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Research article

Research on inventory routing optimization considering multi-temperature joint distribution of mechanical and insulated container

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Abstract: The high logistics cost of cold chain logistics has become a key bottleneck hindering the development of the fresh produce e-commerce industry, where products have multi-temperature characteristics. Two transportation strategies commonly used in this industry are the mechanical and the insulated container multi-temperature joint distributions. For each transportation strategy, an inventory routing optimization model considering the characteristics of multi-temperature products, multi-period, and heterogeneous vehicles is proposed to find the optimal distribution plan to minimize logistics costs. Research results indicate that the threshold for choosing the optimal transportation strategy is related to the ratio of multi-temperature products. When the ratio of multi-temperature products is below 40%, the optimal transportation strategy is the mechanical multi-temperature joint distribution, which can reduce logistics costs by an average of 31.6%. Additionally, the key parameter exerting the predominant impact on logistics costs was identified, and its unit change can increase logistics costs by 3.2%. Furthermore, the laws of logistics cost changes during multi-temperature product distribution have been revealed, being influenced by transportation strategies, the ratio of multi-temperature products, and the total transportation volume. Based on this, valuable management insights have been put forward.

Keywords: multi-temperature products; mechanical multi-temperature joint distribution; insulated container type multi-temperature joint distribution; inventory routing optimization problem
Mathematics Subject Classification: 90B06

1. Introduction

The high logistics cost of cold chain logistics has become a key bottleneck hindering the development of the fresh produce e-commerce industry [1]. In urban distribution operations, exorbitant logistics and warehousing costs further erode corporate profit margins [2]. Fresh products have short shelf lives and are highly perishable, requiring strict temperature-controlled delivery. Additionally, consumers' infrequent purchases, small quantities, and uncertainty have led to widespread high logistics costs at Hema Fresh and Yonghui Superstores. These high logistics costs not only constrain the supply of high-quality fresh products but also severely hamper the broader fresh produce e-commerce industry's development.

In the fresh produce e-commerce industry, the supply chain is a network system that integrates the flows of materials, information, and funds to deliver products from raw materials to end consumers [3]. When the core functions of the network focus on temperature-sensitive products, the network is defined as a "cold chain" [4]. The food/fruit cold chain is designed to deliver perishable food/fruit to consumers [5]. With the rapid development of cold chain logistics, the multi-temperature joint distribution proposed in recent years has become a key component of urban distribution logistics. Multi-temperature joint distribution refers to the simultaneous transportation of products requiring different storage temperatures, such as multi-temperature products, within a single delivery process to meet diversified customer demands.

There are two transportation strategies commonly used in the fresh produce e-commerce industry for multi-temperature products distribution. The first transportation strategy is the mechanical multi-temperature joint distribution, which uses a mechanical compression refrigeration-based multi-compartment vehicle to distribute the multi-temperature products [6]. The second transportation strategy is the insulated container multi-temperature joint distribution, which uses standard ordinary vehicles equipped with refrigerated containers for multi-temperature product distribution [7].

There are four differences between the two transportation strategies. (1) The storage methods for multi-temperature products are different: the first strategy involves allocating multi-temperature zones within a vehicle, while the second transportation strategy involves ordinary vehicles with refrigerated containers. (2) The transportation costs are different: the first transportation strategy requires refrigerated vehicles, which have a high cost, while the second transportation strategy requires ordinary vehicles, which have a low cost. (3) The volume adjustment flexibility is different: in the first transportation strategy, the volume of each temperature zone's compartments is fixed, while in the second transportation strategy, the number of refrigerated containers can be adjusted according to the number of refrigerated products. (4) The applicable scenarios are different: In practical scenarios, when the ratio of refrigerated products is relatively high, managers generally adopt the first transportation strategy; when the ratio of ordinary products is relatively high, managers generally adopt the second transportation strategy. However, how to scientifically select the optimal transportation strategy to minimize logistics cost has become a critical issue that the industry urgently needs to address. Few studies comprehensively compared the differences between the two transportation strategies.

Moreover, research has constructed routing optimization problems to optimize multi-temperature distribution costs, but the efficiency of inventory management has been overlooked. In the fresh produce e-commerce industry, enterprises reserve a safety stock for each product category to satisfy the consumer's demand. Replenishment requests are activated when the inventory level of a certain product falls below the safety stock level. If the replenishment quantity is too large, it will cause overstocking, reduce the freshness of multi-temperature products, and even lead to spoilage, thereby increasing inventory costs. Therefore, the laws of logistics costs for the inventory and routing optimization problem in multi-temperature distribution still need to be explored.

To find the optimal transportation strategy with minimum logistic cost, an inventory and routing optimization model considering the characteristics of multi-temperature products, multi-period, and heterogeneous vehicles is proposed to find the optimal distribution and inventory plan with minimum logistics costs.

The structure of this study is organized as follows: Section 2 reviews the relevant research literature on route optimization for mechanical and insulated container systems in urban distribution scenarios. Section 3 details the research questions addressed in this study. Section 4 constructs multi-temperature products, multi-period inventory routing problem (MMIRP) models to address these questions. Section 5 presents the results and analysis of case studies. Section 6 presents the conclusions.

The innovative aspects of this study are as follows:

(1) Due to the few studies that have comprehensively compared the differences between the two transportation strategies, which are widely applied in the market, for each transportation strategy, this paper proposes an inventory routing problem (IRP) model to explore the laws of logistics cost changes during multi-temperature product distribution.

(2) Based on these proposed models, this paper reveals the laws of logistics cost changes during multi-temperature product distribution, which are influenced by transportation strategies, total transportation volume, and the ratio of multi-temperature products.

(3) The threshold for selecting the optimal transportation strategy is obtained, which is the ratio of multi-temperature products. In the case study, when the ratio of multi-temperature products is below 40%, the optimal transportation strategy is the mechanical multi-temperature joint distribution. Specifically, if the optimal transportation strategy is not chosen, the distribution plan's logistics costs will increase by an average of 31.6%.

(4) The key parameter exerting the predominant impact on logistics costs is identified, and its unit change can increase logistics costs by 3.2%. This can help managers to focus on the variation of the key parameter and adopt appropriate methods to reduce logistics costs.

2. Literature review

2.1. Research on insulated container multi-temperature joint distribution route optimization

For the route optimization problem of insulated container multi-temperature joint distribution, Huang et al. [8] proposed a random route optimization model that combines time window and cargo damage constraints, and verified the model's effectiveness and cost advantages in a dynamic environment. Cho et al. [9] proposed a two-stage heuristic algorithm to solve the multi-temperature zone refrigerated container vehicle routing problem. Hsu et al. [10] constructed a multi-temperature joint distribution binary integer programming model to verify the cost advantages of replaceable insulated container technology in dense networks, providing decision support for precise temperature control and route optimization in cold chain logistics. Kuo and Chen Muchen [7] integrated multi-temperature joint distribution cold chain logistics to construct an insulated container multi-temperature joint distribution model. Chang et al. [11] proposed a route optimization model for the multi-temperature joint distribution mode of an insulated container and developed an improved salvia swarm algorithm to solve this model. Xu et al. [12] considered the influence of factors such as vague demands, reliability of travel time, and energy consumption on path planning and constructed a mathematical model. They then designed and applied the NSGA-II algorithm to solve the problem. Shen et al. [13]

proposed a three-stage optimization model by considering the effective travel time instead of traditional Euclidean distance and other factors, and designed the EABG algorithm to solve it. Shen et al. [14] considered the concepts of "loading reliability", "transportation time reliability", and "transportation risk" and constructed a two-level optimization model. They designed the SA and IHSGA algorithms to solve the model. Francesca et al. [15] conducted a systematic review of vehicle routing problems (VRP) and their integrated issues in perishable product supply chains, with a particular focus on maintaining product quality during the distribution process.

2.2. Research on mechanical multi-temperature joint distribution route optimization

Regarding the route optimization problem for mechanical multi-temperature joint distribution, Zhang and Chen [16] constructed a vehicle scheduling optimization model for mechanical multi-temperature joint distribution to minimize logistics costs. Hübner and Ostermeier [17] proposed an optimization model for mechanical multi-temperature joint distribution targeting multi-temperature zone food delivery problems, designing a large-domain search algorithm for the solution. Mendoza et al. [18] studied the vehicle route problem for mechanical multi-temperature joint distribution under random demand conditions, using a memetic algorithm for solution. Mohamed et al. [19] constructed a vehicle route optimization model for mechanical multi-temperature joint distribution to address the vehicle route problem, employing a hybrid algorithm combining local search with an existing ant colony algorithm for the solution. Qi [20] proposed a mechanical multi-temperature joint distribution strategy for delivering fresh products using multi-temperature refrigerated trucks, constructed a path optimization model, and adopted the improved NSGA-II algorithm to solve this model. Zou et al. [21] developed a new artificial neural network model to accurately estimate the temperature of food stored in multi-temperature refrigerated transport vehicles using the least number of sensors, ensuring the effectiveness of the mechanical multi-temperature joint distribution strategy. Yang et al. [22] constructed a mathematical model for the dual temperature control of the two refrigerated compartments of cold chain logistics refrigerated vehicles, and determined the optimal control gain by using the linear matrix inequality algorithm and the H-2/ h-infinity robust control method. Frank et al. [23] proposed a mechanical multi-temperature joint distribution routing model, jointly optimizing the distribution cycle and vehicle routing. This model reduces costs by 15% compared to step-by-step planning and outperforms the single-temperature zone benchmark by 3%.

2.3. Research on mechanical and insulated container multi-temperature joint distribution routes

In a comparative study of route optimization for mechanical and insulated container multi-temperature joint distribution, Hsu et al. [24] considered different temperature-time window constraints, compared the traditional multi-vehicle distribution system with the insulated container multi-temperature joint distribution system from the cost perspective, and used the simulated annealing algorithm for route optimization. Chen et al. [25] considered delivery plans for multi-temperature food demand that changes over time and constructed mathematical models to compare and analyze traditional multi-vehicle delivery and multi-temperature joint delivery systems. Zhang et al. [26] proposed an innovative solution based on storage-type multi-temperature co-configuration and mechanical multi-temperature co-configuration modes. With the goal of minimizing the total cost, they constructed a path optimization model and designed a genetic algorithm to solve the multi-temperature co-configuration optimization path.

