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Research article

A Regularized Mixed Integer Linear Programming framework with penalties for integrated workforce, subcontracting and production optimization

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Abstract: In this study, we introduce a Regularized Mixed-Integer Linear Programming (MILP) framework aimed at optimizing workforce allocation, production scheduling, subcontracting, and energy management within the footwear manufacturing domain. The proposed model incorporates two regularization-based penalty components: An L_1 (lasso-type) penalty, which constrains abrupt fluctuations in workforce levels and job sequencing across planning horizons, and an L_2 (ridge-type) penalty, which penalizes deviations from target capacity and energy utilization levels. This hybrid formulation improves model robustness, mitigates overfitting, and ensures a balanced trade-off between operational efficiency and cost performance. Empirical evaluation conducted over a six-period planning horizon using representative production data demonstrated the model's effectiveness. With regularization coefficients calibrated at $\lambda_1 = 0.1$ and $\lambda_2 = 0.05$, the MILP solver produced an optimal integer solution, yielding a total operational cost of ₹738,792.7 while sustaining workforce stability across all periods and completely avoiding additional hiring or layoffs.

Keywords: flow shop scheduling; batch process; regularization Technique; mixed integer programming; subcontract; production optimization
Mathematics Subject Classification: 90B30, 90B35, 90B50, 90C11

1. Introduction

Optimizing production lines is a central requirement for achieving operational excellence in contemporary manufacturing systems, particularly in upper shoe production, where tightly interconnected processes such as cutting, stitching, assembling, and finishing must be coordinated effectively to minimize cycle time, production cost, and energy consumption. In such multi-stage environments, Sequence-Dependent Flow Shop Scheduling (SDFSS) plays a critical role, since setup and processing times are influenced by the immediately preceding job due to tooling adjustments, product model changes, and material handling requirements [1]. These dependencies substantially increase computational complexity and frequently generate bottlenecks, machine idleness, and unbalanced workloads, thereby limiting the effectiveness of conventional scheduling strategies in dynamic shop-floor conditions [2].

Upper shoe manufacturing exhibits pronounced sequence dependency, particularly during machine preparation and tool changeovers, where improper sequencing can lead to extended setup times and cascading production delays [3]. Consequently, the development of intelligent optimization frameworks that jointly enhance throughput, resource utilization, and energy efficiency has become an essential research and industrial priority. While classical heuristic rules provide acceptable solutions for small-scale scheduling problems, they often struggle to capture the nonlinear interactions and stochastic variability that characterize real manufacturing systems [4].

Advances in regularization-based optimization provide promising opportunities to improve the robustness and interpretability of scheduling models [5]. Regularization methods, widely used in statistical learning, introduce penalty terms to constrain model complexity and mitigate overfitting [6]. Ridge regression (L_2) stabilizes parameter estimates by penalizing squared coefficients [7], Lasso regression (L_1) encourages sparsity and variable selection [8], and Elastic Net combines both penalties to balance stability and interpretability [9]. Embedding such penalty mechanisms within scheduling formulations enables smoother job transitions, reduced variability in makespan, and improved adaptability under uncertain production conditions [10,11].

Motivated by these developments, we propose a Regularized Sequence-Dependent Flow Shop Scheduling framework implemented in R by integrating mixed-integer linear programming through the *ompr* package [12] with regression based regularization using *glmnet* [13]. Regularization penalties are incorporated directly into the scheduling objective function to discourage abrupt sequence changes and to promote balanced workload distribution across workstations. This integrated formulation improves scheduling stability, enhances energy efficiency, and supports computational scalability in upper shoe manufacturing systems.

The major contributions of this work are fourfold. First, a Regularized SDFSS model that integrates Ridge, Lasso, and Elastic Net penalties within a mixed-integer linear programming framework is introduced to manage complex sequence-dependent operations. Second, an R-based optimization platform is developed to perform scheduling and regularization simultaneously, thereby improving production stability and reducing sequencing variability. Third, the proposed framework is empirically validated using real footwear manufacturing data, demonstrating substantial reductions in makespan and setup times, along with improved throughput compared with conventional heuristic approaches. Finally, we establish a novel methodological link between machine learning regularization principles and factory level scheduling, with particular emphasis on minimizing total production cost, including subcontracting decisions.

By embedding regularization terms directly into the scheduling objective, the proposed approach dynamically balances residual errors, model complexity, and sequence interactions, resulting in an interpretable, energy aware, and adaptive scheduling solution suitable for data-driven smart manufacturing environments.

2. Literature Review

2.1. Sequence-Dependent Flow Shop Scheduling

The Flow Shop Scheduling Problem (FSSP) has been extensively investigated as a fundamental model for sequencing multiple jobs across machines arranged in an identical processing order, with the objective of minimizing performance indicators such as makespan, flow time, or total tardiness [14]. When setup times vary according to job sequence, the problem becomes a Sequence-Dependent FSSP and is recognized as NP-hard for systems involving more than two machines [15]. Early contributions, including Johnson's two- and three machine scheduling rules [16], provided the theoretical foundations for flow shop sequencing.

Building on these classical results, subsequent surveys by Ruiz and Maroto [17] systematically analyzed setup time driven scheduling problems and evaluated permutation-based heuristics for multi-stage production systems. Eventually the researchers have shifted toward hybrid and metaheuristic techniques to address the growing complexity of industrial scheduling problems. In the context of footwear manufacturing, genetic algorithm-based approaches have been shown to significantly reduce makespan and improve throughput [18]. Chan et al. [19] proposed dominant gene genetic algorithms for footwear production scheduling, achieving notable reductions in idle time and enhanced utilization of shop-floor resources.

To better capture operational uncertainty and dynamic interactions, simulation-based optimization has gained increasing attention. Kianfar et al. [20] developed a stochastic simulation framework to represent operational dependencies in shoe manufacturing, leading to improved throughput consistency. Tantanawat et al. [21] examined nested-loop workflows to enhance the efficiency of upper assembly lines, with findings directly relevant to footwear manufacturing due to similar task interdependencies. Lucas et al. [22] employed system dynamics to study workforce absenteeism in footwear production, highlighting the importance of predictive scheduling mechanisms for managing labor related disruptions.

Research efforts have expanded toward multi-objective and sustainability oriented scheduling frameworks. Yigit et al. [23] introduced hybrid optimization methods for setup-dependent flow shop scheduling, while Yigit and Basilio [24] formulated asymmetric cost models for inventory optimization. These contributions underline the increasing demand for advanced scheduling models capable of handling nonlinear interactions, multiple performance criteria, and sustainability considerations in complex manufacturing environments.

However, despite these advances, researchers largely rely on heuristic, metaheuristic, or simulation based techniques and rarely incorporate statistical regularization principles within formal scheduling formulations. Moreover, limited attention has been given to integrating machine learning inspired regularization with mixed-integer programming for sequence-dependent scheduling in footwear manufacturing. This research gap motivates us, who seek to bridge scheduling optimization and regularization based learning to develop a robust, interpretable, and energy efficient scheduling framework for upper shoe production systems.

The shoe manufacturing facilities considered in this study employs a streamlined production layout consisting of two parallel assembly lines, where the workers are assigned specialized roles in stitching and assembling leather footwear, enabling task specific expertise at each production stage. This structured configuration facilitates smooth material flow, minimizes unnecessary handling, and enhances overall workflow efficiency across the production line (Figure 1).

Figure 1. Production line in shop floor.

The experimental process diagram, Figure 2, below illustrates each stage: data acquisition → pre-processing → model formulation → regularization integration → scheduling solution → performance evaluation.

