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Research article

How does green advertising serve as a quality signal in the gray market?

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Abstract: In this study, we investigated how a manufacturer strategically uses green advertising to navigate the complexities of gray markets and enhance overall supply chain profitability when selling through two downstream retailers to two countries (regions). Green advertising, which causes less consumer aversion compared to traditional advertising, is increasingly used as a means to improve corporate reputation. In this framework, the manufacturer first determines the level of green advertising expenditure, then sets the wholesale price, followed by the retailers establishing the retail and gray market product prices, with customers making a final decision on which product to purchase. We found that when consumers do not prefer the green product, green advertising serves as an external signal to address quality information asymmetry, a cost that is solely borne by high-quality manufacturers. The level of investment in green advertising increases with the complexity of the gray market structure. When customers have a green preference, green advertising as an internal signal is a necessary investment for manufacturers and can act as a tool to activate or curb the gray market. Finally, we verified that when the green-gray effect is present, the main conclusions remain valid and green advertising not only affects perceived product quality but also changes consumers' relative valuation of gray products.

Keywords: green advertising; gray market; quality signaling; information asymmetry; supply chain management
Mathematics Subject Classification: 91A80, 91A10, 90B06

1. Introduction

The gray market is created by unauthorized retailers as an alternative (parallel) channel to sell the same genuine branded products. When the price difference of the authorized product between the two markets is greater than the transaction cost, downstream retailers are motivated to arbitrage. Such markets are becoming increasingly common across industries. On average, $58 billion of gray market

products flow through the global economy, and it was thought to be 22% of the $267 billion of the luxury personal goods market, as of 2021. Gray markets are prevalent in the cosmetics, food, pharmaceutical, and electronic device industries. Procter & Gamble's downstream retailers sell gray products in different regions, even though they have authorized retailers in their own region. A total of 70% of mobile phones are sold on the gray market in Malaysia, and two-thirds of personal computers are sold on the gray market in India, where the gray market continues to thrive [1,2]. The gray market is even more prosperous for virtual products. Today, nearly 80% of console game revenue in China is generated by gray market sales. The Xbox Series console does not have regional locks, enabling players to freely switch regions to purchase cheaper games (see Appendix G for details).

Gray markets have significant consequences for firms and supply chains. In the market for watches over $5,000, gray products account for a 20% share. Because gray products are sold by unauthorized retailers rather than designated channel members, they are often not supported by official warranties or after-sales services, which may damage brand reputation and weaken channel governance [3−5].

Even so, the gray market seems to be unstoppable in its rising momentum. Experts project that the number will grow to $1.5 trillion worldwide in the coming years. One reason for the prosperity of the gray market is the lack of internal channel control and the high cost of monitoring and detection [6]. Furthermore, the support for the 'first sale doctrine' by the EU and the US is also an important reason for the prevalence of the gray market. The US Supreme Court once ruled in favor of a speculator who resold textbooks on eBay in the United States. Moreover, EU law stipulates that if a product has been put on the market, it has the right to circulate within the EU region [7,8]. Surprisingly, on the other hand, companies do not always implement management control systems to strictly curb the gray market [9].

Against this background, a central challenge for manufacturers is not only whether to curb the gray market, but also how to manage downstream channels when product quality information is imperfectly observed by retailers and consumers. In practice, manufacturers usually have superior information about product quality, production standards, and brand-related attributes, while downstream market participants can infer such information only indirectly. This information asymmetry is particularly important in gray-market settings, where unauthorized resale may further distort consumers' perceptions of product reliability and service quality. Autrey et al. [10] believe that when there is a gray market, investments such as advertising can benefit downstream retailers and increase their own revenue. This positive externality helps companies establish a foothold when entering new markets. However, advertising, while expanding product awareness and increasing searches for related products, can also cause customer aversion [11].

To address this informational problem, manufacturers may rely on advertising-related investments that shape downstream beliefs about product quality. Unlike traditional advertising that focuses solely on promoting products, green advertising emphasizes conveying the green aspects of products and can significantly reduce customer aversion [12]. Investment in green advertising has increased tenfold in the past two decades [13,14]. In 2022, Audi released a green advertisement titled *"Pioneering the Future"*, and Alipay released *"The Green Lifestyle"*. In 2023, Apple Inc. released an advertisement called "*Nature Mother*", which promises to permanently eliminate carbon footprints, aiming to disclose the company's environmental efforts throughout the product production process.

Why have large companies in various industries invested in green advertising in recent years? Traditional supply chain internal information sharing mechanisms, such as EDI, POS, and VMI cannot convey quality information indirectly to the end of the market, resulting in information asymmetry

between the supply chain and customers. On the contrary, green advertising is a clever investment for companies to convey quality information to downstream retailers and customers at the same time. On the one hand, green advertising has a positive effect on corporate reputation, avoiding the aversion that traditional advertising causes in consumers; on the other hand, information transmission is just a byproduct of green advertising, saving the cost of disclosing quality information to downstream retailers and customers.

Retailers'participation in the buying and selling of gray products is the main reason for the formation of the gray market [15−18]. We have developed a supply chain with a gray market channel, where the manufacturer sells products through retailer 1 and retailer 2 to two separate markets. Customers in the H-market have a higher valuation for the product, while customers in the L-market have a lower valuation for the product. Since the manufacturer has more control over product quality, he will implement green advertising to indirectly convey quality information and thus affect customers' judgment of product quality. In our first model, we consider the cases where no retailer sells gray products, only one retailer sells gray products, and both retailers sell gray products simultaneously, as shown in Figure 1.

We have studied three forms of green advertising as an indirect conveyance of quality signals. In the first part, we consider that customers in all markets do not have green preferences, and at this time, green advertising is a dissipative signal for a company to improve its reputation. The empirical study by Banerjee and Wathieu [19] indicates that due to individual cognitive differences, customers' quality beliefs do not necessarily have an inevitable connection with external signals. We have abandoned the assumption in the quality signal literature that there is a monotonically non-increasing relationship between external signals and product quality [20,21] or the most pessimistic assumption (where beliefs on the off-equilibrium path are valued at zero) [22]. Considering the recent entry of Chinese companies such as Xiaomi and Huawei into the European market, where the valuation of products is lower than in the domestic market but the green concept is more prevalent, we have developed a second model where the manufacturer establishes a presence in a new market where customers who have a preference for green products. In this case, the gray products flow only from the new market (L-market) to the domestic market (H-market), and this green preference can directly increase customers' valuation of the product. Since customers inevitably have some skepticism about green advertising, which reduces the effectiveness of green advertising, we have introduced a customer skepticism coefficient. Finally, we consider the behavior of manufacturers when green advertising affects customers' relative evaluation of gray market product. Note that in our models, green advertising cannot directly improve product quality.

In this paper, we examine how green advertising functions as a quality signal in the presence of gray markets, following the supply-chain decision sequence from the manufacturer's advertising and wholesale pricing to the retailer's channel pricing and consumers' purchase choices. In Section 2, we review related literature. In Section 3, we develop the baseline model and characterize equilibria under alternative gray-market structures and information regimes, establishing the main mechanism. In Section 4, we provide numerical analyses to illustrate key comparative statics. Building on the baseline, in Section 5, we extend the model by incorporating consumers' green preference toward green advertising, and in Section 6, we further introduce the green–gray effect, where green advertising also shapes consumers' perceptions of gray products. In Section 7, we conclude with managerial implications and future research directions.

2. Literature review

2.1. The signaling theory of green advertising

The concept of green advertising originates from environmental advertising [23]. Compared with non-green advertising, green advertising emphasizes the environmental attributes of products [24]. Subsequently, green advertising became closely linked with corporate social responsibility (CSR). Advertisements that promote the concept of a green lifestyle and emphasize an enterprise's environmental protection efforts are generally classified as green advertisements [25,26]. Subsequently, the connotation of green advertising was further extended. Manrai [27] proposed that environmental protection practices throughout the production process effectively enable customers to perceive firms' environmental behavior and enhance green customer satisfaction, thereby constituting qualified green advertising. Banerjee [28] gave three definitions of green advertising in a more comprehensive way: First, the content of the advertisement explicitly or implicitly indicates the relationship between the product or service and the ecology and the physical environment; second, the advertisement advocates a green lifestyle; and third, the advertisement presents the image of the advertiser actively undertaking corporate social responsibility. In essence, this stream of literature suggests that green advertising does more than communicate environmental attributes; it also shapes how consumers evaluate the firm and its products more broadly.

In form, green advertising is much like government-funded public service advertisements, but in fact it is a marketing tool for enterprises to gain consumers' favorable impression by conveying product heterogeneity [22,29]. From the perspective of personal benefit, green advertising can make customers think that the products produced by enterprises are healthier and of better quality [30−32]; from the perspective of society and values, buying the products of enterprises implementing green advertising is a kind of signal transmission, which can show customers' concern for the environment [33]. These two factors affect consumers' purchase intention [34−36]. These findings imply that green advertising can affect not only environmental evaluations, but also consumers' beliefs about product quality and firm credibility.

Signal theory was first used by Spence [37] to solve the problem of information asymmetry in the labor market. In the product market, enterprises also need to use signals to convey their own advantages, even if they have to pay high costs [38]. Researchers extended this signaling perspective to sustainability-related operational settings. For example, Wang et al. [39] showed that green advertising is not merely a communication tool but also interacts with firms' operational and reputational decisions. Zhang et al. [40] investigated how to balance portfolio selection and carbon emissions when climate risk is characterized by ambiguity aversion. Wang and Li [41] demonstrated through empirical research that high-quality green advertisements of enterprises strengthen customers' acceptance of the environmental properties of products, enhance the persuasive effect of advertisements, and consequently influence customers' purchase decisions. Wei et al. [33] studied the effect of signal transmission of green products on payment intention through green perceived quality. In terms of green advertising content, Luchs et al. [42] and Kronrod et al. [43] held that linguistic and semantic implications of green advertising also affect customers' cognition of products and companies in the process of signal transmission. Taken together, these studies indicate that green advertising can function as a costly and influential signal, especially when consumers cannot directly verify relevant product attributes.

Although green advertising is widely studied and used in industrial and academic circles [44−46,13,14], it is abused to a certain extent as a signal to convey a certain idea of the company, that is, green-

washing behavior, which distorts the signaling function of green advertising. When the social atmosphere advocates green consumption, and green products enjoy a good reputation in the country, enterprises are motivated to falsely report their environmental achievements and exaggerate the environmental protection of the product production process, thereby sending out misleading signals to customers [47]. Because an enterprise's green behavior is essentially a credence attribute, consumers often cannot verify these environmental actions even after product usage. This difficulty can motivate companies to send excessive or misleading signals about their environmental efforts [48,49]. Investing in green advertising to send these signals requires additional resources, such as specialized equipment, advertisers, new human resources, and updated management practices within the company. As a result, companies may be motivated to divert resources from improving product quality, in order to offset the costs of signaling [50,51]. Berger [34] argues that such unreliable signals lead to more risks for customers seeking to obtain high quality products. Therefore, the green advertising literature not only highlights its persuasive and reputational value, but also raises a more fundamental question: Under what market conditions can green advertising credibly serve as a quality signal rather than merely an environmental claim? The literature pays less attention to how green advertising operates as a quality signal under severe channel-level information asymmetry. This limitation becomes particularly important in gray-market environments, where product authenticity, service support, and quality perceptions are harder for consumers to evaluate.

2.2. Asymmetric information of quality in the gray market

Since Akerlof's [52] groundbreaking study, the issue of asymmetric information regarding product quality has captured scholarly attention. However, the context of the gray market presents this issue with unique complexities. The gray market, also termed parallel importation, involves the unauthorized distribution of products to consumers, often driven by price discrepancies across different markets [16,15,53]. Here, we address the problems of asymmetric quality information within the gray market and explores strategies to manage these challenges effectively. In particular, we emphasize two issues that are most relevant to our study: Why quality uncertainty is especially severe in gray-market channels, and what mechanisms may be used to mitigate that uncertainty.

