Journal of Industrial and Management Optimization

https://www.aimspress.com/journal/jimo

JIMO, 22(4): 1789–1823.
DOI:10.3934/jimo.2026066
Received: 27 September 2025
Revised: 16 January 2026
Accepted: 04 March 2026
Published: 11 March 2026

Research article

Fair profit distribution in global supply chains under demand uncertainty

Zijing Yang, Songsong Liu* **and Boya Yu**

School of Management, Harbin Institute of Technology, Harbin 150001, China

* **Correspondence:** Email: s.liu@hit.edu.cn

Abstract: Traditional global supply chain models often prioritize profit maximization while neglecting the fair profit distribution among stakeholders and the dynamic challenges of demand uncertainty, leading to cooperative conflicts and inefficiencies. This study bridges this gap by proposing a two-stage stochastic programming framework, which simultaneously addresses demand uncertainty and fair profit distribution. By integrating the Nash bargaining method into the stochastic model, collaborative decision-making that balances fairness and global optimality under uncertain conditions is achieved. To address the computational complexity of large-scale scenario trees, a hybrid strategy combining scenario reduction and Lagrangian relaxation is proposed, which improves the solution efficiency without affecting the accuracy. A real supply chain case study demonstrates the effectiveness of this framework, indicating that fair distribution based on the Nash bargaining method enhances overall performance, stability, and multi-agent coordination in volatile markets. The research results emphasize that fairness based on the Nash bargaining method can enhance multi-agent collaboration in an uncertain environment. This framework provides a practical decision-making tool for the global supply chain, promoting the efficiency and fairness of dynamic markets.

Keywords: supply chain; profit distribution; uncertainty; fairness; stochastic programming; Lagrangian relaxation approach
Mathematics Subject Classification: 90B06, 90C15

1. Introduction

With the rapid globalization of markets and the intensification of competition, modern supply chains have evolved into highly intricate networks involving multiple stakeholders—suppliers, manufacturers, distributors, and retailers—operating in a highly dynamic environment [1]. Traditional supply chains prioritize cost minimization and profit maximization. Such approaches often lead to skewed profit distributions, where powerful players (e.g., large retailers) capture disproportionate gains at the expense of smaller suppliers. This imbalance not only erodes trust but also destabilizes the entire supply chain network. For instance, the 2019 Boeing 737 MAX crisis was partly attributed to cost-cutting pressures on suppliers, resulting in catastrophic system failures [2]. Consequently, fairness has been increasingly recognized as a critical factor for ensuring supply chain sustainability and collaboration.

However, the actual operation of supply chains is fraught with uncertainties, particularly in demand [3], supply disruptions [4], and price fluctuations [5]. This poses fundamental challenges in implementing the concept of fairness within supply chains. A specific example can be drawn from the low-carbon supply chain practices of BYD, a leading electric vehicle manufacturer, where fairness concerns in revenue-sharing contracts significantly influenced carbon emission reduction levels and pricing strategies under demand uncertainty [6]. This case illustrates that fairness serves as a key operational prerequisite for sustaining stable cooperation amid uncertainty. Although existing studies have coordinated economic interests and maintained partnerships in a deterministic environment through a cooperative framework [7–10], the current models are unable to adequately address the issue of fair distribution in an uncertain environment. There is a significant modeling gap between the static concept of fairness and the dynamic, adaptive decision-making in an uncertain environment.

To cope with uncertainties, scholars have developed various modeling frameworks, such as robust optimization [11–14], fuzzy programming [15, 16], chance-constrained programming [17, 18], and notably, two-stage stochastic programming [19–21]. Two-stage stochastic programming, in particular, achieves a good balance between the robustness of network configuration and operational flexibility by explicitly modeling scenario-based pursuit decisions. However, its application is limited by the "dimension disaster": to capture the evolution of uncertainties, supply chain models often need to generate large scenario trees, and incorporating fairness considerations further complicates the problem. To address the computational challenges, techniques such as scenario reduction [22–24] and decomposition algorithms (e.g., Lagrangian relaxation methods [25–27]) are adopted to mitigate this issue. However, in the multi-stage stochastic models that take fairness factors into account, the research on the collaborative integration of scenario reduction and Lagrangian relaxation is still insufficient, and the universality of the existing works is limited [28, 29].

Overall, the existing models have difficulty comprehensively capturing and addressing these challenges, mainly for two reasons. First, current fairness considerations mainly focus on the inability to accommodate fluctuations in the real world (such as demand fluctuations [30]). Second, embedding fairness into stochastic models exacerbates computational intractability, rendering large-scale optimization infeasible [31]. This leads to a stark disconnect between theory and practice: On one side, fairness mechanisms rooted in deterministic assumptions struggle to cope with actual market fluctuations—for instance, static fairness designs may become unsustainable under uncertainty, causing suboptimal resource allocation or system instability [32, 33]; on the other side, uncertain models that neglect fairness risk amplifying power imbalances in supply chains (e.g., demand fluctuations may enable dominant retailers to transfer risks to smaller suppliers via operational flexibility [34]). However, the Nash bargaining method is well-suited to this setting because it inherently incorporates bargaining power and proportional fairness, which can be adapted to dynamic conditions through stochastic programming. For example, in multi-echelon supply chains facing demand shocks, the Nash bargaining method can negotiate equitable profit splits that reflect real-time risk exposure, unlike rigid deterministic models.

Motivated by the work of Liu and Papageorgiou [35] and aiming to address these critical gaps, this paper proposes a framework that integrates the Nash bargaining method into a two-stage stochastic programming model to address fair profit distribution under demand uncertainty. The primary contributions lie in the following: (1) This study provides a unified theoretical framework that integrates fairness theory with uncertain optimization theory. It offers a new benchmark for achieving fairness in a dynamic environment, bridging the gap between fairness and stochastic decision-making models. (2) A two-stage stochastic programming model is constructed, which covers a specific three-echelon structure of suppliers, factories, and markets and considers multiple products, multiple regional markets, multiple price levels, multiple time periods, and multiple scenarios. The embedding of the Nash bargaining method for proportional fairness is highly applicable and scalable in practice. (3) Combining scenario reduction with Lagrangian relaxation, this approach uses a scenario tree to simulate demand uncertainty and employs the Lagrangian relaxation algorithm for large-scale solving. It not only retains the accuracy of uncertainty but also significantly boosts solution efficiency, enabling large-scale stochastic optimization problems to get high-quality solutions within an acceptable time.

The rest of this paper is organized as follows: Section 2 reviews the existing relevant literature. Section 3 describes the considered supply chain planning problem under uncertainties. The detailed mathematical formulation and the proposed approach are given in Section 4 and Section 5, respectively. A numerical case and the corresponding results are presented and discussed in Section 6. Finally, some concluding remarks are given in Section 7.

2. Literature review

This section is divided into two sub-sections: (i) the fairness in supply chain management, and (ii) two-stage stochastic programming in supply chain optimization.

2.1. The fairness in supply chain management

The rapid development of research indicates that people are not only concerned about their profits but also attentive to fairness [36], which plays an important role in supply coordination [37]. Therefore, an increasing body of research is focusing on supply chain fairness. Existing literature considers supply chain fairness in different aspects, with the main approach being to coordinate economic incentives through a cooperative framework and maintain the stability of partnerships [7, 8, 38, 39]. As supply chains become more complex and globalized, issues of fairness, such as power imbalances, unfair pricing practices, and exploitation of vulnerable stakeholders, have become more pronounced. Fairness is a critical concept that affects all aspects of social and economic interactions [30, 40, 41].

In the research of supply chain profit distribution fairness, Hu et al. [31] studied the supply chain coordination problem considering wholesale price constraints and fairness. They derived supply chain coordination conditions by considering the fairness of profit distribution between retailers and suppliers. Zheng et al. [42] studied a three-level closed-loop supply chain where retailers exhibited fairness issues. Three coordination mechanisms based on the Shapley value, nucleolar solution, and other satisfaction levels were proposed to allocate residual profits. Further research was conducted on how the fairness issues of retailers affect profit distribution under three mechanisms. Pan et al. [43] studied the impact of fairness concern on a two-echelon supply chain model. The results indicate that the supply chain can get additional profits from the issue of fair distribution. Jian et al. [44] considered competitive supply chains with revenue-sharing contracts and fairness issues. They found that a focus on fairness in manufacturing can cause an increase in supply chain profits. Lyu and Liu [41] developed a mixed-integer linear programming (MILP) model with dual fairness, using the Nash bargaining method to balance the fairness in profit distribution and emissions reduction. Liu and Papageorgiou [35] considered fairness in a global supply chain. The criteria of proportional fairness and max-min fairness was utilized to establish equitable distribution of profits. Taking into account the bargaining power of participants, game-theoretic-based solution methods were formulated to achieve fair outcomes based on the Nash bargaining method and lexicographic maxi-min methods. However, max-min fairness is more about ensuring that the weakest parties receive the maximum possible benefit. In contrast, the Nash bargaining method aims to negotiate a distribution plan that satisfies all participants, emphasizing effectiveness and relative fairness. Max-min fairness may lead to inefficient allocation of resources, and this method may incur longer computation time [45]. The Nash bargaining method aims to maximize overall utility and satisfaction among all participants, often resulting in higher overall efficiency.

In the context of fairness problems, the Nash bargaining method provides a cooperative negotiation-based framework for fair outcomes, which explains its wide use in fairness issues. For example, Ni et al. [46] developed a two-echelon newsvendor model with the Nash bargaining method as a fair reference, analyzing the impact of fairness on retailer/manufacturer-dominated supply chain performance. Li et al. [47] used it to coordinate wholesale price, buy-back, and revenue-sharing contracts in dual-channel supply chains, showing that fairness concerns boost channel efficiency when aligned with bargaining power. Li et al. [48] integrated it into low-carbon supply chains, illustrating how fairness preferences affect carbon reduction and warranty periods and using revenue-sharing contracts to alleviate adverse fairness impacts. These works highlight the method's flexibility in reconciling stakeholder interests. Further, Du et al. [49] extended its use to cooperative advertising, finding that fairness concerns enhance local advertising investment but lead to inverted U-shaped profits for retailers and manufacturers. This nonlinearity highlights the need to calibrate fairness parameters to avoid inefficiencies—a challenge that our stochastic programming framework addresses via scenario reduction and Lagrangian relaxation.

While most existing supply chain fairness studies focus on deterministic environments, research increasingly incorporates uncertainty into fairness considerations. Gholamian et al. [50] proposed a lexicographic min-max fairness approach for sustainable closed-loop supply chains with uncertain delivery times, yet they prioritized temporal equity over profit distribution fairness. Zhao et al. [51] introduced data-driven robust optimization for network design, but their "balanced uncertainty" concept mainly addresses spatial resource allocation rather than inter-stakeholder profit equity. Wu et al. [52] examined fairness preferences under demand uncertainty but focused on behavioral responses instead of optimization-based profit distribution mechanisms. Yang and Liu [53] established a fuzzy-based optimization model to consider multi-objective optimization problems oriented toward uncertainty and fairness.

Notably, existing studies still lack a unified framework that integrates stochastic programming with fairness mechanisms in dynamic environments—especially one that addresses participants' heterogeneous uncertainty levels while balancing ex-ante equity and ex-post profit distributions.

2.2. *Two-stage stochastic programming in supply chain optimization*

Supply chain uncertainty emanates from the task environment. These manifestations of uncertainty can significantly impede operational efficiency and undermine competitive positioning through mechanisms such as stockouts, delivery delays, and resource underutilization. In an uncertain decision-making environment, two-stage stochastic programming has been extensively applied in the optimization of supply chains [54]. It reflects a methodological progression from basic uncertainty handling toward integrated resilience and computational efficiency [55]. Yaghin and Farmani [56] considered the supply uncertainty caused by COVID-19 and developed a new scenario-based MINLP model. The optimal tactical plan was then derived by applying two-stage stochastic programming and convex analysis, supported by an interval Hessian matrix. Chebeir et al. [57] developed a model to tackle the shale gas supply chain under uncertainties. They introduced a two-stage stochastic programming model that utilizes a scenario-based approach to manage the uncertainties associated with prices. This was achieved by constructing a scenario tree and employing a binomial option pricing model to approximate the stochastic process. Luo et al. [58] advanced uncertainty modeling by combining stochastic and robust techniques, developing a two-stage stochastic-robust optimization framework that mitigates disruption cascades while integrating demand uncertainty through decision-dependent uncertainty sets. Concurrently, Nikzad et al. [59] integrated uncertain demand into the model through a two-stage stochastic programming approach, examining the effects of this uncertainty on supply chain outcomes. They first proposed a scenario-based two-stage stochastic programming model that does not include probability constraints and developed a mathematical programming algorithm to solve it. Furthermore, the Latin hypercube sampling method was employed to generate scenarios for the scenario-based model. For large-scale implementations, Schildbach and Morari [55] pioneered the embedding of scenario-based model predictive control within two-stage stochastic programming, enabling real-time adjustments to lead-time variability. Most recently, Xie et al. [60] applied multi-period two-stage stochastic programming with rolling horizon recourse, reporting significant cost reduction through stochastic biomass allocation in their case study. Marousi et al. [61] proposed a two-stage stochastic model of oligopoly. The model takes the fair allocation of customers and contract types as the first-stage decision.

