta]^2}{\rho l_3(\rho+\alpha)^2} \end{cases} \tag{66}$$

Substituting $k^C$ into Equation (62) gives the jointly optimal feedback strategies stated in Proposition 3. Substituting the corresponding effort levels into the state equation yields:

$$\begin{cases} \dot{x} = \dfrac{dx}{dt} = A^C - \alpha x(t) \\ x(0) = x_0 \end{cases} \tag{67}$$

where $A^C$ denotes the effort-induced accumulation term under the cooperative game:

$$A^C = \lambda_1 E_1^C + \lambda_2 E_2^C + \lambda_3 E_3^C = \frac{\lambda_1\left[\theta_1(\rho+\alpha)+\lambda_1\beta\right]}{l_1(\rho+\alpha)} + \frac{\lambda_2\left[\theta_2(\rho+\alpha)+\lambda_2\beta\right]}{l_2(\rho+\alpha)} + \frac{\lambda_3\left[\theta_3(\rho+\alpha)+\lambda_3\beta\right]}{l_3(\rho+\alpha)}$$

The corresponding trajectory of the new-energy transformation level index is:

$$x^C(t) = \frac{A^C}{\alpha} + \left(x_0 - \frac{A^C}{\alpha}\right)e^{-\alpha t} \tag{68}$$

Using the solved coefficients, the aggregate value function under the cooperative game is:

$$U^c(x) = \frac{\beta}{\rho+\alpha}x^c + \frac{\left[\theta_1(\rho+\alpha)+\lambda_1\beta\right]^2}{\rho l_1(\rho+\alpha)^2} + \frac{\left[\theta_2(\rho+\alpha)+\lambda_2\beta\right]^2}{\rho l_2(\rho+\alpha)^2} + \frac{\left[\theta_3(\rho+\alpha)+\lambda_3\beta\right]^2}{\rho l_3(\rho+\alpha)^2} \tag{69}$$

### *3.4. Comparative analysis*

In this section, we compare the equilibrium strategies, transformation trajectories, and utility outcomes under the Nash non-cooperative, Stackelberg, and cooperative games. Under Assumption 4 and the same initial transformation level, the following propositions compare stakeholder effort levels, the new-energy transformation level index, and stakeholder utilities across the three governance scenarios.

**Proposition 4:** Under Assumption 4, the optimal effort levels satisfy: $E_1^C > E_1^S > E_1^N$, $E_2^C > E_2^S > E_2^N$, and $E_3^C > E_3^S = E_3^N$.

The optimal subsidy coefficients are:

$\sigma_1^S = \dfrac{E_1^S - E_1^N}{E_1^S} = \dfrac{2-3\omega_1-2\omega_2}{2-\omega_1-2\omega_2}$, $\sigma_2^S = \dfrac{E_2^S - E_2^N}{E_2^S} = \dfrac{2-2\omega_1-3\omega_2}{2-2\omega_1-\omega_2}$.

Proof. According to Equations (7), (30), and (58), we obtain:

$$\begin{aligned} E_1^S - E_1^N &= \frac{(2-3\omega_1-2\omega_2)\left[\theta_1(\rho+\alpha)+\lambda_1\beta\right]}{2l_1(\rho+\alpha)} \\ &= E_1^S \cdot \sigma_1^S > 0 \end{aligned} \tag{70}$$

$$E_1^C - E_1^S = \frac{(\omega_1+2\omega_2)\left[\theta_1(\rho+\alpha)+\lambda_1\beta\right]}{2l_1(\rho+\alpha)} > 0 \tag{71}$$

According to Equations (8), (31), and (59), we obtain:

$$
\begin{aligned}
E_2^S - E_2^N &= \frac{(2-2\omega_1-3\omega_2)\left[\theta_2(\rho+\alpha)+\lambda_2\beta\right]}{2l_2(\rho+\alpha)} \\
&= E_2^S \cdot \sigma_2^S > 0
\end{aligned}
\tag{72}
$$

$$
E_2^C - E_2^S = \frac{(2\omega_1+\omega_2)\left[\theta_2(\rho+\alpha)+\lambda_2\beta\right]}{2l_2(\rho+\alpha)} > 0 \tag{73}
$$

According to Equations (9), (32), and (60), we obtain:

$$
E_3^S - E_3^N = 0 \tag{74}
$$

$$
E_3^C - E_3^S = \frac{(\omega_1+\omega_2)\left[\theta_3(\rho+\alpha)+\lambda_3\beta\right]}{l_3(\rho+\alpha)} > 0 \tag{75}
$$

These comparisons prove Proposition 4.

**Corollary 1**. Moving from the Nash non-cooperative game to the government-subsidized Stackelberg game increases the effort levels of ICE vehicle manufacturers and NEV enterprises. Although the local government's own effort level remains unchanged, its cost-sharing subsidies reduce enterprise effort costs and encourage higher enterprise-side transformation efforts.

**Corollary 2.** In the cooperative game, all three stakeholders reach their highest effort levels. This result indicates that coordinated decision-making can strengthen the transformation efforts of ICE vehicle manufacturers, NEV enterprises, and local governments and provides the most effective effort structure among the three game scenarios.

**Proposition 5.** Under Assumption 4 and the same initial transformation level, the trajectories of the new-energy transformation level index satisfy: $x^C(t) > x^S(t) > x^N(t)$.

