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Research article

A general approximate computational framework for basket spread options pricing with and without default risk

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Abstract: In this paper, we propose a synthetic computation framework for basket spread options pricing with and without default risk. Our approach is applicable whenever the underlying assets' joint moment-generating function is known, enabling both model-free pricing based solely on data and accurate closed-form pricing formulas under specified models that are up to a user's calibration. Specifically, we utilize a so-called Lau-Bjerksund-Stensland approach to handle the payoffs of basket spread options with and without default risk. By combining it with the Fourier-sinc method, we first derive a closed-form pricing formula for the non-defaultable basket spread options, which is more efficient than the existing ones in the literature. Furthermore, by measure-change techniques, we extend our model-free pricing framework to basket spread options with default risk. To the best of our knowledge, this is the first model-free pricing for such options, whereas previous research is based on specific models. Numerical examples demonstrate that our pricing formula exhibits high accuracy and robust performance across varying parameters. In particular, our model-free pricing formula still performs well even with limited data.

Keywords: basket spread option; default risk; closed-form pricing; Fourier-sinc method; approximation
Mathematics Subject Classification: 60G51, 62P20, 91B70, 91G20

1. Introduction

Basket spread options are financial derivatives whose terminal payoffs depend on the spread between two underlying baskets. In practice, the underlying assets of these options are diverse, encompassing currencies, securities, commodities, etc. (see [1]). From the perspective of mathematical finance, due to the intricacies associated with the weighted sum of the assets within underlying baskets, it is difficult to determine the exact distribution of this weighted sum, even under the classic Gaussian model. As a result, deriving closed-form pricing formulas for basket spread options is generally intractable.

Although Monte Carlo (MC) simulations serve as a versatile benchmark for pricing these options, it is computationally prohibitive, particularly when underlying assets are modeled by complex processes to better fit market dynamics. Therefore, in this paper, we aim to derive a closed-form approximate pricing formula for basket spread options. Furthermore, as a product typically traded over the counter, a holder of basket spread options is often exposed to default risks, which have been noted as a critical topic since the subprime crisis in 2008. In addition to the high-dimensional correlation of underlying assets, incorporating default risks adds another stochastic factor, further complicating the computations involved in pricing. Therefore, it is essential to extend the computational pricing methodology to account for default risks and develop a general pricing framework for basket spread options with and without default risk. Hereafter, we refer to basket spread options with default risk as defaultable basket spread options.

Before delving into the more complex pricing problems of basket spread options, a substantial body of literature has already explored the pricing methods for non-defaultable single-asset, two-asset, or path-dependent derivatives in depth. Traditional option pricing primarily relies on the Black-Scholes model, which has since been extended to a range of models that account for real-world market characteristics, including jump-diffusion models and stochastic volatility models. Within these frameworks, researchers have proposed various closed-form or numerical approximation methods, such as Fourier transform methods based on characteristic function [2–4], cosine expansion methods [5], etc. These approaches provide a robust theoretical foundation and numerical tools for pricing multi-asset options, such as basket spread options.

There has been considerable research focused on approximate pricing of non-defaultable basket spread options, some of which rely on specific underlying models while others do not. Under the Gaussian model, [6] used the European option pricing based on moment-matching methods to approximate the price of basket spread options. [7] employed comonotonic theory to establish the upper and lower bounds of the basket option prices, assuming that the weights of underlying assets are all positive. Meanwhile, [8] derived an approximate pricing formula for multi-asset basket spread options by extending [9]'s formula. For non-Gaussian scenarios, [10] developed a pricing method for basket options under the multivariate generalized hyperbolic model. [11] generalized the method of [7] to accommodate a stochastic interest rate model.

However, the underlying assets of basket spread options across different markets and asset classes often exhibit distinct price dynamics. Some studies focus on approaches that do not rely on a specific underlying asset dynamic model, such as utilizing Fourier-based methodologies. To name a few, [12] utilized the multidimensional fast Fourier transform (FFT) to price multi-asset options under regular affine models, which excludes important models such as mean-reversion models. [13] extended this approach to facilitate the pricing of options with underlying assets that adhere to a mean-reverting process. Despite these works, a notable disadvantage of multidimensional FFT is the well-known curse of dimensionality, that is, when the number of underlying assets is high, the performance of FFT-based pricing may drop significantly. Although [14] addressed this issue by proposing a lower bound for option prices based on approximate sets, the parameters involved in these methods must be optimized numerically, and appropriate damping factors need to be selected to ensure the existence of the Fourier transform.

In these regards, we propose a three-stage model-free* pricing methodology which extends the existing pricing framework for basket spread options (without default risk) to a significant extent. By contrast with the aforementioned works, our approach only requires the existence of the joint moment-generating function and does not depend on specific market models. More specifically, we first apply a so-called Lau-Bjerksund-Stensland (LBS) method† to decompose the payoff structure of basket spread options, effectively transforming the pricing problem into a sequence of complex expectation calculations. Next, we use the measure-change technique to re-express these expectations of pricing in terms of a series of equivalent probabilities. Finally, we employ the Fourier-sinc method proposed by [17] to calculate these probabilities, thereby deriving closed-form pricing formulas for basket spread options without default risk. Our proposed computational methodology establishes a unified pricing framework for basket spread options in model-free settings. Notably, unlike existing approaches such as that of [14], our framework does not rely on numerical optimization techniques for pricing.

In recent years, there has been a growing body of research on derivative instruments with default risk. Some of these studies adopt the structural model, where default is modeled as a firm's asset value hitting a predetermined barrier, while others employ the reduced-form model that treats default as a Poisson jump process. When default risk is incorporated into basket option pricing, the option value depends not only on the dynamics of the underlying asset prices but also on the correlation between default events and asset prices, thereby providing a more realistic representation of market behaviour. Although research specifically targeting defaultable basket spread options remains relatively limited, several studies have attempted to extend classical option pricing frameworks to incorporate default risk (see, e.g., rainbow option [18], spread option [19], power exchange option [20], lookback option [21] etc.). These efforts offer both theoretical foundations and practical motivations for the defaultable basket spread pricing framework proposed in this paper.

