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Research article

Constructing a new robust bilevel fashion product supply chain network with uncertain demand and transportation cost

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Abstract: In this paper, we address the construction of a multi-period fashion product supply chain network (FSCN) with the incorporation of production discounts and transportation quantity decisions. A bilevel design framework is proposed to capture the decision-making interaction between a fashion product manufacturer (acting as a leader) and retailers (acting as followers) over multiple periods. By utilizing Karush-Kuhn-Tucker (KKT) conditions, the bilevel optimization model can be transformed into a single-level mixed-integer linear programming (MILP) model. Furthermore, uncertainty in unit transportation cost and demand was addressed via a robust optimization framework, with the aim of balancing the optimal decisions of the upper and lower decision makers. To solve the proposed robust model efficiently, we designed a tailored Benders decomposition (BD) algorithm. To validate the effectiveness of our proposed model and BD algorithm, a case study based on ZARA was conducted. Thus, based on extensive experiments, the benefits of the proposed methods are illustrated, and significant insights are obtained for decision makers.

Keywords: fashion industry; Stackelberg game; uncertainty modeling; mixed-integer linear programming; Benders decomposition
Mathematics Subject Classification: Primary 90B06, 90C90; Secondary 91A65

1. Introduction

In recent years, the fashion industry has had considerable growth, leading to intensified competition and an urgent need for efficient supply chain management. The World Trade Organization (WTO) reports that global textile and apparel exports reached 5,777 billion USD in 2022, with China maintaining its position as the leading exporter at an export volume of 1,824.16 billion USD. According to the National

Bureau of Statistics of China, textile, clothing, and apparel enterprises recorded a total asset value of 1,116.04 billion yuan in 2022, along with a total profit of 70.5 billion yuan. These data underscore the fashion industry's crucial role in economic growth, as the ever-increasing demand makes the importance of efficient and reliable supply chains undeniable.

Conventional one-to-one supply chain models between manufacturers and retailers are inadequate to address the intricate operational challenges encountered by the fashion industry. To overcome these limitations, a bilevel supply chain network concept has emerged. In a bilevel supply chain, both cooperation and competition exist between upper- and lower-level members. This dual relationship can foster collaboration and achieve mutual benefits, while encouraging both parties to improve resource utilization efficiency through competition. Additionally, the demands and feedback of lower-level customers can assist upper-level decision-makers in allocating resources more accurately. Moreover, coordination and cooperation between upper and lower-level members enable bilevel supply chains to respond more rapidly to market changes, demand fluctuations, and risk events, thereby enhancing the overall supply chain's flexibility and responsiveness. However, further exploration of upper- and lower-level problems is necessary in the fashion industry. To this end, a comprehensive mathematical model is developed to capture the complex interactions and decision-making processes within a bilevel supply chain network. This model considers factors such as the pivotal role of demand uncertainty, production capacity constraints [1], transportation management [2, 3], inventory planning [4], and discount strategies [5]. In this study, we focus on the bilevel FSCN problem for equilibrium decision making. The decision-making processes and interactions between the upper-level (manufacturers or distributors) and lower-level (retailers or customers) problems of the FSCN are examined. It is important to note that decision makers at each level strive to achieve their own objectives, often leading to competitive relationships among them.

In the FSCN, uncertainty cannot be avoided, owing to factors such as incomplete and unpredictable information, as well as human subjective perception and judgment [6]. The design of the FSCN encompasses a considerable amount of uncertainty, commonly focusing on the uncertainty in unit transportation costs and demand. Stochastic optimization is a standard method used to address this uncertainty [7]. However, insufficient data make it challenging to accurately estimate the probability distributions of uncertain parameters, and deviations from the true probability distributions can create obstacles in decision execution. An alternative popular method is robust optimization (RO) [8, 9], which employs the characteristics of data to describe uncertainty sets and requires the solution to be feasible for all uncertain parameters. Following Ben-Tal et al. [10], the globalized robust optimization (GRO) method is adopted. This method treats the uncertainty set in the RO framework as the outer uncertainty set and introduces a smaller inner uncertainty set. It assumes that both uncertainty sets are convex and that the treatment of the inner uncertainty set is consistent with that in RO. All parameters in the uncertainty set must satisfy the original constraints, whereas parameters that belong to the outer set beyond the inner set are allowed to violate the original constraints to a certain degree. The degree of violation can be quantified using a distance function. By contrast with the RO method, the GRO method offers more flexible decisions based on the definition of uncertainty sets and distance metrics, while reducing conservatism. Consequently, the globalized robust counterpart (GRC) of our FSCN model is derived and presented as a computationally tractable reformulation. The transformed reformulation not only reduces the computational burden but also enhances the flexibility and practicality of the RO framework.

The model formulated within the framework of bilevel programming is an NP-hard problem and therefore cannot be solved easily [11]. As the scale of the proposed problem increases, so does the computational complexity, reflecting the inability to arrive at ideal decisions within a reasonable time frame. In recent years, innovative approaches have been developed to tackle complex optimization and equation-solving problems. Notably, neural network-based methods have shown promising capabilities in obtaining exact analytical solutions for nonlinear partial differential equations [12, 13]. For instance, Zhang and Bilige [14] introduced a bilinear neural network method that can derive exact analytical solutions for nonlinear PDEs, while other researchers have extended this approach to more complex equations [15, 16]. These developments highlight the growing intersection between machine learning and mathematical modeling, offering new perspectives for solving challenging optimization problems. Furthermore, the BD method is well-suited for handling this challenge, as it decomposes the problem into a master problem and a subproblem, enabling an effective iterative solution process and mitigating computational complexity [17]. Given the research status of FSCNs, we aim to leverage robust optimization techniques to effectively address model uncertainties in the design of a bilevel FSCN. Additionally, we analyze the impact of discount pricing on the network structure.

1.1. Research questions

In this paper, we address the following three research questions to guide the investigation:

- *How does the proposed robust bilevel fashion supply chain network effectively manage uncertain demand and unit transportation costs?*
  In a bilevel FSCN, rational transportation and sales allocation decisions are crucial to the financial health of fashion enterprises. Indeed, numerous fashion enterprise managers have invested considerable time and effort in designing transportation and sales allocation strategies within the network to enhance FSCN performance. However, these allocation decisions are frequently affected by uncertain transportation costs and demand, leading to unfulfilled customer needs. Ignoring these uncertainties can result in either under–production or over–production, subsequently affecting sales. Such circumstances may escalate operational risks for businesses and, in extreme cases, threaten their survival [18]. To address this problem, the uncertainties associated with transportation costs and demand are incorporated, leveraging historical data to construct a comprehensive set of uncertainty scenarios. A bilevel globalized robust FSCN (GR-FSCN) model is developed, which is designed to effectively manage the uncertain demand and unit transportation costs. Our model aims to reduce the likelihood of decision-makers making impractical choices under uncertain conditions, thereby ensuring that strategic decisions are made on a strong foundation of data and analysis.
- *How can a product distribution plan be implemented in a multi-period setting to achieve higher economic benefits?*
  Fashion enterprises can achieve considerable economic benefits by strategically implementing tailored product distribution plans across periods of uncertainty. Our bilevel GR-FSCN model can effectively tackle these complexities by incorporating variability in transportation costs and demand into its framework. At the upper level, decision-makers such as corporate headquarters or logistics managers focus on minimizing the overall costs associated with transportation, facility location, and other strategic investments. By leveraging the optimal strategies provided by the model, firms can reduce costs and improve their overall financial performance. At the lower level,

retailers or distribution centers aim to maximize profits by effectively managing inventory levels, pricing strategies, and sales activities [19]. The balance between the upper and lower levels is achieved through an inherent integrated optimization process in the model [20]. By considering the interactions and dependencies between the two levels, the model identifies strategies that not only minimize costs at the upper level but also enable the lower level decision-makers to maximize profits. This synergy creates a mutually beneficial situation in which both levels of the supply chain benefit from an optimized distribution plan.

- *How can the inherent difficulties in solving uncertain bilevel FSCN design problems be overcome in practical applications?*
  In this respect, our GR-FSCN model faces a serious obstacle in that it leads to an infinite dimensional problem that is difficult to handle [21]. To overcome this inherent difficulty, the model is reformulated based on the GRO method, enabling the derivation of an MILP formulation that rendered the problem computationally tractable. However, the bilevel model structure cannot be solved directly [17], and the KKT method is applied to transform it into a single-level model. Consequently, the characteristics of the resulting MILP are structured to design a tailored BD algorithm suited to the problem.

1.2. Contributions of our work

The major contributions of our work can be summarized as the following three aspects:

- We propose an FSCN model that integrates the uncertainties in demand and transportation costs. Based on available historical data, each uncertain parameter in our model is characterized via a pair of inner and outer uncertainty sets. Consequently, a novel GR-FSCN model is developed. For its tractable GRC-FSCN reformulation, the Lagrangian duality theory is adopted.
- The FSCN design problem is studied, and a Stackelberg game framework is constructed with two types of decision-makers. Our model aims to find the equilibrium solution for the fashion product manufacturer (a leader) to minimize costs at the upper level and for retailers (followers) to maximize profits at the lower level. To solve our proposed model, the bilevel model is transformed into a single-level MILP model using KKT conditions.
- The effectiveness of our new model and algorithm is demonstrated using a practical case involving the ZARA company. A new tailored BD algorithm is designed to solve the proposed FSCN model. The algorithm decomposes an MILP model into a master problem and a subproblem to enhance the efficiency of the solution process. Computational results demonstrate the effectiveness of our proposed model and solution algorithm.

The remainder of this paper is organized as follows: In Section 2, we review the relevant literature on FSCN and uncertain FSCN problems. In Section 3, we introduce the notation and present the bilevel FSCN model under uncertainty. Based on the uncertainty sets and optimality conditions, in Section 4, we derive a tractable reformulation of the proposed bilevel FSCN model. In Section 5, we focus on the tailored BD algorithm for solving the single-level GRC-FSCN model. In Section 6, we present the ZARA case study to demonstrate the advantages of the proposed model, and report the findings from the computations. Finally, the paper is concluded in Section 7.

2. Literature review

In this section, the works related to our study are briefly reviewed from two perspectives: Optimization methods of FSCN design and the uncertain FSCN design problem.

2.1. Optimization methods of FSCN design

The FSCN is a complex bilevel structure that involves various stages and participants, such as production, distribution, and sales [19, 11]. Bevilacqua et al. [22] qualitatively analyzed the impact of the interruption in an FSCN. Shi et al. [23] studied pricing and production decisions for fashion apparel brands during the pre-season and regular sales stages by considering the seasonal nature of clothing and the changing demand over time. Longo et al. [1] designed an evolutionary decision support model for the design and optimization of clothing product platforms, addressing the challenges of large-scale customization in the fashion industry. Choi et al. [24] focused on ethical operations, product greenness levels and retail pricing in an FSCN. Wang et al. [25] employed a heuristic algorithm to solve the inventory balancing and scheduling problem for clothing stores in an FSCN that permitted lateral transshipments. Shen et al. [26] built a game-theory model to analyze the behavior of conspicuous consumers to provide guidance and insights for strategic decision-making in luxury supply chains. What distinguishes our research from the literature is the simultaneous consideration of the bilevel structure of production and demand levels. This intricate network structure requires coordination and collaboration among stages and participants to effectively improve profit margins and ensure that consumers' personalized and high-end needs are addressed more accurately.

Although Dehghan et al. [27] discussed the coordination and pricing strategies of the omni-channel supply chain, few researchers have examined the bilevel structure of the FSCN. For example, Fares et al. [28] developed a mathematical model to optimize the distribution network of a complete multi-tier supply chain with four links. Parvasi et al. [29] designed a bilevel model to achieve high profits and low costs, employing explicit enumeration techniques to determine the optimal strategy. Camacho-Vallejo et al. [20] conducted a comprehensive review of the available literature on the subject, categorizing the handling approaches of critical bilevel aspects in metaheuristic algorithms in detail. In addition, Luo et al. [30] and Zhang and Xue [31] formulated a bilevel model at the government and industry levels to balance economic benefits and environmental protection.

Additionally, research addresses the discount sales problem of fashion products, particularly in a multi-period sales model, to assist retailers in formulating better sales strategies to enhance sales performance. Discount sales are a common sales strategy in the fashion industry, aimed at driving sales, clearing inventory, attracting consumers, and enhancing brand awareness. For instance, Li et al. [17] explored a multifaceted inventory system that integrated financial considerations, marketing tactics, and operational processes across a three-tier supply chain architecture, highlighting the importance of the time value of capital by employing discounted cash flow methods to evaluate its financial ramifications. Shen et al. [32] proposed a model by incorporating a detailed analysis of pricing concessions and examining the relationship between consumer optimism indices and the strategic allocation of inventory resources. This innovative perspective sheds light on how consumers' psychological characteristics can inform supply chain optimization strategies. Ma et al. [21] investigated the application of complex dynamic behavior in inventory control systems under pricing and discount promotion policies. Chen et al. [5] studied a randomized promotional policy for two market segments. In our research, we aim to assist retailers in formulating better sales strategies to improve sales performance.