In this study, the IRP was addressed using the maximum level (ML) replenishment strategy in practice. The ML replenishment strategy refers to a situation where, once a customer needs to replenish inventory, the replenishment quantity can range between the customer's current inventory level and the maximum capacity of the inventory [27]. Among studies on the IRP based on the ML replenishment strategy, Li et al. [28] modeled the ML replenishment strategy problem as a two-stage stochastic programming model and solved it using a two-stage heuristic method based on the single-cut decomposition algorithm and the multi-cut decomposition algorithm. Archetti et al. [29] studied the joint optimization problem of inventory routes under discrete time, considering two different replenishment strategies: the order-up-to (OU) and the ML replenishment strategy.

Based on the studies mentioned above, Table 1 summarizes the relevant research and characteristics of the joint optimization of mechanical and insulated container multi-temperature joint distribution inventory routes.

Based on the analysis of the existing literature, we highlight the existing gaps that this study addresses:

(1) First, this paper comprehensively considers two transportation strategies for multi-temperature product distribution, which are widely applied in the industry. Few studies comprehensively compared the two transportation strategies.

(2) Second, existing research on mechanical and insulated container multi-temperature joint distributions is primarily focused on route optimization problems; few studies have considered joint optimization of inventory and route problems.

(3) Third, for each transportation strategy, this paper proposed an IRP model considering the characteristics of multi-temperature products, multi-period, and heterogeneous vehicles, which is more closely aligned with practical applications.

3. Problem description

3.1. Symbol definitions

To make the subsequent mathematical models and algorithms clearer and more concise, all the symbols and definitions used in this chapter are compiled in Table 2.

3.2. Problem definition

To improve the urban distribution efficiency of multi-temperature products, two transportation strategies commonly used in this industry are considered: a mechanical and an insulated container multi-temperature joint distribution. For each transportation strategy, the inventory and routing optimization model considering the characteristics of multi-temperature products, multi-period, and heterogeneous vehicles is proposed, and the branch and cutting exact algorithm is applied to find the optimal distribution plan with minimum logistics costs. The overall framework of this study is shown in Figure 1.

Table 1. Joint optimization study of mechanical and insulated container multi-temperature joint distribution inventory routes.

Related literature	Product		Period		Heterogeneous vehicles		Refrigeration method		Replenishment strategy		Supply chain
	Single	More	Single	More	Ordinary vehicles	Refrigerated vehicles	Mechanical	Insulated container	ML	OU	
Hsu et al. [10]		√			√			√			
Cho et al. [9]		√		√		√	√				
Muyldermans et al. [6]		√				√	√				
Chen et al. [25]		√		√	√			√			
Chen et al. [30]		√		√	√			√			
Chen and Shi [31]		√	√			√	√				
Hsu et al. [24]		√		√	√			√			
Chang et al. [11]		√		√	√			√			
Zhang et al. [32]		√		√		√	√				
Zhang et al. [26]		√		√	√	√	√	√			
Zou et al. [21]		√		√		√	√				
Yang et al. [22]		√		√		√	√				
Voigt et al. [33]		√		√		√	√				
Manso et al. [34]		√		√							food
Soodeh et al. [35]		√		√		√					food
Javier et al. [36]		√		√		√					fruit
Ji et al. [37]		√		√		√					cold
Glock et al. [38]		√						√			food
Lama et al. [39]				√							
Wang et al. [40]		√		√	√						
This study		√		√	√	√	√	√	√		cold

Table 2. Model parameter assumptions.

Sets	Definition
N	The set of all nodes in the network graph, $N=\{1,...,n\}$, where 1 is the distribution center and the rest are the end nodes
N^*	The set of all nodes in the network graph except the distribution center, $N^*=\{2,...,n\}$
E	The set of all edges in a network diagram, $E=\{(1,2),\dots,(i,j)\}$, $i\in N$, $j\in N$
T	Given the time sequence number in the planning time, $T=\{1,2,3\}$
P	A collection of products, $P=\{1,\dots,n\}$
P_1	A collection of product categories that need to be stored at ordinary, $P_1=\{i,\dots,p\}$, $p\in P$
P_2	A collection of products that need to be refrigerated preserved, $P_2=\{j,\dots,p\}$, $p\in P$
K	A collection of all vehicles, $K=\{1,\dots,n\}$
O	Vehicle type (ordinary\refrigerated) collection, $O=\{1,-1\}$, 1 indicates ordinary-type vehicles, -1 indicates refrigerated vehicles
B	A collection of warehouse types, $B=\{0,1\}$, where 0 indicates an ordinary warehouse and 1 indicates a refrigerated warehouse.
Parameters	**Definition**
h_{ip}	Inventory cost of the class p product at the node i, $p\in P$
f_p	The thermal insulation requirements of class p products, the value is 1 or -1, 1 indicates ordinary, and -1 indicates refrigerated, $p\in P$
u_{i1}	The upper limit of the storage capacity of the ordinary inventory on the i node, $i\in N$
u_{i2}	The upper limit of the storage capacity of refrigerated inventory on the i node, $i\in N$
d_{tip}	In the t period, the quantity demanded of the p product at the i node, $t\in T$, $p\in P$, $i\in N$
r_{tp}	During the t period, the total quantity of the type p of product in the distribution center, $t\in T$, $p\in P$
I_{i0p}	In the initial period, the inventory of the p product in the i node, $p\in P$, $i\in N$
q_k	The loading capacity of the vehicle k, $k\in K$
l_c	The cost of using an insulation container
l_d	The cost of using a refrigerated container
q_c	Capacity of the ordinary compartment of a vehicle
q_b	Capacity of the refrigerated compartment of a vehicle
q_d	The capacity of a refrigerated container
e_k	The insulation type of the k vehicle, the value is 1 or -1, 1 indicates ordinary, -1 indicates refrigerated, $k\in K$
v_k	Transport cost per kilometer for a vehicle k, $k\in K$
a_k	The rental cost of the vehicle k, $k\in K$
g_k	The available time of the k vehicle in each period, $k\in K$
c_{ij}	The distance traveled from point i to point j, $(i,j)\in E$

Continued on next page

Parameters	Definition
cor_i	The location coordinates of each node, $i \in E$
S_{level}	Indicates the safety stock level, which is set to 0 during the model testing process
w	Vehicle service time at each node
Decision variables	**Definition**
I_{tip}	Integer, the inventory at the end of the period t of the product p at the i node, $i \in N$, $p \in P$, $t \in T$
x_{tijk}	0 or 1, equal to 1 if the k vehicle accesses from node i to node j in t a period, otherwise equal to 0. $i,j \in N$, $k \in K$, $t \in T$
y_{tipk}	Integer, the number of products of the class p that the k vehicle supplements when it visits a node i in a period t, $i \in N$, $p \in P$, $k \in K$, $t \in T$
z_{tik}	0 or 1; if the k vehicle has visited the i node in t a period, it is equal to 1. Otherwise, it is equal to 0. $i \in N$, $k \in K$, $t \in T$
z_{tuk}	0 or 1; if the k vehicle has visited the i node in t a period, it is equal to 1. Otherwise, it is equal to 0. $S \subseteq N, \|S\| \geq 2, k \in K, t \in T, u \in S$
m_k	0 or 1, whether the k vehicle is used; if it is used, $m_k = 1$; otherwise, $m_k = 0$
s_{kc}	Integer, the number of insulation partitions used by the k vehicle, $k \in K$
s_{kb}	Integer, the number of refrigerated partitions used by the k vehicle, $k \in K$
ρ_p	0 or 1, whether to replenish the p product, $p \in P$
s_{kd}	Integer, the number of refrigerated containers used by the k vehicle, $k \in K$
y	0 or 1, auxiliary decision parameter

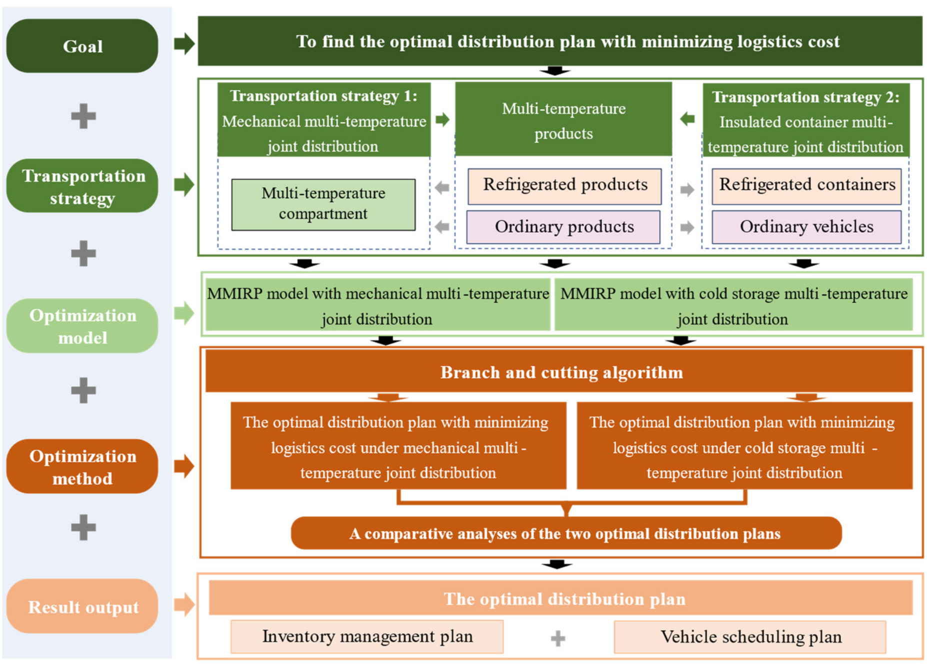

Figure 1. Framework of this research.

