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Research article

Service rivalry reshapes strategic interplay: Manufacturer encroachment and extended warranty provision under retailer-owned extended warranty

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Abstract: This paper studies a service rivalry setting in which the retailer is not a passive product reseller but an incumbent extended warranty (EW) provider. In this environment, a manufacturer must decide whether to open a direct product channel and whether to introduce its own EW, while its service offer competes with the retailer's pre-existing EW. We formulate a manufacturer-led Stackelberg model with four strategy regimes to examine whether manufacturer encroachment and manufacturer-provided EWs function as complementary or substitutive strategies, and how competition from a retailer-owned EW alters this relationship. The analysis shows that the retailer's EW provision fundamentally changes the logic of the manufacturer's two strategies. Encroachment does not necessarily promote EW provision; by raising direct channel operating pressure and intensifying product and service competition, it can lower the manufacturer's willingness to provide EWs. The reverse effect is also conditional: The manufacturer's EW provision encourages encroachment only when the EW selling cost is sufficiently low, but it discourages encroachment once the service cost erodes the channel's benefit. We further identify a cost threshold that separates complementarity from substitutability between encroachment and EW provision, and show how this threshold is reshaped by retailer-owned EWs, the manufacturer's service differentiation, and direct selling cost. The paper therefore extends the encroachment and EW literature from a manufacturer-centered service setting to a competitive service channel setting and provides pricing implications for both the manufacturer's and the retailer's EWs.

Keywords: extended warranty; manufacturer encroachment; retailer-owned service; service competition; dual channel
Mathematics Subject Classification: 90B06, 91A80, 91B24

1. Introduction

Extended warranties (EWs) have become an important source of service revenue in many durable goods markets. They reduce consumers' exposure to repair and replacement losses after the base warranty expires and are especially valuable to risk-averse consumers. For firms, however, EWs are no longer only an after-sales support arrangement. They increasingly operate as a competitive service product that affects demand, differentiation, customer retention, and profit allocation within the supply chain [1, 2].

In many durable goods markets, retailers are not merely product resellers; they also operate their own EW businesses to earn service margins from risk-averse consumers and strengthen customer relationships [3], such as Suning, JD.com, Best Buy, and Amazon. This changes the retailer's role in the channel from a pure product reseller to both a product seller and a service provider. A manufacturer-provided EW is therefore not only an additional service offering but also a direct competitor to the retailer-owned EW. At the same time, manufacturers such as Siemens, Panasonic [4], Haier, and Huawei increasingly sell their own EWs through direct online channels. Huawei's direct online EW sales, for example, illustrate that a manufacturer may bypass authorized retailers in service provision even when product distribution still involves retail channels. Consequently, the manufacturer's EW provision is no longer merely an after-sales service decision; it becomes a strategic decision about whether to compete with an incumbent retailer in the service market.

The manufacturer's channel decision creates a second layer of strategic tension. The digital sales infrastructure allows manufacturers to sell products directly to consumers, obtain demand information, and reduce reliance on downstream intermediaries. Firms such as Sony, Nike, and Adidas have used direct channels for these purposes. Yet product encroachment also brings selling costs, operational responsibilities, and channel conflict. These costs are particularly important when service competition is present, because a direct product channel may later serve as a platform for the manufacturer to also sell its own EW, but it can also intensify competition with the retailer in both the product and service dimensions.

In this complex competitive landscape, manufacturers face a dual strategic challenge: Whether to encroach into the direct channel and whether to offer their own EWs. These two decisions are unlikely to be independent. On the one hand, manufacturer-provided EWs may enhance products' attractiveness for risk-averse consumers and increase product demand. Direct channel encroachment may then become more valuable because it allows the manufacturer to appropriate a larger share of the demand expanded by EW provision. On the other hand, when the retailer already offers its own EW, manufacturer-provided EW introduces service competition in a market where the retailer is an incumbent service provider. This service competition may intensify channel conflict, put pressure on EW margins, and reduce the profitability of simultaneously adopting both strategies. Consequently, the relationship between encroachment and the manufacturer's EW provision is theoretically ambiguous and depends on the balance between demand expansion and service competition effects.

Real practices illustrate this heterogeneity: Electrolux distributes products through retailers while offering EWs on its official website, whereas Huawei combines dual product channels with manufacturer-controlled online EW sales. These observations raise an important question: Under retailer-owned EW competition, do channel encroachment and manufacturer-provided EWs reinforce or substitute for each other? Addressing this question requires a joint analysis of product channel and service channel strategies.

Existing research provides important insights into channel encroachment [5,6] and the EW strategy [7,8], but it leaves open a distinct question: How does a retailer's own EW reshape the manufacturer's joint encroachment and EW strategy? Prior work on encroachment mainly treats downstream competition as product competition, while much EW research takes the product channel structure as fixed. As a result, the literature offers limited guidance on whether a manufacturer's direct channel makes its EW more attractive, whether a manufacturer-provided EW makes encroachment more profitable, and how these relationships change when the retailer already competes in the EW market.

To address this gap, we develop a manufacturer-led Stackelberg model in which the retailer always offers an EW and the manufacturer chooses whether to encroach and whether to offer its own EW. We study four strategy regimes and ask four questions. First, does encroachment increase or decrease the manufacturer's incentive to provide an EW when the retailer has its own EW? Second, does the manufacturer-provided EW strengthen or weaken the incentive to encroach? Third, how do the manufacturer and retailer set EW prices under different channel and service configurations? Fourth, under what conditions are encroachment and EW provision complementary rather than substitutable?

We find that encroachment does not necessarily support manufacturer-provided EWs; when direct selling cost and service rivalry are sufficiently strong, it lowers the acceptable EW selling cost threshold. Manufacturer-provided EWs, in turn, encourage encroachment only when the EW selling cost is low enough for the manufacturer to benefit from the demand expansion effect of service provision. The two strategies are complementary below a critical EW selling cost threshold and substitutable above it. Unlike Qi et al. [9], who assumed that the manufacturer is the sole EW provider, we show that retailer-owned EW competition reshapes the strategic relationship between encroachment and manufacturer-provided EWs by introducing service rivalry and narrowing the manufacturer's dual-strategy region under certain cost conditions.

This study makes three contributions to the literature. First, it extends the manufacturer encroachment literature by incorporating retailer-owned EWs as an incumbent competing service. Unlike prior studies that mainly focus on product channel competition, our model shows how demand for the retailer's EW, the manufacturer's service differentiation, and the direct selling cost jointly reshape the value of encroachment. Second, it contributes to the EW literature by examining the manufacturer's provision of EWs as a decision made jointly with channel encroachment rather than under a fixed product channel structure. We show that encroachment can reduce, rather than enhance, the manufacturer's willingness to launch EWs when the direct selling cost and service rivalry are sufficiently strong. Third, it advances the most recent encroachment and EW literature by comparing the benchmark without retailer-provided EW competition with the retailer-owned EW setting. This comparison reveals that manufacturer-provided EWs encourage encroachment only when the EW selling cost is sufficiently low, and that retailer-owned EW competition attenuates the manufacturer's incentive to adopt both strategies while changing both firms' EW pricing responses.

The remainder of this paper is organized as follows. Section 2 reviews the related literature and clarifies the positioning of this study. Section 3 presents the model's assumptions and notations. Section 4 derives the equilibrium outcomes for the four strategy regimes. Section 5 analyzes the two-way interaction between encroachment and EW provision and examines EW pricing. Section 6 concludes with managerial implications and future research directions.

2. Related literature

This study is related to two streams of supply chain research: Manufacturer encroachment and the EW strategy. Its specific position lies at their intersection, but with an important additional feature: The retailer is an active EW provider. This feature makes the service market competitive and distinguishes the present model from studies in which the manufacturer introduces EWs without facing a retailer-owned service alternative.

The issue of manufacturer encroachment has been extensively studied. Prior research has primarily investigated whether and under what conditions manufacturers should establish direct channels, as well as the effects of manufacturer encroachment on supply chain members. Some studies have found that both the manufacturer and retailer may benefit from manufacturer encroachment [5, 6, 10–12]. For instance, Arya et al. [5], Chiang et al. [6], and Tsay and Agrawal [10] demonstrated that the introduction of a direct channel can mitigate double marginalization by prompting the manufacturer to lower wholesale prices, ultimately improving profitability for both parties. Chiang et al. [6] were among the first to investigate manufacturer encroachment and found that even if the manufacturer's direct channel does not actually sell products, its mere presence can exert a channel control effect, prompting the retailer to reduce retail prices and increase sales, thereby boosting profits in the retail channel. Cattani et al. [11] compared pricing rules for a manufacturer that adds a direct outlet alongside the incumbent retail channel and showed that equal pricing can align the interests of the manufacturer, retailer, and consumers. Arya et al. [5] demonstrated that when the manufacturer is at a competitive disadvantage relative to the retailer, encroachment can benefit both parties. Yang et al. [13] demonstrated that although retailers often suffer profit losses due to manufacturer encroachment, manufacturers with relatively weak bargaining power can benefit from it. Yan et al. [14] considered four different channel structures and explored the influence of logistics service levels on the manufacturer's channel selection. Wan et al. [15] proposed that retail price leadership significantly influences the manufacturer's encroachment decisions.

Furthermore, studies have incorporated quality factors and information asymmetry as pivotal elements in a manufacturer's encroachment decision-making. Zhang et al. [16] investigated the impact of encroachment and the information structure on the quality decisions and profits of supply chain members, revealing that the manufacturer always benefits from encroachment, while the retailer's benefits depend on the manufacturer's direct selling cost. Ha et al. [17] showed that quality differentiation can alleviate the channel conflict caused by encroachment when the manufacturer's selling cost is low. Li et al. [18] analyzed manufacturer encroachment in the presence of asymmetric information and found that it may lead to lose-win or lose-lose outcomes between the manufacturer and retailer. Ha et al. [19] investigated encroachment and information sharing decisions and proved that the encroachment decision and information sharing are complementary. Wang et al. [20] studied manufacturer encroachment with endogenous quality investment decisions in a two-period supply chain and found that the manufacturer always chooses to encroach and it does not always undermine the retailer's profits.

Additionally, several studies have considered encroachment in competitive settings. Liu et al. [21] analyzed a manufacturer's incentive to encroach when selling through multiple retailers, showing that the number of downstream competitors significantly affects encroachment motives and members' performance. Maruthasalam and Balasubramanian [22] explored how asymmetric cost information among competing retailers influences manufacturer encroachment, noting that retail competition may spur supplier encroachment and boost direct sales, with high-efficiency retailers always benefiting.

Huang and Chen [23] investigated the strategic interaction between a manufacturer and multiple competing retail platforms.

Overall, the encroachment literature has generated rich explanations for direct channel entry, but the competitive object is usually the product. Less attention has been paid to settings in which the retailer's service business changes the manufacturer's payoff from direct selling. When a retailer earns margin from EW sales, encroachment may affect not only quantity and wholesale pricing of the product but also the size and profitability of the retailer's service market. This service channel effect is central to our model.

The second related stream of literature concerns the EW strategy, which has attracted considerable scholarly attention. One branch focuses on EW pricing. For example, Bouguerra et al. [24] studied differentiated periodic maintenance strategies and optimal EW pricing from the perspectives of both the manufacturer and the consumer. Tong et al. [25] investigated two-dimensional EW design and pricing, including point-of-sale and end-of-warranty purchase options. Wang et al. [26] proposed a dynamic integration model to address market fluctuations and EW pricing in a changing environment. Zheng et al. [2] examined a manufacturer's optimal EW strategy and pricing decisions between the traditional fixed price and a flexible dynamic price of the EW, concluding that dynamic pricing is superior under certain conditions. Existing research on EW design primarily focuses on how providers determine the EWs' coverage [27–30]. For example, Wang et al. [30] used a multinomial logit model to capture consumers' choice behavior and analyzed an EW provider's portfolio decisions regarding EWs' duration and pricing. Their results show that, under the optimal strategy, all offered duration-price bundles yield equal marginal profitability.