In contrast to the extensive literature on basket spread options without default risk, research on pricing such instruments under credit risk remains relatively limited. Existing studies primarily focus on specific underlying models (see stochastic volatility model [22], heston-nandi garch model [23], hawkes jump-diffusion model [24], stochastic liquidity model [25]) and share a common methodological foundation, i.e. they employ the Vorst-Bjerksund-Stensland (VBS) approximation method to handle the payoff structure of basket spread options. However, as discussed in Section 2.1, the VBS method's approximation exhibits inferior performance compared to the more efficient LBS method. To address this, we further extend the previously developed pricing framework of basket spread options without default risk to the defaultable basket spread options. In doing so, we consider both reduced-form and structural approaches to modeling default risk, as the two are the most widely used in the literature. Such a pricing framework of defaultable basket spread options is again closed-form and model-free.

Compared to existing literature, our work contributes to basket spread option pricing in three ways. First, our pricing framework operates in a model-free setting and is applicable whenever the joint moment-generating function of underlying assets is known. To the best of our knowledge, we are the first to develop a model-free pricing framework for defaultable basket spread options in the literature. Moreover, this model-free pricing framework can also provide general pricing formulas for arbitrary market models, which

*Following the terminology in [14], we use the term "model-free" to indicate that the framework is applicable to a wide range of market settings where the joint moment-generating function exists. Furthermore, our proposed framework offers data-driven pricing capabilities when the empirical joint moment-generating function is used as a proxy based on data. For details, please refer to Section 5.2.

†We name the LBS method and the VBS method (next paragraph) following the approximate pricing methods in [8, 15, 16]. These methods suggest pricing options by adjusting exercise regions. We introduce the two methods in detail in Section 2.1.

is especially valuable in scenarios where the underlying processes are intricate to calibrate. Second, we employ the LBS approach combined with the Fourier-sinc method to price basket spread options, providing a tailor-made pricing procedure for these derivatives. On one hand, compared to existing studies on basket spread option pricing without default risk, integrating the Fourier-sinc method can significantly enhance the computational accuracy and efficiency, compared to the algorithm proposed by [14]. Moreover, the LBS-based payoff approximation avoids optimization methods, making our approach more efficient and tractable. On the other hand, when accounting for default risk, the single-step approximation inherent in the LBS method significantly improves computational accuracy compared to the two-step approximation required in the VBS approach, which has been used in many studies in the literature. In other words, our pricing formulas provide insights into improving the VBS-based formulas. Third, our numerical results demonstrate that our pricing formula exhibits high accuracy and robust performance across parameter changes. When compared to the notable work of [14], our pricing formula exhibits enhanced accuracy, robustness, and computational efficiency, saving almost 50% of the computation time under equivalent computational conditions. Meanwhile, we also demonstrate the effectiveness of our model-free pricing formulas in a data-driven way. Numerical results show that the pricing results obtained using the empirical joint moment-generating function, fitted with market-available data, provide satisfactory outcomes, even when the data is limited.

The rest of this paper is structured as follows. Section 2 introduces the mathematical framework and basic methods, including the VBS method, LBS method, and the Fourier-sinc method. Section 3 presents the proposed model-free pricing formula for basket spread options. Section 4 provides the model-free pricing formulas for defaultable basket spread options, where default risk is modeled by the reduced-form approach and the structured method approach in Section 4.1 and Section 4.2, respectively. Section 5 presents numerical analyses to assess the performance of our pricing formulas. Finally, Section 6 concludes our work.

2. Preliminary

We consider an arbitrage-free financial market consisting of $n+m$ tradable assets $S_i(t)$, $i = 1, 2, \cdots n+m$, which are equipped in a filtered probability space $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\geq 0}, \mathbb{Q})$ in a finite time horizon $[0, T]$. Here, $(\mathcal{F}_t)_{t\geq 0}$ is a filtration that satisfies regular assumptions, and $\mathbb{Q}$ denotes the risk-neutral probability measure. Without loss of generality, we assume that the underlying assets of the basket spread option are long on the first n assets but short on the remaining m assets. Then, the payoff of a basket spread option is given by

$$(\sum_{i=1}^{n} w_i S_i(T) - \sum_{i=n+1}^{n+m} w_i S_i(T) - K)^+, \tag{2.1}$$

where $S_i(T)$ is the price of i-th underlying asset at maturity T, $w_i > 0$ represents the weight of the i-th asset in the entire underlying assets, $(\cdot)^+$ denotes $\max(\cdot, 0)$, and K is the strike price. In particular, when $n = m = 1$ and $w_i = 1$, this payoff reduces to a spread option (see, e.g., [16, 26]); when $m = 0$, this payoff reduces to a basket option (see [7, 11]).

2.1. VBS and LBS approximation method

Both LBS and VBS methods calculate approximate prices of basket options by adjusting the exercise regions involved in the price expectations. We first introduce the VBS method, which is extensively utilized in the relevant literature addressing the pricing of payoffs similar to basket spread options (see, e.g., stochastic volatility model [22], heston-nandi garch model [23], hawkes jump-diffusion model [24], stochastic liquidity model [25], stochastic interest rate model [27]). This approach first approximates the expiration payoff of the basket spread option to align with the payoff structure of a standard spread option, as described in [15]. Subsequently, it approximates this spread option payoff using the method outlined in [16]. Specifically, the VBS method approximates the payoff of the basket spread option in (2.1) as follows:

$$(\prod_{i=1}^{n} S_i(T)^{w_i} - \prod_{i=n+1}^{n+m} S_i(T)^{w_i} - \bar{K}) \cdot I(\hat{A}),$$

where $\bar{K} = K + \mathbb{E}_{\mathbb{Q}}[\prod_{i=1}^{n} S_i(T)^{w_i}] - \mathbb{E}_t^{\mathbb{Q}}[\sum_{i=1}^{n} w_i S_i(T)] - \mathbb{E}_{\mathbb{Q}}[\prod_{i=n+1}^{n+m} S_i(T)^{w_i}] + \mathbb{E}_{\mathbb{Q}}[\sum_{i=n+1}^{n+m} w_i S_i(T)]$, and

$$\hat{A} = \left\{\omega : \frac{\prod_{i=1}^{n} S_i(T)^{w_i}}{(\prod_{i=n+1}^{n+m} S_i(T)^{w_i})^{\alpha}} > \frac{\hat{a}}{\mathbb{E}^{\mathbb{Q}}[(\prod_{i=n+1}^{n+m} S_i(T)^{w_i})^{\alpha}]}\right\},$$

with $\hat{a} = \mathbb{E}_{\mathbb{Q}}[\prod_{i=n+1}^{n+m} S_i(T)^{w_i}] + \bar{K}$, $\alpha = \mathbb{E}_{\mathbb{Q}}[\prod_{i=n+1}^{n+m} S_i(T)^{w_i}]/(\mathbb{E}_{\mathbb{Q}}[\prod_{i=n+1}^{n+m} S_i(T)^{w_i}] + \bar{K})$.