The major concerns involve addressing the asymmetry of quality information inherent in the gray market and leveraging this asymmetry strategically to enhance utility for market participants [54]. Unlike authorized channels, the gray market intrinsically suffers from profound information asymmetry. Because products are sold outside the manufacturer's official control, consumers often lack access to verifiable quality assurances, authorized warranties, or reliable after-sales support. To mitigate this gap, information senders may employ various strategies. One approach is to indirectly signal product quality through mechanisms such as pricing, advertising, after-sales service, return guarantees, and the deliberate creation of scarcity [55−60]. Another approach is to directly disclose quality information, such as revealing inspection results as a structured signal. While these studies collectively suggest that signaling and disclosure are the two major approaches for coping with quality uncertainty, both become significantly more difficult in gray markets, where unauthorized resale weakens the manufacturer's direct control over information transmission.

The persistent prevalence of the gray market further complicates this informational landscape. Its existence is legally supported by the 'Exhaustion Doctrine', and it is often tolerated because it can tap into otherwise inaccessible consumer bases, potentially increasing sales revenues for trademark owners. More importantly, the circulation of genuine products within the gray market offers a level of authenticity that partially reduces customer risk [10]. However, this creates a profound informational

paradox: Products are genuine enough to foster partial consumer trust, yet the lack of official channel support leaves quality assessment fundamentally incomplete. Furthermore, the gray market cannibalizes authorized sales and risks long-term erosion of brand reputation due to absent after-sales services [61,62]. These characteristics make gray markets especially challenging from an informational perspective: products are genuine enough to create partial consumer trust, yet the lack of official channel support leaves quality assessment incomplete.

2.3. Research gaps and motivation

Despite the rich separate streams of literature on green advertising signaling and gray market operations, the logical connection between them remains underexplored. The literature mostly views green advertising either as a tool for CSR or strictly as a signal of ecological attributes. Furthermore, while traditional signaling strategies have been studied to mitigate information asymmetry in gray markets, the strategic use of green advertising specifically as a robust quality signal in this context has been largely overlooked. This paper bridges this critical gap. By explicitly linking green advertising signaling to the severe quality asymmetry inherent in the gray market, we investigate how manufacturers can deploy green advertising not just to cater to environmental preferences, but as a costly, credible mechanism to signal overall high product quality and differentiate themselves from unauthorized alternatives. This integration provides the foundational motivation for our theoretical model.

3. Basic model: Green advertising signals' product quality

Consider a manufacturer sells a single product in two segmented markets with different customers' value, and the market with high (low) customers' value is called the high(low)-end market, which is denoted as the H(L)-market. In both markets, we normalized the measure of customers to one. The manufacturer delegates retailer 1(2) to sell a product in the H(L)-market. Due to factors such as transportation costs, pricing policy, and income level differences, the two retailers adopt different pricing strategies; thus, both retailers have the incentive to sell a gray product in the other's market. Given the manufacturer's strong brand presence and the authorized retailers' local monopoly power within their respective segmented channels, we assume the pricing strategies of the manufacturer and retailers are decided on a take-it-or-leave-it basis.

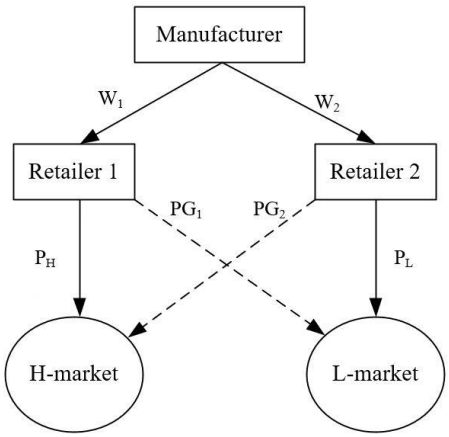


Figure 1. Product flows in a gray market.

The net utility obtained by the customer from consuming a unit of authorized product and gray product in market i are, respectively, $v_i - p_i, \theta v_i - p_G (i = H, L)$, where $\theta \in [0,1]$ is the discount factor. The value of gray market product is perceived as lower than that of authorized products, primarily because the after-sales service for grey market products is not guaranteed. When θ=0, the gray product has no value for customers; when θ=1, the gray product and authorized product are not different for customers. In this model, θ is assumed to be independent of product quality index *j*, because the effect of *j* on consumer utility is captured by the baseline valuation v, whereas θ reflects only the additional discount associated with the gray channel relative to the authorized channel. The utility is zero if customers do not purchase any product. The product can be high quality or low quality. We refer to the manufacturers that produce high(low)-quality products as *h*(*l*)-type manufacturers. Product quality is the manufacturer's private information, and its prior distribution μ_0 is the common knowledge. Customers have different valuation for products of different quality even with the same market (customers always value high-quality products more).

In this case, customers' value is determined by market segmentation and product quality. For brevity, in the following text, we omit the subscript *j* when the product quality is not specified. To capture the heterogeneity in consumers' willingness to pay arising from diverse income levels and personal preferences within each market, customers' value follows a uniform distribution $v_{ij} \sim U[0, v_{ij}](j = h, l)$, where *j* denotes the product quality. This uniform distribution assumption is a standard approach to derive linear demand functions. To concentrate on the role of green advertisements in the context of asymmetric information, we assume that the retailers' sales costs and the manufacturer's production costs are zero [63]. This widely adopted abstraction enables us to isolate the signaling mechanism, ensuring that pricing and advertising strategies are driven by quality perception and market segmentation rather than operational cost efficiencies.

3.1. Symmetric information

In this scenario, retailers and customers can distinguish the type of manufacturer. The manufacturer earns its revenues based on its product quality, which is known as monopolistic separation. The manufacturer has no incentive to signal its type through green advertising (g); hence, the signaling mechanism does not exist. The equilibrium concept we use is the subgame perfect Nash equilibrium (SPE).

In the unilateral gray market case, L-market's customer's valuation below a certain threshold gives retailer 2 an incentive to look for demand in other markets ($0 \le v_L < 2\theta^2 - 6\theta / \theta - 2$ is needed to guarantee only retailer 2 sells a gray product). It is unprofitable for retailer 1 to sell gray products because there is no residual demand in the L-market. Retailer 2 faces no competition while retailer 1 competes against the gray product imported from retailer 2. Given the product quality *j*, the conditions for customers to buy authorized products and gray products are, respectively:

$$\begin{cases} v_{Hj} - p_H > \theta v_{Hj} - p_G, v_{Hj} - p_H > 0 \\ v_{Lj} - p_L > 0 \\ \theta v_{Hj} - p_G > v_{Hj} - p_H, \theta v_{Hj} - p_G > 0 \end{cases}$$

(1) yields the demand for each market:

$$\begin{cases} D_{Hj} = 1 - \dfrac{p_H - p_G}{v_H(1-\theta)} \\ D_{Lj} = 1 - \dfrac{p_L}{v_L} \\ D_{Gj} = \dfrac{p_H - p_G}{v_H(1-\theta)} - \dfrac{p_G}{v_H\theta} \end{cases}$$

(2) Inequality (1) and equation (2) imply the profit functions of the manufacturer and retailers under the unilateral gray-market structure. Due to market segmentation, neither retailer is aware of the other's decisions. Using backward induction, we have

$$\pi_{r1,j} = (p_H - w_1)D_{Hj}$$

$$\pi_{r2,j} = (p_L - w_2)D_{Lj} + (p_G - w_2)D_{Gj}$$

$$\pi_{M,j} = w_1 D_{Hj} + w_2\left(D_{Lj} + D_{Gj}\right)$$

The game is solved as a Stackelberg game: Retailers 1 and 2 first maximize their respective profits, and the manufacturer then maximizes its own profit by choosing the wholesale prices (w1, w2).

Now, we need to guarantee that retailers 1 and 2 can sell gray products in each other's market. The residual demand in the L market is not satisfied, and retailer 1 also sells gray products. By a similar derivation, the profit functions of supply-chain members under the bilateral gray-market structure are given by

$$\pi_{r1,j} = (p_H - w_1)D_{Hj} + (p_{G1} - w_1)D_{G1,j}$$

$$\pi_{r2,j} = (p_L - w_2)D_{Lj} + (p_{G2} - w_2)D_{G2,j}$$

$$\pi_{M,j} = w_1\left(D_{Hj} + D_{G1,j}\right) + w_2\left(D_{Lj} + D_{G2,j}\right)$$

The game is solved in the same way: Retailers 1 and 2 first maximize their respective profits, and the manufacturer then maximizes their own profit by choosing the wholesale prices.

Here, p_{G1} (p_{G2}) represents the price of gray products by retailer 1(2), while D_{G1j} (D_{G2j}) are the corresponding demand of gray products. Intuitively, Equation (4) shows that in a bilateral gray market, both retailers optimize their profit margins by balancing the demand from their primary authorized market against the arbitrage opportunities in the opposing market. The manufacturer, anticipating this cross-market cannibalization, sets the wholesale prices to maximize total supply chain extraction.

The selling price is not only related to the market structure, but also to the discount coefficient of gray products θ. In fact, it is precisely because $\theta \neq 0$ that the retailer's decision is linked to the customers' value of the other market. For example, it might seem counterintuitive that in the unilateral gray market, retailer 1's decisions are correlated with v_L. This is because v_L affects the quantity that retailer 2 sells gray products. The concavity of the profit function reflects the strategic positioning of each entity in the market. The manufacturer, being the 'owner of the market,' follows the law of diminishing marginal returns, while the retailer, acting as a 'speculator,' may experience increasing marginal profits (see Table A.1 in Appendix A for details).

3.2. Asymmetric information

To study the signaling effect of green advertising, we initially assume that customers have no inherent preference for green products (however, in Section 5, we abandon this assumption to study the case of customers' green preference). In this part, green advertising is used only as a signal to disclose potential product quality (such as performance, reliability, and safety). If the manufacturer does not invest in green advertising, neither retailers nor customers can accurately distinguish the product quality. If the manufacturer invests in green advertising, the outcome may be pooling or separating from the market point of view, which is determined by the structure of the belief. Under what circumstances will manufacturers invest in the same level of green advertisements? The h-type manufacturer has a strong incentive to signal its type for a higher profit though green advertising (g); but how much green advertising is necessary to make itself stand out? What is the particularity of the equilibrium solution in a supply chain structure with a gray market? To answer those questions, we consider Perfect Bayesian equilibrium (PBE).

We assume that customers assign a probability μ to the belief that the product is of high quality. Thus, the customers' valuation of the product in market i is $v_i = \mu v_{ih} + (1-\mu)v_{ih}$. Due to the introduction of the behavioral belief system, we can further write Table A.1 as a function of μ, $a_i = v_{ih} - v_{il}, b_i = v_{il}$.

When customers do not have a preference for green products, green advertising indirectly affects only the values of price and profit without changing their functional form.

In the baseline model, we interpret green advertising g primarily as advertisement exposure that can be purchased at an approximately constant unit price (e.g., cost per impression). Hence, we adopt a linear cost specification $C(g) = k_i a, i \in h, l$ as a parsimonious approximation. This baseline choice keeps the equilibrium characterization transparent and enables us to focus on how the gray-market structure and quality-information asymmetry shape signaling incentives and pricing decisions. In the extensions, green advertising also represents credibility-enhancing efforts (e.g., certification and overcoming attention saturation), whose marginal difficulty increases with intensity; therefore, a convex quadratic cost is a standard way to capture escalating marginal costs.