Considering the MMIRP of mechanical and insulated container multi-temperature joint distribution, the inventory levels of multi-temperature products at node i within period t are I_{tip}, and the demand is d_{tip}. To meet the demand and achieve the maximum inventory level, multi-temperature products are distributed from the distribution center $N = 1$ to the retail end nodes $N^* = \{2,...,n\}$ within the T period. The replenishment quantity of products in period t is y_{tipk}. Transportation from the distribution center to the retail outlet is carried out by a vehicle with a capacity of q_k within the period t. On a given route, the vehicle visits the i node, i.e., $z_{tik} = 1$, and travels from node i to node j, i.e., $x_{tijk} = 1$. The initial inventory levels for ordinary and refrigerated products at the urban distribution center and each retail outlet are I_{i0p}, with upper limits of u_{i1} and u_{i2}, respectively. During distribution, the goal is to minimize distribution costs, taking into account inventory costs h_{ip}, transportation costs v_k, rental vehicle costs a_k, refrigeration costs l_c, and insulated container box costs l_d.

This study makes the following assumptions about the MMIRP:

(1) During the vehicle's operation, traffic congestion and other conditions are not considered.

(2) During transportation, refrigeration equipment malfunctions are not considered.

(3) Distribution centers have sufficient supplies and transport capacity to complete delivery tasks.

4. Mathematical model

This study used mathematical modeling methods and market research methods to develop the models. The mathematical modeling methods involve three elements, which are the objective function, constraint conditions, and decision variables.

(1) For the objective function, by using the market research methods, we built the objective function with the goal of minimizing logistics costs, which include inventory cost, transportation cost, refrigeration cost, refrigerated containers cost, and vehicle rental cost.

(2) For the constraint conditions, by using the market research methods, this study constructed the constraint conditions, which include inventory constraints, transportation constraints, replenishment constraints, and strategy constraints.

(3) For the decision variables, by using the market research methods, managers focus on inventory levels, transportation routes, and replenishment quantity.

We take the optimal solution as the criterion for optimization. To find the optimal solution, we adopt the exact branch and cutting algorithm to solve this model. Specifically, we invoke the branch and cutting algorithm in the GUROBI solver and set the gap value between the output result and the optimal solution to 0, which indicates that the output result is the optimal solution.

4.1. MMIRP model for mechanical multi-temperature joint distribution

The mechanical multi-temperature joint distribution involves allocating multi-temperature zones within a vehicle. Based on this strategy, this study constructed an MMIRP model, which is shown below.

4.1.1. Objective function

The objective is to minimize the logistics costs, including inventory costs, transportation costs, temperature control costs, and refrigerated vehicles costs, as shown in Equation (1).

$$minC = \sum_{i\in N}\sum_{p\in P}\sum_{t\in T} h_{ip} * I_{tip} + \sum_{k\in K}\sum_{(i,j)\in E}\sum_{t\in T} v_k * c_{ij} * x_{tijk} + \sum_{k\in K}\sum_{p\in P_1} s_{kc} * l_c * \rho_p + \sum_{k\in K}\sum_{p\in P_2} s_{kb} * l_c * \rho_p + \sum_{k\in K} m_k * a_k \quad (1)$$

4.1.2. Constraint conditions

1) Inventory constraints: The total inventory of refrigerated products must be less than the upper limit of the refrigerated warehouse capacity, as shown in Equation (2). The total inventory of ordinary products must be less than the upper limit of the ordinary warehouse capacity, as shown in Equation (3). The calculation equations for distribution center inventory and end-point inventory are shown in Equations (4) and (5). The inventory levels of all product types in distribution centers and end-points must be greater than or equal to safety stock level, as shown in Equation (6).

$$\sum_{p\in P_1} I_{tip} \le u_{i1} \qquad \forall i \in N^*, t \in T \quad (2)$$

$$\sum_{p\in P_2} I_{tip} \le u_{i2} \qquad \forall i \in N^*, t \in T \quad (3)$$

$$I_{(t-1)ip} + \sum_{k\in K} y_{tipk} - d_{tip} = I_{tip} \qquad \forall i \in N^*, t \in T, p \in P \quad (4)$$

$$I_{(t-1)1p} - \sum_{k\in K}\sum_{i\in N^*} y_{tipk} + r_{t1p} = I_{t1p} \qquad \forall t \in T, p \in P \quad (5)$$

$$I_{tip} \ge S_{level} \qquad \forall i \in N, t \in T, p \in P \quad (6)$$

2) Transportation constraints: The load carried by each vehicle in each period must not exceed its capacity, as shown in Equation (7). Within any cycle, if any vehicle has delivered a certain quantity of a specific product type to an end node, then that vehicle must have visited the end node, as shown in Equation (8). Vehicle usage within any cycle is shown in Equation (9).

$$q_k * z_{t1k} \ge \sum_{i\in N^*}\sum_{p\in P} y_{tipk} \qquad \forall t \in T, k \in K \quad (7)$$

$$y_{tipk} \le u_{ip} * z_{tik} \qquad \forall i \in N^*, k \in K, t \in T, p \in P \tag{8}$$

$$\begin{aligned} &-(m_k - 1) \le My && \forall k \in K && \text{(9-1)} \\ &\sum_{i \in N} z_{tik} - 1 < M(1-y) && \forall t \in T, k \in K && \text{(9-2)} \end{aligned} \tag{9}$$

Constraint (9) uses the auxiliary binary variable y to enforce the following "If-Then" logic: when a vehicle is not used (m_k =0), constraint (9-1) forces y = 1, which in turn forces $\sum_{i \in N} z_{tik} < 1$ in constraint (9-2). This means the vehicle cannot visit any node at any time t. When the vehicle is used (m_k =1), constraint (9-1) imposes no restriction on y, then set y = 0 to relax the constraint (9-1), allowing the vehicle to visit nodes. Thus, this constraint ensures that a vehicle can follow a visiting sequence only if it is actively used.

3) Subtour elimination constraint: Each node in a route must exist in two arcs, as shown in Equation (10). Subtour elimination method, as shown in Equation (11); the following examples will specifically illustrate the function of this equation. During the calculation of the route, only the following two situations may occur: one is a loop including the distribution center, and the other is a loop not including the distribution center. Suppose there is a loop consisting of four nodes. If it includes the distribution center, the left side of equation (11) calculates the number of edges not connected to the distribution center in the loop, which is 2. The right side of equation (11) calculates the value obtained by subtracting 1 from the number of vertices excluding the distribution center, and the right side is 2. Since 2 ⩽ 2, the equation holds. If this loop consisting of four nodes does not include the distribution center, the value calculated on the left side of equation (11) is 4, and the value calculated on the right side is 3. Since 4 ⩽ 3, the equation does not hold.

$$\sum_{j \in N, i \prec j} x_{tijk} + \sum_{j \in N, j \prec i} x_{tijk} = 2z_{tik} \qquad \forall i \in N, k \in K, t \in T \tag{10}$$

$$\sum_{i \in S} z_{tik} - z_{tuk} \ge \sum_{i \in S} \sum_{j \in S, i \prec j} x_{tijk} \qquad S \subseteq N, |S| \ge 2, k \in K, t \in T, u \in S \tag{11}$$

This method of eliminating subtours has certain advantages in terms of computational efficiency compared to the MTZ method (Miller–Tucker–Zemlin) [41], which simultaneously adds all subtour elimination constraints to the model. When the problem size is small, this method has certain advantages in terms of solution efficiency because it does not need to add all subtour elimination constraints to the model before the calculation, but rather determines whether subtours exist in the calculation results after solving the model. If there are no subtours, it outputs the optimal solution. If there are subtours, it adds the subtour elimination constraints to the model and solves the model again. This process is repeated until there are no subtours in the solution results.

4) Replenishment constraints: The replenishment quantity to each terminal node in the urban network must not exceed the available inventory at the distribution center, as shown in equation (12). In any period, the sum of the replenishment quantity of a product class at a terminal node and the

inventory of that product class from the previous period must not exceed the inventory upper limit for that product class at the terminal node, as shown in equation (13).

$$\sum_{k\in K}\sum_{i\in N^*} y_{tipk} \le I_{t1p} \qquad \forall t\in T, p\in P \tag{12}$$

$$\sum_{k\in K} y_{tipk} + I_{(t-1)ip} \le u_{ip} \qquad \forall i\in N^*, t\in T, p\in P \tag{13}$$

5) Strategy constraints: The ratio of the quantity of goods in each cycle to the loading capacity of the partitioned area cannot exceed the number of partitioned areas, as shown in Equation (14).

$$\frac{\sum_{i\in N^*}\sum_{p\in P_1} y_{tipk}}{q_c} \le s_{kc} \qquad \forall t\in T, k\in K \tag{14}$$

$$\frac{\sum_{i\in N^*}\sum_{p\in P_2} y_{tipk}}{q_b} \le s_{kb} \qquad \forall t\in T, k\in K \tag{15}$$

4.2. MMIRP model for insulated container multi-temperature joint distribution

The insulated container multi-temperature joint distribution uses ordinary vehicles equipped with refrigerated containers. Based on this strategy, this study constructed an MMIRP model, which differs from the MMIRP model for mechanical multi-temperature joint distribution in terms of objective functions and constraints, as shown below:

1) Objective function

The objective function aims to minimize logistics costs, including inventory costs, transportation costs, refrigerated containers costs, and ordinary rental vehicle costs, as shown in Equation (15).

$$\min C_1 = \sum_{i\in N}\sum_{p\in P}\sum_{t\in T} h_{ip} * I_{tip} + \sum_{k\in K}\sum_{(i,j)\in E}\sum_{t\in T} v_k * c_{ij} * x_{tijk} + \sum_{k\in K} s_{kd} * l_d + \sum_{k\in K} m_k * a_k \tag{16}$$

2) Constraint conditions

The constraints of this model are similar to some of the constraints in the MMIRP model of the mechanical multi-temperature joints distribution strategy, including (2-13). In addition, the transportation strategy-type constraints of this model are as shown in (17).

$$\frac{\sum_{i\in N^*}\sum_{p\in P_2} y_{tipk}}{q_d} \le s_{kd} \qquad \forall t\in T, k\in K \tag{17}$$

Therefore, the MMIRP model for the insulated container multi-temperature joint distribution consists of the objective function (16) and the constraints (2-13) and (17).

