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Research article

Present value optimization of two-echelon supply chains with trade credit and lot-splitting under a discounted cash flow framework

Tien-Yu Lin[1,2], Xiu-Hua Wei[1,2,*] and Yun-Feng Zheng[1]

[1] Qiaoxing College of Economics and Management, Fujian Polytechnic Normal University, No. 1, Campus New village, Longjiang Street, Fuqing City, Fujian Province 350300, China

[2] China–Indonesia Industrial Cooperation Research Center, Fujian Polytechnic Normal University, No. 1, Campus New village, Longjiang Street, Fuqing City, Fujian Province 350300, China

* **Correspondence:** Email: wxh180124@126.com; Tel: +86-591-85222439.

Abstract: This study develops a rigorous discounted cash flow (DCF) framework for optimizing replenishment decisions in two-echelon supply chains with trade credit and lot-splitting. Traditional models rely on single-cycle present value, which does not provide a theoretically consistent basis for evaluating replenishment policies in an infinite-horizon setting and may bias decisions toward unrealistically short cycles. To address this limitation, this paper formulates the infinite-horizon present value of a stationary replenishment policy and converts it into a discounted cost-rate objective. Within the unified DCF structure, all major cost components are valued according to their exact timing, allowing the model to jointly determine the optimal cycle length and shipment frequency. Numerical experiments show that discounting has a strong influence on optimal policies, that lot-splitting becomes more beneficial when discounted holding costs dominate, and that trade credit significantly reshapes the joint optimality of replenishment timing and shipment frequency. Overall, the study provides a theoretically consistent DCF-based two-echelon inventory model, offering both theoretical refinement and practical guidance for firms operating under capital constraints and time-sensitive replenishment environments.

Keywords: inventory strategy; present value optimization; discounted cash flow; supply chain management; trade credit; lot-splitting

Mathematics Subject Classification: 90B05, 91B06, 90B30

1. Introduction

Supply chains operating under capital constraints and time-sensitive delivery requirements increasingly rely on financial–operational coordination mechanisms such as trade credit, multi-shipment replenishment, and dynamic capital pricing. These mechanisms influence not only payment timing but also the structure of replenishment cycles and the responsiveness of shipment schedules. In two-echelon settings, where a supplier dispatches multiple deliveries within a buyer's replenishment cycle, firms must jointly consider financial liquidity, inventory dynamics, and shipment frequency when determining long-run optimal replenishment policies. As market volatility and financing costs rise, payment timing and delivery timing have become strategic levers for improving operational and financial performance.

Although a rich literature exists on trade credit and multi-shipment inventory systems, most prior studies evaluate system performance using the present value (PV) of a single replenishment cycle. A structured review of the related literature is provided in Section 2. Despite its intuitive appeal, the single-cycle PV formulation contains a fundamental theoretical inconsistency: Holding costs are discounted over the cycle, whereas setup costs are treated as immediate lump-sum expenditures. This asymmetric treatment implicitly favors shorter replenishment cycles and therefore does not represent a valid criterion for infinite-horizon performance. Correctly modeling discounting within multi-shipment and trade-credit environments therefore requires a unified formulation in which all cost components are evaluated under consistent intertemporal principles.

Accordingly, this challenge becomes even more pronounced in two-echelon systems involving both trade credit and lot-splitting. The timing of invoice payments, the number of shipments, and the evolution of inventory levels jointly shape the capital cost of operations, and capturing these interactions within a mathematically coherent discounted cost structure is nontrivial. The derivation of a logically consistent objective function requires reconciling the time value of money with multi-delivery inventory flows, leading to a technically demanding formulation. Despite extensive work on PV-based and credit-based models, the literature still lacks a rigorous discounted cash flow (DCF) representation that simultaneously incorporates payment timing, shipment frequency, and cost discounting in a two-echelon environment.

More broadly, recent studies in optimization-based logistics and supply-chain modeling have also emphasized the importance of integrating operational decision variables with broader system-level considerations in complex supply networks (e.g., [1–3]).

Motivated by these developments, this study develops a discounted cost-rate framework that provides a theoretically consistent evaluation of long-run replenishment policies under continuous discounting.

2. Literature review

Research on inventory and financial coordination has evolved along several major streams, including trade credit arrangements, lot-splitting or multi-shipment delivery policies, discounted cost modeling, and two-echelon integrated inventory systems. This section reviews the most relevant contributions in each domain and highlights the gaps that motivate the present study.

2.1. Trade credit in inventory systems

Trade credit models can be categorized into classic economic order quantity (EOQ)-type, credit-risk-based, and modern financing-integrated models. Trade credit has become one of the most influential mechanisms linking operational decisions to financing conditions in modern supply chains. Under trade credit, suppliers allow buyers to delay payment until a specified credit period, thereby reducing short-term liquidity pressure and altering the cost structure of replenishment decisions. Permissible delay in payment can effectively reduce the buyer's capital opportunity cost and shift the optimal replenishment cycle [4]. This insight has subsequently driven extensive extensions incorporating demand patterns, product characteristics, and financial constraints..

A significant stream of research has expanded trade-credit modeling to demand-dependent structures, showing that credit periods may stimulate demand and alter optimal order quantities [5]. A comprehensive review of credit-linked inventory models has shown that payment timing fundamentally interacts with inventory holding behavior [6]. Building on this foundation, subsequent research further examined situations in which demand is influenced simultaneously by price and credit terms, showing that combining trade credit with pricing decisions can generate nonlinear optimality conditions [7,8]. Another group of models has incorporated trade credit into deteriorating-item environments. It has been shown that deterioration amplifies the effect of the credit period, particularly when holding costs increase over time [9]. Extending this line of research to more complex system structures, trade credit in multi-echelon or multi-shipment settings has also been examined in the literature, although discounting was not modeled consistently [10,11].

Recent research has also focused on financial frictions and capital constraints. The impact of credit risk and default probability on optimal trade-credit policies has also been incorporated into trade-credit models [12]. Working-capital liquidity constraints have also been incorporated into EOQ-type models and showed that firms facing borrowing limits respond more sensitively to credit periods [13]. Hybrid financial–inventory frameworks have also been developed to capture the interaction between trade credit and external financing instruments such as bank loans, highlighting the complex interplay among payment timing, financial cost, and operational decisions [14]. Further studies have examined seller–buyer coordination, where payment schemes are part of a broader contractual mechanism. Permissible delay in payment can be used to coordinate channel decisions under appropriate parameter settings [15,16]. This coordination mechanism has been further extended to supply-chain contracts such as two-part tariffs, wholesale pricing, and quantity discounts [17–19]. These models collectively establish that trade credit can be used as a strategic coordination tool—beyond a mere financing arrangement. More recently, scholars have explored stochastic demand, credit risk, and machine-learning-enhanced screening of buyers. Inventory systems with uncertain buyer repayment probability have been modeled, and data-driven methods have been introduced to predict creditworthiness and integrate credit risk into replenishment decisions [20,21].

Despite these advancements, most of the above studies adopt single-cycle present value (PV) minimization as the objective function, which limits their ability to evaluate long-run performance under continuous discounting. The PV approach discounts holding costs over time but treats ordering/setup costs as lump-sum expenses, generating an asymmetric objective that systematically favors shorter replenishment cycles. As such, although trade credit has been extensively modeled, a unified discounted cash flow framework based on a theoretically consistent discounted cost rate objective is still largely absent—particularly in settings that also include multi-shipment delivery or two-echelon coordination. This gap motivates the development of the present study.

2.2. Lot-splitting and multi-shipment delivery policies

Lot-splitting and multi-shipment delivery policies have long been recognized as effective mechanisms for reducing average inventory and improving delivery responsiveness in coordinated supply chains. Early multi-shipment EOQ-type models show that allowing multiple deliveries within a replenishment cycle can substantially lower holding costs compared with a single-shipment policy [22]. This line of research has been further developed in more realistic coordinated settings [23].

A supply chain with imperfect quality and disposal costs was analyzed in a just-in-time environment, where the supplier adopts a single-setup, multiple-delivery policy to synchronize frequent deliveries with the buyer's usage rate [24]. The model demonstrates that lot-splitting not only reduces the buyer's on-hand inventory but also reshapes the joint cost structure when defective items and disposal expenses are present. In particular, it was extended to a two-stage supply chain with quantity discounts and overlapped deliveries to a two-stage supply chain with quantity discounts and overlapped deliveries, and a coordination policy was proposed to jointly determine order quantity and delivery structure so as to minimize the integrated annual cost under imperfect quality [23]. More recent work considered nondeteriorating and deteriorating items under an advance–loan–deposit scheme, deriving the optimal replenishment time and lot-splitting delivery policy and showing how financial arrangements interact with multi-delivery decisions in a two-echelon context [25].