Proof. From Proposition 4, the effort-induced accumulation terms satisfy $A^C \geq A^S > A^N$. Moreover, the state trajectory under each game scenario is given by

$$
x^j(t) = \frac{A^j}{\alpha} + \left(x_0 - \frac{A^j}{\alpha}\right)e^{-\alpha t}, \quad j = N, S, C \tag{76}
$$

$$
x^C(t) - x^S(t) = \frac{A^C - A^S}{\alpha}\left(1-e^{-\alpha t}\right) > 0 \tag{77}
$$

$$
x^S(t) - x^N(t) = \frac{A^S - A^N}{\alpha}\left(1-e^{-\alpha t}\right) > 0 \tag{78}
$$

Because $x^j(t)$ is increasing in $A^j$ for $j = N, S, C$, it follows that $x^C(t) > x^S(t) > x^N(t)$. This proves Proposition 5.

**Corollary 3.** Government cost-sharing subsidies improve the dynamic trajectory of the new-energy transformation level index, and cooperative decision-making further strengthens this effect. This result shows that the transformation level is not determined by enterprise effort alone, but by the coordinated efforts of ICE vehicle manufacturers, NEV enterprises, and local governments.

**Proposition 6.** Under Assumption 4 and the same initial transformation level, the Stackelberg game improves the individual utilities of all three stakeholders compared with the Nash non-cooperative mechanism, and the cooperative mechanism yields the highest collective utility. Specifically, $U_1^S(x) > U_1^N(x),\quad U_2^S(x) > U_2^N(x),\quad U_3^S(x) > U_3^N(x)$, $U^C(x) > U^S(x) > U^N(x)$

**Proof**: According to the value functions derived under the Nash non-cooperative and Stackelberg games, the utility difference for ICE vehicle manufacturers is:

$$\begin{aligned} U_1^S(x) - U_1^N(x) &= \frac{\omega_1 \beta}{\rho+\alpha}\left[x^S(t) - x^N(t)\right] + \frac{\omega_1(2-3\omega_1-2\omega_2)[\theta_1(\rho+\alpha)+\lambda_1\beta]^2}{4\rho l_1(\rho+\alpha)^2} \\ &+ \frac{\omega_1(2-2\omega_1-3\omega_2)[\theta_2(\rho+\alpha)+\lambda_2\beta]^2}{2\rho l_2(\rho+\alpha)^2} > 0 \end{aligned} \tag{79}$$

For NEV enterprises, the utility difference is:

$$\begin{aligned} U_2^S(x) - U_2^N(x) &= \frac{\omega_2 \beta}{\rho+\alpha}\left[x^S(t) - x^N(t)\right] + \frac{\omega_2(2-2\omega_1-3\omega_2)[\theta_2(\rho+\alpha)+\lambda_2\beta]^2}{4\rho l_2(\rho+\alpha)^2} \\ &+ \frac{\omega_2(2-3\omega_1-2\omega_2)[\theta_1(\rho+\alpha)+\lambda_1\beta]^2}{2\rho l_1(\rho+\alpha)^2} > 0 \end{aligned} \tag{80}$$

For local governments, the utility difference is:

$$\begin{aligned} U_3^S(x) - U_3^N(x) &= \frac{(1-\omega_1-\omega_2)\beta}{\rho+\alpha}\left[x^S(t) - x^N(t)\right] + \frac{(2-3\omega_1-2\omega_2)^2[\theta_1(\rho+\alpha)+\lambda_1\beta]^2}{8\rho l_1(\rho+\alpha)^2} \\ &+ \frac{(2-2\omega_1-3\omega_2)^2[\theta_2(\rho+\alpha)+\lambda_2\beta]^2}{8\rho l_2(\rho+\alpha)^2} > 0 \end{aligned} \tag{81}$$

Therefore, each stakeholder obtains a higher utility under the Stackelberg game than under the Nash non-cooperative game: $U_1^S(x) > U_1^N(x),\quad U_2^S(x) > U_2^N(x),\quad and\ U_3^S(x) > U_3^N(x)$. Adding the three individual inequalities gives $U^S(x) = U_1^S(x) + U_2^S(x) + U_3^S(x) > U_1^N(x) + U_2^N(x) + U_3^N(x) = U^N(x)$ Furthermore, in the cooperative game, the three stakeholders act as a unified decision-making system and maximize aggregate ecosystem utility. The cooperative decision problem includes the feasible effort profiles generated under the Stackelberg game. Therefore, the aggregate cooperative utility is no lower than the aggregate Stackelberg utility. Under the interior optimal solution, the cooperative effort levels are strictly higher than those under the Stackelberg game, which implies $U^C(x) > U^S(x)$. Combining $U^S(x) > U^N(x)$ and $U^C(x) > U^S(x)$, we obtain $U^C(x) > U^S(x) > U^N(x)$. This proves Proposition 6.

**Corollary 4.** The individual and aggregate payoffs of ICE vehicle manufacturers, NEV enterprises, and local governments are higher in the Stackelberg game than in the Nash non-cooperative game. This result indicates that government cost-sharing subsidies can generate Pareto improvements by reducing enterprise effort costs and increasing stakeholder utilities.

**Corollary 5.** The cooperative game generates the highest aggregate utility among the three scenarios. By coordinating the decisions of ICE vehicle manufacturers, NEV enterprises, and local governments, the cooperative structure internalizes stakeholder interactions and maximizes the overall payoff of the tripartite system.