5. Experimental testing

5.1. Data introduction

5.1.1. Extended MMIRP dataset

The IRP is a widely studied issue. Some scholars [42,43] have published some benchma rk datasets to test the issue, which can be downloaded from http://www.leandro-coelho.com/in stances/inventory-routing/. To verify the validity of the model proposed in this paper, we also used these benchmark datasets. However, since the study proposed that the MMIRP is differ ent from the IRP, we added some additional parameters based on benchmark datasets. These additional parameters come from one of the largest fresh food e-commerce enterprises in Chi na. The parameters are as follows:

(1) Upper capacity limits for refrigerated and ordinary inventory, u_{i1} and u_{i2}, have been added;

(2) Added insulation requirement data f_p for multi-temperature products, as well as demand data d_{tip} for multi-temperature products under varying demand conditions;

(3) Added insulation type data e_k, rental cost data a_k, loading capacity data q_k, travel distance data c_{ij}, transportation cost data v_k, and service time data w for different types of vehicles;

(4) Added refrigeration costs and usage costs of thermal insulation boxes l_c and l_d, as well as the loading capacities of refrigerated compartments, ambient temperature compartments, and thermal insulation boxes q_c, q_b, and q_d. In summary, this study constructed an expanded MMIRP dataset from the standard MMIRP dataset. The data expanded based on benchmark datasets can be downloaded here: https://github.com/wbb-wq/DATA-FOR-PAPER.

5.1.2. Enterprise data

In this study, data were collected from one of the largest fresh food e-commerce enterprises in China, which conducts urban distribution business in over 20 cities. The operational data (OD) was collected from its Chongqing distribution area, which is responsible for replenishing multi-temperature products to 12 end nodes, as shown in Figure 2. The selected enterprise and distribution area exhibits typical characteristics of urban fresh food distribution, including multi-temperature products and frequent replenishment tasks, making it well-suited for validating the proposed model.

In Figure 2, point 1 represents the distribution center, while the other numbers indicate the sequence of the end nodes. The OD dataset can be downloaded from the website https://github.com/wbb-wq/DATA-FOR-PAPER. The relevant parameters in the OD dataset are shown in Table 3.

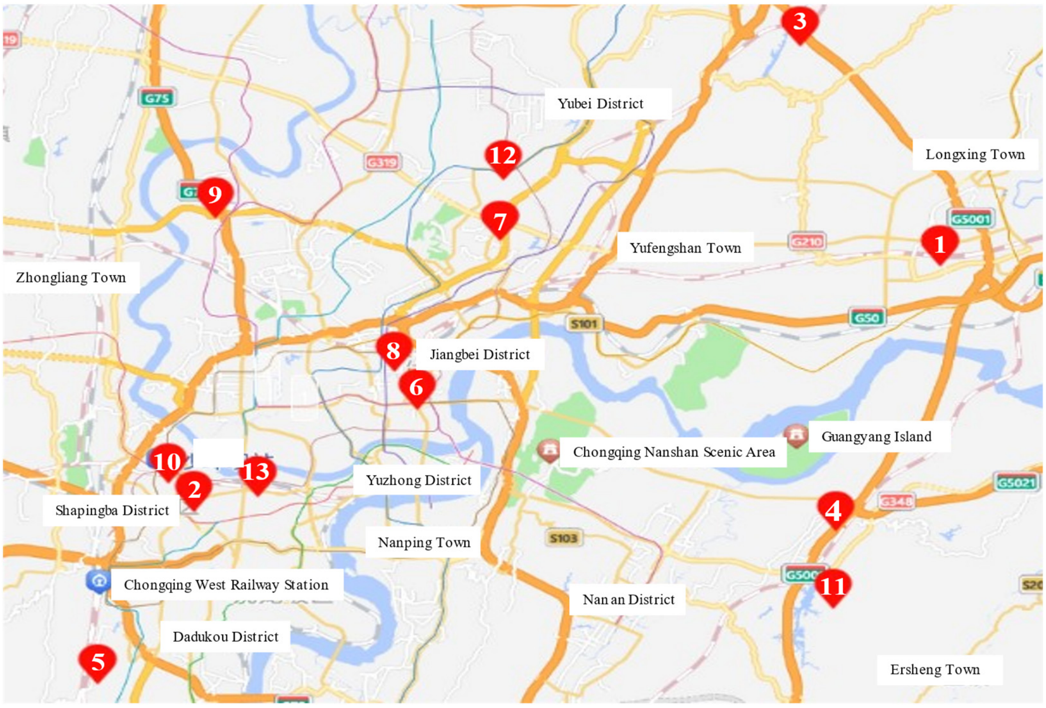

Figure 2. Multi-temperature products in Chongqing's distribution area network.

Table 3. Presentation of key data in the OD.

Parameter	Numerical	Parameter	Numerical
N	N = [44,5,6,7,8,9,10,11,12,13]	h_{ip}	Dataset
N*	N* = [44,6,7,8,9,10,11,12,13]	u_{i1}	Dataset
T	T= {1, 2, 3}	u_{i2}	Dataset
P	P= {1,2,3}	d_{tip}	Dataset
K	K = {1, 2, 3, 4, 5, 6,}	r_{tp}	Dataset
f_p	f_p= {−1,1,−1}	I_{i0p}	Dataset
e_k	e_k= {1,1,1,−1,−1,−1}	q_k	q_k ={800,900,1000, 800,900,1000}
v_k	v_k= {1.8,1.4,1.5,2.6,2.8,2.6}		

5.1.3. Parameter settings

Expand the parameters related to the dataset, as shown in Table 4.

Table 4. Information in the expanded datasets.

Parameter	Numerical	Parameter	Numerical
l_c	80	h_{ip}	Dataset
q_c	120	u_{i1}	Dataset
q_b	100	f_p	[−1,1,−1]
l_d	65	u_{i2}	Dataset
q_d	80	d_{tip}	Dataset
e_k	[1,1,1,−1,−1,−1]	r_{tp}	Dataset
v_k	[1.2,1.1,1.0,1.5,1.3,1.1]	I_{i0p}	Dataset
a_k	[300,320,340,400,420,440]	c_{ij}	Dataset
w	30	cor_i	Dataset
g_k	[4,8,5,6,7,11]	q_k	[600,700,800,600,700,800]

5.2. Experimental results

Based on the extended MMIRP dataset, this study applied the branch and cutting algorithm built into the Gurobi commercial solver to test the validity of the proposed model. The model and algorithm were compiled in the Gurobi 11.0.1 solver, and the entire experimental process was conducted in the Pycharm compilation environment. The branch and cutting algorithm is selected for the following reasons. First, the model has a natural network flow structure, which can be effectively tightened by various cutting planes embedded in the branch and cutting framework. Second, the sub-cycles elimination constraints are added, which significantly reduces the initial model size and improves computational efficiency. Third, compared to other exact methods such as Benders decomposition or column generation, the branch and cutting algorithm does not require problem-specific decomposition structures and can be directly applied to the MMIRP model, without losing optimality.

The experimental results are shown in Table 5. Table 5 shows the optimal solution (C_{Best}), the gap between the optimal solution and the solution (Gap), and the solution time (*T*).

The experimental results indicate that there are significant differences in the logistics costs of the optimal distribution plans under different transportation strategies. For instances 1, 2, 3, 5, 8, 9, 14, 15, 16, 18, and 20, the optimal transportation strategy is the mechanical multi-temperature joint distribution. For instances 4, 6, 7, 10, 11, 12, 13, 17, 19, and 21, the optimal transportation strategy is the insulated container multi-temperature joint distribution. In one-time delivery, our proposed model can save the logistics cost by an average of 2.1%, i.e., $44. This study roughly estimates that in practical applications, if a company delivers five times a week, the logistics cost is reduced by $220; if it delivers 20 times a month, the logistics cost is reduced by $880; and if it delivers 240 times a year, the logistics cost is reduced by $10,560. Moreover, the greater the transportation volume for this enterprise, the more significant the logistics cost savings will be.

Furthermore, as shown in Table 5, when the network size increases to N = 13, the branch and cutting algorithm is unable to obtain the optimal solution for any of the test instances within 3600 seconds. This indicates that the practical application limit of the exact solution method adopted in this paper is approximately 13 nodes (including the distribution center). Beyond this scale, the number of binary decision variables increases exponentially, resulting in a sharp increase in computation time.

Table 5. Optimal solutions under transportation strategies.

Case	N	T	Transportation strategy 1			Transportation strategy 2		
			C_{Best}	Gap	T	C_{Best}	Gap	T
1	5	3	1958	0	2	1963	0	3
2	5	3	1592	0	2	1617	0	2
3	5	3	1329	0	2	1334	0	4
4	5	3	1723	0	2	1572	0	2
5	5	3	1367	0	1	1405	0	2
6	5	3	2079	0	1	1349	0	1
7	5	3	2119	0	3	2101	0	11
8	5	3	1367	0	1	1409	0	3
9	5	3	1433	0	1	1438	0	2
10	5	3	1582	0	2	1444	0	4
11	10	3	2282	0	17	2130	0	24
12	10	3	2370	0	28	2322	0	26
13	10	3	2548	0	22	2536	0	49
14	10	3	2509	0	49	2623	0	668
15	10	3	2526	0	85	2581	0	1133
16	10	3	2659	0	3080	2758	0	3600
17	10	3	2366	0	162	2320	0	109
18	10	3	2649	0	29	2650	0	294
19	10	3	2780	0	305	2717	0	52
20	10	3	2700	0	44	2793	0	52
21	13	3	4013	10%	3600	3970	10%	3600
Average			2188	0.5%	354	2144	0.5%	459

In practical urban distribution, the distribution network usually consists of dozens or even hundreds of nodes. Therefore, exact solution methods are applicable to small-scale scenarios.

The computational performance comparison of the branch and cutting algorithm is shown in Table 6.

As shown in Table 6, since case 3 has an appropriate scale, a moderate number of nodes, a typical number of cuts, comparable strategies, and successful convergence, it is chosen as the representative case for analysis. In the transportation strategy 1, the branch and cutting algorithm explored 1975 nodes and generated 202 cut planes, with simplex iterations 76,537 times. In the transportation strategy 2, it explored 2513 nodes and generated 436 cut planes, with simplex iterations of 124,700 times. This performance indicates that the algorithm can obtain the optimal solution within a reasonable computational cost. Furthermore, a convergence curve graph has also been drawn, as shown in Figure 3.

Table 6. Computational performance comparison of the branch and cutting algorithm.