Based on the aforementioned literature, it is evident that the focus of these researchers on fashion products is predominantly on constructing optimization models to reduce costs and increase profits. Depending on the problem scale, solution methods have been employed, including direct software-based solutions and heuristic algorithm designs. Although a substantial portion of research on FSCN design challenges has focused predominantly on single-level network architectures, bilevel network structure may be more appropriate in real-world scenarios. However, there is limited research on the optimization of an FSCN with a bilevel structure. Furthermore, it is worth noting that the aforementioned literature primarily considers deterministic environments and does not discuss the influence of uncertain factors.

2.2. Uncertainty in the FSCN design problem

A number of studies have illuminated the critical role of uncertainty in shaping the design and optimization strategies of the FSCN. These studies underscore the need of integrating uncertainty considerations into FSCN in single- and multi-level frameworks. For example, Li et al. [33] considered a supply chain model under stochastic market demand. Roudbari et al. [7] proposed a closed-loop supply chain design for the garment industry leveraging stochastic programming techniques. Lin et al. [34] formulated a discrete-time stochastic model based on volatile demand in the fashion industry and introduced a heuristic approach to improve efficiency. Similarly, Delgoshaei et al. [35] developed a scheduling strategy for a multi-market, multi-retailer fashion supply chain that incorporated stochastic demand and utilized a metaheuristic algorithm for empirical validation. Cardona-Valdés et al. [36] constructed a multi-period two-stage stochastic model with demand uncertainty in the fashion industry. These researchers considered the design problems of single- and multi-level supply chain networks in stochastic environments. Additionally, some researchers have considered the use of the RO method in the absence of precise information regarding the distribution of parameters. These researchers focused on developing the RO method to handle uncertainty and variability in supply chain design and decision-making. Yu et al. [37] demonstrated that robust models can balance income and stability and outperform deterministic classification models in the worst-case scenarios. They proposed that robust models can address parameter uncertainty may be more suitable for practical applications. Timonina-Farkas et al. [6] built a bilevel stochastic model including chance constraints and accounting for uncertainty in production disruptions and customer demand distributions. The model utilized a customized BD algorithm for the solution. We focus on the design problem of an FSCN under uncertainty. The uncertain transportation costs in the upper-level problem and the uncertain market demand in the lower-level problem are considered. To tackle the challenges posed by these uncertainties, we employ the RO method, which considers the uncertainties and variations in system parameters and seeks the feasible region that is robust based on all possible uncertain parameters values. The GRO method enables controlled violations of constraints, thereby enhancing the efficiency of the FSCN.

2.3. Distinction of our work from the literature

The primary research gap addressed in this study lies in the comparison and analysis of the literature on FSCN optimization. We focus on the design of a bilevel FSCN under uncertainty, which represents a significant departure from the research.

The single-level studies [17] are insufficient for effectively coordinating different levels of participation, leading to decreased supply chain efficiency and failure to meet consumer demand. Consequently, we propose a bilevel framework that considers the product manufacturer and retailers to improve the FSCN modeling.

Although there has been some research on FSCN problems, it fails to adequately tackle the intricate challenges posed by managing supply chains under uncertainty, leaving room for further investigation and strategies to mitigate these complexities [29, 28]. Ma et al. [21] and Fares et al. [28] analyzed a supply chain network in a deterministic environment, whereas Delgoshaei et al. [35] and Li et al. [33] explored the impact of stochastic demand on supply chains. Additionally, Yu et al. [37] used the RO method to handle uncertain parameters. By contrast, we employ the GRO method to relax the feasibility constraints within specific regions of the uncertainty set in a controlled manner, thereby expanding the flexibility of decision-makers. Through comparisons with the literature, we comprehensively address the design of a multi-period bilevel FSCN and tackle the challenges of supply chain management under uncertainty. Table 1 summarizes representative studies along the key dimensions suggested by the reviewer.

Table 1. Review and analysis of the related FSCN literature.

Literature	Model structure		Uncertainty treatment			Periods	Solution methodology	Discount
	Single-level	Bilevel	Deterministic	Stochastic	Robust			
[1]	√		√			Single	Genetic algorithm	
[5]	√		√			Multi	Software	√
[7]	√			√		Multi	Software	
[17]		√			√	Single	Benders decomposition	
[21]		√			√	Single	Software	
[23]	√		√			Multi	Software	√
[24]	√		√			Single	Software	
[25]	√		√			Multi	Heuristic algorithm	
[26]	√		√			Single	Software	
[28]	√		√			Multi	Software	√
[29]		√	√			Single	Enumeration technique	
[30]		√	√			Multi	Software	
[31]		√	√			Single	Software	
[32]	√		√			Single	Software	
[33]	√			√		Single	Software	
[34]	√			√		Multi	Heuristic algorithm	
[35]	√			√		Multi	Metaheuristic algorithm	√
[36]	√			√		Multi	Software	
[37]	√				√	Single	Software	
This paper		√			√	Multi	Benders decomposition	√

3. Methodology

3.1. Problem statement

As shown in Figure 1, the structure of our FSCN consists of several facilities: a manufacturer, W potential distribution center locations, R retailer locations, and I market locations. These facilities play an important role in the sales of fashion products, but their capacities are limited. We assume that the four periods in a year corresponded to the four seasons. The manufacturer engages in continuous transportation of goods to distribution centers via air freight in each season. After fashion products are produced by the manufacturer, they undergo processing. Once the processing is complete, the fashion products need to be transported from the manufacturer's location to the distribution centers via air freight. Distribution centers serve as crucial nodes in the supply chain and are responsible for storing, managing, and distributing fashion products to retailers via ground transportation. After receiving the products, retailers check the product to ensure that the quality is qualified before selling them to the customers, where customers can make their own purchases. At the end of each period, retailers have the option to offer surplus products at discounted prices to facilitate sales. As a decision-maker, the manufacturer must consider how to optimize the processing and transportation processes to improve the efficiency and reduce costs. Retail decision-makers must take appropriate measures to optimize their operations and achieve profit growth. Moreover, demand and price are parameters calibrated from historical data rather than decision variables in our model, reflecting the market reality that demand does not vary with price in the short term. Our model optimizes operational decisions, including flow quantities from manufacturers to distribution centers, between distribution centers and retailers, inventory holdings, network design regarding distribution center opening, and discounting and sales quantities in each market, with the objective of minimizing the total costs. This setting is consistent with the fast fashion industry. In ZARA, demand for in-season, full-priced products is primarily driven by product novelty. Consumers' purchasing decisions are often influenced by product scarcity and time-to-market, resulting in relatively low price sensitivity during the regular-price selling period [38]. The notations used in our FSCN model and their definitions are provided as follows:

Sets:
W: Set of potential distribution center locations $w \in W$.
R: Set of retail locations $r \in R$.
I: Set of market locations $i \in I$.
T: Set of periods $t \in T$.
Parameters:
MC: The capacity of the manufacturer in each period (per batch).
WC_w: The capacity of distribution center w in each period (per batch).
RC_r: The capacity of retailer r in each period (per batch).
P_r^t: Per batch sale price in retailer r in period t (CNY).
DM_w: Distance from the manufacturer to distribution center w (km).
DW_{wr}: Distance from distribution center w to retailer r (km).
CL_w^t: Per batch inventory cost from distribution center w in period t (CNY).
α_w: Fixed cost of opening distribution center w (CNY).
PA_r^t: Per batch discount price in retailer r in period t (CNY).
PM^t: Per batch production cost of products for the manufacturer in period t (CNY).
Uncertain parameter:
CM_w: Per batch transportation cost of each unit product for the manufacturer to distribution center w (CNY).
CW_{wr}: Per batch transportation cost from distribution center w to retailer r (CNY).
D_i^t: Demand occurring at market i in period t (per batch).

Upper-level decision variables:
x^t_w: Quantity of products flowing from the manufacturer into distribution center w during period t (per batch).
y^t_{wr}: Quantity routed from distribution center w to retailer r in period t (per batch).
h^t_w: Quantity of products held in inventory at distribution center w by the end of period t (per batch).
o_w: Binary variables, 1 if distribution center w is opened, 0 otherwise.
Lower-level decision variables:
a^t_{ri}: Quantity of discounted products of retailer r in market i in period t (per batch).
s^t_{ri}: Quantity of sold products of retailer r to market i in period t (per batch).

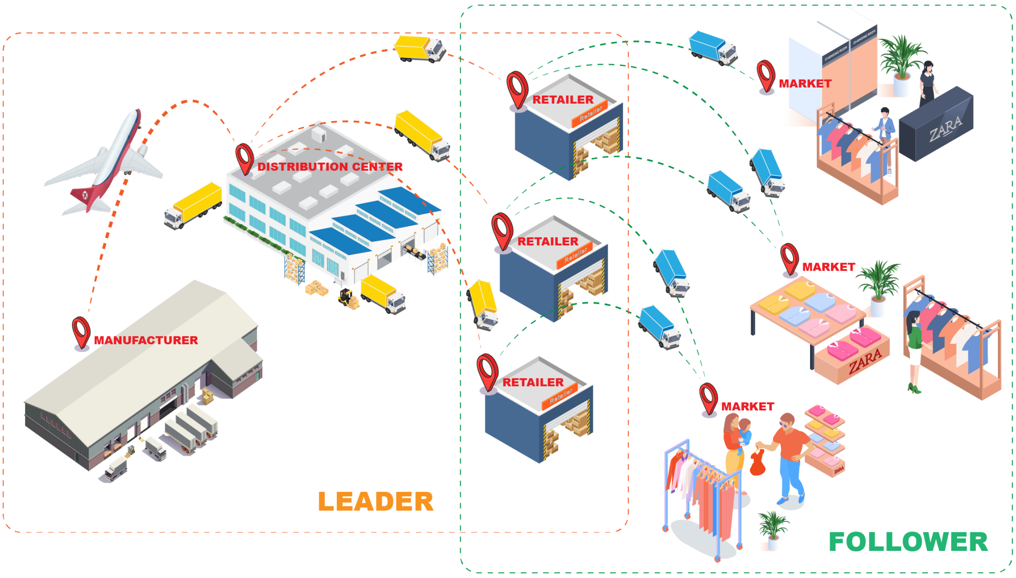

Figure 1. The structure of the investigated FSCN.

The optimization model introduced herein relies on the following underlying assumptions:

A.1. Each distribution center has a fixed opening cost.

A.2. Returns are ignored owing to the relatively small quantity of returns.

A.3. The fixed opening costs are considered for each distribution center.

A.4. The initial inventory at distribution centers is zero.

3.2. Deterministic bilevel FSCN model

Based on the above notations, our hierarchical relationship in FSCN can be characterized by a leader-follower optimization framework. This bilevel FSCN model will be conducted in the following subsection.

Upper-level model:

The upper-level model aims to minimize the leader's total cost, which consists of four parts. The first component is the total transportation cost, covering product shipment from the manufacturer to distribution centers and then from distribution centers to retailers, expressed as $TC = \sum_{w \in W} \sum_{t \in T} CM_w * x_w^t * DM_w + \sum_{w \in W} \sum_{r \in R} \sum_{t \in T} CW_{wr} * y_{wr}^t * DW_{wr}$. The second part is the cost associated with opening distribution centers, represented by $OC = \sum_{w \in W} o_w * \alpha_w$. The third part is the production costs incurred by the manufacturer, which can be expressed as $PC = \sum_{w \in W} \sum_t PM^t * x_w^t$. The fourth part is the inventory costs associated with storing products in distribution centers, which can be defined as $IC = \sum_{w \in W} \sum_{t \in T} CL_w^t * h_w^t$. Thus, minimizing these four parts optimizes the leader's total cost.