## 4. Dynamic simulation and case study

### 4.1. *Dynamic simulation*

The numerical simulation is designed to illustrate the theoretical propositions and compare the dynamic trajectories under different game scenarios, rather than to provide direct econometric estimation. Following related studies on low-carbon supply-chain differential games, green innovation games, and government-subsidy decision models [32−36], the baseline parameters are selected as normalized simulation values.

**Table 1.** Baseline parameter settings for numerical simulation.

| Parameter | Value | Meaning |
|---|---|---|
| $\rho$ | 0.8 | Discount rate |
| $\alpha$ | 0.2 | Natural depreciation rate of the accumulated transformation level |
| $\beta$ | 2 | Effect intensity of the transformation level index on cumulative ecosystem revenue |
| $\lambda_1,\lambda_2,\lambda_3$ | 2, 2, 2 | Marginal contribution coefficients of stakeholder efforts to the transformation stock |
| $l_1,l_2,l_3$ | 2, 2, 2 | Effort cost coefficients of ICE vehicle manufacturers, NEV enterprises, and local governments |

**Table 2.** Revenue and initial-state parameters.

| Parameter | Value | Meaning |
|---|---|---|
| $\theta_1,\theta_2,\theta_3$ | 3, 3, 3 | Marginal revenue coefficients of ICE vehicle manufacturers, NEV enterprises, and local governments |
| $\omega_1,\omega_2$ | 0.3,0.3 | Revenue-sharing coefficient of ICE vehicle manufacturers and NEV enterprises |
| $1-\omega_1-\omega_2$ | 0.4 | Revenue-sharing coefficient of local governments |
| $x_0$ | 5 | Initial value of the new-energy transformation level index |

The parameter setting follows four principles. First, all parameters satisfy the theoretical constraints of the model, including the non-negativity of equilibrium efforts, the admissibility of subsidy coefficients, and the revenue-sharing restrictions in Assumption 4. Second, the relative magnitudes of the parameters are consistent with numerical simulation practices in related differential-game studies, where normalized values are commonly used to verify theoretical propositions and conduct sensitivity analysis. Third, observable automobile-industry information, such as enterprise R&D investment and new-energy transformation practices, is used as qualitative case evidence to support the relative ordering of stakeholder efforts and transformation influence. Fourth, because several parameters, including effort influence coefficients, cost coefficients, and transformation depreciation rates, are latent theoretical parameters and cannot be directly observed, normalized baseline values are adopted. Therefore, the parameter values should be interpreted as normalized values for verifying theoretical trends rather than as direct empirical estimates. The baseline parameter values used in the numerical simulation are reported in Table 1, and the revenue-sharing and initial-state parameters are reported in Table 2.

The subsidy coefficients $\sigma_1$, $\sigma_2$ are not preset simulation parameters. They are endogenously derived in the Stackelberg game as the optimal cost-sharing subsidy coefficients for ICE vehicle manufacturers and NEV enterprises, respectively. The selected values satisfy the parameter restrictions in Assumption 4: $0<\omega_1<1,\quad 0<\omega_2<1,\quad \omega_1+\omega_2<1,\quad 3\omega_1+2\omega_2<2,\quad 2\omega_1+3\omega_2<2$ .

**Table 3.** Comparison of optimal effort intensities across game scenarios.

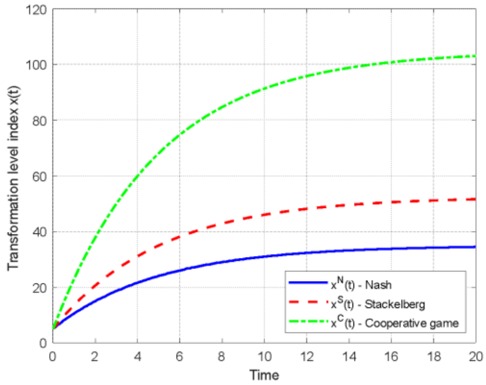
| Game mode | $E_1$ | $E_2$ | $E_3$ |
|---|---|---|---|
| Nash non-cooperative game | 1.05 | 1.05 | 1.40 |
| Stackelberg game | 1.93 | 1.93 | 1.40 |
| Cooperative game | 3.50 | 3.50 | 3.50 |
| Comparison | $E_1^C > E_1^S > E_1^N$ | $E_2^C > E_2^S > E_2^N$ | $E_3^C > E_3^S = E_3^N$ |

Table 3 compares the optimal effort intensities under the three game scenarios. In the Nash non-cooperative game, the effort levels of the three stakeholders are the lowest. In the Stackelberg game, government cost-sharing subsidies increase the effort levels of ICE vehicle manufacturers and NEV enterprises, while the effort level of local governments remains unchanged. In the cooperative game, all three stakeholders reach their highest effort levels. These results are consistent with Proposition 4.

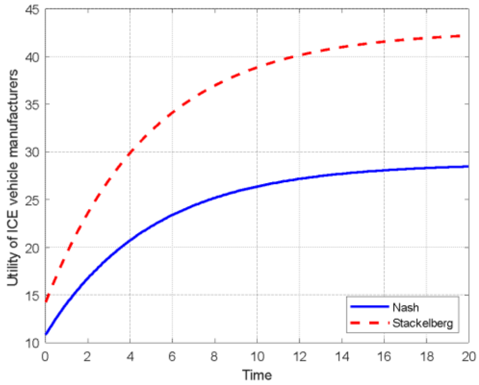
Under the Nash non-cooperative game, the trajectory of the new-energy transformation level index is $x^N(t)=35-30e^{-0.2t}$, and the total ecosystem utility is $U^N(t)=95.42-60e^{-0.2t}$. Under the Stackelberg game, the corresponding trajectory is $x^S(t)=52.50-47.50e^{-0.2t}$, and the total ecosystem utility is $U^S(t)=139.22-95e^{-0.2t}$ Under the cooperative game, the trajectory is $x^C(t)=105-100e^{-0.2t}$, and the aggregate ecosystem utility is $U^C(t)=255.94-200e^{-0.2t}$. These results are illustrated in Figure 3.