Stochastic programming can effectively handle demand uncertainty. However, as the number of scenario trees increases, its computational complexity grows exponentially. This challenge requires advanced solution strategies, such as scenario reduction [29, 62] and the Lagrangian relaxation method [63–65]. However, existing literature exhibits limitations in systematically integrating scenario reduction with Lagrangian relaxation. For instance, Tang et al. [66] applied Lagrangian relaxation with incremental proximal methods to wind power dispatch but omitted scenario reduction, limiting scalability for large scenario sets. Papavasiliou et al. [67] employed parallel Lagrangian relaxation for stochastic unit commitment but relied on scenario sampling rather than reduction, which constrained their ability to handle correlated uncertainties. Qian et al. [68] developed a scenario reduction-based approach for transportation auctions using CPLEX but excluded Lagrangian decomposition, resulting in suboptimal handling of multi-stage stochastic constraints. Similarly, Wu et al. [69] addressed long-term security-constrained unit commitment via Lagrangian relaxation and Monte Carlo simulation but overlooked scenario reduction, leading to computational inefficiency.

From the literature review, it can be seen that the Lagrangian relaxation algorithm can effectively solve large-scale problems. In contrast, there has been extensive research on fairness in deterministic supply chain environments, but there is a lack of models that explicitly incorporate uncertainty into fairness considerations, especially in global supply chains, which often face higher levels of uncertainty. It is necessary to develop a fair model that takes into account the dynamic environment in which supply chain participants face varying degrees of uncertainty.

3. Problem description

A three-echelon supply chain network consisting of several suppliers (h), several factories (j) at different places, and markets (k) is considered in this work, as shown in Figure 1.

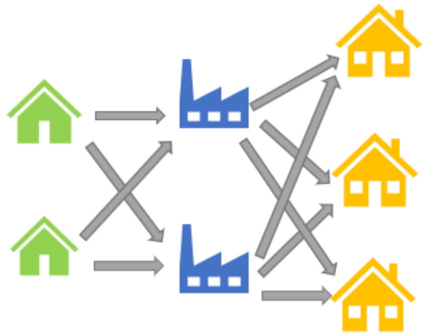


Figure 1. The three-echelon supply chain network example.

This study focuses on product production, distribution, and capacity expansion strategies of a three-echelon supply chain network. Suppliers are accountable for various costs, including raw material expenses, production costs, inventory costs for primary products, and the costs of capital investment. The costs incurred by factories encompass payments to suppliers for the procurement of primary products, the costs of raw material for primary products, the formulation cost of products, the costs of inventory for primary and final products, the costs of transportation from suppliers to factories, capital investment costs, and duties associated with the importation of final products. Each market bears the costs associated with purchasing final products from factories, maintaining final product inventory, transporting final products from factories, paying duties on final product imports, and handling unmet demand.

The revenues generated by suppliers and factories stem from transfer payments made by factories and markets, respectively. Consequently, the transfer prices of primary products from suppliers to factories, as well as the transfer prices of final products from factories to markets, significantly impact the profits of every supply chain participant. This problem focuses on optimizing transfer pricing decisions, which are contingent upon the price levels and the bargaining power of supply chain members. Moreover, it is assumed that the final selling prices in each market are predetermined.

Due to unpredictable events, demand is inherently uncertain. The exact demand cannot be determined before the end of the planning period. Underestimating overall demand can lead to sales losses and customer dissatisfaction. However, overestimation of product demand leads to an increase in production and inventory costs. Therefore, demand is treated as a random parameter in each period. A set of scenarios is considered as a scenario tree, which is used to simulate stochastic parameters.

This study aims to achieve fair distribution of profit among supply chain members by adopting a widely accepted standard of fairness: proportional fairness. Overall, this optimization problem can be articulated as follows:

Given are:

- a three-echelon supply chain including suppliers, factories in different regions, and markets;
- final product groups and products;
- capacities and the capabilities of factories;
- the low, medium, and high weekly product demands at markets and their corresponding probabilities;
- minimum and maximum production levels of the products and their corresponding flows;
- penalty for final product sales loss.

to determine:

- production of the final products;
- distribution of the primary and final products;
- the capacity expansion of the factories;
- the levels of inventory of the final products in different factories;
- the levels of inventory of the final products in different markets;
- the sales of final products in different markets.

aim as to maximize the total profit of the supply chain with proportional fairness concerns for a fair profit distribution among its members.

4. Model formulation

The considered optimization model for addressing the fair distribution of profit issue is built upon an existing supply chain optimization model from the literature [35]. This study focuses on a three-echelon supply chain consisting of suppliers, factories, and markets, where each member operates as an independent profit center. This section presents the constraints and objective functions incorporated into the proposed model. Notation for indices, parameters, and variables is summarized in the Appendix.

4.1. Production

If the primary product r is produced at supplier h during time period t in scenario s, the production quantity, P^S_{rhts}, is subject to specified minimum ($MinP^S_{rh}$) and maximum ($MaxP^S_{rh}$) production constraints, and W^S_{rhts} is a binary variable that indicates whether the primary product r is produced by supplier h during time period t at scenario s.

$$MinP^S_{rh} \cdot W^S_{rhts} \le P^S_{rhts} \le MaxP^S_{rh} \cdot W^S_{rhts}, \quad \forall r, h, t, s. \tag{4.1}$$

The production quantity of final product i produced at factory j during time period t at different scenarios s, denoted as P^P_{ijts}, is constrained by minimum and maximum production constraints ($MinP^P_{ij}$ and $MaxP^P_{ij}$), and the binary variable W^P_{ijts} equals 0 if the final product i is not produced at factory j during time period t:

$$MinP^P_{ij} \cdot W^P_{ijts} \le P^P_{ijts} \le MaxP^P_{ij} \cdot W_{ijts}, \quad \forall g \in G_j, j, i \in \bar{I}_g, t. \tag{4.2}$$

Here, G_j represents the set of product groups g that can be produced in factory j, and $\bar{I}_g$ denotes the set of products that fall into product group g.

4.2. Capacity

The overall production allowed at each supplier and factory is restricted by its current capacity (Cap_r^S and Cap_j^F), as well as any relevant capacity increments (ΔCap_r^S and ΔCap_j^F, respectively), which need to be optimized.

$$\sum_h P_{rhts}^S \leq Cap_r^S + \Delta Cap_r^S, \quad \forall r, t, s, \tag{4.3}$$

$$\sum_{g \in G_j} \sum_{i \in \bar{I}_g} P_{ijts}^P \leq Cap_j^F + \Delta Cap_j^F, \quad \forall j, t, s. \tag{4.4}$$

Since expanding production capacity usually requires a large amount of capital investment, a long delivery time, and fixed contractual commitments, capacity expansion is always considered as strategic and a "here-and-now" decision and treated as a first-stage variable. In the hierarchical structure of supply chain planning, such decisions are clearly positioned at the long-term strategic level and must be committed to before addressing the uncertainty of demand [64].

4.3. Flow

When there exists a transportation of the primary product r from supplier h to factory j in time period t under different scenarios s, indicated by the binary variable $Y_{rhjts}^S = 1$, the quantity of the shipped flow (F_{rhjts}^S) must remain within its minimum ($MinF_{rhj}^S$) and maximum ($MaxF_{rhj}^S$) limits. The upper limit is generally determined by physical constraints, while the lower thresholds for shipments ensure operational feasibility and maintain minimum contractual obligations or service levels.

$$MinF_{rhj}^S \cdot Y_{rhjts}^S \leq F_{rhjts}^S \leq MaxF_{rhj}^S \cdot Y_{rhjts}^S, \quad \forall r, h, j, t, s. \tag{4.5}$$

For the quantity of final product i from factory j to market k in time period t under different scenarios s, F_{ijkts}^P, the following constraints are established:

$$MinF_{ijk}^P \cdot Y_{ijkts}^P \leq F_{ijkts}^P \leq MaxF_{ijk}^P \cdot Y_{ijkts}^P, \quad \forall j, k, g \in G_g, i \in \bar{I}_g \cap I_k, t, s. \tag{4.6}$$

4.4. Inventory

The primary product r's inventory at supplier h during time period t under different scenarios s (IV_{rhts}^{PS}) can be determined by taking the previous time period's inventory ($IV_{r,h,t-1,s}^{PS}$), adding primary product's production (P_{rhts}^S), and subtracting the total flows of the primary product to factories (F_{rhjts}^S):

$$IV_{hts}^{PS} = IV_{h,t-1,s}^{PS} + P_{hts}^S - \sum_j F_{hjts}^S, \quad \forall h, t, s. \tag{4.7}$$

The inventory of primary product r at factory j during time period t under different scenarios s (IV_{rjts}^{PP}) can be calculated at the end of the previous time period's inventory ($IV_{rj,t-1,s}^{PP}$), the incoming flow of primary products from suppliers ($F_{rhj,t-\bar{\tau}_{hj},s}^S$), minus the consumption of primary products for final product production (P_{ijts}^P):

$$IV_{rjts}^{PP} = IV_{rj,t-1,s}^{PP} + \sum_h F_{rhj,t-\bar{\tau}_{hj},s}^S - \sum_{i \in \bar{I}_j} \sum_{g \in G_j} \beta_{ij} \cdot P_{ijts}^P, \quad \forall n, j, t, s, \tag{4.8}$$

where $\bar{\tau}_{hj}$ means the time from supplier h to factory j.

The final product i's inventory at factory j by the end of each time period in different scenarios s (IV^{FP}_{ijts}) is represented at the end of the previous time period's inventory ($IV^{FP}_{ij,t-1,s}$) plus final product production (P^{P}_{ijts}) and minus the total flows to the markets (F^{P}_{ijkts}) during that time period:

$$IV^{FP}_{ijts} = IV^{FP}_{ij,t-1,s} + P^{P}_{ijts} - \sum_{k \in K_i} F^{P}_{ijkts}, \quad \forall j, g \in G_j, i \in \bar{I}_g, t, s. \tag{4.9}$$

The final product i's inventory at market k during the time period t in scenario s (IV^{FM}_{ikts}) is determined by taking the inventory at the end of the previous time period ($IV^{FM}_{ik,t-1,s}$), adding any incoming flows from factories ($F^{P}_{ijk,t-\tau_{jk},s}$), and subtracting the local sales that occur in the same time period (SA_{ikts}):

$$IV^{FM}_{ikts} = IV^{FM}_{ik,t-1,s} + \sum_{j \in J_g} \sum_{g \in \bar{G}_i} F^{P}_{ijk,t-\tau_{jk},s} - SA_{ikts}, \quad \forall k, i \in I_k, t, \tag{4.10}$$

where τ_{jk} denotes the transportation time from factory j to market k.

The aforementioned inventories are constrained by their lower and upper bounds. The lower bounds are generally regarded as safety stocks to account for demand uncertainty, while the upper bounds typically arise from storage capacity limitations.

$$MinIV^{PS}_{h} \leq IV^{PS}_{hts} \leq MaxIV^{PS}_{h}, \quad \forall h, t, s, \tag{4.11}$$

$$MinIV^{PP}_{j} \leq IV^{PP}_{jts} \leq MaxIV^{PP}_{j}, \quad \forall j, t, s, \tag{4.12}$$

$$MinIV^{FP}_{ij} \leq IV^{FP}_{ijts} \leq MaxIV^{FP}_{ij}, \quad \forall j, g \in G_j, i \in \bar{I}_g, t, s, \tag{4.13}$$

$$MinIV^{FM}_{ik} \leq IV^{FM}_{ikts} \leq MaxIV^{FM}_{ik}, \quad \forall k, i \in I_k, t, s. \tag{4.14}$$

4.5. *Transfer price*

Transfer prices for primary products from suppliers to factories (TP^{S}_{rh}), as well as for final products from factories to markets (TP^{P}_{ij}), are treated as decision variables. It is assumed that all factories pay the same transfer price to each supplier, and likewise, each factory sets a uniform transfer price for each final product across all markets [35,70,71]. Considering that in the actual supply chain, transfer prices are usually agreed upon through long-term agreements, frequent short-term adjustments will undermine cooperative trust and increase negotiation costs. These parameters are defined as the decision variable in the first stage and remain fixed in the subsequent stages.