These studies provide a rich foundation for understanding how lot-splitting and overlapped deliveries can be used to coordinate vendor–buyer relationships, particularly in the presence of imperfect quality or contract-based incentives. However, none of these models evaluates policies under a rigorously derived discounted cost rate; instead, they typically rely on average or annual cost criteria without fully incorporating the time value of money. Moreover, lot-splitting has been incorporated into advance–loan–deposit schemes in prior studies [25]. However, such formulations are not developed within a DCF-rate framework, and discounting is not consistently applied across all cost components. Consequently, a unified DCF-based model that jointly integrates trade credit, lot-splitting, and a theoretically consistent discounted cost rate objective in a two-echelon setting remains lacking.

2.3. Two-echelon inventory models

Two-echelon inventory systems, typically composed of an upstream supplier and a downstream buyer or retailer, have been extensively investigated due to their importance in coordinated replenishment, production–delivery synchronization, and supply-chain financing. Early formulations of such systems focused on the vendor–buyer integrated cost minimization problem, demonstrating that coordination through shipment policies or joint lot-sizing can significantly reduce channel-wide costs [26,27]. Stochastic or demand-dependent structures have been incorporated into two-echelon inventory models, highlighting the importance of aligning shipment frequency with downstream demand variability [28,29]. Research on two-echelon systems with multi-shipment or lot-splitting arrangements forms another important stream. Dispatching multiple shipments or partial lots can reduce the buyer's on-hand inventory and lead to more synchronized ordering [30,31]. More recent contributions (such as [27,32,33]) demonstrated that multi-shipment delivery is particularly beneficial when demand is uncertain or when the supplier seeks to smooth production rates. These models, however, do not incorporate financial timing considerations such as trade credit or discounting.

A third line of research incorporates trade credit into the two-echelon framework. A vendor–buyer system with permissible delay in payment has been proposed to explore how credit terms influence joint optimal lot sizes [10]. This line of work was extended to multi-shipment delivery with partially discounted cost structures [11]. Trade credit across multi-echelon supply chains has also been investigated [34–36]. These works highlight the growing recognition that financial arrangements fundamentally influence replenishment behavior in multi-stage systems.

Table 1. Comparison of related literature.

Study	Trade credit	Lot-splitting	Discount-ing	Objective function	Difference from present study
[4]	✓	✗	✗	PV of cycle	No DCF rate; no multi-shipment
[5]	✓	✗	✗	PV	No lot-splitting
[6]	✓	✗	Limited	PV	No DCF derivation
[22]	✗	✓	✗	Average/annual cost	No trade credit; no discounting
[24]	✗	✓	✗	Joint annual cost	JIT lot-splitting with defective items; no trade credit; no DCF rate
[23]	✗	✓	✗	Annual integrated cost	Two-stage coordination; no trade credit; no discounting
[7]	✗	✗	✗	PV	No financial mechanism
[9]	✓	✗	✗	PV	No lot-splitting
[37]	✗	✗	Partial	PV	Carbon tax & imperfect quality; no trade credit
[10]	✓	✗	✗	PV	Two-echelon credit only; no lot-splitting
[16]	✓	✗	✗	PV	No discount-rate objective
[35]	✓	✗	✓	PV	Includes discounting but no DCF rate
[8]	✓	✗	✗	PV	No multi-shipment
[33]	✗	✓	✗	Average cost	Random demand; no financial integration
[14]	✓	✗	Partial	PV	Hybrid financial–inventory; no unified DCF framework
[36]	✓	✗	✗	PV	Multi-echelon credit; no lot-splitting
[11]	✗	✓	Partial	PV	Multi-shipment + partial discounting; no DCF rate
[25]	✓	✓	✗	Annual cost	Lot-splitting + financing; no fully derived DCF rate
This study	✓	✓	✓	DCF rate	Integrates trade credit + lot-splitting + full DCF + two-echelon model

Quality- and deterioration-related two-echelon models also constitute a substantial segment of the literature. Imperfect quality, carbon emissions, and processing costs have also been incorporated into two-echelon models [37]. Two-echelon systems with deterioration and partial backlogging have also been examined [38,39]. Yet, despite these advances, these models remain grounded in average-cost or PV-cycle formulations and do not adopt a fully discounted objective. In recent years, coordination mechanisms—including revenue-sharing, quantity discounts, two-part tariffs, and consignment contracts—have also been integrated into the two-echelon setting (for example, [40–42]). While these studies primarily focus on contractual coordination, they further underscore the importance of aligning inter-firm incentives—yet discounted cash flow timing and DCF-rate evaluation remain largely unexplored in such settings. Overall, these streams motivate a closer examination of how contractual coordination interacts with discounting when the objective is formulated as a rigorously derived DCF-rate function.

While these studies substantially advance sustainability-oriented and resilience-focused supply-chain modeling, they primarily emphasize environmental trade-offs, uncertainty modeling, or policy evaluation. In contrast, the present study addresses a complementary yet distinct methodological gap by establishing a theoretically consistent discounted cash flow rate framework for long-run replenishment decisions with trade credit and lot-splitting. By focusing on the timing of cash flows and infinite-horizon valuation, our model provides financial consistency that is not explicitly examined in the above sustainability-oriented formulations. Taken together, the existing literature reveals a clear methodological gap. Existing two-echelon models rarely provide a financially consistent, discounted evaluation of long-run replenishment decisions that jointly incorporate trade credit and lot-splitting.

The literature reviewed across Sections 2.1 to 2.3 highlights substantial progress in the study of trade credit, lot-splitting, discounted cost formulations, and two-echelon replenishment systems, yet several fundamental limitations remain unresolved. Research on trade credit has demonstrated the importance of payment timing and capital cost, while studies on lot-splitting emphasize shipment frequency and inventory-reduction benefits. Work on discounted models shows how intertemporal valuation affects ordering decisions, and two-echelon models illustrate the complexity of coordinating upstream and downstream operations. However, these streams of research have largely evolved in parallel rather than in an integrated manner, resulting in a lack of a unified analytical framework that jointly captures financial and operational interactions. To clarify the methodological gaps among these four domains, Table 1 provides a consolidated comparison of the major studies discussed in Sections 2.1–2.3, summarizing how each contribution addresses (or fails to address) trade credit, multi-shipment policies, discounting assumptions, and system structure. This comparative synthesis facilitates a clearer understanding of the fragmented development in the existing literature and highlights the absence of a unified framework that jointly incorporates financing mechanisms, delivery scheduling, and theoretical consistency in discounted cost evaluation within a two-echelon system. As shown in Table 1, three overarching gaps emerge from prior research. First, trade credit and lot-splitting have been widely examined but almost always in isolation; very few studies embed both mechanisms within a single optimization model. Second, models relying on the present value (PV) of a single replenishment cycle apply discounting asymmetrically—typically discounting holding costs over time while treating setup and purchasing costs as lump-sum expenses—thereby lacking theoretical consistency for evaluating long-run replenishment decisions under continuous discounting. Third, despite the extensive literature on two-echelon and multi-shipment systems, no study to date has jointly integrated trade credit and lot-splitting within a two-echelon system under a rigorously derived discounted cost-rate objective. Motivated by these unresolved methodological gaps, the present study develops a unified DCF-based two-echelon framework that consistently reformulates all major cost components—purchasing, receiving, holding, and setup—under continuous-time discounting and analytically characterizes the joint optimization of cycle length and shipment frequency. The proposed model thus resolves the long-standing inconsistency in PV-based formulations and offers a coherent approach for analyzing financially and operationally integrated replenishment policies.

3. Model formulation

This section formally develops the unified discounted cash flow rate framework promised in the Introduction. We consider an infinite-horizon two-echelon supply chain with trade credit and coordinated shipments, and explicitly reformulate all major cost components according to their exact cash-flow timing. By converting the infinite-horizon present value of a stationary replenishment policy into a discounted cost-rate objective, the model provides a theoretically consistent basis for jointly optimizing the cycle length and shipment frequency under continuous-time discounting.

3.1. Notations and assumptions

3.1.1. Notations

Table 2. All symbols and their definitions.

