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Research article

Mitigation effect on the efficiency mismeasurement of deterministic data envelopment analysis through statistical noise correction

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Abstract: This study proposes a potential method to improve the efficiency measurement accuracy of deterministic data envelopment analysis (DEA) by correcting for the effects of statistical noise on individual decision-making units. Because the solution to a DEA model is obtained at the boundary of the feasible domain, it is susceptible to small statistical noise. Several methods exist for estimating statistical noise; however, this work utilizes the statistical error estimates of stochastic frontier analysis (SFA). The proposed mismeasurement mitigation method is motivated by a stylized equivalence in that, under constant returns to scale with a single output and a single input with no statistical noise, the dual DEA formulation can be conceptualized as a projection of SFA. The results of Monte Carlo simulations based on the Cobb–Douglas model and actual datasets indicate that the proposed method outperforms previous approaches in terms of accuracy under increasing statistical noise.

Keywords: Data envelopment analysis (DEA); stochastic frontier analysis (SFA); mismeasurement mitigation; statistical noise correction; efficiency mismeasurement
Mathematics Subject Classification: 90C08, 90B50

1. Introduction

Data envelopment analysis (DEA) was initially introduced as a linear programming model by Charnes et al. [1] to determine frontier production and estimate efficiencies of decision-making units (DMUs). Because DEA is nonparametric, it does not require a priori specification of an unknown frontier line function; therefore, it remains unaffected by the personal judgment of the modeler. Furthermore, it facilitates multiple inputs and outputs. Owing to these advantages, DEA has been used as an effective tool for efficiency estimation, along with its competing efficiency estimation model, stochastic frontier analysis (SFA), which was originally proposed independently by Aigner et al. [2] and Meeusen and van den Broeck [3].

The classical DEA is a deterministic model, wherein all deviations from the frontier line are measured in terms of one-sided components and assumed to represent the inefficiency of DMU. Practically, these deviations are caused by true inefficiency and statistical noise, such as measurement errors. Because deterministic DEA selects solutions at the boundary of the feasible domain, it is highly sensitive to statistical noise such as outliers, often leading to biased efficiency estimates. Efforts have been made to accommodate statistical noise by extending the classical DEA to a stochastic model, and two main theoretical approaches have been proposed.

The first is the chance-constrained DEA (CCDEA) proposed in [4], which applies chance-constrained programming [5] to deterministic DEA. The second approach is stochastic DEA (SDEA) proposed by Banker [6], which introduces a symmetric two-sided error component accommodating the possibility of random factors. Similar to SFA, these two approaches aim to extend the deterministic DEA such that the frontier line can incorporate inefficiencies and stochastic errors. The extreme outliers are considered stochastic error terms, thereby positioning the frontier line closer to the region wherein most DMUs are clustered. Consequently, the measured efficiency of any DMU tends to be higher than that of the deterministic DEA. Notably, to implement these approaches, CCDEA and SDEA must predetermine stochastic setting parameters (e.g., regression quantile or the probability of the best practice output exceeding the observed outputs). If the correct parameter values are defined or known a priori, the models will exhibit superior performance; otherwise, their performance will be subpar. Consequently, the usefulness of CCDEA and SDEA depends on the ability to choose appropriate parameters, which is difficult to achieve in some cases. For instance, in SDEA, the regression quantile (i.e., the coefficient of the linear combination in the objective function) should be obtained by solving a separate optimization problem requiring additional information.

In [7], Chen employed CCDEA and SFA to measure the technical efficiency of banks in Taiwan, indicating that the efficiency score difference between the two models is significant. Wu and Olson reformulated the original nonlinear programming CCDEA into an equivalent linear programming problem, and applied it to the vendor selection problem in supply chains [8]. They reported that CCDEA was advantageous for performing efficiency analysis by handling inefficiency and stochastic errors. Similarly, Zha et al. applied CCDEA to measure regional energy and carbon dioxide emissions efficiency in China, demonstrating the effectiveness of DEA under stochastic conditions [9]. More recently, a chance-constrained directional DEA framework was developed to incorporate stochastic elements directly into efficiency analysis in [10]. Furthermore, Jradi and Ruggiero simplified Banker's original nonparametric SDEA model into a semiparametric model that identifies the most likely quantile based on the assumptions of the composed error terms [11]. Thereafter, the Monte Carlo analysis was used to compare their SDEA model with the econometric SFA model under different distributional assumptions. Reportedly, the translog SFA model outperforms the SDEA model in all scenarios.

In addition to CCDEA and SDEA, several studies have focused on improving DEA estimators in noisy environments. See, for example, [12] for early attempts to integrate stochastic noise into DEA, and [13] for a formalization of stochastic DEA estimators in a nonparametric setup. Furthermore, the statistical properties and bias correction techniques for DEA have been extensively studied as well. For more details, we refer the reader to [14] and [15–17]. Recent work, such as [18], has explored alternative DEA models like the slacks-based measure (SBM), which aims to better handle inefficiencies related to input slacks and output shortfalls. While SBM addresses a different aspect of efficiency measurement, our study focuses on mitigating the impact of statistical noise in DEA from SFA.

Recent studies continue to advance SDEA methodologies and applications, particularly in the healthcare [19], environmental, and energy domains [20, 21]. Recognizing the complexity of modern organizations, research has also extended stochastic approaches to network structures. For example, Vishghaei et al. proposed an inverse stochastic two-stage DEA model to address resource planning, estimating necessary input changes while preserving efficiency in the presence of stochastic data and undesirable outputs [22].

Beyond stochastic models that primarily address statistical noise, extensive research focuses on handling data imprecision and ambiguity when exact data is unavailable. Various frameworks have been employed to manage these uncertainties. Uncertainty theory has been utilized to develop generalized DEA models for imprecise data [23] and has been integrated with chance-constrained programming for uncertain two-stage network analysis [24]. Robust optimization offers another approach, demonstrating improved reliability for opened-network DEA models under uncertain inputs and outputs [25]. Other methodologies include rough set theory, applied to construct unified production frontiers with rough data [26], and fuzzy set theory, often used to handle ambiguous data, capture decision-makers' risk preferences (e.g., [27, 28]), or model partially controllable factors [29]. While these methods effectively address various forms of data imprecision and ambiguity, our study specifically focuses on correcting for statistical noise.

Efforts have been made to compare DEA and SFA as nonparametric and parametric approaches, respectively, and create a novel methodology using the complementarity of the two methods. Ferrier and Lovell argued that the disagreements between the results of deterministic DEA and SFA are substantial owing to their distinct structure and implementation [30]. Oh and Shin measured the impact of stochastic noise in performance benchmarking using a Monte Carlo comparison of SFA and DEA with different multi-period budgeting [31]. They reported that SFA outperformed DEA for multi-period budgeting performance in the presence of small measurement errors; however, as the measurement errors increased, DEA exhibited superior performance. For optimization and policy making of electricity distribution units, Azadeh et al. proposed a novel algorithm that runs DEA and SFA parallel and integrates the results [32].

However, to improve the efficiency measurement accuracy when the stochastic noise is unknown, the benefit of statistical noise estimation of SFA on deterministic DEA remains to be investigated. This study proposes a statistical noise correction method to mitigate the inaccuracy in the efficiency measurement of DEA's deterministic setting by adopting the error estimates of SFA. The contribution of the paper is to develop, within the standard radial DEA formulation, a unit-level SFA-based noise correction procedure and to examine when this procedure reduces efficiency mismeasurement. The theoretical motivation is given by a stylized relationship between the dual model of DEA and an error-free SFA frontier. In particular, Theorem 3.3 is established for the special case of a single input under constant returns to scale with no statistical noise and serves as conceptual motivation. Accordingly, the proposed method is intended for radial technical efficiency and does not address slack-based or mix inefficiencies within the current framework. To validate the proposed method, it was compared with the classical DEA and CCDEA models using the Cobb–Douglas model and actual datasets, which were generated by incrementally adding statistical noise for the Monte Carlo simulation.

The remainder of this paper is organized as follows. Section 2 reviews the literature on DEA models, including the stochastic setting. Section 3 discusses the special relationship between the dual model of DEA and SFA, establishing the theoretical foundation for incorporating SFA error estimates to

mitigate deterministic DEA model error. Section 4 proposes a procedure to mitigate the inaccuracy in the efficiency measurement of DEA's deterministic setting and validates the proposed methodology using Monte Carlo simulations by comparing it with classical DEA and CCDEA, wherein the simulation scenarios are generated based on the Cobb–Douglas function and actual datasets. Finally, Section 5 concludes the study and provides essential directions for future research.

2. DEA models considering statistical noise

Assume the existence of N DMUs, where the ith DMU_i produces one output y_i using a vector of M inputs $X_i = (x_{i1}, ..., x_{iM})$ for $i = 1, \cdots, N$. Furthermore, assume that the outputs are stochastic and the inputs are predetermined; hence, the production outputs can be expressed using Eq. (2.1).

$$y_i = \hat{y}_i + \epsilon_i = f(X_i) + \epsilon_i, \quad i = 1, 2, \cdots, N, \tag{2.1}$$

where $\hat{y}_i$ is the frontier production for X_i such that $\hat{y}_i = f(X_i)$. Depending on the value of ϵ_i, Eq. (2.1) can be differentiated into three models. First, if ϵ_i is purely statistical noise and follows a symmetric two-sided distribution with a zero mean, Eq. (2.1) becomes a regression model. Second, if $\epsilon_i \leq 0$ and it represents only inefficiency, Eq. (2.1) becomes a deterministic production frontier. Third, if ϵ_i is a composite of statistical noise and inefficiency, Eq. (2.1) becomes a stochastic frontier.

When $\epsilon_o \leq 0$ for all observations $o = 1, 2, \cdots, N$, $\hat{y}_o$ becomes a deterministic production frontier obtained by solving the following linear programming problem for each observed DMU_o, which is a constant returns to scale (CRS) DEA model.

$$\begin{aligned} \underset{\gamma, \theta_o}{\text{minimize}} \quad & \theta_o && \text{(2.2a)} \\ \text{subject to} \quad & \gamma Y \geq y_o && \text{(2.2b)} \\ & \gamma X \leq \theta_o X_o && \text{(2.2c)} \\ & \gamma \geq 0 && \text{(2.2d)} \end{aligned}$$

where $\gamma = (\gamma_1, \gamma_2, \cdots, \gamma_N)$ and $Y = (y_1, y_2, \cdots, y_N)^T$ are the row and column vectors, respectively, and X is an $N \times M$ matrix whose ith row vector is X_i. Vector inequalities are utilized when each component of the vector satisfies the corresponding inequalities. For instance, (2.2c) represents

$$\sum_{i=1}^{N} \gamma_i x_{ij} \leq \theta_o x_{oj} \quad \text{for all } j = 1, 2, \cdots, M.$$

The objective value $\hat{\theta}_o$ for model (2.2) is the Farrell efficiency of the oth DMU.