Constraints (3.1) state that the quantity of products delivered from the manufacturer to distribution center w should not be less than the quantity of products delivered from the distribution center to the retailer in period t,

$$x_w^t \geq \sum_{r \in R} y_{wr}^t, \ \forall \ w, t. \tag{3.1}$$

Constraints (3.2) show that the aggregate quantity of products dispatched by the distribution center w to retailer r must be no less than the cumulative sales volume of products handled by retailer r, including products sold at regular prices and sold at discounted prices in period t,

$$\sum_{i \in I} a_{ri}^t + \sum_{i \in I} s_{ri}^t \leq \sum_{w \in W} y_{wr}^t, \ \forall \ r, t. \tag{3.2}$$

Constraints (3.3) impose that the total quantity supplied to all distribution centers cannot exceed the manufacturer's production capacity,

$$\sum_{w \in W} x_w^t \leq MC, \forall \ t. \tag{3.3}$$

Constraints (3.4) limit the inbound quantity at each distribution center w during period t to its respective handling capacity,

$$x_w^t \leq WC_w * o_w \ , \forall \ w, t. \tag{3.4}$$

Constraints (3.5) ensure that the quantity delivered to any retailer r in period t should not surpass that retailer's intake capacity,

$$\sum_{w \in W} y_{wr}^t \leq RC_r, \forall \ r, t. \tag{3.5}$$

Constraints (3.6) stipulate that the inventory level held at distribution center w in period t remains within its storage capacity,

$$h_w^t \leq WC_w * o_w, \forall \ w, t. \tag{3.6}$$

Constraints (3.7) balance the inventory flow of distribution centers. The inventory at center w in a given period equals its prior-period inventory, plus incoming shipments from the manufacturer, minus outgoing shipments to retailers,

$$\sum_{t=1}^{T} h_w^t = \sum_{t=1}^{T} h_w^{t-1} + \sum_{t \in T} x_w^t - \sum_{t \in T} \sum_{r \in R} y_{wr}^t, \forall \ w. \tag{3.7}$$

Constraints (3.8) express the flow relationship between quantity and inventory levels. In period t, the inventory at distribution center w must be sufficient to cover all shipments from that center to retailers,

$$h_w^t \geq \sum_{r \in R} y_{wr}^t, \forall\ w, t. \tag{3.8}$$

On the basis of the above analysis, we can build the deterministic upper-level model as follows:

$$\begin{aligned} &\min TC + OC + PC + IC \\ &\text{s.t. constraints (3.1)} - (3.8), \end{aligned}$$

where the upper decision variables x_w^t, y_{wr}^t, and h_w^t are nonnegative variables, where $x_w^t \in \mathbb{R}_+^{W \times T}$, $y_{wr}^t \in \mathbb{R}_+^{W \times R \times T}$, and $h_w^t \in \mathbb{R}_+^{W \times T}$. In addition, o_w is a binary variable. The upper-level represents the manufacturer in the supply chain system, aiming to minimize the total costs ($F_1 + F_2 + OC + PC + IC$).

Lower-level model:

The lower-level model seeks to maximize the follower's total profit. This profit comprises two distinct parts. First, there is the profit derived from selling products at their regular price P_r^t, which is denoted as $RP = \sum_{i \in I} \sum_{r \in R} \sum_{t \in T} P_r^t * s_{ri}^t$. Second, there is the additional profit garnered from offering a discounted price PA_r^t, which is represented by $DP = \sum_{i \in I} \sum_{r \in R} \sum_{t \in T} PA_r^t * a_{ri}^t$.

Constraints (3.9) ensure that the demand of each market is met in any period. Demand is met by quantity sold at regular prices and discounted prices in the lower-level model,

$$\sum_{r \in R} a_{ri}^t + \sum_{r \in R} s_{ri}^t \geq D_i^t,\ \forall\ t, i. \tag{3.9}$$

where s_{ri}^t represents regular-price sales. This constraint indicates that market demand is met by the combination of regular-price and discounted sales. Therefore, although the demand parameter D_i^t does not vary with price, the discount mechanism plays a substantive role in operational decisions by regulating the supply quantity a_{ri}^t, thereby affecting how demand is realized and how profits are distributed across the supply chain.

Constraints (3.10) guarantee that the total quantity of products sold to markets by retailer r at regular and discounted prices does not exceed the capacity of retailer r in the lower-level model,

$$\sum_{i \in I} a_{ri}^t + \sum_{i \in I} s_{ri}^t \leq RC_r,\ \forall\ r, t. \tag{3.10}$$

Constraints (3.11) represent that in each period t, the quantity a_{ri}^t of discounted products that retailer r can sell in market i cannot exceed the quantity a_{ri}^{t-1} from the previous period $t-1$, minus the discounted price PA_r^{t-1} divided by the unit selling price P_r^{t-1}, multiplied by a constant ρ^{t-1} and the time interval Δt. This equation serves to limit the quantity of discounted products, control changes in sales strategy, and ensure the stability in sales profitability. As the periods increase, the quantity of discounted products sold decreases. In addition, we set the time interval Δt equal to 1 for two consecutive periods,

$$a_{ri}^t \leq a_{ri}^{t-1} - PA_r^{t-1} / P_r^{t-1} * \rho_r^{t-1} * \Delta t,\ \forall\ r, i, t \geq 2. \tag{3.11}$$

This formulation ensures that: (i) Discounted quantities decrease as product age increases; (ii) sales strategy transitions are controlled and stable; and (iii) profitability is protected by limiting excessive discounting. This modeling approach draws on relevant literature regarding inventory-driven discount strategies [39, 40].

Finally, we can formally construct the deterministic lower-level model as follows:

$$\begin{aligned} \max \quad & RP + DP \\ \text{s.t.} \quad & \text{constraints} \quad (3.9)-(3.11), \end{aligned}$$

where the lower decision variables a_{ri}^t and s_{ri}^t are nonnegative variables, where $a_{ri}^t \in \mathbb{R}_+^{R\times I\times T}$ and $s_{ri}^t \in \mathbb{R}_+^{R\times I\times T}$. The lower-level represents the retailers, aiming to maximize their profits ($RP + DP$).

3.3. Construction of the bilevel GR-FSCN model

In this section, a bilevel GR-FSCN model is established with uncertain unit transportation costs CM_w and CW_{wr} of the upper-level objective and the uncertain demand D_i^t of the lower-level constraints.

***Source of uncertain unit transportation costs*:**

Changes in national policies and the outbreak of wars have become important factors affecting oil prices, which in turn introduce uncertainty in unit transportation prices. Furthermore, the repair and maintenance of the means of transportation also cause changes in unit transportation costs. Based on the historical data on unit transportation costs between facilities, the nominal unit transportation costs CM_w^0 and CW_{wr}^0 over the planning period can be estimated. We use the affine form to represent the uncertain unit transportation costs, $CM_w = CM_w^0 + \zeta_{CM}\widetilde{CM}_w$ and $CW_{wr} = CW_{wr}^0 + \zeta_{CW}\widetilde{CW}_{wr}$. Here, ζ_{CM} and ζ_{CW} are perturbation vectors, and the perturbation coefficient is presented as $\widetilde{CM}_w$, implying the influence degree of the uncertain unit transportation cost for the manufacturer to distribution center w. The perturbation coefficient $\widetilde{CW}_{wr}$ implies the influence degree of uncertain unit transportation cost for distribution center w to retailer r.

To address uncertainty, the concept of inner-outer uncertainty set has been employed. The uncertainty sets of unit transportation costs CM_w and CW_{wr} can be expressed in the following form:

$$\begin{aligned} U_{CM}^l &= \{CM_w = CM_w^0 + \zeta_{CM}\widetilde{CM}_w \,|\, \zeta_{CM} \in Z_{CM}^l\},\ l = 1,2, \\ U_{CW}^l &= \{CW_{wr} = CW_{wr}^0 + \zeta_{CW}\widetilde{CW}_{wr} \,|\, \zeta_{CW} \in Z_{CW}^l\},\ l = 1,2, \end{aligned}$$

where Z_{CM}^l and Z_{CW}^l represent perturbation sets about uncertain unit transportation costs, and U_{CM}^l and U_{CW}^l represent uncertainty sets. Z_{CM}^1 and Z_{CW}^1 are given nonempty, convex, and compact sets with $0 \in ri(Z_{CM}^1)$ and $0 \in ri(Z_{CW}^1)$. The sets Z_{CM}^2 and Z_{CW}^2 are convex in nature. However, they are not assumed to be compact, enabling a potentially unbounded range of values. We assume that $Z_{CM}^1 \subset Z_{CM}^2$ and $Z_{CW}^1 \subset Z_{CW}^2$, implying that $U_{CM}^1 \subset U_{CM}^2$ and $U_{CW}^1 \subset U_{CW}^2$.

To model uncertain parameters, we use the GRO method to reduce the conservatism of classical robust optimization and reduce the requirements for feasibility. The GRO method incorporates two distinct convex uncertainty sets: Inner uncertainty sets U_{CM}^1 and U_{CW}^1 that are contained within the outer uncertainty sets U_{CM}^2 and U_{CW}^2. Different uncertainty sets control the conservatism of the uncertain parameters to different degrees. This mechanism ensures that while the outer set enables a broader exploration of uncertainty, the solution remains constrained by the proximity to the feasible region defined by U^1 [10].

We build the following GR representation of TC in the upper-level model by introducing variables F_1 and F_2,

$$\begin{aligned} \min \quad & F_1 + F_2 + OC + PC + IC \\ \text{s.t.} \quad & \sum_{w\in W}\sum_{t\in T} CM_w * x_w^t * DM_w - F_1 \le \min_{CM_w^{'}\in U_{CM}^1} \phi(CM_w, CM_w^{'}), \forall CM_w \in U_{CM}^2, \quad (3.12) \\ & \sum_{w\in W}\sum_{r\in R}\sum_{t\in T} CW_{wr} * y_{wr}^t * DW_{wr} - F_2 \le \min_{CW_{wr}^{'}\in U_{CW}^1} \phi(CW_{wr}, CW_{wr}^{'}), \forall CW_{wr} \in U_{CW}^2. \quad (3.13) \end{aligned}$$

In inequalities (3.12) and (3.13), $\phi(\cdot,\cdot)$ measures the distance between parameters. The distance measure function $\phi(\cdot,\cdot)$ is presumed to possess three key properties: Nonnegativity, closedness, and joint convexity in its two arguments, and $\phi(\cdot,\cdot) = 0$ for the same parameters.

***Uncertain demand*:**

Fashion products are subject to a high degree of demand uncertainty owing to changing fashion trends and consumer preferences. Sales of fashion products are also affected by transportation factors, such as varying epidemic control policies across regions and adverse weather factors. In our FSCN, it is common to have censored data. It is also difficult to obtain precise information regarding these uncertain parameters. We reformulate the uncertain parameters in an affine form that is linearly related to the original data and the sources of uncertainty. In our work, we use the affine form to represent the uncertain demand, $D_i^t = D_i^{t0} + \zeta_D \widetilde{D}_i^t$. The presented formulation underscores the parameter that variations in demand are fundamentally driven by the perturbation vector ζ_D. The nominal value D_i^{t0} represents the nominal value for demand, whereas the perturbation coefficient $\widetilde{D}_i^t$ indicates the relative influence of the uncertain factor on the actual demand realization.

The inner and outer uncertainty sets of the uncertain demand are affected by the fluctuation vector ζ_D. In the inner uncertainty set $\overline{U}_D$, we have $\zeta_D \in \overline{Z}_D$. In the outer uncertainty set U_D, there is $\zeta_D \in Z_D$. The fluctuation vector ζ_D determines the extent of the uncertain demand within the inner uncertainty set $\overline{U}_D$ (where $\zeta_D \in \overline{Z}_D$) and the outer uncertainty set $\zeta_D \in \overline{Z}_D$ (where $\zeta_D \in \overline{Z}_D$). Set $\overline{Z}_D$ is nonempty, convex, and compact with $0 \in ri(\overline{Z}_D)$, while set Z_D is convex but not necessarily compact. We assume that $\overline{Z}_D \subset Z_D$, implying that $\overline{U}_D \subset U_D$.

By the GRO method, we can build the GR form of the lower-level constraints (3.9):

$$D_i^t - \sum_{r\in R} a_{ri}^t - \sum_{r\in R} s_{ri}^t \le \min_{\overline{D}_i^t \in \overline{U}_D} \phi(D_i^t, \overline{D}_i^t), D_i^t \in U_D, \ \forall\, i, t. \quad (3.14)$$

If we substitute (3.12)-(3.14) in the FSCN model, then we can get the following equivalent reformulation of the FSCN model:

$$\begin{aligned} \min \quad & F_1 + F_2 + OC + PC + IC \\ \text{s.t.} \quad & \text{constraints} \quad (3.1) - (3.8), (3.12) - (3.13), \\ \max \quad & RP + DP \\ \text{s.t.} \quad & \text{constraints} \quad (3.10) - (3.11), (3.14). \end{aligned} \quad (3.15)$$

The uncertain parameters in the upper- and lower-level constraints of the model fluctuate within the uncertainty set, causing model (3.15) to yield an infinite number of constraints. To solve the proposed bilevel model, the model must be reformulated in a computationally tractable form. The process of deriving a tractable reformulation of model (3.15) can be carried out in the following section.