Symbol	Definition
D	Constant demand rate of the downstream buyer (units per unit time)
T	Length of a replenishment cycle (decision variable)
L	Initial inventory level at t =0 (after the first sub-delivery)
k	Number of sub-deliveries (shipments) from supplier to buyer per cycle (decision variable)
T/k	Time interval between consecutive sub-deliveries
C_s	Setup (ordering) cost incurred at the beginning of each cycle
C	Purchasing cost per unit paid by the buyer
C_g	Receiving cost per sub-delivery
h	Holding cost rate per unit of inventory per unit time
R	Continuous discount rate (financial discount factor)
W	Trade credit period offered by the supplier
$I(t)$	Inventory level at time $t \in [0,T]$ under lot-splitting; $I(t)$ decreases linearly between deliveries and increases by DT/k at each delivery time
$g(T, k)$	Discounted cost-rate function under the infinite-horizon DCF formulation
$\exp(-Rt)$	Continuous discounting factor applied to all time-dependent costs

To develop the discounted cash flow based two-echelon inventory model with trade credit and lot-splitting, the notation and modeling assumptions are summarized below. All symbols and their definitions are listed in Table 2.

3.1.2. Assumptions

We consider a two-echelon inventory system consisting of an upstream supplier and a downstream retailer. The following assumptions describe the operational and financial environment under which the replenishment policy is defined.

(1) The retailer faces a deterministic and constant demand rate $D > 0$ for a single product. The product is non-deteriorating, non-perishable, and homogeneous. All units are identical, and quality issues are not present in the system.

(2) The retailer adopts a cyclic replenishment policy with cycle length $T > 0$. In each cycle, the retailer orders a total quantity DT, which exactly satisfies demand during the cycle. No shortages or backorders are allowed, implying that inventory must remain non-negative over [0,T].

(3) To reduce the buyer's inventory burden, the supplier agrees to split each order into $k \in Z^+$ equal-sized shipments. These shipments arrive at evenly spaced times

$$t_i = iT/k,\ i = 0, 1, \ldots, k-1,$$

and each increases the retailer's inventory by DT/k (measured in the same time unit as T). Between two arrivals, inventory depletes linearly at rate D. No capacity limits or shipment delays occur.

(4) The supplier offers a trade-credit period of length $W > 0$. Payment for the purchasing cost CDT is made at time W. Therefore, if $W \leqslant T$, payment occurs within the cycle; if $W > T$, payment is deferred beyond the cycle boundary. The credit terms apply identically to every cycle. In repeated cycles, the payment timing affects the discount factor applied to $C \cdot DT$.

(5) The system incurs the following costs in each cycle: (i) setup cost $C_s > 0$, paid at time 0; (ii) purchasing cost CDT, paid at time 0 or W depending on Assumption 4; (iii) receiving cost C_g per shipment, incurred at times $t_i = iT/k$; (iv) holding cost h per unit of inventory per unit time, applied continuously as inventory remains positive.

(6) All monetary flows are discounted continuously at rate $R > 0$. If a payment of nominal amount X occurs at time t, its discounted value is Xe^{-Rt}. Accordingly, continuous holding cost is evaluated under discounting through the integral

$$h\int_0^T I(t)e^{-Rt}dt .$$

(7) The replenishment policy (T, k) is repeated indefinitely, and the objective is to evaluate the long-run discounted economic cost. Because single-cycle present value does not correspond to a long-run performance criterion under discounting, the appropriate objective is the discounted cost rate, derived in Section 3.4.

(8) The retailer chooses the cycle length $T > 0$ and the shipment frequency $k \in Z^+$ to minimize the long-run discounted cost rate over an infinite horizon.

3.2. Problem description

This study considers a two-echelon inventory system consisting of a single upstream supplier and a single downstream retailer facing deterministic and constant demand. The retailer adopts a cyclic replenishment policy with cycle length T, ordering a total quantity DT in each cycle. Instead of delivering the order in a single shipment, the supplier splits it into k equal-sized sub-deliveries dispatched at evenly spaced intervals. The replenishment policy is therefore characterized by two decision variables: the cycle length T and the shipment frequency k.

Operationally, inventory increases instantaneously by DT/k at each delivery epoch and decreases continuously at the constant demand rate between deliveries. No shortages are permitted. Increasing the shipment frequency reduces average inventory and holding costs but increases total receiving costs. The model must therefore balance the inventory-reduction benefit of lot-splitting against the additional operational cost generated by more frequent deliveries.

In addition to operational decisions, the supplier offers a trade-credit period W, which determines the timing of the purchasing payment. If $W \leqslant T$, payment occurs within the cycle; if $W > T$, payment is deferred beyond the cycle boundary. Although the inventory trajectory depends solely on (T, k), the purchasing cash flow depends on the relationship between W and T. Consequently, the retailer's decision simultaneously affects both operational cost components and the discounted value of purchasing expenditures.

All cash flows—including setup, purchasing, receiving, and holding costs—are evaluated under continuous discounting at rate R. Because the replenishment policy is repeated indefinitely, performance must be assessed using a long-run discounted cost-rate objective rather than a single-cycle present value. The retailer's problem is therefore to determine $T > 0$ and $k \in Z^+$ that jointly minimize the infinite-horizon discounted cost rate derived in the following subsections.

3.3. Integrated inventory dynamics and discounted present-value formulation under lot-splitting and trade credit

The inventory evolution in a two-echelon system with lot-splitting and discounting is driven jointly by (i) the shipment frequency k, (ii) the cycle length T, and (iii) the credit period W. Although the total order quantity in each cycle is fixed at DT, its temporal distribution—both physically (through k sub-deliveries) and financially (through payment timing)—creates distinct patterns of inventory trajectories and cash-flow timing. Inventory trajectories depend on (T, k), whereas cash-flow timing depends solely on the relation between W and T. These patterns are fundamental to the subsequent derivation of discounted cycle costs and the discounted cost-rate objective developed in this section, as well as the convexity analysis in the subsequent section.

To illustrate the operational and financial behavior under different contractual structures, Figures 1–3 provide graphical representations of the inventory path over two consecutive cycles under three economically relevant regimes, which are introduced below.

3.3.1. Non-trade-credit regime: Baseline inventory dynamics

In the absence of trade credit, all purchasing costs are paid at the beginning of each replenishment cycle, and the temporal structure of the system is governed solely by the physical flow of inventory. The supplier delivers a total quantity equal to the cycle demand DT, but instead of dispatching it in a single shipment, the order is divided into k equal sub-deliveries. These shipments occur at evenly spaced times

$$t_i = \frac{iT}{k}, \quad i = 0,1,\ldots,k-1,$$

and each sub-delivery instantaneously increases the retailer's on-hand inventory by DT/k. Between two consecutive delivery epochs, inventory depletes at the constant demand rate D, producing a sequence of linear downward segments. This makes the inventory path a periodic piecewise-linear function whose shape depends jointly on (T, k).

Between two consecutive delivery epochs, inventory decreases continuously at the constant demand rate D, and thus satisfies the differential equation $dI(t)/dt = -D$, where $t_i = iT/k$ denotes the arrival time of the i-th sub-delivery. Solving over $t \in [t_i, t_{i+1})$ yields $I(t) = I(t_i^+) - D(t - t_i)$. Under equal lot-splitting, each delivery increases inventory by DT/k, and the demand over one interval is $D(t_{i+1} - t_i) = D(T/k) = DT/k$. Substituting gives the unified expression consistent with Eq (1):

$$I(t) = \frac{DT}{k} - D(t - t_i),\ t \in [t_i, t_{i+1}),\ i = 0,1,\dots,k-1, \tag{1}$$

where each segment corresponds to the time interval between the i-th and $(i+1)$-th shipments. Because the system operates under stationary demand and identical cycles, the inventory pattern repeats itself indefinitely over the infinite horizon.

Under a single-shipment policy ($k = 1$), inventory rises sharply to DT at the cycle origin and subsequently declines to zero, yielding a higher average inventory level. Increasing the shipment frequency ($k > 1$) spreads the same total quantity across multiple smaller deliveries, thereby lowering the amplitude of the sawtooth pattern and reducing discounted holding costs.