Note that the VBS method requires two steps of approximation; it thus might perform less effectively in high-dimensional cases. Therefore, we propose the LBS approximation method to approximate the payoff of the basket spread option, which only requires one approximation step. In particular, the LBS method approximates the payoff of the basket spread option in (2.1) as follows:

$$(\sum_{i=1}^{n} w_i S_i(T) - \sum_{i=n+1}^{n+m} w_i S_i(T) - K) \cdot I(A),$$

where

$$A = \left\{\omega : \frac{\prod_{i=1}^{n}(w_i S_i(T))^{b_i}}{\mathbb{E}_{\mathbb{Q}}[\prod_{i=1}^{n}(w_i S_i(T))^{b_i}]} \geq \frac{a \cdot \prod_{i=n+1}^{n+m}(w_i S_i(T))^{b_i}}{\mathbb{E}_{\mathbb{Q}}[\prod_{i=n+1}^{n+m}(w_i S_i(T))^{b_i}]}\right\} \tag{2.2}$$

is the LBS exercise region that can be obtained by applying an identity transformation to [8]'s pricing formula under the Gaussian model, and

$$a = \frac{\sum_{i=n+1}^{n+m} w_i S_i(0) e^{rT} + K}{\sum_{i=1}^{n} w_i S_i(0) e^{rT}}, \quad b_i = \begin{cases} w_i S_i(0) e^{rT} / (\sum_{i=1}^{n} w_i S_i(0) e^{rT}), & i = 1, 2, \cdots, n, \\ w_i S_i(0) e^{rT} / (\sum_{i=n+1}^{n+m} w_i S_i(0) e^{rT} + K), & i = n+1, n+2, \cdots, n+m. \end{cases}$$

Using the LBS method, the price of the basket spread option breaks down to several complex expectations. Then, we can use the measure-change technique to convert these expectations into one-dimensional probabilities, which is particularly helpful in tackling the pricing of high-dimensional basket spread options. We then utilize the Fourier-sinc method to calculate these probabilities.

Remark 2.1. Although initially proposed within a Gaussian framework, the effectiveness of the VBS and LBS methods as approximation techniques for multi-asset options under non-Gaussian models has been demonstrated by extensive research, see, e.g., stochastic volatility model [22], heston-nandi garch model [23], hawkes jump-diffusion model [24], stochastic liquidity model [25], stochastic interest rate model [27].

2.2. *Fourier-sinc method*

The Fourier-sinc method, similar to the Fourier-cosine method proposed by [28], is a technique suitable for calculating one-dimensional probabilities. This approach builds upon the Shannon sampling theorem, adapted for functions with bounded support. The numerical results from [17] demonstrate that the Fourier-sinc method outperforms both the Fourier-cosine method and the one-dimensional Fourier transform using the FFT method. We present the following proposition on using the Fourier-sinc method to calculate one-dimensional probabilities.

Proposition 2.1. Let g be the moment-generating function of X, and let there exist a constant x. Then, based on the Fourier–sinc method outlined by [17], we can compute the probability that X exceeds x, i.e., $\mathbb{Q}(X > x)$, as follows:

$$\Psi_g(x) = \frac{1}{2} - \frac{2}{\pi}\sum_{n=1}^{N/4} \frac{1}{2n-1}\left[\sin(2\pi v_{2n-1}x)\,\mathrm{Re}(g(2\pi i v_{2n-1})) - \cos(2\pi v_{2n-1}x)\,\mathrm{Im}(g(2\pi i v_{2n-1}))\right], \quad v_n = \frac{n}{X_h - X_l},$$

where i is the imaginary unit, $\mathrm{Re}(\cdot)$ denotes the real part, and $\mathrm{Im}(\cdot)$ denotes the imaginary part. $[X_l, X_h]$ is the artificial truncation interval for X. Following [29], we select $[X_l, X_h] = [\gamma_1 - c\sqrt{\gamma_2},\ \gamma_1 + c\sqrt{\gamma_2}]$ and $\gamma_1 = g'(0)$, $\gamma_2 = g''(0)$, where c is an empirical constant, and g' and g'' denote the first and second derivatives of g, respectively.

In the following sections, we first investigate the basic problem of pricing basket spread options without default risk, and then we delve into the more complex task of pricing defaultable basket spread options.

3. Pricing basket spread options without default risk

With the payoff outlined in equation (2.1), the price of a basket spread option can be denoted by

$$C_0 = e^{-rT}\mathbb{E}_{\mathbb{Q}}[(\sum_{i=1}^{n} w_i S_i(T) - \sum_{i=n+1}^{n+m} w_i S_i(T) - K)^+]. \tag{3.1}$$