To accommodate the efficiency measurement and statistical errors, stochastic variation can be incorporated into the linear model (2.2). Based on the chance-constraint mechanism introduced in [4], the CCDEA model is formulated as follows:

$$\begin{aligned} \underset{\gamma, \theta_o}{\text{minimize}} \quad & \theta_o \\ \text{subject to} \quad & \text{Prob}\left[\gamma Y - y_o \geq 0\right] \geq \alpha \\ & \gamma X \leq \theta_o X_o \\ & \gamma \geq 0 \end{aligned} \tag{2.3}$$

The first constraint indicates that the probability corresponding to the best practice output exceeds the observed output by at least α. If α=0.95, then 5% DMUs are set as the best performers and the constraint is equivalent to the following form:

$$\mathbb{E}[\gamma Y - y_0] - 1.645\sigma \geq 0, \tag{2.4}$$

where $\mathbb{E}$ is the expectation and standard deviation $\sigma = \sigma[\gamma Y - y_0]$ is computed as

$$\sigma = \sqrt{\sum_{i=1}^{N}\sum_{j=1}^{N} \delta_i \delta_j \operatorname{Cov}(y_i, y_j)},$$

$$\begin{aligned} \delta_i &= \gamma_i, \qquad i \neq o, \\ \delta_i &= \gamma_o - 1, \quad i = o. \end{aligned}$$

The number 1.645 is the z-score in a standard normal distribution setting,

$$\operatorname{Prob}[\gamma Y - y_o \geq 0] \geq 0.95.$$

For the derivation, see [4] and the references therein.

Furthermore, Jradi and Ruggiero extended the SDEA model developed in [6] to estimate the production frontier for a given quantile [11]. The extended SDEA is a variant of parametric quantile regression as follows:

$$\begin{aligned} \text{minimize} \quad & \sum_{i=1}^{N} (\tau e_{1i} + (1-\tau) e_{2i}) \\ \text{subject to} \quad & \ln y_i = \alpha + \beta \ln x_i + e_{1i} - e_{2i} \\ & \beta \geq 0, e_{1i} \geq 0, e_{2i} \geq 0, \quad \forall i = 1, \ldots, N \end{aligned} \tag{2.5}$$

The SDEA (2.5) has a limitation in that the quantile τ (similar to a weight value in the goal programming setting objective function) should be calculated by solving a separate optimization problem requiring additional information. For instance, if the error term ϵ is normally distributed with zero mean, then Prob $[\epsilon \leq 0] = 0.5$, where the appropriate quantile τ becomes 0.5. However, this approach is difficult to implement if such information is not provided in advance. Therefore, in this study, the classical DEA and CCDEA were selected as benchmarking models to validate the proposed method.

3. Relation between the DEA and SFA models

This section discusses the special relationship between the dual model of DEA and SFA, utilizing SFA's error estimates to correct DEA's mismeasurement issues. First, the dual problem of the CRS DEA model (2.2) is required, which is easily derived as follows:

$$\underset{\mathbf{w}^o \in \mathbb{R}^M, h^o \in \mathbb{R}}{\text{maximize}} \quad h^o y_o \tag{3.1a}$$

$$\text{subject to} \quad X_o \mathbf{w}^o \le 1 \tag{3.1b}$$
$$h^o Y \le X \mathbf{w}^o \tag{3.1c}$$
$$h^o, \mathbf{w}^o \ge 0 \tag{3.1d}$$

Recall that $Y = (y_1, y_2, \cdots, y_N)^T$, $X_i = (x_{i1}, x_{i2}, \cdots, x_{iM})$ for $i = 1, \cdots, N$, and X is an $N \times M$ matrix whose ith row is X_i. Note that $h^o \in \mathbb{R}$ and column vector $\mathbf{w}^o = (w_1^o, w_2^o, \cdots, w_M^o)^T$ depend on the target unit DMU_o as indicated by the superscript.

As y_o is fixed, the objective function (3.1a) is equivalent to

$$\underset{\mathbf{w}^o, h^o}{\text{maximize}} \; h^o. \tag{3.1a'}$$

Furthermore, one can show that the maximum is attained when

$$X_o \mathbf{w}^o = 1. \tag{3.1b'}$$

Indeed, if the maximum were attained at $\mathbf{w}^o = \hat{\mathbf{w}}^o, h^o = \hat{h}^o$ and $X_o \hat{\mathbf{w}}^o = c$ for some $0 < c < 1$, then

$$X_o \frac{\hat{\mathbf{w}}^o}{c} = 1,$$
$$\frac{\hat{h}^o}{c} Y \le X \frac{\hat{\mathbf{w}}^o}{c}.$$

That is, $\hat{\mathbf{w}}^o/c$ and $\hat{h}^o/c$ satisfy the constraints (3.1b)-(3.1d). Because $\hat{h}^o$ is assumed to be the maximum of the objective function (3.1a′) with (3.1b)-(3.1d), we have

$$\hat{h}^o \ge \frac{\hat{h}^o}{c},$$

which contradicts the assumption $0 < c < 1$. Thus, (3.1b′) can replace (3.1b). Additionally, the solution to (3.1) is obtained when $h^o > 0, \mathbf{w}^o \ge 0$ instead of (3.1d), assuming $X, Y > 0$. See also [33] for a derivation of the dual problem.

We define

$$\beta^o := \frac{1}{h^o} \mathbf{w}^o,$$
$$\mathbf{e}^o := X\beta^o - Y.$$

As $h^o > 0$ and (3.1b′), one can see that (3.1a′) is equivalent to

$$\text{minimize} \, \frac{1}{h^o} = \text{minimize} \, \frac{X_o \mathbf{w}^o}{h^o} = \text{minimize} \, X_o \beta^o = \text{minimize} (\mathbf{e}^o + Y)_o.$$

Minimizing $(\mathbf{e}^o + Y)_o$ is equivalent to minimizing $(\mathbf{e}^o)_o$ since Y is fixed. By taking into account that $\beta^o \ge 0$ and $\mathbf{e}^o \ge 0$, the dual problem (3.1) or (3.1a′),(3.1b′),(3.1c)-(3.1d) can be rewritten as

$$\underset{\beta^o \ge 0}{\text{minimize}} \quad (X\beta^o - Y)_o \tag{3.2a}$$
$$\text{subject to} \quad X\beta^o - Y \ge 0 \tag{3.2b}$$

Now, the connection between (3.2) and the SFA model is investigated. In SFA, the parametric model for (2.1) is

$$y_i = f(X_i;\beta) + \epsilon_i, \quad i = 1, 2, \cdots, N.$$

If we assume that the functional form is linear, we can estimate the parameter β by quadratic programming such as

$$\underset{\beta\in\mathbb{R}^M, b_0\in\mathbb{R}}{\text{minimize}} \quad \|X\beta - Y + \mathbf{b}_0\|^2 \tag{3.3a}$$

$$\text{subject to} \quad X\beta - Y + \mathbf{b}_0 \geq 0 \tag{3.3b}$$

where $\mathbf{b}_0 = b_0(1, 1, \cdots, 1)^T$ is the bias term. Throughout the paper, $\|\cdot\|$ denotes the Euclidean norm (L_2-norm) on the appropriate $\mathbb{R}^n$. We note that it is natural to set

$$b_0 = 0, \tag{3.4}$$

assuming that the model is additive and has a constant returns to scale.

The objective function (3.2a) can be written as

$$\underset{\beta^o\geq 0}{\text{minimize}} \left[(X\beta^o - Y)_o\right]^2. \tag{3.2a$'$}$$

Evidently, the dual problem (3.2) minimizes the difference in frontier production and measurement component-wise, whereas the parametric model (3.3) considers all components simultaneously. The following indicates their relationships more precisely:

Proposition 3.1. *Suppose that the system is strictly productive such that $\beta > 0$ and a constant returns to scale,* (3.4)*. Then, the DEA* (3.2) *and parametric frontier models* (3.3) *are equivalent if and only if the solutions $\hat{\beta}^o$ to* (3.2) *are identical for all the target units $o = 1, 2, \cdots, N$.*

Proof. Because the forward direction is obvious, only the opposite direction will be shown. For a given X, Y, define $D_0 := \{\beta \in \mathbb{R}^M : X\beta - Y \geq 0\}$. Let $\hat{\beta}^o$ ($o = 1, 2, \cdots, N$) and $\hat{\beta}$ be the solutions to (3.2) and (3.3), respectively. Then, $\hat{\beta}^o, \hat{\beta} \in D_0$ for each o and

$$\left|(X\hat{\beta} - Y)_o\right|^2 \geq \left|(X\hat{\beta}^o - Y)_o\right|^2, \quad o = 1, 2, \cdots, N,$$

or

$$\|X\hat{\beta} - Y\|^2 \geq \sum_{o=1}^{N} \left|(X\hat{\beta}^o - Y)_o\right|^2.$$

If $\hat{\beta}^o$ is identical for all o, then

$$\sum_{o=1}^{N} \left|(X\hat{\beta}^o - Y)_o\right|^2 = \|X\hat{\beta}^o - Y\|^2.$$

That is, $\hat{\beta}^o$ solves (3.3) as well. As D_0 and the objective function in (3.3) are convex, it has a unique solution. Therefore,

$$\hat{\beta} = \hat{\beta}^o.$$

□

As pointed out in Section 2, the stochastic model assumes that the difference between ideal production $X\beta + \mathbf{b}_0$ and observation Y consists of inefficiency u and measurement error v, i.e.,

$$\mathbf{e} := (X\beta + \mathbf{b}_0) - Y = u - v. \tag{3.5a}$$

Furthermore, it is widely assumed that the random variables v and u are distributed normally and half-normally, respectively.

$$v \sim N(0, \sigma_v^2),\ u \sim N^+(0, \sigma_u^2). \tag{3.5b}$$

The SFA model estimates the unknown parameters β, b_0, σ_u, and σ_v using the maximum likelihood principle instead of constraint (3.3b), i.e., the SFA model considered herein is formulated as

$$\begin{aligned} \underset{\beta, b_0, \sigma_u, \sigma_v}{\text{maximize}} \quad & l(X, Y; \beta, b_0, \sigma_u, \sigma_v) \\ \text{subject to} \quad & v \sim N(0, \sigma_v^2) \\ & u \sim N^+(0, \sigma_u^2) \end{aligned} \tag{3.6}$$

where l denotes the corresponding log-likelihood function.