4. Major results of the bilevel GR-FSCN model

In this section, we reformulate the bilevel GR-FSCN model as a computationally tractable form. In subsection 4.1, we propose the appropriate uncertainty sets to make the GR-FSCN model computationally tractable. In subsection 4.2, the bilevel optimization model is transformed into a single-level form by the optimality conditions.

4.1. Computationally tractable GRC

In this subsection, we derive the computationally tractable form of the bilevel GR-FSCN model considering inner and outer uncertainty sets and distance functions.

Uncertain unit transportation costs

Let Z^1_{CM} and Z^1_{CW} be the intersection of ball-budget-box perturbation sets,

$$Z^1_{CM} = \left\{ \zeta_{CM} \mid \|\frac{\zeta_{CM}}{\sigma'_{CM}}\|_2 \le \kappa'_{CM}, \|\frac{\zeta_{CM}}{\sigma'_{CM}}\|_1 \le \mu'_{CM}, \|\zeta_{CM}\|_\infty \le \sigma'_{CM} \right\},$$
$$Z^1_{CW} = \left\{ \zeta_{CW} \mid \|\frac{\zeta_{CW}}{\sigma'_{CW}}\|_2 \le \kappa'_{CW}, \|\frac{\zeta_{CW}}{\sigma'_{CW}}\|_1 \le \mu'_{CW}, \|\zeta_{CW}\|_\infty \le \sigma'_{CW} \right\},$$

and let Z^2_{CM} and Z^2_{CW} be the intersection of budget-ball perturbation sets, i.e.,

$$Z^2_{CM} = \left\{ \zeta_{CM} \mid \|\frac{\zeta_{CM}}{\sigma_{CM}}\|_1 \le \mu_{CM}, \|\frac{\zeta_{CM}}{\sigma_{CM}}\|_2 \le \kappa_{CM} \right\},$$
$$Z^2_{CW} = \left\{ \zeta_{CW} \mid \|\frac{\zeta_{CW}}{\sigma_{CW}}\|_1 \le \mu_{CW}, \|\frac{\zeta_{CW}}{\sigma_{CW}}\|_2 \le \kappa_{CW} \right\},$$

where $0 < \mu'_{CM} \le \mu_{CM}$, $0 < \mu'_{CW} \le \mu_{CW}$, $0 < \kappa'_{CM} \le \kappa_{CM}$, and $0 < \kappa'_{CW} \le \kappa_{CW}$. σ_{CW}, σ_{CM}, σ'_{CW}, and σ'_{CM} are control parameters that characterize the fluctuation levels of the outer and inner uncertainty sets, respectively. We take σ_{CM} and σ'_{CM} as examples to illustrate, similar to σ_{CW} and σ'_{CW} in our model. Control parameter σ'_{CM} serves as the boundary parameter for the box constraint $\|\zeta_{CM}\|_\infty \le \sigma'_{CM}$ in the inner perturbation set Z^1_{CM}, controlling the maximum fluctuation range of each perturbation component. As σ'_{CM} increases, the box constraint relaxes, enabling larger perturbations for each component and thus expanding the volume of the inner perturbation set Z^1_{CM}. Control parameter σ_{CM} serves as the scaling parameter for the normalization constraints $\|\zeta_{CM}/\sigma_{CM}\|_1 \le \mu_{CM}$ and $\|\zeta_{CM}/\sigma_{CM}\|_2 \le \kappa_{CM}$ in the outer perturbation set Z^2_{CM}, functioning to scale the perturbation vector ζ_{CM}. As σ_{CM} increases, the normalized values $\|\zeta_{CM}/\sigma_{CM}\|$ become smaller for the same ζ_{CM}, making the constraints easier to satisfy, which effectively enables ζ_{CM} to take larger values, thereby expanding the coverage of the outer perturbation set Z^2_{CM}. Therefore, larger values of $(\sigma_{CM}, \sigma'_{CM})$ lead to larger volumes of the inner and outer perturbation sets Z^1_{CM} and Z^2_{CM}, meaning the model enables a wider range of parameter fluctuations. Expanding uncertainty sets implies that the model must account for more extreme perturbation scenarios (e.g., sharp increases in transportation costs and sudden drops in demand). To ensure constraint feasibility under all possible scenarios, the robust feasible region contracts, forcing decision variables to take more conservative values. Consequently, larger values of $(\sigma_{CM}, \sigma'_{CM})$ result in higher model conservatism, reflected in increased total costs; conversely, smaller values reduce conservatism. In addition, we adopt the function $\phi(CM_w, CM_w^{'})$ to measure the distance between the uncertainty parameters $CM_w^{'} \in U_{CM}^1$ and $CM_w^{'} \in U_{CM}^2$. Similarly, we use the function $\phi(CW_{wr}, CW_{wr}^{'})$ to measure the distance between the uncertainty parameters $CW_{wr}^{'} \in U_{CW}^1$ and $CW_{wr} \in U_{CW}^2$. We represent the two distance functions as follows:

$$\phi(CM_w, CM_w^{'}) = \alpha(\|CM_w - CM_w^{'}\|_1),\ with\ \alpha(a) = \theta_{CM} a,\ a \geq 0,\ \theta_{CM} \geq 0,$$
$$\phi(CW_{wr}, CW_{wr}^{'}) = \alpha(\|CW_{wr} - CW_{wr}^{'}\|_1),\ with\ \alpha(a) = \theta_{CW} a,\ a \geq 0,\ \theta_{CW} \geq 0.$$

We take θ_{CM} as an example to illustrate, similar to θ_{CW} and θ_D in our model. Global sensitivity parameter θ_{CM} controls solution conservatism without altering the geometric structure of uncertainty sets. When $\theta_{CM} = 0$, our model reduces to the classical robust optimization model, where the outer uncertainty set U_{CM}^2 must be fully satisfied. As θ_{CM} increases, the model no longer requires full satisfaction of the outer uncertainty set U_{CM}^2; instead, it requires only that decisions satisfy the smaller inner uncertainty set U_{CM}^1, while exceeding U_{CM}^1 is also permitted as long as the decisions remain within the bounds of the outer set U_{CM}^2. Compared to the case of strictly satisfying the outer uncertainty set U_{CM}^2, this represents a relaxation of constraints, thereby reducing conservatism and enabling decision-makers to achieve a more flexible trade-off between economic efficiency and robustness, which is as tolerance to extreme fluctuations.

We treat θ_{CM} as a flexible parameter chosen by the decision-maker, rather than as a fixed value. By adjusting θ_{CM} within a reasonable range, decision-makers can balance solution conservatism and cost efficiency according to their risk preferences and the specific market environment. Specifically, θ_{CM} and θ_{CW} influence the allowable degree of constraint violation in constraints (4.1) and (4.2), respectively, thereby affecting the optimal values of the auxiliary variables F_1 and F_2, respectively, and, consequently, affecting the total objective value,

$$\begin{aligned}
\min \quad & F_1 + F_2 + OC + PC + IC \\
\text{s.t.} \quad & \sum_{w\in W}\sum_{t\in T} CM_w * x_w^t * DM_w - F_1 \leq \theta_{CM} \min_{CM_w^{'}\in U_{CM}^1} \|CM_w - CM_w^{'}\|_1, \\
& \qquad \forall CM_w \in U_{CM}^2, \qquad (4.1) \\
& \sum_{w\in W}\sum_{r\in R}\sum_{t\in T} CW_{wr} * y_{wr}^t * DW_{wr} - F_2 \leq \theta_{CW} \min_{CW_{wr}^{'}\in U_{CW}^1} \|CW_{wr} - CW_{wr}^{'}\|_1, \\
& \qquad \forall CW_{wr} \in U_{CW}^2. \qquad (4.2)
\end{aligned}$$

Since constraints (3.12) and (3.13) contain uncertain parameters CM_w and CW_{wr}, respectively, the GRC of constraints (3.12) and (3.13) can be reformulated as computationally tractable systems under perturbation sets Z_{CM}^1 and Z_{CW}^1, Z_{CM}^2 and Z_{CW}^2 and distance measure functions $\phi(CM_w, CM_w^{'})$ and $\phi(CW_{wr}, CW_{wr}^{'})$, which are summarized in Theorems 4.1-4.2.

Theorem 4.1. *Based on perturbation sets Z_{CM}^1 and Z_{CM}^2 and the distance measure function $\phi(CM_w, CM_w^{'})$, constraint (3.12) can be represented as the following equivalent computationally* *tractable system:*

$$\begin{cases} \sum_{w\in W}\sum_{t\in T} CM_w^0 * x_w^t * DM_w + \kappa'_{CM}\sqrt{(\sigma'_{CM}m_{CM})^2} + \mu'_{CM}|\sigma'_{CM}l_{CM}| \\ + \sigma'_{CM}|u_{CM}| + \mu_{CM}|\sigma_{CM}c_{CM}| + \kappa_{CM}\sqrt{(\sigma_{CM}d_{CM})^2} \le F_1, \\ m_{CM} + l_{CM} + u_{CM} = -\sum_{w\in W}\sum_{t\in T}\eta_w^t * \widetilde{CM}_w, \\ c_{CM} + d_{CM} = -\sum_{w\in W}\sum_{t\in T}(x_w^t * DM_w - \eta_w^t) * \widetilde{CM}_w, \\ \sum_{w\in W}\sum_{t\in T}|\eta_w^t| \le \theta_{CM}, \end{cases} \tag{4.3}$$

where m_{CM}, l_{CM}, u_{CM}, c_{CM}, d_{CM}, and η_w^t are auxiliary decision variables.

Theorem 4.2. *Based on perturbation sets Z_{CW}^1 and Z_{CW}^2 and the distance measure function $\phi(CW_{wr}, CW'_{wr})$, constraint (3.13) can be represented as the following equivalent computationally tractable system:*

$$\begin{cases} \sum_{w\in W}\sum_{r\in R}\sum_{t\in T} CW_{wr}^0 * y_{wr}^t * DW_{wr} + \kappa'_{CW}\sqrt{(\sigma'_{CW}m_{CW})^2} + \mu'_{CW}|\sigma'_{CW}l_{CW}| \\ + \sigma'_{CW}|u_{CW}| + \mu_{CW}|\sigma_{CW}c_{CW}| + \kappa_{CW}\sqrt{(\sigma_{CW}d_{CW})^2} \le F_2, \\ m_{CW} + l_{CW} + u_{CW} = -\sum_{w\in W}\sum_{r\in R}\sum_{t\in T}\eta_{wr}^t * \widetilde{CW}_{wr}, \\ c_{CW} + d_{CW} = -\sum_{w\in W}\sum_{r\in R}\sum_{t\in T}(y_{wr}^t * DW_{wr} - \eta_{wr}^t) * \widetilde{CW}_{wr}, \\ \sum_{w\in W}\sum_{r\in R}\sum_{t\in T}|\eta_{wr}^t| \le \theta_{CW}, \end{cases} \tag{4.4}$$

where m_{CW}, l_{CW}, u_{CW}, c_{CW}, d_{CW}, and η_{wr}^t are auxiliary decision variables.

Uncertain demand

Let $\overline{Z}_D$ and Z_D be the budget perturbation set, i.e., $\overline{Z}_D = \{\zeta_D \mid \|\frac{\zeta_D}{\sigma'_D}\|_1 \le \mu'_D\}$, $Z_D = \{\zeta_D \mid \|\frac{\zeta_D}{\sigma_D}\|_1 \le \mu_D\}$, with $0 < \mu'_D \le \mu_D$. The distance function $\phi(D_i^t, \overline{D}_i^t)$ measures the distance between the uncertainty parameters $\overline{D}_i^t \in \overline{U}_D$ and $D_i^t \in U_D$, and represent them as follows:

$$\phi(D_i^t, \overline{D}_i^t) = \alpha(\|D_i^t - \overline{D}_i^t\|_1), \ \textit{with } \alpha(a) = \theta_D a, \ a \ge 0, \ \theta_D \ge 0.$$

Since constraints (3.14) contain uncertain demand D_i^t, the GRC of constraints (3.14) can be reformulated as a computationally tractable system under perturbation sets $\overline{Z}_D$ and Z_D and the distance measure function $\phi(D_i^t, \overline{D}_i^t)$, which is summarized in Theorem 4.3.

Theorem 4.3. *Based on perturbation sets $\overline{Z}_D$ and Z_D and the distance measure function $\phi(D_i^t, \overline{D}_i^t)$, constraints* (3.14) *can be represented as the following equivalent computationally tractable system:*

$$\begin{cases} D_i^{t0} + \mu'_D\sigma'_D\widetilde{D}_i^t + \mu_D\sigma_D\widetilde{D}_i^t * \eta_D \le \sum_{r\in R} a_{ri}^t + \sum_{r\in R} s_{ri}^t, \ \forall\, i, t, \\ 0 \le \eta_D \le \theta_D, \end{cases} \tag{4.5}$$

where l_D, c_D, and η_D are auxiliary decision variables.