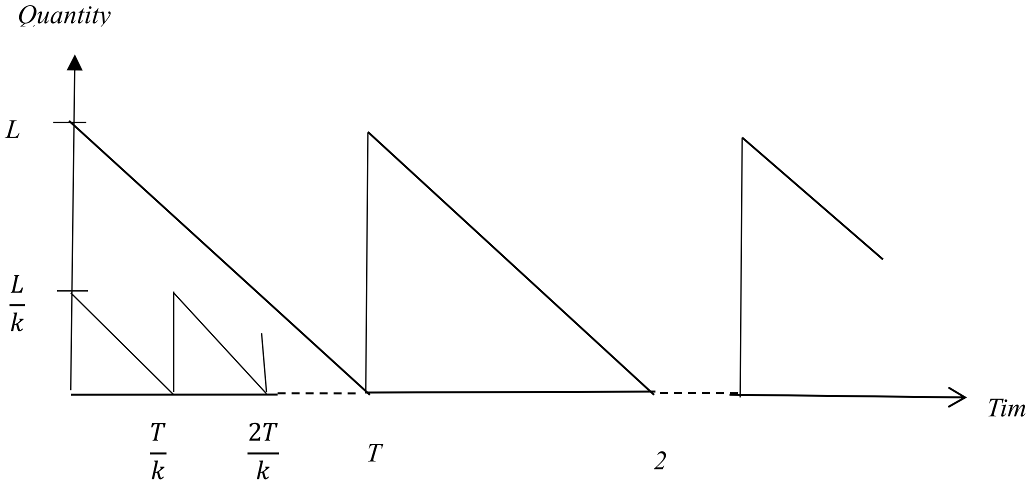

Figure 1. Inventory trajectory with lot-splitting and time discounting under a non-trade-credit policy.

3.3.2. Trade credit with early payment: $W \leqslant T$

When the supplier offers a trade-credit period W that does not exceed the retailer's replenishment cycle length T, the purchasing outflow occurs at an interior point of the cycle. In this regime the condition $W \leqslant T$ implies that the retailer settles the purchasing cost before the current cycle is completed. Operationally, the supplier continues to dispatch k equal-sized shipments at times $t_i = iT/k$, and the retailer's inventory evolves according to the same piecewise-linear trajectory as in (1). Hence, the physical inventory path remains governed by (T, k) and is independent of W.

The key difference lies in the timing of the purchasing cash flow under discounting. Instead of paying at the cycle origin $t = 0$, the retailer now pays at $t = W$. Under continuous discounting, this shifts the purchasing outflow by a factor exp($-RW$), while inventory-related costs remain unaffected because they depend on the physical trajectory determined by (T, k).

Figure 2 illustrates the early-payment trade-credit regime. The sawtooth inventory trajectory is identical to that in Figure 1, while the vertical marker at $t = W$ within the first cycle indicates the payment date. This visualization clarifies that the physical and financial dimensions are partially decoupled: the inventory profile follows (1), whereas the purchasing-cost term in the present-value expression is shifted in time.

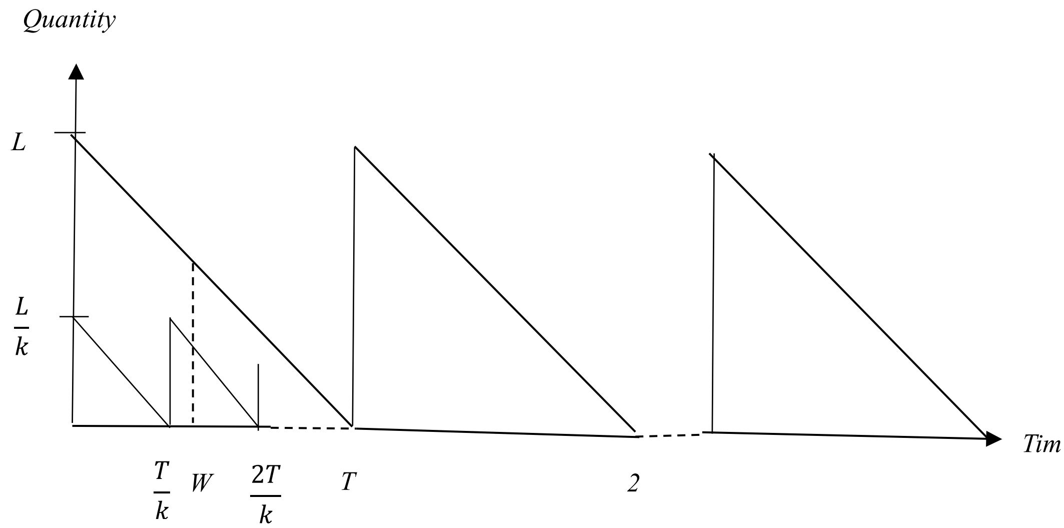

Figure 2. Inventory trajectory with lot-splitting and time discounting under trade credit when the credit period does not exceed the cycle length ($W \leq T$).

3.3.3. Trade credit with deferred payment: $W > T$

A qualitatively different regime arises when the credit period exceeds the replenishment cycle length, that is, $W > T$. In this case, the retailer completes an entire cycle—and possibly initiates the next one—before the purchasing cost associated with the first cycle is actually paid. Operationally, the lot-splitting structure remains unchanged: The supplier still delivers k equal-sized shipments per cycle at times $t_i = iT/k$, and the inventory trajectory within each cycle is given by Equation (1). The physical behavior of the inventory system is therefore identical to that in the non-credit and early-payment regimes.

Financially, because the payment date W occurs strictly after the end of the cycle, the purchasing outflow is postponed beyond the cycle horizon. Under continuous discounting, this further delay reduces the present value of the purchasing cost through the factor exp($-RW$) with $W > T$, thereby amplifying the financing benefit relative to the $W \leqslant T$ regime.

Figure 3 depicts the deferred-payment regime. The inventory trajectory again follows (1), but the payment date is shown at a time W beyond the first cycle boundary. This contrast with Figure 2 highlights how the same physical inventory model can yield substantially different financial timing structures depending on the relation between W and T.

Taken together, Figures 1–3 show that the inventory trajectory is invariant across the three regimes (determined solely by T and k), while the timing of the purchasing cash flow differs according to whether payment occurs at $t = 0$, within the cycle ($W \leqslant T$), or beyond the cycle ($W > T$).

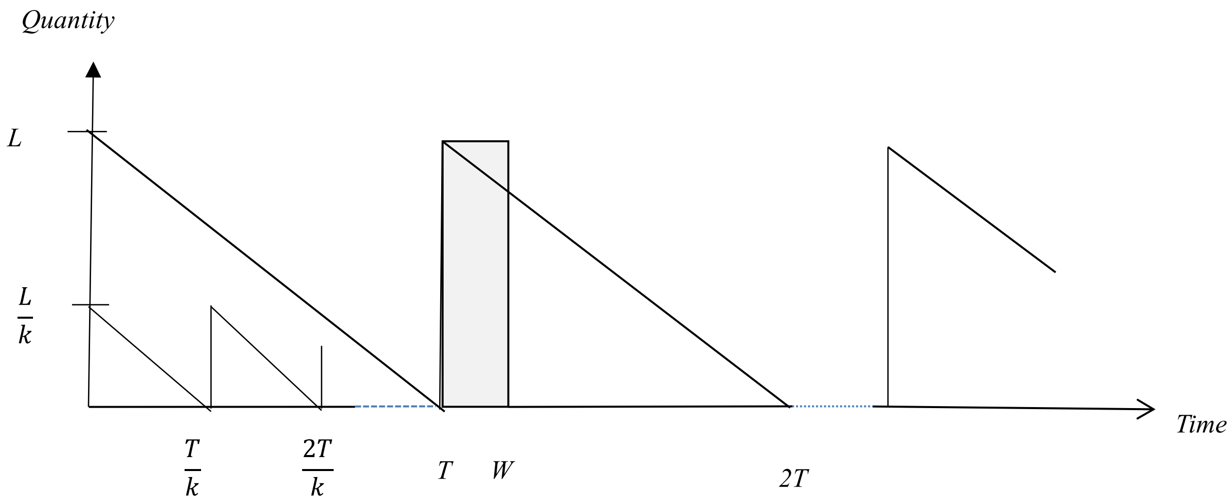

Figure 3. Inventory trajectory with lot-splitting deliveries and time discounting under trade credit with deferred payment $(W > T)$.

3.3.4. Present-value formulation of single-cycle costs

The system operates in cycles of length *T*, each consisting of *k* equally spaced sub-deliveries of size *DT*/*k*. All costs—setup, purchasing, receiving, and holding—are evaluated under continuous discounting at rate *R*. This section derives the present value (PV) of each component cost in a single cycle, explicitly accounting for the cash-flow timing structures established in Sections 3.3.1–3.3.3.