In the absence of green preferences, g does not enter consumers' utility directly and therefore has no direct effect on demand. Instead, g operates purely as a Spence-type costly signal chosen by a privately informed manufacturer. Because product quality is not directly observable ex ante, consumers rationally use the observed advertising level to update beliefs about the manufacturer's type and, hence, the expected quality. A higher g can credibly convey high quality because its payoff is realized only through this belief-updating channel: If consumers assign a higher posterior probability to h-type after observing g, the expected revenue term increases; conversely, a low-quality manufacturer would find it unprofitable to mimic a high g because it would incur the advertising cost without being able to sustain the corresponding increase in the cost. Hence, even without green preferences, green advertising transmits quality information through Bayesian updating [37]. When there is a preference for green products, the level of green advertising directly influences customers' valuation of the product, thereby changing the structure of the price and profit functions. To ensure the existence of equilibrium, we set the cost of green advertising as a quadratic function. In fact, we need to take only the square root of the equilibrium g here to ensure the consistency of g across the two models.

Corollary 3.1

The greater the likelihood that customers perceive the product to be of high quality, the higher the retail price, wholesale price, and profits of supply chain members. $\frac{\partial w_1}{\partial \mu} > 0$, $\frac{\partial w_2}{\partial \mu} > 0$, $\frac{\partial p_H}{\partial \mu} > 0$, $\frac{\partial p_G}{\partial \mu} > 0$, $\frac{\partial p_L}{\partial \mu} > 0$, $\frac{\partial \pi_{R1}}{\partial \mu} > 0, \frac{\partial \pi_{R2}}{\partial \mu} > 0, \frac{\partial R}{\partial \mu} > 0$

When the signaling cost is not taken into account, the improvement of customers' posterior belief, which is positively correlated with the willingness to pay, boosts the manufacturer's pricing power and thus makes the supply chain better off.

3.2.1. Pooling equilibria

We first derive on-equilibrium posteriori belief in pooling equilibria and explain why it is an uneconomical outcome. By the definition of pooling equilibrium, we have $g = g^{p^*}, \mu = \mu_0$.

To mathematically sustain these equilibria, we must define how customers interpret unexpected (off-equilibrium) levels of green advertising. The following conditions ensure that any deviation from the equilibrium advertising level is interpreted in a way that makes the deviation unprofitable for the manufacturer.

Lemma 3.1

If $g^{p^*} \neq 0$, then $\pi_M(0) = R(\mu_0) - k_l g^{p^*}, \pi_M(1) = R(\mu_0) - k_h g_h^{s^*}$ can be supported as a pooling equilibrium by specifying off-equilibrium brief as follows:

(i) If $0 < g < g^{p^*}$, then μ satisfies $R(\mu) \leq R(\mu_0) - k_l(g^{p^*} - g)$

(ii) If $g^{p^*} < g$, then μ satisfies $R(\mu) \leq R(\mu_0) - k_h(g^{p^*} - g)$

if $g^{p^*} = 0, \pi_M(0) = \pi_M(1) = R(\mu_0)$ can be supported as a pooling equilibrium by specifying off-equilibrium brief as following:

(iii) If $g > 0$, then μ satisfies $R(\mu) \leq R(\mu_0) + k_h g$

k_h, k_l denote the cost coefficients of green advertising for h-type and l-type manufacturers, respectively.

To a certain extent, Lemma 3.1 relaxes the restrictions on customers' beliefs in the signaling games literature. Customers' acceptance of green advertisements is unlimited when pooling. This belief structure is due to the competitive behavior of the manufacturer, which makes it impossible for customers to distinguish the approximate quality of products in the market and can rely only on prior beliefs for judgment. This is uneconomical for the supply chain and customers. We will prove that this kind of equilibrium can be ruled out by intuitive criteria in Section 3.2.3.

Proposition 3.1

(i) $g^{p^*} \in [0, \frac{R(\mu_0) - R(0)}{k_l}]$

(ii) $\forall g_0 \in \left(\frac{R(\mu_0) - R(0)}{k_l}, +\infty \right)$, g_0 *will not be a pooling equilibrium.*

In fact, (ii) is a corollary of (i). Proposition 3.1 has identified the set of all pooling equilibria. Compared to the symmetric case in Table A.1, the profit of the *l*-type manufacturer has increased while the profit of the *h*-type manufacturer has decreased. Customers are exposed to the same level of green

advertisements, which leads to identical beliefs, raising the customers' evaluation of low-quality products and lowering the evaluation of high-quality products. In other words, green advertising fails to separate product quality in a pooling equilibrium: Although the two manufacturers differ in their true quality, they choose the same signal, so customers cannot infer quality from advertising and must instead rely on their prior belief. Because customers' belief directly affects their willingness to pay, and hence market prices and profits, this compression of beliefs distorts the market's assessment of product quality. It is precisely because of customers' belief that the *l*-type manufacturer is able to engage in free-riding behavior in this equilibrium. The *l*-type manufacturer can mimic the h-type manufacturer's advertising behavior and benefit from the favorable reputation associated with the h-type without possessing the same underlying quality. As a result, part of the informational advantage that should belong to the h-type manufacturer is effectively transferred to the l-type manufacturer. As a result, the *h*-type manufacturer, which suffers a loss in profit, is motivated to invest more in green advertisements. However, it is evident that markets, in reality, often cannot always achieve separation. This is because the maximum value of the pooling equilibrium $[R(\mu_0)-R(0)]/k_l$ is less than the minimum value of the separating equilibrium $[R(1)-R(0)]/k_l$. It is challenging to achieve separation when k_l is very low.

3.2.2. Separating equilibrium

We first derive on-equilibrium posteriori belief in pooling equilibria. By the definition of separating equilibrium, we have $g=g_l^{s*},\mu=0; g=g_h^{s*},\mu=1$ .Without loss of generality, we set $g_l^{s*}=0$.

Lemma 3.2

$\pi_M(0)=R(0),\pi_M(1)=R(1)-k_h g_h^{s*}$ can be supported as a separating equilibrium by specifying off-equilibrium brief as follows:

(i) If $0<g<\hat{g}$, then μ satisfies $R(\mu)\le R(0)+k_l g$

(ii) If $\hat{g}\le g<g_h^{s*-}$, then μ satisfies $R(\mu)\le R(1)-k_h(g_h^{s*}-g)$

(iii) If $g_h^{s*+}<g$, then μ is arbitrary.

$$\hat{g}=\frac{g_h^{s*}k_h+R(1)-R(0)-k_h g_h^{s*}}{k_l-k_h}$$

g_h^{s*}, g_l^{s*} denote the separating equilibrium green advertising levels for h-type and l-type. This indicates that the irregular beliefs on the left of the off-equilibrium path are constrained, while the belief on the right side has no restriction. Lemma 3.2 reveals two points: (i) More green advertising does not necessarily lead customers to believe that the product quality is higher. Customers' quality beliefs cannot be completely captured, but at least we can identify the region where beliefs must reside. (ii) The extent to which customers accept green advertising is limited with a threshold g_h^{s*}. If the threshold is exceeded, the level of green advertisements reaches saturation, and we cannot characterize the constraints on customers' belief. This aligns with real-world scenarios: Even companies that produce high-quality product will not invest indefinitely in green advertising.

Proposition 3.2

(i) $g_h^{s*} \in \left[\frac{R(1)-R(0)}{k_l}, \frac{R(1)-R(0)}{k_h} \right]$

(ii) $\forall g_0 \notin [\frac{R(1)-R(0)}{k_l}, \frac{R(1)-R(0)}{k_h}]$, g_0 *will not be a separating equilibrium of h-type manufacturer.*

Proposition 3.2 characterizes all possible separating equilibria. The profit of the *l*-type manufacturer remains unchanged while the profit of the *h*-type manufacturer has decreased compared to Table A.1 (see Appendix A). The *h*-type manufacturer cannot endure earning the same profit as the *l*-type manufacturer and, thus, chooses a high level of green advertising to prevent the *l*-type manufacturer from free-riding. It is easy to achieve separation when k_l is very high.

3.2.3. The most reasonable equilibrium

We demonstrate the existence of a least-cost separation equilibrium that can survive intuitive criterion refinement in three types of gray markets without restricting off-equilibrium belief of customers, also known as the Riley outcome. When customers have green preference, we will prove that the Riley outcome holds when there is a significant difference in the cost coefficients of green advertising investments.

Proposition 3.3

(i) Lemma 3.3 (see Appendix C) kills all pooling equilibria.

(ii) In all separating equilibria, only $g_l^{s*} = 0$, $g_h^{s*} = \frac{R(1)-R(0)}{k_l}$ survived Lemma 3.3.

This exalting result means that both markets can reach separating equilibrium with the least cost. However, under the premise of separation, how does the welfare of supply chain members change compared with symmetric information?

Proposition 3.4

Only h-type manufacturers have loss of profit in separating equilibrium.

$\pi_{r1}(0) = \pi_{r1}^s(l), \pi_{r2}(0) = \pi_{r2}^s(l), \pi_M(0) = \pi_M^s(l)$

$\pi_{r1}(1) = \pi_{r1}^s(h), \pi_{r2}(1) = \pi_{r2}^s(h), \pi_M(1) < \pi_M^s(h)$

Proposition 3.4 indicates that in the gray market, conveying a quality signal is not enough to induce l-type manufacturer to invest in green advertisements. In our setting, green advertisements are not just cheap talk but a signal that genuinely requires certain green practices in the product manufacturing process. Therefore, the quantity of green advertisements in society is insufficient at this time. We will discuss how to encourage l-type manufacturer to invest in green advertising in Sections 5 and 6. It can be proven that *h*-type manufacturers may not bear all the cost of green advertisements in following analysis.

4. Numerical experiment

4.1. The influence of belief on the supply chain model

To study the action logic of each supply chain member in detail and verify the above discussion, we set the parameter values as follows: $a_H = 0.2; b_H = 0.8;$ $a_L = 0.4; b_L = 0.3; \theta = 0.6.$ We illustrate

every type of equilibria of manufacturer and supply chain performance derived in Section 3.2 in Figure 2 and Figure 3. In Appendix D, we provide a detailed discussion of the logic of retailer's behavior.

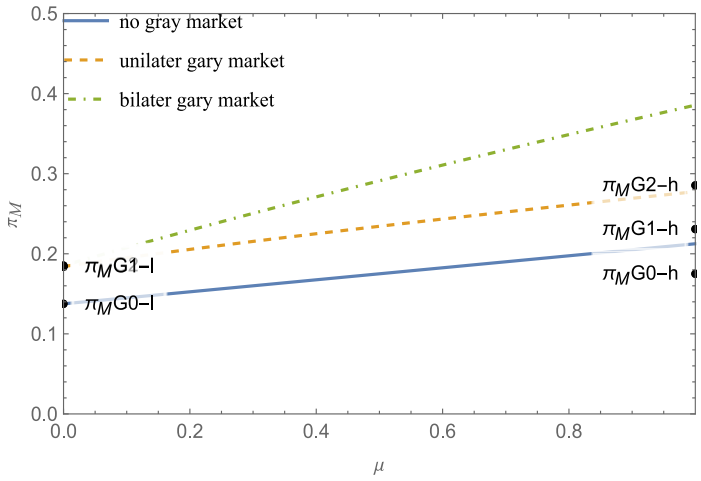


Figure 2. Manufacturer's profit.