The number of transfer price levels is determined based on market conditions and negotiation power, and reflects market practices and discrete pricing tiers. Each supplier can only choose one level of transfer price for each primary product (TPL^{S}_{rhl}), provided that production takes place at that supplier. Similarly, each factory can only select one level of transfer price to produce final products (TPL^{P}_{ijl}). O^{S}_{rhl} and O^{P}_{ijl} are two binary variables that indicate whether price level l is chosen, and E^{S}_{rhl} and E^{P}_{ijl} represent the allocation of production for the respective products.

$$TP^S_{rh} = \sum_l TPL^S_{rhl} \cdot O^S_{rhl}, \quad \forall r, h, \tag{4.15}$$

$$TP^P_{ij} = \sum_l TPL^P_{ijl} \cdot O^P_{ijl}, \quad \forall j, g \in G_j, i \in \bar{I}_g, \tag{4.16}$$

$$\sum_l O^S_{rhl} = E^S_{rh}, \quad \forall r, h, \tag{4.17}$$

$$\sum_l O^P_{ijl} = E^P_{ij}, \quad \forall j, g \in G_j, i \in \bar{I}_g. \tag{4.18}$$

Treating the transfer price as the first-stage decision is based on economic and operational practical considerations. In practice, transfer pricing in cooperative supply chains is usually governed by long-term contracts or strategic partnership agreements [72]. These agreements aim to maintain stability within a strategic planning cycle, reduce the cost of repeated negotiations, ensure the predictability of accounting for enterprises, and maintain the trust of the partnership. From an economic perspective, the transfer pricing mechanism is a pre-strategic decision aimed at adjusting incentives before operational decisions are made. Therefore, determining the transfer price in the first stage reflects the pursuit of stability while allowing all other operational variables to best adapt to the revealed demand.

4.6. Lost sales

Final product i's lost sales (LS_{ikts}) in market k during time period t in different scenarios s cannot exceed the difference between corresponding demands (D_{ikts}) and sales amount (SA_{ikts}).

$$LS_{ikts} = D_{ikts} - SA_{ikts}, \quad \forall k, i \in I_k, t, s. \tag{4.19}$$

4.7. Logical constraints

If supplier h does not choose to produce the primary product r, its production amount is 0.

$$\sum_t W^S_{rhts} \leq |\, t \,| \cdot E^S_{rh}, \quad \forall r, h, s. \tag{4.20}$$

Similarly, if there is no shipment link from supplier h to factory j for the transportation of primary product r, then there will be no production flow along this link during any time period.

$$\sum_t Y^S_{rhjts} \leq |\, t \,| \cdot X^S_{rhj}, \quad \forall r, h, j, s. \tag{4.21}$$

When supplier h is not designated for the production of primary product r, the link from supplier h to any factory j will not be utilized for the shipment of the primary product.

$$\sum_j X^S_{rhj} \leq |\, j \,| \cdot E^S_{rh}, \quad \forall r, h. \tag{4.22}$$

If the final product i is not assigned to be produced in factory j, then the factory j will not produce the final product.

$$\sum_{t} W^{P}_{ijts} \leq | t | \cdot E^{P}_{ij}, \quad \forall j, g \in G_j, i \in \bar{I}_g, s. \tag{4.23}$$

When there is no link from factory j to market k for the shipment of final product i, the flow of final product i through this link will be zero during all time periods.

$$\sum_{ts} Y^{P}_{ijkts} \leq | t | \cdot X^{P}_{ijk}, \quad \forall j, k, g \in G_j, i \in \bar{I}_g \cap I_k, s. \tag{4.24}$$

If the final product i is not produced in factory j, then there will be no final product i's flow between factory j and market k.

$$\sum_{t} X^{P}_{ijk} \leq | k | \cdot E^{P}_{ij}, \quad \forall j, g \in G_j, i \in \bar{I}_g. \tag{4.25}$$

4.8. Profit

The supply chain's total profit is determined by summing the profits generated by suppliers (Pr^{S}_{h}), factories(Pr^{F}_{j}), and markets (Pr^{M}_{k}).

$$TotPr = \sum_{h} Pr^{S}_{h} + \sum_{j} Pr^{F}_{j} + \sum_{k} Pr^{M}_{k}. \tag{4.26}$$

4.8.1. Suppliers

The revenue of supplier h (Re^{S}_{h}) is derived from the total transfer payment received from all factories. This is calculated by multiplying the transfer prices of primary products by the corresponding shipment quantities between the supplier and each factory, and then by the probabilities associated with all scenarios.

$$Re^{S}_{h} = \sum_{t}\sum_{j}\sum_{s}\sum_{n} TP^{S}_{rh} \cdot F^{S}_{rhjts} \cdot \psi_s, \quad \forall h. \tag{4.27}$$

However, Eq. (4.27) contains nonlinear terms due to the product of decision variables, which will lead to a nonlinear optimization problem and complicate the solution process. To maintain linearity and reduce computational expenses, Eq. (4.27) is reformulated into a linear form by introducing auxiliary variables and constraints as follows:

$$\overline{OF}^{S}_{rhjlts} \leq MaxF^{S}_{rhj} \cdot O^{S}_{rhl}, \quad \forall r, h, j, l, t, s, \tag{4.28}$$

$$\sum_{l} \overline{OF}^{S}_{rhjlts} = F^{S}_{rhjts}, \quad \forall r, h, j, t, s, \tag{4.29}$$

$$Re^{S}_{h} = \sum_{t}\sum_{j}\sum_{l}\sum_{n}\sum_{s} TP^{S}_{hl} \cdot \overline{OF}^{S}_{hjlts} \cdot \psi_s, \quad \forall h. \tag{4.30}$$

The costs incurred by a supplier consist of the raw materials cost for primary products (RMC^{S}_{h}), cost of production (PC^{S}_{h}), cost of inventory (IVC^{S}_{h}), and cost of capital investment(CIC^{S}_{h}):

$$RMC^{S}_{h} = \sum_{t}\sum_{n}\sum_{s} MC^{S}_{rh} \cdot P^{S}_{rhts} \cdot \psi_s, \quad \forall h, \tag{4.31}$$

$$PC^S_h = \sum_n FPC^S_{rh} \cdot E^S_{rh} + \sum_t \sum_n \sum_s VPC^S_{rh} \cdot P^S_{rhts} \cdot \psi_s, \quad \forall h, \tag{4.32}$$

$$IVC^S_h = \sum_t \sum_n \sum_s IC^{PS}_{rh} \cdot IV^{PS}_{rhts} \cdot \psi_s, \quad \forall h, \tag{4.33}$$

$$CIC^S_h = crf \cdot CC^S_h \cdot \Delta Cap^S_h, \quad \forall h. \tag{4.34}$$

The profit of a supplier (Pr^S_h) is calculated as its revenue subtracted by all the associated costs mentioned above.

$$Pr^S_h = Re^S_h - RMC^S_h - PC^S_h - IVC^S_h - CIC^S_h, \quad \forall h. \tag{4.35}$$

4.8.2. Factories

A factory's revenue (Re^F_j) consists of all transfer payments obtained from markets, calculated based on the transfer prices of final products and the corresponding flows from the factory to the markets, multiplied by the probabilities associated with each scenario.

$$Re^F_j = \sum_{g\in G_j} \sum_t \sum_k \sum_{i\in I_g\cap I_k} \sum_s TP^p_{ijk} \cdot F^p_{ijkts} \cdot \psi_s, \quad \forall j. \tag{4.36}$$

The nonlinear Eq. (4.36) can be linearized using the same approach as applied to Eq. (4.27):

$$\overline{OF}^P_{ijklts} \leq MaxF^P_{ijk} \cdot O^P_{ijl}, \quad \forall g \in G_j, j, k, i \in \bar{I}_g \cap I_k, l, t, s, \tag{4.37}$$

$$\sum_l \overline{OF}^P_{ijklts} = F^P_{ijkts}, \quad \forall g \in G_j, j, k, i \in \bar{I}_g \cap I_k, t, s, \tag{4.38}$$

$$Re^F_j = \sum_t \sum_k \sum_{g\in G_j} \sum_{i\in I_g\cap I_k} \sum_l \sum_s TPL^P_{ijl} \cdot \overline{OF}^P_{ijklts} \cdot \psi_s, \quad \forall j. \tag{4.39}$$

The costs associated with factories include the cost of transfer payment to suppliers (TPC^F_j), primary product's raw materials cost (RMC^F_j), cost of formulation (FOC^F_j), cost of inventory (IVC^F_j), cost of product transportation (TRC^F_j), cost of capital investment (CIC^F_j) and duties (DUC^F_j).

$$TPC^F_j = \sum_t \sum_h \sum_l \sum_s TPL^S_{hl} \cdot \overline{OF}^S_{hjlts} \cdot \psi_s, \quad \forall j, \tag{4.40}$$

$$RMC^F_j = \sum_t \sum_{g\in G_j} \sum_{i\in \bar{I}_g} \sum_s MC^P_{ij} \cdot P^P_{ijts} \cdot \psi_s, \quad \forall j, \tag{4.41}$$

$$FOC^F_j = \sum_{g\in G_j} \sum_{i\in \bar{I}_g} FFC^P_{ij} \cdot E^P_{ij} + \sum_t \sum_{g\in G_j} \sum_{i\in \bar{I}_g} \sum_s VFC^P_{ij} \cdot P^P_{ijts} \cdot \psi_s, \quad \forall j, \tag{4.42}$$

$$IVC^F_j = \sum_t \sum_r \sum_s IC^{PP}_{rj} \cdot IV^{PP}_{njts} \cdot \psi_s + \sum_t \sum_{g\in G_j} \sum_{i\in \bar{I}_g} \sum_s IC^{FP}_{ij} \cdot IV^{FP}_{ijts} \cdot \psi_s, \quad \forall j, \tag{4.43}$$

$$TRC_j^F = \sum_r \sum_h FTC_{rhj}^S \cdot X_{rhj}^S + \sum_t \sum_r \sum_h \sum_s VTC_{rhj}^S \cdot F_{rhjts}^S \cdot \psi_s, \quad \forall j, \tag{4.44}$$

$$CIC_j^F = crf \cdot CC_j^F \cdot \Delta Cap_j^F, \quad \forall j, \tag{4.45}$$

$$DUC_j^F = \sum_t \sum_h \sum_l \sum_r \sum_s DC_{rhj}^S \cdot TPL_{rhl}^S \cdot \overline{OF}_{rhjlts}^S \cdot \psi_s, \quad \forall j. \tag{4.46}$$

Then, the profit of a factory (Pr_j^F) can be presented as follows:

$$Pr_j^F = Re_j^F - TPC_j^F - RMC_j^F - FOC_j^F - IVC_j^F - TRC_j^F - CIC_j^F - DUC_j^F, \quad \forall j. \tag{4.47}$$

4.8.3. Markets

The market's revenue (Re_k^M) is determined by the sales amounts of final product i in market k multiplied by the corresponding selling price (V_{ik}) and the probabilities in all scenarios:

$$Re_k^M = \sum_t \sum_{i \in I_k} \sum_s V_{ik} \cdot SA_{ikts} \cdot \psi_s, \quad \forall k. \tag{4.48}$$

The costs associated with each market include the cost of transfer payment to factories (TPC_k^M), the cost of inventory (IVC_k^M), the transportation cost of final product (TRC_k^M), duties (DUC_k^M), and the cost of lost sales (LSC_k^M).

$$TPC_k^M = \sum_t \sum_j \sum_{g \in G_j} \sum_{i \in \bar{I}_g \cap I_k} \sum_l \sum_s TPL_{ijl}^P \cdot \overline{OF}_{ijklts}^P \cdot \psi_s, \quad \forall k, \tag{4.49}$$

$$IVC_k^M = \sum_t \sum_{i \in I_k} \sum_s IC_{ik}^{FM} \cdot IV_{ikt}^{FM} \cdot \psi_s, \quad \forall k, \tag{4.50}$$

$$TRC_k^M = \sum_j \sum_{g \in G_j} \sum_{i \in \bar{I}_g \cap I_k} FTC_{ijk}^P \cdot X_{ijk}^P + \sum_s \sum_j \sum_{g \in G_j} \sum_{i \in \bar{I}_g \cap I_k} \sum_t VTC_{ijk}^P \cdot F_{ijkts}^P \cdot \psi_s, \quad \forall k, \tag{4.51}$$

$$DUC_k^M = \sum_s \sum_j \sum_{g \in G_j} \sum_{i \in \bar{I}_g \cap I_k} \sum_l \sum_t DC_{ijk}^P \cdot TPL_{ijl}^P \cdot \overline{OF}_{ijklts}^P \cdot \psi_s, \quad \forall k, \tag{4.52}$$

$$LSC_k^M = \sum_t \sum_{i \in I_k} \sum_s PC_{ik} \cdot LS_{ikts} \cdot \psi_s, \quad \forall k. \tag{4.53}$$

The profit of a market (Pr_k^M) can be represented by Eq. (4.54):

$$Pr_k^M = Re_k^M - TPC_k^M - IVC_k^M - TRC_k^M - DUC_k^M - LSC_k^M, \quad \forall k. \tag{4.54}$$

The proposed MILP model comprises Eq. (4.26) as the objective function, along with Eq. (4.1)–(4.25), (4.28)–(4.35), and (4.37)–(4.54) serving as constraints. To develop a fair profit distribution strategy focused on maximizing total profit, the Nash bargaining method will be discussed in the following section.