Assuming the absence of measurement error in (3.6), i.e., $\sigma_v = 0$, the log-likelihood function can be derived as

$$l(X, Y; \beta, b_0, \sigma_u) = -\frac{N}{2} \log \frac{\pi\sigma_u^2}{2} - \frac{\|X\beta - Y + \mathbf{b}_0\|^2}{2\sigma_u^2},$$

and

$$\begin{aligned} \frac{\partial l}{\partial \beta} &= -\frac{1}{\sigma_u^2} X^T (X\beta - Y + \mathbf{b}_0), \\ \frac{\partial l}{\partial \sigma_u} &= -\frac{N}{\sigma_u} + \frac{\|X\beta - Y + \mathbf{b}_0\|^2}{\sigma_u^3}. \end{aligned} \tag{3.7}$$

We see that $\partial l / \partial \beta = 0$ is a normal equation for $X\beta + \mathbf{b}_0 = Y$. Furthermore, each component of $X\beta - Y + \mathbf{b}_0$ is half-normally distributed, i.e.,

$$X\beta - Y + \mathbf{b}_0 \geq 0.$$

Thus, the solution to the error-free SFA model solves the parametric frontier model (3.3) and vice versa.

Proposition 3.2. *Suppose that the system has no measurement errors. Then, the parametric frontier model* (3.3) *and the SFA model* (3.6) *for the system are equivalent.*

We notice that the assumptions $\beta_0 = 0$ and $\beta > 0$ are unnecessary in Proposition 3.2. Further, the solution is obtained at the boundary $D = \{(\beta, b_0) \in \mathbb{R}^{M+1} : X\beta - Y + \mathbf{b}_0 \geq 0\}$ because the solution to the normal equation is not in the interior of the domain D. This can be deduced from the Karush-Kuhn-Tucker (KKT) conditions for a general log-likelihood function. Indeed, we can see that there is no maximum of the likelihood function when $\sigma_v = 0$, since it diverges as σ_v goes to 0. Thus, the solution to the error-free SFA model is obtained at the boundary of D.

Now, it is claimed that this occurs for the DEA model (3.2), i.e., the solution to (3.2), is obtained at the boundary. For a given β in the interior of the domain D_0, we can find $\delta \in \mathbb{R}^M$ such that $(X(\beta + \delta))_o < (X\beta)_o$ and $X(\beta^o + \delta) - Y \geq 0$. Thus, interior point β cannot be a solution to (3.2). This can

be demonstrated more precisely using a slack variable. Let $S = (s_1^2, \cdots, s_N^2)^T$ be a slack variable and $\lambda = (\lambda_1, \cdots, \lambda_N)^T$ be the Lagrange multiplier. Subsequently, the Lagrangian for (3.2) (replacing (3.2a) with (3.2a′)) is given by

$$\mathcal{L}^o = (X_o\beta^o - y_o)^2 - \lambda^T(X\beta^o - Y - S).$$

The KKT conditions yield

$$\frac{\partial \mathcal{L}^o}{\partial \beta^o} = 2(X_o\beta^o - y_o)X_o - \lambda^T X = 0, \tag{3.8a}$$

$$\frac{\partial \mathcal{L}^o}{\partial \lambda} = -(X\beta^o - Y - S)^T = 0, \tag{3.8b}$$

$$\frac{\partial \mathcal{L}^o}{\partial s_i} = 2\lambda_i s_i = 0, \quad i = 1, \cdots, N. \tag{3.8c}$$

Suppose that the solution $\hat{\beta}^o$ of the DEA model satisfies $X\hat{\beta}^o - Y > 0$. Then, (3.8b) implies that

$$S > 0. \tag{3.9}$$

It follows from (3.8c) that $\lambda = 0$ and (3.8a) implies that

$$(X_o\hat{\beta}^o - y_o)X_o = 0.$$

Together with (3.8b), the following is obtained:

$$\begin{aligned} 0 &= -(X\hat{\beta}^o - Y - S)_o \\ &= -(X_o\hat{\beta}^o - y_o - s_o^2)X_o \\ &= s_o^2 X_o \end{aligned}$$

As $X_o \neq 0$, we have $s_o = 0$, which contradicts the assumption (3.9). Thus, the solution is attained at the boundary of D as claimed.

Define the index set $J := \{j : s_j = 0\}$ using the slack variables. Clearly, J is nonempty because $o \in J$ and

$$(X\beta^o - Y)_j = 0, \quad j \in J.$$

As a special case, consider $M = 1$. Then,

$$x_{j1}\beta^o - y_j = 0, \quad j \in J,$$

or

$$\beta^o = \frac{y_j}{x_{j1}}.$$

As the slack variable $s_i^2 \geq 0$, (3.8b) yields

$$\beta^o \geq \frac{y_i}{x_{i1}} \quad \text{for all } i,$$

or

$$\beta^o = \max_i \frac{y_i}{x_{i1}}.$$

We notice that this is independent of the observation index o, so β^o is the same for all o. From Propositions 3.1 and 3.2, the following theorem is obtained:

Theorem 3.3. *Suppose that the system is productive ($\beta \geq 0$) and a constant returns to scale ($\beta_0 = 0$). Then, the solutions to the DEA model* (3.2)*, parametric frontier model* (3.3)*, and error-free SFA model* (3.6) *are attained at the boundary of the feasible domain. Furthermore, all the solutions are identical, assuming $M = 1$.*

In the parametric frontier and SFA models, the coefficient β and inefficiency are calculated by simultaneously considering all the units, whereas they depend on the peers in the DEA model. In the CRS DEA model, the number of peers is at most $M + q - 1$ when the number of outputs is q (see, e.g., [34]). In Theorem 3.3, assuming $M = 1$ implies that the coefficients β^o are the same for all $o = 1, \cdots, N$, which is the steepest slope of the line passing through the origin and (x_i, y_i). Because the number of peers is at most 1, all β^o are the same and the peer of all units is the best performing. Thus, the inefficiency of each unit can be calculated only from the best DMU.

Furthermore, the feasible domains for (3.2) and (3.3) are the same, and the objective function for (3.2) is the oth entry of that for (3.3). Thus, the DEA model can be understood as a projection of a parametric frontier model or error-free SFA model.

Theorem 3.3 establishes an exact DEA–SFA correspondence only in the stylized setting of CRS with a single input and no statistical noise. Although the theorem does not easily extend to the general case of multiple inputs, it serves as the conceptual motivation for combining SFA-based noise estimation with deterministic DEA. We observe that multi-input DEA operates analogously to the single-input case in the dual view, because DEA converts multiple inputs into a scalar virtual composite input via optimal weights [35]. Indeed, for any target DMU_o, the optimal input weights $\mathbf{w}^o$ convert each multi-input vector X_i into a scalar virtual input $X_i\mathbf{w}^o$. Under these weights, the dual feasibility constraints take the single-input form $h^o y_i \leq X_i\mathbf{w}^o$ for all i, so each supporting hyperplane used by DEA induces an effectively one-dimensional representation of the data. Theorem 3.3 clarifies that, in a truly one-dimensional CRS setting, DEA and an error-free frontier formulation coincide, which makes it natural to use a frontier model to estimate the two-sided noise component and move the observations toward a noise-free setting before applying DEA. This insight suggests that correcting the data toward the true frontier using estimates from a general SFA model can mitigate the mismeasurement issues inherent in deterministic DEA, even when multiple inputs are present.

4. Mismeasurement mitigation method

Section 3 provides a motivating link between DEA and an error-free frontier formulation. Furthermore, we show that the solution to the DEA model is attained at the boundary of the domain. Thus, the measurement errors, particularly on the peer units, may produce significantly distorted results. For this reason, we modify the deterministic DEA model to mitigate mismeasurement.

$$\underset{\beta^o}{\text{minimize}} \quad (X\beta^o - Y + \xi)_o \tag{4.1a}$$

$$\text{subject to} \quad X\beta^o - Y + \xi \geq 0 \tag{4.1b}$$

Here, $\xi = (\xi_1, \xi_2, \cdots, \xi_N)^T$ represents the errors of the units, which are known. When the errors are non-negligible, it is necessary to estimate them. Several attempts have been made to overcome this difficulty, such as CCDEA and SDEA, as we discuss in Section 2. Herein, the error of each unit is determined

from the prescribed quantile of a certain probability distribution. However, the setting of quantile is questionable. In the previous section, we demonstrate that the DEA model can be conceptualized as a projection of an error-free SFA model. In light of this connection between DEA and SFA, we propose employing the SFA model to estimate the error associated with each individual unit. The proposed method consists of the following steps:

Algorithm 1 Mismeasurement mitigation (MM) method

(1) Given data $\{(X, Y)\}$, solve the SFA model (3.6).
(2) Estimate the individual error terms $\hat{v} = (\hat{v}_1, \hat{v}_2, \cdots, \hat{v}_N)^T$.
(3) Set $\xi = \hat{v}$ and solve the linear programming model (4.1) to estimate the inefficiency of each unit.