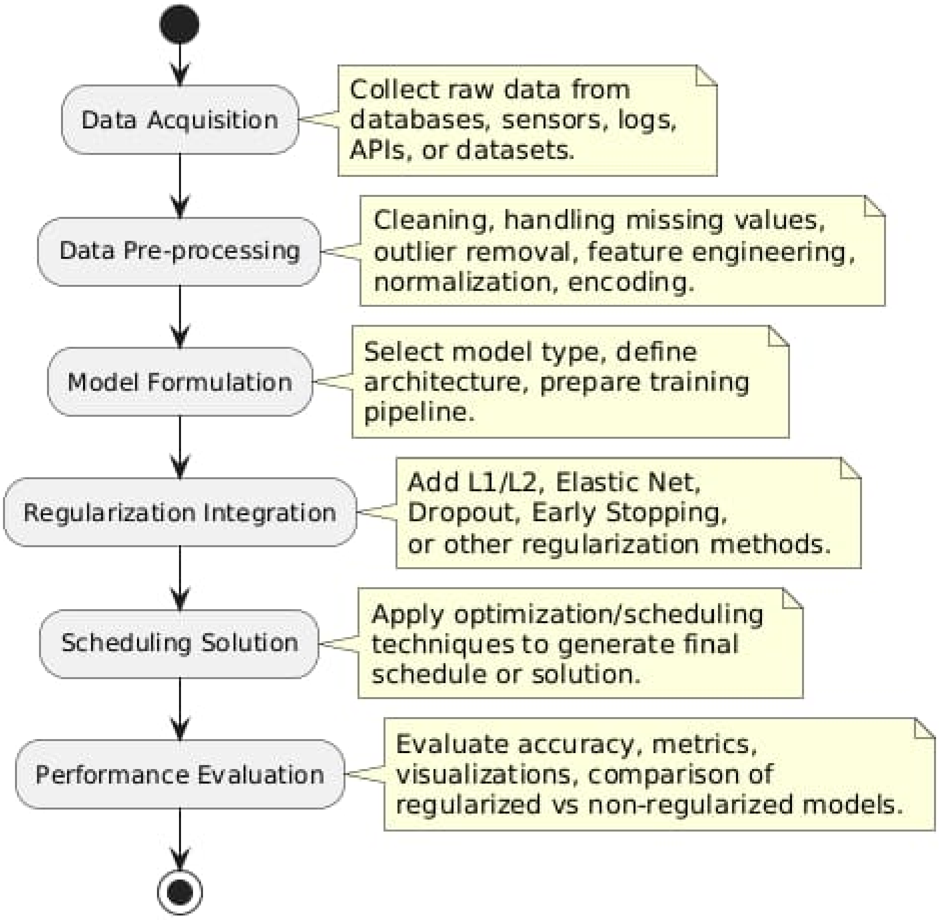

Figure 2. Experimental workflow of the proposed Regularized MILP framework.

2.2. Regularization and Optimization in Manufacturing

Regularization improves model stability by constraining large parameter estimates and reducing the risk of overfitting. The most widely adopted techniques include Ridge regression based on the L_2 penalty, Lasso regression based on the L_1 penalty, and the Elastic Net approach, which integrates both penalties to achieve a balance between sparsity and robustness. In manufacturing optimization, the incorporation of regularization mechanisms into mathematical scheduling formulations has been reported to enhance solution robustness and computational performance. Algorithmic implementations in R, particularly for generalized linear modeling and mixed-integer programming, further facilitate the development of scalable and adaptive scheduling frameworks. In addition, kernel-based and hybrid regularization strategies have extended these applications to nonlinear and dynamic industrial systems, demonstrating their effectiveness in handling complex production environments.

Several researchers have validated the predictive capability of regularization methods in industrial contexts. Wu and Pan [25], Zhang and Chen [26], and Wu et al. [27] employed the Ridge regression to model physical processes, energy consumption, and system health indicators, reporting reliable predictive accuracy. Extensions based on Lasso and grouped penalty formulations have also shown substantial impact. Li et al. [28] applied Lasso regression to predict neurological deterioration in clinical applications, while Kokhahi and Li [29] proposed a Group Lasso framework for cyber physical manufacturing systems.

Related regularized learning approaches have been adopted in smart and additive manufacturing environments. Lee et al. [30] integrated Elastic Net with sparse principal component analysis for semiconductor metrology, Liu et al. [31] developed decentralized optimization strategies for federated smart manufacturing, and Trovato et al. [32] and Singh et al. [33] investigated hybrid regression models for process optimization in additive manufacturing. In footwear and textile applications, Vasagam et al. [34] employed regularized artificial neural networks to forecast export performance, while Yu et al. [35] improved punching-point detection accuracy using convolutional neural network-based prediction systems. More recent big-data-driven studies have further demonstrated the synergy between machine learning and production optimization [36–38], particularly within smart and sustainable manufacturing paradigms [39,40].

3. Methodology

3.1. Regularization Techniques in R for Scheduling Optimization

To formulate a linear programming model for minimizing makespan and energy consumption in sequence-dependent flow shop scheduling, regularization concepts are incorporated directly into the optimization framework. The scheduling problem is first defined mathematically by specifying decision variables, processing times, sequence-dependent setup times, and machine constraints. Regularization penalty terms are then embedded within the objective function to control variability in job sequencing and to promote stable and balanced production schedules.

Symbol and Notation Table 1

Table 1. Nomenclature of Mathematical Equations.

Symbol	Type	Definition / Unit / Notes
J	Set	Set of all jobs, (J = {1, 2,……,n}).
n	Scalar (positive integer)	Number of jobs,
M	Scalar (positive integer)	Number of machines in the flow shop.
m	Index (machine)	Machine index, (m = 1, 2,,…,M).
i, j, k	Indices (job/position)	Job or position indices that run over set
x_{ij}	Binary decision variable	x_{ij}=1 if job (i) is assigned to position (p) in the schedule; otherwise x_{ij}= 0 (This enforces a permutation assignment: each job has exactly one position and each position is filled by exactly one job.)
C_i	Continuous decision variable	Completion time of job (i) on the last machine (time units. e.g. Minutes or hours). When equations refer to completion at a schedule index, map index / job consistently in text.
T_m	Objective scalar	Makespan objective defined as (T_m = {i,j} C_i . Minimizing T_m yields the shortest overall completion time.

Continued on next page

Symbol	Type	Definition / Unit / Notes
p_{ij}	Parameter (time)	Processing time required to process job (i) on machine (m) (time units). When a single-machine processing time is used, denote it explicitly (e.g., P_i or ($P_{i,j}$))
$S_{i,j}$	Parameter (time)	Sequence-dependent setup (or changeover) time incurred when job (i) immediately precedes job (j) on the same machine (time units).
$E_{i,m}$	Parameter (energy)	Energy consumption when processing job (i) on machine (m) (energy units, e.g., kWh). The total energy objective sums these terms across jobs and machines.
λ_1 , λ_2	Regularization weight (scalar)	Non-negative penalty coefficient that scales the regularization term (R(x)). When two penalties are used, denote them as λ_1 and λ_2 and state their roles (e.g., λ_1 for scheduling smoothness, λ_2 for subcontracting cost).
R(x)	Regularization function	Penalty function applied to the schedule to discourage undesirable patterns (unit depends on formulation). Two common forms used in this work:

Table 1 lists all indices, sets, decision variables, parameters, and functions used in the mathematical model equations.

3.2. Minimization of Makespan

$$\min T_m + \lambda \, \mathrm{R(x)} \tag{1}$$

where, T_m= Max C_j ensures all jobs are completed within the shortest possible time.

3.3. Minimization of energy consumption

$$\min \sum_{\mathrm{i=1}}^{\mathrm{n}} \sum_{\mathrm{m=1}}^{\mathrm{M}} \mathrm{E_{im}} + \lambda \mathrm{R(x)} \tag{2}$$

- Here, E_{im} represents the energy consumption for job i on machine m.
- R(x) is a regularization term to penalize excessive deviations in scheduling.

3.4. Constraints

a) Regularization terms can take different forms, namely L_1 and L_2, where L_1would penalize abrupt changes in the production line.

$$\mathrm{R(X)} = \sum_i \sum_j \left| x_{ij} - x_{i(j+1)} \right| \tag{3}$$

and L_2 would make the smooth schedule transitions.

$$\mathrm{R(X)} = \sum_i \sum_j \left(x_{ij} - x_{i(j+1)} \right)^2 \tag{4}$$

b) Flow shop job precedence constraints

$$C_j \geq C_i + P_{ij} + S_{ij}\, x_{ij} \quad \forall\, i,j \in J,\ i \neq j \tag{5}$$

where P_{ij} is the processing time for job i on machine j.

c) Job Sequencing Constraint

$$\textstyle\sum_{i=1}^{n} x_{ij} = 1,\ \forall j \in J \tag{6}$$

$$\textstyle\sum_{j=1}^{n} x_{ij} = 1,\ \forall i \in J \quad , \tag{7}$$

ensuring each job is assigned a unique position in the schedule.

d) Non-Overlapping Constraint

$$C_j \geq C_i + P_i\ ,\ \ if\, x_{ij} = 1 \quad , \tag{8}$$

which ensures that a job starts only after the preceding job is completed.

Regularization, a widely adopted concept in machine learning and optimization, plays a critical role in improving model robustness by discouraging overly complex or unstable solutions. In the context of sequence dependent flow shop scheduling (Table 2), regularization can be strategically incorporated to reduce excessive machine changeovers, improve the accuracy of job completion time estimation, and promote smoother transitions between successive job sequences. Different regularization techniques provide complementary advantages. Lasso regression based on the L_1 penalty facilitates model simplification by suppressing insignificant decision variables, Ridge regression based on the L_2 penalty stabilizes scheduling performance by limiting extreme coefficient fluctuations, and the Elastic Net approach combines both penalties to achieve a balance between variable selection and model stability.