4.2. Reformulation of the single-level model via KKT conditions

Bilevel optimization models are generally NP-hard and cannot be solved directly by standard optimization solvers. A common approach to address this challenge is to transform the bilevel model into a single-level optimization problem by applying the Karush-Kuhn-Tucker (KKT) conditions to the lower-level problem [19]. When the lower-level problem is a convex optimization problem, the KKT conditions are necessary and sufficient for global optimality [41]. This means any feasible solution satisfying the KKT conditions is guaranteed to be an optimal solution to the lower-level problem. The key idea is that by incorporating the KKT conditions, namely primal feasibility, dual feasibility, and complementary slackness, as constraints in the upper-level problem, we ensure that any feasible solution to the single-level model corresponds to the optimize solutions of the lower-level model.

For the lower-level problem, the corresponding dual is given by:

$$\min \sum_{i\in I}\sum_{t\in T}\beta_i^t*(-D_i^{t0}-\mu'_D\sigma'_D\widetilde{D}_i^t)+\sum_{r\in R}\sum_{t\in T}\beta_r^t*RC_r+\beta\theta_D$$

$$+\sum_{r\in R}\sum_{i\in I}\sum_{t=2}^{4}\beta_{ri}^t*(-PA_r^{t-1}/P_r^{t-1}*\rho^{t-1}*\Delta t)$$

$$\text{s.t.} -\beta_i^t+\beta_r^t-\beta_{ri}^{t+1}\ge PA_r^t,\ \forall\ r,i,t=1, \tag{4.6a}$$

$$-\beta_i^t+\beta_r^t+\beta_{ri}^t-\beta_{ri}^{t+1}\ge PA_r^t,\ \forall\ r,i,t=2,3, \tag{4.6b}$$

$$-\beta_i^t+\beta_r^t+\beta_{ri}^t\ge PA_r^t,\ \forall\ r,i,t=4, \tag{4.6c}$$

$$-\beta_i^t+\beta_r^t\ge P_r^t,\ \forall\ r,i,t, \tag{4.6d}$$

$$\beta+\sum_{i\in I}\sum_{t\in T}\beta_i^t\mu_D\sigma_D\widetilde{D}_i^t\ge 0, \tag{4.6e}$$

$$\beta_i^t,\beta_r^t,\beta\ge 0,\ \forall\ r,i,t,\beta_{ri}^t\ge 0,\ \forall\ r,i,t\ge 2. \tag{4.6f}$$

Constraints (3.10) and (4.5) are associated with dual variables β_r^t, β_i^t, and β, respectively. The non-negative variables β_{ri}^t (for any $r,i,t\ge 2$) correspond to constraints (3.11).

Consequently, the KKT conditions for the lower-level problem are derived as follows,

$$\beta_r^t*(\sum_{i\in I}a_{ri}^t+\sum_{i\in I}s_{ri}^t-RC_r)=0,\ \forall\ r,t, \tag{4.7a}$$

$$\beta_{ri}^t*(a_{ri}^t-a_{ri}^{t-1}+PA_r^{t-1}/P_r^{t-1}*\rho^{t-1}*\Delta t)=0,\ \forall\ r,i,t\ge 2, \tag{4.7b}$$

$$\beta_i^t*(D_i^{t0}+\mu'_D\sigma'_D\widetilde{D}_i^t+\mu_D\sigma_D\widetilde{D}_i^t*\eta_D-\sum_{r\in R}a_{ri}^t-\sum_{r\in R}s_{ri}^t)=0,\ \forall\ i,t, \tag{4.7c}$$

$$\beta*(\eta_D-\theta_D)=0, \tag{4.7d}$$

$$a_{ri}^t*(-\beta_i^t+\beta_r^t-\beta_{ri}^{t+1}-PA_r^t)=0,\ \forall\ r,i,t=1, \tag{4.7e}$$

$$a_{ri}^t*(-\beta_i^t+\beta_r^t+\beta_{ri}^t-\beta_{ri}^{t+1}-PA_r^t)=0,\ \forall\ r,i,t=2,3, \tag{4.7f}$$

$$a_{ri}^t*(-\beta_i^t+\beta_r^t+\beta_{ri}^t-PA_r^t)=0,\ \forall\ r,i,t=4, \tag{4.7g}$$

$$s_{ri}^t*(-P_r^t-\beta_i^t+\beta_r^t)=0,\ \forall\ r,i,t, \tag{4.7h}$$

$$\eta_D*(\sum_{i\in I}\sum_{t\in T}\beta_i^t\mu_D\sigma_D\widetilde{D}_i^t+\beta)=0, \tag{4.7i}$$

$$\text{constraints}\quad (3.10)-(3.11),(4.5)-(4.6).$$

By introducing binary auxiliary variables $\gamma_r^t \in \{0,1\}^{R\times T}$, $\gamma_{ri}^{t0} \in \{0,1\}^{R\times I\times T}$, $\gamma_i^t \in \{0,1\}^{I\times T}$, $\gamma, \gamma_D \in \{0,1\}$, $\gamma_{ri}^{t1} \in \{0,1\}^{R\times I\times T}$, and $\gamma_{ri}^{t2} \in \{0,1\}^{R\times I\times T}$ and a sufficiently large positive number A, nonlinear constraints (4.7) are linearized as follows:

$$\beta_r^t \le A * \gamma_r^t,\ \forall\ r,t, \tag{4.8a}$$

$$\sum_{i\in I} a_{ri}^t + \sum_{i\in I} s_{ri}^t - RC_r \le A * (1-\gamma_r^t),\ \forall\ r,t, \tag{4.8b}$$

$$\beta_{ri}^t \le A * \gamma_{ri}^{t0},\ \forall\ r,i,t \ge 2, \tag{4.8c}$$

$$a_{ri}^t - a_{ri}^{t-1} + PA_r^{t-1}/P_r^{t-1} * \rho^{t-1} * \Delta t \le A * (1-\gamma_r^{t0}),\ \forall\ r,i,t \ge 2, \tag{4.8d}$$

$$\beta_i^t \le A * \gamma_i^t,\ \forall\ i,t, \tag{4.8e}$$

$$D_i^{t0} + \mu_D'\sigma_D'\widetilde{D}_i^t + \mu_D\sigma_D\widetilde{D}_i^t * \eta_D - \sum_{r\in R} a_{ri}^t - \sum_{r\in R} s_{ri}^t \le A * (1-\gamma_i^t),\ \forall\ i,t, \tag{4.8f}$$

$$\beta \le A * \gamma, \tag{4.8g}$$

$$\eta_D - \theta_D \le A * (1-\gamma), \tag{4.8h}$$

$$a_{ri}^t \le A * \gamma_{ri}^{t1},\ \forall\ r,i,t=1, \tag{4.8i}$$

$$-\beta_i^t + \beta_r^t - \beta_{ri}^{t+1} - PA_r^t \le A * (1-\gamma_{ri}^{t1}),\ \forall\ r,i,t=1, \tag{4.8j}$$

$$a_{ri}^t \le A * \gamma_{ri}^{t1},\ \forall\ r,i,t=2,3, \tag{4.8k}$$

$$-\beta_i^t + \beta_r^t + \beta_{ri}^t - \beta_{ri}^{t+1} - PA_r^t \le A * (1-\gamma_{ri}^{t1}),\ \forall\ r,i,t=2,3, \tag{4.8l}$$

$$a_{ri}^t \le A * \gamma_{ri}^{t1},\ \forall\ r,i,t=4, \tag{4.8m}$$

$$-\beta_i^t + \beta_r^t + \beta_{ri}^t - PA_r^t \le A * (1-\gamma_{ri}^{t1}),\ \forall\ r,i,t=4, \tag{4.8n}$$

$$s_{ri}^t \le A * \gamma_{ri}^{t2},\ \forall\ r,i,t, \tag{4.8o}$$

$$-P_r^t - \beta_i^t + \beta_r^t \le A * (1-\gamma_{ri}^{t2}),\ \forall\ r,i,t, \tag{4.8p}$$

$$\eta_D \le A * \gamma_D, \tag{4.8q}$$

$$\sum_{i\in I}\sum_{t\in T} \beta_i^t \mu_D\sigma_D\widetilde{D}_i^t + \beta \le A * (1-\gamma_D), \tag{4.8r}$$

$$\gamma_i^t \in \{0,1\}^{I\times T}, \gamma_r^t \in \{0,1\}^{R\times T}, \gamma, \gamma_D \in \{0,1\}, \gamma_{ri}^{t0}, \gamma_{ri}^{t1}, \gamma_{ri}^{t2} \in \{0,1\}^{R\times I\times T}. \tag{4.8s}$$

Through the above procedure, our original bilevel model can be equivalently reformulated as a single-level MILP, where the upper-level cost minimization objective is preserved, and the lower-level revenue maximization requirement is fully enforced through the KKT conditions. The single-level GRC-FSCN model has been reformulated as the following MILP model:

$$\begin{aligned} \min \quad & F_1 + F_2 + OC + PC + IC \\ \text{s.t.} \quad & \text{constraints} \quad (3.1)-(3.8),(3.10)-(3.11),(4.3)-(4.6),(4.8). \end{aligned} \tag{4.9}$$

Based on the structural characteristic of the above MILP model, we design a tailored BD algorithm to model our FSCN problem in Section 5.

5. Design of the tailored BD algorithm

Despite the capability of optimization solvers to solve deterministic MILP problems, there remains a challenge for large-scale instances. To enhance computational efficiency, a tailored BD method is designed. Introduced by Benders [42], BD algorithm effectively divides the complex MILP model into a master problem (MP) and a subproblem (SP), which facilitates the search for optimal solutions more efficiently.

In our FSCN problem, by reasonably selecting and adjusting the algorithm parameters, we can design an efficient, accurate, and decomposed solution method. In this section, we elaborate on the principles and implementation process of our BD solution algorithm to provide practical solutions for the optimization of the supply chain network.

5.1. Division of MP and SP

In general, when designing the BD algorithm, complex integer or discrete decision variables are typically placed in the MP, while continuous decision variables are placed in the SP. This practice helps separate decision variables, making the problem structure clearer and the solution process more efficient [43]. However, it should be noted that placing binary variables in the MP and continuous variables in the SP is not a universally applicable approach for our MILP model. Because it leads to the lower level problem being too complicated. To overcome this difficulty, we treat the continuous and binary variables from the lower-level model as variables for the MP, while those from the upper-level model serve as variables for the SP. This partitioning approach avoids the issues of model intractability that arise with traditional partitioning methods. As a result, our MP and the SP are constructed as follows:

The subproblem

$$
\begin{aligned}
\min \quad & F_1 + F_2 + PC + IC \\
\text{s.t.} \quad & \sum_{i \in I} \overline{a}^t_{ri} + \sum_{i \in I} \overline{s}^t_{ri} \leq \sum_{w \in W} y^t_{wr}, \ \forall\ r, t. \\
& x^t_w \leq WC_w * \overline{o}_w\ , \forall\ w, t, \\
& h^t_w \leq WC_w * \overline{o}_w, \forall\ w, t, \\
& \text{constraints} \quad (3.1), (3.3), (3.5), (3.7)-(3.8), (4.3)-(4.4).
\end{aligned}
$$

The decision variables $\overline{o}_w$, $\overline{a}^t_{ri}$, and $\overline{s}^t_{ri}$ are set as fixed values in the SP.

The master problem

$$
\begin{aligned}
\min \quad & \sum_{w \in W} o_w * \alpha_w + Q(o_w) \\
\text{s.t.} \quad & o_w \in \{0, 1\}^W, \\
& \text{constraints} \quad (3.2), (3.4), (3.6), (3.10)-(3.11), (4.5)-(4.6), (4.8).
\end{aligned}
$$

In the iterative process of our BD algorithm, we ensure that feasibility and optimality of the solution are paramount. For this purpose, the MP incorporates feasibility and optimality cuts derived from the outcomes of its associated SP. Based on whether the feasible domain of the subproblem is empty, indicating the feasibility of SP, we include the following two types of cuts in MP.

1. **Feasibility cut**: In instances where SP is deemed infeasible, feasibility cuts (5.1) are integrated into MP to guarantee the feasibility of the explored solution within MP. The structure of feasibility cut

is as follows:

$$\begin{aligned} &WC_w * \bar{o}_w * d_1 + WC_w * \bar{o}_w * d_2 + MC * d_3 + RC_r * d_4 \\ &+\left(-\sum_{i\in I}\overline{a}_{ri}^t - \sum_{i\in I}\overline{s}_{ri}^t\right) * d_5 + \theta_{CM} * d_6 + \theta_{CW} * d_7 \geq 0, \end{aligned} \tag{5.1}$$

where $d_1, d_2 \in \mathbb{R}^W$, $d_4 \in \mathbb{R}^R$, $d_3, d_6, d_7 \in \mathbb{R}$, and $d_5 \in \mathbb{R}^{R\times T}$ are introduced auxiliary decision variables.