Case	Transportation strategy 1			Transportation strategy 2		
	Nodes	Cuts	Simplex iterations	Nodes	Cuts	Simplex iterations
1	1968	485	170941	1914	467	185635
2	4582	271	138402	3824	254	182657
3	1975	202	76537	2513	436	124700
4	1391	323	77144	430	283	29923
5	2387	318	89481	4129	269	216729
6	6242	303	123143	273	101	9470
7	8272	393	469437	11543	406	697986
8	2170	228	102263	4129	174	106662
9	1634	265	65405	573	204	26259
10	5481	210	151557	3694	320	189740
11	10043	1201	802844	8818	1116	640153
12	48418	190	663034	26892	159	628266
13	7032	1103	514491	13871	961	1257307
14	62076	825	1208957	543625	1313	11698264
15	92973	1205	2977511	303324	1613	13983461
16	2108044	1339	38512689	1582968	3057	64612788
17	180365	1113	3306211	56248	904	1646672
18	47869	1047	1899603	194494	1423	10837495
19	163289	1157	3520186	63452	1039	2398353
20	51319	1027	1915067	27254	966	1329395
21	527093	5120	46873220	391404	6541	55360799

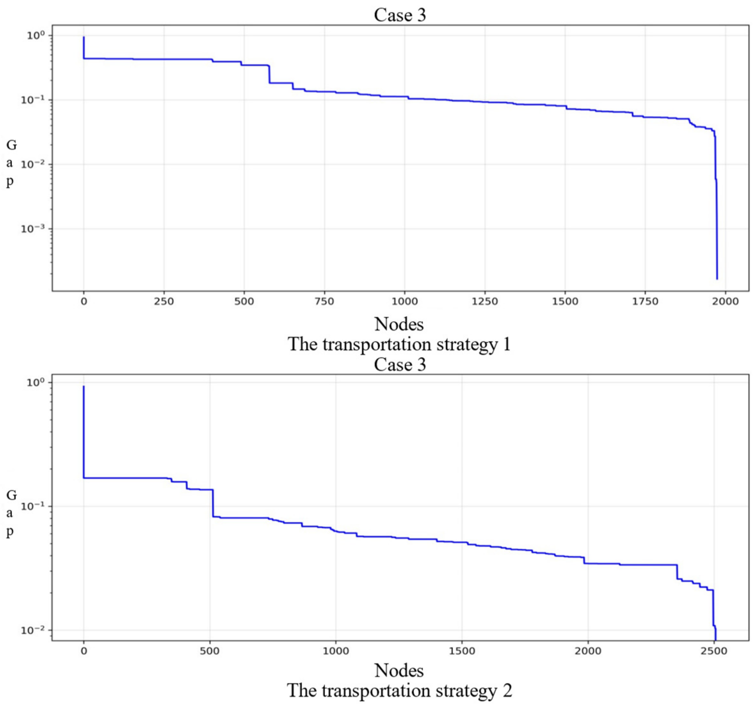

Figure 3. Convergence curve of the branch and cutting algorithm.

5.2.1. Logistics costs analysis with optimal distribution plans

To further illustrate the logistics cost of the optimal distribution plans under different transportation strategies, this study selects the optimal distribution plans of case 13 to present the inventory costs, rental vehicle costs, temperature control costs, refrigerated container costs, and transportation costs, as shown in Table 7 and Figure 4.

Table 7. Cost analysis of the optimal distribution plans of case 13.

Various costs	Transportation strategy 1	Transportation strategy 2
Logistics costs	2548	2536
Inventory costs	1413	1427
Transportation costs	375	509
Rental vehicle costs	440	340
Temperature control costs	320	-
Refrigerated container costs	-	260

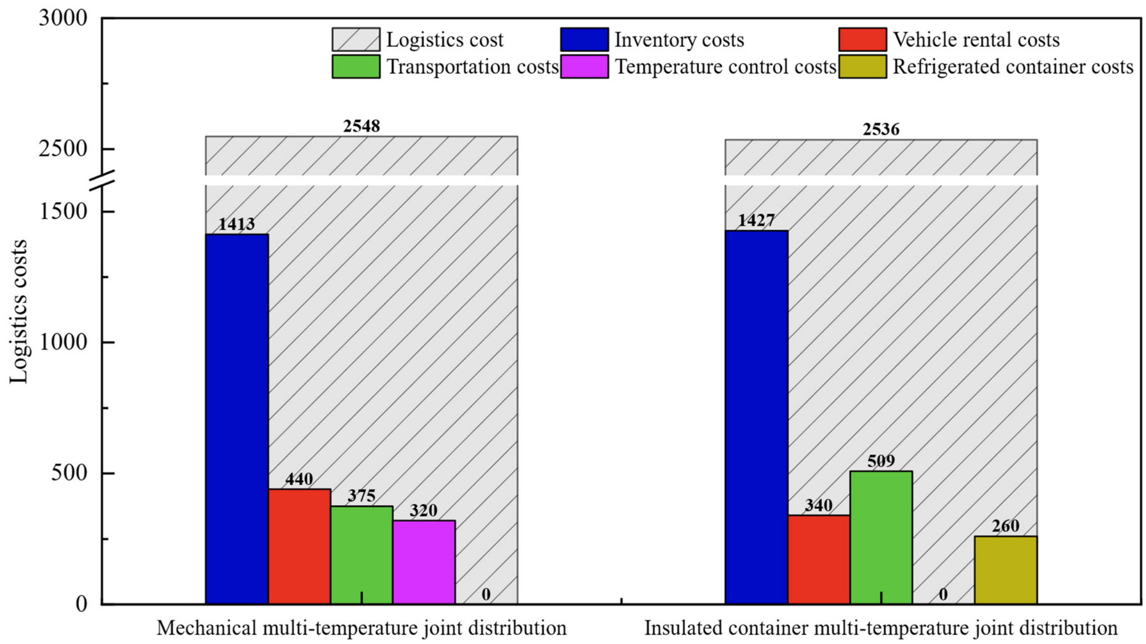

Figure 4. Cost analysis of the optimal distribution plans of case 13.

As shown in Figure 4, the inventory cost has a high proportion by an average of 55.9%, explained by the high inventory cost of fresh products, the cumulative effect of multi-period inventory, and the scale effect of transportation costs. The inventory costs of the optimal distribution plans under different transportation strategies are relatively similar, as both strategies adopt the same ML replenishment strategy, face the same demand pattern, and use the same inventory cost coefficient. However, transportation costs differ significantly, explained by two reasons: first, the partitioning of mechanical vehicles can be flexibly adjusted, allowing for the loading of more multi-temperature products in a single transportation, thereby reducing the total mileage of the vehicle. Second, the average loading rate of transportation strategy 1 (85%) is higher than that of transportation strategy 2 (68%), and the transportation cost per unit product is lower. The transportation cost of mechanical multi-temperature joint distribution is the lowest, reduced by 26.33%. The rental vehicle cost of the insulated container multi-temperature joint distribution is the lowest, reduced by 22.73%. Additionally, regardless of the optimal distribution plans for each transportation strategy, the combined inventory and transportation costs consistently account for over 70% of the total logistics cost. Therefore, enterprises need to identify their own cost structure type and select the appropriate transportation strategy.

5.2.2. Vehicle efficiency analysis with optimal distribution plans

To further demonstrate the utilization efficiency of vehicles in the optimal distribution plans for each transportation strategy, this study selects the optimal distribution plans of case 11 to present the vehicle loading rate (LR), delivery mileage (DM), and delivery route (R) information from the optimal solution, as shown in Table 8. The formulas for calculating LR and DM are shown in Equations (18) and (19).

$$LR = \frac{\sum_{i \in N} \sum_{p \in P} y_{tipk}}{q_k} \qquad \forall t \in T, k \in K \tag{18}$$

$$\mathrm{DM} = \sum_{\mathrm{i} \in \mathrm{N}} \sum_{\mathrm{j} \in \mathrm{N}} \mathrm{c_{ij}} * \mathrm{x_{tijk}} \qquad \forall t \in T, k \in K \tag{19}$$

Table 8. Efficiency of vehicles with optimal distribution plans.

	Mechanical multi-temperature joint distribution			Insulated container multi-temperature joint distribution		
T	1	2	3	1	2	3
e_k	[−1]	[−1]	[−1]	[1]	[1]	[1]
R	[1, 7, 8, 10,2, 5, 1]	[1, 9, 6,3, 1]	[1, 2, 4,5, 1]	[1, 2, 10, 8, 7, 9, 1]	[1, 6, 3, 5, 1]	[1, 5, 4, 2, 1]
DM	163.5	172.4	104.6	202.1	140.7	104.6
LR	100%	85%	100%	77%	69%	68%

As shown in Table 8, the optimal distribution plans for each transportation strategy exhibit short distribution routes and an average vehicle loading rate of 83%. Among these, the mechanical multi-temperature joint distribution achieves the highest vehicle loading rate of 100%, with distribution routes [1, 7, 8, 10, 2, 5, 1] and [1, 2, 4, 5, 1], and delivery route distances of 163.5 and 104.6 kilometers. The insulated container multi-temperature joint distribution has the shortest delivery route distances of 104.6 km, with delivery routes [1, 5, 4, 2, 1] and a vehicle loading rate of 68%. The difference in the vehicle loading rate between the two transportation strategies is 12%. The optimal delivery routes for delivery vehicles in each period can be effectively obtained, which has certain value in improving vehicle loading rates and reducing transportation costs.

To intuitively illustrate the vehicle delivery routes, this study plots the routes from the optimal distribution plan, as shown in Figure 5, where retail nodes are marked in blue circles, distribution center(s) are designated by red squares, and delivery routes are traced with blue solid lines.