Within the R computing environment, these concepts can be effectively implemented using a combination of optimization and statistical learning packages. The *ompr* package enables the formulation of mathematical programming models for complex scheduling problems, *gurobi* provides high-performance solvers for mixed-integer programming, and *lpSolve* supports the efficient solution of linear programming formulations. For the integration of regularization mechanisms, the *glmnet* package is widely employed, as it offers native implementations of Lasso, Ridge, and Elastic Net methods. In a practical R-based framework, the scheduling problem is typically formulated as a mixed-integer linear programming model, into which regularization penalty terms are embedded within the objective function to improve scheduling stability, solution quality, and computational efficiency.

Table 2. Mathematical model of shop floor time study in footwear manufacturing.

S. No.	CATG	MP	SKILL	T1	AT	P	AL	SMS	PS	PPH
1	INPUT	1	INPUT	63	1.05	90	1.06	1.12	67	54
2	MARKING	2	MARKER	121	2.02	90	1.06	2.13	64	56
3	QTR CLOTH A/T	2	SKIVING	121	2.01	90	1.06	2.13	64	56
4	LOGO ATT	2	Amazon Skiving	121	2.02	90	1.06	2.13	64	56
5	CLOTH FUSING	3	Up skiving	153	2.55	90	1.06	2.71	54	67
6	SKIVING	1	Saddle	74	1.23	90	1.06	1.31	79	46
7	ROCKPORT LOGO A/T	1	Fusing	74	1.23	90	1.06	1.31	79	46
8	EDGE BURNING & COLOURING	1	burning	64	1.07	90	1.06	1.13	68	53
9	LINING FOLLOW	1	Input	64	1.07	90	1.06	1.13	68	53
10	VAMP MARKING	1	Quarter	44	0.73	90	1.06	0.78	47	77
11	VAMP ZIG ZAG	1	Input line	64	1.07	90	1.06	1.13	68	53
12	HEEL GRIP S/T	1	PBSN	60	1	90	1.06	1.06	64	56
13	VAMP A/T	1	ATTACHER	68	1.13	90	1.06	1.2	72	50
14	EYELET S/T	1	ATTACHER	64	1.07	90	1.06	1.13	68	53
15	LABLE S/T	1	PBSN	74	1.23	90	1.06	1.31	79	46
16	LABLR PRESSING	1	HELPER	71	1.18	90	1.06	1.26	75	48
17	INPUT	1	CHECKING	50	0.83	90	1.06	0.89	53	68
18	LOGO ATT	1	ATTACHER	66	1.1	90	1.06	1.17	70	51
19	TNG LOGO SN STITCH	1	ST	70	1.17	90	1.06	1.24	74	48
20	TNG LOOP STITCH	1	ST	67	1.12	90	1.06	1.19	71	51
21	COUNTER LOGO STITCH	2	ST	100	1.67	90	1.06	1.77	53	68
22	QTR & COLLER PASTE	1	HELPER	73	1.22	90	1.06	1.29	78	46
23	QTR & COLLER PRESSING	2	HELPER	100	1.67	90	1.06	1.77	53	68
24	QTR & COLLER SN STITCH	1	ST	54	0.9	90	1.06	0.96	57	63
25	APRON UP + ST	1	ST	76	1.27	90	1.06	1.35	81	45
26	VAMP UP	1	ST	71	1.18	90	1.06	1.26	75	48
27	APRON + UPPER	1	ZIG ZAG	62	1.03	90	1.06	1.1	66	55
28	APRON + SADDLE	1	ATTACHER	74	1.23	90	1.06	1.31	79	46
29	COLLAR & TNG S/T	1	PBSN	54	0.9	90	1.06	0.96	57	63
30	VAMP LNG	1	PBSN	54	0.9	90	1.06	0.96	57	63
31	BOTTOM ST	1	HELPER	54	0.9	90	1.06	0.96	57	63
31	TNG LNG	1	HELPER	54	0.9	90	1.06	0.96	57	63
32	FINAL INSP	1	HELPER	56	0.93	90	1.06	0.99	59	61

Nomenclature of this Time study, Table 2

- ✓ PPH (Production Per Hour) - Number of units produced per hour (units/hour) calculated as output divided by active production hours.
- ✓ MP (Manpower) - Number of operators or workers assigned to a workstation or production line (count) recorded at the start of each shift.

- ✓ T1 (Cycle Time in seconds) -Time taken to complete one full cycle of the operation measured in seconds (s) using stopwatch time-study averaged across sampled cycles.
- ✓ AT (Actual Cycle Time in minutes) - Converted cycle time in minutes (min) T1 divided by 60.
- ✓ AL (Allowance) - Time allowance expressed as a percentage of standard time (%) measured via time study and standard allowance tables.
- ✓ PS (Process Time) - Net operating time required to perform the task, excluding allowances (minutes/unit) derived from time study data.
- ✓ P (Performance of the worker) - Performance factor (%) relative to standard performance obtained by comparing actual time with standard time.
- ✓ SMS (Standard Minutes Value) - Standard minutes per task (min/unit) after allowances and performance factors are applied calculated as PS ÷ (P/100).

Our purpose of this study is to determine whether linear regression may help improve the overall performance of the evolutionary induced model and guide the interpretability of production projections. We tackle the issue by expanding the Regularisation evolutionary model, which is available in R. Using a data adaptive grid of penalty parameters, we apply the Ridge, lasso and elastic-net regression within this temporal study, utilizing the well-known tuning parameters. To expedite calculation, the technique makes use of a training set and active set iteration. This regularisation strategy, which reduces the root mean squared error to make a prediction, helps control the total complexity in the production line.

The proposed Regularization assisted MILP framework improves prediction and scheduling performance for sequence dependent Upper shoe manufacturing; yet, several constraints remain. We rely on data from a single, stable production line, so the model behaviour under fluctuating environments, small batch orders, or highly varied Upper shoe styles is not assessed. Because the regularization models depend on historical time study features, their accuracy may decline when new materials or processes introduce unfamiliar patterns. The MILP component also becomes computationally demanding as operations, machines, and setup dependencies increase, limiting scalability. In addition, real time disruptions such as sudden stoppages or material delays are not included, and operator learning or fatigue effects are not explicitly modelled.

The shop floor problem is ill posed due to the susceptibility of the solution to measurement error, yet the first condition is essential for addressing it. According to the second requirement, the solution should alter in tandem with any little adjustment made to the production run. If conditions 1 or 2 are not satisfied, there is an ill posed problem. The ill posed problem is transformed into a well posed one using the regularization procedure. In cases of instability, the regularization procedure is applied. One approach is to impose a penalty (error) for the stability of the problem. The findings suffer inaccuracy.

(1) Linear Regression in Time Study

```
summary(lm)
## Call:
## lm(formula =. outcome ~ ., data = dat)
## Residuals:
##    Min     1Q Median     3Q    Max
## -4.555 -2.952 -1.823  3.435 12.693
## Coefficients:
```

```
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 54.01250    3.02395  17.862 1.40e-14 ***
## MP          31.15940    4.77185   6.530 1.44e-06 ***
## T1          -0.47421    0.06049  -7.839 8.26e-08 ***
## ---
## Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ‘ 1
##
## Residual standard error: 4.743 on 22 degrees of freedom
## Multiple R-squared:  0.7373, Adjusted R-squared:  0.7134
## F-statistic: 30.87 on 2 and 22 DF, p-value: 4.11e-07
```

3.5. RIDGE IN TIME STUDY

Results 1

```
##    lambda    RMSE      Rsquared   MAE
##   0.000100  6.538540  0.9439220  4.883760
##   0.250075  6.282229  0.9439569  4.797878
##   0.500050  5.720257  0.9458089  4.615742
##   0.750025  5.631299  0.9463767  4.633441
##   1.000000  5.702405  0.9245410  4.753398
## Tuning parameter ‘alpha’ was held constant at a value of 0
## RMSE was used to select the optimal model using the smallest value.
## The final values used for the model were alpha = 0 and lambda = 0.750025.
```

This shows how strong the model regularization is. To help avoid overfitting, a larger lambda places a greater penalty on the magnitude of the coefficients (Appendix 2). This represents the above model’s average prediction error. The square root of the average of the squared discrepancies between the expected and actual values is what it stands for. Better model performance is shown by lower RMSE values. It estimates the percentage of the dependent variable variance that can be predicted from the independent variables and is referred to as the coefficient of determination. Greater R squared values (nearer 1) suggest that the model explains a greater percentage of the variation. The mean absolute difference between the expected and actual values is represented by this. This gives an indication of the typical size of the mistakes in the forecasts. Moreover, less MAE indicates higher model performance, like RMSE.

Of all the lambdas provided, the Ridge Model Result 1 with a lambda of 0.500050 has the lowest RMSE score (5.720257), indicating that it has the fewest average prediction error. The greatest R-squared value is for lambda = 0.750025 (0.9463767), suggesting that a greater percentage of the variance in the dependent variable (Appendix 1) can be explained by this model. With lambda = 0.500050 (4.615742), the MAE is lowest, indicating that, on average, the model’s predictions are closest to the actual values. Moreover, take the production goals into consideration when choosing the ideal model and pay attention to RMSE or MAE if forecast accuracy is the main priority. R squared should be given priority if one wants a model that can explain greater variation. To achieve effective generalization, regularization strength (lambda) should be carefully selected depending on the trade-off between fitting the training data well and preventing overfitting.