(i) Holding-cost present value

Let *I*(*t*) denote the inventory trajectory over a cycle, as defined in Equation (1). The instantaneous holding cost at time t is hI(t), where *h* denotes the holding cost rate per unit of inventory per unit time. The holding cost itself refers to the accumulated and discounted holding expenditure obtained by integrating this instantaneous cost over the replenishment cycle. Under continuous discounting, the present value of holding cost over one cycle is obtained by discounting and integrating the instantaneous holding cost rate applied to the inventory trajectory, and is given by:

$$HC(T,k) = \int_0^T hI(t)e^{-Rt}dt. \tag{2}$$

Substituting the piecewise-linear inventory function and integrating over the k subintervals yields

$$HC(T,k) = \sum_{i=0}^{k-1} \int_{t_i}^{t_{i+1}} hI(t)e^{-Rt}dt\ , \tag{3}$$

which forms the basis for the numerical evaluation used later.

(ii) Receiving-cost present value

Each sub-delivery incurs a receiving cost C_g at time $t_i = iT/k$. Because the deployment schedule is deterministic, the present value of receiving costs in one cycle is

$$RC(T,k) = \sum_{i=0}^{k-1} C_g e^{-Rt_i}. \tag{4}$$

(iii) Purchasing-cost present value under three credit regimes

Let C be the unit purchasing cost. The nominal purchasing expenditure in a cycle is CDT, but its timing depends on the credit structure. The PV of purchasing cost is therefore:
(a) No trade credit (non-credit case): Payment occurs at the beginning of each cycle:

$$PC_0(T) = CDT \tag{5}$$

(b) Early-payment trade credit ($W \leqslant T$): Payment occurs at time W within the cycle:

$$PC_E(T, W) = CDTe^{-RW}. \tag{6}$$

(c) Deferred-payment trade credit ($W > T$): Payment occurs after the cycle ends:

$$PC_D(T, W) = CDTe^{-RW}. \tag{7}$$

Although (6) and (7) share the same algebraic form, they correspond to different timing regimes (inside vs. outside the cycle), which leads to different interpretations and subsequent discounted cost-rate behavior.

(iv) Setup-cost present value

Setup cost is incurred at the beginning of each cycle:

$$SC = C_s. \tag{8}$$

(v) Total single-cycle present value

Combining all components yields the single-cycle present value:

$$PV(T, k) = SC + PC(T, W) + RC(T, k) + HC(T, k), \tag{9}$$

where:

$$PC(T,\mathrm{W}) = \begin{cases} CDT, & \text{non-credit (i.e. } PC_0(T)\text{)}, \\ CDTe^{-RW}, & W \le T \ \text{ (i.e. } PC_E(T,W)\text{)}, \\ CDTe^{-RW}, & W > T \ \text{ (i.e. } PC_D(T,W)\text{)}. \end{cases} \tag{10}$$

Note that although the purchasing-cost expressions for $W \leqslant T$ and $W > T$ share the same algebraic form, their economic interpretations differ because the discount factor is evaluated at different financial regimes, as illustrated in Figures 2 and 3.

3.3.5. Infinite-horizon valuation and the discounted cost rate objective

Based on the discounted cash flow principle, the infinite-horizon present value of a stationary replenishment policy can be expressed as the discounted sum of identical single-cycle present values:

$$PV(T, k) + PV(T, k)e^{-RT} + PV(T, k)e^{-R(2T)} + \cdots. \quad (11)$$

Summing the geometric series yields the infinite-horizon present value:

$$\text{Total } PV(T,k) = \frac{PV(T,k)}{1-e^{-RT}}. \quad (12)$$

Accordingly, the long-run discounted cost-rate objective function under a stationary replenishment policy with cycle length T and shipment frequency k is defined as follows:

$$g(T,k) = \frac{PV(T,k)R}{1-e^{-RT}}, \quad (13)$$

which provides a theoretically consistent long-run performance measure under continuous discounting and enables meaningful comparison across replenishment policies with different cycle lengths. Here, discounted cash flow refers to the valuation principle under which all cash flows are discounted according to their timing, whereas the discounted cost rate $g(T,k)$ is the corresponding long-run objective obtained by transforming the infinite-horizon discounted cash flow of a stationary policy into an equivalent cost rate. The retailer's optimization problem is therefore formulated as

$$\min_{T>0,\, k\in Z^+} g(T,k), \quad (14)$$

subject to feasibility and contractual constraints. The mathematical properties of the objective function, including convexity and the uniqueness of the optimal solution, are examined in the subsequent section.

3.4. Mathematical properties of the discounted cost-rate function

Based on the discounted cash flow formulation developed in Section 3.3, the discounted cost-rate function $g(T,k)$ defined in Equation (14) determines the retailer's optimal replenishment cycle length for any fixed shipment frequency k. This section studies the fundamental mathematical properties of $g(T, k)$, including coercivity, convexity, and the existence and uniqueness of the optimal solution $T^*(k)$. These properties ensure that the optimization problem is well posed

3.4.1. Limiting behavior and coercivity of the discounted cost-rate function

We first examine the limiting behavior of the discounted cost rate. Recall that

$$g(T,k)=\frac{PV(T,k)R}{1-e^{-RT}}. \quad (15)$$

To characterize the behavior of the cost-rate function at the boundaries of the cycle length, we analyze the limits as $T\to 0^+$ and $T\to\infty$.

$$\lim_{T\to 0^+} g(T,k) = \lim_{T\to 0^+} \frac{PV(T,k)R}{1-e^{-RT}} = \lim_{T\to 0^+} \frac{PV(T,k)}{\left(\frac{1-e^{-RT}}{RT}\right)T} = \lim_{T\to 0^+} \frac{PV(T,k)}{T} = \infty, \tag{16}$$

since

$$\lim_{T\to 0^+} \frac{1-e^{-RT}}{RT} = 1,$$

by L'Hospital's Rule.

Similarly,

$$\lim_{T\to\infty} g(T,k) = \lim_{T\to\infty} \frac{PV(T,k)R}{1-e^{-RT}} = \lim_{T\to\infty} PV(T,k)R = \infty, \tag{17}$$

since $PV(T,k)$ grows without bound as $T \to \infty$.

Equations (16) and (17) characterize the asymptotic behavior of the discounted cost-rate function as the cycle length varies.

Together, Equations (16)–(17) show that $g(T,k)$ is coercive in T, which guarantees the existence of at least one interior minimizer for each fixed shipment frequency k. This existence result is independent of the convexity analysis that follows and ensures that the optimization problem is well posed. In particular, as $T \to 0$, the setup and receiving cost components dominate and cause the discounted cost rate to diverge, whereas as $T \to \infty$, the discounted holding-cost term grows without bound. Numerical curvature checks and optimization results are therefore reported in Section 4 to support the uniqueness of the optimal cycle length under the tested parameter settings.

3.4.2. Convexity of the single-cycle present value

The discounted cost rate inherits key shape properties from the present value PV(T, k). Consider its decomposition:

$$PV(T, k) = C_s + PC(T, W) + RC(T, k) + HC(T, k), \tag{18}$$

where

$PC(T, W)$ is linear in T;

$RC(T, k)$ is a discounted finite sum independent of T except through the factor $\exp(-RT/k)$;

$HC(T, k)$ is an integral of a piecewise-linear nonnegative function multiplied by $\exp(-Rt)$.

Differentiating (18) with respect to the cycle length T yields

$$\frac{\partial PV}{\partial T} = CD\alpha(W,T) + \frac{\partial RC}{\partial T} + \frac{\partial HC}{\partial T}, \tag{19}$$

where $\alpha(W, T) = 1$ for the non-credit case and $\alpha(W, T) = e^{-RW}$ for trade-credit cases.

A second differentiation gives

$$\frac{\partial^2 PV}{\partial T^2} = \frac{\partial^2 RC}{\partial T^2} + \frac{\partial^2 HC}{\partial T^2}. \tag{20}$$

For the holding-cost term,

$$HC(T,k)=h\sum_{i=0}^{k-1}\int_{t_i}^{t_i+1} I(t)e^{-Rt}dt.$$

Applying Leibniz's rule and simplifying yields the result reported in Eq (21); the detailed derivation is provided in Appendix A. Hence,

$$\frac{\partial^2 HC(T,k)}{\partial T^2}=\sum_{i=0}^{k-1}\frac{hDR}{k^2}e^{-RiT/k}\left(e^{-RT/k}-1+\frac{RT}{k}\right)>0.\quad \forall T>0,$$

which proves Eq (21). We therefore have

$$\frac{\partial^2 HC(T,k)}{\partial T^2}>0. \tag{21}$$

For the receiving-cost term,

$$RC(T,k)=c_g\sum_{i=0}^{k-1}e^{-RiT/k}.$$

Direct differentiation yields

$$\frac{\partial RC(T,k)}{\partial T}=-\sum_{i=0}^{k-1}\frac{Ri}{k}c_g e^{-RiT/k},$$

and

$$\frac{\partial^2 RC(T,k)}{\partial T^2}=\sum_{i=0}^{k-1}\left(\frac{Ri}{k}\right)^2 c_g e^{-RiT/k}=\frac{c_g}{k^2}\sum_{i=0}^{k-1}R^2 i^2 e^{-RiT/k}\geq 0,\quad \forall T>0. \tag{22}$$

Because each term $\left(\frac{Ri}{k}\right)^2 c_g e^{-RiT/k}$ is non-negative for $T>0$, the second derivative of $RC(T,k)$ with respect to T is non-negative.