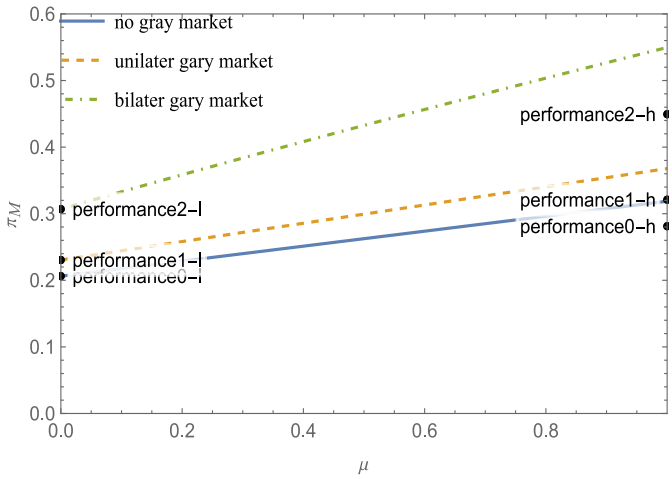


Figure 3. Supply chain's performance.

4.1.1. Optimal profit of the manufacturer

The black dots in Figure 2 represent the net profit of the manufacturer under the three gray market structures. The preferences of the manufacturer are divided, bilateral gray markets are strictly better than unilateral, and there are no gray markets in the separating equilibrium, which is consistent with the gray market literature.

As the number of gray channels increases, the profit of the manufacturer and the supply chain increase. In Figures 2 and 3, the ends of the dashed line represent the profits obtained by the manufacturers (supply chains) at the separating equilibrium. The points on the dashed line represent the manufacturer's profits at the pooling equilibrium for any prior belief μ_0. The black dots on both sides represent the net profits of the manufacturers (supply chains) at the separating equilibrium. It can be seen that the black dot on the left side of the dashed line coincides with the end of the line, indicating that the *l*-type manufacturer (supply chain) will not invest in green advertising, while the black point

on the right side of the dashed line is always below the points sorted by their respective types, indicating that the high type manufacturer (supply chain) will always invest in green advertising. It is worth noting that in the separating equilibrium, the profits (net profits) of the low type manufacturer are almost the same in the unilateral gray market (G1) and the bilateral gray market (G2), while the profits (net profits) of the supply chain increase significantly from G1 to G2, which can be inferred from Section 3 due to only retailer 1 obtaining excessive profits. From Proposition 3.2, it is known that as the number of gray channels increases, the level of green advertisement investment by the high type manufacturer must increase, i.e., as the order quantity increases, the manufacturer will increase the advertising investment.

4.1.2. Supply chain performance

The black dots in Figure 3 represent the net profit of the two types of supply chain under the three gray market structures. The performances of the *l*-type supply chain under the three gray market structures are 0.21, 0.23, and 0.31, respectively. The performances of the *h*-type supply chain under the three gray market structures are 0.28, 0.32, and 0.45, respectively.

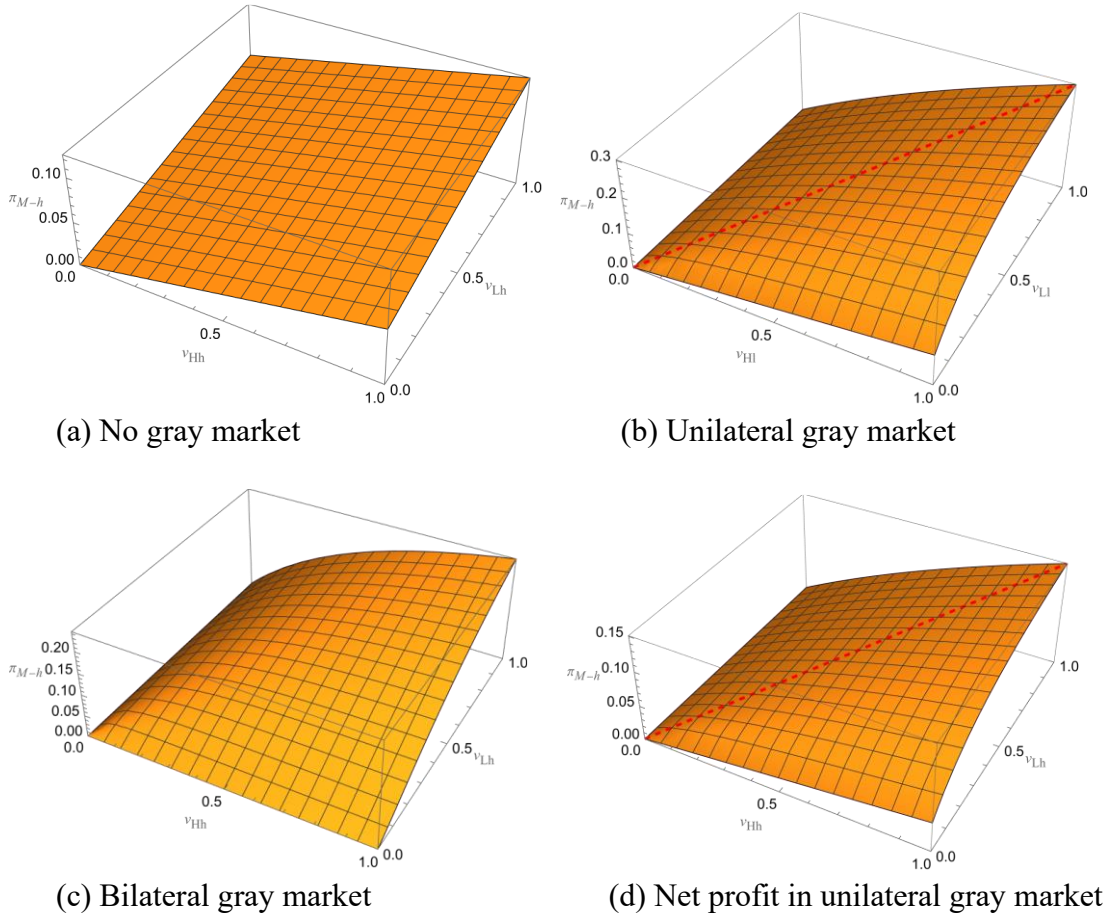
Table 1 The improvement of change for the gray market structure and belief on supply chain performance.

	No→unilateral	unilateral→bilateral	No→bilateral
l-type supply chain	11.73%	33.14%	48.76%
h-type supply chain	14.14%	39.95%	59.75%
Type change	*l*→*h* (no)	*l*→*h* (unilateral)	*l*→*h* (bilateral)
increase in profit	36.3548%	39.3059%	46.4321%

For a given type of supply chain, more gray markets are always better off than fewer; for a given gray market structure, an *h*-type supply chain is always better off than an *l*-type one. In a *h*-type supply chain, gray market can bring more performance. The improvement of product quality can make the supply chain performance better in a supply chain with more gray market.

4.2. The influence of willingness to pay on a supply chain model

In this section, we take $\mu = 1$ and $\mu = 0$ to simulate the choice and profit of the manufacturer in the most reasonable equilibrium. Due to space limitation, only the *h*-type manufacturer is shown. According to Section 3.2, the shape of the function for the *l*-type manufacturer is the same.



(a) No gray market (b) Unilateral gray market

(c) Bilateral gray market (d) Net profit in unilateral gray market

Figure 4. Manufacturer's profit.

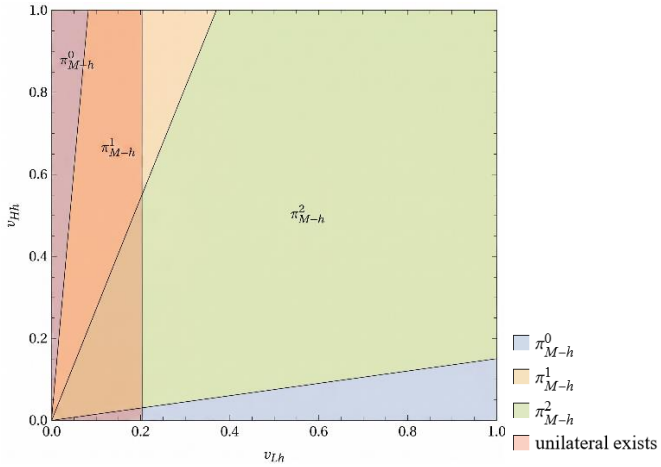
4.2.1. Gray market structure

We consider only the case where market segmentation exists, so we examine only the region to the right of the red line in Figure 4. The direction of the increase in the profit function is the same as the direction of the increase in willingness to pay (in fact, this is true for all variables except the profit function of retailer 2 in all supply chain structures). In Figure 4, we can observe the following findings: When there is no gray market, the manufacturer's profit function is a convex cone, indicating that the manufacturer's wholesale price decision is separating. However, as retailers begin to sell gray products, the manufacturer's profit becomes increasingly curved, indicating that the manufacturer should take increasingly consideration of the interwoven influence between the high and low markets when setting the wholesale price. This impact has two aspects: On the one hand, it fills the demand of the vacancy in the two markets and increases the maximum profit that the manufacturer can obtain; on the other hand, it enhances the interdependence between the high and low markets, resulting in the emergence of extreme conditions. For example, in a bilateral gray market, if no one is willing to buy the product at the L-market, the supply chain may collapse. To sum up, we believe that although the gray market improves the performance of the supply chain, it also introduces greater instability into supply chain decision-making.

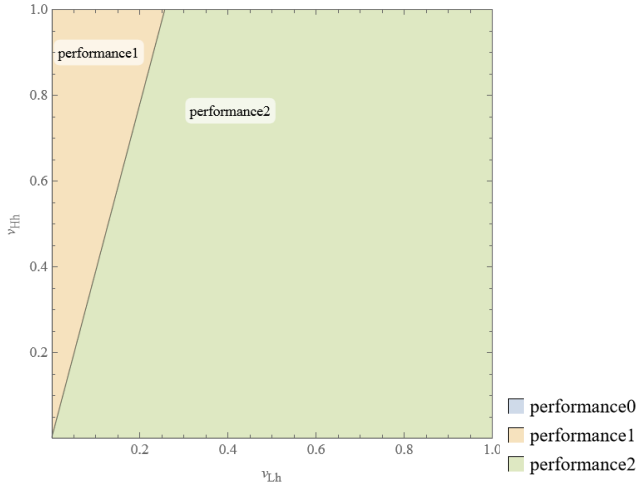
4.2.2. Asymmetric information

Comparing Figure 4(b) with Figure 4 (d), we can find that information symmetry does not alter the shape of profit function; it affects only its magnitude. The profit of a high-quality manufacturer at the least-cost separating equilibrium is less than that under the conditions of information symmetry. Therefore, the supply chain structure influences the form of the profit function, whereas market information solely modifies the profit's scale.

4.2.3. Customers' valuation



(a) Manufacturer's optimal gray market structure



(b) Supply chains optimal gray market structure

Figure 5. The optimal choice under different gray market structures.

In Appendix E, we show profit increase as customers' valuation increase in both markets (retailer 2's profit is decreasing with respect to buyers' value in the high-end market). The profit functions for retailer 1 in bilateral gray markets is convex, while all other variables are concave. This indicates that changes in wholesale prices, retail prices, manufacturer' profit, and the overall performance of the

supply chain are highly sensitive to the customers' valuation. However, when the customers' valuation in both markets are similar, the supply chain tends to stabilize

We focus only on the region where $v_{Hh} > v_{Lh}$ in Figure 5. When the customer value of the L -market is very low, no gray market is the most beneficial for the h-type manufacture. When the marketer's customer value is relatively low, the unilateral gray market is the most beneficial for the h-type manufacture. As the customer value of the L-market gradually increases, the bilateral gray market is the most beneficial for the h-type manufacture, and this trend is expected to continue. We can see that v_{Lj} determines which market structure h-type manufacturers prefer. The essential reason behind this pattern is the shape of the manufacturer's revenue function, so the least-cost separating equilibrium advertisement investment and the profit of the manufacturer follow this pattern. The only difference is that when the customer value of the L-market is low, the supply chain performance of the unilateral gray market is always better than that of no gray market.