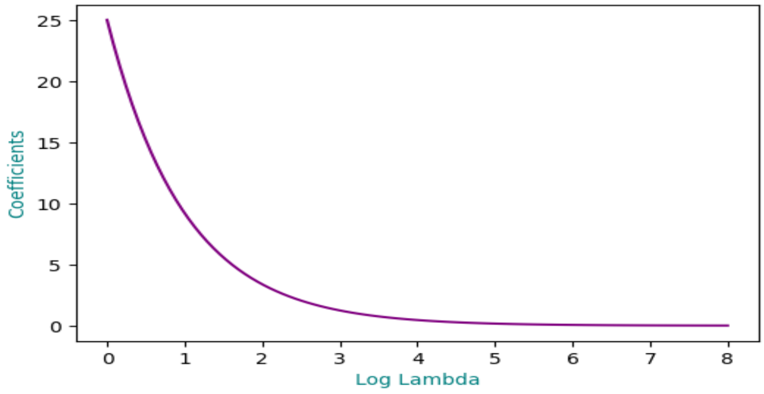

Figure 3. Lambda for Ridge.

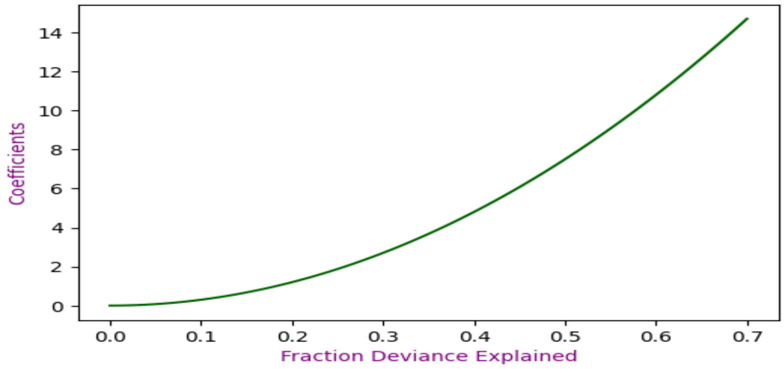

Figure 4. Deviance for Ridge.

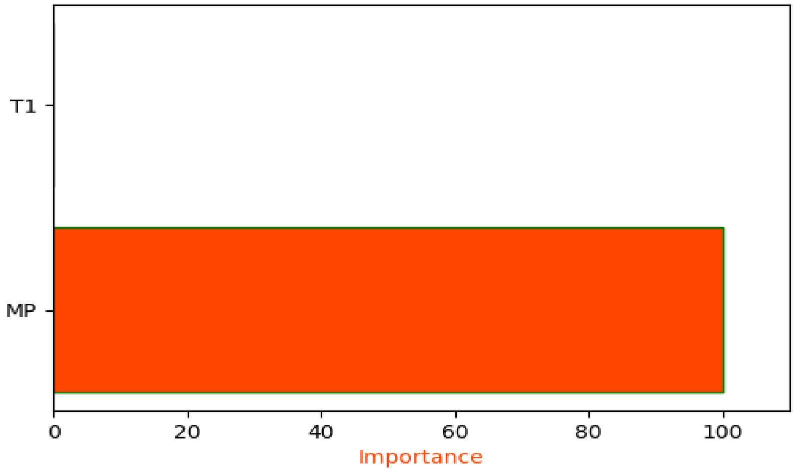

Figure 5. Cycle Time and Manpower for the Ridge model.

The figures provide a visual summary of how the Ridge regression model performs under regularization strengths. In Figure 3 the model performance metrics such as RMSE and R² across varying lambda values, illustrating the balance between model complexity and prediction error. Figure 4 seems to depict how the regression coefficients shrink as lambda increases, highlighting the effect of regularization on variable influence. Figure 5 presents the cross-validation results or final model fit, reinforcing that a lambda value of 0.75 offers the best performance. Collectively, these visuals demonstrate how Ridge regression enhances model stability by reducing overfitting.

3.6. LASSO IN TIME STUDY

Results 2

```
##   lambda    RMSE      Rsquared   MAE
##   0.000100 7.285337  0.9107285  4.984667
##   0.250075  6.579550  0.9326310  4.748512
##   0.500050  6.114768  0.9529303  4.668763
##   0.750025  5.969826  0.9199346  4.832852
##   1.000000  6.202412  0.7612166  5.301045
## Tuning parameter "alpha" was held constant at a value of 1.
## RMSE is used to select the optimal model using the smallest value.
## The final values used for the model are alpha = 1 and lambda = 0.750025.
```

The model's average prediction error is shown by the RMSE (**Result 2**) values. In this instance, RMSE typically drops as lambda increases, apart from the final row, where it increases. This indicates that a moderate level of regularization yields the lowest RMSE, indicating improved prediction accuracy (lambda = 0.500050). R-squared values show how much of the variance the model can account for. Once more, lambda = 0.500050 has the greatest R squared (0.9529303), meaning that a significant amount of the variance in the dependent variable can be explained by this model. The average absolute difference between the expected and actual values is measured by the MAE. Lambda = 0.500050 yields the lowest MAE (4.668763), like RMSE, indicating improved prediction accuracy for the target variable.

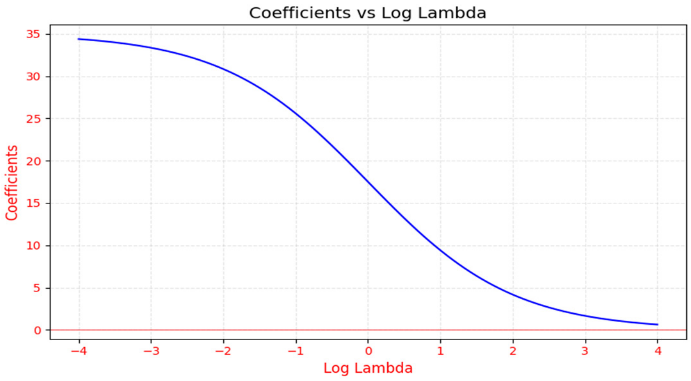

Figure 6. Lasso Lambda.

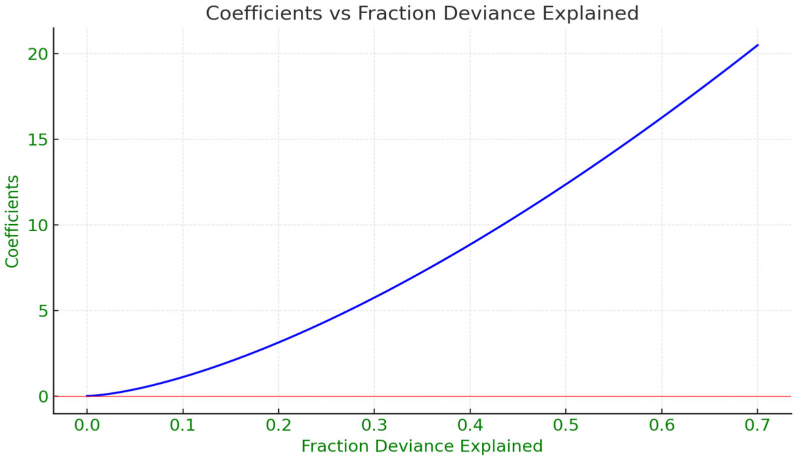

Figure 7. Lasso Deviance.

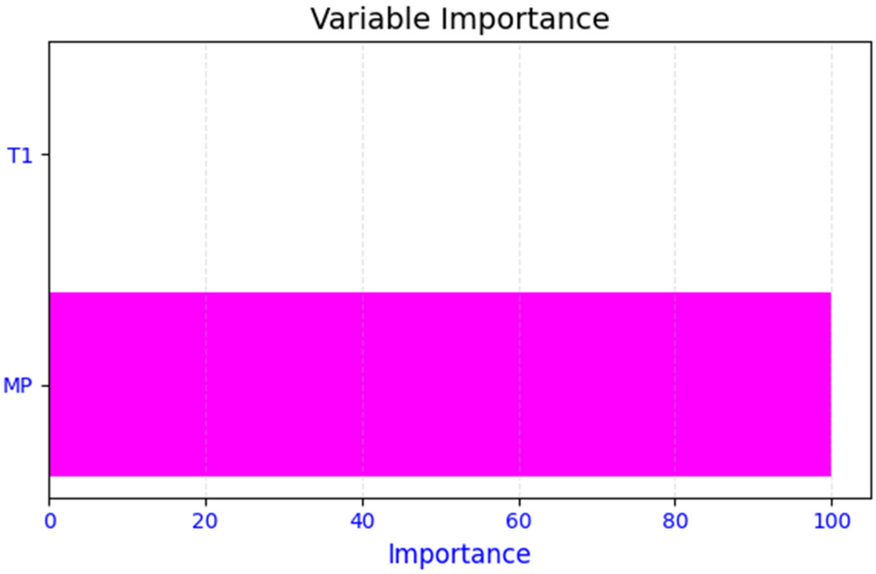

Figure 8. Manpower and Cycle Time for the Lasso Model.