Thus,

$$\frac{\partial^2 PV}{\partial T^2}>0, \tag{23}$$

meaning the single-cycle present value is strictly convex in T.

3.4.3. Convexity of the discounted cost rate

To examine the curvature of the discounted cost-rate function, we differentiate the objective function defined in (15) with respect to the cycle length T for any fixed shipment frequency k. Recall that the discounted cost-rate function can be written as

$$g(T,k)=\frac{PV(T,k)R}{1-e^{-RT}}.$$

For notational convenience, define

$f(T)\equiv PV(T,k)$, $\Delta(T)\equiv 1-e^{-RT}$, so that $g(T,k)=Rf(T)/\Delta(T)$.

Differentiating $g(T, k)$ with respect to the cycle length T yields

$$\frac{\partial g(T,k)}{\partial T} = R\frac{f'(T)d(T) - f(T)d'(T)}{d(T)^2}, \tag{24}$$

and

$$\frac{\partial^2 g(T,k)}{\partial T^2} = \frac{R}{\left(1-e^{-RT}\right)^3}\Big[\left(f''(T)\left(1-e^{-RT}\right) + f(T)R^2 e^{-RT}\right)\left(1-e^{-RT}\right)$$

$$-2\left(f'(T)\left(1-e^{-RT}\right) - f(T)R\cdot e^{-RT}\right)R\cdot e^{-RT}\Big], \tag{25}$$

where the above expressions are obtained by applying the quotient rule.

Equation (25) provides an explicit analytical representation of the curvature of the discounted cost-rate function. The first and third terms in the numerator are strictly positive for all $T>0$, while the sign of the remaining term depends on the marginal behavior of the single-cycle present-value function. Consequently, global strict convexity cannot be guaranteed analytically for all parameter values. Nevertheless, the coercivity results established in (16)–(17) ensure that the discounted cost-rate function attains at least one interior minimizer $T^*(k) > 0$ for any fixed shipment frequency k. The uniqueness of the optimal cycle length under the tested parameter settings is further supported by the numerical curvature checks reported in Section 4.

For the parameter settings considered in this study, we further examine the curvature of $g(T, k)$ numerically over the feasible domain of T. The numerical curvature checks reported in Section 4 consistently show that the second-order derivative remains strictly positive for all $T > 0$. This empirical verification supports the strict convexity of $g(T, k)$ with respect to T under the tested conditions and is sufficient to ensure the uniqueness of the optimal cycle length in the numerical analysis conducted in this study. Accordingly, we obtain

$$\frac{\partial^2 g(T,k)}{\partial T^2} > 0 \quad \text{, for all T > 0,} \tag{26}$$

which confirms the strict convexity of g(T, k) with respect to T and ensures the uniqueness of the optimal cycle length under the tested conditions.

Differentiating twice yields the explicit second-derivative expression in Eq (25). The expression is obtained directly from the quotient rule and involves only the derivatives of $f(T) = PV(T, k)$ and the exponential term $e{-}RT$. The curvature checks in Section 4 evaluate Eq (25) directly across the tested grid, providing an independent verification of the derived expression. We now proceed to examine the curvature behavior and the resulting optimal cycle length under representative parameter settings.

3.4.4. Existence and uniqueness of the optimal cycle length

Combining the results above: (1) g(T, k) is coercive by (16)–(17); (2) g(T, k) is strictly convex by (26). Therefore, the optimization problem (14):

$$\min_{T>0} g(T,k) \tag{27}$$

admits a unique global minimizer T*(k) > 0 for each fixed shipment frequency k. In other words, the first-order condition

$$g'\left(T^*(k),k\right)=0 \tag{28}$$

has exactly one solution, and that solution is the global minimum of $g(T, k)$.

Based on the strict convexity of the discounted cost-rate function $g(T, k)$ with respect to the cycle length T for any fixed shipment frequency k, the optimization problem possesses a unique global minimizer. This analytical property ensures that the function decreases monotonically before the optimal point and increases thereafter, thereby preventing the existence of multiple local minima. As a result, the model exhibits stable numerical behavior and guarantees that the one-dimensional search procedure described in Section 3.5 consistently identifies the unique optimal cycle length $T^*(k)$ for every shipment frequency. This structural characteristic also ensures the robustness of the joint optimization over (T, k) in subsequent numerical analysis.

3.5. Solution procedure for determining the optimal policy (T^*, k^*)

The optimization problem formulated in the previous sections requires identifying the pair (T^*,k^*) that minimizes the discounted cost rate $g(T, k)$ over all feasible cycle lengths $T > 0$ and positive integers k. Because the shipment frequency k is a discrete variable while T is continuous, the problem naturally decomposes into two nested components: (i) for any fixed shipment frequency, determine the unique optimal cycle length $T^*(k)$, and (ii) compare the minimized discounted cost rates across the feasible values of k. The mathematical properties established in Section 3.4 ensure that this procedure is well posed: The objective is coercive, continuously differentiable, and strictly convex in T, guaranteeing that a unique minimizer exists for every fixed k.

For a given integer k, the optimal cycle length solves the first-order condition

$$g'\left(T^*(k),k\right)=0 \tag{29}$$

where $g(T, k)$ is defined in Equation (13) and its derivative is provided in Equation (24). Because the derivative changes sign exactly once by strict convexity (Equation (26)), the solution of (29) is not only necessary but also sufficient for optimality. The unique minimizer may be characterized implicitly, but a closed-form expression does not exist due to the complex interaction between discounting, receiving costs, and the piecewise-linear inventory trajectory. Consequently, numerical root-finding is necessary. Any monotonic bracketed search such as Newton–Raphson, bisection, or Brent's method converges reliably because the curvature of g is positive and its derivative is continuous. In practice, the solution for each k is obtained by solving

$$T^*(k)=\arg\min_{T>0}\left(T,k\right). \tag{30}$$

Once $T^*(k)$ is computed, the corresponding minimized discounted cost rate is

$$g^*(k)=g\left(T^*(k),k\right). \tag{31}$$

Because the shipment frequency k must be a positive integer and is typically bounded by operational constraints, the global optimum of the two-dimensional problem is obtained by enumerating feasible k values. The optimal policy is therefore

$$\left(T^{*},k^{*}\right)=\arg\min_{k\in Z_{+}} g^{*}\left(k\right), \tag{32}$$

subject to any upper bound on k imposed by practical considerations such as receiving capacity, transportation cost structure, or supplier coordination limits. Owing to the strict convexity in T, each evaluation of $g^*(k)$ is computationally inexpensive, and the dimensionality of the search over k is small. This structure makes the integrated DCF problem tractable even when embedded in sensitivity analysis or scenario simulation.

The numerical implementation used in Section 4 follows this solution logic. For each admissible shipment frequency, a dense grid search or root-finding procedure is applied to identify the unique optimal cycle length. The minimized cost rates are then compared across k, ensuring that the selected policy pair (T^*, k^*) is the global minimizer of the long-run discounted cost rate. This two-stage optimization is consistent with the theoretical structure of the model and guarantees convergence to the optimal replenishment strategy

4. Numerical examples

This section presents numerical experiments to illustrate how the proposed discounted cash flow (DCF) model determines the optimal replenishment cycle and shipment frequency. Unless otherwise specified, all computations are based on the following baseline parameter set:

D = 20,000 units/year, C_s = 500 \$/order, C = 20 \$/unit, C_g = 50 \$/shipment,

h = 2 \$/unit/year, R = 0.05 (per year), W=0.5 years.