In contrast, the EW distribution channel has received relatively limited attention in the literature, with most studies focusing on channel selection and optimization. Desai and Padmanabhan [8] first investigated three distribution channels, indirect (through retailers), direct (via the manufacturer), and dual (both), and examined their impact on EW sales. They found that double marginalization and complementary product effects influence the manufacturer's channel selection, and that manufacturers benefit most from indirect channels when the EW selling costs are low and consumer appeal is high. Li et al. [31] investigated distribution channel structures for EWs, examining not only base models in which manufacturer or retailer offer their own EWs, but also scenarios involving resale through third parties.

Substantial research has investigated who should provide the EW: The manufacturer, retailer, or a third party. Jiang and Zhang [32] showed that when the customers can evaluate the product's quality, the manufacturer's base warranty will be negatively affected by the retailer's service plan. He et al. [7] examined three scenarios with no EW, manufacturer-provided EW, and retailer-provided EW, concluding that the EW generally improves supply chain profits and that consumers' channel preference influences the optimal provider. Zhang et al. [33] studied the manufacturer's outsourcing strategies for EWs in a dual-channel supply chain, comparing retailer-undertaken, third-party-undertaken, and hybrid provision models, and showed that retailer-undertaken EW creates a win-win outcome and improves total supply chain profit when the base market is sufficiently large. Recent work extends this to platform supply chains. Li et al. [34] analyzed the conditions under which the manufacturer and platform offer EWs, showing that both provide EWs when the scale of potential market is large, neither does when it is small, and only one offers EWs in a market of moderate size. Liu et al. [35] studied the optimal pricing and strategy selection for EWs on a retail platform in the context of two competing manufacturers, and

they found that the retail platform's EW provision harms the profit of the EW-bundled manufacturer but benefits the standalone product manufacturer. Zhang et al. [36] analyzed optimal EW strategies in an online platform for both platforms and manufacturers within agency and resale sales modes.

The aforementioned studies often assumed one manufacturer and one retailer, although some work has examined EW competition arising from multiple firms or multiple EW types. For instance, Liu et al. [4] showed that manufacturers can benefit from retailer-offered EWs under moderate trade-in costs by examining EW competition between channels in a closed-loop supply chain with new and remanufactured products.

Nevertheless, few studies integrate encroachment and service decisions, and even fewer simultaneously model manufacturer encroachment and EW strategies in a competitive service environment. Chai et al. [37] investigated whether the retailer should offer its own EW or resell the manufacturer's EW when confronting manufacturer encroachment, finding that as long as the retailer provides the EW, both the manufacturer and retailer will benefit from it. Zhang et al. [38] examined how a dominant retailer counters manufacturer encroachment through investments in retail service and show that such investment can result in Pareto improvements for both supply chain members and consumers.

The literature therefore leaves three issues unresolved. First, it is unclear whether encroachment remains an effective support for manufacturer-provided EWs when the retailer already sells its own EW. Second, the reverse effect from manufacturer-provided EWs to encroachment has not been characterized in a setting where the EW demand is split between two providers. Third, few studies explicitly compare the manufacturer's joint strategy with and without the retailer's service competition, even though this comparison is needed to assess the incremental contribution of retailer-owned EW. By incorporating the retailer's EW as a standing market feature, this paper identifies how incumbent retailer service changes the manufacturer's channel and service incentives, the pricing of competing EWs, and the complementarity or substitutability between the two manufacturer strategies.

3. The model

We consider a two-echelon supply chain with one manufacturer and one retailer. The retailer purchases a repairable product from the manufacturer, resells it to consumers, and, importantly, owns an EW offer in the retail channel. This assumption allows us to focus on how an existing retailer-owned EW business affects the manufacturer's channel and EW decisions. Accordingly, the retailer-provided EW is treated as an exogenous market feature rather than a strategic service entry decision. The product fails with probability $\alpha \in (0, 1)$ and works with probability $1-\alpha$. Following Desai and Padmanabhan [8] and Mai et al. [39], the manufacturer's unit production cost is specified as $(1-\alpha)c$, where c denotes the production cost associated with a perfectly reliable product (i.e., a no-failure state during the EW period). This linear specification reflects the widely adopted assumption that higher product reliability requires greater production and quality control investment, thereby increasing the manufacturing costs. Against the retailer's service position, the manufacturer makes two decisions: Whether to sell the product directly to consumers and whether to launch a manufacturer-provided EW. Direct product selling incurs a unit cost S, whereas the retailer's selling cost is normalized to zero to capture its incumbent advantage in retail execution and customer access [5, 40].

The retailer's EW reduces the consumer's loss from product failure. If the manufacturer introduces its own EW, consumers choose between two service providers rather than between having the service and

no service only. The responsibility for repair is borne by the firm whose EW is purchased. Consistent with the EW literature, the base warranty period is normalized to zero and EW durations are treated as exogenous, allowing us to isolate the strategic effects of EW provision and competition from the contract design problem of warranty length determination. Specifically, the EW durations offered by the retailer and manufacturer are fixed at β_r and β_m, respectively [39]. Furthermore, we assume that the manufacturer's EW is more protective than the retailer's EW, i.e., $\beta_m < \beta_r$. This assumption reflects the manufacturers' advantages in product-specific repair expertise, direct access to the original components, and greater authority in product diagnosis and replacement. It also introduces vertical differentiation between the two EW services, thereby avoiding homogeneous service offerings and enabling a meaningful analysis of consumers' warranty choices and the resulting competitive interactions between EW providers. We assume at most one failure during the EW period. The manufacturer incurs a unit selling cost s for direct EW sales, with $s < S$, reflecting that delivering a service contract is less costly than fulfilling a physical product order. The four regimes in Figure 1 differ only in the manufacturer's product channel and EW decisions; in all regimes, the retailer remains an EW provider.

- Model NNS. The manufacturer relies on the retail product channel and leaves EW provision to the retailer.
- Model NMS. The manufacturer relies on the retail product channel but competes in the EW market through direct EW sales.
- Model ENS. The manufacturer opens a direct product channel while the retailer remains the sole EW provider.
- Model EMS. The manufacturer operates both a direct product channel and a direct EW channel, competing with the retailer in product-related services.

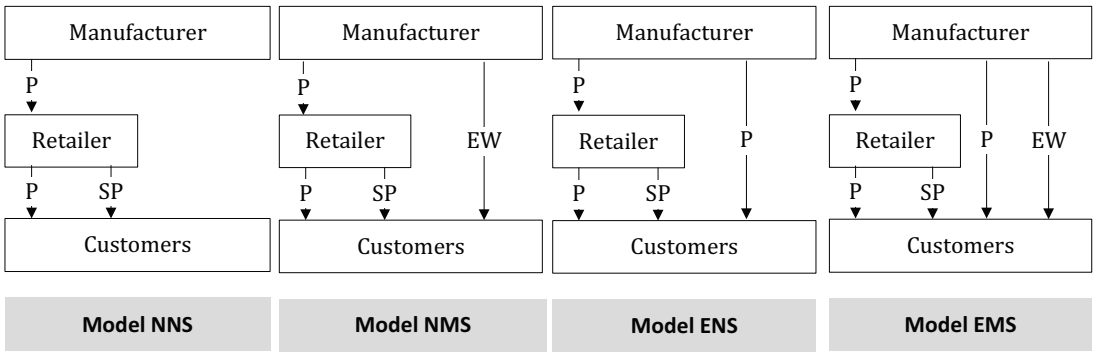


Figure 1. The manufacturer's distribution strategies, where P, EW, and SP represents the product and the manufacturer's and retailer's EWs, respectively.

Following He et al. [7], market size is normalized to 1. Each consumer buys at most one product and at most one EW. A consumer first decides whether to buy the product; conditional on a product purchase, the consumer then chooses no EW, the retailer's EW, or, when available, the manufacturer's EW. The

product yields value v in the no-failure state and imposes a loss r in the failure state if no EW is held. Under provider i's EW, the compensation is $r(1-\beta_i)$, where $0 \le \beta_i \le 1$ is exogenous [8]. Thus, a consumer with the retailer's EW bears the residual loss $r\beta_r$ after failure, whereas a consumer with the manufacturer's EW bears the residual loss $r\beta_m$. Positive product demand requires $(v-c)(1-\alpha) - r\alpha - s - S > 0$. We assume that the customers are heterogeneous in their risk aversion, and the customer utility function exhibits constant absolute risk aversion, denoted by $\theta \in (0,1)$ [8, 32]. For notational simplicity and to facilitate the subsequent demand and equilibrium comparisons, we introduce the auxiliary terms A, B_m, and B_r. Customers with sufficiently high risk aversion may opt out of purchasing the product. Accordingly, the customer's utility of buying the product without any EW can be expressed as

$$u_p = v(1-\alpha) - r\alpha - p_p - A\theta\alpha(1-\alpha), \tag{1}$$

where $A = (v+r)^2$ and p_p is the product price. The first three terms capture the expected net value for a risk-neutral consumer [41]; the final term captures risk disutility, with $A\alpha(1-\alpha)$ measuring the variance of the consumer's product payoff [39]. If the consumer buys the product together with the retailer's EW, the utility becomes

$$u_{ps} = v(1-\alpha) - r\alpha\beta_r - p_p - p_s - \theta B_r\alpha(1-\alpha), \tag{2}$$

where $B_r = (v+\beta_r r)^2$ and p_s is the retailer's EW price. The utility of a customer purchasing both the product and the manufacturer's EW can be formulated as

$$u_{pe} = v(1-\alpha) - r\alpha\beta_m - p_p - p_e - \theta B_m\alpha(1-\alpha), \tag{3}$$

where $B_m = (v+\beta_m r)^2$ and p_e is the manufacturer's EW price. Because manufacturers usually possess product-specific repair expertise and brand authority, their EWs can be perceived as more protective than retailer-provided EWs [42]. Together with the earlier assumption $\beta_m < \beta_r < 1$, these definitions imply $B_m < B_r < A$.

The consumer choice cutoffs are obtained by pairwise indifference. Setting $u_p = u_{ps}$ gives $\theta_1 = \frac{p_s - r\alpha(1-\beta_r)}{(A-B_r)\alpha(1-\alpha)}$, below which consumers buy only the product. Setting $u_{pe} = u_{ps}$ gives $\theta_2 = \frac{p_e - p_s - r\alpha(\beta_r-\beta_m)}{(B_r-B_m)\alpha(1-\alpha)}$, so consumers between θ_1 and θ_2 choose the retailer's EW. Finally, $u_{pe} = 0$ yields $\theta_3 = \frac{v(1-\alpha) - r\alpha\beta_m - p_p - p_e}{B_m\alpha(1-\alpha)}$; consumers between θ_2 and θ_3 choose the manufacturer's EW, while those above θ_3 do not buy. The consumers' choice structure is valid when the cutoffs satisfy $0 \le \theta_1 \le \theta_2 \le \theta_3 \le 1$, which ensures non-negative demands for the product, the retailer's EW, and the manufacturer's EW. In the subsequent equilibrium analysis, we focus on the parameter region in which these feasibility conditions hold. Hence, the manufacturer-provided EW demand is $q_e = \theta_3 - \theta_2$, the retailer-provided EW demand is $q_s = \theta_2 - \theta_1$, and total product demand is $q_p = \theta_3$. In Models ENS and EMS, $q_p = q_{pr} + q_{pm}$, where q_{pr} and q_{pm} denote the retail channel and direct channel product quantities, respectively. The resulting product demand and EW inverse demand functions are

$$p_p = v(1-\alpha) - p_e - \alpha(1-\alpha)B_m q_p - r\alpha\beta_m, \tag{4}$$

$$q_e = \frac{p_e - p_s - \alpha(1-\alpha)(B_r - B_m)q_p - r\alpha(\beta_r - \beta_m)}{(1-\alpha)\alpha(B_m - B_r)} \text{ and} \tag{5}$$

$$q_s = \frac{r\alpha(B_r - B_m) - (A - B_m)(p_s + r\alpha\beta_r) + (A - B_r)(p_e + r\alpha\beta_m)}{(1-\alpha)\alpha(A - B_r)(B_r - B_m)}. \tag{6}$$

4. Equilibrium results

This section characterizes the equilibrium outcomes under the four strategy regimes determined by the manufacturer's channel and EW choices. In each regime, the manufacturer acts as the Stackelberg leader and the retailer as the follower. By applying backward induction, we derive the equilibrium decisions and corresponding profit functions. To streamline the presentation, only the key equilibrium results are reported here, whereas detailed proofs and derivations are relegated to Appendix A.