Figure 3 summarizes the dynamic simulation results under the three game scenarios. Figure 3(a) shows that the new-energy transformation level index increases over time and converges to a steady state in all scenarios. The cooperative game generates the highest trajectory, followed by the Stackelberg game and the Nash non-cooperative game, which is consistent with Proposition 5. The Stackelberg game also improves the transformation trajectory relative to the Nash benchmark, indicating that government cost-sharing subsidies stimulate enterprise-side transformation efforts.

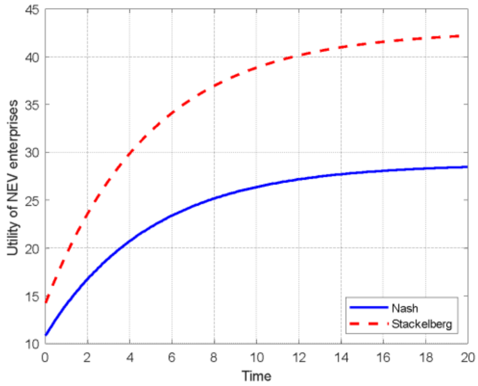
Figures 3(b)-3(d) compare stakeholder utilities under the Nash and Stackelberg scenarios. The utilities of ICE vehicle manufacturers, NEV enterprises, and local governments are higher under the Stackelberg game than under the Nash non-cooperative game. Figure 3(e) further shows that total ecosystem utility is highest under the cooperative game. These results are consistent with Proposition 6 and show that government-led subsidies and cooperative decision-making improve stakeholder payoffs relative to decentralized decision-making.



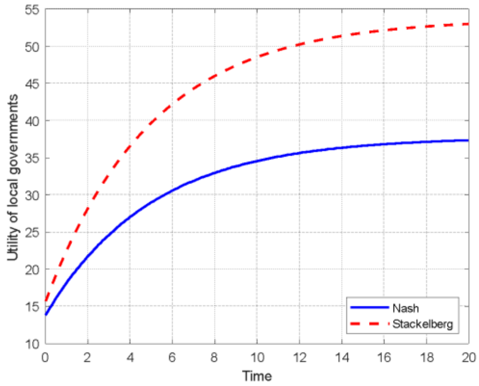

(a) Evolutionary trajectories of the new-energy transformation level index $x(t)$.



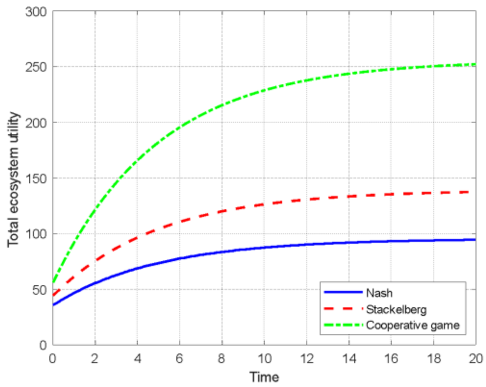

(b) Utilities of ICE vehicle manufacturers under the Nash and Stackelberg scenarios.




(c) Utilities of NEV enterprises under the Nash and Stackelberg scenarios.




(d) Utilities of local governments under the Nash and Stackelberg scenarios.

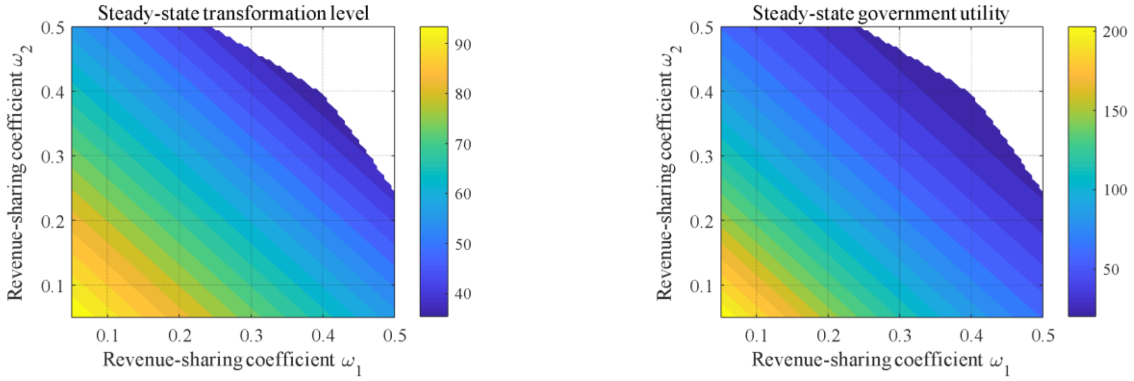



(e) Total ecosystem utilities under the Nash, Stackelberg, and cooperative scenarios.

**Figure 3.** Dynamic simulation results under different game scenarios.

## *4.2. Sensitivity analysis*

A sensitivity analysis is conducted for the revenue-sharing coefficients $\omega_1$ and $\omega_2$ within the feasible parameter region specified in Assumption 4. For each feasible pair of $\omega_1$ and $\omega_2$, the endogenous subsidy coefficients $\sigma_1$ and $\sigma_2$ are recalculated according to the Stackelberg equilibrium conditions. The results are shown in Figure 4.