Under the SFA specification described in (3.5), the inefficiency component is estimated using the conditional mean estimator of Jondrow et al. [36],

$$\hat{u}_i = \hat{\sigma}_* \left(\frac{\phi(\hat{m}_i)}{\Phi(\hat{m}_i)} + \hat{m}_i \right), \quad \hat{m}_i = \frac{e_i \hat{\lambda}}{\hat{\sigma}}$$

where $\hat{\lambda} = \hat{\sigma}_u / \hat{\sigma}_v$, $\hat{\sigma} = \sqrt{\hat{\sigma}_u^2 + \hat{\sigma}_v^2}$, and $\hat{\sigma}_* = \hat{\sigma}_u \hat{\sigma}_v / \hat{\sigma}$, and ϕ and Φ denote the standard normal pdf and cdf, respectively. The statistical noise component in Step (2) is then recovered as

$$\hat{v}_i = \hat{u}_i - e_i.$$

First, Algorithm 1 is simulated using the Cobb–Douglas model. Consider the Cobb–Douglas function with one output y_i from two inputs (x_{i1}, x_{i2}) for ith DMU, where $i = 1, 2, \cdots, N$ such that

$$y_i = 2 x_{i1}^{0.4} x_{i2}^{0.6}, \quad i = 1, 2, \cdots, N.$$

Each element of the input X_i is randomly generated from a uniform distribution $U(5, 15)$. Taking the logarithm and adding the inefficiency u_i and error v_i, the following is obtained:

$$\log y_i = \log 2 + 0.4 \log x_{i1} + 0.6 \log x_{i2} - u_i + v_i, \quad i = 1, 2, \cdots, N. \tag{4.2}$$

We assume that inefficiency u_i is half-normally distributed by $|N(0, \sigma_u^2)|$ and error v_i is normally distributed by $N(0, \sigma_v^2)$. Herein, 72 scenarios are simulated with $\sigma_u = 0.05, 0.10, 0.15, \cdots, 0.4$ and $\sigma_v = 0.01, 0.10, 0.15, \cdots, 0.45$.

Because the proposed method assumes the constant returns to scale, we consider 1 as a fixed input for each unit such as

$$\log y_i = (\log 2) \cdot 1 + 0.4 \log x_{i1} + 0.6 \log x_{i2} - u_i + v_i. \tag{4.3}$$

That is, $X_i = (1, \log x_{i1}, \log x_{i2})$ and $\log y_i$ are the input and output for ith DMU, respectively.

For the Monte Carlo simulation, we assume that there are 100 DMUs (i.e., $N = 100$) and generate 100 replication datasets for $\{(x_{i1}, x_{i2}), u_i, v_i\}_{i=1}^{100}$ for each scenario (σ_u, σ_v). The output $\log y_i$ is given by (4.3), and the best practice $\log \hat{y}_i$ is computed as follows:

$$\log \hat{y}_i = (\log 2) \cdot 1 + 0.4 \log x_{i1} + 0.6 \log x_{i2}, \quad i = 1, 2, \cdots, 100.$$

Accordingly, the best practice output, the simulation inefficiency target, and all accuracy measures (e.g., MAE) in the Monte Carlo study are defined and computed in the log-transformed output space. Thus, the actual inefficiency is

$$\theta_i^{ACT} = \frac{\log y_i}{\log \hat{y}_i}.$$

We begin with implementing the DEA using formulation (2.2) for the dataset $\{(X_i, \log y_i)\}_{i=1}^{100}$. Subsequently, the inefficiencies θ_i^{DEA} obtained from the DEA model are compared to the actual inefficiencies θ_i^{ACT} using the mean absolute error (MAE), which is one of the standard indicators (see [37] and the references therein),

$$MAE^D = \frac{1}{100}\sum_{i=1}^{100}\left|\theta_i^{ACT} - \theta_i^{DEA}\right|.$$

Using the same dataset, θ_i^{MM} is computed according to Algorithm 1. Note that in the third step of Algorithm 1, the inefficiency θ^{MM} is estimated using the DEA model (2.2) by replacing Y with $Y - \xi$. Then, the corresponding MAE is given by

$$MAE^M = \frac{1}{100}\sum_{i=1}^{100}|\theta_i^{ACT} - \theta_i^{MM}|.$$

To compare the proposed method with a known stochastic DEA model, the CCDEA method was simulated using the same data. As noted in Section 2, a limitation of the CCDEA model is determining the noise level. In this study, we evaluate two versions of CCDEA. First, ξ is assumed to be an independent normally distributed random variable, i.e., $\xi_i \sim N(0, \sigma_v^2)$ using the prescribed true σ_v^2 in the evaluation (2.4). This serves as an idealized, best-case benchmark, denoted as CCDEA. Second, for an equal-footing comparison with MM, we implemented CCDEA using the SFA-estimated noise variance $\hat{\sigma}_v^2$ denoted as CCDEA_est. We define their mean absolute errors as MAE^C and $MAE^{C_{est}}$, respectively.

The simulation is repeated using 100 replication datasets to compute $\{(MAE_k^D, MAE_k^M, MAE_k^C, MAE_k^{C_{est}})\}_{k=1}^{100}$ for each scenario.

Figure 1 shows boxplots of the MAEs for each scenario. For all choices of σ_u, the accuracy of classical DEA decreases significantly as the variance of noise σ_v^2 increases. As expected, the idealized CCDEA using the true σ_v outperforms CCDEA_est. Notably, the proposed MM method achieves lower MAEs compared to DEA, CCDEA_est, and the idealized CCDEA in most cases.

To formally substantiate these observations, we performed paired Wilcoxon signed-rank tests on the replication-wise MAE differences for each scenario, applying Holm's procedure to adjust p-values and control the family-wise error rate at 5% [38]. Figure 2 presents the resulting significance maps. The left panel confirms that while DEA is statistically superior at the lowest noise level ($\sigma_v = 0.01$, indicated in red), the MM method significantly outperforms DEA in all scenarios where $\sigma_v \geq 0.1$ (indicated in blue). The middle and right panels compare MM to the idealized CCDEA and CCDEA_est, respectively. The dominance of blue and gray regions indicates that MM yields significantly lower MAE or shows no statistically significant difference compared to both chance-constrained baselines across most of the scenario grid.

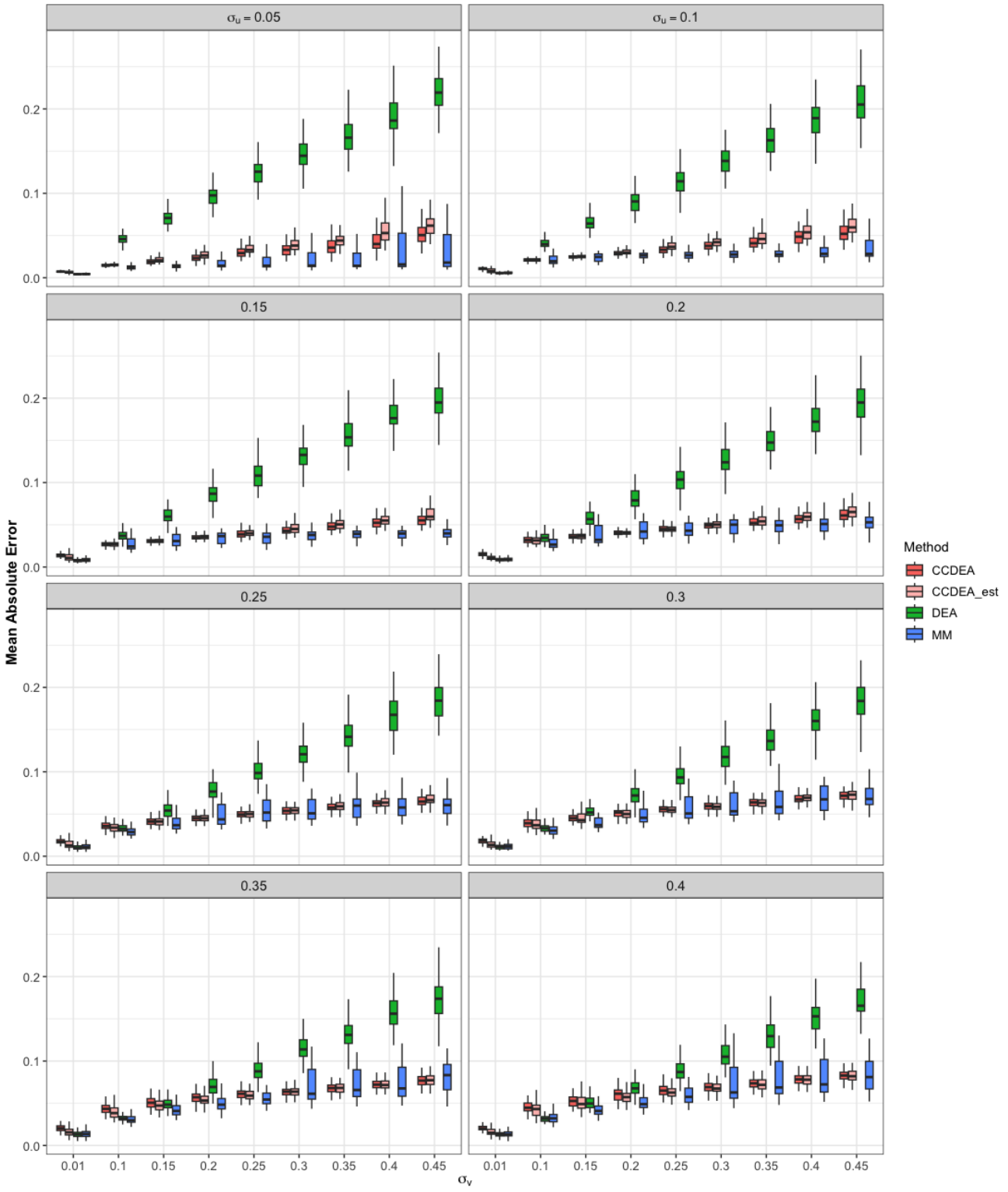

Figure 1. The boxplots show the MAEs with true inefficiency in various scenarios. CCDEA utilizes the true noise variance, while CCDEA_est utilizes the SFA-estimated noise variance. The figures show that the proposed method yields lower MAE as σ_v increases.