4.3. Robustness and sensitivity analysis

4.3.1. Robustness with respect to the valuation-gap parameter

To further verify the robustness of our results, we refine the numerical simulation by considering a more comprehensive set of parameter settings. For market i, where $i \in \{H, L\}$, the expected quality valuation of consumers is written as

$$\tilde{v} = \mu v_{ih} + (1-\mu)v_{il} = \mu(v_{ih} - v_{il}) + v_{il} = a_i\mu + b_i,$$

where $a_i = v_{ih} - v_{il}$ and $b_i = v_{il}$. Since $v_{Hl} > v_{Ll}$, we have $b_H > b_L$. In the original analysis, we consider only the case $a_H < a_L$. To supplement this, we further examine the case $a_H > a_L$. The simulation results show that the manufacturer's profit follows the same qualitative pattern as in Figure 2, indicating that our parameter setting for a_i is robust.

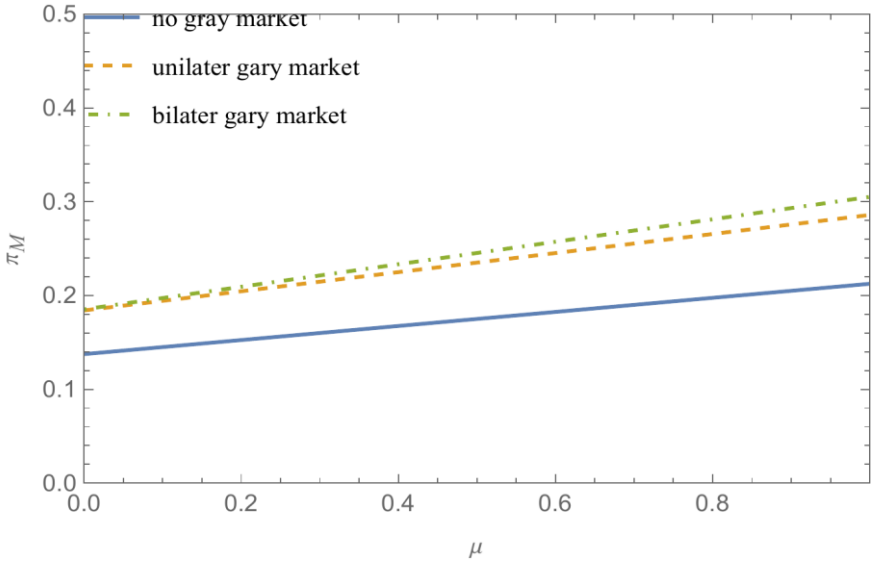


Figure 6. The manufacturer's profit when $a_H > a_L$.

As shown in Figure 6, the manufacturer's profit exhibits the same qualitative pattern as discussed

above when $a_H > a_L$.

4.3.2. The effect of cost coefficient on equilibrium green advertising

We also examine the impact of the cost coefficient on the equilibrium level of green advertising. Based on the analysis in Section 3, the Riley equilibrium is given by

$$g_l^{s*} = 0, \qquad g_h^{s*} = \frac{R(1) - R(0)}{k_l}.$$

Since a low-quality manufacturer has no incentive to invest in green advertising in the absence of green preferences and the gray-green effect, we focus on this equilibrium. Numerical results under different gray-channel structures exhibit the same qualitative trend; for brevity, we report only the unilateral gray-channel case.

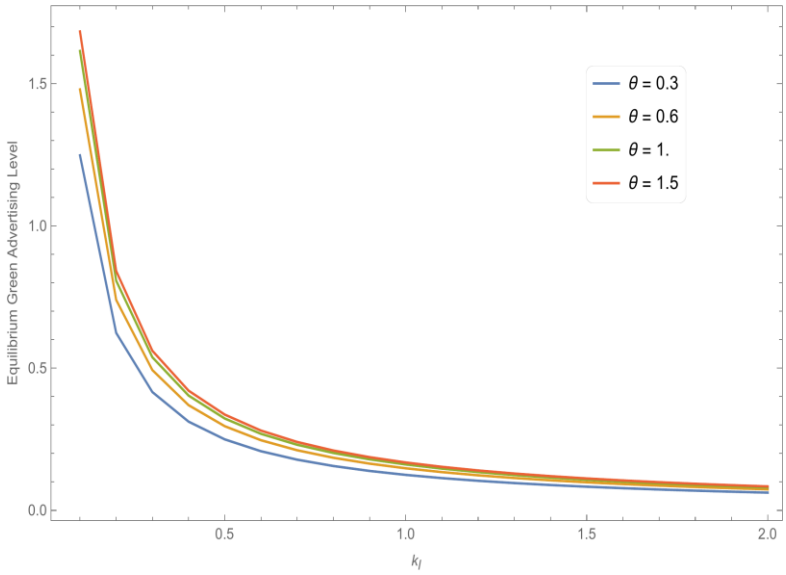


Figure 7. The comparative static of cost coefficient on equilibrium green advertising level.

As shown in Figure 7, the equilibrium level of green advertising decreases as the cost coefficient k_l increases, which is consistent with the analytical results.

4.3.3. Comparative statics of gray-product discount factor and sensitivity analysis

Furthermore, we conduct a comparative-static analysis with respect to the gray-product discount factor θ. The results show that the equilibrium level of green advertising increases with θ. This suggests that not only the expansion of gray channels, but also a higher valuation of gray products can increase firms' green advertising investment.

Finally, we perform a sensitivity analysis with respect to the cost coefficient k_l. The results confirm that the equilibrium level of green advertising is negatively related to k_l and positively related to θ. Moreover, k_l exhibits submodularity with respect to θ: A higher cost coefficient weakens the marginal positive effect of θ on the equilibrium green advertising level. This reflects the standard trade-off between the marginal benefit and marginal cost of signaling.

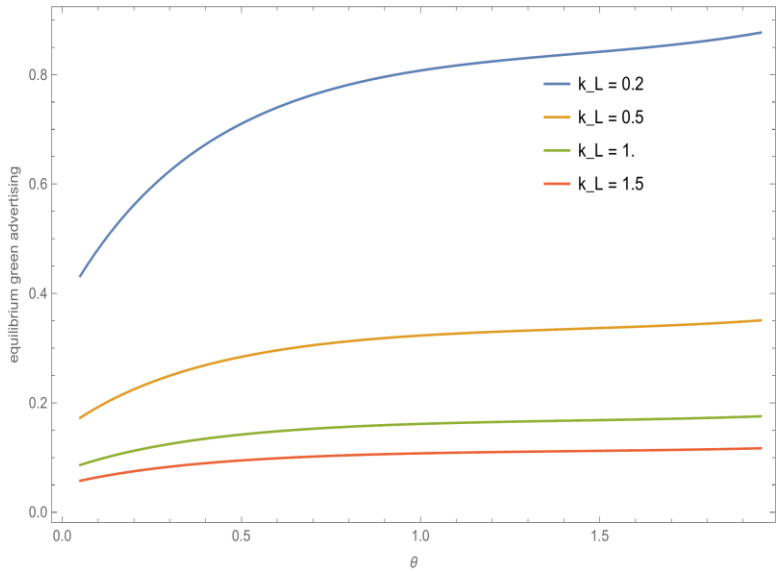


Figure 8. The comparative static of discount factor on the equilibrium green advertising level.

As shown in Figure 4, the equilibrium level of green advertising increases with the gray-product discount factor θ.

5. Green preference

Having established the fundamental signaling mechanism of green advertising under different gray market structures, we now turn our attention to a critical demand-side factor: Environmental awareness. The preceding analyses assumed a generalized consumer valuation without explicitly isolating the impact of consumers' preference for sustainable products. To better reflect the reality of green marketing, we extend the baseline model by incorporating consumer green preference.

Our model includes two markets: A H-market and L-market. Consider this scenario: The H-market is the domestic mature market, the L-market is the emerging market, and a customer has green preference, so we consider only the unilateral gray market case for simplicity.

As a manufacturer who wants to blaze emerging markets, they consider only the impact of L-market's green preference. When modeling the customer valuation structures, we account for the distinct consumption characteristics of emerging markets. The customers in the L-market perceive green products as having higher value than non-green product, choosing low-quality products mainly to meet their fundamental uses, since these products act as basic commodities fulfilling standard needs, and consumers' willingness to pay for their functional attributes is relatively uniform. Thus, the pricing of low-quality products in the market is more concentrated, while the pricing of high-quality products is more dispersed [64]. Therefore, we assume that customers' valuation of low-quality goods is homogeneous, i.e., $v_{Ll} = v_{L0} + bg$. The customers' valuation of high-quality products is always heterogeneous $v_{Lh} \sim U[v_{L0} + bg, 1 + v_{L0} + bg]$, driven by varying levels of disposable income, environmental awareness, and status-seeking behaviors among emerging market consumers. In the H-market, customers value low-quality products as $v_{Hl} = v_{H0}$, and the customers' evaluation of high-quality products is $v_{Hh} \sim U[v_{H0}, 1 + v_{H0}]$. v_{i0} $(i=H,L)$ is the utility brought by the functional attributes of the product. To reflect the macroeconomic reality that mature domestic markets (H-market) possess stronger overall purchasing power and a higher baseline willingness to pay compared to emerging markets, we assume that the purchase intention of customers in the H-market is much greater than that

in the L-market, i.e., $v_{H0} > v_{L0} + bg$, v_{ih} is independently distributed, $v_1 = v_{ih} - v_{il}$, and b is the customers' preference behavior for green advertising, $b \in [0,1]$. b>0 indicates that customers prefer green products, b=1 indicates customers have 100% trust in green advertisements, and $b \in (0,1)$ indicates that customers prefer green products but have certain skepticism toward green advertisements. The closer b is to 1, the less skeptical customers are toward green advertisements.

To capture the economic reality of diminishing marginal returns in marketing and the escalating difficulty of achieving higher signal intensity, we adopt a convex quadratic cost function for green advertising, $C_j(g) = k_j g^2, k_j > 1, j = h,l$. This structure implies that while initial advertising efforts are relatively inexpensive, reaching broader audiences, securing rigorous third-party green certifications, and overcoming consumer attention saturation become disproportionately more costly as the advertising level increases. Moreover, according to Ni et al. [65], enterprises producing high-quality products annotate altruistic attribute information on the content of green advertisements and highlight the egoistic functional attributes of green products, so that customer can feel the benefits brought by green products. Wang & Li [41] found that enterprises producing high-quality products have more authoritative, accurate, and reliable advertising information with strong perceived attractiveness, which has a more significant impact on customers' purchase intention. Base on the empirical study above, we assume $k_l > k_h$. The small k_h represents the high effectiveness of sending it.

The expected utility in the H-market is $\mu v_{Hh} + (1-\mu)v_{H0}$, and the customer chooses the authorized product when $v_H - p_H \geq \theta v_H - p_G$, i.e. $v_1 \geq \gamma - v_{H0}/\mu$, $v_1 = v_{ih} - v_{il}$ $\gamma = p_H - p_G/(1-\theta)$. $\gamma \in [v_{H0}, v_{H0} + \mu]$ is the conversion ratio of authorized products to gray products. Similarly, when $\gamma - v_{H0}/\mu \geq v_1 \geq 0$, the customer buys a gray product. When $\mu = 0$, the prices are take-it-or-leave-it for customers. Thus, the demand for authorized products is $D_H = 1 - \gamma - v_{H0}/\mu$, and the demand for gray products is $D_G = \gamma - v_{H0}/\mu$. Considering the after-sales service of an authorized product, customers will choose authorized products if $\gamma = v_{H0}$ [66].