(a) Steady-state transformation level (b) Steady-state government utility

**Figure 4.** Steady-state sensitivity to revenue-sharing coefficients.

Figure 4 presents the sensitivity analysis of the steady-state outcomes with respect to the revenue-sharing coefficients $\omega_1$ and $\omega_2$. The blank region represents infeasible parameter combinations outside the feasible region specified in Assumption 4. For each feasible pair of $\omega_1$ and $\omega_2$, the endogenous subsidy coefficients $\sigma_1$ and $\sigma_2$ are recalculated according to the Stackelberg equilibrium conditions.

As shown in Figure 4(a), the steady-state transformation level is higher when both revenue-sharing coefficients are relatively low. As $\omega_1$ and $\omega_2$ increase, the steady-state transformation level decreases. This pattern indicates that a larger enterprise revenue share reduces the residual revenue share of local governments, thereby weakening the government's incentive to provide cost-sharing subsidies. As a result, enterprise-side transformation efforts decline and the accumulated new-energy transformation level decreases.

Figure 4(b) shows a similar pattern for government utility. Government utility is higher in the region where $\omega_1$ and $\omega_2$ are relatively low, and it decreases as the enterprise revenue-sharing coefficients increase. This result suggests that effective subsidy policy depends not only on the subsidy rates themselves, but also on the underlying revenue-sharing structure. In low-NEV-penetration markets, a balanced revenue-sharing mechanism helps preserve government policy incentives, sustain cost-sharing subsidies, and promote long-term new-energy transformation.

Overall, these results show that the revenue-sharing structure is an important determinant of the endogenous subsidy mechanism. A balanced allocation of ecosystem revenue helps maintain government subsidy incentives and improves the long-term transformation performance of the tripartite system.

### *4.3. Case study*

To reflect the low-NEV-penetration context emphasized in this study, the case analysis uses China's automobile market after 2019 as the empirical background. In 2019, NEV sales accounted for only 4.68% of total automobile sales in China, indicating that the market was dominated by conventional vehicles. Under this condition, the key transition problem was not only the expansion of NEV production, but also the transformation of incumbent ICE-oriented manufacturers under policy incentives, technological investment, and changing market demand.

Changan Automobile is selected as a representative incumbent manufacturer because it has a long-established conventional vehicle and engine production base while actively promoting new-energy and intelligent transformation. This dual feature is consistent with the proposed model, in which ICE vehicle manufacturers face transition pressure, NEV enterprises provide new-energy development momentum, and local governments guide the transition through subsidy and regulatory incentives. The Changan case is therefore used not as a direct econometric test of the normalized model parameters, but as case-based evidence illustrating how the theoretical mechanisms operate in a low-penetration transition context. The correspondence between the theoretical model and the Changan case is summarized in Table 4.

**Table 4.** Mapping between the theoretical model and the Changan case.

| Model element | Case-study interpretation | Case indicator |
|---|---|---|
| Local governments | Policy actors supporting industrial transition | Subsidies, dual-credit regulation, low-carbon transport policy |
| Effort level $E_i$ | Enterprise-side transformation input | R&D investment, R&D personnel |
| Transformation level index $x(t)$ | Accumulated new-energy transformation level | Contextual NEV sales trend, NEV sales share, product expansion |

Changan's transformation illustrates how an incumbent automaker responds to policy incentives and market transition pressure. The company was originally dominated by conventional vehicle manufacturing, but it gradually shifted toward electrified and intelligent mobility. Its early exploration of new-energy vehicles included hybrid prototypes in 2001 and electric vehicle models in 2009. In 2017, the company launched the "Shangri-La" new-energy strategy, followed by the "Beidou Tianshu" intelligent-vehicle strategy and the "Haina Baichuan" global expansion strategy. These initiatives show that Changan's transformation was a continuous adjustment process involving product electrification, intelligent-vehicle technology, organizational investment, and external policy support.

To link the case evidence with the theoretical model, we focus on two groups of observable indicators. Revenue and sales growth describe the market-side outcome of Changan's transformation, while R&D investment and R&D personnel serve as proxies for enterprise-side transformation effort. These indicators do not directly estimate the latent model parameters. Instead, they illustrate how sustained enterprise effort and government incentives jointly support new-energy transformation.

(1) Revenue and sales volume growth. Changan's revenue and sales data show the market-side outcome of its transformation. As shown in Table 5, the company's business revenue increased from 70.6 billion yuan in 2019 to 159.7 billion yuan in 2024, representing a total increase of approximately 126.2%. Over the same period, automobile sales increased from 1.76 million units to 2.68 million units, with a total increase of approximately 52.3% and an average annual growth rate of about 8.8%. These results show that Changan maintained market expansion during its new-energy transition.

(2) R&D investment and transformation effort. R&D investment and R&D personnel are used as observable indicators of Changan's enterprise-side transformation effort. As shown in Table 6, Changan's total R&D investment increased from 4.478 billion yuan in 2019 to 10.1 billion yuan in 2024, while the number of R&D personnel remained at a relatively high level during the same period. The ratio of R&D investment to net assets increased from 6.34% in 2019 to 6.36% in 2024, despite a

temporary decline between 2020 and 2022. These data show that Changan continued to allocate substantial resources to technological development during its new-energy transition. In relation to the theoretical model, these indicators correspond to the enterprise effort variables, but they are not used to directly estimate the normalized model parameters. Instead, they support the case-based interpretation that sustained technological input is necessary for incumbent ICE-oriented manufacturers to move toward new-energy transformation.