4.9. *Fair profit distribution*

Due to the different concerns reflected by different objectives, some managers may, depending on the situation, prioritize one objective excessively over others. Over time, this practice may also expose many vulnerabilities of the enterprise, which affects the long-term development of the supply chain. A number of examples show that the decision-makers of enterprises are very concerned about fairness in real life [73, 74]. Considering the supply chain optimization problem addressed in this work, the Nash bargaining method is employed as a normative fairness criterion to determine a mutually acceptable profit allocation. It provides an axiomatic standard for a fair division of the cooperative surplus, balancing overall efficiency with proportional equity among members based on their bargaining power, rather than simulating a dynamic negotiation process. The central element of this formulation is the disagreement point pr_m^{min}, which defines the minimum acceptable profit (or reservation profit) for supply chain member m to participate in cooperation. Conceptually, it represents the outcome each member will obtain if the agreement fails to be reached. A fair solution based on the Nash bargaining method can be achieved by maximizing the product of the excess profits, i.e., the profits in excess of the disagreement points, of all supply chain members.

$$\Psi = (Pr_m - Pr_m^{min})^{a_m}, \tag{4.55}$$

where a_m is the supply chain member m's bargaining power and Pr_m^{min} is the minimum profit of supply chain member m.

The aforementioned nonlinear Eq. (4.55) can be reformulated as follows by applying the logarithmic transformation:

$$\ln \Psi = \sum_{a_m} a_m \cdot \ln(Pr_m - Pr_m^{min}), \tag{4.56}$$

$$\ln \bar{\Psi} = \sum_{m} \sum_{q=1}^{Q} a_m \cdot \mu_q \cdot \ln(Pr_m - Pr_m^{min}), \tag{4.57}$$

$$\sum_{q=1}^{Q} \mu_q = 1, \tag{4.58}$$

$$Pr_m = \sum_{q=1}^{Q} \mu_q \cdot Pr_{mq}, \quad \forall m. \tag{4.59}$$

Overall, the proposed model for fair strategies can be described by Eqs. (4.1)–(4.25), (4.28)–(4.35), (4.37)–(4.54), (4.58), and (4.59) as constraints, and Eq. (4.57) as the objective function.

5. Solution approach

5.1. *Scenario tree construction*

In order to address the demand uncertainty, a scenario tree method is proposed, which discretizes the stochastic demand process while maintaining its temporal dynamics. This method discretizes continuous uncertainty into a finite set of representative scenarios, thereby making the optimization computationally manageable. The scenario tree can be regarded as a time-dependent probability tree that arises from discrete, known demand fluctuations. Starting from the initial or baseline demand, all possible fluctuations can be defined, resulting in various realizations of stochastic demand, which are represented by a specific number of potential scenarios. The demand changes sequentially over time within a defined range, and each movement is associated with an estimated probability. Every arc in the scenario tree corresponds to a specific probability, reflecting the likelihood of that particular scenario. This process is then repeated in the subsequent stages of the scenario tree [75]. Importantly, decisions made after the initial stage can be considered contingent plans that depend on future realizations of demand.

Let the demand for product i in market k in time period t and under scenario s be denoted by D_{ikts}. The scenario tree (Figure 2) is constructed as follows: The root node D_{ikt1} represents the initial demand. At each stage t, the demand evolves according to $D_{ik,t+1,s} = \zeta_{ts} \cdot D_{ikts}$, where ζ_{ts} is a three-way branching factor corresponding to low, medium, and high demand, with corresponding probabilities $p_s = \{0.25, 0.5, 0.25\}$ for each scenario s, respectively. This ensures that demand fluctuates symmetrically around its baseline value.

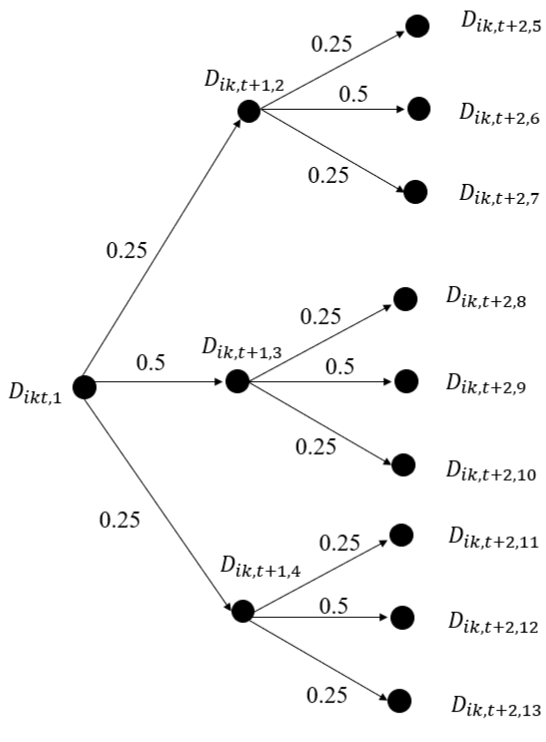


Figure 2. Scenario tree for demand uncertainty.

- Beginning with the initial value D_{ikts}, that is, the initial scenario.
- As a result, all the realizations of the stochastic demand, depicted by a specific number of potential scenarios, can be expressed as follows: for instance, $D_{ik,t+1,2} = 0.8D_{ik,t,1}$ and $D_{ik,t+1,4} = 1.2D_{ikt,1}$.
- $p(s)$ denotes the probability associated with scenario s, for example, $p(D_{ik,t+1,2}) = p(D_{ik,t+1,4}) = 0.25$.

5.2. Scenario reduction

Efficiently minimizing the number of scenarios is crucial to avoiding computationally intractable stochastic programs. There are two reasons to use the scenario reduction method [76]: First, it can obtain the best approximation as the target. Second, it can effectively solve the difficulties caused by too many scenarios, which often emerges in multi-stage optimization problems where numerous time steps and various scenarios make the optimization task computationally unmanageable. This highlights the attractiveness of methods that create discrete approximations using sparse scenario trees.

This section discusses two approaches: forward and backward construction, derived from successive forward and backward scenario reduction methods across multiple periods [75]. The goal of scenario reduction is to eliminate some scenarios from the original finite scenario set S while preserving the distribution characteristics of the remaining scenarios to minimize approximation errors. The probability distribution P of the original scenario set S is discrete with corresponding weights $p_e > 0$ for $e = 1, \ldots, S$, and $\sum_{e=1}^{S} p_e = 1$. Let $B \in \{1, \ldots, S\}$, and calculate the weighted distance between each candidate scenario $e \in \mathcal{S} \setminus B$:

$$\text{Impact}(e) = \sum_{f \neq e} p_f \cdot c_{|T|}(\zeta_e, \zeta_f), \quad \forall e \in \mathcal{S} \setminus B, \tag{5.1}$$

where ζ_e is the complete demand trajectory for scenario e and $c_{|T|}(\zeta_e, \zeta_f)$ is the distance measure (such as Euclidean distance) between scenario e and f over $|T|$ time periods.

Choose the scenario with the least importance e^*, and move it from candidate set $\mathcal{S} \setminus B$ to reserved set B. The probability of the remaining scenarios $f \in \mathcal{S} \setminus B$ is adjusted as follows:

$$q_f = p_f + \sum_{e \in B} p_e \cdot \frac{c_{|T|}(\zeta_e, \zeta_f)}{\sum_{ti \in \mathcal{S} \setminus B} c_{|T|}(\zeta_e, \zeta_c)}, \quad \forall f \in \mathcal{S} \setminus B, \tag{5.2}$$

where t_i is the temporary index for summing over all retained scenarios and $c_{|T|}(\zeta_e, \zeta_f)$ is the distance between removed scenario e and retained scenario f.

Thus, the optimal scenario reduction problem is formulated as

$$D_k(P, Q) = \min_B \sum_{i=1}^{S} p_i \cdot \min_{j \in \mathcal{S} \setminus B} c_{|T|}(\zeta_i, \zeta_j). \tag{5.3}$$

Therefore, scenario reduction algorithms utilizing this concept or similar ideas have been incorporated into software modules, such as those found in GAMS (see, for example, [77]).

5.3. Lagrangian relaxation approach

To tackle the proposed model with a substantial number of scenarios within a reasonable timeframe, an exact solution algorithm based on Lagrangian relaxation techniques is proposed for this problem. Lagrangian relaxation techniques [78] are a significant and widely-used tool in discrete optimization, serving as an exact algorithm for solving large-scale MILP and linear programming (LP) problems with specific structures. This method employs an iterative procedure that relaxes a set of complicating constraints, aiming to derive upper and lower bounds on the optimal solution of the original problem.

In this model, the constraint specified by Eq. (4.10) is relaxed, as doing so results in a more easily solvable model. By relaxing this constraint, the following equation can be obtained:

$$\begin{aligned}\ln\bar{\Psi} = &\sum_{m}\sum_{q=1}^{Q} a_m \cdot \mu_q \cdot \ln(pr_m - pr_m^{min}) \\ &+\mu \cdot (\sum_{n}\sum_{t}\sum_{k}\sum_{i} IV_{ikts}^{FM} - IV_{ik,t-1,s}^{FM} - \sum_{j\in J_g}\sum_{g\in\bar{G}_j} F_{ijk,t-\tau_{jk},s}^{P} + SA_{ikts}),\end{aligned} \tag{5.4}$$

where μ represents the Lagrangian multipliers that arise from relaxing the constraints defined by Eq. (4.10).

A successful technique for this problem is the Lagrangian relaxation approach. Therefore, the algorithm can be summarized as Algorithm 1.

Algorithm 1. Algorithm of Lagrangian relaxation approach.

Input: Initial upper bound L^*, Initial Lagrangian multipliers for relaxed constraints $\mu_0 \geq 0$, Initial step size parameter θ_0, maximum iterations j_{max}.

Set iteration counter j=1 best upper bound L^*
for $j \in [1, j_{max}]$ **do**
  $curUB = \ln\bar{\Psi}$
  if $curUB \leq L^*$ **then**
    $L^* = curUB$
  end if
  $\gamma^j = \sum_{s}\sum_{t}\sum_{k}\sum_{i} IV_{ikts}^{FM} - IV_{ik,t-1,s}^{FM} - \sum_{j\in J_g}\sum_{g\in\bar{G}_j} F_{ijk,t-\tau_{jk},s}^{P} + SA_{ikts}$ {subgradient}
  $t_j = \theta_j(L^* - curUB)/\left\|\gamma^j\right\|$ {step size}
  Update multipliers $\mu^{j+1} = max\{0, \mu^j + t_j\gamma^j\}$
  if $\left\|\mu^{j+1} - \mu^j\right\| \leq \varepsilon$ **then**
    Stop
  end if
  if no progress in more than K iterations **then**
    $\theta_{j+1} = \theta_j/2$
  else
    $\theta_{j+1} = \theta_j$
  end if
  $j = j + 1$
end for
return Outputs

where L^* represents the best upper bound obtained, $curUB$ refers to the current upper bound achieved at current iteration count, and θ is a parameter that is halved if there is no improvement in $curUB$ after several iterations. Numerous variants are possible regarding the calculation of the step size and the updating of the parameter θ [79]. As the algorithm iterates, both the Lagrangian multipliers and the step sizes are updated; subsequently, the upper and lower bounds are determined to compute the optimality gap.

6. Case study

In this section, a real-world supply chain network is considered, following the framework established by Liu and and Papageorgiou [80]. The network consists of one supplier (H1), eight factories (F1–F8), and 32 products (P1–P32) organized into 10 groups (G1–G10) with weekly demands distributed across 10 distinct regional markets (R1–R10). Additionally, 10 transfer price levels (L1–L10) are empirically chosen to balance solution flexibility and computational tractability. It's important to note that each final product is sold in select markets at varying prices. In this analysis, the currency unit is cu, and the mass unit is mu. It is assumed that all members of the supply chain possess equal bargaining power. For simplicity, it is assumed that $a_m = 1$.