5.2.3. Inventory management efficiency analysis with optimal distribution plans

To further demonstrate the management efficiency of inventory in optimal distribution plans for each transportation strategy, this study selects the optimal distribution plans of case 2 to present the inventory levels (EL), inventory turnover rates (ITO), and average values (A) of each transportation strategy, as shown in Table 9. The formula for calculating ITO [44] is shown in (20):

$$ITO = \frac{D_{tip}}{I_{tip}} \qquad \forall i \in N, t \in T, \ p \in P \tag{20}$$

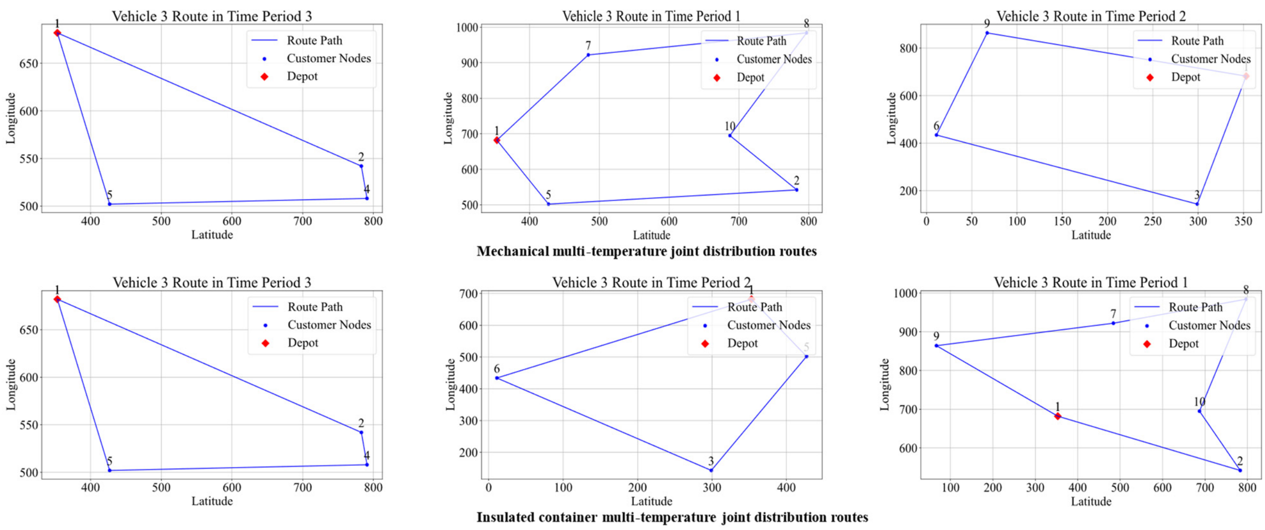

Figure 5. Routings of the optimal distribution plan.

Table 9 indicates that the inventory levels and inventory turnover rates are different for each transportation strategy. Among these, the mechanical multi-temperature joint distribution has the highest average inventory level of 38.1, while the insulated container multi-temperature joint distribution has an average inventory level of 32.1. The insulated container multi-temperature joint distribution also has the highest average inventory turnover rate of 1.1, while the mechanical multi-temperature joint distribution has an average inventory turnover rate of 1.

Table 9. Efficiency of inventory management with optimal distribution plans.

*N**	*T*	*P*	Transportation strategy 1		Transportation strategy 2	
			EL	ITO	EL	ITO
2	1	1	60	0.56	60	0.56
		2	101	0.27	101	0.27
		3	97	0.29	97	0.29
	2	1	58	0.22	47	0.22
		2	28	0.75	25	0.75
		3	36	0.85	15	0.85
	3	1	0	1	0	1.23
		2	0	1	0	1.12
		3	0	1	0	2.40
3	1	1	36	0.68	36	0.68
		2	55	0.57	55	0.57
		3	60	0.54	60	0.54
	2	1	88	1.53	88	1.53
		2	93	0.78	93	0.78
		3	66	1.33	66	1.33
	3	1	0	1	0	1
		2	0	1	0	1
		3	0	1	0	1
4	1	1	45	0.68	45	0.68
		2	85	0.65	48	0.65
		3	57	0.56	57	0.56
1	2	1	64	1.87	0	1.87
		2	31	0.82	0	1.46
		3	47	1.04	0	1.04
	3	1	0	1	0	1.04
		2	0	1	0	1.04
		3	0	1	0	1.04
5	1	1	25	0.79	25	0.79
		2	100	0.14	100	0.14
		3	81	0.39	81	0.39
	2	1	0	3.36	0	3.36
		2	12	0.88	12	0.88
		3	46	0.43	46	0.43
	3	1	0	0.43	0	0.43
		2	0	6.50	0	6.50
		3	0	2.04	0	2.04
A			38.1	1.0	32.1	1.1

5.3. Analysis of application cases

5.3.1. Optimal distribution plans

This study is based on the OD dataset to calculate the optimal distribution plans in urban distribution. Considering that the size of the dataset is not large, the branch and cutting algorithm is used to solve the MMIRP model.

In the optimal distribution plan of the MMIRP model, the minimum costs of the mechanical multi-temperature joint distribution and the insulated container multi-temperature joint distribution are \$3929 and \$3638. The optimal replenishment times and quantities for each terminal outlet are shown in Figure 6.

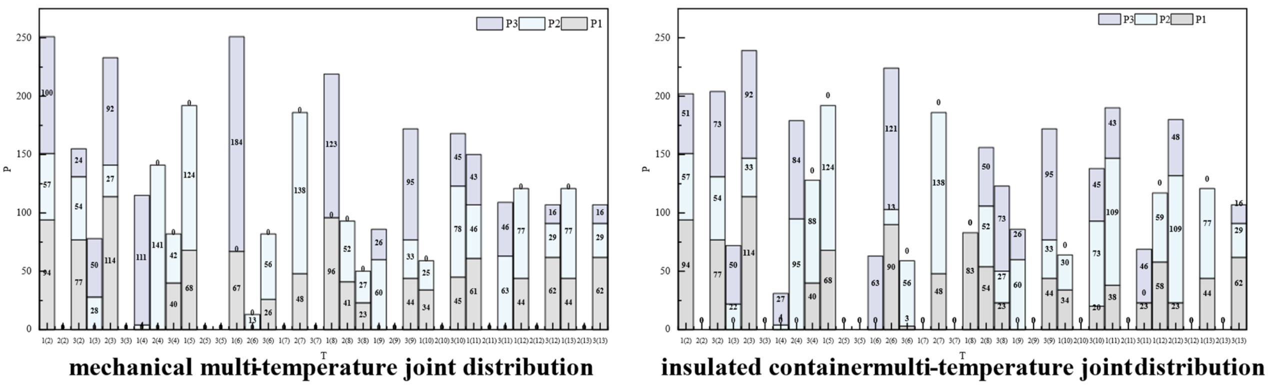

Figure 6. Replenishment time and quantity in the optimal solution.

Figure 6 shows that the inventory and routing joint optimization method proposed in this study for urban distribution scenarios, which considers the heterogeneity of ordinary and refrigerated products based on the ML replenishment strategy, can effectively obtain the optimal replenishment times and quantities for the end node in each period. The mechanical multi-temperature joint distribution's average inventory turnover rate is 2.15. The insulated container multi-temperature joint distribution's average inventory turnover rate is 1.64.

5.3.2. Optimal replenishment routes under distribution plans

During distribution, due to the different requirements of multi-temperature products for types of transport vehicles, as well as the differences in the maximum loading capacities of various types of transport vehicles, the optimal distribution plan of the MMIRP model provides the optimal replenishment routes considering the heterogeneities of vehicles. The information about the replenishment routes in the optimal solution is presented in Table 10.

As shown in Table 10, the optimal distribution plans under different transportation strategies exhibit high vehicle utilization efficiency, short distribution routes, and an average vehicle loading rate of 68.5% and 86%. From the table, it can be seen that the optimal replenishment route not only meets the replenishment needs of the end-point outlets at the minimum cost but also maintains a high loading rate of the delivery vehicles. To more intuitively display the replenishment path of the vehicles, the replenishment path in the optimal solution is drawn in Figure 7, where the black dots represent the end-point outlets, and the red dots represent the distribution center.

Table 10. Efficiency of vehicles with optimal distribution plans.

Transportation strategy 1					
T	1	1	2	2	3
e_k	[−1]	[−1]	[−1]	[−1]	[−1]
q_k	1000	900	900	1000	1000
R	[1,3,12,9,10,2,5,13,8,6,1]	[1,4,11,1]	[1,4,1]	[1,6,8,7,12,3,1]	[1,4,11,5,2,10,9,8,6,1]
$Load$	999	208	178	1000	1000
LR	99%	23%	20%	100%	100%
Transportation strategy 2					
T	1	1	2		3
e_k	[1]	[1]	[1]		[1]
q_k	900	1000	1000		1000
R	[1,11,4,1]	[1,6,8,13,5,2,10,9,12,3,1]	[1, 6, 8,7,12,3,1]		[1,4,11,5,2,10,13,8,6,1]
$Load$	448	1000	937		1000
LR	50%	100%	94%		100%

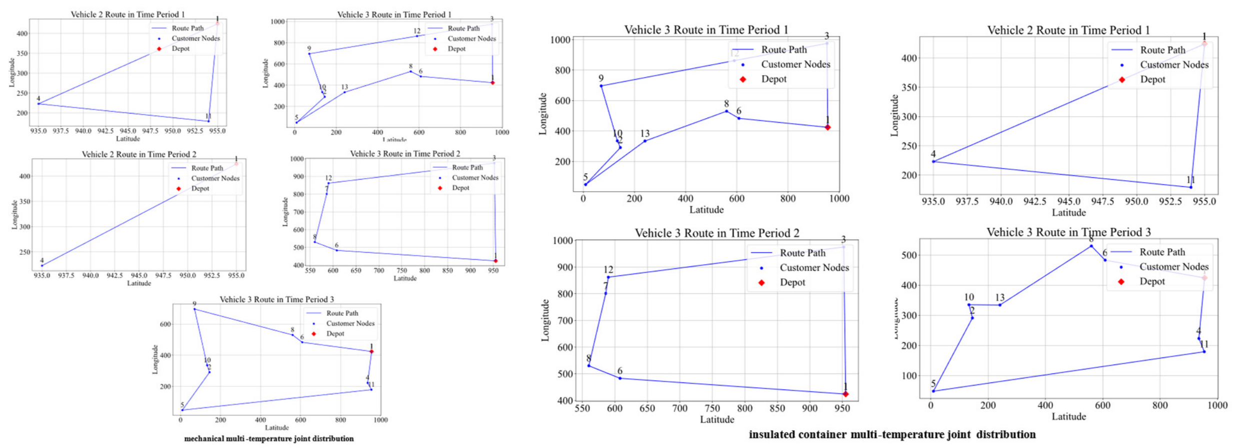

Figure 7. Routings of the optimal distribution plan.