In the L-market, a customer chooses to purchase the product if $v_1 \geq p_L - (v_{L0} + bg)/\mu$, $D_L = 1 - p_L - (v_{L0} + bg)/\mu$, $p_L \in [v_{L0} + bg, v_{L0} + bg + \mu]$. As with the H-market, the customers' purchase decision depends on the magnitude of p_L and $v_{L0} + bg$: i.e., $p_L \leq v_{L0} + bg$, $D_L = 1$; $p_L > vL0 + bg$, $D_L = 0$.[1]

5.1. Symmetric information

In the symmetric case, the *h*-type manufacturer faces heterogeneous demand, while the *l*-type manufacturer faces homogeneous demand.

Proposition 5.1. *In the h-type supply chain with complete information, both retailers have positive profit, while in the l-type supply chain, all profit of both retailers are deprived by the manufacturer, which it is concluded as follows*:

(i) The green advertising level, wholesale pric,e and profit of the two types of suppliers in equilibrium are

1 indicate customers' distrust of low-quality products. When they determine that the product is of low quality, they choose the gray product on the premise that the gray product is cheaper than the licensed product, and the licensed product can win the market as long as it is not more expensive than the gray product.

$$g_h^* = \frac{b(3+v_{L0})}{8k_h - b^2}, g_l^* = \frac{b}{k_l}$$

$$w_{1-h} = \frac{1}{2}\left(5 + v_{H0} + v_{L0} + bg_h^* - (2+v_{H0})\theta\right), w_{2-h} = \frac{4k_h g_h^*}{b},$$

$$w_{1-l} = v_{H0} + bg_l^* + v_{H0}(1-\theta), w_{2-l} = v_{L0} + bg_l^*$$

$$\pi_{M-h} = \frac{1}{8}\left(15 + (b^2 - 8k_h)\ g_h^{*2} + 2bg_h^*(3+v_{L0}) + vL0(6+v_{L0}) - 6\theta\right),$$

$$\pi_{M-l} = (1-\theta)v_{H0} + 2(v_{L0} + bg_l^*)$$

（ii）The retail price under the two supply chains in equilibrium are

$$p_{Lh} = \frac{1}{4}\left(5 + 3bg_h^* + 3v_{L0}\right)$$

$$p_{Hh} = \frac{1}{6}\left(17 + 3bg_h^* + 4v_{H0} + 3v_{L0} - 4(2+v_{H0})\theta\right),\ \ p_{Gh} = \frac{1}{6}\left(13 + 3bg_h^* + 3v_{L0} + v_{H0}(-1+\theta) - 4\theta\right)$$

$$p_{Ll} = w_{2-l}, p_{Hl} = w_{1-l}$$

（iii）The retailer's profit under the two supply chains in equilibrium are

$$\pi_{R1-h} = \frac{1}{36}(2+v_{H0})^2(1-\theta), \pi_{R1-l} = \pi_{R2-l} = 0$$

$$\pi_{R2-h} = \frac{27b^3 g_l^{*2} - 6g_l^*(3+3v_{L0})(3b^2 - 8k_h) - +b\left(99 - 144k_h g_l^{*2} + 54v_{L0} + 27{v_{L0}}^2 - 36\theta\right)}{144b}$$

There is no gray market in the *l*-type supply chain. Unlike in Section 3.2, green advertisements become necessary for the manufacturer in the asymmetric case.

5.2. Asymmetric information case

Before discussing the asymmetric information case, we can obtain that the signal cost coefficient of green advertisements is determined by the true type of manufacturer, and the structure of profit function is determined by the customers' cognition. According to the above conclusions, the manufacturer's profit functions under different circumstances can be obtained:

$$\pi_{hl}^M(g) = \pi_{M-l}(g;k_h),\ \ \pi_{lh}^M(g) = \pi_{M-h}(g;k_l),$$

$$\pi_h^M(g) = \pi_{M-h}(g;k_h),\ \ \pi_l^M(g) = \pi_{M-l}(g;k_l) \tag{6}$$

Sending excessive green advertisements induces customers to have heterogeneity in product valuation, thus improving customer valuation while creating the demand for circulation of gray products and obtaining more profit. Conversely, if the green advertisements are insufficient, the customer's valuation will degenerate into being homogeneous. This not only blocks the generation of gray market but also deprives downstream retailers of all revenue, compromising the supply chain.

Lemma 5.1 *（i）$\pi_{lh}^M(g)$ is maximized at $g_{lh}^* = \frac{b(3+v_{L0})}{8k_l - b^2}$.* *（ii）$\pi_{hl}^M(g)$ is maximized at $g_{hl}^* = \frac{b}{2k_h}$.*

To characterize separating equilibrium, we assume that there is a non-empty signaling set about green advertising $\Theta \in R$, such that the following belief structure exists:

$$u(g) = \begin{cases} 1, g \in \Theta \\ 0, otherwise \end{cases}$$

The *l*-type manufacturer faces the same decision problems as in the full information case, so

$g_l^{s*} = g_l^*$ [58]. According to the definition of separation equilibrium, the posterior belief of the customer is $\mu(g) = 0$ for the green advertising level g_l^* chosen by the *l*-type manufacturer. From the property of posterior belief and the definition of separating equilibrium, we know that $g_l^{s*} < g_h^{s*}$.

We have dictated the structure of separating equilibrium if it exists. Next, we show that there is a pure-strategy separation equilibrium. A necessary condition for its existence is that the customer believes there are two kinds of products with different quality in the H(L)-market.

Proposition 5.2

Θ *can be characterized as* $[\bar{g}', \bar{g}]$

(i) If $\bar{g}' > g_h^*, g_h^{s*} = \bar{g}'$

(ii) If $\bar{g}' \le g_h^*, g_h^{s*} = g_h^*$

Proposition 5.2 demonstrates that the *h*-type manufacturer can achieve separation under green preferences, and this separating equilibrium outcome survives intuition criteria and the lexicographically maximum sequential equilibrium (LMSE) refinement introduced by Mailath et al. [67]. Next, we seek the conditions under which they can achieve costless separation.

Proposition 5.3

Under certain conditions, the costlessly separation equilibrium is achieved when there is a significant difference between k_h *and* k_l*. Conversely, the costly separation equilibrium is achieved when the difference is less pronounced.*

Compared with Section 3.2, green advertising plays an additional role in Section 5, which not only induces customers' belief in product quality, but also directly affects customers' evaluation of products in the L-market. Furthermore, due to the particularity of the gray market structure, manufacturers can use the level of investment in green advertising as a tool to control whether downstream retailers sell gray products. If the manufacturer wants to curb the flow of gray products between the high and low markets, then he can choose a lower level of green advertising, where the customers' belief in the equilibrium generated by Bayes' rule makes the valuation of the product homogeneous, and this homogeneity in the equilibrium inhibits the flow of gray products, as shown by Proposition 5.1. A manufacturer choosing a higher level of green advertisements can activate the circulation of gray products between the high and low markets. Here, the intuition is that a customer in the H-market can judge the type of manufacturer by observing whether there are gray products, but the event sequence we set rules out this possibility (customers' beliefs are generated after observing green advertisements).

6. Model extension: green-gray effect

In the preceding sections, we systematically analyzed the baseline model to demonstrate how green advertising functions as a quality signal in gray markets, and subsequently examined how consumer green preferences influence these signaling strategies. In those analyses, the valuation discount of gray products was treated as an exogenous constant. Building upon these foundations, we introduce a further extension by relaxing this assumption to capture potential spillover effects across channels.

The gray products, originating from the same manufacturer as the authorized products, possess identical quality [68]. Cao and Zhang [69] and Altug [66] regard gray products and authorized products as perfect substitutes. However, due to the lack of official channel support, the after-sales service of many gray products lag behind that of authorized products. In this paper, the discount factor of gray

products reflects the customers' estimation of gray product quality before purchase, which is independent of the quality. Therefore, as the level of green advertising increases, customers have greater trust in the manufacturer. However, this increase in trust diminishes as the level of advertising continues to rise. Following Zhao [70] and Nian and Sundararajan [71], we use $\theta(g) = g/1+g$ to characterize this relationship. This satisfies $\theta(0^+) = 0, \theta(\infty) = 1$, and $\theta'(g) \geq 0, \theta''(g) \leq 0$, where the boundary condition indicates that the manufacturer cannot invest in green advertising to the extent that customers perceive no difference between gray and authorized products, and the derivative condition indicates that the role of green advertising in increasing consumer trust is decreasing.

With the green--gray effect, green advertising affects not only perceived product quality but also consumers' relative evaluation of gray products through $\theta(g)$. Because g now changes the demand allocation and the cross-channel substitution margin, we maintain a convex cost $C(g) = k_j g^2/2$ to reflect diminishing marginal returns and increasing marginal difficulty of strengthening the signal, and to keep the manufacturer's objective well behaved when g interacts with $\theta(g)$.[2] Following the step in Section 3.1 and Section 5, we can arrive at a conclusion similar to that in Section 5.

Proposition 6.1

In the presence of a green-gray effect, even if customers do not have a green preference, l-type manufacturers will invest in green advertising regardless of whether customers accurately know the product quality.

As g increases, customers tend to perceive the manufacturer as having a stronger sense of social responsibility or being more altruistic, which makes the quality of the gray product more reliable [10,41,12]. We refer to this as the green-gray effect. Unlike in Section 3, when the green-gray effect is taken into account, green advertising is no longer wasteful but can influence the relative value of authorized products and gray products.

Here, a manufacturer faces a trade-off: As known from Section 3, increasing θ will alleviate the double marginalization effect while increasing costs. It is worth pointing out that although the manufacturer's green advertising behavior is similar to that in Section 5, the mechanism is different.[3] Specifically, with the green–gray effect, green advertising no longer acts only as a wasteful signal of quality, as it also changes the relative attractiveness of authorized versus gray products through *θ(g)*. This introduces a richer trade-off. Increasing g raises *θ(g)*, which: (1) Mitigates double marginalization by intensifying downstream competition and reducing retail markups (Appendix F(i)); (2) strengthens cannibalization by making gray products closer substitutes for authorized products in the high-end market, eroding the authorized channel's pricing power and market share (Appendix F(ii)); and (3) expands residual demand by serving lower-valuation consumers via gray-channel substitution without fully displacing high-valuation demand (Appendix F(iii)), but this residual-demand expansion effect becomes weaker as green advertising increases (i.e., it weakens with a higher *g* through *θ(g)*). Because *θ(g)* is increasing but saturating (diminishing marginal trust gains) while advertising costs are convex, the marginal benefit of raising *g* declines, whereas the marginal cost increases, implying that the optimal green advertising level is typically interior and depends on the relative strength of coordination gains versus cannibalization losses.

If customers accurately know product quality, the structure of the SPE is similar to the results in

[2] If the advertising cost is a linear function $C(g) = k_j g$, the conclusion remains the qualitatively same. The reason we assume the advertising cost function to be quadratic is to make the subsequent graphs more pronounced.

[3] The former originates from the customer's preference for gray products, and the latter comes from the customer's preference for green products.

Section 5. We mainly focus on the situation where customers cannot accurately know product quality. When $k_h < k_l$, following the process of Proposition 5.2, it can be handily proven that $g_l^{s*} = g_l^*$ $g_h^{s*} = \max\{g_h^*, \bar{g}'\}$ still holds and is the unique equilibrium that survives intuitive criterion[4]. When $k_h = k_l$, the manufacturer's profit is related only to the perceived type and not related to the true type.

Proposition 6.2

When the marginal cost of green advertising is the same, the pooling equilibrium is the most reasonable equilibrium.

When $k_h = k_l$, the signaling technology becomes cost-symmetric, so green advertising is no longer a differentially costly signal across types. As a result, the single-crossing property fails, and the economic foundation for a separating equilibrium disappears. In particular, the low-type manufacturer no longer faces any intrinsic cost disadvantage in mimicking the high type's advertising choice. To sustain separation, the high type would therefore have to choose an excessively high advertising level, which is distortionary.