The random parameters related to demand are generated into three possible values: $0.8D_{ikt}$, D_{ikt}, and $1.2D_{ikt}$. The probability of D_{ikt} is assigned as 0.5, and the probability of each of $0.8D_{ikt}$ and $1.2D_{ikt}$ is assigned as 0.25. Table 1 illustrates the transportation times (in weeks) from factories to markets, with the associated transportation costs in currency units equal to the transportation time in weeks. Table 2 provides the consumption details for each final product at each factory. The fixed formulation cost for transportation is set at 10 times both the unit variable production cost and the unit transportation cost. Additional data can be found in Liu and Papageorgiou [80].

Table 1. The transportation times from each factory to each market (week).

	R1	R2	R3	R4	R5	R6	R7	R8	R9	R10
F1	1	2	3	4	4	5	5	6	6	7
F2	1	2	3	4	4	5	5	6	6	7
F3	3	2	1	2	2	4	4	5	6	6
F4	4	4	2	1	1	3	3	4	5	5
F5	4	4	2	1	1	3	3	4	5	5
F6	4	4	2	1	1	3	3	3	5	5
F7	6	5	4	3	3	3	2	1	2	3
F8	6	6	5	5	5	4	3	2	1	2

Table 2. The consumption of each final product at each factory (cu/mu).

	F1	F2	F3	F4	F5	F6	F7	F8
P1	0.23	0.23	0.23	0.23	0.23	0.23	0.23	0.23
P2	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
P3	0.22	0.22	0.22	0.22	0.22	0.22	0.22	0.22
P4	0.20	0.20	0.20	0.20	0.20	0.20	0.2	0.20
P5	0.18	0.18	0.18	0.18	0.18	0.18	0.18	0.18
P6	0.27	0.27	0.27	0.27	0.27	0.27	0.27	0.27
P7	0.28	0.28	0.28	0.28	0.28	0.28	0.28	0.28
P8	0.13	0.13	0.13	0.13	0.13	0.13	0.13	0.13
P9	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24
P10	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30
P11	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24
P12	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15
P13	0.17	0.17	0.17	0.17	0.17	0.17	0.17	0.17
P14	0.23	0.23	0.23	0.23	0.23	0.23	0.23	0.23
P15	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16
P16	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24
P17	0.20	0.20	0.20	0.20	0.20	0.20	0.2	0.20
P18	0.22	0.22	0.22	0.22	0.22	0.22	0.22	0.22
P19	0.21	0.21	0.21	0.21	0.21	0.21	0.21	0.21
P20	0.22	0.22	0.22	0.22	0.22	0.22	0.22	0.22
P21	0.17	0.17	0.17	0.17	0.17	0.17	0.17	0.17
P22	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16
P23	0.18	0.18	0.18	0.18	0.18	0.18	0.18	0.18
P24	0.23	0.23	0.23	0.23	0.23	0.23	0.23	0.23
P25	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16
P26	0.27	0.27	0.27	0.27	0.27	0.27	0.27	0.27
P27	0.22	0.22	0.22	0.22	0.22	0.22	0.22	0.22
P28	0.19	0.19	0.19	0.19	0.19	0.19	0.19	0.19
P29	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2
P30	0.22	0.22	0.22	0.22	0.22	0.22	0.22	0.22
P31	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24
P32	0.19	0.19	0.19	0.19	0.19	0.19	0.19	0.19

The study takes into account a total of eight weeks and employs scenario fans comprising of 3,280 scenarios. To mitigate potential errors in custom implementations, publicly available software packages for moment matching and scenario reduction were used. The reduction scenario trees were generated by scenred.gms, which is provided with GAMS, allowing users to select the desired size of the tree. All implementations described in this work were carried out using GAMS 27.3 on a 64-bit Windows 11-based machine featuring a 3.2 GHz Intel Core i9 processor and 32 GB of RAM. The optimal gap was configured to 1%, and a CPU time limit of 7,200 seconds was imposed for each model execution.

6.1. Assessing the performance of the solution approach

In this section, the performance of the stochastic model for the Nash bargaining method (SNash) and the stochastic Lagrangian relaxation model for the Nash bargaining method (SLagNash) is first investigated by using several scenario trees. We set $\epsilon = 0.01$, $K = 3$, $\mu_0 = 0.5$, and $\theta_0 = 2$. Considering that a small number of reduced scenarios can result in poor solution results while a large number can lead to an exponential increase in solution time and complexity, we ultimately chose a range of 100 to 160 for comparison. In order to rigorously evaluate the proposed method, we evaluated the actual effectiveness of the solution through total profit. However, since the logarithmic results obtained under the Nash bargaining method in different scenarios did not differ significantly, they are not presented. Table 3 reports the optimization results of total profit under different numbers of generated scenario trees.

Table 3. Results for Nash bargaining method under different scenario reduction sizes.

	SLagNash		SNash	
Scenarios	Total profit (cu)	CPU time(s)	Total profit (cu)	CPU time(s)
100	253959	2160	203143	1320
110	279883	3462	229071	3310
120	292019	4520	283085	4657
130	320346	5756	307213	6689
140	346468	6284	No solution return	-
150	331619	7200	No solution return	-
160	279086	7200	No solution return	-

The results in Table 3 show a clear trade-off regarding the impact of scenario reduction. The use of a simplified tree containing 100 scenarios leads to suboptimal profits. When the number of retained scenarios increases to 130–140, profits increase significantly, indicating that a richer scenario set can better capture the tail risks and opportunities of demand uncertainty. However, after more than 140 scenarios, SLagNash's profits begin to fluctuate or even decline, which can be attributed to the increased complexity of the Lagrangian algorithm within the time limit. This highlights the crucial role of scenario simplification: It must find an "optimal point" that retains sufficient random information without making the problem computationally difficult to handle. The 130 scenarios selected for subsequent analysis represent this balance, generating high-quality solutions within an acceptable time frame.

The effectiveness of Lagrangian relaxation is demonstrated by the comparison of CPU time between SNash and SLagNash. For problems involving up to 130 scenarios, SNash can find solutions, although it requires longer computing time. However, for larger problems (greater than 140 scenarios), due to memory limitations, SNash cannot return solutions, while SLagNash is still feasible. This verifies the scalability of the Lagrangian relaxation method. Furthermore, by relaxing the inventory balance constraint, the Lagrangian dual problem yields an easily solvable decomposition, enabling the algorithm to effectively explore the solution space of large-scale instances and handle up to 160 scenarios.

As a result, the solutions derived from the reduced scenario trees tend to be overly optimistic. From the preceding results, we can infer that the outcomes based on larger trees are more representative of reality compared to those based on smaller trees, as larger trees encompass a more diverse range of scenarios. Consequently, the profits vary significantly when different scenario trees are utilized, suggesting that the accuracy of solutions deteriorates due to excessive reduction of the scenario tree. However, increasing the initial size results in a notable increase in computation time, making it challenging to achieve optimal results within the specified time. Therefore, it is essential to consider an appropriately sized scenario for reduction.

Next, the capacities of supplier and factories are examined as shown in Figure 3. Compared with stochastic programming, H1 exhibits greater capacity expansion under the Lagrangian relaxation model. F2, F5, and F6 also exhibit varying degrees of capacity increase. F2 has the largest capacity (over 380 mu/week), followed by F5 (over 350 mu/week). In the SLagNash model, due to the relaxation of inventory balance constraints, there is a greater tendency toward further expansion to maximize total profit.

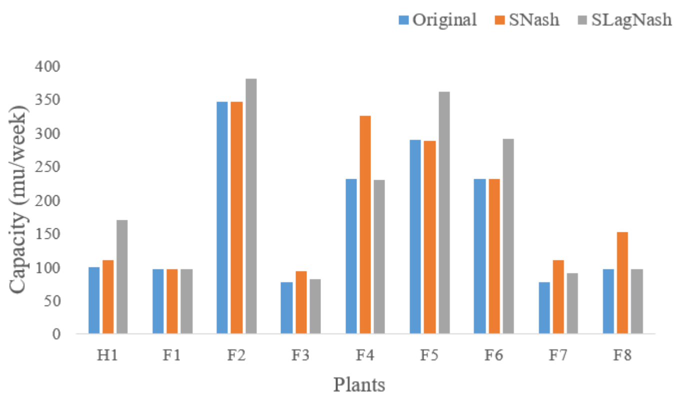


Figure 3. Capacities of each supply chain members under SNash and SLagNash model.

After obtaining the optimal number of reduced scenarios, Monte Carlo (MC) simulation is used to compare the results of the SNash and SLagNash models. MC simulation is used to evaluate the performance of the two models in relation to fair strategies. The MC simulation analysis is applied to the solutions derived from the Nash bargaining method. By fixing the design variable and using the capacity increments determined via the Nash bargaining method, the optimal values of the operational parameters are calculated in each iteration, taking into account random realizations of uncertain demands that follow specified distributions. The mean values of the total profit of the supply chain members are utilized to assess the performance of the supply chain in the MC simulation analysis. The procedure for the MC simulation is illustrated in Figure 4.

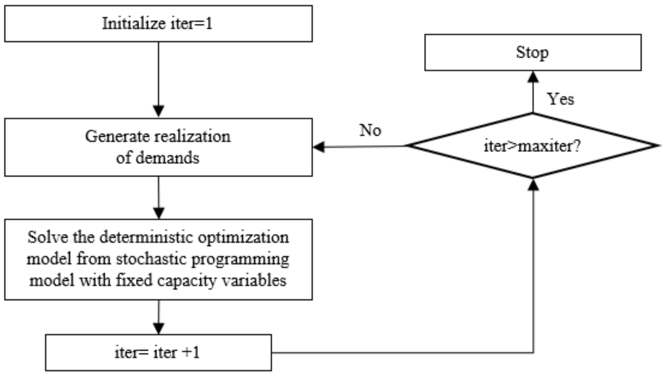


Figure 4. The procedure of MC simulation.

From Figure 5, it can be seen that, the mean total profit obtained through the Lagrangian relaxation algorithm is higher than that obtained through MILP.

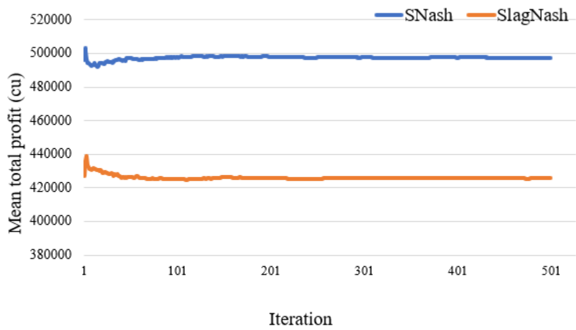


Figure 5. MC simulation of SNash model and SlagNash model.

Next, the influence of different parameters on the results is examined, as shown in Figures 6 and 7. As shown in Figure 6, the parameter ϵ controls the convergence accuracy, thereby affecting the CPU time and the quality of the final solution. A smaller ϵ enhances proximity to the true optimal solution but incurs excessive computational overhead: Near the optimal region, marginal improvements demand numerous additional iterations. Under the 7,200-second computational budget, an overly small ϵ is counterproductive; algorithms may waste resources on final refinement, risking timeout before convergence or insufficient time for high-quality feasible solutions. Conversely, a large ϵ causes premature termination and suboptimal solutions despite shorter runtime. Among tested values, ϵ=0.001 yields the highest solution quality but demands longer computation. ϵ=0.01 is more practical, representing the optimal compromise: It is sufficiently small to ensure convergence to a high-quality solution plateau while remaining large enough to reliably reach this plateau within the 7,200-second limit. This balance guarantees robust, convergent numerical results.

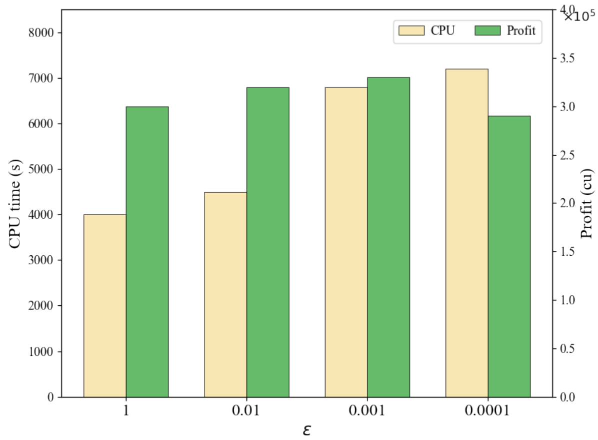


Figure 6. Results for SLagNash model under different ϵ values.