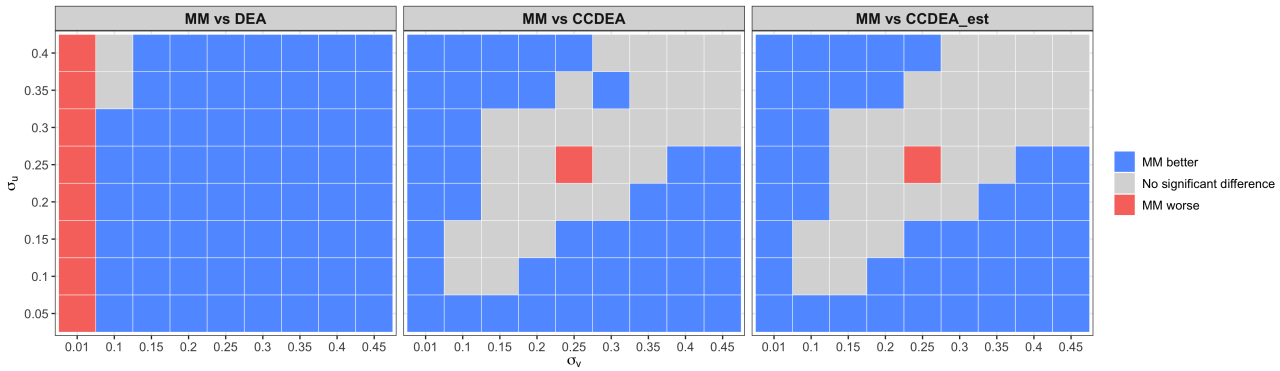

Figure 2. Scenario-wise significance maps comparing MM with DEA, CCDEA, and CCDEA_est over the (σ_u, σ_v) grid. Statistical significance was determined using paired Wilcoxon signed-rank tests (two-sided) on the replication-wise MAE differences. P-values were adjusted using Holm's procedure across all scenarios to control the family-wise error rate at 5%. Cells are classified into three categories: MM has significantly lower MAE (blue), no significant difference (gray), or MM has significantly higher MAE (red).

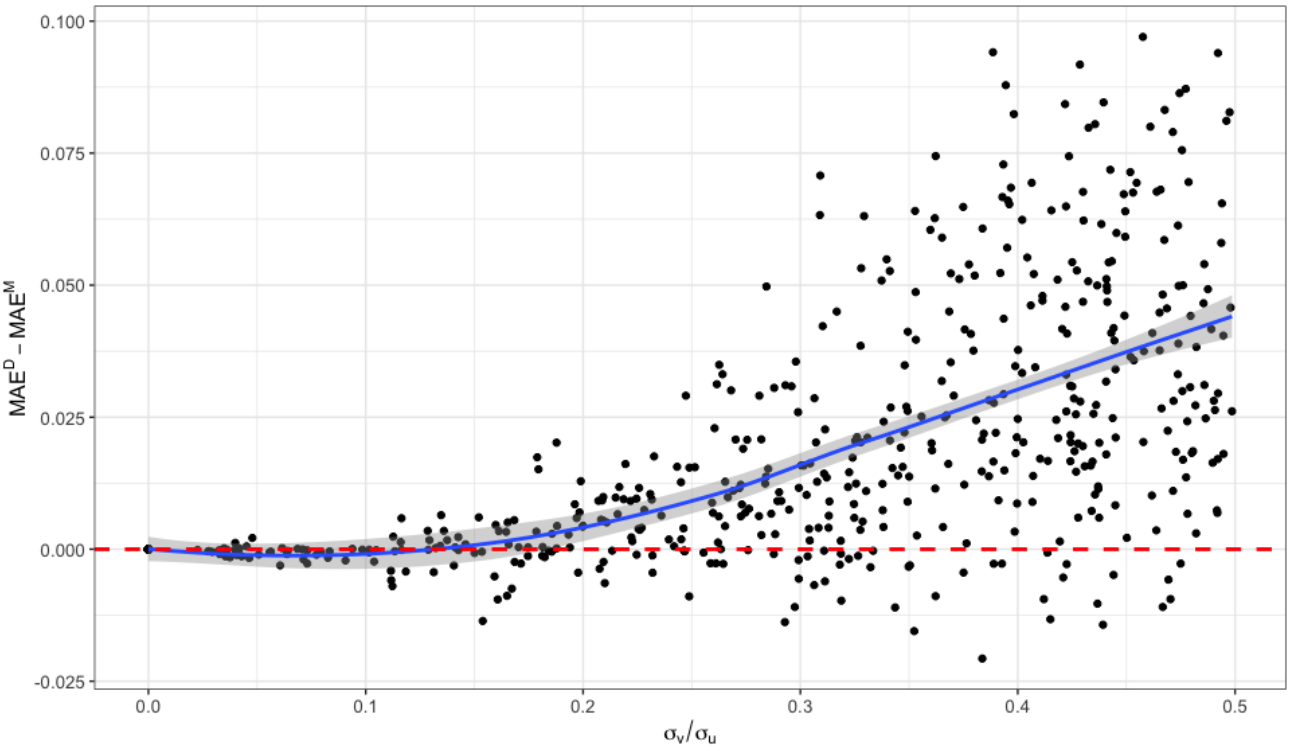

Figure 3. The figure shows the relationship between $MAE^D - MAE^M$ and the ratio σ_v/σ_u. In our simulation design, the crossover occurs at a small ratio (approximately 0.15), below which DEA can be slightly more accurate and above which MM tends to be more accurate.

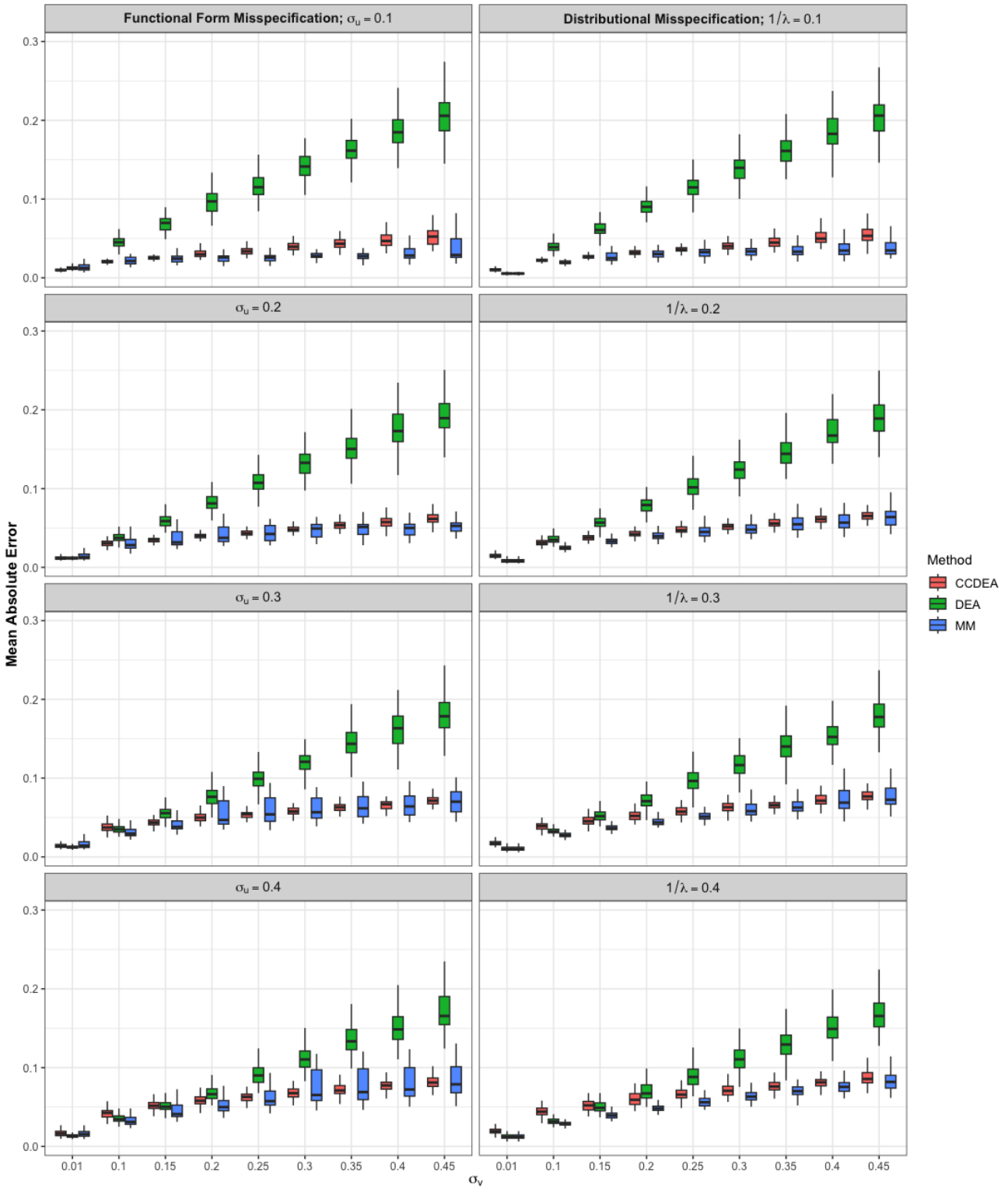

Figure 4. Boxplots of MAEs under SFA misspecification. The left panels show functional-form misspecification (true translog frontier), and the right panels show distributional misspecification (true exponential inefficiency). In both cases, the proposed MM method employs a Cobb–Douglas SFA with a half-normal assumption and delivers improvement over classical DEA when noise is moderate to high.

These results are notable given that the idealized CCDEA utilizes prior knowledge of the true noise level σ_v^2 in the simulations – a condition that is rarely achievable in practice. As pointed out in Introduction, estimating the noise level in practice is challenging, which is one of the limitations of CCDEA and reflected in the larger errors of CCDEA_est in Figure 1.

Although CCDEA is implemented using the true noise variance in the simulations, it corrects for stochasticity through a global chance constraint. In contrast, the proposed MM approach estimates and removes unit-level noise prior to DEA, which directly mitigates noise-induced changes in peer selection. This structural difference explains why MM can outperform CCDEA even under idealized CCDEA implementations. Moreover, from a computational perspective, MM consists of a single SFA estimation step followed by standard DEA computations on adjusted data, and can therefore be implemented efficiently using widely available SFA and DEA software.

Interestingly, for scenarios with small σ_v, the classical DEA method demonstrates better performance than MM. This behavior is closely related to the ratio σ_v/σ_u. Indeed, when σ_v is small relative to σ_u, the deterministic nature of DEA allows it to produce more accurate results. This is because in low-noise settings, even minor estimation errors from the initial SFA stage can make the noise-adjusted data less accurate than the original data, which explains why classical DEA can perform slightly better. However, as σ_v/σ_u increases, the effect of noise becomes more pronounced, causing DEA's accuracy to degrade. Under these conditions, the MM method proves more robust and reliable, as the magnitude of the true noise begins to dominate any SFA estimation error, making the noise-correction process highly beneficial. Figure 3 illustrates the relationship between σ_v/σ_u, computed using SFA, and $\mathrm{MAE}^{\mathrm{D}} - \mathrm{MAE}^{\mathrm{M}}$ for each dataset, which are plotted as dots, along with fitted curves. Three hundred data points were randomly sampled from each scenario and plotted near the y-axis. The results indicate that for small σ_v/σ_u, the classical DEA method remains reliable. However, as the ratio increases, the proposed MM method surpasses DEA in accuracy, demonstrating its robustness under higher noise levels.