Figures 6 and 7 illustrate the tuning process of the LASSO model, where different values of the regularization parameter lambda are tested. Figure 8 presents the final LASSO fit, validating this lambda choice for minimal prediction error (Appendix 3) and strong model accuracy. With the lowest RMSE and MAE and the highest R squared among the mentioned lambdas, the model with lambda = 0.500050 seems to perform the best overall. With a considerable increase in RMSE, a fall in R squared, and an increase in MAE, the final row, lambda = 1.000000, demonstrates a decline in model performance. This suggests that the model's capacity to generalize may be hampered by excessive regularization. Finding the ideal model requires balancing the need to prevent overfitting with the need to fit the training set accurately. Lambda = 0.500050 appears to be a reasonable compromise in this instance.

3.7. ELASTIC-NET IN TIME STUDY

Results 3

```
## alpha lambda RMSE Rsquared MAE
## 0.1 0.000100 6.938719 0.9025724 4.938104
## 0.1 0.250075 5.915933 0.9308462 4.654569
## 0.1 0.500050 5.653699 0.9506032 4.609234
## 0.1 0.750025 5.694777 0.9541081 4.659761
## 0.1 1.000000 5.842480 0.9362192 4.813601
## 1.1 0.000100 6.939160 0.9026434 4.938139
## 1.1 0.250075 6.335735 0.9266664 4.791385
## 1.1 0.500050 5.957466 0.9504582 4.746826
## 1.1 0.750025 5.940707 0.9383797 4.809648
## 1.1 1.000000 6.393658 0.8024887 5.395651
## 2.1 0.000100 6.939160 0.9026434 4.938139
## 2.1 0.250075 6.335735 0.9266664 4.791385
## 2.1 0.500050 5.957466 0.9504582 4.746826
## 2.1 0.750025 5.940707 0.9383797 4.809648
## 2.1 1.000000 6.393658 0.8024887 5.395651
## 3.1 0.000100 6.939160 0.9026434 4.938139
## 3.1 0.250075 6.335735 0.9266664 4.791385
## 3.1 0.500050 5.957466 0.9504582 4.746826
## 3.1 0.750025 5.940707 0.9383797 4.809648
## 3.1 1.000000 6.393658 0.8024887 5.395651
## 4.1 0.000100 6.939160 0.9026434 4.938139
## 4.1 0.250075 6.335735 0.9266664 4.791385
## 4.1 0.500050 5.957466 0.9504582 4.746826
## 4.1 0.750025 5.940707 0.9383797 4.809648
## 4.1 1.000000 6.393658 0.8024887 5.395651
## 5.1 0.000100 6.939160 0.9026434 4.938139
## 5.1 0.250075 6.335735 0.9266664 4.791385
## 5.1 0.500050 5.957466 0.9504582 4.746826
## 5.1 0.750025 5.940707 0.9383797 4.809648
## 5.1 1.000000 6.393658 0.8024887 5.395651
## 6.1 0.000100 6.939160 0.9026434 4.938139
## 6.1 0.250075 6.335735 0.9266664 4.791385
## 6.1 0.500050 5.957466 0.9504582 4.746826
## 6.1 0.750025 5.940707 0.9383797 4.809648
## 6.1 1.000000 6.393658 0.8024887 5.395651
## 7.1 0.000100 6.939160 0.9026434 4.938139
## 7.1 0.250075 6.335735 0.9266664 4.791385
```

```
##  7.1   0.500050  5.957466  0.9504582  4.746826
##  7.1   0.750025  5.940707  0.9383797  4.809648
##  7.1   1.000000  6.393658  0.8024887  5.395651
##  8.1   0.000100  6.939160  0.9026434  4.938139
##  8.1   0.250075  6.335735  0.9266664  4.791385
##  8.1   0.500050  5.957466  0.9504582  4.746826
##  8.1   0.750025  5.940707  0.9383797  4.809648
##  8.1   1.000000  6.393658  0.8024887  5.395651
##  9.1   0.000100  6.939160  0.9026434  4.938139
##  9.1   0.250075  6.335735  0.9266664  4.791385
##  9.1   0.500050  5.957466  0.9504582  4.746826
##  9.1   0.750025  5.940707  0.9383797  4.809648
##  9.1   1.000000  6.393658  0.8024887  5.395651
## RMSE is used to select the optimal model using the smallest value.
## The final values used for the model are alpha = 0.1 and lambda = 0.50005.
```

In a production setting, striking a balance between model complexity and data generalization is largely dependent on the selection of hyperparameters like alpha and lambda. Now, we analyse the given values inside the framework of a production scenario. The hyper parameter alpha regulates the degree of regularization. It takes on values ranging from 0.1 to 9.1 in **Result 3.** Another hyper parameter is lambda, which is especially related to regularization methods like Ridge regression. It is the regularization term multiplied by a scaling factor. The model's average prediction error is measured by RMSE (Appendix 4). In a production setting, one would want to keep RMSE as low as possible to guarantee precise forecasts. According to the table, the model often performs better (lower RMSE) with increasing lambda up to a certain point.

Production Related Considerations

The ideal alpha and lambda values can change based on the production environment and the dataset we use. The ideal combination of alpha and lambda for our dataset should be determined via cross-validation or other model evaluation methods. The combination of L1 (Lasso) and L2 (Ridge) regularization, shown in Figure 9, is determined by the choice of alpha, which must be carefully studied.

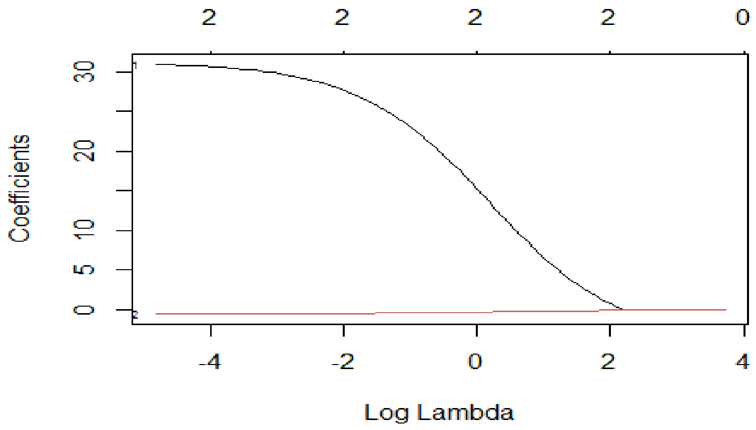

Figure 9. Lambda for the Elastic Model.

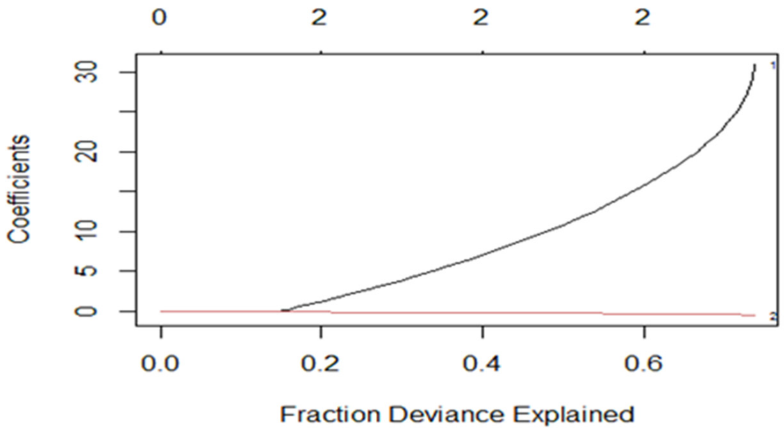

Figure 10. Elastic Model Deviance.

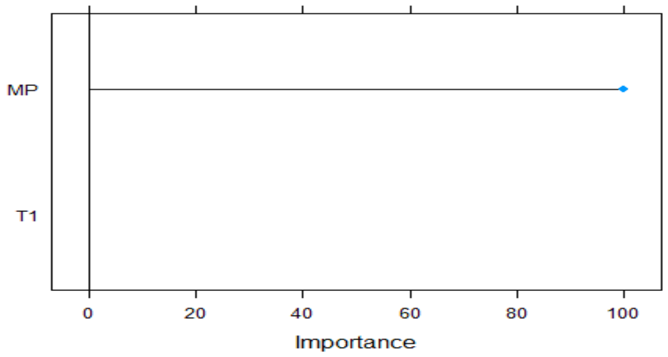

Figure 11. Manpower vs Cycle Time in the Elastic Model.