**Table 5.** Revenue and sales volume of Changan Automobile from 2019 to 2024.

| Years | 2019 | 2020 | 2021 | 2022 | 2023 | 2024 |
|---|---|---|---|---|---|---|
| Business revenue (100 million yuan) | 706 | 846 | 1051 | 1213 | 1513 | 1597 |
| Increase rate of business revenue (%) | 6.5% | 19.8% | 24.3% | 15.3% | 24.8% | 5.6% |
| Automobile sales volume (10,000 units) | 176 | 200 | 230 | 235 | 255 | 268 |

**Table 6.** R&D investment of Changan Automobile from 2019 to 2024.

| Years | 2019 | 2020 | 2021 | 2022 | 2023 | 2024 |
|---|---|---|---|---|---|---|
| Number of R&D personnel | 7829 | 6636 | 7269 | 7899 | 10972 | 10159 |
| Total R&D investment (100 million yuan) | 44.78 | 38.77 | 48.27 | 56.78 | 90.1 | 101 |
| Ratio of R&D investment to net assets (%) | 6.34 | 4.58 | 4.59 | 4.68 | 5.95 | 6.36 |

Overall, the Changan case is consistent with the theoretical mechanism proposed in this study. In a low-NEV-penetration transition context, incumbent automakers face transformation pressure and cost constraints. Government subsidies and regulatory incentives can reduce part of the transition burden, while sustained R&D investment supports the shift from conventional vehicle production toward new-energy and intelligent mobility. Although the case evidence does not estimate the normalized model parameters, it illustrates how coordinated policy incentives and enterprise-side technological effort jointly promote the accumulated transformation process described by the model.

## 5. Conclusions and policy implications

### 5.1. *Major findings*

In this study, we develop a tripartite differential game framework to examine how ICE vehicle manufacturers, NEV enterprises, and local governments interact during the new-energy transformation of the automobile industry. By solving the HJB equations, we compare equilibrium effort levels, subsidy strategies, stakeholder utilities, and the trajectory of the new-energy transformation level index x(t) under Nash non-cooperative, government-led Stackelberg, and cooperative game structures.

The results lead to three major findings. First, the government-led Stackelberg game improves the effort levels of ICE vehicle manufacturers and NEV enterprises compared with the Nash non-cooperative benchmark. This result is consistent with the effort-level comparison in Proposition 4 and shows that cost-sharing subsidies can reduce enterprise transformation burdens and induce higher new-energy investment.

Second, the cooperative game generates the highest effort levels, the strongest improvement in the NEV transformation level index, and the highest aggregate ecosystem utility. This finding is consistent with Propositions 4-6 and indicates that decentralized decision-making cannot fully internalize the benefits of coordinated transformation among the three stakeholders.

Third, the sensitivity analysis shows that the revenue-sharing structure affects the endogenous subsidy mechanism and steady-state outcomes. When the enterprise revenue-sharing coefficients are relatively low, local governments retain a higher residual revenue share, which supports stronger cost-sharing incentives and leads to higher steady-state transformation level and government utility. This result indicates that the effectiveness of subsidy-based transformation policies depends not only on the subsidy rates, but also on the underlying allocation of ecosystem revenue.

### *5.2. Policy implications*

The model results provide policy implications that are directly linked to the equilibrium comparisons and sensitivity analysis.

First, local governments should design cost-sharing subsidies as targeted transformation incentives rather than as general financial support. The comparison between the Nash non-cooperative and government-led Stackelberg games shows that subsidies linked to enterprise transformation costs can increase the effort levels of ICE vehicle manufacturers and NEV enterprises. Therefore, subsidy policies should prioritize R&D investment, production-system upgrading, battery and powertrain technology transformation, and new-energy infrastructure deployment. Such subsidies can reduce the marginal burden of enterprise effort and guide firms toward higher levels of new-energy transformation.

Second, policy makers should coordinate subsidy design with revenue-sharing arrangements. The sensitivity analysis shows that the revenue-sharing coefficients affect steady-state transformation outcomes through the endogenous subsidy mechanism. If enterprise revenue shares are excessively high, the residual revenue share of local governments declines, weakening their ability to sustain cost-sharing subsidies. Therefore, in low-NEV-penetration markets, subsidy policy should not be designed independently of revenue allocation. A balanced revenue-sharing structure can preserve government policy incentives, reduce the transformation burden of incumbent ICE vehicle manufacturers, and support the expansion of NEV enterprises. This also implies that firms should not focus only on maximizing short-term revenue shares; maintaining a cooperative allocation structure may improve long-term transformation performance for the whole system.

Third, policy makers should promote cooperative mechanisms that move the three stakeholders closer to the cooperative-game outcome. The cooperative game produces the highest aggregate ecosystem utility and the strongest improvement in $x(t)$, indicating that subsidies alone are insufficient if stakeholders remain in a purely decentralized decision-making structure. Local governments should therefore encourage joint R&D platforms, shared battery and powertrain development systems, coordinated supply-chain upgrading, and collaborative infrastructure construction. These mechanisms can transform subsidies from isolated cost compensation into coordination instruments that align enterprise efforts, reduce duplicated investment, and accelerate the automobile industry's new-energy transformation.