By directly integrating the payoff with the exercise region in (2.2), similarly to the approach for spread options outlined by [16], we can initially derive a lower bound for the price of the basket spread option expressed as $e^{-rT}\mathbb{E}^{\mathbb{Q}}[(\sum_{i=1}^{n} w_i S_i(T) - \sum_{i=n+1}^{n+m} w_i S_i(T) - K)I(A)]$. However, this lower bound might be negative under specific parameter conditions associated with low-probability events, as noted by [30]. Thus, we improve the lower bound by applying Jensen's inequality, which ensures that the approximated price remains non-negative:

$$C_0 \geq e^{-rT}\Big[\Big(\mathbb{E}_{\mathbb{Q}}[(\sum_{i=1}^{n+m} \epsilon_i w_i S_i(T) - K)I(A)]\Big)^+ + \Big(\mathbb{E}_{\mathbb{Q}}[(\sum_{i=1}^{n} \epsilon_i w_i S_i(T) - K)I(A^c)]\Big)^+\Big] \triangleq C_0^{LB},$$

where $\epsilon_i = 1$ for $i = 1, 2, \cdots, n$ and $\epsilon_i = -1$ for $i = n+1, n+2, \cdots, n+m$. To derive a model-free pricing formula, we require the existence of the joint moment-generating function. We define $f(\phi)$ as the moment-generating function of the underlying asset returns:

$$f(\phi) = \mathbb{E}_{\mathbb{Q}}[e^{\sum_{i=1}^{n+m} \phi_i \ln(S_i(T))}],$$

where $\phi = [\phi_1, \phi_2, \cdots, \phi_{n+m}]$. Applying the measure-change technique in conjunction with Proposition 2.1, we derive the pricing formula for the basket spread call option in Theorem 3.1.

Theorem 3.1. The approximated price of basket spread call option in (3.1) is given by

$$C_0^{LB} = LB^+ + (F - LB)^+,$$

where

$$LB = \sum_{k=1}^{n+m} \epsilon_k w_k S_k(0)\Psi_{f_k}(\hat{L}) - Ke^{-rT}\Psi_{f_0}(\hat{L}), \quad F = \sum_{k=1}^{n+m} \epsilon_k w_k S_k(0) - Ke^{-rT},$$
$$f_0(u) = f(u\boldsymbol{b})e^{-u\boldsymbol{b}\otimes\ln(\boldsymbol{S}(0))}, \quad f_k(u) = f(u\boldsymbol{b} + \boldsymbol{e}_k)e^{-(u\boldsymbol{b}+\boldsymbol{e}_k)^T\otimes\ln(\boldsymbol{S}(0))-rT},$$

with $\hat{L} = \ln(\frac{L}{\boldsymbol{b}\cdot\ln(\boldsymbol{S}(0))})$, $\boldsymbol{b} = [b_1, \cdots b_n, -b_{n+1}, \cdots, -b_{n+m}]$, and $\boldsymbol{e}_k$ is a vector with 1 in the k-th position and 0 in all other positions. $\otimes$ denotes the dot product of vectors, and the expression of $\Psi_{\cdot}(\cdot)$ is given in Proposition 2.1.

Proof. See Appendix A.

□

Remark 3.1. It is noteworthy that, by employing the LBS method to address the execution region of the options, we successfully convert the original multidimensional execution region into a one-dimensional execution region. In this process, we do not require the optimization of specific parameters, which is essential in the work of [14].

4. Pricing basket spread options with default risk

In this section, we turn our attention to pricing defaultable basket spread options. We employ two popular models to incorporate default risk: the reduced-form model, which is discussed in Section 4.1, and the structural model, covered in Section 4.2.

4.1. Reduced-form model

In this section, we capture the default risk using the reduced-form model, which is introduced by [31]. Specifically, the default time τ of option issuers is modeled as the first jump time of a Cox process with the intensity process $\{\lambda_t\}_{0\leq t\leq T}$. We postulate a recovery rate of δ, signifying that in the event of a credit loss, the option holder is entitled to a fraction δ of the option value. Then, the payoff of defaultable basket spread options under the reduced-form model is adjusted as

$$(\sum_{i=1}^{n} w_i S_i(T) - \sum_{i=n+1}^{n+m} w_i S_i(T) - K)^+ \cdot (I(\tau > T) + \delta I(0 \leq \tau \leq T)). \tag{4.1}$$

We rewrite the payoff in (4.1) as

$$\delta(\sum_{i=1}^{n} w_i S_i(T) - \sum_{i=n+1}^{n+m} w_i S_i(T) - K)^+ + (1-\delta)I(\tau > T)(\sum_{i=1}^{n} w_i S_i(T) - \sum_{i=n+1}^{n+m} w_i S_i(T) - K)^+.$$

Then, we obtain the lower bound of defaultable basket spread options under the reduced-form model,

$$C_r^{LB} = \delta \cdot C_0^{LB} + (1-\delta)C_r, \tag{4.2}$$

where

$$C_r = e^{-rT}[(\mathbb{E}_{\mathbb{Q}}[(\sum_{i=1}^{n+m} \epsilon_i w_i S_i(T) - K) \cdot I(A, \tau > T)])^+ + (\mathbb{E}_{\mathbb{Q}}[(\sum_{i=1}^{n+m} \epsilon_i w_i S_i(T) - K) \cdot I(A^c, \tau > T)])^+].$$

In order to investigate the pricing of defaultable basket spread options within a model-free framework, an additional stochastic source driven by default risk must be incorporated into the joint moment-generating function. Mathematically, we consider the joint moment-generating function, $f^r(\boldsymbol{\phi}, \psi)$, defined as

$$f^r(\boldsymbol{\phi}, \psi) = \mathbb{E}_{\mathbb{Q}}[e^{\sum_{k=1}^{n+m} \phi_k \ln(S_k(T)) + \psi \int_0^T \lambda_u du}],$$

where $\boldsymbol{\phi} = [\phi_1, \phi_2, \cdots, \phi_{n+m}]$. Using the measure-change technique along with Proposition 2.1, we have the following theorem.