For each candidate shipment frequency k, the unique optimal cycle length $T^*(k)$ is obtained by minimizing the discounted cost rate $g(T, k)$ in (13). The global optimum (T^*, k^*) is found by comparing minimized values across feasible k. In the numerical implementation, trade credit is modeled by shifting the purchasing cash flow to time W, i.e., multiplying CDT by $\exp(-RW)$. The distinction between $W \leqslant T$ and $W > T$ affects the economic interpretation of payment timing (within vs. beyond the cycle), although the purchasing cost is discounted by the same factor $\exp(-RW)$ in both cases.

4.1. Baseline parameter settings and computation

This section describes the baseline parameter settings and the numerical procedure used to determine the optimal replenishment policy. Unless otherwise stated, all numerical experiments are conducted using the baseline parameters specified at the beginning of this section.

Under the baseline parameter values, we consider shipment frequencies $k = 1, 2, \ldots, K$, where K represents a realistic upper bound on the number of deliveries that can be handled within a replenishment cycle (for example, K = 8). For each fixed shipment frequency k, the optimal cycle length is obtained by solving the first-order condition $g'\left(T,k\right)=0$ given in Equation (29). This one-dimensional optimization problem is solved numerically using a standard root-finding procedure with a positive initial bracket.

As established in Section 3, the discounted cost-rate function is coercive with respect to the cycle length T, which guarantees the existence of at least one interior minimizer for each shipment frequency. Moreover, the objective function is smooth over the feasible domain of T, ensuring stable convergence of the numerical solution procedure.

Before presenting the numerical optimization results, we first verify the curvature property of the discounted cost-rate function under the baseline parameter settings. Although Section 3 derives the explicit analytical expression for the second-order derivative, strict convexity cannot, in general, be guaranteed analytically for all parameter values due to the ratio structure of the objective function. We therefore conduct numerical curvature checks based on Equation (25) by evaluating the sign of $\partial^2 g(T, k)/\partial T^2$ over the feasible range of the cycle length T for representative shipment frequencies k. This verification step confirms that the discounted cost-rate function is strictly convex with respect to T under the parameter settings considered, thereby ensuring the uniqueness of the optimal cycle length obtained in the subsequent numerical analysis.

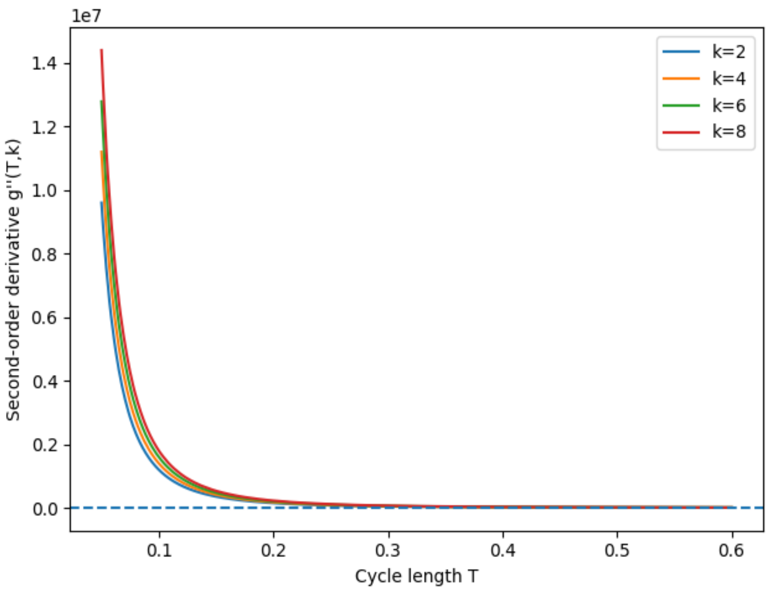

Figure 4. Numerical curvature check of $\partial^2 g(T,k)/\partial T^2$ under baseline parameters.

4.2. Baseline results without trade credit

Having verified the strict convexity of the discounted cost-rate function with respect to the cycle length under the baseline parameter settings (Figure 4), we now present the numerical optimization results for the baseline case without trade credit. This scenario serves as a benchmark for assessing the effects of shipment coordination and discounting in the absence of payment delays.

For each shipment frequency k, the optimal cycle length $T_0^*(k)$ is obtained by numerically solving the first-order condition $g'(T,k)=0$, and the corresponding minimized discounted cost rate is evaluated at the optimal solution. Since the numerical curvature check in Section 4.1 confirms strict convexity with respect to T under the baseline parameters, the solution obtained for each k corresponds to a unique global minimum.

Table 3 reports the minimized discounted cost rate for each shipment frequency in the baseline case without trade credit. As the shipment frequency k increases, the optimal cycle length $T_0^*(k)$ increases, while the lot-splitting structure still reduces the buyer's inventory exposure within each cycle. However, increasing k also raises total receiving costs. As a result, the minimized discounted cost rate initially declines and then increases as the receiving-cost burden becomes dominant.

The numerical results indicate that the interior optimum is achieved at k = 4, with an optimal cycle length T_0^* = 0.2166 and a minimum discounted cost rate of 406,499.2086. This outcome highlights the joint effect of inventory reduction through shipment coordination and the increasing receiving-cost burden associated with higher delivery frequencies.

Table 3. Optimal cycle length and minimized discounted cost rate under the baseline setting without trade credit.

k	Cycle time $T_o^*(k)$	Minimum cost $g(T_o^*(k), k)$
1	0.1352	408142.3839
2	0.1729	406946.1518
3	0.1967	406600.5656
4	0.2166	406499.2086
5	0.2305	406500.2731
6	0.2424	406551.0636
7	0.2563	406631.1852
8	0.2703	406728.2637

Note: For each shipment frequency k, the optimal cycle length $T_0^*(k)$ is obtained by numerically solving the first-order condition $g'(T,k)=0$. The minimized discounted cost rate is evaluated at the corresponding optimal solution. The interior optimum is identified by comparing the minimized cost across all considered shipment frequencies.

4.3. Results with trade credit

To assess the impact of payment timing on the optimal replenishment policy, we now report the numerical results under the trade-credit setting. All parameter values and computational procedures are identical to those used in Section 4.2, ensuring that any observed differences are attributable solely to the introduction of trade credit rather than to changes in the underlying operational structure.

For each shipment frequency k, the optimal cycle length $T_1^*(k)$ is obtained by numerically solving the first-order condition $g'(T,k)=0$ under the trade-credit policy, and the corresponding minimized discounted cost rate is evaluated at the optimal solution. As verified by the numerical curvature check in Section 4.1, the discounted cost-rate function remains strictly convex with respect to the cycle length T under the tested parameter settings. Consequently, the computed solution for each k again corresponds to a unique global minimum.

Table 4 reports the numerical results for the trade-credit case and completes the structural comparison between the non-credit and trade-credit regimes. The results indicate that trade credit improves the minimized discounted cost rate while preserving the qualitative structure of the optimal shipment frequency and cycle length. This comparison isolates the institutional effect of payment timing under otherwise identical operational parameters. Having established this baseline policy contrast, we next examine how the optimal solution responds to parameter variations within the trade-credit environment.

Table 4. Optimal cycle length and minimized discounted cost rate under the trade-credit policy.

k	Cycle time $T_1^*(k)$	Minimum cost $g(T_1^*(k), k)$
1	0.1352	398232.9403
2	0.1729	397027.2792
3	0.1987	396675.5465
4	0.2166	396569.5955
5	0.2325	396566.8250
6	0.2484	396614.1163
7	0.2583	396691.3621
8	0.2703	396785.3532

Note: The numerical procedure and parameter values are identical to those used in Table 3, except for the introduction of trade credit. For each shipment frequency k, the optimal cycle length $T_1^*(k)$ is obtained by solving the same first-order condition $g'(T,k)=0$. Differences in the minimized discounted cost rate reflect changes in payment timing rather than alterations in the operational structure.

5. Numerical insights and managerial implications

Section 4 establishes the optimal replenishment structure under baseline parameters and verifies the curvature properties of the discounted cost-rate function. Building on those results, this section investigates the robustness of the optimal policy under parameter variations. The sensitivity analysis focuses on how the optimal solution responds to economically meaningful changes in cost and financial parameters, building on the structural results established in Sections 3 and 4. Under the tested parameter configurations, lot-splitting continues to serve as an operational adjustment mechanism even when all cash flows are evaluated using a rigorous discounted cost-rate objective. For a given credit arrangement, increasing the shipment frequency reduces average inventory levels and hence discounted holding costs, but at the expense of higher receiving and coordination costs. Across the tested parameter ranges, the optimization results yield interior optimal solutions, indicating that extreme policies—such as single-shipment or excessively frequent deliveries—are not favored under the examined settings. From a managerial perspective, this highlights the importance of balancing marginal inventory savings against additional logistics costs, rather than relying on simplistic "more frequent is always better" heuristics. These sensitivity results complement the analytical findings in Section 3 and the numerical verification in Section 4 by illustrating policy responsiveness, rather than introducing new theoretical claims.