### 5.3. Limitations and future research

This study has several limitations that provide directions for future research. First, for analytical tractability, key structural parameters, such as cost coefficients, effort-influence coefficients, revenue coefficients, and transformation depreciation rates, are assumed to be time-invariant. In practice, technological learning, economies of scale, market competition, and policy adjustment may cause these parameters to change over time. In future studies, researchers can extend the model by introducing time-varying or stochastic parameters to capture more complex transformation dynamics.

Second, the new-energy transformation level index $x(t)$ is modeled as a latent stock variable that represents accumulated transformation capacity. This specification is suitable for the differential game framework used in this study, but it does not explicitly capture heterogeneous consumer behavior, demand elasticity, charging-infrastructure accessibility, or regional market differences. Researchers can also incorporate consumer-side and region-specific variables into the state equation to better explain the interaction between industrial transformation and market adoption.

Third, the numerical simulation and case discussion are mostly based on the context of China's automobile industry and the Changan Automobile case. Although this context is appropriate for examining low-NEV-penetration transition stages, the generalizability of the findings to other regions and market structures requires further examination. In future studies, researchers can use multi-case comparisons, regional panel data, or firm-level longitudinal data to test how institutional conditions, market maturity, supply-chain structure, and carbon-emission constraints affect the equilibrium outcomes of the tripartite game.

## Author contributions

Xiaojuan Li: Conceptualization, methodology, formal analysis, writing−original draft; JinZhou He: Investigation, resources, data curation; Zhenfei Zhan: Methodology, validation, supervision, writing−review & editing; Shanshan Li: Investigation, data curation, writing−original draft; Yechao Cheng: Formal analysis, visualization; Zhengzhi Ma: Formal analysis, writing−review & editing; Zufeng Zhang: Writing−review & editing, project administration. All authors have read and approved the final version of the manuscript.

## Use of Generative-AI tools declaration

The authors declare that no artificial intelligence (AI) tools were used in the creation of this manuscript.

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## Conflict of interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

## References


1. L. Pei, R. Zhai, M. Shou, J. Luo, Modeling the diffusion of new energy vehicles in China considering subsidy policies, *J. Syst. Sci. Inf.*, **12** (2024), 457–475. https://doi.org/10.21078/JSSI-2023-0170
2. S. Jia, Y. Gao, Y. Guo, H. Ma, Y. Li, H. Yu, Energy-saving and $CO_2$ reduction strategies for new energy vehicles based on the integration approach of voluntary advocacy and system dynamics, *Environ. Sci. Pollut. Res.*, **31** (2024), 14804–14819. https://doi.org/10.1007/s11356-024-32172-w
3. A. D. Chandler, T. Hikino, *Scale and Scope: The Dynamics of Industrial Capitalism*, Harvard University Press, Cambridge, MA, 1990. https://doi.org/10.4159/9780674029385
4. L. Minford, D. Meenagh, Testing a model of UK growth: A role for R&D subsidies, *Econ. Model.*, **82** (2019), 152–167. https://doi.org/10.1016/j.econmod.2019.01.002
5. Y. Cheng, Z. Yao, X. Wang, Differential game analysis for cooperation models in automotive supply chain under low-carbon emission reduction policies, In: *Frontiers of Algorithmics*, IJTCS-FAW 2023, Macau, China, 2023, 147–159. https://doi.org/10.1007/978-3-031-39344-0_11
6. W. Zhang, S. Zhao, X. Wan, Industrial digital transformation strategies based on differential games, *Appl. Math. Model*., **98** (2021), 90–108. https://doi.org/10.1016/j.apm.2021.05.001
7. F. Liu, L. Wang, S. Xie, Effects of subsidy cancellations on investment strategies of local governments and new energy vehicle manufacturers: A study based on differential game, *Sustainability*, **14** (2022), 12324. https://doi.org/10.3390/su141912324
8. S. Yin, B. Li, A stochastic differential game of low carbon technology sharing in collaborative innovation system of superior enterprises and inferior enterprises under uncertain environment, *Open Math*., **16** (2018), 607–622. https://doi.org/10.1515/math-2018-0056
9. D. Liu, T. Chen, X. Liu, Y. Yu, Do more subsidies promote greater innovation? Evidence from the Chinese electronic manufacturing industry, *Econ. Model*., **80** (2019), 441–452. https://doi.org/10.1016/j.econmod.2018.11.027
10. A. Garcia Calvo, Production networks and innovation in the semi-periphery: The transition to electric vehicles in South Korea and Spain, *Stud. Comp. Int. Dev.*, **59** (2024), 690–711. https://doi.org/10.1007/s12116-024-09436-0
11. OECD, The future of the automotive value chain: Implications for FDI-SME linkages, *OECD SME and Entrepreneurship Papers*, No. 64, OECD Publishing, Paris, 2024. https://doi.org/10.1787/cb730d65-en
12. Y. Xu, New explorations in autonomous low-altitude area defense decision-making and control methods of UAV swarms, *Guid. Navig. Control*, **5** (2025), 1–5. https://doi.org/10.1142/S2737480725030013
13. Z. Xiang, M. Xu, Dynamic cooperation strategies of the closed-loop supply chain involving the internet service platform, *J. Clean. Prod.*, **220** (2019), 1180–1193. https://doi.org/10.1016/j.jclepro.2019.01.310
14. J. T. Hong, P. Huang, Dynamic decision-making model of collaborative knowledge innovation in the enterprise value chain, *Chin. J. Manag. Sci.*, **19** (2011), 130–136. https://doi.org/10.16381/j.cnki.issn1003-207x.2011.04.007
15. H. X. Li, H. Si, Control for intelligent manufacturing: A multiscale challenge, *Engineering*, **3** (2017), 608–615. https://doi.org/10.1016/j.eng.2017.05.016
16. International Energy Agency, *Global EV Outlook 2025: Expanding sales in diverse markets.* Paris: IEA, 2025. Available from: https://www.iea.org/reports/global-ev-outlook-2025.