2. **Optimality cut**: In the case that SP generates a feasible and optimal solution, the optimality cuts (5.2) are incorporated into MP. These cuts enhance the lower bound of MP by leveraging the optimal solution derived from SP. The structure of the optimality cut is defined as follows:

$$\begin{aligned} &WC_w * \bar{o}_w * q_1 + WC_w * \bar{o}_w * q_2 + MC * q_3 + RC_r * q_4 \\ &+\left(-\sum_{i\in I}\overline{a}_{ri}^t - \sum_{i\in I}\overline{s}_{ri}^t\right) * q_5 + \theta_{CM} * q_6 + \theta_{CW} * q_7 \geq q, \end{aligned} \tag{5.2}$$

where q is the optimal value in SP, q_1, $q_2 \in \mathbb{R}^W$, $q_4 \in \mathbb{R}^R$, $q_3, q_6, q_7 \in \mathbb{R}$, and $q_5 \in \mathbb{R}^{R\times T}$ are auxiliary decision variables.

5.2. *Pseudocode of the tailored BD algorithm*

Upon solving the formulated SP, Benders optimality and feasibility cuts are derived from the obtained optimal solution. These cuts are then incorporated into the MP for the subsequent iteration. The refined MP with the new Benders cuts undergoes another round of solution. This iterative process repeats until the SP is unable to identify any additional cuts, indicating convergence toward an optimal or near-optimal solution. The pseudocode for this algorithm is as follows:

Algorithm 1: BD algorithm for model (4.9).

Input : Set the iteration counter $k = 1$ and a tolerance value ε, and set decision variables o_w, a_{ri}^t, s_{ri}^t as initial values $\overline{o}_w$, $\overline{a}_{ri}^t$, $\overline{s}_{ri}^t$

$LB \leftarrow -\infty$;
$UB \leftarrow \infty$;
while $UB - LB \geq \varepsilon$ **do**
    Solve SP;
    if *Unbounded* **then**
        Generation of Benders feasibility cuts (5.1);
    else
        Generation of Benders optimality cuts (5.2);
    end
    Solve reformulated MP;
end

6. Method implementation in a real case

In this section, the computational experiments conducted to validate the proposed bilevel GR-FSCN model and tailored BD algorithm are described. We consider ZARA as the case study to investigate the characteristics of its supply chain network. All numerical experiments are performed using Python in Visual Studio Code, leveraging Gurobi 10.0.0 solver callbacks. The solution environment is a personal computer equipped with an Intel(R) Core(TM) i5-8400 processor (2.80GHz), 16 GB of RAM, and running Windows 11.

6.1. Problem description and data source

ZARA, established in 1975, is positioned as Inditex's primary retail clothing subsidiary. It holds the top position in Spain in terms of sales and brand popularity, based on data from its official platform (https://www.zara.com/). ZARA is a pioneer in the fashion industry, specializing in fashion products. Our proposed method is applied to the problem of designing a supply chain network for ZARA fashion products. We consider the transportation structure and volume design from Spain to China as the design of the product transportation structure and volumes. The sole manufacturer (M) is in La Colonia, Spain. Three potential distribution centers are considered: W1 in Beijing, and W2 and W3 in Shanghai. We consider five retailers in Beijing (R1), Shanghai (R2), Chengdu (R3), Shenzhen (R4), and Guangzhou (R5). There are ten market locations in Beijing (I1, I2), Shanghai (I3, I4), Chengdu (I5, I6), Shenzhen (I7, I8), and Guangzhou (I9, I10). Derived from the online mapping platform AMap (https://www.amap.com), the distances between facilities are compiled in Tables 2–3. Model parameters are presented separately in Table 4. Furthermore, demand projections for the ten markets across the four periods are calculated using official census data (https://www.stats.gov.cn/).

Table 2. Locations of the specific facilities.

Facility	Symbol	Location Coordinate
Manufacture	M	(-8.411594, 43.362337)
Distribution center	W1	(116.459146, 39.922294)
	W2	(121.461347, 31.23342)
	W3	(121.485656, 31.241672)
Retailer	R1	(113.32942, 23.13835)
	R2	(121.461347, 31.23342)
	R3	(116.417292, 39.920835)
	R4	(104.090765, 30.658172)
	R5	(113.943168, 22.522995)

Table 3. The transportation distances among facilities (1×10^3km).

Facility	W1	W2	W3
M	9.252	10.3	10.301
R1	1.884	1.204	1.207
R2	1.064	0.001	0.002
R3	0.003	1.066	1.066
R4	1.521	1.659	16.611
R5	1.943	1.219	1.221

Table 4. Parameter configurations used in this case study.

Parameter	Value	Parameter	Value
T	4	V	2
MC	2000	WC_w	[2,400, 2,700]
RC_r	[1,500, 1,700]	VC_v	[800, 1,000]
CM_w^v	[0.006, 0.009] CNY	CW_wr^v	[0.0001, 0.0005] CNY
P_i^t	[12,000, 13,000] CNY	PA_r^t	[11,000, 12,000] CNY
α_w	2,000 CNY	β_r	3,000 CNY
λ_i^t	[0.01, 0.03]		

Uncertain unit transportation costs are affected by factors, such as weather, national policies, and pandemic situations. Transportation costs are an important factor in the design of an FSCN, and they directly affect a company's profitability. An efficient supply chain not only ensures operational efficiency but also improves competitiveness. The variations in the uncertainty parameters CM_w and CW_{wr} are fixed at 10% of their nominal values. The fluctuation ranges of the unit transportation costs are $[CM_w^0 - 0.1 * CM_w^0, CM_w^0 + 0.1 * CM_w^0]$ and $[CW_{wr}^0 - 0.1 * CW_{wr}^0, CW_{wr}^0 + 0.1 * CW_{wr}^0]$.

Customer demands are also highly uncertain. ZARA's fashion products are designed and produced according to customer orders. Therefore, to obtain high profits, decision-makers must identify customer needs. Positioned as a fast-fashion leader, ZARA focuses on serving metropolitan consumers aged 20 to 35. This group exhibits a keen awareness of style trends while typically operating within a modest budget. To curate its offerings, the brand draws on diverse sources: It tracks haute couture presentations, consolidates broad-based trend reports, and, crucially, incorporates direct feedback from shoppers at its retail outlets (https://www.zara.com/). Rapid changes in fashion trends make it difficult for customers to form consistent preferences regarding fashion products. Additionally, store location, employee attitudes, and other factors influence customer demand. A symmetric ±10% variation is applied to the baseline demand D_i^{t0}, resulting in a range of $[0.9D_i^{t0}, 1.1D_i^{t0}]$.

6.2. *Assessing the performance of the GR-FSCN model*

We present the computational results obtained by solving the single-level GRC model using our tailored BD algorithm. Then, we analyze the parameter sensitivity and discuss the influence of parameter values on the bilevel GR-FSCN model results. This analysis helps identify the parameters that have a major influence on the decision-making results to better support the adjustment and management of the FSCN.

6.2.1. Computational results

We compute the results of the single-level GRC model via the tailored BD algorithm by setting the global sensitivity parameters $\theta_D = \theta_T = 10$. The uncertain control parameters are set as $\mu'_D = \mu'_D = 5$, $\sigma_D = 0.3$, $\sigma'_D = 0.1$, $\mu'_{CM} = \mu'_{CW} = 10$, $\mu_{CM} = \mu_{CW} = 10$, $\sigma_{CM} = \sigma_{CW} = 0.6$, $\sigma'_{CM} = \sigma'_{CW} = 0.5$, $\kappa_{CM} = \kappa_{CW} = 10$, and $\kappa'_{CM} = \kappa'_{CW} = 10$. The optimal objective value of the globalized robust FSCN model is 49,915,235.77 CNY. The results suggest that decision-makers should open distribution center 1 and retailers 2-5 for operation in the situation. The shipments from the manufacturer to distribution center 1 are as follows: 1,035 batches in the first period, 2,000 batches in the second period, 411 batches in the third period, and 129 batches in the fourth period. The inventory levels at distribution center 1 are 1,035 batches in the first period, 1,955 batches in the second period, 328 batches in the third period, and 129 batches in the fourth period. Subsequently, the shipments from distribution center 1 to retailers 2-5 are shown in Figure 2. The customer allocation strategy with a discount price for R3 is presented in Figure 3. The allocation of customer with the sale price for R3 is shown in Figure 4.

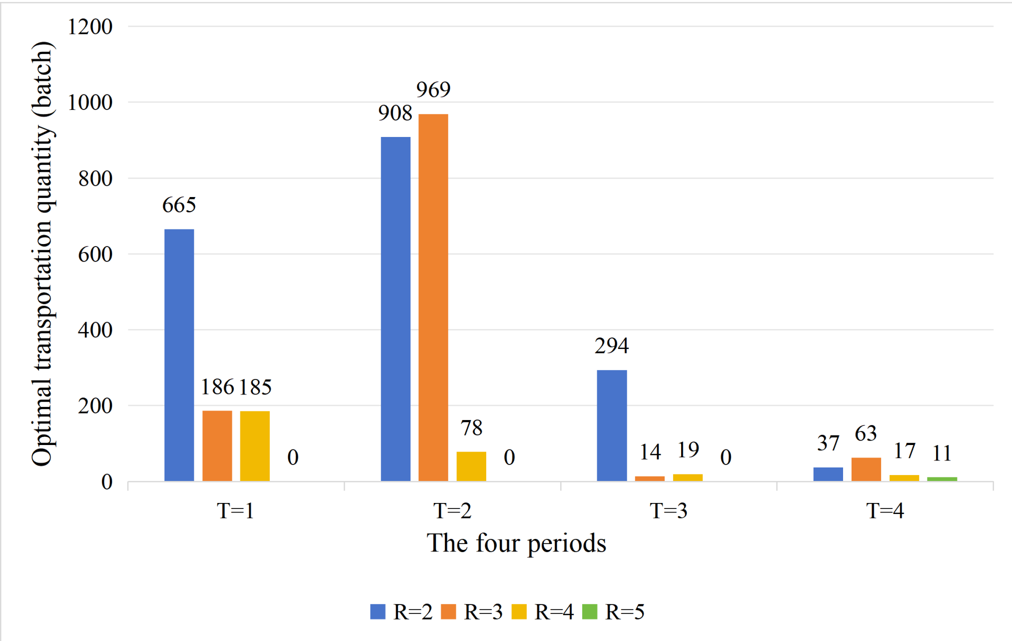

Figure 2. Optimal transportation decisions from a DC to retailers during four periods.

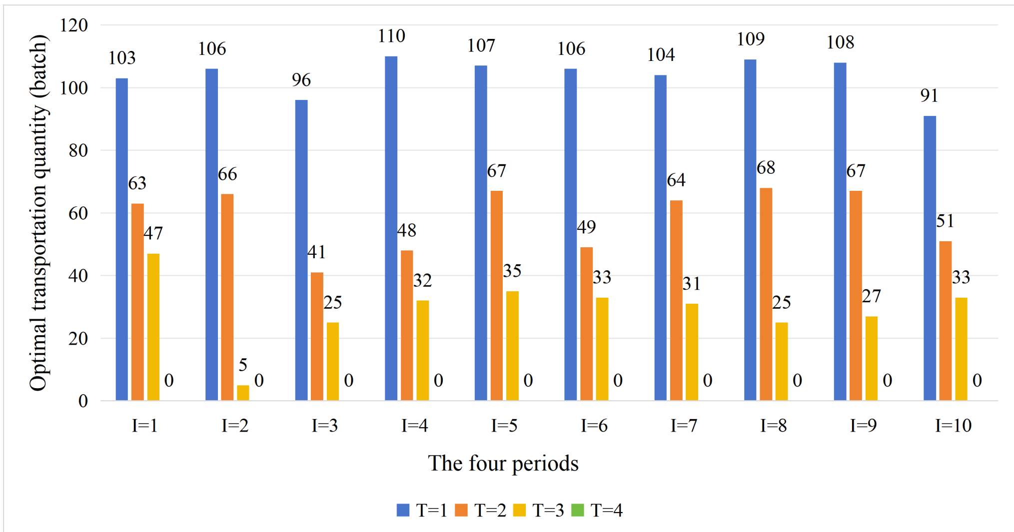

Figure 3. Optimal transportation decisions from retailers to customers with a discounted price.