Theorem 4.1. The approximated price of the defaultable basket spread call option under the reduced-form model is given by

$$C_r^{LB} = \delta \cdot C_0^{LB} + (1-\delta)C_r,$$

where

$$C_r = LB_r^+ + (F_r - LB_r^+), \quad F_r = e^{-rT}(\sum_{k=1}^{n+m} \epsilon_k w_k f^r(\boldsymbol{e}_k, -1) - K f^r(\boldsymbol{0}, -1)),$$

$$LB_r = e^{-rT}(\sum_{k=1}^{n+m} \epsilon_k w_k f^r(\boldsymbol{e}_k, -1)\Psi_{f_k^r}(\hat{L}) - K f^r(\boldsymbol{0}, -1)\Psi_{f_0^r}(\hat{L})),$$

$$f_0^r(u) = f^r(u\boldsymbol{b}, -1)e^{-u\boldsymbol{b}\otimes\ln(\boldsymbol{S}(0))}, \quad f_k^r(u) = f^r(u\boldsymbol{b} + \boldsymbol{e}_k, -1)e^{-u\boldsymbol{b}\otimes\ln(\boldsymbol{S}(0))}/f^r(\boldsymbol{e}_k, -1),$$

and the expression of $\hat{L}$, $\boldsymbol{b}$, and $\boldsymbol{e}_k$ are given in Theorem 3.1, $\Psi_{\cdot}(\cdot)$ is given by Proposition 2.1.

Proof. See Appendix B. □

4.2. Structural model

In this section, we consider the default risk with the structural model as in [32]. That is, the option issuer defaults when the market value of the option issuer's asset $V(T)$ is less than a specific threshold of $D^{*‡}$. If a credit loss occurs, $(1-z)V(T)$ is paid to compensate its creditors, where z indicates the cost ratio of the insolvency proceedings. If the liabilities of the option issuer are of the same priority, the option holder receives the original option value of $\frac{(1-z)V(T)}{D}$. Therefore, under the structured model, the maturity payoff of the defaultable basket spread option is

$$(\sum_{i=1}^{n} w_i S_i(T) - \sum_{i=n+1}^{n+m} w_i S_i(T) - K)^+ \cdot (I(V(T) > D^*) + \frac{(1-z)V(T)}{D} I(V(T) \le D^*)).$$

‡The threshold D^* is equal to the liabilities of the option issuing company D, or maybe less than D, because the company may continue to operate even if its asset value is less than its liability value.

Then, with the same handle as in formula (4.2), we obtain the lower bound of defaultable basket spread options under the structural model:

$$\begin{aligned}C_s^{LB} = e^{-rT}[(\mathbb{E}_{\mathbb{Q}}[(\sum_{i=1}^{n+m} \epsilon_i w_i S_i(T) - K) \cdot I(A, V(T) > D^*)])^+ + (\mathbb{E}_{\mathbb{Q}}[(\sum_{i=1}^{n+m} \epsilon_i w_i S_i(T) - K) \cdot \frac{(1-z)V(T)}{D} I(A, V(T) \le D^*)])^+ \\ + (\mathbb{E}_{\mathbb{Q}}[(\sum_{i=1}^{n+m} \epsilon_i w_i S_i(T) - K) \cdot I(A^c, V(T > D^*))])^+ + (\mathbb{E}_{\mathbb{Q}}[(\sum_{i=1}^{n+m} \epsilon_i w_i S_i(T) - K) \cdot \frac{(1-z)V(T)}{D} I(A^c, V(T \le D^*))])^+].\end{aligned}$$

Similarly to Section 4.1, we incorporate the stochastic component of default risk into the moment-generating function, denoted as $f^s(\phi, \psi)$, which is defined as

$$f^s(\phi, \psi) = \mathbb{E}_{\mathbb{Q}}[e^{\sum_{i=1}^{n+m} \phi_i \ln(S_i(T)) + \psi \ln(V(T))}],$$

where $\phi = [\phi_1, \phi_2, \cdots, \phi_{n+m}]$. Using the measure-change technique again and according to Proposition 2.1, we have the following theorem.

Theorem 4.2. The approximated price of the defaultable basket spread call option under the structural model is given by

$$C_s^{LB} = LB_s^+ + LB_{sc}^+ + LB_{sv}^+ + LB_{svc}^+,$$

where

$$LB_s = e^{-rT}(\sum_{k=1}^{n+m} \epsilon_k w_k G_k^s - K G_0^s), \qquad LB_{sc} = e^{-rT}(\sum_{k=1}^{n+m} \epsilon_k w_k G_k^{sc} - K G_0^{sc}),$$

$$LB_{sv} = e^{-rT} \frac{(1-z)}{D}(\sum_{k=1}^{n+m} \epsilon_k w_k G_k^{sv} - K G_0^{sv}), \qquad LB_{svc} = e^{-rT} \frac{(1-z)}{D}(\sum_{k=1}^{n+m} \epsilon_k w_k G_k^{svc} - K G_0^{svc}),$$

and the expression of $G_\cdot^\cdot$ is given by (C.1) in Appendix C.

Proof. See Appendix C.

□

Remark 4.1. The pricing formulas in Theorems 3.1, 4.1, and 4.2 can be utilized in both scenarios with and without default risk, serving as a synthetic pricing framework for these types of derivatives. By selecting specific parameters, it can also be applied to price a range of options, both with and without default risk, including European options, exchange options, spread options, and basket options.

Remark 4.2. These pricing formulas rely solely on the joint moment-generating function, allowing them to be applied not only under specific model assumptions but also in a data-driven manner. By constructing an empirical joint moment-generating function from available data, our approach facilitates genuinely model-free pricing.

5. Numerical illustration

In practice, there are instances where we can gain deep insight into the characteristics of the underlying assets of the options and confidently select an appropriate model. However, there are also situations where such confident model calibration is not available. Notably, our framework enables effective pricing regardless of whether the market model is deterministic, as discussed in Remark 4.2. Consequently, we divide our numerical illustration of the pricing formulas into two parts, with MC simulations serving as the benchmark in both cases. First, we evaluate the performance of the pricing formulas based on known specific models (see Section 5.1 for details) and compare them with existing methods. Second, in the absence of specific models, we validate the accuracy of our model-free pricing formulas empirically, relying solely on the data (see Section 5.2 for details).

5.1. Numerical illustration based on specific model

First, in Sections 5.1.1 and 5.1.2, we introduce two widely used models for pricing defaultable basket spread options. When the default component is omitted, it can be applied to price options without default risk. The first is the stochastic volatility model with default risk under the reduced model (see [22, 33]), and the second is the stochastic liquidity model with default risk under the structural model (see [25, 34]). These two models capture market dynamics from different perspectives and are particularly important for pricing defaultable basket spread options. For each model, we provide a brief introduction along with its corresponding joint moment-generating function.