4.1. Model NNS

In Model NNS, the manufacturer does not enter either the direct product market or the EW market. The following lemma summarizes the equilibrium outcomes.

Lemma 1. *In Model NNS, the manufacturer and retailer's optimal profits are* $\pi_m^{NNS*} = \frac{\Phi^2}{8(1-\alpha)\alpha B_r}$ *and* $\pi_r^{NNS*} = \frac{\Phi^2}{16(1-\alpha)\alpha B_r}$, *respectively, where* $\Phi = (v-c)(1-\alpha) - r\alpha$.

4.2. Model NMS

In Model NMS, the manufacturer still relies on the retailer for product sales but enters the EW market directly. The following lemma summarizes the equilibrium outcomes.

Lemma 2. *In Model NMS, the manufacturer and the retailer's optimal profits are* $\pi_m^{NMS*} = \frac{\Delta_m^{NMS}}{4(1-\alpha)\alpha(B_r-B_m)(A-2B_m+B_r)^2(5B_m+3B_r)}$ *and* $\pi_r^{NMS*} = \frac{\Delta_r^{NMS}}{8(1-\alpha)\alpha(B_r-B_m)(A-2B_m+B_r)^2(5B_m+3B_r)^2}$, *respectively.*

4.3. Model ENS

In Model ENS, the manufacturer opens a direct product channel while the retailer remains the only EW provider. The following lemma summarizes the equilibrium outcomes.

Lemma 3. *In Model ENS, the manufacturer and the retailer's optimal profits are*

$$\pi_m^{ENS*} = \begin{cases} \frac{\Delta_m^{ENS}}{12(1-\alpha)\alpha B_r(A+B_r)^2}, & \text{if} \quad S \le \frac{\Phi(A+5B_r)}{3A+7B_r} \\ \frac{(\Phi-S)(A+3B_r)(S(A+2B_r)-\Phi B_r)}{2(1-\alpha)\alpha B_r(A+B_r)^2}, & \text{if} \quad \frac{\Phi(A+5B_r)}{3A+7B_r} < S < \Phi \end{cases} \quad \text{and}$$

$$\pi_r^{ENS*} = \begin{cases} \frac{\Delta_r^{ENS}}{36(1-\alpha)\alpha B_r(A+B_r)^2}, & \text{if} \quad S \le \frac{\Phi(A+5B_r)}{3A+7B_r} \\ \frac{(\Phi-S)^2\left(A^2+3AB_r+4B_r^2\right)}{4(1-\alpha)\alpha B_r(A+B_r)^2}, & \text{if} \quad \frac{\Phi(A+5B_r)}{3A+7B_r} < S < \Phi \end{cases},$$

respectively.

4.4. Model EMS

In Model EMS, the manufacturer encroaches on the market to sell both the product and the EW directly to customers. Meanwhile, the retailer resells the product from the manufacturer and offers its own EW. The following lemma summarizes the equilibrium outcomes.

Lemma 4. *In Model EMS, the retailer and manufacturer's equilibrium profits are*

$$\pi_r^{EMS*} = \begin{cases} \dfrac{\Delta_{r,1}^{EMS}}{72(1-\alpha)\alpha B_r(A-2B_m+B_r)^2(B_r-B_m)}, \\ if \quad \frac{\Phi}{2} \le S < \frac{3B_r\Phi}{2B_m+3B_r} and s \le \frac{(A-2B_m+B_r)(6B_r\Phi-2(2B_m+3B_r)S)}{B_m(A-13B_r)+6B_r(A+B_r)} \\ \dfrac{\Delta_{r,2}^{EMS}}{8(1-\alpha)\alpha B_m^2(B_r-B_m)(A-2B_m+B_r)}, \\ if \quad \frac{\Phi}{2} \le S < \frac{3B_r\Phi}{2B_m+3B_r} and \Phi < s < (v-c)(1-\alpha) - r\alpha - S \end{cases} \quad and$$

$$\pi_m^{EMS*} = \begin{cases} \dfrac{\Delta_{m,1}^{EMS}}{48(1-\alpha)\alpha B_m B_r(A-2B_m+B_r)^2(B_r-B_m)}, \\ if \quad \frac{\Phi}{2} \le S < \frac{3B_r\Phi}{2B_m+3B_r} \text{ and } s \le \frac{(A-2B_m+B_r)(6B_r\Phi-2(2B_m+3B_r)S)}{B_m(A-13B_r)+6B_r(A+B_r)} \\ \dfrac{\Delta_{m,2}^{EMS}}{16(1-\alpha)\alpha B_m^2(A-2B_m+B_r)^2(B_r-B_m)}, \\ if \quad \frac{\Phi}{2} \le S < \frac{3B_r\Phi}{2B_m+3B_r} \text{ and } \frac{(A-2B_m+B_r)(6B_r\Phi-2(2B_m+3B_r)S)}{B_m(A-13B_r)+6B_r(A+B_r)} < s < \Phi - S \end{cases}$$

respectively.

5. Equilibrium analysis

This section analyzes the strategic interplay between the manufacturer's encroachment decision and its EW provision strategy under competition from a retailer-owned EW. We first examine the two directions of influence between the decisions. We then study the EW pricing set by the manufacturer and the retailer under different market configurations. By comparing profit differences across the four regimes, we identify the threshold conditions under which encroachment and EW provision function as complements or substitutes. Finally, we compare the results with the benchmark without retailer-provided EW competition to show how the retailer-owned service changes the manufacturer's strategic incentives.

5.1. The impact of encroachment on the manufacturer's EW provision decision

We begin by examining how the manufacturer's decision to open a direct channel for the product influences its incentive to offer an EW, under both encroachment and non-encroachment settings. By comparing the manufacturer's profits between Model NNS (no encroachment, without EW) and Model NMS (no encroachment, with EW), we characterize the manufacturer's optimal EW provision decision without manufacturer encroachment, as presented in the following proposition.

Proposition 1. *Assume the following:*

$$s_1 = \frac{\Phi(B_r-B_m)\begin{pmatrix} 4B_r\left(3A^2+16B_m^2+8AB_r+5B_r^2-2B_m(7A+9B_r)\right) \\ -2(A-2B_m+B_r)\sqrt{2B_r(5B_m+3B_r)(A^2-4B_m^2-2B_m(A-5B_r)-5B_r^2)} \end{pmatrix}}{2\left(2B_r\left(4B_m^3+2B_m^2(A+9B_r)+B_r\left(3A^2+12AB_r+13B_r^2\right)-B_m\left(A^2+18AB_r+33B_r^2\right)\right)\right)} \quad and$$

$$s_2 = \frac{\Phi(B_r-B_m)\begin{pmatrix} 4B_r\left(3A^2+16B_m^2+8AB_r+5B_r^2-2B_m(7A+9B_r)\right) \\ +2(A-2B_m+B_r)\sqrt{2B_r(5B_m+3B_r)(A^2-4B_m^2-2B_m(A-5B_r)-5B_r^2)} \end{pmatrix}}{2\left(2B_r\left(4B_m^3+2B_m^2(A+9B_r)+B_r\left(3A^2+12AB_r+13B_r^2\right)-B_m\left(A^2+18AB_r+33B_r^2\right)\right)\right)}, \text{ and suppose}$$

$\frac{\sqrt{5}A}{5} < B_r$. *Without encroachment, it is optimal for the manufacturer to offer the EW when one of the following conditions holds:*

- $B_m \leqslant \frac{(5B_r - A) - \sqrt{5}(A - B_r)}{4}$ *and* $s < \Phi$,
- $\frac{(5B_r - A) - \sqrt{5}(A - B_r)}{4} < B_m < B_r$ *and* $s \leqslant s_1$, *or*
- $\frac{(5B_r - A) - \sqrt{5}(A - B_r)}{4} < B_m < B_r$ *and* $s_2 \leqslant s < \Phi$.

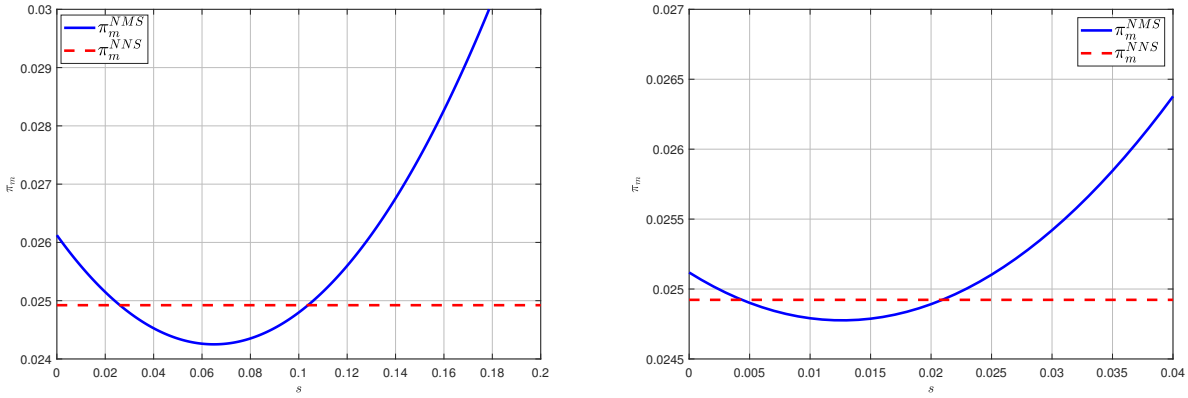
Otherwise, the manufacturer abstains from offering the EW.

Proposition 1 shows that without product encroachment, the manufacturer's incentive to provide an EW depends mainly on two factors: The EW selling cost s and the manufacturer's service protection level, represented by B_m. Since $B_m = (v + \beta_m r)^2$, a lower B_m corresponds to a lower consumer co-payment rate β_m and thus a more protective manufacturer-provided EW. When B_m is sufficiently small, the manufacturer's EW can be interpreted as a premium service guarantee: Consumers bear a smaller residual loss after product failure, while the manufacturer assumes greater service responsibility. This stronger protection clearly differentiates the manufacturer EW from the retailer's EW and makes it more attractive to risk-averse consumers. In this case, the manufacturer can profitably enter the EW market as long as the EW selling cost remains within the feasible range. Therefore, the manufacturer-provided EW is optimal for $s < \Phi$.

When B_m is relatively high, the manufacturer's EW becomes less differentiated from the retailer's EW, so the manufacturer can no longer rely solely on the service quality advantage. The underlying mechanism is the interaction between the manufacturer's wholesale price adjustment and the service margin effect. After introducing its own EW, the manufacturer lowers the wholesale price relative to the NNS benchmark. This wholesale price reduction stimulates the retailer's product order quantity and enlarges the potential customer base for EW sales, because EW demand is derived from product demand. At the same time, the manufacturer must choose an EW price that balances service demand against the unit margin from EW sales.

This trade-off explains why the profitable EW provision region is not monotonic in s. When s is low, the manufacturer can price its EW more flexibly and use the service offer to attract risk-averse consumers, so EW provision is profitable. When s is high but still within the feasible range, the manufacturer may still provide EW if the higher EW price generates enough unit service margin to compensate for the reduced EW demand. However, for intermediate values of s, neither the demand expansion effect nor the unit margin effect is strong enough to dominate the cost burden and service competition. In this region, the manufacturer abstains from offering EWs.