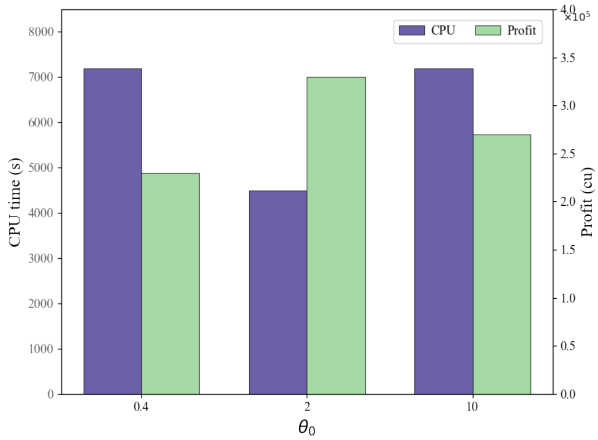


Figure 7. Results for SLagNash model under different θ_0 values.

Figure 7 demonstrates the impact of different initial step size θ_0 values on both CPU time and the final profit. Although the subgradient method is theoretically convergent within a certain step size range under asymptotic conditions, the 7200-second time limit renders the initial step size crucial. A large initial step size may cause severe oscillations in the early iterations. The algorithm has too many iterations to recover from the initial overshoot, resulting in delayed stable convergence. On the contrary, an excessively small initial step size may result in slow convergence speed. Although stable, small updates to Lagrangian multipliers will require a large number of iterations to fully penalize constraint violations and approach the optimal dual value, potentially leaving the algorithm far from convergence at termination and resulting in a suboptimal outcome. Therefore, setting $\theta_0 = 2$ achieves a robust balance, enabling positive initial progress toward the optimal region while maintaining stability.

As for the initial Lagrangian multipliers μ_0, since the initial multipliers are iteratively updated, the algorithm is not very sensitive to them and will not significantly change the quality of the final solution, although they may affect the convergence speed. Therefore, their results are no longer shown.

6.2. Assessing the performance of the stochastic programming solution

To facilitate further discussion, a reduction scenario tree for the stochastic demand is created comprising of 130 scenarios. This is accomplished by solving both the deterministic maximization model (DMax) and the stochastic maximization model (SMax). The DMax maximizes total profit under the assumption of fixed, known demand (e.g., historical averages). It ignores both demand uncertainty and fairness constraints, serving as a theoretical upper bound on profit. By comparing DMax with SNash and SLagNash, the trade-offs between profit, robustness, and equity in supply chain planning are quantified.

6.2.1. Optimal results without considering fairness

First, the optimal profit of each supply chain member under different models without considering fairness is obtained, as shown in Figure 8. From Figure 8, we can see that some supply chain members who do not consider fairness have negative profits, indicating a highly skewed profit distribution.

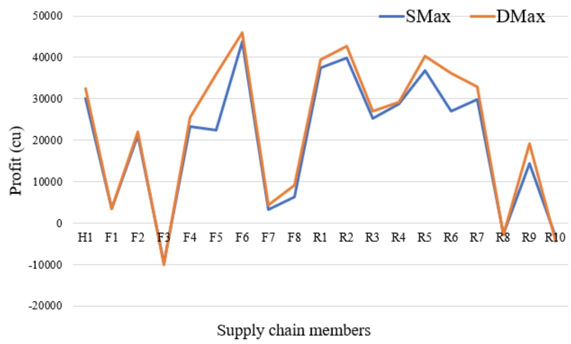


Figure 8. Results of DMax model and SMax model.

6.2.2. Optimal results considering fairness

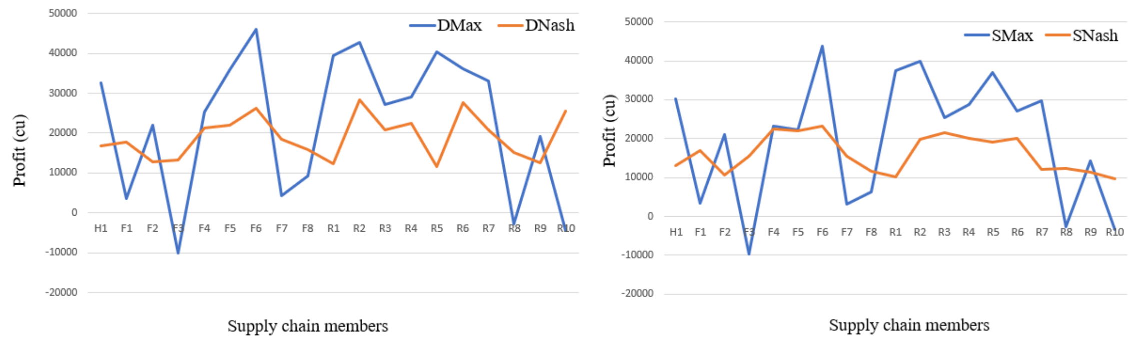
To assess the fairness of allocation as a crucial performance metric, Jain et al. [81] introduced a formula that offers a quantitative measure of the fairness in resource-sharing allocations: Jain's fairness index. This index is now widely recognized in the literature as a significant indicator of fairness [82, 83].

The formula of Jain's fairness index is as follows:

$$J(Pr_1, Pr_2, ..., Pr_n) = \frac{(\sum_{i=1}^{n} Pr_i)^2}{n \times \sum_{i=1}^{n} Pr_i^2}. \tag{6.1}$$

where $Pr_1, Pr_2, ..Pr_n$ represent the profit distribution of the i-th member. A larger value of J reflects a higher level of fairness.

After determining the fairness indicators and adopting the Nash bargaining method, the results from deterministic model (DNash) and the stochastic model (SNash) for the Nash bargaining method are obtained as shown in Figure 9 and Table 4.



(a) (b)

Figure 9. Comparison of (a) DMax model and DNash model and (b) SMax model and SNash model.

Table 4. Results of before and after using the Nash bargaining method.

	SMax	SNash	DMax	DNash
H1	30116	13145	30505	16858
F1	3377	16881	3415	17682
F2	21021	10742	21204	12779
F3	-9589	15581	-10100	13349
F4	23277	22375	23348	21250
F5	22336	21924	23045	21924
F6	43746	23262	44021	26135
F7	3173	15425	4100	18425
F8	6343	11506	8037	15771
R1	37402	10095	38463	12432
R2	39964	19815	44776	28250
R3	25338	21493	26052	20698
R4	28777	20146	29083	22346
R5	36914	19221	37181	11567
R6	27088	20079	28034	27588
R7	29763	12030	29867	20865
R8	-2579	12411	-3276	15191
R9	14276	11480	15001	12675
R10	-3276	9602	-4570	25609
Jain's index	0.61	0.93	0.60	0.92

Figure 9 shows that after applying the Nash bargaining method, the profit changes of supply chain members become smoother, and none of them have negative profits. In addition, Table 4 shows the profit of each supply chain member before and after considering fairness. The Jain's fairness index results in Table 4 provide quantitative evidence for addressing the core research questions of this study, which focuses on integrating the Nash bargaining method into a stochastic framework. The introduction of the Nash bargaining method enhances the fairness of profit distribution: Under the deterministic setting, the index rises from 0.60 (DMax) to 0.92 (DNash), and under the stochastic setting, from 0.61 (SMax) to 0.93 (SNash). First, comparison between SMax (0.61) and SNash (0.93) demonstrates that integrating the Nash bargaining method into the stochastic model drastically improves distributive justice, representing a substantial increase. More critically for our main contribution, SNash (0.93) achieves a marginally higher level of fairness than DNash (0.92). This indicates that the proposed stochastic framework does not merely preserve but can slightly enhance the fairness outcomes of the Nash bargaining method under uncertainty.

Next, the capacities of supplier and factories are examined, as shown in Figure 10. Both H1 and F7 increase their capacities under two models. F4 has the largest capacity of all under the stochastic programming model, which is more than 320mu/week, followed by F8, more than 150mu/week. Since only the first eight weeks of the plan (not the full 52 weeks) are considered in this study, and the demands for the first eight weeks are relatively small compared to those for the remaining weeks, some factories do not need to expand capacity to meet the demands.

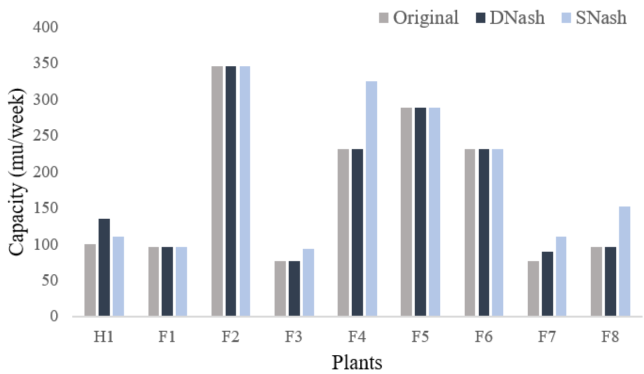
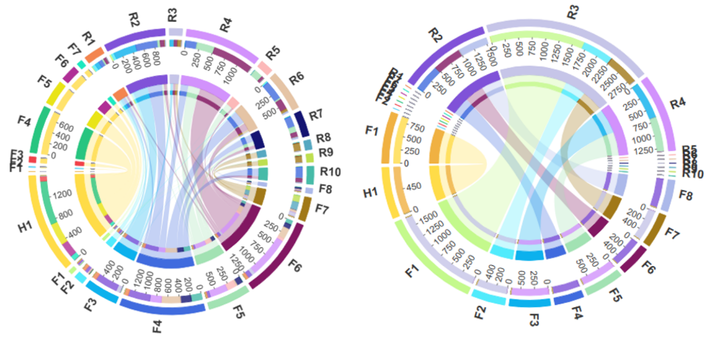


Figure 10. Capacities of each supply chain members under the Nash bargaining method.

The optimal transfer payments among all 19 supply chain members under the SNash and DNash models are illustrated using Circos in Figure 11. In this figure, the supply chain members are arranged in a circular layout, with colored links representing the transfer payments between each pair of members. In Figure 11(a), it is evident that the largest payment from factories to the supplier originates from F4, accounting for approximately a quarter of the total revenue generated by H1. The most significant payment links between factories and markets are observed from R2 to F3, R4 to F6, and R6 to F4, respectively.

Conversely, the optimal results from the DNash model are presented in Figure 11(b). The largest payment links between factories and markets are from R3 to F1, R2 to F8, and R4 to F3, respectively. Employing the Nash bargaining method enables fair profit distribution, as evidenced by increased payments between specific factories and markets compared to the original unfair profit allocations (e.g., enhanced payments involving F7, R8, and R10). This is demonstrated by F3 increasing payments to R8 and R9, F7 increasing payments to R8 and R9, and F4 increasing payments to R10.



(a) (b)

Figure 11. Optimal transfer payments under (a) SNash and (b) DNash, in thousand cu.

Next, the MC simulation is utilized to compare the results of the SNash and DNash models. As depicted in Figure 12, the total profit generated by the SNash model exceeds that of the DNash model during the MC simulation, indicating that the stochastic model is more adept at reflecting real-world conditions.

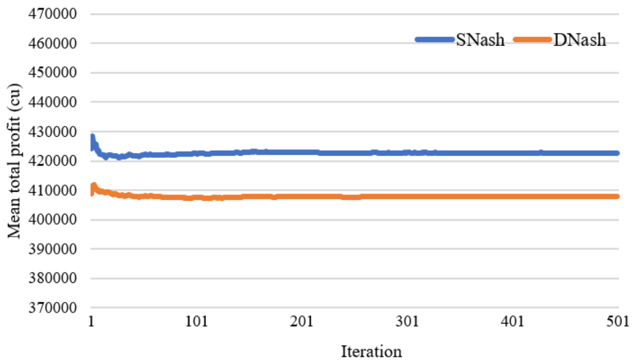


Figure 12. MC simulations of the SNash model and DNash model.

Meanwhile, the MC simulations of the two models under Jain's fairness index are also compared, as shown in Figure 13. Figure 13 shows that the two models exhibit similar fairness performance, around 0.93, confirming the effectiveness of the Nash bargaining method in addressing fairness issues.

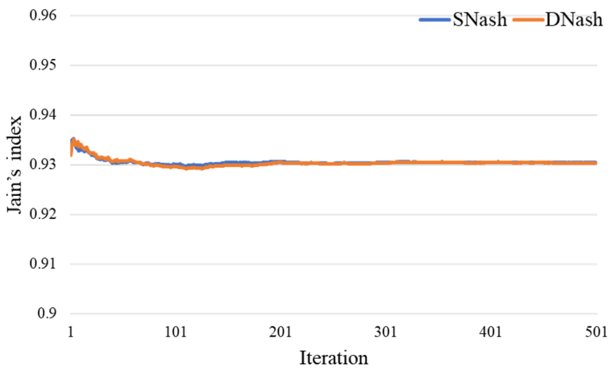


Figure 13. MC simulations of the SNash model and DNash model.