To address finite-sample effects, we repeated the Monte Carlo design using a varying number of DMUs ($N = 30, 50, 70, 100$), with 100 replications per (σ_u, σ_v) scenario. For each scenario, we determined statistical significance using the same methodology, paired Wilcoxon signed-rank tests with Holm's adjustment, as in Figure 2. Figure 5 summarizes the share of scenarios in which the proposed MM method outperforms the benchmarks. In the comparison with DEA (left panel), MM maintains a superior performance rate (blue bars) exceeding 60% even at $N = 30$. The scenarios where MM performs worse (red bars) are strictly confined to low-noise settings, a pattern consistent with our findings for $N = 100$. In the comparison with CCDEA (right panel), we observe that with a small number of DMUs ($N = 30$), the method is more sensitive to estimation errors; MM performs worse than the idealized CCDEA in approximately 20% of scenarios (red bar). However, this gap closes rapidly as N increases. As N grows from 30 to 100, the accuracy of the SFA noise estimation improves, and the MM method's performance advantage becomes statistically distinct, with the share of 'MM Better' scenarios increasing to approximately 50% at $N = 100$, while the share of 'MM Worse' scenarios vanishes.

The proposed method inherits the assumptions of the SFA model, since the estimated noise is derived from the SFA specification. This implies that the noise estimates may be compromised if the chosen functional form or distributional assumptions are incorrect. To examine robustness to such misspecification, we conduct two stress tests. First, for functional-form misspecification, the true data

are generated from a CRS translog frontier,

$$\log y_i = \log 2 + 0.4 \log x_{i1} + 0.6 \log x_{i2} + \tfrac{\gamma}{2}(\log x_{i1} - \log x_{i2})^2 - u_i + v_i,$$

with $\gamma = 0.3$, and u_i follows a half-normal distribution. Second, for distributional misspecification, the true inefficiency u_i is generated from an exponential distribution with means $1/\lambda = 0.1, 0.2, 0.3, 0.4$ under a Cobb-Douglas frontier. We note that the exponential distribution with mean $1/\lambda$ has a variance of $1/\lambda^2$, whereas the half-normal distribution has a variance of $\sigma_u^2(1 - 2/\pi)$. In both tests, Step 1 of Algorithm 1 fits a Cobb–Douglas SFA assuming a half-normal inefficiency distribution. We use the same grids for σ_v as in the design for Figure 1.

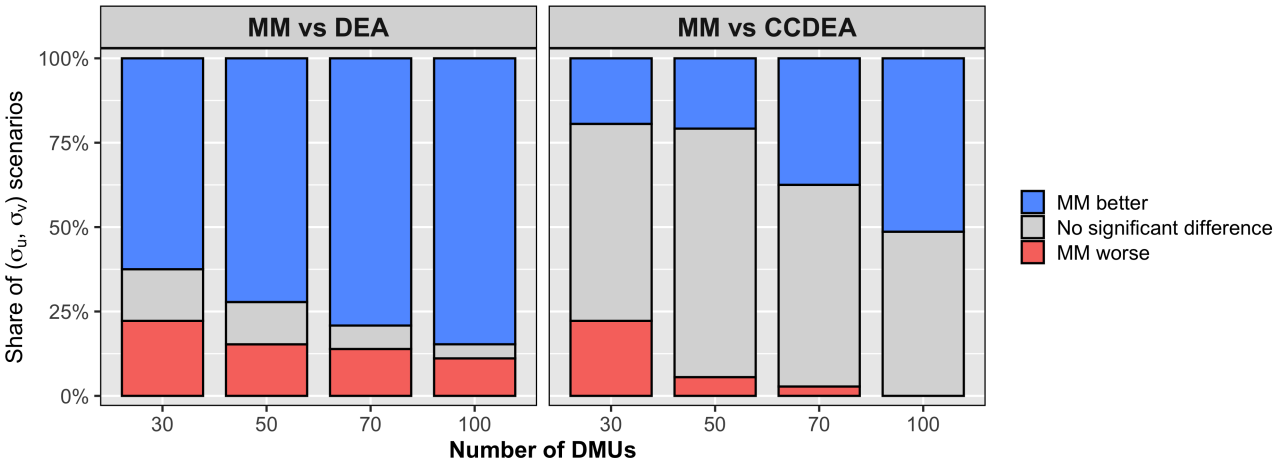

Figure 5. The figure shows the performance of MM method across varying sample sizes. The bar charts display the proportion of simulation scenarios over the defined (σ_u, σ_v) grid where the proposed MM method performs significantly better (blue), shows no significant difference (gray), or performs significantly worse (red) compared to classical DEA (left panel) and CCDEA (right panel). Significance was determined using paired Wilcoxon signed-rank tests with Holm's adjustment.

These results, as shown in Figure 4, closely mirror Figure 1. In very low noise, classical DEA can be slightly more accurate, whereas for moderate to high noise the MM method consistently reduces mismeasurement relative to DEA. To formally assess the performance differences under SFA misspecification, we applied paired Wilcoxon signed-rank tests with Holm's adjustment to these results. Figure 6 presents the corresponding significance maps. The statistical results show that the MM method reduces mismeasurement relative to classical DEA when noise is moderate to high. Against CCDEA, the MM method maintains its advantage or shows no significant difference across the most of the grids. This finding confirms that, while the proposed method necessarily depends on the assumptions of the SFA model used in the first step, its practical performance does not require perfectly specified SFA assumptions. Let $\hat{v}$ denote the estimated statistical noise from Step 1. Since DEA is applied to the adjusted output $Y - \hat{v}$, the noise correction procedure is beneficial whenever the mean squared error of the noise estimate is less than the variance of the true noise, that is, when $\mathbb{E}[(v - \hat{v})^2] < \mathrm{Var}(v)$. The stress test demonstrates that the MM approach retains its advantage under mild SFA misspecification,

continuing to reduce mismeasurement when noise is moderate to high. We remark that a comprehensive sensitivity study of how perturbations in the estimated noise affect the corrected DEA scores is beyond the scope of this paper and is left for future research.

In the first simulation, artificial data is used because the true inefficiency is required to measure the accuracy of the proposed MM method in Algorithm 1. Next, the DEA and MM methods are simulated and compared with actual data by treating the original dataset as an unperturbed baseline and using the corresponding DEA scores as a reference for assessing robustness under injected noise. We note that the DEA scores computed from the original (unperturbed) data are used as a pragmatic reference for noise-sensitivity comparisons, as true efficiencies are unobservable in empirical datasets. To quantify the inherent uncertainty of this pragmatic reference, we applied the bootstrap procedure by Simar and

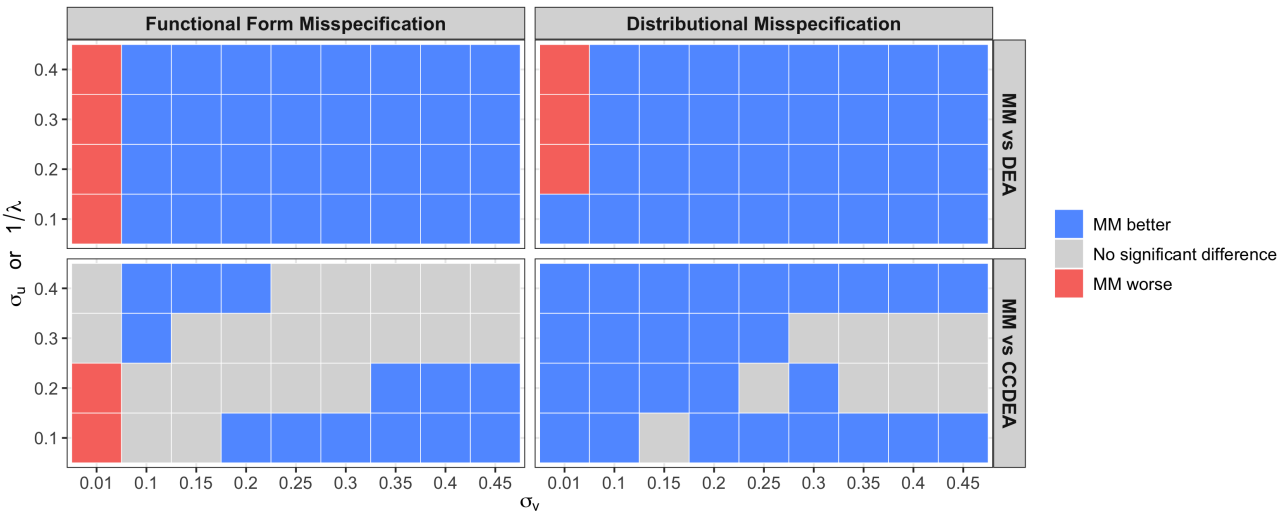

Figure 6. Scenario-wise significance maps for the misspecification experiments. Each cell reports a paired Wilcoxon signed-rank test based on replication-wise MAE differences, with Holm-adjusted p-values. The upper row compares MM with DEA, and the lower row compares MM with CCDEA. The left column reports functional-form misspecification, and the right column reports distributional misspecification. Blue cells indicate significantly lower MAE for MM, gray cells indicate no significant difference, and red cells indicate significantly higher MAE for MM.