For the case $B_r \leq \frac{\sqrt{5}A}{5}$, the analytical comparison between π_m^{NMS*} and π_m^{NNS*} does not yield a clear conclusion. Therefore, to examine the manufacturer's EW provision decision in this region, we conduct a numerical analysis by setting $v = 3$, $r = 2$, $c = 0.5$, $\alpha = 0.4$, and $\beta_r = 0.1$. This region corresponds to the case in which the retailer-owned EW provides relatively strong protection, because a lower β_r implies a lower consumer co-payment rate and hence makes the retailer's EW more attractive. Figure 2 reports the results for $\beta_m = 0.04$ and $\beta_m = 0.09$, respectively. The numerical results are consistent with the findings of Proposition 1, the equilibrium EW provision decision exhibits the same non-monotonic pattern, when s is sufficiently low or sufficiently high, the manufacturer chooses to provide EWs, whereas it tends not to provide EWs when s takes intermediate values. Specifically, the EW selling cost s remains the key determinant of the manufacturer's EW provision decision.



(a) $\beta_m = 0.04$ **(b)** $\beta_m = 0.09$

Figure 2. Comparison of the manufacturer's profits with and without EW provision under no encroachment.

Through comparative analysis of Models ENS and EMS, we investigate the manufacturer's EW provision decision under encroachment. Our core findings are outlined in the ensuing proposition. Given the parametric complexity, detailed mathematical expressions are relegated to the appendix.

Proposition 2. *Suppose* $\frac{A}{3} \leqslant B_r < A$ *and* $B_m \leqslant \frac{3B_r(A+B_r)\left((A+5B_r)-2\sqrt{B_r(A+3B_r)}\right)}{2\left(A^2+6AB_r+13B_r^2\right)}$, *and the manufacturer encroaches on the market. The manufacturer optimally provides EWs if and only if*

- $\frac{\Phi}{2} \leqslant S \leqslant \frac{\Phi(A+5B_r)}{3A+7B_r}$ *and* $s \leqslant s_3$, *or*
- $\frac{\Phi(A+5B_r)}{3A+7B_r} < S < \frac{\Phi\left((A+3B_r)^2+(A+B_r)\sqrt{B_r(A+3B_r)}\right)}{2(A+2B_r)(A+3B_r)}$ *and* $s \leqslant s_4$, *where* $s_3 < s_4$.

Proposition 2 shows that once the manufacturer encroaches into the product market, its EW provision decision is jointly shaped by the direct product selling cost S and the EW selling cost s. Different from the no encroachment case, the manufacturer now obtains revenue from the direct product channel but also bears the direct channel operating cost and faces stronger product and service competition. Therefore, the manufacturer-provided EW is profitable only when the service margin and the demand enhancing effect of EWs can compensate for the additional cost and competitive pressure created by encroachment.

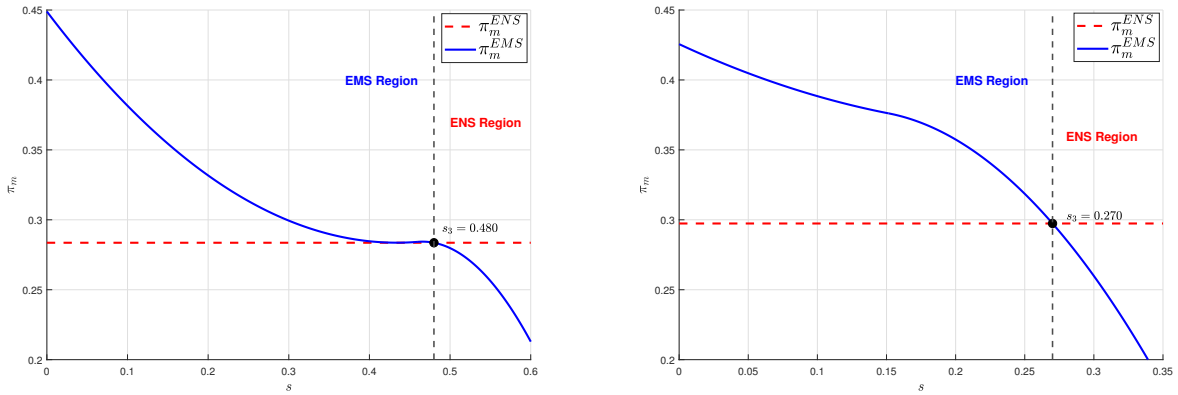
When S is relatively low, the direct product channel is already profitable. In this case, the manufacturer does not need to rely heavily on EW provision to support the value of encroachment; the EW mainly functions as an additional service product for risk-averse consumers. Because the retailer is also an incumbent EW provider, however, the manufacturer's EW entry intensifies service rivalry and may trigger service price responses. The manufacturer therefore offers an EW only when the EW selling cost is sufficiently low, $s \leq s_3$. A low s allows the manufacturer to price its EW competitively while still preserving the service margin.

When S is higher but still within the feasible encroachment region, the direct channel profit becomes more constrained. Under this condition, EW provision becomes more strategically valuable because it can help the manufacturer recover part of the profit pressure from costly encroachment. By attracting risk-averse consumers, the manufacturer-provided EW strengthens consumers' willingness to purchase

the product and creates an additional service revenue source. These demand expansion and service margin effects make the manufacturer willing to tolerate a higher EW selling cost threshold, s_4, where $s_3 < s_4$. Nevertheless, EW provision remains profitable only when $s \leq s_4$; once s exceeds this threshold, the service margin is eroded and the manufacturer abstains from offering EWs.

Thus, the mechanism behind Proposition 2 is not that a higher direct selling cost is inherently beneficial. Rather, when encroachment becomes more costly, the manufacturer has a stronger need to use EW provision as a complementary service tool, but only if the EW selling cost remains low enough to preserve profitability. This explains why the acceptable EW selling cost threshold varies with S.

To further illustrate Proposition 2, we compare the manufacturer's profits under Models EMS and ENS through numerical examples. We set $v = 2.05$, $r = 2$, $c = 0.2$, $\alpha = 0.1$, $\beta_r = 0.75$, and $\beta_m = 0.06$. Figure 3 reports two cases with different direct selling costs. In Panel (a), $S = 0.8$ falls into the first interval of Proposition 2, and Model EMS yields a higher manufacturer profit than Model ENS when s is below the threshold; once s exceeds the threshold, Model ENS becomes more profitable. In Panel (b), $S = 1.05$ falls into the second interval of Proposition 2, and a similar threshold structure is observed. These numerical results support Proposition 2 by showing that, under manufacturer encroachment, the manufacturer provides EWs only when the EW selling cost is low; otherwise, it abstains from providing EWs.



(a) $S = 0.8$ **(b)** $S = 1.05$

Figure 3. Comparison of the manufacturer's profits with and without EW provision under encroachment.

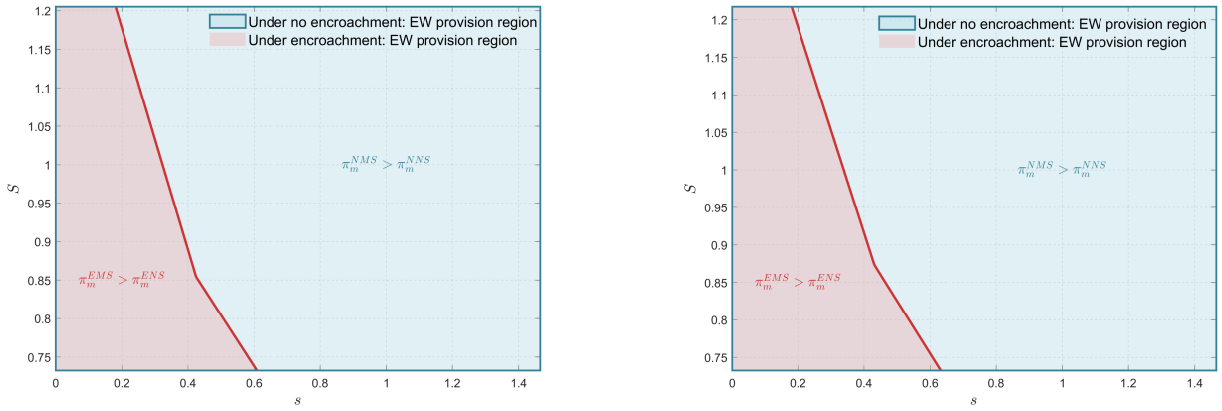
Propositions 1 and 2 allow us to examine how manufacturer encroachment influences the manufacturer's incentive to provide EWs. Because the EW offering thresholds are algebraically complex, we compare the two decisions under parameter conditions that preserve tractability. The comparison is based on the EW provision region, namely the range of the EW selling cost s under which the manufacturer-provided EW is profitable. If encroachment expands this region, it strengthens the manufacturer's incentive to offer an EW; if it narrows this region, it weakens the incentive.

Remark 1. Suppose that $\frac{\sqrt{5}A}{5} < B_r < A$ and $B_m \leqslant \frac{(5B_r - A) - \sqrt{5}(A - B_r)}{4}$. When $\frac{\Phi}{2} \leqslant S \leqslant \frac{\Phi(A+5B_r)}{3A+7B_r}$ exists, the encroachment decision weakens the motivation for manufacturer to offer the EW.

Remark 1 shows that encroachment does not automatically facilitate manufacturer-provided EWs. This result follows from comparing the EW provision regions in Propositions 1 and 2. Under the

parameter conditions in Remark 1, Proposition 1 shows that without encroachment, the manufacturer-provided EW is profitable over the entire feasible range $s < \Phi$. In contrast, Proposition 2 shows that under encroachment, when $\frac{\Phi}{2} \leq S \leq \frac{\Phi(A+5B_r)}{3A+7B_r}$, EW provision is profitable only if $s \leq s_3$, where $s_3 < \Phi$. Hence, encroachment narrows the EW provision region from $s < \Phi$ to $s \leq s_3$, indicating that EW provision remains profitable under fewer cost conditions.

The result reflects that encroachment changes the role of manufacturer-provided EWs. Without encroachment, the manufacturer relies on the retailer for product sales, and EW provision mainly works as a service-based demand expansion tool. A more protective manufacturer EW attracts risk-averse consumers and enlarges the potential product and service demand base, while the manufacturer avoids the direct selling cost S. Once the manufacturer encroaches, however, EW provision must be evaluated together with the direct product channel. Although encroachment improves customer access, it also creates direct channel cost pressure, intensifies product channel competition, and, because the retailer already offers its own EW, strengthens service competition. These forces compress the incremental profit from EW provision and make the manufacturer more sensitive to the EW selling cost. Therefore, only a sufficiently low s can allow the demand expansion benefit of EWs to offset the additional cost burden and competitive pressure.



(a) $\beta_r = 0.80$ **(b)** $\beta_r = 0.95$

Figure 4. EW provision regions with and without manufacturer encroachment.

To provide a more intuitive illustration of Remark 1, Figure 4 plots the regions in which the manufacturer provides an EW in the s-S plane. The comparison is conducted within the finite region where the two scenarios are jointly comparable. The parameter values are set as $v = 2.05$, $r = 2$, $c = 0.2$, $\alpha = 0.1$, and $\beta_m = 0.01$. Panels (a) and (b) correspond to $\beta_r = 0.80$ and $\beta_r = 0.95$, respectively. In each panel, the blue region represents the EW provision region without manufacturer encroachment, where $\pi_m^{NMS*} > \pi_m^{NNS*}$, while the red region represents the EW provision region under manufacturer encroachment, where $\pi_m^{EMS*} > \pi_m^{ENS*}$. The labels and profit inequalities in the figure identify the corresponding equilibrium outcomes in each region. Comparing the two regions within each panel shows that the EW provision region under encroachment is smaller than that without encroachment. This graphical evidence is fully consistent with Remark 1 and supports the conclusion that manufacturer encroachment weakens the manufacturer's incentive to provide EWs.

5.2. The impact of EW provision on the manufacturer's encroachment decision

In this subsection, we reverse the perspective to investigate how the manufacturer's decision to offer an EW influences its incentive to encroach when facing a retailer that provides its EW through its distribution channel. We conduct a comprehensive analysis of the encroachment decision across two distinct scenarios: (1) When the manufacturer elects not to offer an EW, and (2) when it chooses to provide an EW. By comparing the manufacturer's profits between Model NNS (no encroachment, without an EW) and Model ENS (encroachment, without an EW), we characterize the manufacturer's optimal encroachment decision without offering an EW, as presented in the following proposition.