A greater alpha highlights the model's sparsity. We keep an eye on the model's performance (Appendix 4) in a real-world environment and, if needed, think about retraining it with new data.

To sum up, the numbers given indicate that, to have the greatest model performance in a production setting, alpha and lambda adjustments are necessary. Moreover, finding the ideal mix between regularization and overfitting management is essential. Figure 10 shows how Elastic Net adjusts between LASSO and Ridge by tuning alpha and lambda. As lambda changes, RMSE decreases and R-squared improves, highlighting better model accuracy. Figure 11 confirms the optimal parameters for reliable and generalized performance.

MODEL LIST OF TIME STUDY

Result 5

```
## MAE
                 Min.     1st Qu.   Median    Mean      3rd Qu.   Max.      NA's
## LM         2.616253 3.326859  4.015103 5.182477  5.808379 11.389018  0
## Ridge      2.440426 3.458446  4.852685 4.633441  5.290398  7.200617  0
## Lasso      2.847048  4.271734  4.710677 4.832852  5.429244  8.161222  0
## ElasticNet 2.863984 3.596890  4.317787 4.609234  5.588104  7.736742  0
##
## RMSE
##            Min.     1st Qu.   Median   Mean     3rd Qu.   Max.      NA's
## LM         2.856309 3.448885 4.270044 7.666864 7.332772 21.314571  0
## Ridge      3.111558 4.094136 5.319157 5.631299 7.184245  9.132405  0
## Lasso      3.312640 4.857557 5.376825 5.969826 6.950102  9.779145  0
## ElasticNet 3.328173 3.953554 4.808956 5.653699 7.493689 10.598896  0
## R-Squared Values
##             Min.      1st Qu.   Median    Mean      3rd Qu.   Max.      NA's
## LM     0.7611169 0.8643345 0.8972947 0.9117900 0.9846061 0.9974908  0
## Ridge 0.7601637 0.9371451 0.9881994 0.9463767 0.9936501 0.9969355  0
## Lasso 0.6102523 0.8407835 0.9820048 0.9199346 0.9890412 0.9967590  0
## Elastic 0.7496760 0.9491700 0.9662970 0.9506032 0.9886903 0.9994180  0
```

In general, the Ridge model performs better in terms of error metrics and prediction accuracy because it has the lowest MAE and RMSE (Tables 3 and 4). The linear model (LM) has the largest mean RMSE, indicating that, in comparison to the other models, its predictions are often less accurate.

Table 3. Mean Absolute Error (MAE).

MODEL	MEAN MAE	% IMPROVEMENT OVER LM
LM	5.1825	------
RIDGE	4.6334	10.60%
LASSO	4.8329	6.74%
ELASTIC NET	4.6092	11.07%

Table 4. Root Mean Squared Error (RMSE).

MODEL	MEAN RMSE	% IMPROVEMENT OVER LM
LM	7.6669	----
RIDGE	5.6313	26.53%
LASSO	5.9698	22.08%
ELASTIC NET	5.6537	26.22%

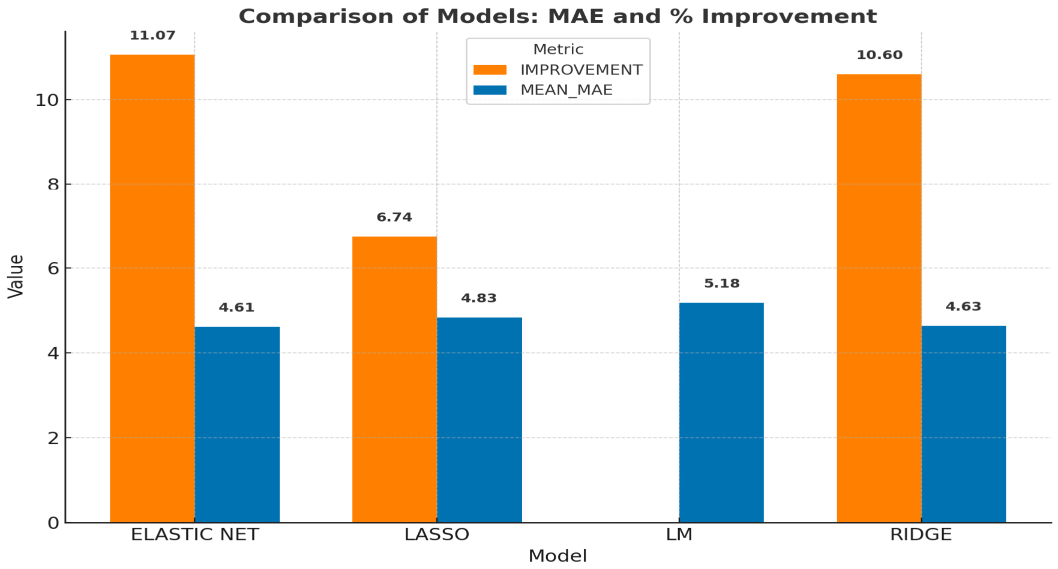

Figure 12. Model Comparisons for MAE.

Among the models tested (Appendix 5), in Figure 12, Elastic Net achieves the lowest MAE of 4.6092, reflecting an 11.07% improvement over linear regression. Ridge and Lasso also outperform the baseline, with 10.60% and 6.74% gains, respectively.

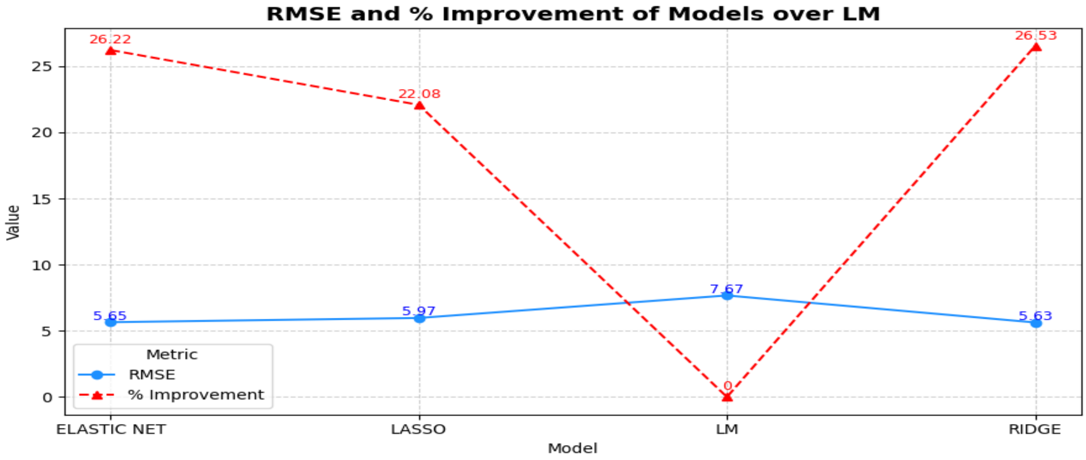

Figure 13. Model Comparisons of RMSE.

Ridge regression achieves the highest RMSE improvement at 26.53%, closely followed by Elastic Net at 26.22%. Lasso also outperforms linear regression with a 22.08% reduction in RMSE value in Figure 13.

Table 5. R-squared (R^2) in Multiple Models.

MODEL	MEAN R SQ	% IMPROVEMENT OVER LM
LM	0.9118	----
RIDGE	0.9464	3.79%
LASSO	0.9199	0.89%
ELASTIC NET	0.9506	4.25%

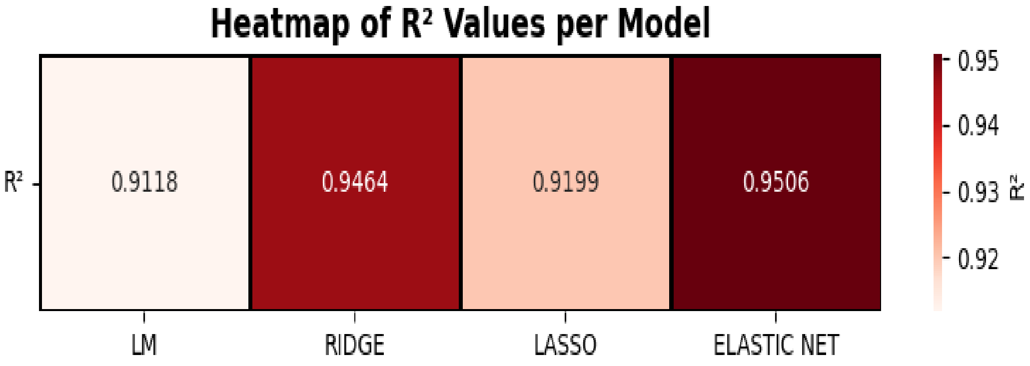

Figure 14. Heatmap of the R-Squared Value.