The result shows that the proposed method can help the enterprise to find the optimal distribution strategy and the optimal distribution plan with minimum logistics.

(1) The optimal transportation strategy for this enterprise is obtained: the insulated container multi-temperature joint distribution. Compared with the other transportation strategy, the optimal transportation strategy can save approximately \$69,840 in logistics costs per year.

(2) The optimal distribution plan under the optimal transportation strategy is obtained. Compared with the optimal distribution plan under other transportation strategies, the optimal distribution plan under the optimal transportation strategy can save approximately 7.4%.

(3) In the optimal distribution plan under the optimal transportation strategy, the loading efficiency of vehicles and the inventory turnover rate are significantly enhanced. The average vehicle loading rate increased 17.5%. The average inventory turnover rate increased by 0.51.

5.4. Sensitivity analysis

In urban distribution, variations in rental vehicle costs, refrigerated container costs, temperature control costs, and demand for multi-temperature products can significantly impact logistics costs. This study used the expanded MMIRP dataset to explore the impact of the above data changes on logistics costs.

5.4.1. The ratio of multi-temperature products' impact on logistics cost

To demonstrate changes in the ratio of multi-temperature products, this study selects the optimal distribution plan of case 2. Under the condition of maintaining the total demand for multi-temperature products unchanged, the study adjusts the ratio of multi-temperature products to the logistics cost of the optimal distribution plans for each transportation strategy. The experimental results are shown in Table 11, where the minimized logistics cost (C_{Best}) is presented.

Table 11. Logistics costs under different ratios of multi-temperature products.

Ratio	Transportation strategy 1			Transportation strategy 2			Logistics cost difference	
	C_{Best}	T	Gap	C_{Best}	T	Gap	Value	Proportion
1:9	**1380**	1	0	1634	2	0	254	18.4%
2:8	**1317**	1	0	1469	1	0	152	11.5%
3:7	**1592**	2	0	1617	2	0	25	1.6%
4:6	1505	1	0	**1297**	1	0	208	16%
5:5	1771	1	0	**1271**	1	0	500	39.3%
6:4	1860	1	0	**1298**	1	0	562	43.3%
7:3	1954	1	0	**1321**	1	0	633	47.9%
8:2	2132	1	0	**1349**	1	0	783	58%
9:1	2181	1	0	**1470**	1	0	711	48.4%
Average								31.6%

Table 11 shows the logistics cost of the optimal distribution plans under different ratios of multi-temperature products for each transportation strategy, identifying the ratio of multi-temperature products of 4:6 as the threshold. Specifically, when the ratio of ordinary products is below 40%, the mechanical multi-temperature joint distribution has the lowest logistics cost, with reductions of 15.5%, 10.3%, and 1.5%. Once the ratio of ordinary products exceeds 40%, the cold-storage multi-temperature joint distribution has the lowest logistics cost, with reductions of 13.8%, 28.2%, 30.2%, 32.3%, 36.7%, and 32.6%. If the optimal transportation strategy is not chosen, the distribution plan's logistics cost will increase by an average of 31.6%. Figure 8 shows a bar chart to provide a more intuitive comparison of the variations in logistics costs under different demand ratios.

From Table 12 and Figure 8, it can be seen that through the analysis of case 2, the threshold for choosing the optimal transportation strategy is 40%. In this study, the threshold's theoretical expression was derived through indifference point methods. According to Equations (1) and (16), the costs of the two transportation strategies are simplified, as shown in the following Equations (21) and (22):

$$C_1 = CI_1 + CT_1 + a_{-1} + CN_1 \tag{21}$$

$$C_2 = CI_2 + CT_2 + a_1 + l_d * \frac{(1-\alpha) * Q}{q_d} \tag{22}$$

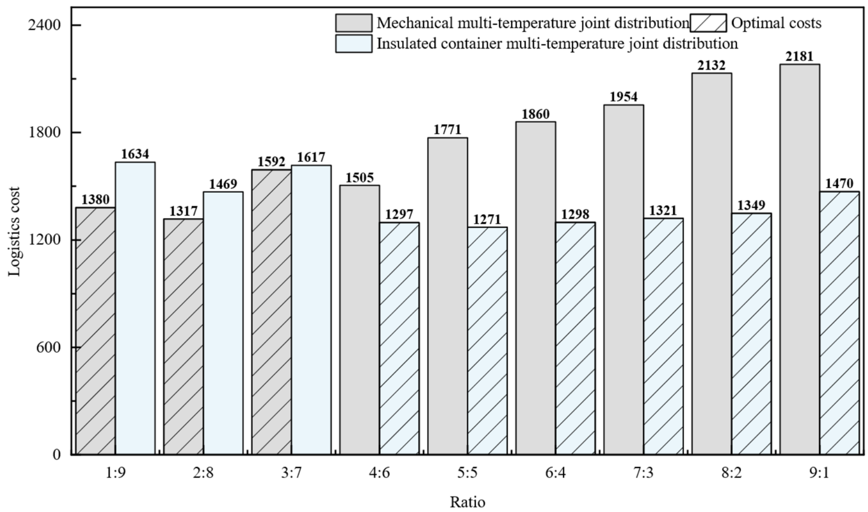

Figure 8. Logistics costs under different ratios of multi-temperature products.

α represents the proportion of ordinary products, Q represents the total transportation volume, C_1 represents the cost of the transportation strategy 1, C_2 represents the cost of the transportation strategy 2, CI represents the inventory cost, CT represents the transportation cost, *a* represents the rental cost, CN_1 represents the temperature control cost, and $l_d * \frac{(1-\alpha)*Q}{q_d}$ represents the cost of the refrigerated container. Letting $C_1 = C_2$, the threshold's theoretical expression is derived:

$$\alpha = 1 - \frac{q_d[(CI_1 - CI_2) + (CT_1 - CT_2) + (a_{-1} - a_1) - CN_1]}{l_d * Q} \tag{23}$$

5.4.2. The influence of vehicle cost on logistics cost

To demonstrate changes in rental vehicle costs, this study selects the optimal distribution plan of case 3. Under all other conditions constant, the study adjusts rental vehicle costs to the logistics cost of the optimal distribution plans for each transportation strategy. The experimental results are shown in Table 12, where the minimized logistics cost (C_{Best}) is presented.

Table 12 shows that when rental vehicle costs increase, the logistics costs of the optimal distribution plans in different transportation strategies also increase. Specifically, when the rental vehicle cost changes, the average change in the logistics cost of the optimal distribution plan is 10.1% for the mechanical multi-temperature joint distribution. When the rental vehicle cost changes, the average change in the logistics cost of the optimal distribution plan is 8.9% for the insulated container multi-temperature joint distribution. In conclusion, changes in rental vehicle costs have a high impact on logistics costs.

Table 12. Logistics costs under different rental vehicle costs.

a_{-k}	Transportation strategy 1	Logistics cost difference		a_k	Transportation strategy 2	Logistics cost difference	
	C_{Best}	Value	Proportion		C_{Best}	Value	Proportion
−60%	1065	264	19.9%	−60%	1130	204	15.3%
−40%	1153	176	13.2%	−40%	1198	136	10.2%
−30%	1197	132	9.9%	−30%	1155	179	13.4%
−20%	1241	88	6.6%	−20%	1266	68	5.1%
−10%	1285	44	3.3%	−10%	1223	111	8.3%
0	1329	0	0	0	1334	0	0
10%	1369	40	3%	10%	1342	8	0.6%
20%	1409	80	6%	20%	1402	68	5.1
30%	1449	120	9%	30%	1358	24	1.8%
40%	1489	160	12%	40%	1470	136	10.2%
60%	1569	240	18.1%	60%	1583	249	18.7%
Average		138.4	10.1%	Average		118.3	8.9%

5.4.3. The influence of temperature control costs on logistics costs

To demonstrate changes in temperature control costs and refrigerated container costs, this study selects the optimal distribution plan of case 3. Under all other conditions constant, the study adjusts temperature control costs and refrigerated container costs to the logistics cost of the optimal distribution plans for each transportation strategy. The experimental results are shown in Table 13, where the minimized logistics cost (C_{Best}) is presented.

Table 13. Logistics costs under different temperature control and container costs.

l_c	Transportation strategy 1	Logistics cost difference		l_d	Transportation strategy 2	Logistics cost difference	
	C_{Best}	Value	Proportion		C_{Best}	Value	Proportion
−80%	1201	128	9.6%	−80%	1159	175	13.1%
−60%	1233	96	7.2%	−60%	1225	109	8.2%
−40%	1265	64	4.8%	−40%	1270	64	4.8%
−30%	1281	48	3.%	−30%	1219	115	8.6%
−20%	1297	32	2.4%	−20%	1231	103	7.7%
−10%	1313	16	1.2%	−10%	1245	89	6.7%
0	1329	0	0	0	1334	0	0
10%	1345	16	1.2%	10%	1271	63	4.7%
20%	1361	32	2.4%	20%	1283	51	3.8%
30%	1377	48	3.6%	30%	1297	37	2.8%
40%	1393	64	4.8%	40%	1386	52	3.9%
60%	1425	96	7.2%	60%	1412	78	5.8%
80%	1457	128	9.6%	80%	1438	104	7.8%
Average		64	4.8%	Average		87	6.5%

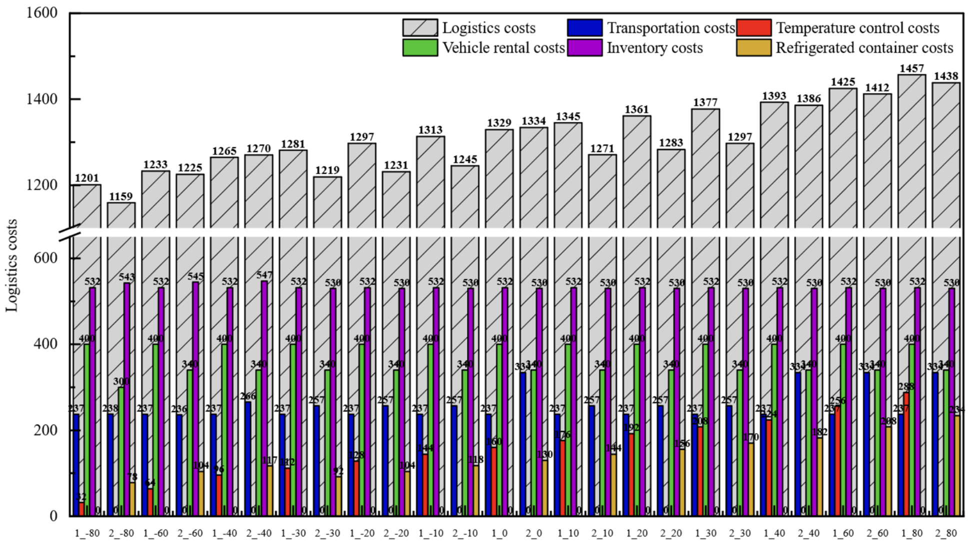

Figure 9. Logistics costs under different temperature control and container costs.