Proposition 3. *When the manufacturer's EW is not available, the manufacturer prefers to encroach on the market if any one of the following three conditions is satisfied:*

- $B_r < \frac{(4\sqrt{2}-3)A}{23}$ *and* $\frac{\Phi\left((A+3B_r)^2-(A+B_r)\sqrt{B_r(A+3B_r)}\right)}{2(A+2B_r)(A+3B_r)} \leqslant S \leqslant \frac{\Phi\left((A+3B_r)^2+(A+B_r)\sqrt{B_r(A+3B_r)}\right)}{2(A+2B_r)(A+3B_r)}$,
- $\frac{(4\sqrt{2}-3)A}{23} \leqslant B_r < \frac{\sqrt{5}A}{5}$ *and* $\frac{\Phi\left(8B_r(A+2B_r)+(A+B_r)\sqrt{6\left(A^2-5B_r^2\right)}\right)}{2\left(3A^2+12AB_r+13B_r^2\right)} \leqslant S < \frac{\Phi\left((A+3B_r)^2+(A+B_r)\sqrt{B_r(A+3B_r)}\right)}{2(A+2B_r)(A+3B_r)}$,
- $\frac{(2\sqrt{3}-1)A}{11} \leqslant B_r < \frac{\sqrt{5}A}{5}$ *and* $S \leqslant \frac{\Phi\left(8B_r(A+2B_r)-(A+B_r)\sqrt{6\left(A^2-5B_r^2\right)}\right)}{2\left(3A^2+12AB_r+13B_r^2\right)}$, *or*
- $\frac{\sqrt{5}A}{5} \leqslant B_r < A$ *and* $S \leqslant \frac{\Phi\left((A+3B_r)^2+(A+B_r)\sqrt{B_r(A+3B_r)}\right)}{2(A+2B_r)(A+3B_r)}$.

Otherwise, the manufacturer will not encroach on the market.

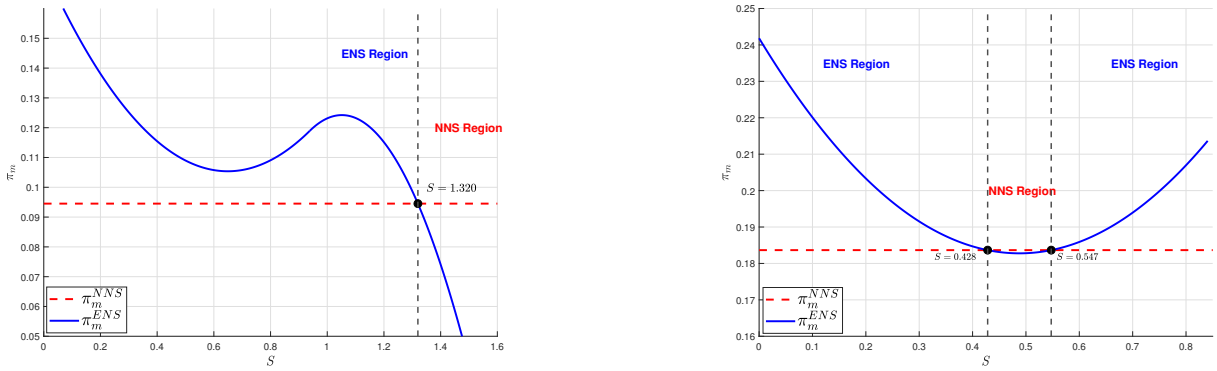
Proposition 3 characterizes the manufacturer's encroachment decision when the manufacturer does not provide its own EW. In this case, the incentive to encroach depends mainly on the direct selling cost S and the attractiveness of the retailer's EW, represented by B_r. Since $B_r = (v + \beta_r r)^2$, a lower B_r corresponds to a lower consumer co-payment rate β_r, meaning that the retailer's EW provides stronger protection and is more attractive to risk-averse consumers.

When B_r is relatively small, the retailer's EW can stimulate product demand by reducing consumers' expected loss from product failure. This service-induced demand expansion matters to the manufacturer even though the manufacturer does not provide an EW, because the manufacturer can capture part of the enlarged product demand after opening a direct channel. At the same time, encroachment changes the manufacturer's pricing incentives. The manufacturer may reduce the wholesale price to alleviate double marginalization and stimulate total product sales, while also earning the direct channel margin from consumers who purchase through the direct channel. Therefore, encroachment is attractive when the demand expansion benefit, wholesale price adjustment, and direct channel margin are sufficient to cover the direct selling cost.

The role of S reflects the trade-off between channel expansion and direct selling burden. When S is low or moderate, the direct channel can generate additional sales without imposing excessive cost pressure, so the manufacturer benefits from both product market expansion and direct access to consumers. As S increases, however, the direct channel becomes more costly, and encroachment remains profitable only if the manufacturer can compensate for the higher selling cost through improved margins or sufficient demand expansion. Otherwise, the manufacturer stays with the retail channel only.

When B_r becomes larger, the retailer's EW is less protective and less effective in stimulating product demand from risk-averse consumers. The manufacturer then relies less on the retailer's service-induced demand expansion and more on the standalone benefits of encroachment, such as expanded market coverage, improved product availability, and the direct channel margin. This explains why the acceptable range of S changes with B_r, that is, retailer-owned EW affects the manufacturer's encroachment incentive by changing the demand base that the manufacturer can capture through the direct channel.

Thus, Proposition 3 shows that retailer-owned EW affects encroachment even when the manufacturer does not provide an EW. The retailer's EW changes product demand through consumer risk protection, while the direct selling cost determines whether the manufacturer can profitably capture that demand through encroachment.



(a) $\beta_r = 0.80$ **(b)** $\beta_r = 0.15$

Figure 5. The manufacturer's encroachment decision without EW provision.

Figure 5 provides a graphical illustration of Proposition 3 by comparing the manufacturer's profits under Models NNS and ENS when the manufacturer does not provide an EW. The common parameters are $v = 3$, $r = 2$, $c = 0.5$, and $\alpha = 0.2$. Panel (a) considers $\beta_r = 0.80$, under which the retailer's EW provides relatively weak protection. In this case, the profit of Model ENS remains above the benchmark of NNS over a broad range of S and falls below Model NNS only when the direct selling cost becomes sufficiently high. Panel (b) considers $\beta_r = 0.15$, under which the retailer's EW provides stronger protection. In this case, the comparison between Models ENS and NNS becomes non-monotonic: Encroachment is profitable when S is low, becomes unprofitable over an intermediate range of S, and becomes profitable again over another feasible range.

When the manufacturer offers the EW, a comparison of its profits in Models NMS and EMS leads to the following findings.

Proposition 4. *Suppose that* $\frac{\sqrt{5}A}{5} < B_r < A$, $B_m \leqslant \frac{(5B_r - A) - \sqrt{5}(A - B_r)}{4}$ *and* $s < \frac{6\Phi B_r(A - 2B_m + B_r)}{B_m(5A - 21B_r) + 12B_r(A + B_r) - 8B_m^2}$. *If the manufacturer offers its own EW, only when*

$$S \leqslant \frac{\begin{pmatrix} (5B_m + 3B_r)\begin{pmatrix} 2\Phi(A - 2B_m + B_r)(B_m + 3B_r) + \\ s\left(4B_m^2 - 6B_r(A + B_r) + B_m(-3A + 11B_r)\right) \end{pmatrix} \\ +B_m\left(4\Phi(A - 2B_m + B_r) - s(3A - 8B_m + 5B_r)\right)\sqrt{2B_m(5B_m + 3B_r)} \end{pmatrix}}{6(A - 2B_m + B_r)(B_m + B_r)(5B_m + 3B_r)},$$

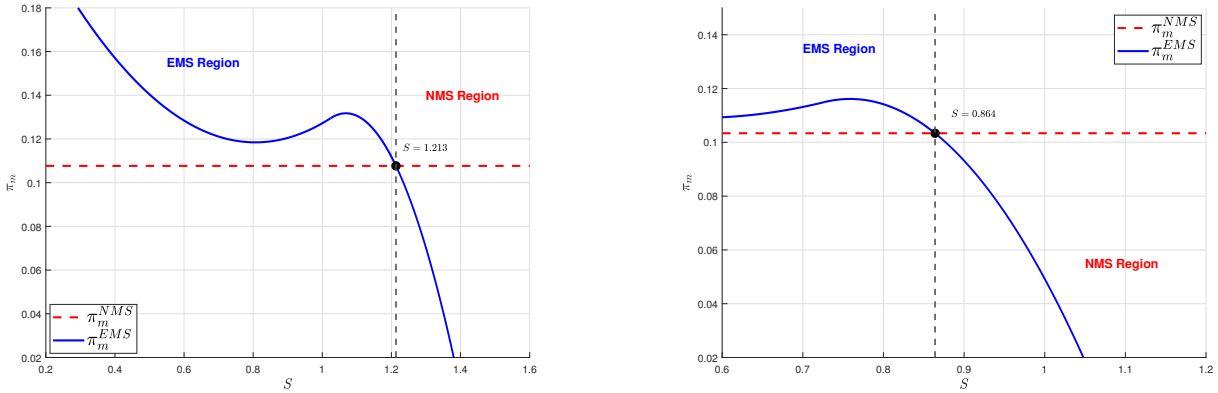
the manufacturer will choose to encroach on the market.

Proposition 4 characterizes the manufacturer's encroachment decision when the manufacturer already provides its own EW. In this case, encroachment is not only a product channel decision, because the manufacturer's EW demand is derived from product demand. Opening a direct product channel changes the product demand base and therefore affects the value of the manufacturer's EW business. Thus, encroachment is profitable only when the additional direct channel margin and the induced EW demand can compensate for the direct selling cost S.

When S is relatively low, encroachment is attractive because the manufacturer can obtain a direct channel product margin without bearing a large operating burden. At the same time, the manufacturer can adjust the wholesale price to mitigate double marginalization and maintain the retail channel product demand. The coexistence of retail and direct product channels expands the product demand base, which, in turn, enlarges the potential market for the manufacturer's EW. In this region, product sales and EW sales reinforce each other: Direct access to consumers increases the manufacturer's ability to capture product demand, while EW provision helps attract risk-averse consumers and strengthens the value of product sales.

As S increases, the manufacturer faces a sharper trade-off between channel expansion and cost burden. A higher direct selling cost reduces the net margin from the direct channel and weakens the benefit of expanding the product volume. Under moderate values of S, encroachment can still be profitable if the manufacturer can use EW provision to support product demand and use product sales to expand the EW customer base. EW pricing then serves as an adjustment tool; that is, the manufacturer may price the EW more competitively to maintain demand or preserve its service margin, depending on the cost condition.

However, when S becomes sufficiently high, the cost of operating the direct product channel dominates the benefit from product and EW complementarity. The additional direct channel margin and the EW demand generated by product sales are no longer sufficient to offset the cost burden. In this case, encroachment weakens rather than improves the manufacturer's overall profitability, and the manufacturer optimally stays with the no encroachment strategy.



(a) $s = 0.20$ **(b)** $s = 0.60$

Figure 6. The manufacturer's encroachment decision with EW provision.

Thus, Proposition 4 shows that the manufacturer-provided EW supports encroachment only when the direct selling cost is below a critical threshold, which reflects the complementarity between product

demand and EW demand: EW provision increases the value of direct customer access, but this benefit disappears when the direct product channel becomes too costly.

Figure 6 illustrates Proposition 4 by comparing the manufacturer's profits under Models NMS and EMS when the manufacturer provides an EW. The common parameters are $v = 3$, $r = 2$, $c = 0.5$, $\alpha = 0.2$, $\beta_r = 0.80$, and $\beta_m = 0.20$. Panel (a) considers a low EW selling cost, $s = 0.20$. In this case, Model EMS remains above the benchmark of Model NMS over a relatively wide range of S. Panel (b) considers a higher EW selling cost, $s = 0.60$. The profitable encroachment region becomes substantially smaller, and Model EMS falls below Model NMS once S exceeds a lower threshold.