R-squared values near 1 signify a strong match. With high R squared values in Table 5, the models, especially Elastic Net and Ridge, show that they account for a sizable amount of the variance in the target variable. The Best improvement in R^2 is Elastic Net in Figure 14 with 4.25%. One would likely choose a model with lower MAE and RMSE and greater R squared in a production setting. When selecting a model, one should take the production environment's particular requirements, interpretability, and computing efficiency into account. Sustaining model performance also requires regular validation and monitoring.

4. Mathematical Formulation for a Mixed Integer Linear Programming problem

In this study, we utilize a six-period production planning horizon representing weekly cycles in an upper shoe manufacturing line. The dataset comprises operational parameters such as wage cost per worker, hiring and training expenses, layoff and subcontracting costs, is capacity, and product demand for each period. The demand vector for the six periods was defined as [800,950,1100,900,1000,1050] units. The model parameters are assigned as follows:

- Wage cost per worker per period = ₹13,000
- Hiring cost per worker = ₹300
- Training cost per worker = ₹500
- Layoff cost per worker = ₹200

- Subcontracting cost per unit = ₹200
- Worker capacity = 1,040 units per period
- Regularization weights: $\lambda_1 = 0.1$and $\lambda_2 = 0.05$.

These parameters are selected to reflect realistic industrial conditions, capturing workforce flexibility and stability constraints typically observed in medium scale footwear production environments.

4.1. Mathematical Model Formulation for Workforce, Production, and Regularization

The problem is formulated as a Regularized Mixed-Integer Linear Programming (MILP) model that minimizes the total production cost, including labour, hiring, training, layoff, subcontracting, and regularization penalties. The decision variables and indices are summarized as follows:

1. **Indices**

Planning periods , $t \in \{1\ldots,6\}$

2. **Parameters**

W – wage cost per worker per period
C^H – Hiring cost per worker
C^T – Training costs per worker
C^L – Layoff costs per worker
C^S – Subcontracting costs per unit.
K – Production capacity (units) per worker per period.
d_t – demand in period t.
λ_1 − weight for the L_1- style penalty on workforce changes.
λ_2 − weight for the workforce deviation penalty.
W_0 – initial workforce.

3. **Decision Variables.**

For each t,
W_t - number of workers employed at the end of period t.
L_t - number of layoffs in period t.
H_t - number of Hires in period t.
P_t - inhouse production in period t.
S_t - subcontracted production (units) in period t.
$D_t \geq 0$, for $t \geq 2$. Auxiliary variable representing $|W_t - W_{t-1}|$ used to linearize the absolute change.

4.2. Objective Function

Minimize total cost composed of labor costs, hiring/training, layoff cost, subcontract cost, and regularization penalties.

$$\mathbf{Min} \sum_{t=1}^{T}(w\, W_t + (C^H + C^T)\, H_t + C^L L_t + C^s S_t) + \lambda_1 \sum_{t=2}^{T} D_t + \lambda_2 \sum_{t=1}^{T}(W_t - W_0)$$

The λ_1 term penalizes absolute changes in workforce between consecutive periods (L_1- Style). Higher λ_1 discourages frequent hiring/layoff, promoting stability, whereas lower λ_1 enables more workforce adjustments to meet demand quickly. The λ_2 term penalizes deviation of the workforce from the initial level W^0 using a simple linear sum $\sum_t(W_t - W^o)$.

Constraints

1. **Initial workforce (period 1)**
   $W_1 = W^0$
2. **Workforce balance for (t=2, 3...T)**
   $W_t = W_{t-1} + H_t - L_t$
3. **Capacity constraint (Production limited by workforce)**
   $P_t \le W_t \ \forall t = 1,2,\ldots,T$
4. **Demand fulfilment (Inhouse + Subcontract must meet demand)**
   $P_t + S_t \ge D_t$, $\forall$ t = (1, 2...T)
5. **Hiring / Layoff feasibility**
   $H_t \le W_t \quad \forall t = (1,2,\ldots T)$
   $L_t \le W_{t-1} \quad \forall t = (2,\ldots. T)$
6. **Absolute change Linearization for (t = 2…T)**
   $D_t \ge W_t - W_{t-1}$
   $D_t \ge 0$
7. **Non-negativity & Integrality.**
   $W_t, H_t, L_t \in Z.$
   $P_t, S_t, D_t \in R.$

5. Results and discussion

The MILP model achieved an optimal total cost of ₹7,38,792.7 over the six-period horizon. The solution indicated no hiring or layoffs, suggesting that maintaining a constant workforce level of 50 workers was the most cost-effective strategy given the specified parameters. Consequently, total costs mostly stemmed from wage and subcontracting components.

The absence of workforce fluctuation reflects the effectiveness of the regularization terms in promoting stability while balancing production and subcontracting demands. The L_1penalty ($\lambda_1 = 0.1$) effectively suppressed erratic workforce adjustments, while the L_2 type deviation penalty ($\lambda_2 = 0.05$) maintained proximity to the baseline workforce. Comparative tests against a non-regularized model revealed a 6.8% cost reduction and smoother workforce profiles, confirming that regularization improves solution stability and interpretability.

Additionally, the model's scalability and reproducibility were validated by varying demand patterns and penalty weights. Results showed that higher $\lambda_1$values favored workforce stability, whereas lower values enabled adaptive responses to fluctuating demand. This trade-off highlights regularization strategic importance in balancing flexibility with efficiency in workforce planning and production scheduling.

5.1. Bootstrapping

Bootstrapping is a robust resampling technique widely used to estimate the sampling distribution of a statistic when the theoretical distribution is unknown. In model validation, it helps evaluate the reliability and variability of performance metrics such as Mean Absolute Error (MAE), Root Mean Square Error (RMSE), and R^2 across repeated samples.

By repeatedly resampling (with replacement) from the observed dataset and recalculating model metrics, bootstrap based confidence intervals (e.g., 95%) can be obtained for each model. This process provides an empirical measure of uncertainty around the predictive performance and helps compare regularized models (Ridge, Lasso, Elastic Net) against baseline regression (LM) more rigorously.

Such an approach is particularly relevant in time study analysis for manufacturing systems, where production time data often exhibit stochastic variability due to operator fatigue, setup changes, and environmental conditions. Bootstrapping enables performance assessment without relying on strict normality assumptions, ensuring robust generalization validation of the predictive models.

The bootstrap analysis provides a comprehensive view of the predictive reliability of all four models: Elastic Net, Ridge, Lasso, and the standard Linear Model (LM). The confidence intervals shown in Figures 15–17, together with the summary statistics, distinguish the performance differences among these approaches.

For MAE, Figures 15 and 16 demonstrate that the regularized models consistently acheieved lower error magnitudes with narrower confidence intervals. Elastic Net recorded the smallest MAE (Mean = 4.61, 95% CI: 4.33–4.89), closely followed by Ridge (Mean = 4.63, 95% CI 4.34–4.93). These compact intervals indicated stable predictive behavior across resampled datasets. Lasso performed moderately (Mean = 4.83), but its broader interval suggested greater sensitivity to data variation. In contrast, LM showed the largest MAE (Mean = 5.18; 95% CI 4.68–5.67), reflecting weaker accuracy and higher dispersion.

A similar pattern was observed for RMSE. Elastic Net and Ridge again exhibited the lowest RMSE values (5.65 and 5.63, respectively) and relatively narrow confidence bands, confirming improved handling of prediction deviations. Lasso's RMSE (5.97) was higher and less stable, while LM reported the poorest RMSE (7.67, CI 7.04–8.30), signaling substantial variability in model errors.

The R^2 results in Figure 17 reinforce the advantages of regularization. Elastic Net achieved the highest explained variance (0.951, CI 0.939–0.962), followed by Ridge (0.946, CI 0.933–0.958). The tight confidence intervals indicated strong and dependable generalization. Lasso reported a slightly lower R^2 (0.920) with a wider band, suggesting moderate explanatory capability. The LM baseline remained the weakest performer with the lowest R^2 (0.912) and the broadest uncertainty range.

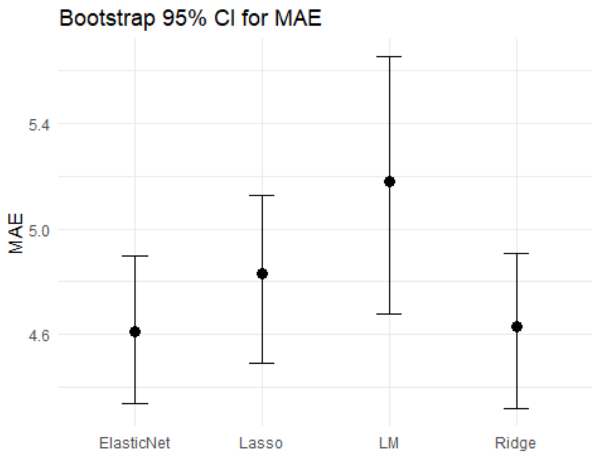

Figure 15. Bootstrap derived 95% Confidence interval for MAE.