Then, in Section 5.1.3, we compare the pricing formula in Theorem 3.1 for basket spread options without default risk with the notable work of [14]. In Section 5.1.4, we compare the pricing formulas in Theorems 4.1 and 4.2 for the defaultable basket spread options with those obtained using the VBS methods, which are widely utilized in the literature (see, e.g., stochastic volatility model [22], heston-nandi garch model [23], hawkes jump-diffusion model [24], stochastic liquidity model [25], stochastic interest rate model [27]).

5.1.1. Stochastic volatility model with default risk under the reduced model

On the filtered probability space $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\geq 0}, \mathbb{Q})$, for $k = 1, \cdots, n+m$, the spot price $S_k(t)$ is modeled by

$$\frac{dS_k(t)}{S_k(t)} = rdt + \beta_k(\frac{dM(t)}{M(t)} - rdt) + \sqrt{Y_k(t)}dB_k(t), \quad \frac{dM(t)}{M(t)} = rdt + \sqrt{Y_0(t)}dB_0(t),$$

where $M(t)$ denotes the value of the market index at time t. All assets are exposed to the same market risk, and their sensitivities are measured by β_k. The standard Brownian motion $B_k(t)$ characterizes the idiosyncratic risk of the k-th asset and is independent of all other $B_j(t)$ for $j \neq k$, as well as independent of $B_0(t)$. The volatility process $Y_k(t)$ is modeled by the following Cox-Ingersoll-Ross (CIR) process:

$$dY_k(t) = (\gamma_k - \alpha_k Y_k(t))dt + \sigma_k \sqrt{Y_k(t)}dL_k(t),$$

where γ_k represents the long-term mean, α_k is the speed of mean reversion, σ_k denotes the instantaneous volatility of $Y_k(t)$, and $L_k(t)$ is a standard Brownian motion. Moreover, the correlation coefficient between $L_k(t)$ and $B_k(t)$ is ρ_k for $k = 0, 1, \cdots, n$. The default intensity process is modeled by

$$\lambda(t) = \beta Y_0(t) + X(t), \quad dX(t) = (\gamma - \alpha X(t))dt + \sigma\sqrt{X(t)}dL(t),$$

where $Y_0(t)$ indicates that the default intensity is exposed to the same market risk as the underlying asset; $X(t)$ is a CIR process used to characterize the issuer's idiosyncratic risk, and $L(t)$ is independent of all other Brownian motions. By referring to the characteristic function of the CIR process in [35], we derive the following joint moment-generating function:

$$\begin{aligned} f^r(T,\phi,\psi) &= \mathbb{E}_{\mathbb{Q}}[e^{\sum_{k=1}^{n+m}\phi_k lnS_k(T)+\psi\int_0^T\lambda_u du}] \\ &= e^{\sum_{k=1}^{n+m}\phi_i\left(lnS_k(0)+rT\right)}h(\psi)g_0(\sum_{k=1}^{n+m}\phi_k\beta_k, \psi\beta - \frac{1}{2}\sum_{k=1}^{n+m}\phi_k\beta_k^2)\prod_{k=1}^{n+m}g_k(\phi_k, -\frac{1}{2}\phi_k), \end{aligned}$$

where $h(\cdot)$ and $g_k(\cdot,\cdot)$ are provided in (D.1) in Appendix D.

5.1.2. Stochastic liquidity model with default risk under the structural model

On the filtered probability space $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\geq 0}, \mathbb{Q})$, for $k = 1, \cdots, n+m$, the spot price $S_k(t)$ is modeled by

$$\frac{dS_k(t)}{S_k(t)} = rdt + \vartheta_k\sigma_k dB_k(t) + \beta_k a(t)dW(t), \quad da(t) = \tilde{\kappa}_a(\tilde{\theta}_a - a(t))dt + \sigma_a dM(t),$$

where $a(t)$ represents the market liquidity measure, the non-negative constant β_k signifies the liquidity risk associated with the k-th stock, and $B_k(t)$ and $W(t)$ are independent standard Brownian motions. The correlation coefficient between $B_i(t)$ and $B_j(t)$ is ρ_{ij} for $i, j = 1, 2, \cdots, n+m$. The standard Brownian motion $M(t)$ has a correlation coefficient of ρ with $W(t)$ and is independent of $B_k(t)$ for $k = 1, 2, \cdots, n+m$. The value of the option-issuing company is given by

$$\frac{dV(t)}{V(t)} = rdt + \vartheta_0\sigma_0 dB_0(t) + \beta_0 a(t)dW(t),$$

where the parameters ϑ_k, σ_k, $\tilde{\theta}_a$, σ_a, ϑ_0, and σ_0 are all constants. For $k = 1, \cdots, n+m$, the correlation coefficient between $B_k(t)$ and $B_0(t)$ is ρ_k. After some derivation, we derive the following joint moment-generating function:

$$f^s(T,\phi,\psi) = \mathbb{E}_{\mathbb{Q}}[e^{\sum_{k=1}^{n+m}\phi_k lnS_k(T)+\psi ln(V(T))}] = e^{H_0(\phi,\psi)}H_1(\phi,\psi)H_2(\phi,\psi),$$

where $H_0(\cdot,\cdot)$, $H_1(\cdot,\cdot)$, and $H_2(\cdot,\cdot)$ are given by (D.2) in Appendix D.

5.1.3. Basket spread options without default risk

The parameters for the stochastic volatility model, as detailed in Section 5.1.1, are sourced from references [22] and [33]. A comprehensive summary of these parameters is presented in Table 1. Similarly, the parameters for the stochastic liquidity model, discussed in Section 5.1.2, are derived from references [25] and [34], with their values compiled in Table 2. For a comprehensive understanding of the full truncation scheme in MC simulations, refer to [36] for more details.

Table 1. Parameter values for stochastic volatility model in Section 5.1.1.