7. Conclusions

In this work, a production, distribution and capacity planning problem of a global supply chain with uncertain demands is addressed. A scenario-based two-stage stochastic programming optimization model is proposed. Furthermore, to effectively solve large-scale problems, a Lagrangian relaxation approach is introduced. The model's primary objective is to maximize total supply chain profit. Additionally, to address practical concerns about fairness in supply chain performance from multiple perspectives, the study incorporates the Nash bargaining method to evaluate fairness among supply chain members, utilizing the proportional fairness criterion as a metric.

The optimal solution of the model is analyzed through case studies, and the MC simulation is conducted to observe the model's performance in practical scenarios. The results demonstrate that explicitly embedding fairness mechanisms into uncertain decision-making environments significantly enhances supply chain resilience. Unlike deterministic models, which risk marginalizing vulnerable partners under demand shocks, the proposed framework dynamically adjusts profit allocation based on realized scenarios. Besides, the Lagrangian relaxation method can solve larger test problems. The results show that integrating the Nash bargaining method under a stochastic framework solves the problem of fair profit distribution under uncertain conditions. In addition, the stochastic model integrated with the Nash bargaining method slightly outperforms the deterministic counterpart in terms of profit and also shows a slight advantage in terms of fairness, according to the Jain's index.

This work also provides some management insights for supply chain managers. First, fairness is a key criterion for achieving sustainable supply chain development and fostering long-term cooperative trust. Based on the fairness of the Nash bargaining method, profit balance among all members of the supply chain can be achieved, as demonstrated in the case study. Managers must recognize that fairness is strategic, as it can mitigate destructive chain reactions and maintain partnerships in volatile markets. In addition, construction of large scenario trees can improve the accuracy of supply chain description and enable the supply chain to better cope with uncertainty. However, an excessively large number of scenarios can lead to an exponential increase in model-solving time and consume more manpower and resources, which is unnecessary. Therefore, determining an appropriate reduced size for the scenario tree is crucial. Finally, integrating uncertainty with fairness reduces the vulnerability to supply disruptions exacerbated by unfair practices (e.g., supplier bankruptcy during shortages), thereby establishing the robustness of the entire system. Managers should prioritize the proposed integrated model that combines fairness and uncertainty considerations.

This work does have several limitations. First, the study focuses solely on a single stochastic parameter. Future research could enhance the stochastic model by integrating additional planning decisions and uncertain parameters, such as cost and time, resulting in a more comprehensive framework for supply chain. Moreover, exploring alternative algorithms for solving complex problems (e.g., machine learning approaches) may yield valuable insights and solutions. Finally, applying alternative fairness criteria, such as α fairness and the Leximax criterion, could further enrich the research on fair supply chain planning.

Author contributions

Zijing Yang: Conceptualization, Data curation, Formal analysis, Investigation, Methodology, Software, Visualization, Writing - Original Draft. Songsong Liu: Conceptualization, Funding acquisition, Investigation, Methodology, Project administration, Resources, Supervision, Validation, Writing - Review & Editing. Boya Yu: Formal analysis, Investigation, Writing - Original Draft. All authors have read and approved the final version of the manuscript for publication.

Use of Generative-AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

Acknowledgments

This work is supported by the National Natural Science Foundation of China under Grants 72071055 and 72121001.

Conflict of interest

All authors declare no conflicts of interest in this paper.

References

1. S. Zhang, C. Zhang, S. Zhang, M. Zhang, Discrete switched model and fuzzy robust control of dynamic supply chain network, *Complexity*, **2018** (2018), 3495096. https://doi.org/10.1155/2018/3495096
2. J. Herkert, J. Borenstein, K. Miller, The Boeing 737 MAX: lessons for engineering ethics, *Sci. Eng. Ethics*, **26** (2020), 2957–2974. https://doi.org/10.1007/s11948-020-00252-y
3. C. X. Chen, J. Liang, S. Yang, J. Zhu, The bullwhip effect, demand uncertainty, and cost structure, *Contemp. Account. Res.*, **41** (2024), 195–225. https://doi.org/10.1111/1911-3846.12908
4. N. Jain, K. Girotra, S. Netessine, Recovering global supply chains from sourcing interruptions: the role of sourcing strategy, *Manuf. Serv. Oper. Manag.*, **24** (2022), 846–863. https://doi.org/10.1287/msom.2021.0967
5. R. Rajput, S. V. Venkataraman, A violent market price contract for agribusiness supply chain, *Ann. Oper. Res.*, **315** (2022), 1971–1996. https://doi.org/10.1007/s10479-021-04068-2
6. L. Wang, X. Wang, Y. Jin, Research on supply chain carbon emission reduction decisions based on fairness concerns and revenue sharing contracts, *Energy Rep.*, **14** (2025), 5756–5773. https://doi.org/10.1016/j.egyr.2025.11.113
7. Q. Feng, C. Li, M. Lu, J. G. Shanthikumar, Implementing environmental and social responsibility programs in supply networks through multiunit bilateral negotiation, *Manag. Sci.*, **68** (2022), 2579–2599. https://doi.org/10.1287/mnsc.2021.4034

8. J. Zarei, M. Reza Amin-Naseri, F. Safa Erenay, A. Elkamel, Subsidized and unsubsidized price competition in a multi-echelon natural gas supply chain with governmental and private members, *Comput. Ind. Eng.*, **164** (2022), 107894. https://doi.org/10.1016/j.cie.2021.107894
9. A. Sharma, S. Singh, Coordinating socially responsible supply chain with fairness via simple wholesale price contract, *J. Clean. Prod.*, **376** (2022), 134131. https://doi.org/10.1016/j.jclepro.2022.134131
10. N. M. Modak, S. Panda, S. Sinha, D. Ghosh, Implications of contract-bargaining mechanisms for coordination and profit sharing in a distribution channel, *Math. Probl. Eng.*, **2021** (2021), 3544374. https://doi.org/10.1155/2021/3544374
11. E. Dehghani, M. S. Jabalameli, A. Jabbarzadeh, Robust design and optimization of solar photovoltaic supply chain in an uncertain environment, *Energy*, **142** (2018), 139–156. https://doi.org/10.1016/j.energy.2017.10.004
12. J. Ghahremani, S. H. R. Pasandideh, S. T. A. Niaki, A robust optimization approach for multi-objective, multi-product, multi-period, closed-loop green supply chain network designs under uncertainty and discount, *J. Ind. Prod. Eng.*, **37** (2020), 1–22. https://doi.org/10.1080/21681015.2017.1421591
13. S. Mirzapour Al-E-Hashem, H. Malekly, M. B. Aryanezhad, A multi-objective robust optimization model for multi-product multi-site aggregate production planning in a supply chain under uncertainty, *Int. J. Prod. Econ.*, **134** (2011), 28–42. https://doi.org/10.1016/j.ijpe.2011.01.027
14. H. Mirzagoltabar, B. Shirazi, I. Mahdavi, A. A. Khamseh, Integration of sustainable closed-loop supply chain with reliability and possibility of new product development: a robust fuzzy optimisation model, *Int. J. Syst. Sci.: Oper. Logist.*, **10** (2023), 2119112. https://doi.org/10.1080/23302674.2022.2119112
15. K. Karimabadi, A. Arshadi-khamseh, B. Naderi, Optimal pricing and remanufacturing decisions for a fuzzy dual-channel supply chain, *Int. J. Syst. Sci.: Oper. Logist.*, **7** (2020), 248–261. https://doi.org/10.1080/23302674.2019.1570381
16. R. Gupta, I. Biswas, S. Kumar, Pricing decisions for three-echelon supply chain with advertising and quality effort-dependent fuzzy demand, *Int. J. Prod. Res.*, **57** (2019), 2715–2731. https://doi.org/10.1080/00207543.2018.1547434
17. A. M. Aguirre, S. Liu, L. G. Papageorgiou, Optimisation approaches for supply chain planning and scheduling under demand uncertainty, *Chem. Eng. Res. Des.*, **138** (2018), 341 – 357. https://doi.org/10.1016/j.cherd.2018.08.021
18. S. Zandkarimkhani, H. Mina, M. Biuki, K. Govindan, A chance constrained fuzzy goal programming approach for perishable pharmaceutical supply chain network design, *Ann. Oper. Res.*, **295** (2020), 425–452. https://doi.org/10.1007/s10479-020-03677-7
19. F. Darvishi, R. G. Yaghin, A. Sadeghi, Integrated fabric procurement and multi-site apparel production planning with cross-docking: A hybrid fuzzy-robust stochastic programming approach, *Appl. Soft Comput.*, **92** (2020), 106267. https://doi.org/10.1016/j.asoc.2020.106267
20. M. A. Quddus, S. Chowdhury, M. Marufuzzaman, F. Yu, L. Bian, A two-stage chance-constrained stochastic programming model for a bio-fuel supply chain network, *Int. J. Prod. Econ.*, **195** (2018), 27–44. https://doi.org/10.1016/j.ijpe.2017.09.019

21. M. Feitó-Cespón, Y. Costa, M. S. Pishvaee, R. Cespón-Castro, A fuzzy inference based scenario building in two-stage optimization framework for sustainable recycling supply chain redesign, *Expert Syst. Appl.*, **165** (2021), 113906. https://doi.org/10.1016/j.eswa.2020.113906

22. G. Gangil, A. Saraswat, S. K. Goyal, A new hybrid distance and similarity based scenario reduction approach for stochastic economic operation of microgrid, *e-Prime - Adv. Electr. Eng. Electron. Energy*, **11** (2025), 100905. https://doi.org/10.1016/j.prime.2025.100905

23. R. Jia, K. Du, Z. Song, W. Xu, F. Zheng, Scenario reduction-based simulation method for efficient serviceability assessment of earthquake-damaged water distribution systems, *Reliab. Eng. Syst. Saf.*, **246** (2024), 110086. https://doi.org/10.1016/j.ress.2024.110086

24. M. Kim, T. Park, J. Jeong, H. Kim, Stochastic optimization of home energy management system using clustered quantile scenario reduction, *Appl. Energy*, **349** (2023), 121555. https://doi.org/10.1016/j.apenergy.2023.121555

25. M. Zheng, W. Li, Y. Liu, X. Liu, A lagrangian heuristic algorithm for sustainable supply chain network considering co2 emission, *J. Clean. Prod.*, **270** (2020), 122409. https://doi.org/10.1016/j.jclepro.2020.122409

26. M. Song, L. Cheng, B. Lu, Solving the multi-compartment vehicle routing problem by an augmented lagrangian relaxation method, *Expert Syst. Appl.*, **237** (2024), 121511. https://doi.org/10.1016/j.eswa.2023.121511

27. Z. Rezaali, A. Ghodratnama, M. Amiri-Aref, R. Tavakkoli-Moghaddam, N. Wassan, Lagrangian relaxation method for solving a new time-dependent production–distribution planning model, *Expert Syst. Appl.*, **255** (2024), 124669. https://doi.org/10.1016/j.eswa.2024.124669

28. M. Yin, M. Huang, X. Wang, L. H. Lee, Fourth-party logistics network design under uncertainty environment, *Comput. Ind. Eng.*, **167** (2022), 108002. https://doi.org/10.1016/j.cie.2022.108002

29. M. Yin, H. Wang, Q. Liu, X. Qian, H. Zhang, X. Lang, A sampling-based winner determination model and algorithm for logistics service procurement auctions under double uncertainty, *Sci. Rep.*, **15** (2025), 12054. https://doi.org/10.1038/s41598-025-94748-x

30. J. Liu, H. Ke, R. Zhang, K. Duan, Who exhibits fairness concern is better for supply chain stability and the environment, *J. Clean. Prod.*, **386** (2023), 135645. https://doi.org/10.1016/j.jclepro.2022.135645

31. B. Hu, C. Meng, D. Xu, Y. J. Son, Supply chain coordination under vendor managed inventory-consignment stocking contracts with wholesale price constraint and fairness, *Int. J. Prod. Econ.*, **202** (2018), 21–31. https://doi.org/10.1016/j.ijpe.2018.05.009

32. P. S. Chow, T. M. Choi, T. C. E. Cheng, Impacts of minimum order quantity on a quick response supply chain, *IEEE Trans. Syst. Man, Cybern. A,*, **42** (2012), 868–879. https://doi.org/10.1109/TSMCA.2012.2183351

33. S. Cheng, F. Zhang, X. Chen, Optimal contract design for a supply chain with information asymmetry under dual environmental responsibility constraints, *Expert Syst. Appl.*, **237** (2024), 121466. https://doi.org/10.1016/j.eswa.2023.121466

34. A. Sohal, A. Bhattacharya, A. A. Nand, G. Croy, Broken food supply chains: priority norms for exchange partnerships in developing countries, *J. Clean. Prod.*, **374** (2022), 133964. https://doi.org/10.1016/j.jclepro.2022.133964