Propositions 3 and 4 allow us to examine how the manufacturer-provided EW provision affects the manufacturer's encroachment incentive. The comparison is based on the encroachment region, namely the range of the direct selling cost S under which encroachment is profitable. If EW provision expands this region, it strengthens the manufacturer's incentive to encroach; if it narrows this region, it weakens the incentive.

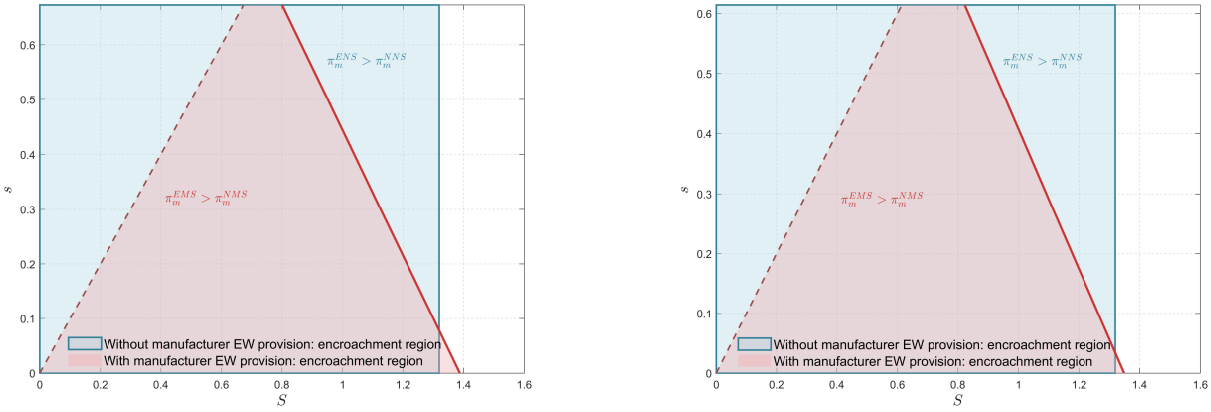
Remark 2. The provision of an EW enhances the manufacturer's incentive to encroach when the EW selling cost is low. Conversely, a high EW selling cost diminishes the manufacturer's motivation for encroachment.

As outlined in Remark 2, the manufacturer's decision to offer an EW can either strengthen or weaken its incentive to encroach, depending on the EW's selling cost. This result follows from comparing the encroachment regions in Propositions 3 and 4. Proposition 3 characterizes the manufacturer's encroachment region when the manufacturer does not provide an EW, whereas Proposition 4 characterizes the corresponding region when the manufacturer provides an EW. When the EW selling cost s is low, EW provision expands the range of the direct selling cost S under which encroachment remains profitable. In other words, the manufacturer can tolerate a higher direct channel cost after introducing the EW, so EW provision strengthens the incentive to encroach. When s is high, however, the encroachment region shrinks, indicating that direct channel entry becomes profitable under fewer cost conditions.

This pattern reflects the dual role of manufacturer-provided EWs. When the manufacturer can provide an EW at a low selling cost, it obtains pricing flexibility in the EW market and can offer a more attractive service option to risk-averse consumers. These consumers become more willing to purchase the product together with the manufacturer's EW, so EW provision expands product demand and strengthens the complementarity between product sales and EW sales. After opening a direct channel, the manufacturer can capture a larger share of this expanded demand, which helps absorb the direct selling cost and makes encroachment more attractive.

By contrast, when the EW selling cost is high, the manufacturer loses pricing flexibility and must either raise the EW price or accept a lower service margin. A higher EW price weakens service demand and limits the ability of EW provision to support product sales, while a lower service margin reduces the direct profit contribution of EWs. Moreover, because the retailer already offers its own EW, a costly manufacturer EW faces stronger service competition and contributes less to product market expansion. As a result, the manufacturer-provided EW no longer sufficiently compensates for the additional cost of direct selling, and the incentive to encroach is weakened.

To further illustrate Remark 2, Figure 7 compares the manufacturer's encroachment regions in the S-s plane with and without manufacturer-provided EWs. The common parameters are $v = 3$, $r = 2$, $c = 0.5$, $\alpha = 0.2$, and $\beta_r = 0.80$. Panels (a) and (b) correspond to $\beta_m = 0.20$ and $\beta_m = 0.60$, respectively.



(a) $\beta_m = 0.20$ **(b)** $\beta_m = 0.60$

Figure 7. Encroachment regions with and without manufacturer-provided EWs.

In each panel, the blue region represents the encroachment region without manufacturer-provided EWs, where $\pi_m^{ENS*} > \pi_m^{NNS*}$, while the red region represents the encroachment region with manufacturer-provided EWs, where $\pi_m^{EMS*} > \pi_m^{NMS*}$. The labels and profit inequalities in the figure identify the corresponding equilibrium outcomes in each region. Comparing the two regions within each panel shows that the manufacturer-provided EW narrows the manufacturer's encroachment region under the considered parameter settings. Moreover, the shrinkage is more pronounced when $\beta_m = 0.20$ than when $\beta_m = 0.60$, indicating that the degree of EW differentiation affects the extent to which the manufacturer-provided EW substitutes for channel encroachment. When the manufacturer's EW is substantially more protective than the retailer's EW, it provides greater value to risk-averse consumers, thereby reducing the manufacturer's reliance on channel encroachment to increase profit.

5.3. Strategic pricing of extended warranties

This section examines the equilibrium pricing strategies for both the manufacturer's and retailer's EWs, revealing how each party adapts its tactics in response to the other's moves. In the absence of encroachment, we evaluate how the manufacturer's introduction of EW affects the retailer's EW pricing. By comparing the retailer's equilibrium EW prices in Models NMS and NNS, we obtain the following results.

Lemma 5. *When the manufacturer does not encroach, the introduction of an EW by the manufacturer always leads the retailer to increase its EW price, and* $p_s^{NMS} > p_s^{NNS}$.

As Lemma 5 indicates, in the absence of manufacturer encroachment, the retailer responds to the manufacturer's direct channel EW offering by raising its own EW price. In the absence of encroachment, since the manufacturer relies exclusively on the retail channel for product sales, the availability of a manufacturer-provided EW enhances the overall product value, attracting more consumers and increasing retail product sales. This rise in product demand subsequently stimulates demand for the retailer's EW. To further encourage retail sales, the manufacturer may lower the wholesale price of the product, thereby increasing the retailer's margins and offering greater flexibility in EW pricing. In pursuit of profit maximization, the retailer opts to raise the EW price. Ultimately, this pricing strategy

not only improves the retailer's profitability but also strengthens channel coordination between the manufacturer and the retailer. When the manufacturer encroaches on the market, we examine how the manufacturer's decision to offer an EW affects the retailer's EW pricing by comparing scenarios with and without the manufacturer-provided EW.

Through an analysis of the retailer's equilibrium EW prices in Models EMS and ENS, we obtain the following insights.

Lemma 6. *Suppose* $\frac{A}{3} \leqslant B_r < A$, $B_m \leqslant \frac{3B_r(A+B_r)\left((A+5B_r)-2\sqrt{B_r(A+3B_r)}\right)}{2\left(A^2+6AB_r+13B_r^2\right)}$ *and* $\frac{\Phi}{2} \leqslant S < \frac{\Phi\left((A+3B_r)^2+(A+B_r)\sqrt{B_r(A+3B_r)}\right)}{2(A+2B_r)(A+3B_r)}$. *When* $s < \frac{(\Phi-S)(A-2B_m+B_r)}{A+B_r}$, *we have* $p_s^{EMS} < p_s^{ENS}$; *however, when* $\frac{(\Phi-S)(A-2B_m+B_r)}{A+B_r} \leqslant s < \Phi - S$, *we have* $p_s^{EMS} \geqslant p_s^{ENS}$.

Lemma 6 reveals that under manufacturer encroachment, the retailer's EW pricing strategy varies with the manufacturer's EW selling cost. When the manufacturer offers an EW at a low selling cost, the retailer responds by reducing its EW price relative to the scenario without the manufacturer-provided EW. This occurs because when the manufacturer offers an EW at a lower selling cost, this results in the manufacturer's EW being perceived as more attractive to customers due to its lower price and enhanced appeal, thereby capturing a significant portion of the EW market demand. Consequently, the retailer faces increased competitive pressure and must adjust its EW pricing strategy to remain competitive and maintain its market share, leading to a reduction in the sales price.

Conversely, when the manufacturer's EW selling cost is high, the retailer increases its EW price compared with the setting without the manufacturer-provided EW. Higher manufacturer EW costs lead to less competitive pricing, making the retailer's EW more appealing to price-sensitive consumers. This enhanced competitive position allows the retailer to set higher prices, thereby improving profitability without a significant loss of demand.

When the manufacturer does not offer an EW, we examine how the manufacturer's encroachment decision influences the retailer's EW pricing strategy by comparing scenarios with and without manufacturer encroachment. An analysis of the retailer's equilibrium EW prices in Models NNS and ENS yields the following results.

Lemma 7. *In a supply chain where only the retailer offers an EW, the retailer's optimal EW price under manufacturer encroachment (Model ENS) is always higher than in the no encroachment scenario (Model NNS).*

Lemma 7 indicates that the retailer increases its EW price following manufacturer encroachment. This pricing response can be attributed to intensified inter channel competition after the manufacturer establishes a direct sales channel for the product, which typically offers a more convenient shopping experience to consumers. The resulting competitive pressure reduces the retailer's profit margin from product sales. To offset this profit loss in product sales, the retailer leverages its monopolistic position in the EW market by raising the EW price, thereby restoring overall profitability despite heightened market competition.

When the manufacturer offers an EW, we analyze how the manufacturer's encroachment decision impacts the retailer's EW pricing strategy by comparing scenarios with and without encroachment. On the basis of the retailer's equilibrium EW prices in Models NMS and EMS, we derive the following results.

Lemma 8. *When the manufacturer offers an EW through the direct channel, if the manufacturer encroaches on the market, the retailer's optimal EW price remains unchanged relative to the no encroachment scenario, i.e.,* $p_s^{EMS} = p_s^{NMS}$.

Lemma 8 indicates that the retailer does not alter its EW price in response to manufacturer encroachment when the manufacturer already offers an EW. As the Stackelberg game leader, the manufacturer holds pricing authority and can strategically adjust both the product's wholesale price and its own EW price to maximize profit. Manufacturer encroachment and differentiated EW provision intensify market competition, placing additional pressure on the retailer. To maintain consumers' trust, preserve brand image, and sustain retail channel competitiveness, the retailer adopts a cautious pricing approach—prioritizing stability in EW pricing rather than engaging in aggressive price adjustments. This allows the retailer to protect its market share and foster long-term customer relationships despite heightened competition.

By comparing the manufacturer's equilibrium EW prices in Models EMS and NMS, we examine how the manufacturer's EW pricing strategy changes under market encroachment relative to the no encroachment scenario. Our comparative results are summarized as follows.

Lemma 9.