Figure 7 shows the results with 'milkProd' and 'pigdata' datasets in Benchmarking package for R. In 'milkProd', the output is the milk production and the input consists of the energy expenses, veterinary expenses, and number of cows with 108 observations. We take the logarithms to the dataset and add random noise generated from the normal distribution $N(0, \sigma_v^2)(\sigma_v = 0.25, 0.5, \cdots, 2)$ to the output. For 'pigdata', the costs for 'fertilizer', 'feedstuff', 'land', 'labor', 'machinery', and 'other capital' are considered as inputs and the total revenue as output. The dataset consists of 248 observations. In simulations, the inefficiencies for unperturbed data obtained from the DEA method are regarded as the reference inefficiencies for each observation. The noise from the normal distribution $N(0, \sigma_v^2)$ was added and the problem was solved using the DEA and MM methods. The MAEs with reference inefficiencies are presented as boxplots for 100 repetitions. The red dashed horizontal lines in Figure 7 represent the average half-widths of the 95% bootstrap confidence intervals, which are 0.0111 for 'milkProd' and 0.0107 for 'pigdata'. We remark that the bootstrap intervals quantify uncertainty in the unperturbed DEA reference scores. They are not bootstrap intervals for the MAE difference between DEA and MM under injected noise. The results show that the MM method produces inefficiencies closer to the reference than classical DEA as σ_v increases. However, because the true efficiencies are unknown, it remains to be determined which method is better for smaller noise, such as $\sigma_v = 0.25, 0.5$ for 'pigdata'. Generally, the DEA offers no diagnostic statistics to determine whether a model is misspecified [39]. However, the averages of σ_v/σ_u for each scenario are
$(11.97, 39.69, 44.47, 50.01, 45.10, 50.56, 60.40, 40.02)$, which are relatively large. Therefore, we may interpret that the MM method is more reliable than the classical DEA method, even for cases with a smaller σ_v.

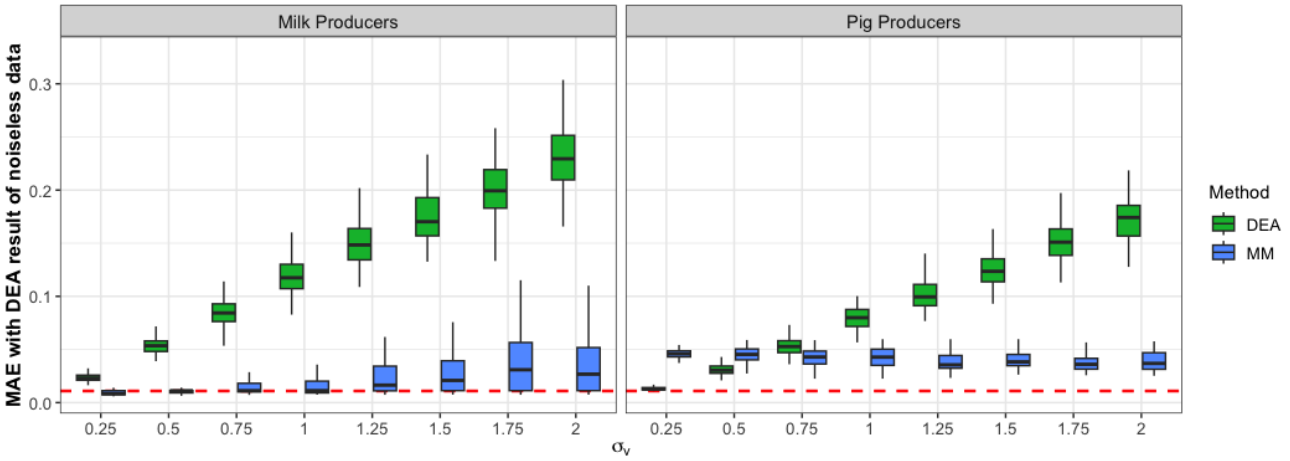

Figure 7. The figures show the MAEs of inefficiencies for DEA and MM methods on noisy empirical data. The noise from the normal distribution $N(0, \sigma_v^2)$ is included in the output for each scenario. The DEA result of the original unperturbed data serves as the pragmatic reference. The red dashed horizontal lines indicate the average half-widths of the 95% bootstrap confidence intervals for the reference scores [14]. The data in the left and right panels correspond to the 'Milk Producers' and 'Pig Producers', respectively. They show that the MM method produces inefficiencies closer to the reference inefficiencies than the classical DEA as σ_v increases.

5. Conclusion and further discussion

This study proposes a potential method to mitigate the efficiency mismeasurement of deterministic DEA by correcting the statistical noise for individual decision-making units. As the solution to the DEA is obtained at the boundary of the feasible domain, the DEA is susceptible to small statistical noise such as outliers, allowing for the occurrence of mismeasurement that may yield significantly erroneous results. To overcome this, the SFA model is used to measure individual measurement errors and mitigate mismeasurement. In contrast, the existing stochastic DEA models require predetermining their stochastic parameters, such as the regression quantile or the probability that the best practice exceeds the observed outputs. The proposed method, however, corrects the statistical errors in the observations prior to analysis, thereby maintaining the nonparametric modeling advantages of DEA. The Monte Carlo simulation study shows that the proposed method is more accurate than the classical DEA and CCDEA in general. Because the proposed method relies on the SFA model to estimate statistical noise, its performance inherently depends on how well the estimated noise $\hat{v}$ approximates the true noise v. While our stress tests confirm robustness against omitted nonlinear terms and an exponentially distributed inefficiency, performance may deteriorate under deeper structural departures such as heavy-tailed noise or heteroscedasticity, which can bias the separation of noise and inefficiency. As pointed out in Section 4, the correction remains beneficial provided the mean squared error of the noise estimate is less than the true noise variance, i.e., $\mathbb{E}[(v - \hat{v})^2] < \mathrm{Var}(v)$. This condition typically holds under mild misspecification, explaining the method's effectiveness in moderate-to-high noise regimes. This indicates that the method's practical usefulness extends beyond perfectly specified SFA settings.

The motivation for the approach is the relationship between CRS DEA and a noise-free SFA frontier stated in Theorem 3.3. Although the theorem is established for a single input, it frames the proposed MM procedure as a hybrid heuristic. The SFA stage is used solely to estimate the noise component, not to impose a parametric frontier shape on the DEA stage. Consequently, improvements are expected when output noise is non-negligible and the noise estimate $\hat{v}$ is sufficiently accurate. However, this single-input analogy may break down in certain multiple-input configurations due to structural differences between DEA and SFA. In DEA, optimal weights are determined locally for each target unit, and some inputs may be assigned zero weights. In contrast, standard SFA applies a single global parametric function to all inputs simultaneously. If the true production technology has local structures that the SFA functional form cannot capture, the SFA projection may not align with the localized DEA hyperplanes. In such cases, the SFA-estimated noise term might absorb structural misspecification errors instead of statistical noise. Extending the theoretical mapping to multiple inputs remains a valuable direction for future research.

A primary avenue for future work is to extend the MM method to variable returns to scale (VRS). Under VRS, the efficient frontier is a convex hull rather than a ray from the origin, and the dual formulation requires the convexity constraint. Adapting the SFA-based correction to this setting will require revisiting the projection interpretation and examining how the correction interacts with the decomposition of technical and scale efficiencies. This extension is particularly relevant in sectors such as banking, healthcare, and education where VRS is the prevailing assumption. To provide empirical insight, an exploratory simulation under a VRS setting is presented in the Appendix. The results indicate that the proposed method continues to reduce mismeasurement relative to classical DEA, suggesting its practical relevance beyond the CRS setting.

Another important direction is to move beyond the single-output framework. While the DEA step accommodates multiple outputs, the first-stage SFA would need a multi-output specification. Developing and validating a multi-output version of the procedure is a natural next step.

For noise-free data or data with very small errors, the noise-correction step is unnecessary and classical DEA can be preferable. Therefore, method selection depends on the relative magnitudes of the statistical noise and inefficiency components. Although a formal decision rule is beyond the current scope, the simulation results provide a reference point. In applications, the ratio σ_v/σ_u is estimated rather than known, and the precision of this estimate should be considered. A practical diagnostic is to examine $\hat{\sigma}_v/\hat{\sigma}_u$ together with evidence on the noise component from the SFA estimation, such as a likelihood-ratio test when available and the standard error or relative standard error of $\hat{\sigma}_v$. If the estimated noise component is not statistically distinguishable from zero or is imprecisely estimated, classical DEA is preferable. If the estimated noise component is non-negligible and precisely estimated, and $\hat{\sigma}_v/\hat{\sigma}_u$ is above the simulation crossover range, the MM approach is more appropriate. This interpretation is consistent with the crossover pattern reported in Figure 3.

It is also worth noting that, while this study employs a parametric SFA approach for noise estimation in the first stage, recent developments in continuous optimization and machine learning provide robust alternatives for frontier estimation. Techniques such as multiple kernel learning (MKL) and infinite kernel learning (IKL) (see, e.g., [40]) offer flexible kernel-based representations of complex, nonlinear production technologies. In addition, conic multivariate adaptive regression splines (CMARS) and their robust extensions, such as robust conic multivariate adaptive regression splines (RCMARS), robust conic generalized partial linear models (RCGPLM), and robust conic generalized additive models (RCGAM) (see e.g., [41] and [42]), have demonstrated significant potential in regression and classification under uncertainty. While the proposed method in this study primarily targets statistical noise in the response variable (output), integrating these advanced methodologies into the first stage noise estimation to explicitly address non-determinism in input variables, as well as outputs, represents a promising avenue for future research. Developing a formal statistical test for model choice and extending the model to a multi-output framework remain important topics for further investigation.

Finally, while this study adopts the standard radial DEA formulation to maintain consistency with the proposed SFA-based correction approach, extending the noise-correction procedure to non-radial or additive DEA formulations remains an important topic for future research. In particular, integration with slack-based models, such as the pure technical virtual gap analysis (PT VGA) framework (e.g., [43]), may be considered to account for mix inefficiencies in the presence of statistical noise.

Author contributions

These authors contributed equally to this work.

Use of Generative-AI tools declaration

During the preparation of this manuscript, the authors used ChatGPT for language editing, improvement of expression, and bibliography formatting assistance. All AI-assisted content was carefully reviewed and revised by the authors, who take full responsibility for the final content of the manuscript.

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Conflict of interest

The authors declare that they have no conflict of interest.