Parameters in $M(t)$						
Initial Price	Initial Variance	Reversion Rate	Long-Term Level	Volatility	Correlation Coefficient	Interest Rate
$M(0) = 10$	$Y_0(0) = 0.02$	$\alpha_0 = 1.15$	$\gamma_0/\alpha_0 = 0.035$	$\sigma_0 = 0.39$	$\rho_0 = -0.64$	$r = 0.04$
Parameters in $S_k(t)$						
Initial Price	Initial Variance	Reversion Rate	Long-Term Level	Volatility	Correlation Coefficient	Sensitivity
$S_k(0) = 10(k \leq n); 8(k > n)$	$Y_k(0) = 0.037$	$\alpha_k = 2.0$	$\gamma_k/\alpha_k = 0.01$	$\sigma_k = 0.10$	$\rho_k = -0.50$	$\beta_k = 0.80$
Parameters in $\lambda(t)$						
Initial Intensity	Sensitivity	Reversion Rate	Long-Term Level	Volatility	Recovery Rate	Maturity Date
$\lambda(0) = 0.062$	$\beta = 1.25$	$\alpha = 2.0$	$\gamma/\alpha = 0.01$	$\sigma = 0.10$	$\alpha = 0.40$	$T = 0.5$

Table 2. Parameter values for stochastic liquidity model in Section 5.1.2.

Parameters in $a(t)$						
Initial value	Volatility	Reversion Rate	Long-Term Level	Interest Rate	Maturity Date	Correlation Coefficient
$a(0) = 0$	$\sigma_a = 0.888$	$\tilde{\kappa}_a = 0.309$	$\tilde{\theta}_a = 0.220$	$r = 0.02$	$T = 2.0$	$\rho = -0.80$
Parameters in $V(t)$						
Initial Price	Volatility	Sensitivity	Recovery Rate	Liability Value	Correlation Coefficient	
$V(0) = 100$	$\vartheta_0\sigma_0 = 0.107$	$\beta_0 = 0.491$	$1 - z = 0.4$	$D = 85$	$\rho_k = -0.50(k \leq n);0.50(k > n)$	
Parameters in $S_k(t)$						
Initial Price	Volatility	Sensitivity	Correlation Coefficient			
$S_k(0) = 10(k \leq n); 8(k > n)$	$\vartheta_k\sigma_k = 0.107$	$\beta_k = 0.491$	$\rho_{ij} = 0.75(i, j \leq n; i, j > n); -0.75(i \leq n, j > n; i > n, j \leq n)$			

Table 3. Price of the basket spread options without default risk.

	Stochastic Volatility Model				Stochastic Liquidity Model			
Moneyness	0.8	0.9	1	1.1	0.8	0.9	1	1.1
Case 1: $n = 5, m = 0$								
MC	2.1736	1.2523	0.4948	0.0993	3.1522	2.5048	1.9317	1.4419
$Merror$	6.80E-06	2.48E-05	1.03E-04	4.78E-04	1.98E-06	2.70E-06	4.14E-06	6.58E-06
C	73.8666	73.8647	73.8592	73.8584	32.9000	32.8984	32.8970	32.8976
CF	2.1697	1.2486	0.4926	0.0986	3.1520	2.5044	1.9315	1.4414
C	0.3570	0.3680	0.3770	0.3480	0.1540	0.1570	0.1600	0.2210
C_0^{LB}	2.1735	1.2522	0.4945	0.0990	3.1522	2.5048	1.9317	1.4419
C	0.1950	0.2010	0.1950	0.2000	0.1140	0.1170	0.1180	0.1150
Case 2: $n = 10, m = 5$								
MC	0.5096	0.3694	0.2533	0.1632	1.2573	1.1608	1.0708	0.9860
$Merror$	3.49E-03	3.01E-03	2.11E-03	8.38E-04	1.27E-02	1.42E-02	1.59E-02	1.79E-02
C	271.4788	271.4997	271.4744	271.4750	93.4405	93.4536	93.4415	93.4462
CF	0.4949	0.3559	0.2413	0.1528	1.1973	1.0899	0.9885	0.8932
C	2.0190	2.0980	2.0310	2.1350	0.4700	0.5630	0.4640	0.4680
C_0^{LB}	0.5091	0.3687	0.2526	0.1625	1.2524	1.1562	1.0659	0.9814
C	1.1740	1.1040	1.1030	1.1010	0.3770	0.3840	0.3850	0.3630

The $Merror$ is the error of MC, which is measured by the ratio of the 95% confidence interval length to the mean value. The C represents the computation time. The option price, denoted by CF, is calculated using the method of [14], with $N = 1024$ terms in the FFT and a damping parameter of 0.75. Consistent with [29], we set $c = 10$ in Proposition 2.1 and use $N = 1024$ terms for the Fourier-sinc approach.

We consider two cases of basket spread options. The first case, denoted as Case 1, involves $n = 5$ and $m = 0$. The second case, denoted as Case 2, involves $n = 10$ and $m = 5$ and is also considered in references [25]. Table 3 presents option prices at various levels of moneyness. The moneyness of the options is represented by the ratio $K/B(0)$, where K signifies the strike price, and $B(0)$ denotes the initial price of the underlying basket.

Clearly, our approximate formulas not only demonstrate high accuracy but also require less computational time than [14]. As an approximation pricing method, its robustness is also of particular interest. We investigate the impact of variations in model parameters and dimensions (i.e. the number of underlying assets) on the approximation error. Except for the parameters under investigation, all other parameters of the underlying assets are consistent with those shown in Table 1 and 2, with the moneyness level set at 1. Specifically for Case 2, when considering various dimensions, we select $n = m$ and allow it to change from 1 to 50. That is, we examine the scenario where the dimension of the options ranges up to 100. We focus on the relative error between the prices produced by the approximate formulas and the MC simulations. Figures 1 and 2 illustrate that, in various parameter sets, the relative approximation error of our formula remains nearly stable at 0. This stability indicates that our proposed approximation pricing method exhibits significant robustness to both parameter and dimensional changes.