35. S. Liu, L. G. Papageorgiou, Fair profit distribution in multi-echelon supply chains via transfer prices, *Omega*, **80** (2018), 77–94. https://doi.org/10.1016/j.omega.2017.08.010

36. T. Cui, P. Mallucci, Fairness ideals in distribution channels, *J. Mark. Res.*, **53** (2016), 969–987. https://doi.org/10.1509/jmr.12.0093

37. F. Qin, F. Mai, M. J. Fry, A. S. Raturi, Supply-chain performance anomalies: fairness concerns under private cost information, *Eur. J. Oper. Res.*, **252** (2016), 170–182. https://doi.org/10.1016/j.ejor.2016.01.033

38. S. Sarkar, S. Bhala, Coordinating a closed loop supply chain with fairness concern by a constant wholesale price contract, *Eur. J. Oper. Res.*, **295** (2021), 140–156. https://doi.org/10.1016/j.ejor.2021.02.052

39. A. Adhikari, A. Bisi, Collaboration, bargaining, and fairness concern for a green apparel supply chain: An emerging economy perspective, *Transport. Res. Part E: Logist. Transp. Rev.* , **135** (2020), 101863. https://doi.org/10.1016/j.tre.2020.101863

40. Y. Liu, Y. Huang, Y. Luo, Y. Zhao, How does justice matter in achieving buyer–supplier relationship performance? *J. Oper. Manag.*, **30** (2012), 355–367. https://doi.org/10.1016/j.jom.2012.03.003

41. C. Lyu, S. Liu, Dual-fairness optimization for supply chain profit and emissions reduction distribution under carbon cap-sharing mechanism, *Int. J. Prod. Econ.*, **294** (2026), 109809. https://doi.org/10.1016/j.ijpe.2025.109809

42. X. X. Zheng, Z. Liu, K. W. Li, J. Huang, J. Chen, Cooperative game approaches to coordinating a three-echelon closed-loop supply chain with fairness concerns, *Int. J. Prod. Econ.*, **212** (2019), 92–110. https://doi.org/10.1016/j.ijpe.2019.01.011

43. K. Pan, Z. Cui, A. Xing, Q. Lu, Impact of fairness concern on retailer-dominated supply chain, *Comput. Ind. Eng.*, **139** (2020), 106209. https://doi.org/10.1016/j.cie.2019.106209

44. J. Jian, Y. Zhang, L. Jiang, J. Su, Coordination of supply chains with competing manufacturers considering fairness concerns, *Complexity,*, **2020** (2020), 4372603. https://doi.org/10.1155/2020/4372603

45. H. S. Bin-Obaid, T. B. Trafalis, Fairness in resource allocation: foundation and applications, in *Network Algorithms, Data Mining, and Applications*, Springer International Publishing, 2020, 3–18. https://doi.org/10.1007/978-3-030-37157-9_1

46. S. Ni, C. Feng, H. Gou, Nash-bargaining fairness concerns under push and pull supply chains, *Mathematics*, **11** (2023), 4719. https://doi.org/10.3390/math11234719

47. Z. P. Li, J. J. Wang, S. Perera, J. Shi, Coordination of a supply chain with nash bargaining fairness concerns, *Transport. Res. Part E: Logist. Transp. Rev.* , **159** (2022), 102627. https://doi.org/10.1016/j.tre.2022.102627

48. S. Li, S. Qu, M. I. M. Wahab, Y. Ji, Low-carbon supply chain optimisation with carbon emission reduction level and warranty period: Nash bargaining fairness concern, *Int. J. Prod. Res.*, **62** (2024), 6665–6687. https://doi.org/10.1080/00207543.2024.2333108

49. X. Du, S. Jiang, S. Tao, S. Wang, Cooperative advertising and coordination in a supply chain: the role of Nash bargaining fairness concerns, *RAIRO Oper. Res.*, **58** (2024), 1–18. https://doi.org/10.1051/ro/2023179

50. N. Gholamian, I. Mahdavi, N. Mahdavi-Amiri, R. Tavakkoli-Moghaddam, Hybridization of an interactive fuzzy methodology with a lexicographic min-max approach for optimizing a multi-period multi-product multi-echelon sustainable closed-loop supply chain network, *Comput. Ind. Eng.*, **158** (2021), 107282. https://doi.org/10.1016/j.cie.2021.107282

51. Y. Zhao, M. R. Ghiasvand, B. M. Tosarkani, Balanced uncertainty sets for closed-loop supply chain design: A data-driven robust optimization framework with fairness considerations, *Expert Syst. Appl.*, **276** (2025), 127170. https://doi.org/10.1016/j.eswa.2025.127170

52. X. Wu, J. A. Niederhoff, Fairness in selling to the newsvendor, *Prod. Oper. Manag.*, **23** (2014), 2002–2022. https://doi.org/10.1111/poms.12222

53. Z. Yang, S. Liu, Fairness-oriented multi-objective optimization of supply chain planning under uncertainties, *Socio-Econ. Plan. Sci.*, **99** (2025), 102198. https://doi.org/10.1016/j.seps.2025.102198

54. K. Govindan, M. Fattahi, Investigating risk and robustness measures for supply chain network design under demand uncertainty: A case study of glass supply chain, *Int. J. Prod. Econ.*, **183** (2017), 680–699. https://doi.org/10.1016/j.ijpe.2015.09.033

55. G. Schildbach, M. Morari, Scenario-based model predictive control for multi-echelon supply chain management, *Eur. J. Oper. Res.*, **252** (2016), 540–549. https://doi.org/10.1016/j.ejor.2016.01.051

56. R. Ghasemy Yaghin, Z. Farmani, Planning a low-carbon, price-differentiated supply chain with scenario-based capacities and eco-friendly customers, *Int. J. Prod. Econ.*, **265** (2023), 108986. https://doi.org/10.1016/j.ijpe.2023.108986

57. J. Chebeir, A. Geraili, J. Romagnoli, Development of shale gas supply chain network under market uncertainties, *Energies*, **10** (2017), 246. https://doi.org/10.3390/en10020246

58. L. Luo, X. Li, Y. Zhao, A two-stage stochastic-robust model for supply chain network design problem under disruptions and endogenous demand uncertainty, *Transp. Res. E Logist. Transp. Rev.*, **196** (2025), 104013. https://doi.org/10.1016/j.tre.2025.104013

59. E. Nikzad, M. Bashiri, F. Oliveira, Two-stage stochastic programming approach for the medical drug inventory routing problem under uncertainty, *Comput. Ind. Eng.*, **128** (2019), 358–370. https://doi.org/10.1016/j.cie.2018.12.055

60. C. Xie, Y. Tian, L. Wu, Multiperiod two-stage stochastic programming of supply chain for co-processing system of bio-oil and heavy oil, *ACS Sustainable Chem. Eng.*, **11** (2023), 3850–3860. https://doi.org/10.1021/acssuschemeng.2c07081

61. A. Marousi, J. M. Pinto, L. G. Papageorgiou, V. M. Charitopoulos, A stochastic programming framework for Nash bargaining in oligopolistic industrial gas markets with customer contracts, *AIChE J.*, **71** (2025), e70046. https://doi.org/10.1002/aic.70046

62. K. M. R. Pothireddy, S. Vuddanti, A hybrid lagrangian and improved class topper optimization for optimal sizing of battery energy storage system, *Smart Grids Sustain. Energy*, **10** (2025), 8. https://doi.org/10.1007/s40866-024-00234-0

63. E. Aghezzaf, Capacity planning and warehouse location in supply chains with uncertain demands, *J. Oper. Res. Soc.*, **56** (2005), 453–462. https://doi.org/10.1057/palgrave.jors.2601834

64. B. S. Pimentel, G. R. Mateus, F. A. Almeida, Stochastic capacity planning and dynamic network design, *Int. J. Prod. Econ.*, **145** (2013), 139–149. https://doi.org/10.1016/j.ijpe.2013.01.019

65. M. Yin, M. Huang, D. Wang, S. C. Fang, X. Qian, X. Wang, Multi-period fourth-party logistics network design with the temporary outsourcing service under demand uncertainty, *Comput. Oper. Res.*, **164** (2024), 106564. https://doi.org/10.1016/j.cor.2024.106564

66. C. Tang, J. Xu, Y. Tan, Y. Sun, B. Zhang, Lagrangian relaxation with incremental proximal method for economic dispatch with large numbers of wind power scenarios, *IEEE Trans. Power Syst.*, **34** (2019), 2685–2695. https://doi.org/10.1109/TPWRS.2019.2891227

67. A. Papavasiliou, S. S. Oren, B. Rountree, Applying high performance computing to transmission-constrained stochastic unit commitment for renewable energy integration, *IEEE Trans. Power Syst.*, **30** (2015), 1109–1120. https://doi.org/10.1109/TPWRS.2014.2341354

68. X. Qian, F. T. Chan, M. Yin, Q. Zhang, M. Huang, X. Fu, A two-stage stochastic winner determination model integrating a hybrid mitigation strategy for transportation service procurement auctions, *Comput. Ind. Eng.*, **149** (2020), 106703. https://doi.org/10.1016/j.cie.2020.106703

69. L. Wu, M. Shahidehpour, T. Li, Stochastic security-constrained unit commitment, *IEEE Trans. Power Syst.*, **22** (2007), 800–811. https://doi.org/10.1109/TPWRS.2007.894843

70. J. Gjerdrum, N. Shah, L. G. Papageorgiou, Fair transfer price and inventory holding policies in two-enterprise supply chains, *Eur. J. Oper. Res.*, **143** (2002), 582–599. https://doi.org/10.1016/S0377-2217(01)00349-6

71. G. P. Cachon, M. A. Lariviere, Supply chain coordination with revenue-sharing contracts: strengths and limitations, *Manag. Sci.*, **51** (2005), 30–44. https://doi.org/10.1287/mnsc.1040.0215

72. R. F. Göx, U. Schiller, An economic perspective on transfer pricing, vol. **2** of *Handbooks of Management Accounting Research*, Elsevier, 2006, 673–695. https://doi.org/10.1016/S1751-3243(06)02009-8

73. Y. Liu, Y. Chen, G. Yang, Developing multiobjective equilibrium optimization method for sustainable uncertain supply chain planning problems, *IEEE Trans. Fuzzy Syst.*, **27** (2018), 1037–1051. https://doi.org/10.1109/TFUZZ.2018.2851508

74. P. Jarumaneeroj, N. Laosareewatthanakul, R. Akkerman, A multi-objective approach to sugarcane harvest planning in thailand: Balancing output maximization, grower equity, and supply chain efficiency, *Comput. Ind. Eng.*, **154** (2021), 107129. https://doi.org/10.1016/j.cie.2021.107129

75. J. Dupačová, N. Groewe-Kuska, W. Roemisch, Scenario reduction in stochastic programming: an approach using probability metrics, *Math. Program.*, **95** (2003), 493–511. https://doi.org/10.1007/s10107-002-0331-0

76. H. Heitsch, W. Römisch, Scenario reduction algorithms in stochastic programming, *Comput. Optim. Appl.*, **24** (2003), 187–206. https://doi.org/10.1023/A:1021805924152

77. H. Heitsch, W. Römisch, Scenario tree modelling for multistage stochastic programs, *Math. Program.*, **118** (2009), 371–406. https://doi.org/10.1007/s10107-007-0197-2

78. M. L. Fisher, An applications oriented guide to lagrangian relaxation, *Interfaces*, **15** (1985), 10–21. https://doi.org/10.1287/inte.15.2.10

79. M. Held, P. Wolfe, H. P. Crowder, Validation of subgradient optimization, *Math. Program.*, **6** (1974), 62–88. https://doi.org/10.1007/BF01580223

80. S. Liu, L. G. Papageorgiou, Multiobjective optimisation of production, distribution and capacity planning of global supply chains in the process industry, *Omega*, **41** (2013), 369–382. https://doi.org/10.1016/j.omega.2012.03.007

81. R. K. Jain, D. M. W. Chiu, W. R. Hawe, A quantitative measure of fairness and discrimination for resource allocation in shared computer systems, *Tech. Rep.* TR-301, Digital Equipment Corporation, Maynard, MA, 1984. Available: `https://www.cs.wustl.edu/~jain/papers/ftp/fairness.pdf`

82. Z. Chen, C. Zhang, A new measurement fornetwork sharing fairness, *Comput. Math. Appl.*, **50** (2005), 803–808. https://doi.org/10.1016/j.camwa.2005.03.015

83. C. Guo, M. Sheng, Y. Zhang, X. Wang, A jain's index perspective on α-fairness resource allocation over slow fading channels, *IEEE Commun. Lett.*, **17** (2013), 705–708. https://doi.org/10.1109/LCOMM.2013.021913.130025