- *When* $s \leqslant \frac{12\Phi B_r(A-2B_m+B_r)}{B_m(25A-33B_r)+24B_r(A+B_r)-40B_m^2}$, *if* $S \leqslant \frac{12\Phi B_r(A-2B_m+B_r)-s(B_m(5A-29B_r)+12B_r(A+B_r))}{4(A-2B_m+B_r)(5B_m+3B_r)}$ *or* $\frac{6\Phi(A-2B_m+B_r)(B_m+B_r)+s\left(12B_m^2-7B_m(A-B_r)-6B_r(A+B_r)\right)}{2(A-2B_m+B_r)(5B_m+3B_r)} < S$, *we have* $p_e^{EMS} < p_e^{NMS}$; *however, if* $\frac{\begin{pmatrix}12\Phi B_r(A-2B_m+B_r)-\\ s(B_m(5A-29B_r)+12B_r(A+B_r))\end{pmatrix}}{4(A-2B_m+B_r)(5B_m+3B_r)} < S \leqslant \frac{\begin{pmatrix}6\Phi(A-2B_m+B_r)(B_m+B_r)+\\ s\left(12B_m^2-7B_m(A-B_r)-6B_r(A+B_r)\right)\end{pmatrix}}{2(A-2B_m+B_r)(5B_m+3B_r)}$, *we have* $p_e^{EMS} \geqslant p_e^{NMS}$.
- *When* $\frac{12\Phi B_r(A-2B_m+B_r)}{B_m(25A-33B_r)+24B_r(A+B_r)-40B_m^2} < s < \frac{6\Phi B_r(A-2B_m+B_r)}{B_m(5A-21B_r)+12B_r(A+B_r)-8B_m^2}$, *if* $S \leqslant \frac{6\Phi(A-2B_m+B_r)(B_m+B_r)+s\left(12B_m^2-7B_m(A-B_r)-6B_r(A+B_r)\right)}{2(A-2B_m+B_r)(5B_m+3B_r)}$, *we have* $p_e^{EMS} \geqslant p_e^{NMS}$; *however, if* $\frac{6\Phi(A-2B_m+B_r)(B_m+B_r)+s\left(12B_m^2-7B_m(A-B_r)-6B_r(A+B_r)\right)}{2(A-2B_m+B_r)(5B_m+3B_r)} < S$, *we have* $p_e^{EMS} < p_e^{NMS}$.

Lemma 9 reveals that the manufacturer's optimal EW price is systematically influenced by its encroachment decision, depending on both the EW selling cost and the product selling cost. When the manufacturer's EW selling cost is low, the manufacturer sets a lower EW price under encroachment compared with no encroachment when the product's selling cost is either very low or very high. In these cases, the manufacturer leverages economies of scale and cost advantages to enhance competitiveness and sustain profitability even with reduced EW prices. In contrast, for a moderate product selling cost, the manufacturer increases the EW price under encroachment. The flexibility afforded by moderate cost conditions allows the manufacturer to raise the price without significantly impairing competitiveness, thereby capturing additional revenue. When the EW selling cost is high, the manufacturer increases the EW price under encroachment when the product's selling cost is low. This is because with a low product sales cost, the manufacturer can achieve high marginal profits by encroaching on the market. Thus, the manufacturer is motivated to increase the EW price to further boost overall profits and offset the high sales cost of EWs, ensuring profit maximization. Conversely, when the product sales cost is high, the

manufacturer lowers the EW price under encroachment. Intense market competition and heightened consumer price sensitivity in this regime motivate a volume-based strategy, where the manufacturer chooses to stimulate demand by reducing the EW price and supports overall profitability, even at the expense of lower per-unit EW margins.

5.4. The interaction: Complements or substitutes?

To formally define the relationship between manufacturer encroachment and manufacturer-provided EWs, we follow the idea of strategic complementarity [43] and compare the manufacturer's incremental profit from adopting one strategy when the other strategy is present or absent. Let $I_M^E = \pi_m^{EMS*} - \pi_m^{ENS*}$ denote the incremental value of the manufacturer-provided EW when encroachment is present, and let $I_M^N = \pi_m^{NMS*} - \pi_m^{NNS*}$ denote the incremental value of the manufacturer-provided EW when encroachment is absent. Manufacturer encroachment and manufacturer EW provision are complementary if $I_M^E \geq I_M^N$, which means that encroachment increases the manufacturer's marginal gain from EW provision. The two strategies are substitutable if $I_M^E < I_M^N$, which means that encroachment reduces the manufacturer's marginal gain from EW provision.

Equivalently, let $I_E^M = \pi_m^{EMS*} - \pi_m^{NMS*}$ denote the incremental value of encroachment when the manufacturer-provided EW is present, and let $I_E^N = \pi_m^{ENS*} - \pi_m^{NNS*}$ denote the incremental value of encroachment when the manufacturer-provided EW is absent. The two strategies are complementary if $I_E^M \geq I_E^N$ and substitutable if $I_E^M < I_E^N$. Both definitions are equivalent to evaluating the interaction term

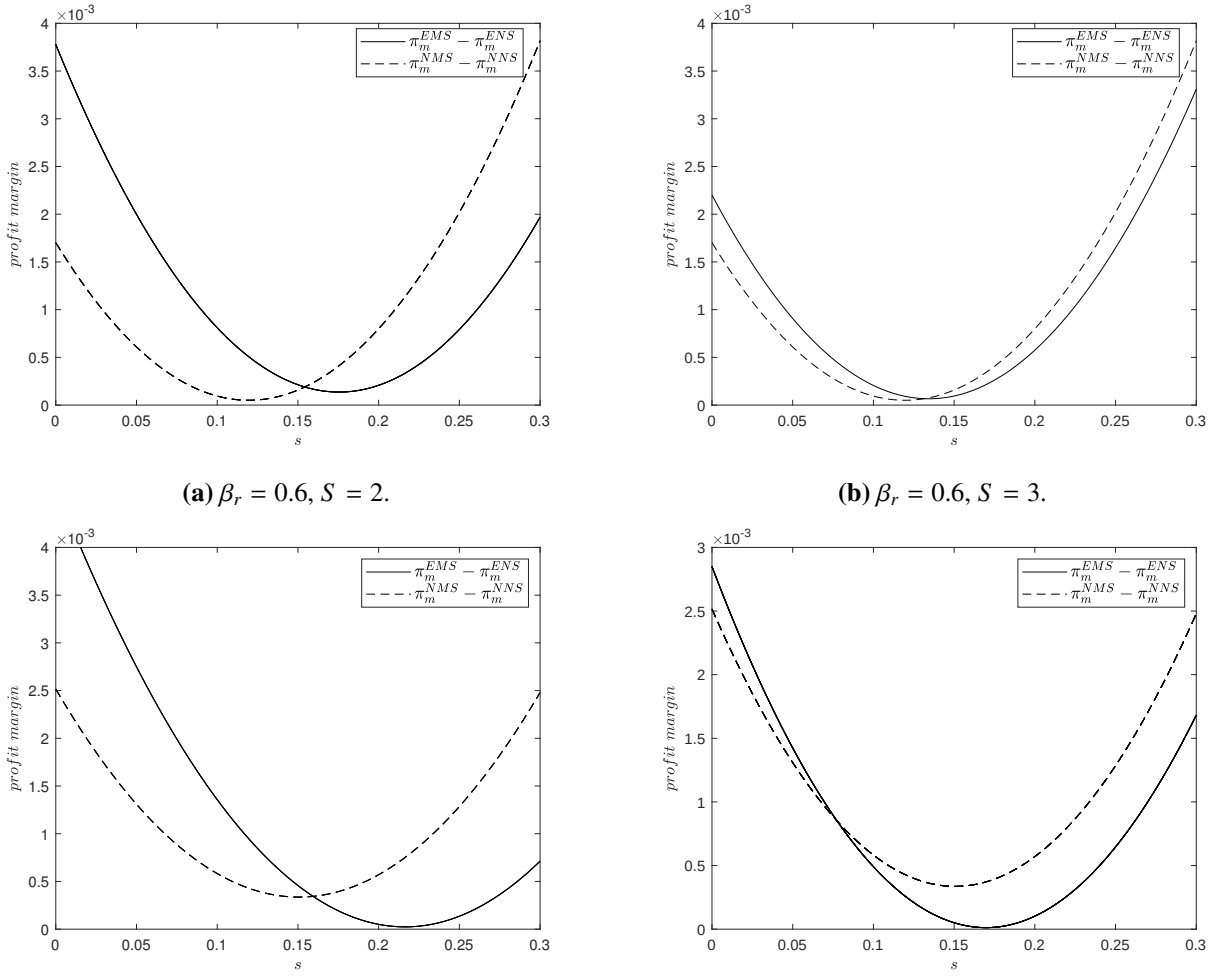
$$K = \pi_m^{EMS*} + \pi_m^{NNS*} - \pi_m^{ENS*} - \pi_m^{NMS*}.$$

Equivalently, K can be written as

$$K = (\pi_m^{EMS*} - \pi_m^{NNS*}) \quad - \left[(\pi_m^{ENS*} - \pi_m^{NNS*}) + (\pi_m^{NMS*} - \pi_m^{NNS*})\right].$$

This expression compares the profit gain from jointly adopting both strategies with the sum of the gains from adopting each strategy alone. If $K \geq 0$, the joint adoption of encroachment and manufacturer-provided EWs generates a super-additive profit effect, so the two strategies are complementary. If $K < 0$, the joint adoption generates a subadditive profit effect, so the two strategies are substitutable.

Given the complexity of comparing closed-form solutions for equilibrium profits across the four scenarios, we conduct numerical analysis with fixed parameter values: $v = 15$, $r = 0.5$, $c = 0.6$, $\alpha = 0.5$, and $\beta_m = 0.2$. We compute the profit differentials for offering an EW under encroachment versus the no encroachment condition separately. As we can see from Figure 8, there are some key findings.



(a) $\beta_r = 0.6, S = 2$. **(b)** $\beta_r = 0.6, S = 3$.

(c) $\beta_r = 0.8, S = 2$. **(d)** $\beta_r = 0.8, S = 3$.

Figure 8. Profit margin of the manufacturer's EW offering: Under encroachment and no encroachment.

First, there is a threshold for the EW selling cost, below which the incremental profit from offering an EW under encroachment exceeds that under no encroachment. This confirms complementarity that manufacturers can leverage synergies between encroachment and EW provision to maximize profits when the EW costs are kept within a reasonable range. However, when the EW selling cost exceeds this threshold, the manufacturer offering an EW under encroachment yields lower incremental profits than under no encroachment, indicating substitutability between these two decisions. Manufacturers should prioritize one strategy or reallocate resources to one of them.

Second, comparative analysis between (a) and (b) shows that a higher direct channel sales cost for the product reduces the threshold below which the complementary relationship takes effect. This implies that complementarity between these two decisions is viable only under stricter cost control for encroachment, highlighting the need for precise resource allocation. Third, the comparison of (a) and (c) reveals that greater differentiation in EW offerings between the manufacturer and retailer–specifically, a higher consumer co-payment rate for the retailer's EW–expands the range of feasibility for the complementary relationship by increasing the sales cost threshold of the product. This is because a higher co-payment

rate for the retailer's EW enhances the relative attractiveness of the manufacturer's EW to consumers. Consequently, the manufacturer maintains strategic flexibility even amid sales costs, as it can capitalize on synergies between encroachment and EW provision by strengthening the competitiveness of their own EW offerings.

5.5. The impact of retailer-owned EWs on the strategic interaction between the manufacturer's decisions

This section builds on and differs from our earlier study [9], which examines the manufacturer's encroachment and EW provision decisions when the manufacturer is the only EW provider. In contrast, the present paper changes the service market structure by making the retailer an incumbent EW provider in every regime. This modification is not a cosmetic extension: It introduces direct EW competition, changes consumer segmentation, endogenizes the retailer's EW pricing response, and allows us to evaluate how the retailer-owned service reshapes the manufacturer's joint encroachment and EW provision incentives. By comparing the equilibrium results with the benchmark without retailer-provided EW competition [9], we identify how retailer-owned EW competition changes the incentive thresholds and complementarity gains.

Proposition 5.