This distortion is especially costly under the green–gray effect. A higher advertising level not only increases the direct signaling cost, but also raises $\theta(g)$, making gray products closer substitutes for authorized products and intensifying gray-market cannibalization. Hence, relative to pooling, separation imposes a higher signaling cost and a stronger channel-conflict cost on the high type. For this reason, the pooling outcome weakly Pareto dominates the separating candidate in our setting, and stronger refinements such as lexicographically maximum sequential equilibrium (LMSE) select pooling while eliminating separation [67].

To achieve separation, the *h*-type manufacturer needs to invest in excessive green advertising, while the h-type gains more profit in the pooling. Furthermore, the pooling makes the *l*-type manufacturer share part of the h-type's prior beliefs, which also increases its profit. Therefore, the pooling equilibrium strictly Pareto dominates to the separating equilibrium[5].

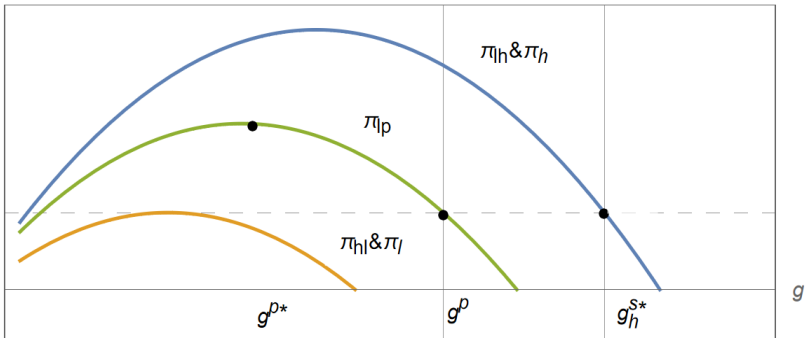


Figure 9. The profit of the manufacturer.

In the separating equilibrium, the incentive compatibility principle requires $\pi_h \le \pi_h\left(g_h^{s*}\right)$, while the profit of the *l*-type manufacturer is π_l^*. In the pooling equilibrium, the profit of the manufacturer is $\pi_{lp}\left(g^{p*}\right)$. The separating equilibrium still satisfies the intuitive criterion, but it is not a rational

[4] $g_h^*, \bar{g}', g_h^{s*}$ is qualitative same as Section5.

[5] $\pi_{lp}\left(g^{p*}\right)$ also survives the LMSE and UE refinement.

equilibrium. We can rule out this equilibrium using the *LMSE*. $\pi_{lp}\left(g^{p*}\right)-\pi_h\left(g_h^{s*}\right)\geq 0$ is the additional profit for the *h*-type manufacturer in the pooling equilibrium, while $\pi_{lp}\left(g^{p*}\right)-\pi_l^*\geq 0$ represents the additional profit for the *l*-type manufacturer.

From a managerial perspective, this result implies that when green advertising is easy to imitate, firms should not rely on it alone to signal superior quality. Instead, high-quality manufacturers may need harder-to-mimic and verifiable instruments, such as third-party certification, traceability, warranties, or authorized-channel-only service bundles, to achieve credible differentiation and mitigate gray-market encroachment.

Taking into account that customers' evaluation of gray products depends on the level of green advertising, namely the green-gray effect, and the equilibrium we obtain is similar to that in Section 5. However, there is a difference: When customers have a preference for green, an increase in green advertising investment by manufacturers will increase the customers' valuation, thereby increasing the expected demand; when the green-gray effect exists, an increase in green advertising investment by manufacturers will increase the customers' valuation. This increase intensifies the cannibalization effect (see Appendix D&F), causing gray products to encroach on a portion of the market share of licensed products in the H market.

7. Conclusion and discussion

In this paper, we investigate how manufacturers use green advertising to convey private product quality information to downstream retailers and consumers in a supply chain with gray channels. Our analysis shows that green advertising can play different roles depending on market conditions. When consumers do not have a green preference, green advertising mainly functions as an external quality signal that helps high-quality manufacturers distinguish themselves from low-quality manufacturers. In this case, as the number of gray channels in the supply chain increases, high-quality manufacturers need to strengthen their green advertising investment r to maintain separation. When consumers do have a green preference, however, green advertising not only affects quality perception but also generates a green premium, which changes firms' incentives and may induce low-quality manufacturers to invest in green advertising. Furthermore, when the green-gray effect is taken into account, green advertising influences perceived quality and consumers' relative valuation of gray products. Under some conditions, especially when signaling costs are similar across manufacturer types, pooling may become the more favorable equilibrium.

This study makes three major theoretical contributions. First, we establish green advertising as a novel quality-signaling mechanism in gray markets. Unlike traditional signals, green advertising combines the functions of reputation enhancement and lower consumer aversion, enabling it to serve as a costly external signal to mitigate severe quality information asymmetry in unauthorized distribution channels. Second, we enrich the operations and signaling literature by incorporating heterogeneous consumer green preferences into the signaling game. Our results show that green advertising can shift from a purely dissipative external signal to an internal operational necessity, depending on whether consumers value green attributes. This insight extends signaling models by relaxing the conventional monotonic belief assumptions. Third, we contribute to the gray market literature by conceptualizing and formalizing the green-gray effect. By showing that green advertising may affect quality beliefs and the relative attractiveness of gray products, we provide a new explanation for why pooling equilibria may arise and become optimal under certain signaling-cost conditions.

These findings also generate several important managerial insights. First, firms should not treat green advertising merely as a promotional or reputational tool. In gray markets, it can also serve as a strategic instrument for quality disclosure and channel governance. In particular, high-quality manufacturers facing more complex gray-market structures should strengthen green advertising investment to preserve quality differentiation. Second, firms should carefully assess consumer green preferences before making advertising decisions or entering new markets, because green preference changes whether green advertising primarily serves as a separating signal, a demand-enhancing investment, or both. Third, managers should recognize that green advertising is not always an effective signaling mechanism. When different manufacturer types face similar marginal costs of green advertising, imitation becomes easier, and the signaling role of green advertising weakens. In such cases, firms may need to rely more on alternative mechanisms, such as price, warranty and after-sales service, direct quality disclosure, third-party certification, or other credible signals, to reduce quality information asymmetry and better manage gray-market competition.

This study has several limitations that suggest directions for future research. First, in our model, only manufacturers invest in green advertising; future work may entail settings in which retailers also participate in advertising investment. Second, researchers may compare the signaling effectiveness of green advertising with that of general advertising or other alternative signals. Third, because our study relies on an analytical modeling framework centered on transaction volumes and profitability, researchers could use empirical data or behavioral experiments to test the theoretical predictions developed here, especially those related to the green-gray effect. Such research may also explore whether green advertising in gray markets influences not only purchase intentions but also broader value co-creation behaviors, such as post-purchase feedback, online reviews, and brand advocacy on social media. Finally, extending the model to different industries, such as luxury goods, consumer electronics, and pharmaceuticals, may yield more contextualized insights, since the signaling effectiveness of green advertising and the severity of gray-market cannibalization may vary substantially across sectors.

Author contributions

Yafei Yu: Conceptualization, methodology, formal analysis, validation, investigation, visualization, writing – original draft, writing – review & editing; Decheng Wen: Supervision, resources, funding acquisition, writing – review & editing; Xiao Chen: Supervision, resources, validation, project administration, writing – review & editing.

Use of Generative-AI tools declaration

The authors declare that they did not utilize any artificial intelligence (AI) tools in the creation of this article.

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Conflict of interest

The authors declare that there is no conflict of interest.

References

1. N. Singal, BT Exclusive: Customers flock to the grey market to purchase cheaper iPhones 15 Pro, Pro Max in India, *Business Today*, 2023. Available from: https://www.businesstoday.in/technology/news/story/bt-exclusive-customers-flock-to-the-gray-market-to-purchase-cheaper-iphones-15-pro-pro-max-in-india-398215-2023-09-13.
2. K. D. Antia, M. E. Bergen, S. Dutta, R. J. Fisher, How does enforcement deter gray market incidence? *J. Mark.*, **70** (2006), 92−106. https://doi.org/10.1509/jmkg.2006.70.1.92
3. S. Shannon, The billion-dollar grey market in watches upsets big brands, *Financial Times*, 2017. Available from: https://www.cnbc.com/amp/2017/09/15/the-billion-dollar-grey-market-in-watches-upsets-big-brands.html.
4. H. Huang, N. H. Lee, Y. C. Ho, Consumer attitude toward gray market goods, *Int. Mark. Rev.*, **21** (2004), 598−614. https://doi.org/10.1108/02651330410568033
5. J. Chen, L. Liang, S. Mukhopadhyay, D. Yao, Price or quantity? An analysis of strategies in the presence of gray markets, *Nav. Res. Logist.*, **70** (2023), 558−573. https://doi.org/10.1002/nav.22109
6. Deloitte, When channel incentives backfire: strategies to help reduce gray market risks and improve profitability, Deloitte, 2011.
7. G. Shapiro, Supreme Court gives American consumers victory over copyright owners in Kirtsaeng vs. John Wiley & Sons, *Forbes*, 2013. Available from: https://www.forbes.com/sites/garyshapiro/2013/03/20/supreme-court-gives-american-consumers-victory-over-copyright-owners-in-kirtsaeng-vs-john-wiley-sons/.
8. T. M. Valletti, S. Szymanski, Parallel trade, international exhaustion and intellectual property rights: a welfare analysis, *J. Ind. Econ.*, **54** (2006), 499−526. https://doi.org/10.1111/j.1467-6451.2006.00298.x
9. R. Lewis, D. Nguyen, Wasn't that ad for an iPad? Display advertising's impact on advertiser- and competitor-branded search, *Yahoo Research Working Paper*, 2012. Available from: https://www.ftc.gov/sites/default/files/documents/public_events/fifth-annual-microeconomics-conference/lewis.pdf.
10. R. L. Autrey, F. Bova, D. A. Soberman, When gray is good: gray markets and market-creating investments, *Prod. Oper. Manag.*, **24** (2015), 547−559. https://doi.org/10.1111/poms.12254
11. L. Lv, M. Huang, Can personalized recommendations in charity advertising boost donation? The role of perceived autonomy, *J. Advert.*, **53** (2024), 36−53. https://doi.org/10.1080/00913367.2022.2109082
12. A. A. Bailey, A. Mishra, M. F. Tiamiyu, Green advertising receptivity: an initial scale development process, *J. Mark. Commun.*, **22** (2016), 327−345. https://doi.org/10.1080/13527266.2014.904812
13. W. J. Kim, Y. Ryoo, S. Yoon, K. Kim, Ethical dissonance in environmental advertising: moderating effects of self-benefit versus other-benefit appeals, *Int. J. Advert.*, **40** (2021), 1320−1342. https://doi.org/10.1080/02650487.2021.1988218
14. Y. K. Kim, M. Y. C. Yim, E. Kim, W. Reeves, Exploring the optimized social advertising strategy that can generate consumer engagement with green message on social media, *J. Res. Interact. Mark.*, **15** (2021), 30−48. https://doi.org/10.1108/JRIM-10-2019-0171
15. F. Iravani, S. Dasu, R. Ahmadi, Beyond price mechanisms: how much can service help manage the competition from gray markets? *Eur. J. Oper. Res.*, **252** (2016), 789−800. https://doi.org/10.1016/j.ejor.2016.02.006