References

1. A. Charnes, W. W. Cooper, E. Rhodes, Measuring the efficiency of decision making units, *Eur. J. Oper. Res.*, **2** (1978), 429–444. https://doi.org/10.1016/0377-2217(78)90138-8
2. D. Aigner, C. A. K. Lovell, P. Schmidt, Formulation and estimation of stochastic frontier production function models, *J. Econom.,* **6** (1977), 21–37. https://doi.org/10.1016/0304-4076(77)90052-5
3. W. Meeusen, J. van den Broeck, Efficiency estimation from Cobb–Douglas production functions with composed error, *Int. Econ. Rev.,* **18** (1977), 435–444. https://doi.org/10.2307/2525757
4. K. C. Land, C. A. K. Lovell, S. Thore, Chance-constrained data envelopment analysis, *Manag. Decis. Econ.,* **14** (1993), 541–554. https://doi.org/10.1002/mde.4090140607
5. A. Charnes, W. W. Cooper, Chance-constrained programming, *Manag. Sci.,* **6** (1959), 73–79. https://doi.org/10.1287/mnsc.6.1.73
6. R. D. Banker, *Stochastic data envelopment analysis,* Working paper, Carnegie Mellon University, 1988.
7. T.-Y. Chen, A comparison of chance-constrained DEA and stochastic frontier analysis: bank efficiency in Taiwan, *J. Oper. Res. Soc.,* **53** (2002), 492–500. https://doi.org/10.1057/palgrave.jors.2601318
8. D. Wu, D. L. Olson, A comparison of stochastic dominance and stochastic DEA for vendor evaluation, *Int. J. Prod. Res.,* **46** (2008), 2313–2327. https://doi.org/10.1080/00207540601039775
9. Y. Zha, L. Zhao, Y. Bian, Measuring regional efficiency of energy and carbon dioxide emissions in China: A chance constrained DEA approach, *Comput. Oper. Res.,* **66** (2016), 351–361. https://doi.org/10.1016/j.cor.2015.07.021
10. V. J. Bolós, R. Benítez, V. Coll-Serrano, Chance constrained directional models in stochastic data envelopment analysis, *Oper. Res. Perspect.,* **12** (2024), 100307. https://doi.org/10.1016/j.orp.2024.100307
11. S. Jradi, J. Ruggiero, Stochastic data envelopment analysis: A quantile regression approach to estimate the production frontier, *Eur. J. Oper. Res.,* **278** (2019), 385–393. https://doi.org/10.1016/j.ejor.2018.11.017
12. J. K. Sengupta, Data envelopment analysis for efficiency measurement in the stochastic case, *Comput. Oper. Res.,* **14** (1987), 117–129. https://doi.org/10.1016/0305-0548(87)90004-9
13. L. Simar, How to improve the performances of DEA/FDH estimators in the presence of noise? *J. Prod. Anal.,* **28** (2007), 183–201. https://doi.org/10.1007/s11123-007-0057-3

14. L. Simar, P. W. Wilson, Sensitivity analysis of efficiency scores: How to bootstrap in nonparametric frontier models, *Manag. Sci.,* **44** (1998), 49–61. https://doi.org/10.1287/mnsc.44.1.49

15. A. Kneip, B. U. Park, L. Simar, A note on the convergence of nonparametric DEA estimators for production efficiency scores, *Econometric Theory,* **14** (1998), 783–793. https://doi.org/10.1017/S0266466698146042

16. A. Kneip, L. Simar, P. W. Wilson, A computationally efficient, consistent bootstrap for inference with non-parametric DEA estimators, *Comput. Econ.,* **38** (2011), 483–515. https://doi.org/10.1007/s10614-010-9217-z

17. A. Kneip, L. Simar, P. W. Wilson, When bias kills the variance: Central limit theorems for DEA and FDH efficiency scores, *Econometric Theory,* **31** (2015), 394–422. https://doi.org/10.1017/S0266466614000413

18. M. Zarrin, J. O. Brunner, Analyzing the accuracy of variable returns to scale data envelopment analysis models, *Eur. J. Oper. Res.,* **308** (2023), 1286–1301. https://doi.org/10.1016/j.ejor.2022.12.015

19. G. P. Afonso, J. R. Figueira, D. C. Ferreira, Dealing with uncertainty in healthcare performance assessment: a fuzzy network-DEA approach with undesirable outputs, *Int. Trans. Oper. Res.,* **32** (2025), 2732–2766. https://doi.org/10.1111/itor.13490

20. A. Hadi-Vencheh, M. Khodadadipour, Y. Tan, H. Arman, D. Roubaud, Cross-efficiency analysis of energy sector using stochastic DEA: Considering pollutant emissions, *J. Environ. Manage.,* **364** (2024), 121319. https://doi.org/10.1016/j.jenvman.2024.121319

21. O. Abdolazimi, M. Sepehrifar, D. Shishebori, I. Banicescu, S. Azad, K. Pishini, et al., A stochastic DEA-GARMA integration for dynamic forecasting of undesirable outputs applied to the temporal efficiency analysis of water and sewerage services, *J. Clean. Prod.,* **521** (2025), 146177. https://doi.org/10.1016/j.jclepro.2025.146177

22. Y. Z. Vishghaei, S. Kordrostami, A. Amirteimoori, S. Shokri, An inverse stochastic two-stage DEA model to deal with resource planning, *J. Ind. Manag. Optim.,* **21** (2025), 3031–3053. https://doi.org/10.3934/jimo.2025002

23. J. Wu, W. Xie, Y. Sheng, A generalized DEA model with imprecise data, *J. Ind. Manag. Optim.,* **20** (2024), 2617–2639. https://doi.org/10.3934/jimo.2024018

24. S. Yang, X. Yang, A new uncertain two-stage network DEA model based on chance-constrained programming, *J. Ind. Manag. Optim.,* **21** (2025), 2032–2051. https://doi.org/10.3934/jimo.2024161

25. H. Hosseini-Nasab, V. Ettehadi, Development of opened-network data envelopment analysis models under uncertainty, *J. Ind. Manag. Optim.,* **19** (2023), 1963–1982. https://doi.org/10.3934/jimo.2022027

26. A. Arya, A. Hatami-Marbini, A new efficiency evaluation approach with rough data: An application to Indian fertilizer, *J. Ind. Manag. Optim.*, **19** (2023), 5183–5208. https://doi.org/10.3934/jimo.2022168

27. K. K. Mohanta, D. S. Sharanappa, A novel technique for solving intuitionistic fuzzy DEA model: an application in Indian agriculture sector, *Manag. Syst. Eng.,* **2** (2023), 12. https://doi.org/10.1007/s44176-023-00022-7

28. M. Majdi, A. Ebrahimnejad, A. Azizi, Common-weights fuzzy DEA model in the presence of undesirable outputs with ideal and anti-ideal points: development and prospects, *Complex Intell. Syst.,* **9** (2023), 6223–6240. https://doi.org/10.1007/s40747-023-01030-6

29. C.-K. Hu, F.-B. Liu, H.-M. Chen, C.-F. Hu, Network data envelopment analysis with fuzzy non-discretionary factors, *J. Ind. Manag. Optim.,* **17** (2021), 1795–1807. https://doi.org/10.3934/jimo.2020046

30. G. D. Ferrier, C. A. K. Lovell, Measuring cost efficiency in banking: Econometric and linear programming evidence, *J. Econom.,* **46** (1990), 229–245. https://doi.org/10.1016/0304-4076(90)90057-Z

31. S.-C. Oh, J. Shin, The impact of mismeasurement in performance benchmarking: A Monte Carlo comparison of SFA and DEA with different multi-period budgeting strategies, *Eur. J. Oper. Res.,* **240** (2015), 518–527. https://doi.org/10.1016/j.ejor.2014.07.026

32. A. Azadeh, S. F. Ghaderi, H. Omrani, H. Eivazy, An integrated DEA-COLS-SFA algorithm for optimization and policy making of electricity distribution units, *Energy Policy,* **37** (2009), 2605–2618. https://doi.org/10.1016/j.enpol.2009.02.021

33. R. D. Banker, A. Charnes, W. W. Cooper, Some models for estimating technical and scale inefficiencies in data envelopment analysis, *Manag. Sci.*, **30** (1984), 1078–1092. https://doi.org/10.1287/mnsc.30.9.1078

34. P. Bogetoft, L. Otto, *Benchmarking with DEA, SFA, and R,* Springer New York, New York, 2011. https://doi.org/10.1007/978-1-4419-7961-2

35. R. D. Banker, A. Charnes, W. W. Cooper, J. Swarts, D. A. Thomas, An introduction to data envelopment analysis with some of its models and their uses, In: *Research in Governmental and Nonprofit Accounting,* vol. 5, JAI Press, Greenwich, CT, 1989, 125–163.

36. J. Jondrow, C. A. K. Lovell, I. S. Materov, P. Schmidt, On the estimation of technical inefficiency in the stochastic frontier production function model, *J. Econom.,* **19** (1982), 233–238. https://doi.org/10.1016/0304-4076(82)90004-5

37. S. Kohl, J. O. Brunner, Benchmarking the benchmarks–comparing the accuracy of data envelopment analysis models in constant returns to scale settings, *Eur. J. Oper. Res.*, **285** (2020), 1042–1057. https://doi.org/10.1016/j.ejor.2020.02.031

38. M. Hollander, D. A. Wolfe, E. Chicken, *Nonparametric Statistical Methods*, 3rd edition, John Wiley & Sons, Hoboken, NJ, 2013. https://doi.org/10.1002/9781119196037

39. F. Pedraja-Chaparro, J. Salinas-Jiménez, P. C. Smith, On the quality of the data envelopment analysis model, *J. Oper. Res. Soc.,* **50** (1999), 636–644. https://doi.org/10.1057/palgrave.jors.2600741

40. S. Özöğür-Akyüz, G.-W. Weber, On numerical optimization theory of infinite kernel learning, *J. Glob. Optim.,* **48** (2010), 215–239. https://doi.org/10.1007/s10898-009-9488-x

41. G.-W. Weber, İ. Batmaz, G. Köksal, P. Taylan and F. Yerlikaya-Özkurt, CMARS: A new contribution to nonparametric regression with multivariate adaptive regression splines supported by continuous optimization, *Inverse Probl. Sci. Eng.,* **20** (2012), 371–400. https://doi.org/10.1080/17415977.2011.624770

42. A. Özmen, G.-W. Weber, İ. Batmaz and E. Kropat, RCMARS: Robustification of CMARS with different scenarios under polyhedral uncertainty set, *Commun. Nonlinear Sci. Numer. Simul.,* **16** (2011), 4780–4787. https://doi.org/10.1016/j.cnsns.2011.04.001

43. F.-H. F. Liu, S.-C. Shih, Virtual gap analysis procedures for multi-criteria decision-making and efficiency analysis problems, *Int. J. Inf. Technol. Decis. Mak.,* **25** (2026), 671–710. https://doi.org/10.1142/S0219622025500646

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