The role of financing decisions becomes particularly pronounced when trade credit is incorporated into the replenishment problem. When the credit period is relatively short and payment occurs within the operating cycle ($W \leqslant T$), the purchasing cost benefits from moderate discounting, while the physical inventory trajectory remains unchanged. In this regime, the optimal cycle length may increase slightly compared with the non-credit case, reflecting firms' incentives to delay payment while keeping inventory costs under control. By contrast, when the credit period exceeds the cycle length ($W > T$), purchasing payments are effectively deferred beyond the completion of the cycle. This reduces the discounted procurement cost in the tested scenarios and may shift the optimal policy toward longer cycles and, in some cases, different shipment frequencies. For firms facing tight liquidity constraints or high opportunity costs of capital, the results provide quantitative guidance for evaluating alternative credit periods and their alignment with operational cycle decisions.

Table 5. Sensitivity analysis summary (trade-credit setting).

Parameter (varied)	*Level*	*Value*	Optimal k^*	Optimal T^* (years)	Min. cost-rate g^*
	Low	0.03	6	0.2939	399,494.66
Discount rate R	Base	0.05	5	0.2333	396,565.87
	High	0.08	4	0.1851	391,898.13
	Low	1.00	3	0.2226	395,975.30
Holding cost rate h	Base	2.00	5	0.2333	396,565.87
	High	3.00	6	0.2325	397,013.46
	Low	30	6	0.2276	396,107.86
Receiving cost C_g	Base	50	5	0.2333	396,565.87
	High	80	4	0.2355	397,099.88
	Low	0.30	5	0.2323	400,509.54
Credit period W	Base	0.50	5	0.2333	396,565.87
	High	0.80	5	0.2345	390,723.67

Note: All results are obtained under the trade-credit setting. In each parameter block, only the indicated parameter is varied, while all other parameters are held at their baseline values. "Base" denotes the benchmark parameter value used in Section 4. The optimal policy (T^*, k^*) is obtained by minimizing the discounted cost-rate function.

Table 5 extends the analysis by conducting a parameter-sensitivity study under the trade-credit regime. Unlike Tables 3–4, which compare institutional settings, Table 5 focuses on how the optimal policy (T^*, k^*) adjusts when key economic parameters vary. The purpose is not to reassess the structural benefit of trade credit, but to evaluate the robustness and responsiveness of the optimal policy within that regime.

Table 5 reports the sensitivity analysis of the optimal policy with respect to key model parameters. The managerial insights derived from these results can be summarized as follows:

(1) Discount rate R: When the discount rate R increases, the optimal policy shifts toward shorter cycle lengths (T^* decreases) and fewer shipments (k^* decreases), reflecting a stronger preference for earlier cost timing under heavier discounting. In other words, both the replenishment horizon and the degree of lot-splitting contract as the discount rate becomes larger.

(2) Holding-cost rate h: As h increases, the model recommends more frequent shipments (larger k^*) to limit inventory exposure; T^* adjusts moderately because it reflects a balance between holding and receiving costs.

(3) Receiving cost C_g: A larger C_g reduces the attractiveness of lot-splitting, and thus the optimal solution moves toward fewer shipments (smaller k^*) with a slightly longer T^*.

(4) Credit period W: Extending W lowers the minimized discounted cost rate g^* and typically increases T^* slightly, while the optimal shipment frequency remains largely unchanged in the tested setting.

Overall, Table 5 shows that the optimal policy responds systematically to parameter changes in economically intuitive ways, clarifying when trade credit mainly improves financial timing (through W and R) and when operational trade-offs dominate (through h and C_g).

In addition, a conceptual comparison between the discounted cost-rate objective and traditional single-cycle present-value formulations highlights an important measurement distinction. Because replenishment cycles recur indefinitely, evaluating policies on a per-cycle present-value basis may bias decisions toward shorter cycles, whereas expressing performance in discounted cost per unit of time provides a more consistent indicator of long-run efficiency under continuous discounting.

From a managerial perspective, the integrated framework provides a tractable decision-support tool. Buyers can jointly calibrate order size, shipment frequency, and acceptable credit terms, while suppliers can evaluate how alternative credit policies influence ordering behavior and financial exposure within a consistent valuation framework.

In addition to the operational insights discussed above, the proposed discounted cost-rate framework also opens several avenues for further mathematical investigation. From an optimization perspective, the present formulation converts the infinite-horizon discounted cash flow of a stationary replenishment policy into a tractable cost-rate objective, thereby providing a consistent basis for analyzing long-run inventory decisions under continuous discounting. Future research may extend this framework to more general modeling environments, including stochastic demand processes, multi-product systems, and multi-echelon coordination structures with financial interactions.

More broadly, integrating financial timing with operational decision variables suggests potential links with recent developments in optimization-based logistics and supply-chain modeling. In particular, extensions incorporating uncertainty, dynamic pricing, or network-level coordination could benefit from combining discounted valuation principles with modern optimization approaches. Related perspectives can be found in recent studies on optimization and supply-chain modeling (e.g., [1–3,43]), which explore advanced modeling techniques for complex decision environments. Embedding the discounted cost-rate formulation within such broader optimization frameworks may therefore provide a promising direction for future research.

6. Conclusion

This study develops a unified and theoretically consistent discounted cash flow (DCF) framework for analyzing replenishment decisions in a two-echelon inventory system with trade credit and lot-splitting. A key contribution of this research is the identification and correction of a long-standing methodological flaw in the literature: the widespread reliance on single-cycle present-value objectives, which for more than twenty-five years have biased optimal solutions toward unrealistically short cycles by ignoring the infinite repetition of replenishment decisions. By deriving the correct discounted cost-rate (DCF-rate) objective and rigorously establishing its analytical properties—continuity, coerciveness, and strict convexity—this study provides a solid foundation for determining a unique and globally optimal policy pair (T^*, k^*).

The proposed framework fully reformulates all major cost components—setup, purchasing, receiving, and holding—according to their exact timing under continuous discounting. Within this structure, trade credit fundamentally alters the joint optimization of T^* and k^* by shifting the purchasing outflow in time, thereby modifying the intertemporal trade-off between holding cost, receiving cost, and discounted procurement expenditure. The model further shows that lot-splitting and the DCF-rate objective interact in a complementary manner, an insight not documented in previous studies. Numerical experiments confirm that lot-splitting meaningfully reduces discounted holding cost, while trade credit substantially mitigates the financial burden of procurement and can reshape both the optimal cycle length and shipment frequency. Sensitivity analyses reinforce the robustness of the model by demonstrating economically intuitive responses to changes in discount rate, credit period, receiving cost, and holding cost.

From a managerial standpoint, the findings highlight the importance of viewing operational and financial decisions as an integrated system. Firms relying on EOQ-type or single-cycle PV metrics may underestimate the value of trade credit or misidentify the efficient level of lot-splitting. In contrast, the proposed DCF-rate framework offers a computationally tractable and conceptually coherent tool for negotiating credit terms, structuring delivery schedules, and designing replenishment strategies that remain optimal over an infinite horizon. By jointly capturing operational flexibility and financial timing, the model provides actionable guidance for supply-chain partners operating under capital constraints, high discount rates, or the need for frequent and coordinated replenishment decisions.

Future research may extend the proposed framework to incorporate stochastic demand, multi-product interactions, and dynamic or endogenous credit terms—where credit periods are determined jointly within the model rather than specified exogenously—as well as contractual coordination mechanisms between buyers and suppliers. In addition, integrating behavioral or risk-sensitive financial considerations represents a promising direction for further investigation. Overall, this study establishes a rigorous theoretical baseline for future research at the intersection of inventory theory and supply-chain finance, while offering practical insights for organizations operating under time-dependent and financially constrained environments.

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Use of Generative-AI tools declaration

During the preparation of this manuscript, the authors used ChatGPT for language polishing and expression improvement. The authors take full responsibility for the final published version.

Author contributions

Tien-Yu Lin: Conceptualization, Methodology, Formal analysis, Investigation, Writing – original draft, Visualization. **Xiu-Hua Wei:** Supervision, Validation, Writing – review & editing, Project administration. **Yun-Feng Zheng:** Data curation, Software, Validation, Writing – review & editing.

Conflict of interest

The authors declare that they have no conflicts of interest.

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