16. R. Ahmadi, F. Iravani, H. Mamani, Coping with gray markets: the impact of market conditions and product characteristics, *Prod. Oper. Manag.*, **24** (2015), 762−777. https://doi.org/10.1111/poms.12319
17. J. Shao, H. Krishnan, S. T. McCormick, Gray markets and supply chain incentives, *Prod. Oper. Manag.*, **25** (2016), 1807−1819. https://doi.org/10.1111/poms.12578
18. R. Ahmadi, F. Iravani, H. Mamani, Supply chain coordination in the presence of gray markets and strategic consumers, *Prod. Oper. Manag.*, **26** (2017), 252−272. https://doi.org/10.1111/poms.12635
19. S. Banerjee, L. Wathieu, Corporate social responsibility and product quality: complements or substitutes? *Int. J. Res. Mark.*, **34** (2017), 734−745. https://doi.org/10.1016/j.ijresmar.2017.06.006
20. S. Knox, S. Maklan, P. French, Corporate social responsibility: exploring stakeholder relationships and programme reporting across leading FTSE companies, *J. Bus. Ethics*, **61** (2005), 7−28. https://doi.org/10.1007/s10551-005-0303-4
21. Y. H. Li, D. B. Ni, X. W. Tang, Signaling quality in supply chains by corporate social responsibility, *Chin. J. Manag. Sci.*, **25** (2017), 38−47. https://doi.org/10.16381/j.cnki.issn1003-207x.2017.07.005
22. H. Ki, J. Y. Kim, Sell green and buy green: a signaling theory of green products, *Resour. Energy Econ.*, **67** (2022), 101266. https://doi.org/10.1016/j.reseneeco.2021.101266
23. W. Schramm, *The process and effects of mass communication*, Urbana: University of Illinois Press, 1954.
24. J. S. Johar, M. J. Sirgy, Value-expressive versus utilitarian advertising appeals: when and why to use which appeal, *J. Advert.*, **20** (1991), 23−33. https://doi.org/10.1080/00913367.1991.10673345
25. W. E. Kilbourne, Green advertising: salvation or oxymoron? *J. Advert.*, **24** (1995), 7−20. https://doi.org/10.1080/00913367.1995.10673472
26. J. Sun, X. Miao, Short-term sight vs. long-term vision: a study on the effectiveness of green advertising from the perspective of construal level, *Nankai Bus. Rev.*, **21** (2018), 195−205.
27. L. A. Manrai, A. K. Manrai, D. N. Lascu, J. K. Ryans Jr, How green-claim strength and country disposition affect product evaluation and company image, *Psychol. Mark.*, **14** (1997), 511−537. https://doi.org/10.1002/(SICI)1520-6793(199708)14:5<511::AID-MAR5>3.0.CO;2-B
28. S. Banerjee, C. S. Gulas, E. Iyer, Shades of green: a multidimensional analysis of environmental advertising, *J. Advert.*, **24** (1995), 21−31. https://doi.org/10.1080/00913367.1995.10673473
29. I. Rahman, D. Reynolds, The influence of values and attitudes on green consumer behavior: a conceptual model of green hotel patronage, *Int. J. Hosp. Tour. Adm.*, **20** (2019), 47−74. https://doi.org/10.1080/15256480.2017.1359729
30. S. Lockie, K. Lyons, G. Lawrence, K. Mummery, Eating "green": motivations behind organic food consumption in Australia, *Sociol. Ruralis*, **42** (2002), 23−40.
31. N. Michaelidou, L. M. Hassan, The role of health consciousness, food safety concern and ethical identity on attitudes and intentions towards organic food, *Int. J. Consum. Stud.*, **32** (2008), 163−170. https://doi.org/10.1111/j.1470-6431.2007.00619.x
32. J. Pickett-Baker, R. Ozaki, Pro-environmental products: marketing influence on consumer purchase decision, *J. Consum. Mark.*, **25** (2008), 281−293. https://doi.org/10.1108/07363760810890516
33. Q. Wei, D. Lv, Y. Lin, D. Zhu, S. Liu, Y. Liu, Influence of utilitarian and hedonic attributes on willingness to pay green product premiums and neural mechanisms in China: an ERP study, *Sustainability*, **15** (2023), 2403. https://doi.org/10.3390/su15032403

34. J. Berger, Signaling can increase consumers' willingness to pay for green products: theoretical model and experimental evidence, *J. Consum. Behav.*, **18** (2019), 233−246. https://doi.org/10.1002/cb.1760
35. S. Costa, L. Zepeda, L. Sirieix, Exploring the social value of organic food: a qualitative study in France, *Int. J. Consum. Stud.*, **38** (2014), 228−237. https://doi.org/10.1111/ijcs.12100
36. R. Elliott, The taste for green: the possibilities and dynamics of status differentiation through "green" consumption, *Poetics*, **41** (2013), 294−322. https://doi.org/10.1016/j.poetic.2013.03.003
37. M. Spence, Job market signaling, *Q. J. Econ.*, **87** (1973), 355−374. https://doi.org/10.2307/1882010
38. S. Kannan, Multiple market entry, cost signalling and entry deterrence, *Manage. Sci.*, **37** (1991), 1539−1555. https://doi.org/10.1287/mnsc.37.12.1539
39. X. Wang, Y. Li, S. Zhang, The interplay between green product production and advertising investment under green reputation, *IEEE Trans. Eng. Manag.*, **72** (2025), 2680−2699. https://doi.org/10.1109/TEM.2025.3582257
40. S. Zhang, T. Wang, X. Wang, Optimal investment portfolio and carbon emissions under climate risk ambiguity aversion, *J. Oper. Res. Soc.,* 2026, 1−31. https://doi.org/10.1080/01605682.2026.2627328
41. J. Wang, A. Li, The impact of green advertising information quality perception on consumers' response: an empirical analysis, *Sustainability*, **14** (2022), 13248. https://doi.org/10.3390/su142013248
42. M. G. Luchs, R. W. Naylor, J. R. Irwin, R. Raghunathan, The sustainability liability: potential negative effects of ethicality on product preference, *J. Mark.*, **74** (2010), 18−31. https://doi.org/10.1509/jmkg.74.5.018
43. A. Kronrod, A. Grinstein, L. Wathieu, Go green! Should environmental messages be so assertive? *J. Mark.*, **76** (2012), 95−102. https://doi.org/10.1509/jm.10.0416
44. M. Zinkhan, L. Carlson, Green advertising and the reluctant consumer, *J. Advert.*, **24** (1995), 1−6. https://doi.org/10.1080/00913367.1995.10673471
45. K. L. Keller, *Building customer-based brand equity: a blueprint for creating strong brands*, Marketing Science Institute, Cambridge, MA, 2001.
46. R. Y. K. Chan, L. B. Y. Lau, The effectiveness of environmental claims among Chinese consumers: influences of claim type, country disposition and ecocentric orientation, *J. Mark. Manag.*, **20** (2004), 273−319. https://doi.org/10.1362/026725704323080425
47. G. Bounds, Misleading claims on "green" labeling, *Wall Street Journal*, 2010. Available from: https://www.wsj.com/articles/SB10001424052702303467004575574521710082414.
48. D. P. Baron, Private politics, corporate social responsibility, and integrated strategy, *J. Econ. Manag. Strateg.*, **10** (2001), 7−45. https://doi.org/10.1111/j.1430-9134.2001.00007.x
49. T. J. Feddersen, T. W. Gilligan, Saints and markets: activists and the supply of credence goods, *J. Econ. Manag. Strateg.*, **10** (2001), 149−171.
50. L. A. Mohr, D. J. Webb, The effects of corporate social responsibility and price on consumer responses, *J. Consum. Aff.*, **39** (2005), 121−147. https://doi.org/10.1111/j.1745-6606.2005.00006.x
51. A. McWilliams, D. Siegel, Corporate social responsibility: a theory of the firm perspective, *Acad. Manag. Rev.*, **26** (2001), 117−127. https://doi.org/10.5465/AMR.2001.4011987
52. G. A. Akerlof, The market for "lemons": quality uncertainty and the market mechanism, *Q. J. Econ.*, **84** (1970), 488−500. https://doi.org/10.2307/1879431

53. M. B. Myers, D. A. Griffith, Strategies for combating gray market activity, *Bus. Horiz.*, **42** (1999), 2−8. https://doi.org/10.1016/S0007-6813(99)80032-9
54. K. Kogan, C. S. Tapiero, Coordination of co-investments in supply chain infrastructure, *J. Intell. Manuf.*, **23** (2012), 2471−2475. https://doi.org/10.1007/s10845-011-0513-9
55. K. Bagwell, M. H. Riordan, High and declining prices signal product quality, *Am. Econ. Rev.*, **81** (1991), 224−239.
56. M. Spence, Consumer misperceptions, product failure and producer liability, *Rev. Econ. Stud.*, **44** (1977), 561−572. https://doi.org/10.2307/2296908
57. S. J. Grossman, The informational role of warranties and private disclosure about product quality, *J. Law Econ.*, **24** (1981), 461−483. https://doi.org/10.1086/466995
58. N. A. Lutz, Warranties as signals under consumer moral hazard, *RAND J. Econ.*, **20** (1989), 239−255.
59. O. Board, Competition and disclosure, *J. Ind. Econ.*, **57** (2009), 197−213. https://doi.org/10.1111/j.1467-6451.2009.00369.x
60. E. Kamenica, M. Gentzkow, Bayesian persuasion, *Am. Econ. Rev.*, **101** (2011), 2590−2615. https://doi.org/10.1257/aer.101.6.2590
61. S. Dasu, R. Ahmadi, S. M. Carr, Gray markets, a product of demand uncertainty and excess inventory, *Prod. Oper. Manag.*, **21** (2012), 1102−1113. https://doi.org/10.1111/j.1937-5956.2012.01347.x
62. T. M. Valletti, Differential pricing, parallel trade, and the incentive to invest, *J. Int. Econ.*, **70** (2006), 314−324. https://doi.org/10.1016/j.jinteco.2005.07.010
63. E. Bolandifar, Z. Chen, P. Kouvelis, W. Zhou, Quality signaling through crowdfunding pricing, *Manuf. Serv. Oper. Manag.*, **25** (2023), 668−685. https://doi.org/10.1287/msom.2022.1177
64. Y. Qiu, R. C. Rao, Can merchants benefit from entry by (Amazon-like) platform if multiagent prices signal quality? *Mark. Sci.*, **43** (2024), 778−796. https://doi.org/10.1287/mksc.2021.0227
65. K. Ni, Y. Lin, S. Ye, Z. Lin, Y. Liu, Using strengths to attack weaknesses: the effect of comparative advertising on purchasing intention of green products, *Front. Psychol.*, **13** (2022), 1051856. https://doi.org/10.3389/fpsyg.2022.1051856
66. M. S. Altug, The dynamics of domestic gray markets and its impact on supply chains, *Prod. Oper. Manag.*, **26** (2017), 525−541. https://doi.org/10.1111/poms.12645
67. G. J. Mailath, M. Okuno-Fujiwara, A. Postlewaite, Belief-based refinements in signaling games, *J. Econ. Theory*, **60** (1993), 241−276. https://doi.org/10.1006/jeth.1993.1043
68. Y. Lu, C. R. H. Foropon, D. Wang, S. Xu, Arbitrage in gray markets and its impact on supply chain decisions, *J. Enterp. Inf. Manag.*, **34** (2021), 382−398. https://doi.org/10.1108/JEIM-02-2020-0069
69. Q. Cao, J. Zhang, Gray market's product quality in the circular economy era, *Int. J. Prod. Res.*, **58** (2020), 308−331. https://doi.org/10.1080/00207543.2019.1638980
70. H. Zhao, Raising awareness and signaling quality to uninformed consumers: a price-advertising model, *Mark. Sci.*, **19** (2000), 390−396. https://doi.org/10.1287/mksc.19.4.390.11788
71. T. Nian, A. Sundararajan, Social media marketing, quality signaling, and the Goldilocks principle, *Inf. Syst. Res.*, **33** (2022), 540−556. https://doi.org/10.1287/isre.2021.1067

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