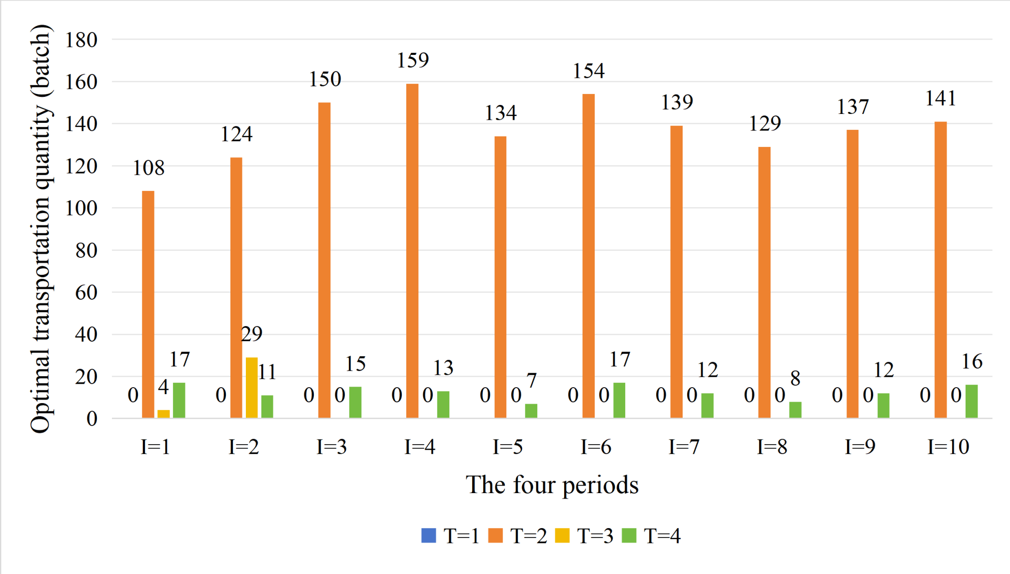

Figure 4. Optimal transportation decisions from retailers to customers with sale price.

According to the results, in the supply chain with 1 manufacturer, 3 distribution centers, and 5 retailers, prioritizing one-to-one transportation is more cost-effective. This indicates that transportation costs can be effectively controlled while meeting customer demands. We also observe that the volume of transportation from the distribution centers to retailers is lower than from the manufacturer to distribution centers. This suggests the occurrence of product loss during transportation. Moreover, this finding reminds decision-makers to pay attention to the transportation process and take measures to reduce or avoid product losses. The computational results show that discount sales quantities are zero for most retailers in most periods. This does not indicate that we disregard discount strategies. Rather, it reflects that under the optimal decision of our model, most products are sold without discounts. This result aligns closely with ZARA's fast-fashion strategy, as ZARA continuously introduces new products with rapid iteration and no replenishment after sell-out [44]. Under this strategy, most products sell out quickly at regular prices without needing discounts to maintain profit, while a small proportion of production is cleared through the discount mechanism [38]. Based on ZARA's operational strategy, this further illustrates that even when firms possess discount capabilities, the optimal pricing strategy for most products may not include a discount under a rapid turnover, which is a scarcity driven business model.

6.2.2. Sensitivity analysis

To examine the influence of the uncertainty-control parameters that govern the size of uncertainty sets, we perform a comprehensive sensitivity analysis across their values using the same data. The effect of the global sensitivity parameters are also investigated. Moreover, the evaluation results of the discount price PA_r^t and initial inventory level are displayed.

Evaluation of uncertain control parameters

We set the values of control parameters $\sigma_D = \sigma_{CM} = \sigma_{CW} = 0.4, 0.5, 0.6, 0.7, 0.8$, $\sigma'_D = \sigma'_{CM} = \sigma'_{CW} = 0.3, 0.4, 0.5, 0.6, 0.7$. By comparing the results across values of the uncertain control parameters $(\sigma_{CM}, \sigma'_{CM})$, the influence of these parameters on the FSCN design can be explored. As $(\sigma_{CM}, \sigma'_{CM})$ increase from $(0.3, 0.4)$ to $(0.7, 0.8)$, the uncertainty sets gradually expand, requiring the model to enable a wider range of perturbation, including more extreme perturbation scenarios. This expansion forces decision variables to adopt more conservative values, leading to an increase in the total cost. For decision-makers, lower $(\sigma_{CM}, \sigma'_{CM})$ values are suitable when the operating environment is stable, and historical data suggest limited parameter fluctuations; higher values are appropriate when facing highly volatile markets or when the enterprise has a low risk tolerance and must prepare for worst-case scenarios. As shown in Figure 5, the objective value exhibits a monotonically increasing trend. Specifically, the objective value rises from 49,914,531.17 CNY at $(0.3, 0.4)$ to 49,916,226.91 CNY at $(0.7, 0.8)$. This pattern reveals that expanding the uncertainty set forces our model to enable a wider range of parameter fluctuations, including more extreme scenarios, leading to more conservative optimal solutions and, consequently, higher total costs. For decision-makers, the choice of control parameters $(\sigma_{CM}, \sigma'_{CM})$ represents a fundamental trade-off between conservative and cost efficiency. Smaller $(\sigma_{CM}, \sigma'_{CM})$ values are suitable for stable operating environments with limited parameter fluctuations based on historical data, where the model achieves better cost performance. Larger $(\sigma_{CM}, \sigma'_{CM})$ values are appropriate for highly volatile markets or when enterprises have low risk tolerance and must prepare for the worst-case scenario, requiring decision-makers to accept the associated cost premium as insurance against extreme events. In practice, decision-makers should calibrate $(\sigma_{CM}, \sigma'_{CM})$ based on the attitude toward risk to determine reasonable value ranges.

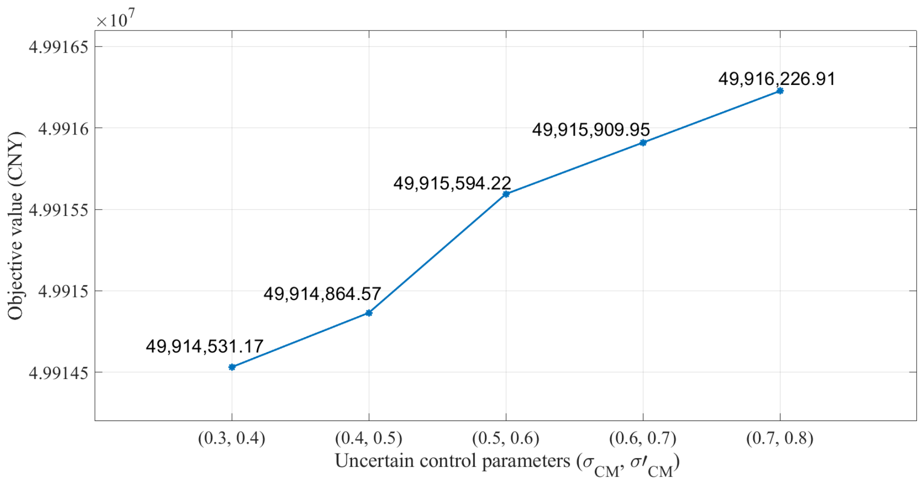

Figure 5. Change in objective value of the bilevel GR-FSCN model w.r.t. uncertain control parameters.

Evaluation of global sensitivity parameters

Global sensitivity parameter θ_{CM} governs the trade-off between the economic efficiency and robustness by controlling the allowable degree of constraint violation. This structure enables decision-makers to flexibly adjust conservatism levels according to their risk preferences, financial conditions, and market forecasts. Low θ_{CM} values enforce strict robustness requirements, limiting constraint violations and ensuring strong protection against extreme fluctuations. This results in higher total costs but provides reliable performance guarantees, making low θ_{CM} suitable for risk-averse decision-makers in highly uncertain environments exposed to supply chain disruptions or geopolitical risks. As θ_{CM} increases, our model progressively relaxes feasibility requirements, permitting greater constraint violations in exchange for reduced cost buffers. This yields lower total costs but diminished protection against worst-case scenarios, making high θ_{CM} appropriate for cost-prioritized strategies in stable markets where decision-makers prioritize efficiency over extreme-case resilience. By selecting θ_{CM} along this spectrum, decision-makers can calibrate the model to reflect their attitude toward uncertainty and strategic objectives.

The influence of global sensitivity parameters is shown in Figure 6, which illustrates the impact of global sensitivity parameters on the objective value. We set $\theta_{CM} = \theta_{CW} = \theta_D$ in our model. When $\theta_{CM} = 0$, the model reduces to a classical robust optimization model, yielding the highest cost of 49,915,689,56 CNY. Increasing the global sensitivity parameters to 5, 10, 15, and 20 leads to changes in total cost. This pattern demonstrates the value of globalized robustness, enabling controlled constraint violations (i.e., $\theta_{CM} > 0$) to mitigate the cost of over-conservatism compared to classical robust optimization ($\theta_{CM} = 0$). However, once θ_{CM} exceeds a threshold, the objective value of further increasing θ_{CM} diminishes, as the model has already captured the most cost-effective trade-off between feasibility and efficiency. Parameter θ_{CM} serves as a control parameter for decision-makers to balance robustness and cost efficiency. Lower θ_{CM} values correspond to higher conservatism, which is suitable for risk-averse decision-makers or highly uncertain environments where feasibility guarantees under extreme scenarios justify accepting higher costs as risk insurance. Moderate θ_{CM} values correspond to moderate conservatism, which is applicable to operations fluctuations, where decision-makers accept cost increases in exchange for enhanced risk. Higher θ_{CM} values correspond to lower conservatism, which is appropriate for cost-sensitive environments with stable supply chains, where decision-makers prioritize operational efficiency and enable controlled violations to achieve better cost performance.

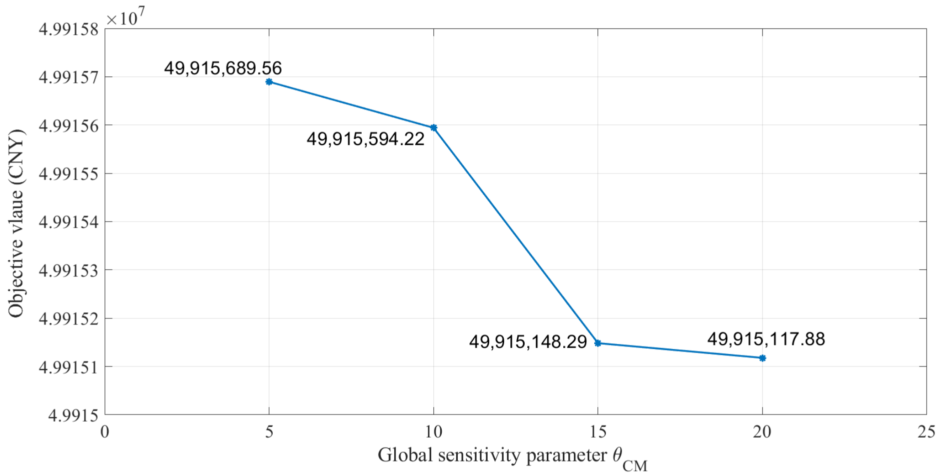

Figure 6. Change in objective value of the bilevel GR-FSCN model w.r.t. global sensitivity parameters.

Evaluation of unit discount price

Discount prices can attract customers to purchase products, thereby increasing sales volume and facilitating economies of scale. However, excessively high or low discount prices may lead to a decrease in the seller's profits and reduce their willingness to promote and sell products, resulting in financial losses. The unit discount price PA_r^t can significantly impact the performance and profitability of the FSCN. It should be carefully determined based on a comprehensive evaluation of multiple factors to ensure the optimal balance between attracting customers, maximizing sales volume, and maintaining profitability. We set the parameters in different ranges PA_r^t = $(8,500, 10,000)$, $(10,000, 11,500)$, $(11,500, 13,000)$, $(13,000, 14,500)$, and $(14,500, 16,000)$and show the influence of discount price on the objective function value for different ranges in Figure 7.

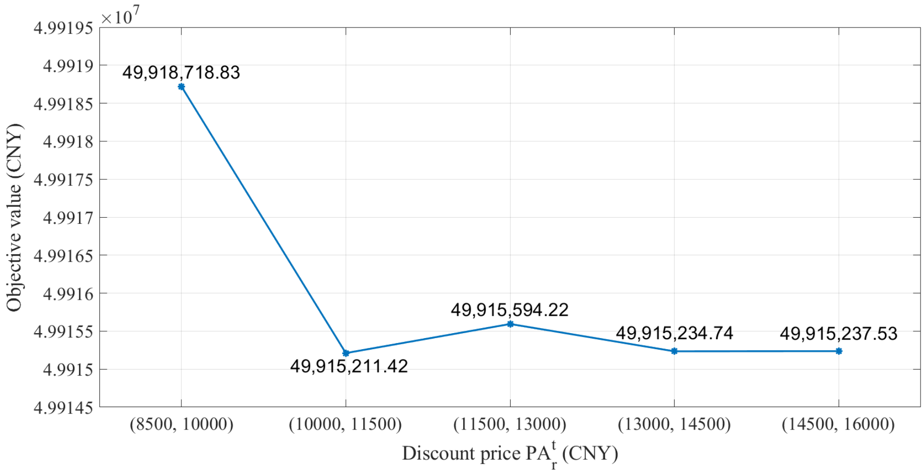

Figure 7. Change in the objective value of the bilevel GR-FSCN model w.r.t. discount price.