Table 13 shows that changes in temperature control costs and refrigerated container costs affect the logistics costs of the optimal distribution plans in different transportation strategies. Specifically, when the temperature control costs change, the average change in the logistics cost of the optimal distribution plan is 4.8% for the mechanical multi-temperature joint distribution. When the refrigerated container costs change, the average change in the logistics cost of the optimal distribution plan is 6.5% for the insulated container multi-temperature joint distribution. In conclusion, changes in temperature control costs and refrigerated container costs have a small impact on logistics costs. Figure 9 shows a bar chart to provide an intuitive visualization of the impact of temperature control costs and container costs on logistics costs.

5.4.4. The influence of container volume on logistics costs

To demonstrate changes in the volume of partitioned compartments and refrigerated containers, this study selects the optimal distribution plan of case 3. Under all other conditions constant, the study adjusts the volume of partitioned compartments and refrigerated containers to the logistics cost of the optimal distribution plans for each transportation strategy. The experimental results are shown in Table 14, where the minimized distribution cost (C_{Best}) is presented.

Table 14 shows that changes in the volume of the partitioned compartments and refrigerated containers affect the logistics costs of the optimal distribution plans in different transportation strategies. Specifically, when the partition compartment volume changes, the average change in the logistics cost of the optimal distribution plan is 2.9% for the mechanical multi-temperature joint distribution. When the refrigerated container volume changes, the average change in the logistics cost of the optimal distribution plan is 5.3% for the insulated container multi-temperature joint distribution. In conclusion, changes in partition compartment volume and refrigerated container volume have a small impact on logistics costs. Figure 10 shows a bar chart to provide an intuitive visualization of the impact of the partitioned zone and refrigerated container volume on logistics costs.

Table 14. Logistics costs under different partitions and refrigerated container volumes.

q_c	q_b	Transportation strategy 1	Logistics cost difference		q_d	Transportation strategy 2	Logistics cost difference	
		C_{Best}	Value	Proportion		C_{Best}	Value	Proportion
−60%	−60%	1518	189	14.2%	−60%	1529	195	14.6%
−50%	−50%	1440	111	8.4%	−50%	1464	130	9.7%
−40%	−40%	1360	31	2.3%	−40%	1425	91	6.8%
−30%	−30%	1360	31	2.3%	−30%	1350	16	1.2%
−20%	−20%	1358	29	2.2%	−20%	1373	39	2.9%
−10%	−10%	1311	18	1.4%	−10%	1305	29	2.2%
0	0	1329	0	0	0	1334	0	0
10%	10%	1329	0	0	10%	1269	65	4.9%
20%	20%	1322	7	0.52%	20%	1268	66	4.9%
30%	30%	1322	7	0.52%	30%	1284	50	3.7%
40%	40%	1315	14	1.1%	40%	1284	50	3.7%
50%	50%	1315	14	1.1%	50%	1283	51	3.8%
60%	60%	1315	14	1.1%	60%	1259	75	5.6%
	Average		39	2.9%		Average	71	5.3%

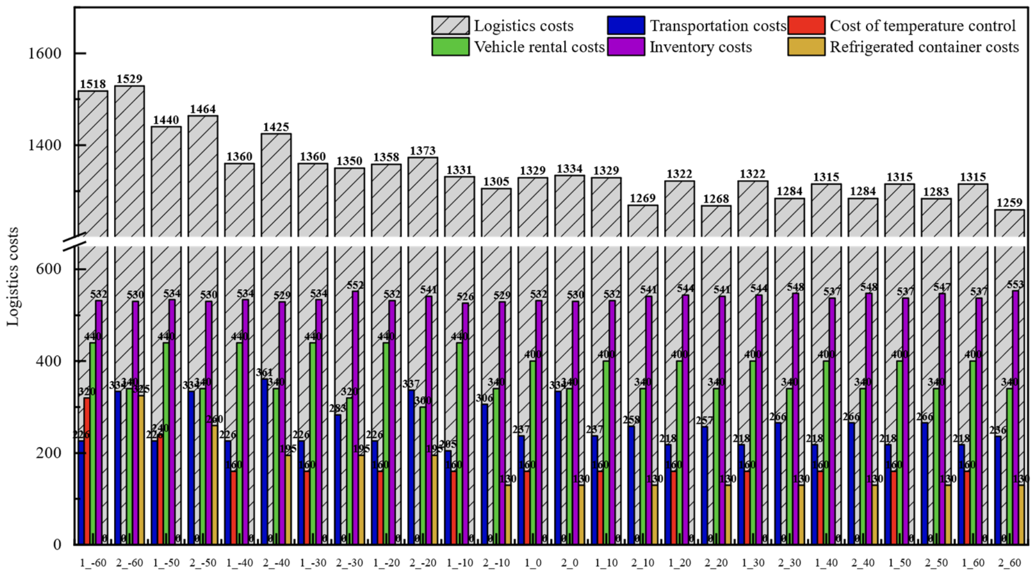

Figure 10. Logistics costs under different partitions and refrigerated container volumes.

6. Conclusions

The high logistics cost of cold chain logistics has become a key bottleneck hindering the development of the fresh produce e-commerce industry, where products have multi-temperature characteristics. Two transportation strategies commonly used in this industry are considered: the mechanical and the insulated container multi-temperature joint distributions. For each transportation strategy, an inventory and routing optimization model considering the characteristics of multi-temperature products, multi-period, and heterogeneous vehicles is proposed, and the branch and cutting algorithm is applied to find the optimal distribution plan to minimize logistics costs.

Based on this research, this paper reveals the laws of logistics cost changes during multi-temperature product distribution, which are influenced by transportation strategies, total transportation volume, and the ratio of multi-temperature products. The threshold for selecting the optimal transportation strategy is found, which is the ratio of multi-temperature products. In the case study, as the ratio of ordinary products was below 40%, the optimal transportation strategy was the mechanical multi-temperature joint distribution; otherwise, the optimal transportation strategy would be the other. Specifically, if the optimal transportation strategy is not chosen, the distribution plan's logistics cost will increase by an average of 31.6%. Moreover, parameters closely correlated to logistics cost were found, which include rental vehicle cost, temperature control costs, refrigerated container costs, partition compartment volume, and refrigerated container volume. In the case study, the average impact of these parameters on logistics costs was 1.35%, 0.25%, 1.6%, 3.2%, and 2.5%. The partition compartment volume parameter exerts the predominant impact on logistic costs. Based on these findings, valuable management insights are recommended to managers as follows:

(1) For the fresh e-commerce industry, each enterprise should select the optimal transportation strategy based on its multi-temperature products' ratio. The proposed methodology can help managers to select the optimal transportation strategy with minimal logistics cost. If the optimal transportation strategy is not chosen, the distribution plan's logistics cost will increase by an average of 31.6%.

(2) For the distribution of multi-temperature products, managers must extend their primary concern beyond distribution efficiency to encompass equally critical inventory management efficacy. The inventory cost has a high proportion of, on average, 55.9%. Combined with the methods proposed in this study, this can help managers find the distribution and inventory plan with minimum logistics costs.

(3) Managers should focus on the variations of the key parameter in the market environment. The variations of the key parameter will exert the predominant impact on logistics cost. Combining the sensitivity method adopted in this study can help managers find the key parameter, such as the partition compartment volume parameter, which exerts the predominant impact on logistics costs (3.2%).

This study has open-sourced all the data, models, and code on GitHub (https://github.co m/wbb-wq/DATA-FOR-PAPER), but managers still need to re-collect data and adjust paramete rs according to their own scenarios. Moreover, the research has the following limitations:

(1) It does not take into account the perishability of refrigerated products, and it does not account for carbon emissions.

(2) The model of this study assumes that vehicles are not affected by dynamic factors such as traffic congestion, road gradients, and weather changes during the delivery process. However, the operational data used in this study comes from the delivery area in Chongqing, which is known for its complex mountainous terrain and severe traffic congestion. In a practical scenario, the time-varying nature of traffic conditions can lead to extended travel times and fluctuating speeds, thus increasing the workload and energy consumption of refrigeration equipment and significantly raising the logistics cost. At the same time, extending the transportation time will accelerate the deterioration of chilled products, affecting their freshness and shelf life.

Future research will incorporate more uncertain factors into the MMIRP framework, including time-varying travel speeds, stochastic traffic conditions, environmental changes, and individual differences among drivers, in order to better reflect real-world operational scenarios. Special attention will be given to evaluating the impact of dynamic traffic and complex terrain (e.g., in cities such as Chongqing) on refrigeration energy consumption and product freshness degradation. This will enhance the model's applicability in mountainous and congested urban environments. Additionally, more efficient and accurate heuristic algorithms will be developed to solve large-scale instances of the problem. Future research will also focus on establishing a standard benchmark instance set.

Author contributions

Author 1 contributed to the study conception and design, data collection and analysis. The data visualization was completed by Author 1. The first draft of the manuscript was written by Author 1. Author 2 read and approved the final manuscript. Author 2 obtained financial support.

Use of Generative-AI tools declaration

AI-assisted tools (ChatGPT provided by OpenAI) were used solely to improve grammar and language clarity, without contributing to the scientific content.

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Conflicts of interest

The authors certify that they have no affiliations with or involvement in any organization or entity with any financial interest or non-financial interest in the subject matter or materials discussed in this manuscript.

Data availability statement

The data of problem instances are available upon reasonable request from the corresponding author.

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