17. Z. Tao, B. Wang, L. Shu, Analysis on the procurement cost of construction supply chain based on evolutionary game theory, *Arab. J. Sci. Eng.*, **46** (2021), 1–16. https://doi.org/10.1007/s13369-020-05261-4
18. N. Firdaus, A. Mori, M. T. Sambodo, Dynamic capabilities and strategic responses to transition risks in Indonesian coal companies, *Discov. Sustain.*, **7** (2026), 151. https://doi.org/10.1007/s43621-025-02303-5
19. T. Başar, J. Moon, Riccati equations in Nash and Stackelberg differential and dynamic games, *IFAC-PapersOnLine*, **50** (2017), 9547–9554. https://doi.org/10.1016/j.ifacol.2017.08.1625
20. F. W. Geels, Technological transitions as evolutionary reconfiguration processes: A multi-level perspective and a case-study, *Res. Policy*, **31** (2002), 1257–1274. https://doi.org/10.1016/S0048-7333(02)00062-8
21. F. Kley, C. Lerch, D. Dallinger, New business models for electric cars: A holistic approach, *Energy Policy*, **39** (2011), 3392–3403. https://doi.org/10.1016/j.enpol.2011.01.036
22. H. Xu, L. Chen, J. Yang, L. Ma, Research on the cost-sharing mechanism of supply-chain product innovation and government subsidy mechanism based on dynamic differential game, *Operations Research and Management Science*, **33** (2024), 42–48.
23. M. Górajski, D. Machowska, Optimal double control problem for a PDE model of goodwill dynamics, *Math. Methods Oper. Res.*, **85** (2017), 425–452. https://doi.org/10.1007/s00186-017-0577-1
24. F. W. Geels, R. Kemp, Dynamics in socio-technical systems: Typology of change processes and contrasting case studies, *Technol. Soc.*, **29** (2007), 441–455. https://doi.org/10.1016/j.techsoc.2007.08.009
25. E. L. Plambeck, Reducing greenhouse gas emissions through operations and supply chain management, *Energy Econ.*, **34** (2012), S64–S74. https://doi.org/10.1016/j.eneco.2012.08.031
26. T. Başar, G. J. Olsder, *Dynamic Noncooperative Game Theory*, 2nd ed., SIAM, Philadelphia, 1999. https://doi.org/10.1137/1.9781611971132
27. Q. Zuo, Z. Zhang, Q. Wu, Y. Ji, J. Ma, Revealing environmental inequality based on three-dimensional extended input-output analysis: An integrated perspective of embodied water consumption, fossil energy use and carbon emissions, *Energy*, **330** (2025), 136898. https://doi.org/10.1016/j.energy.2025.136898
28. P. Dai, R. Zhang, L. Wang, Unveiling multi-agent dynamics in digital transformation: A niche-based evolutionary game analysis of high-end manufacturing, *Int. J. Ind. Eng. Comput.*, **17** (2026), 211–232. https://doi.org/10.5267/j.ijiec.2025.10.002
29. X. Cao, Y. Li, H. Wen, Closed-loop supply chain decision-making considering patent licensing and government subsidies, *J. Syst. Eng.*, **40** (2025), 119–130. https://doi.org/10.13383/j.cnki.jse.2025.01.009
30. R. Liao, L. Wang, F. Deng, A tripartite differential game approach to understanding intelligent transformation in the wastewater treatment industry, *Systems*, **13** (2025), 960. https://doi.org/10.3390/systems13110960
31. A. Sedghamiz, M. R. Nikoo, M. Heidarpour, M. Sadegh, Developing a non-cooperative optimization model for water and crop area allocation based on leader-follower game, *J. Hydrol.*, **567** (2018), 51–59. https://doi.org/10.1016/j.jhydrol.2018.10.034
32. M. Breton, A. Alj, A. Haurie, Sequential Stackelberg equilibria in two-person games, *J. Optim. Theory Appl.*, **59** (1988), 71–97. https://doi.org/10.1007/BF00939867
33. J. Wang, Y. Song, M. Li, C. Yuan, F. Guo, Study on low-carbon technology innovation strategies through government-university-enterprise cooperation under carbon trading policy, *Sustainability*, **14** (2022), 9381. https://doi.org/10.3390/su14159381

34. Y. Zu, L. Chen, Y. Fan, Research on low-carbon strategies in supply chain with environmental regulations based on differential game, *J. Clean. Prod.*, **177** (2018), 527–546. https://doi.org/10.1016/j.jclepro.2017.12.220
35. W. Wang, S. Hao, W. He, M. A. Mohamed, Carbon emission reduction decisions in construction supply chain based on differential game with government subsidies, *Build. Environ.*, **222** (2022), 109149. https://doi.org/10.1016/j.buildenv.2022.109149
36. A. K. M. Mohsin, S. F. A. Hossain, H. Tushar, M. M. Iqbal, A. Hossain, Differential game model and coordination model for green supply chain based on green technology research and development, *Heliyon*, **7** (2021), e07811. https://doi.org/10.1016/j.heliyon.2021.e07811