In the case of ZARA, there are risks involved in the transportation process, such as product loss. Therefore, when designing an FSCN, we need to strike a balance between economic benefits and risks. By analyzing the results under different parameter settings, we can optimize the design of the FSCN. Moreover, strategy choices and parameter settings have an impact on supply chain network design, and our model can improve the efficiency and economic benefits. These findings contribute to improvements in supply chain network design practices. This analysis enables us to make informed decisions and recommendations to improve the robustness and efficiency of FSCN operations.

When the discount price is at a low level, the objective function value is at its highest. This indicates that although excessive price cuts may stimulate sales volume, the profit loss is too substantial, thereby reducing overall profitability. For fashion products, frequent deep discounting damages brand image and cultivates a "wait-for-discount" purchasing habit among consumers, which is detrimental to long-term corporate profitability. When the discount price is raised to a reasonable range, profit losses decrease significantly while sales volume is not materially negatively affected. At this point, the enterprise achieves a better balance between profit and sales volume. Managers can flexibly adjust prices within the 10,000-16,000 range based on market demand, inventory levels, or transportation risks without significant concern for major impacts on overall profitability. Thus, fashion enterprises should adhere to the principle of "moderate discounting" to attract consumers while safeguarding the overall economic performance of the supply chain.

Evaluation of the initial inventory level

To evaluate the impact of the model to the initial inventory level, we conduct sensitivity analysis. Specifically, we consider experimental scenarios where the initial inventory is set to 5%, 10%, 20%, and 30% of total demand volume, and compare the resulting optimal decisions and objective values against the benchmark scenario with zero initial inventory. The results indicate that total cost exhibits a decreasing trend as initial inventory increases, which is primarily attributed to the mitigation of stockout risks during peak demand periods, which is shown in Figure 8.

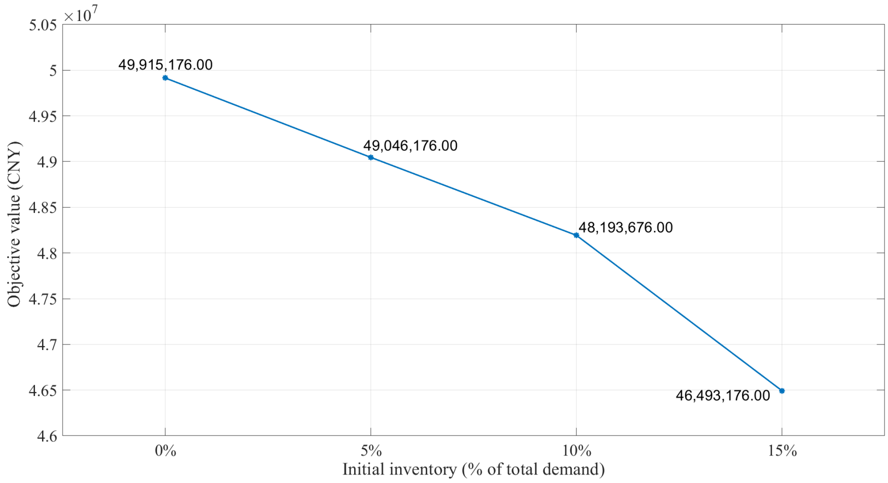

Figure 8. Change in objective value of the bilevel GR-FSCN model w.r.t. initial inventory.

The initial inventory level significantly influences the transportation network structure [45, 46]. We take the scenarios with initial inventories of 5% and 10% of demand as examples to illustrate this point. Under the 5% initial inventory level, regarding the transportation structure, retailer R1 does not supply discounted products to market I6 in the third period. However, under the 10% level, such a supply does occur. In the second period, with a 5% initial inventory, retailer R2 supplies products to market I8 but not to market I10. Conversely, under the 10% initial inventory, the pattern is opposite. The transportation quantities also differ between the two initial inventory levels. In the second period, under the 5% initial inventory scenario, retailer R3 supplies 105 batches of product to market I8, whereas under the 10% scenario, it supplies 129 batches. Additionally, with a 5% initial inventory, retailer R3 supplies 141 batches to market I10, compared to 117 batches under the 10% initial inventory scenario.

6.2.3. Computational efficiency of the BD algorithm

In this section, we discuss key performance metrics such as computation time, number of iterations, and convergence behavior, as follows: (i) A comparison of computation time between the BD algorithm and direct MILP solving; (ii) a convergence plot showing the evolution of upper and lower bounds across iterations; and (iii) experiments on instances of varying sizes to demonstrate scalability.

We supplement the paper with computational experiments on instances of varying scales. As shown in Table 5, we test 7 instances ranging from $3 \times 3 \times 2$ (3 distribution centers, 3 retailers, 2 markets) to $20 \times 5 \times 40$ (20 distribution centers, 5 retailers, 40 markets). The largest instance contains 8,796 variables, effectively validating the algorithm's performance on large-scale problems. Our results demonstrate that, as problem size increases, the time advantage of the Benders decomposition algorithm becomes more pronounced. In the largest instance ($20 \times 5 \times 40$), Gurobi requires 1.54 seconds, while the BD algorithm takes 0.78 seconds, achieving a 49% time improvement. Moreover, variables reduce from 8,796 to 6,355 (a 26% reduction). These results demonstrate the effectiveness and scalability of the proposed method in handling large-scale problems. Table 5 shows a computational time comparison between the BD algorithm and direct MILP solving using Gurobi across instances of varying sizes, demonstrating the efficiency gains of our approach. Furthermore, we conduct experiments on problem instances with increasing dimensions (e.g., varying numbers of distribution centers, retailers, and markets), and the results are summarized in Table 5, showing that the BD algorithm maintains computational advantages as problem size grows. These additions provide a comprehensive and transparent evaluation of the algorithm's performance. Figure 9 presents a convergence plot illustrating the evolution of upper and lower bounds over iterations of $|W| \times |R| \times |I|$=$3 \times 5 \times 20$, confirming the steady convergence behavior of the algorithm.

Table 5. Comparison between the Gurobi and BD algorithm

$\|W\| \times \|R\| \times \|I\|$	Gurobi			BD			Improvement
	Obj	Gap	Time (s)	Obj	Gap	Time (s)	Time (%)
$3 \times 3 \times 2$	9,906,721	0.00%	0.12	9,906,721	0.00%	0.06	50%
$3 \times 3 \times 5$	24,896,445	0.00%	0.19	24,896,449	0.00%	0.12	37%
$3 \times 3 \times 10$	49,915,164	0.00%	0.27	49,915,340	0.00%	0.12	56%
$3 \times 5 \times 10$	49,915,176	0.00%	0.91	49,915,236	0.00%	0.31	66%
$3 \times 5 \times 20$	99,828,539	0.00%	0.96	99,828,782	0.00%	0.36	63%
$20 \times 5 \times 20$	99,828,579	0.00%	1.23	99,828,670	0.00%	0.40	67%
$20 \times 5 \times 40$	199,657,445	0.00%	1.54	199,656,933	0.00%	0.78	49%

*Note: Obj: Objective value; Gap: percentage of relative optimality gap; Time (s): Running time in seconds; Improvement percentage is calculated as (Gurobi value - BD value) / Gurobi value × 100%.

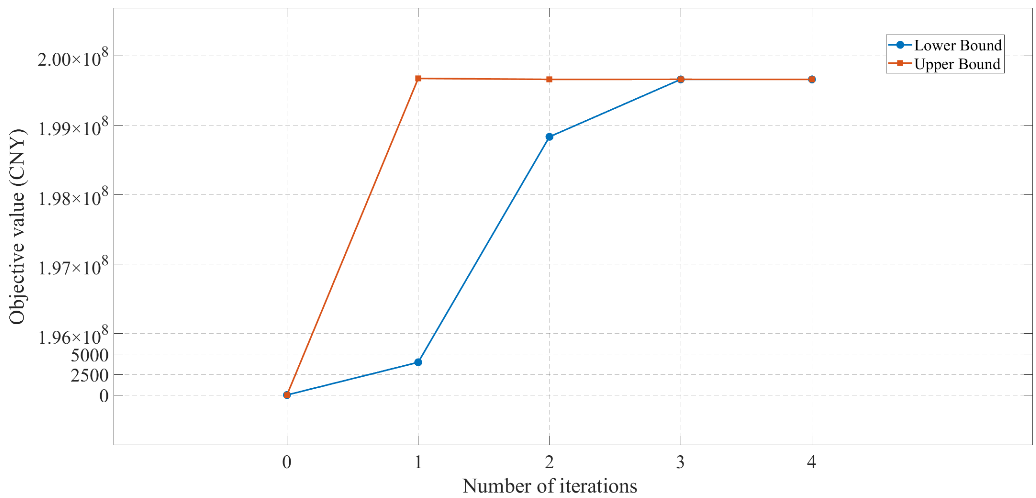

Figure 9. Convergence of the Benders decomposition algorithm ($|W| \times |R| \times |I|$=$3 \times 5 \times 20$).

7. Management insights and conclusions

In this work, we propose a new bilevel GR-FSCN model to address cost control and resource constraint challenges. By adopting the GRO method, our model evaluates the impact on enterprises' economic performance under fluctuations in transportation costs and market demand. In terms of model solution techniques, an efficient BD algorithm is customized for the MILP. As for the innovative application of the GR-FSCN model, we illustrate the superiority of our proposed model through a case study of the ZARA company, providing effective strategic recommendations for enterprises to achieve sustained growth and competitive advantages in an uncertain environment.

Based on the computational results, we observe the following insights and practical implications:

i) Uncertainty is inevitable in FSCN design, and decision-makers should carefully consider uncertainties such as demand fluctuations and transportation cost uncertainties and employ the GRO method to handle these uncertainties (see subsection 4.1). By constructing inner and outer uncertainty sets, the performance of the supply chain network can be evaluated under various uncertainty levels, supporting appropriate decision-making strategies (see subsection 6.2). Our GRO method, applied to handle uncertain parameters, can effectively reduce decision risks and provide more robust decision solutions. This is crucial for ensuring the stable operations of FSCN.
ii) Cooperation and competition exist between the upper and lower levels in a bilevel supply chain network. Thus, decision-makers should actively foster collaboration and cooperation between the upper and lower levels to achieve mutual benefits (see subsection 4.2). During the decision-making process, the mutual influence and information sharing between the upper and lower levels should be taken into account to enhance the flexibility and responsiveness of the supply chain network. Our proposed bilevel FSCN model can assist decision-makers in formulating the optimal strategies, taking into account the hierarchical relationship between upper- and lower-level decision-makers to achieve better decision coordination and enhance the efficiency and profitability of the supply chain network.
iii) The design of an FSCN aims to maximize overall benefits. Thus, decision-makers should strike a balance between the efficiency and effectiveness of the FSCN. By optimizing the decision strategies, the total cost of the FSCN can be reduced, and the profits of all stakeholders can be maximized (see subsection 6.2.1). When making decisions, factors such as production capacity constraints, transportation management, and discount strategies should be considered holistically.

Our proposed robust bilevel FSCN model and solution approach have significant practical implications and insights for supply chain management in the fashion industry. In future studies, researchers can further expand and improve our models and methods to better address the requirements of real-world applications and provide further insights into FSCN optimization, such as by:

i) Incorporating product returns into the model, given their prevalence in the fashion industry and their potential impacts on inventory dynamics and cost structures.
ii) Investigating alternative strategies uncertainty and time-related decisions, with particular attention to scenarios where demand is treated as a decision variable.
iii) Examining disruption events, such as equipment failures and unforeseen logistical blockages.

Author contributions

Shanshan Gao: Conceptualization, methodology, writing-original draft preparation, software, data curation, validation; Meiyu Liu: Conceptualization, methodology, writing-review & editing, software, validation; Yankui Liu: Conceptualization, methodology, writing-review & editing, validation, supervision, funding acquisition.

Use of Generative-AI tools declaration

In the preparation of this work, the authors used Generative AI tools such as ChatGPT to assist in improving the clarity of the language. All AI-assisted content was carefully reviewed and revised by the authors, who take full responsibility for the final version of the manuscript.

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Conflict of interest

All authors declare no conflicts of interest in this paper.

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