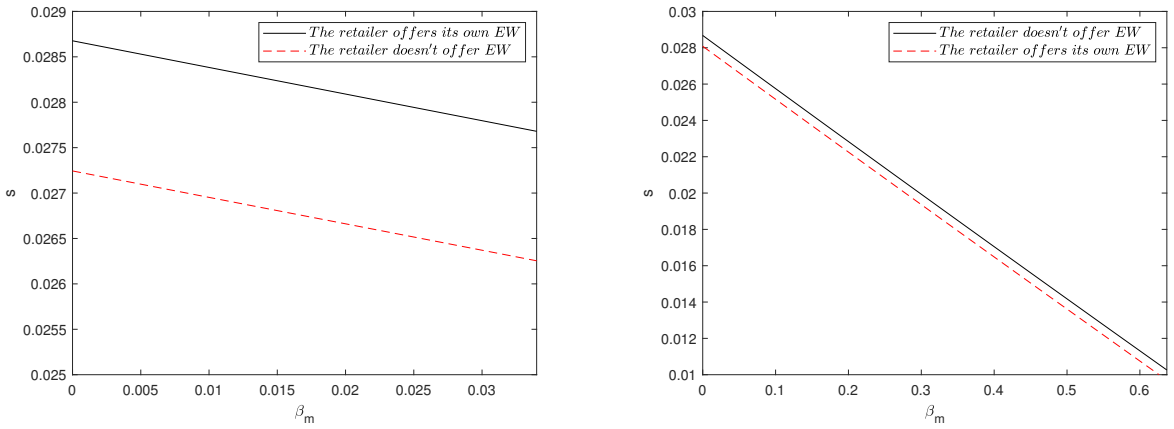
- *The availability of a retailer-offered EW does not alter the impact of encroachment on the manufacturer's EW provision incentives: Encroachment always dampens the manufacturer's incentive to offer EW;*
- *Irrespective of whether the retailer offers an EW, the manufacturer's EW provision decision can either encourage or discourage encroachment, whereas the retailer's EW competition modifies the magnitude of incentives.*

Proposition 5 compares the strategic interaction between manufacturer encroachment and manufacturer-provided EWs across two service environments: The benchmark without retailer-provided EW competition and the retailer-owned EW setting in this paper. The comparison focuses on whether retailer-owned EW competition changes the direction or strength of two effects: The effect of encroachment on manufacturer-provided EWs, and the effect of manufacturer-provided EWs on encroachment.

The first result shows that retailer-owned EW competition does not change the direction of the effect of encroachment on manufacturer-provided EWs. In both service environments, encroachment weakens the manufacturer's incentive to offer EWs. This is because opening a direct product channel changes EW provision from a relatively independent service decision into a channel and service joint decisions. After encroachment, the manufacturer must bear the direct selling cost and faces stronger product channel competition with the retailer. These forces reduce the incremental profit from adding the EW and make the manufacturer more sensitive to the EW selling cost. When the retailer also provides its own EW, the same mechanism remains, but the pressure becomes stronger: The manufacturer's EW must compete with the retailer's incumbent EW for risk-averse consumers, and part of the service demand is captured by the retailer. Therefore, retailer-owned EW competition does not reverse the negative effect of encroachment on EW provision; instead, it reinforces the cost and competition channels that make the manufacturer more cautious about offering EWs.

The second result shows that retailer-owned EW competition changes the strength, rather than the basic direction, of the effect of manufacturer-provided EWs on encroachment. Regardless of whether the retailer offers an EW, the manufacturer's EW offering can either promote or inhibit encroachment—a relationship contingent upon the EW sales costs and consumer co-payment rates, with the magnitude of the incentive varying across EW-related competitive contexts. Crucially, the retailer-owned EW systematically weakens this incentive effect, which is driven by differentials in consumer co-payment rates between manufacturer-provided and retailer-provided EWs. Because the retailer's EW is already available, the manufacturer's EW no longer serves the entire risk-averse segment. Instead, the two EWs split the service demand and compete in service pricing. As a result, part of the demand expansion benefit generated by EW provision is captured or controlled by the retailer's service offer. This reduces the manufacturer's ability to convert EW-induced demand expansion into direct channel profit. Hence, although manufacturer-provided EWs can still promote encroachment when its selling cost is sufficiently low, retailer-owned EW competition narrows the range and lowers the threshold under which this positive effect operates.

In short, Proposition 5 shows that retailer-owned EW competition reshapes the interplay between encroachment and EW provision in an asymmetric way. It does not change the qualitative conclusion that encroachment discourages manufacturer-provided EWs, but it attenuates the extent to which the manufacturer-provided EW provision can encourage encroachment.



(a) $\beta_r = 0.47$ **(b)** $\beta_r = 0.8$

Figure 9. The incentive effect of manufacturer-provided EWs on channel encroachment.

To further illustrate the second part of Proposition 5, Figure 9 compares the EW selling cost threshold below which the manufacturer-provided EW encourages encroachment with and without retailer-owned EW competition. We set $v = 15$, $c = 0.6$, $\alpha = 0.5$, and $r = 0.5$. The dashed line represents the threshold in the retailer-owned EW setting, while the solid line represents the corresponding threshold in the benchmark without retailer-provided EW competition. The regions below these lines indicate the parameter intervals in which the manufacturer-provided EW increases the manufacturer's incentive to encroach.

Figure 9 shows that the dashed line lies below the solid line, meaning that retailer-owned EW competition lowers the maximum EW selling cost under which manufacturer-provided EWs can encourage encroachment. Thus, the presence of the retailer's EW narrows the range in which the positive

effect of manufacturer-provided EWs on encroachment remains active. This weakening arises because retailer-owned EW competition limits the manufacturer's ability to appropriate the demand expansion benefit generated by its own EW. Therefore, compared with the benchmark without retailer-provided EW competition, the retailer-owned EW setting requires a lower EW selling cost for manufacturer-provided EWs to encourage encroachment.

6. Conclusions

6.1. Research results

This study examines manufacturer encroachment and the manufacturer's provision of EWs in a market where the retailer already owns an EW business. This distinction is central: The retailer is not only a downstream product seller but also a service competitor. We build four manufacturer-led Stackelberg regimes, Models NNS, NMS, ENS, and EMS, to analyze how the manufacturer's two strategic choices interact when EW demand can be captured by either the retailer or the manufacturer. The comparison also allows us to identify how retailer-owned service competition changes the conclusions that would arise in a setting without retailer-provided EWs.

First, the manufacturer's EW decision is channel dependent. Without encroachment, a manufacturer-provided EW can benefit from retail channel product demand and may be profitable across a wider range of EW selling costs. With encroachment, however, the manufacturer faces direct channel operating costs and stronger product and service competition, so the acceptable EW selling cost threshold becomes more restrictive. Thus, the direct channel is not automatically a service launch platform.

Second, the encroachment decision depends on the service environment. When only the retailer offers EWs, the attractiveness of the retailer's service can enlarge product demand and affect the manufacturer's gain from direct selling. When the manufacturer also sells EWs, encroachment is attractive only if the product's direct selling cost and the EW's selling cost are jointly manageable. A low-cost manufacturer-provided EW can make encroachment more valuable by expanding service-related demand, whereas a costly EW absorbs the benefit of direct access and weakens the case for encroachment.

Third, retailer-provided EW pricing is shaped by whether competition occurs in products, services, or both. If there is no manufacturer encroachment, the manufacturer's introduction of an EW can increase the retailer's EW price because the manufacturer still relies on retail product sales. Under encroachment, the retailer's pricing response depends on the manufacturer's EW cost and the induced pressure in the service market. When the retailer is the only EW provider, encroachment raises the retailer's incentive to rely on the EW margin to compensate for product channel pressure. When both firms sell EWs, the retailer's EW price may become less sensitive to the manufacturer's product channel choice because the service competition already controls the market.

Fourth, the two manufacturer strategies switch from complements to substitutes at a threshold level of the EW selling cost. Below the threshold, direct access to consumers and the manufacturer's service provision reinforce each other and create superadditive profit. Above the threshold, service provision consumes the margin and managerial resources, so the manufacturer benefits more from prioritizing one strategy. The feasible complementarity region narrows when direct product selling becomes costly and expands when the manufacturer's EW is more differentiated from the retailer's EW.

Finally, comparison with the benchmark without the retailer-provided EW shows that retailer-owned service competition dampens the manufacturer's incentive to combine encroachment with EW provision.

The retailer's EW captures part of the risk-averse consumer segment and changes the pricing response in the service market, thereby reducing the incremental value of the manufacturer's dual strategy.

6.2. Managerial insights

Our findings offer several managerial implications for manufacturers and retailers that operate in markets where retailers already provide extended warranty services.

First, manufacturers should not regard channel encroachment as universally beneficial. When the direct selling cost is low, opening a direct channel can be attractive because the manufacturer can appropriate a larger share of product demand and better capture the demand enhancing effect of its own EW. In this case, the manufacturer may consider jointly adopting channel encroachment and EW provision, especially when the EW selling cost is also low. By contrast, when the direct selling cost is high, encroachment becomes less attractive because the additional channel profit may not compensate for the channel's operating cost and the intensified competition with the retailer. Under such conditions, the manufacturer should be cautious about entering the direct channel and may instead rely on the retailer's product channel while selectively offering an EW only if the service cost is sufficiently low.

Second, the manufacturer's EW strategy should depend critically on the EW selling cost. When the EW selling cost is low, the manufacturer-provided EW can serve as a demand expansion tool by attracting risk-averse consumers and increasing the value of product sales. This makes EW provision particularly useful when the manufacturer can capture the expanded demand through a direct channel. However, when the EW selling cost is high, providing an EW may no longer be profitable, especially in the presence of retailer-owned EW competition. In this case, the manufacturer should avoid duplicating the retailer's service offering unless it can create clear service advantages or reduce service delivery costs.

Third, the degree of service differentiation determines whether the manufacturer's and the retailer's EWs can coexist profitably. When service differentiation is high, the manufacturer's EW and the retailer's EW appeal to different consumer segments, which weakens direct service competition and allows the manufacturer to use its EW as a differentiated value-added service. Therefore, the manufacturer may benefit from emphasizing superior repair quality, original parts, faster replacement, or stronger post-sales support. In contrast, when service differentiation is low, the two EWs become close substitutes. Service competition then erodes the EW margins and may also intensify channel conflict. In this case, the manufacturer should avoid aggressive EW introduction and may instead coordinate with the retailer through service authorization, revenue sharing, or differentiated warranty scopes.

For retailers, our results suggest that an incumbent EW business can play a defensive role against manufacturer encroachment. When the manufacturer's direct selling cost or EW selling cost is high, the retailer can preserve its advantage by maintaining a competitive EW price and strengthening its service relationship with risk-averse consumers. However, when the manufacturer has both a low direct selling cost and a low EW selling cost, the retailer faces stronger pressure from the manufacturer's dual strategy. In this situation, the retailer should increase service differentiation, improve customers' convenience, bundle EWs with retail services, or negotiate cooperative arrangements with the manufacturer to reduce destructive product and service competition.

6.3. Future research directions

This study offers a number of promising avenues for future research. First, while our model focuses on a supply chain consisting of a single manufacturer and a single retailer, real-world markets often feature multiple competing manufacturers and retailers. Extending the analysis to settings with multiple players at either the upstream or downstream level would enrich the understanding of competitive interactions and strategic decision-making in a more realistic industrial context. Such extensions could also incorporate the formation of a coalition or competitive alignment among channel members, which represents a key issue in channel management.

Second, the introduction of manufacturer encroachment and direct channel EW provision inevitably heightens channel conflict. Future research could explore the design of coordination mechanisms–such as revenue sharing contracts, service cooperation agreements, or joint pricing strategies–to mitigate conflict and improve overall supply chain efficiency. Examining how these contracts can be tailored to align incentives between manufacturers and retailers would provide valuable managerial insights.

Additionally, other dimensions worth exploring include the impact of consumer heterogeneity in EW preferences, the role of digital platforms in EW sales and service delivery, and the effect of risk attitudes on the EW strategy's design. Empirical and experimental studies would also help validate the theoretical predictions and enhance the practical relevance of the findings.

Author contributions

Yue Qi: Conceptualization, methodology and writing original draft, formulating the research question, developing the analytical framework, deriving the model results, and drafting the manuscript.

Taofeng Ye: Supervision, review and editing, overseeing the study, revising the manuscript, enhancing the academic presentation and language quality, and providing scholarly suggestions.

Guoqing Zhang: Supervision, review and editing, advising on the research design, reviewing the manuscript, improving the exposition and language quality, and providing professional feedback.

All authors have read and approved the final version of the manuscript.

Use of Generative-AI tools declaration

The authors declare that no artificial intelligence (AI) tools were used in the creation of this manuscript.

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Conflict of interest

All authors declare no conflicts of interest in this paper.

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