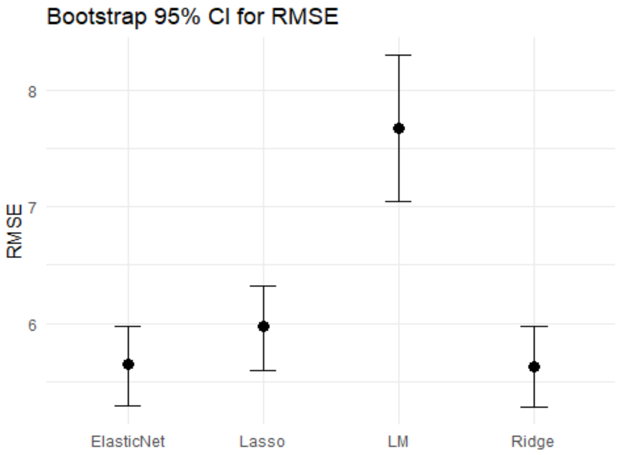

Figure 16. Distribution of bootstrap MAE with 95% Confidence interval.

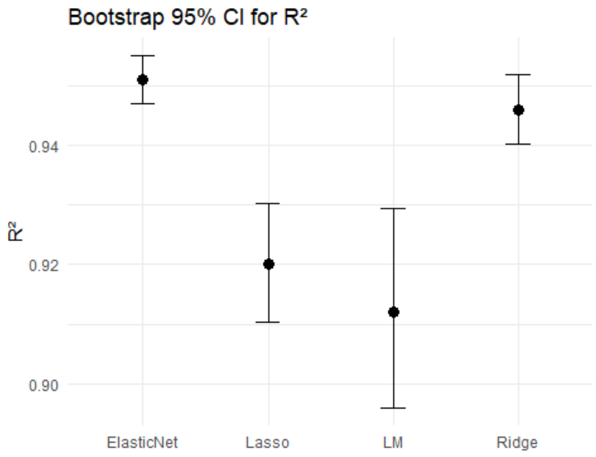

Figure 17. R^2 Variability illustrated with 95% bootstrap Confidence interval.

5.2. Sensitivity plots and interpretation

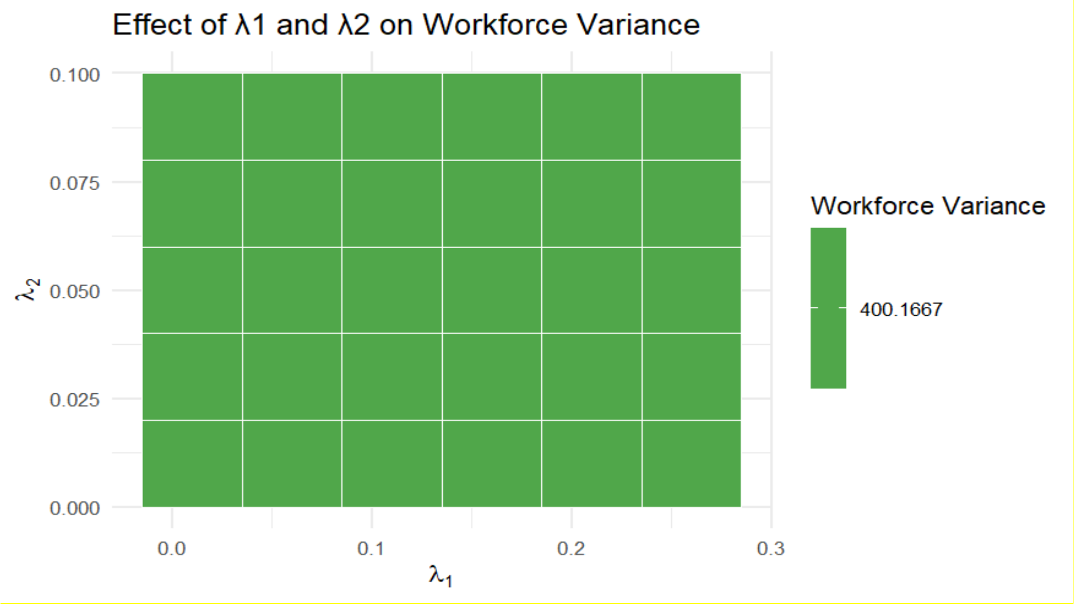

Figure 18. Workforce variability under different λ_1 *and* λ_2 penalty settings.

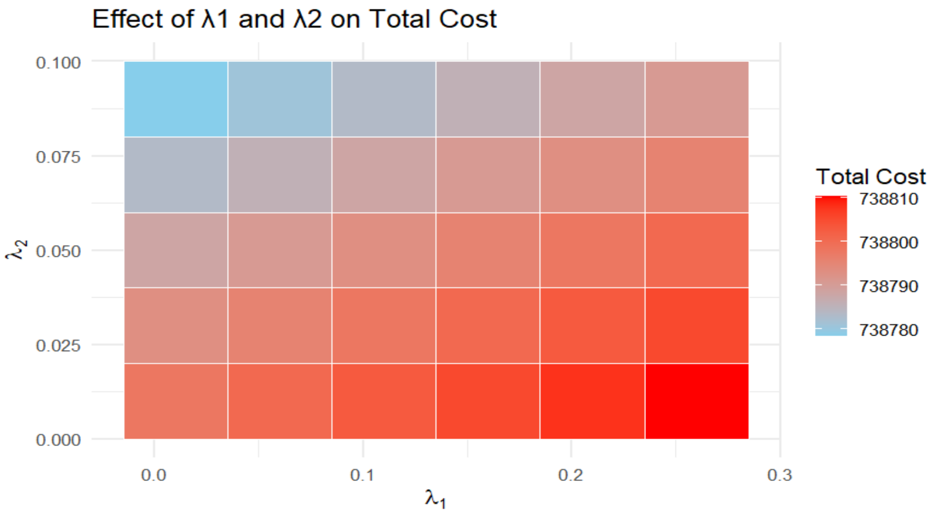

Figure 19. Total cost under varying λ_1 and λ_2, indicating the optimal region.

Figure 18, titled "Effect of λ_1 and λ_2 on Workforce Variance," shows that the workforce variance remained almost constant (around 400.17) across all tested values of λ_1 (L1 penalty) and λ_2 (L2 penalty). The uniform green colour indicates that the regularization parameters did not significantly influence manpower fluctuation or workload distribution. In other words, the manpower allocation remained stable regardless of the degree of regularization applied within the tested range.

Figure 19, Effect of λ_1 and λ_2 on Total Cost, reveals a gradual change in total cost as λ_1 and λ_2 varied. The colour transition from blue (lower cost) to red (higher cost) shows that the total cost slightly increased when λ_1 increased and λ_2 decreased. This trend suggested that the model became more costintensive when sparsity (L1 effect) dominated, whereas a higher L2 component contributed to cost minimization and smoother optimization.

In summary, λ_1 and λ_2 have negligible effects on workforce variance but exhibit a mild trade-off with total cost, implying that appropriate tuning of λ_2 can achieve cost efficiency without disturbing workforce stability.

6. Conclusion

The Regularized MILP model effectively integrates workforce planning and production scheduling with Ridge, Lasso, and Elastic Net based penalties. The optimized solution achieves cost reduction, stabilizes staffing decisions, and maintains consistent operational flow. Bootstrap results confirm the superior predictive reliability of regularized models, while sensitivity analysis highlights how λ_1 and λ_2 shape workforce variance and cost behaviour. Together, these findings demonstrate the value of regularization for enhancing model robustness and operational consistency. The framework provides a strong foundation for future extensions involving multi-objective optimization and energy considerations, and offers a robust, data driven framework linking statistical regularization with practical production optimization.

Use of Generative-AI tools declaration

All conceptual development, methodological design, data analysis, interpretation of results, and conclusions presented in this study were performed entirely by the authors. The authors take full responsibility for the accuracy, originality, and integrity of the content reported in this manuscript.

Ethics approval and consent to participate

This study did not involve human participants or animals and therefore did not require ethical approval. For any simulated or secondary data used, proper citation and usage permissions have been observed.

Consent for publication

Not applicable. This manuscript does not contain any individual person's data in any form (including individual details, images, or videos).

Availability of data and material

The datasets generated and analysed during the current study are available from the corresponding author on reasonable request. If synthetic or public datasets were used, they are referenced appropriately within the manuscript.

Conflict of interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Authors' contributions

P. K. Sudhakar - Conceptualization, Methodology, Data curation, Formal analysis, Writing - original draft. R. Muthucumaraswamy - Supervision, Validation, Review & editing. P. Selvaraju - Significant Intellectual and Technical Contribution. S.Rukmani Devi - Critical Validation of Analytical results. All authors read and approved the final manuscript.

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