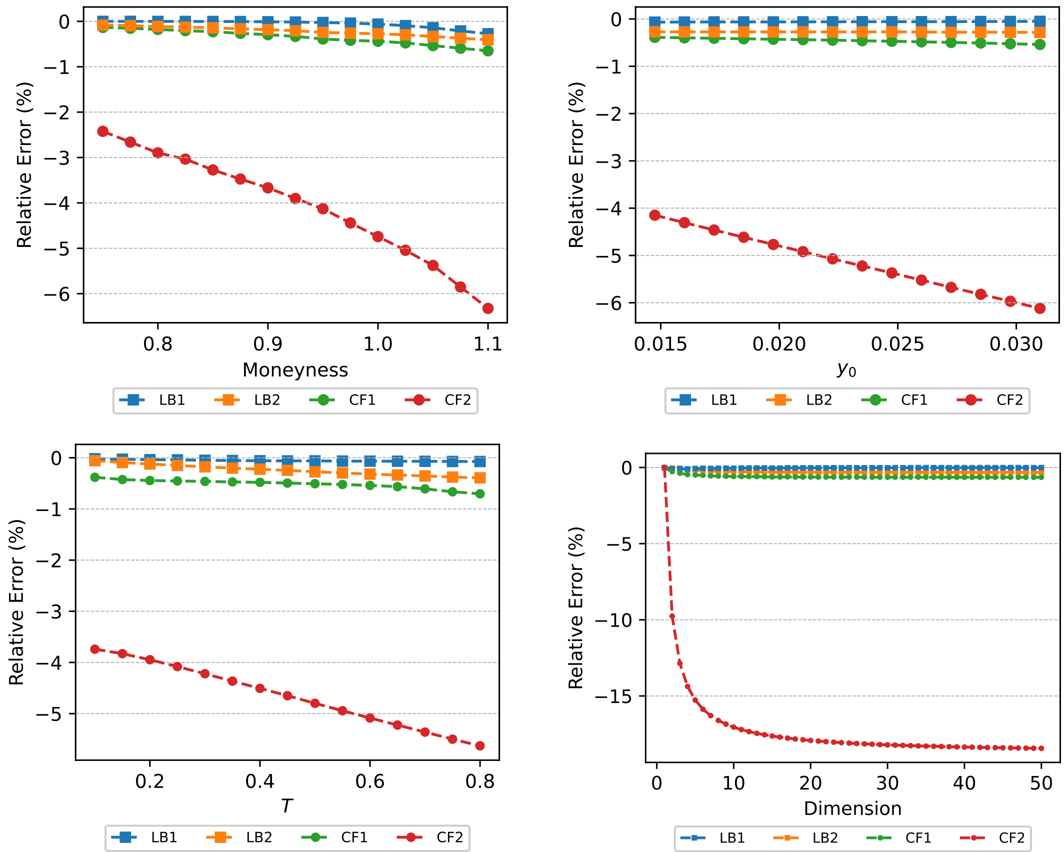

Figure 1. Comparison of approximation errors for options under stochastic volatility model.

The symbols $LB1$ and $LB2$ represent the option price obtained through our pricing formula in Theorem 3.1 for the cases 1 and 2. Similarly, $CF1$ and $CF2$ denote the prices obtained using the method in [14] for the same case.

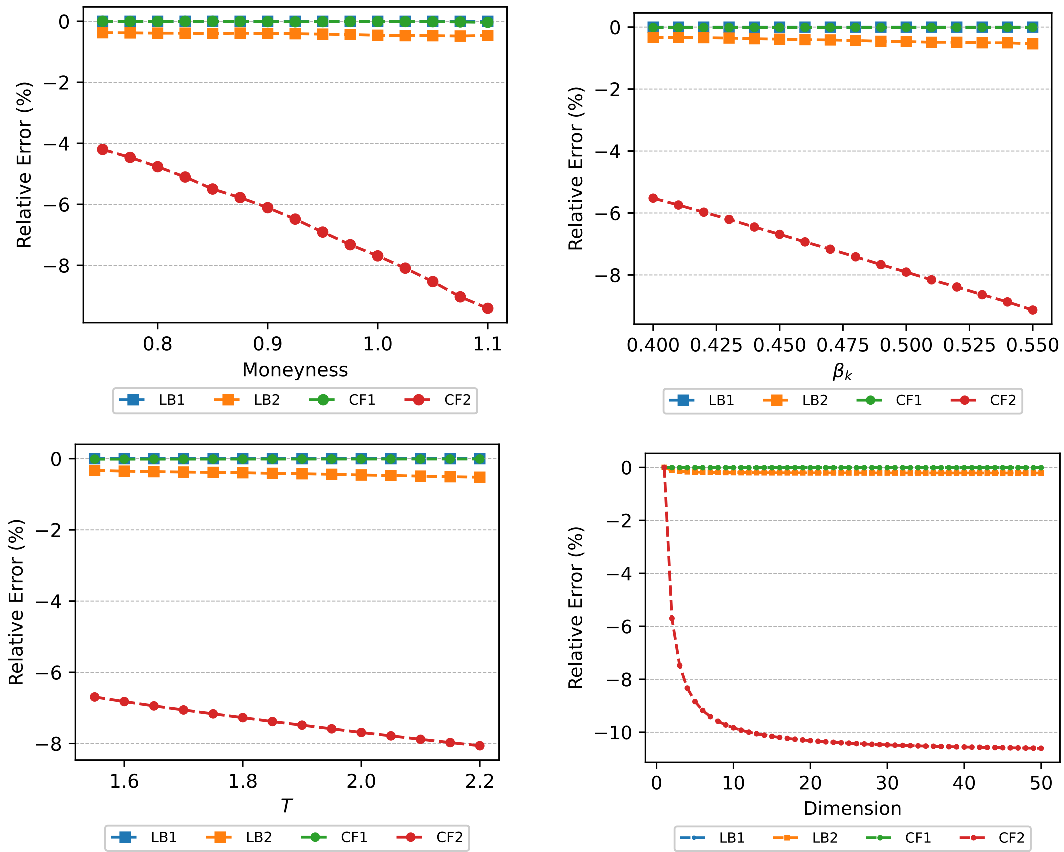

Figure 2. Comparison of approximation errors for options under the stochastic liquidity model.

The symbols $LB1$ and $LB2$ represent the option price obtained through our pricing formula in Theorem 3.1 for the cases 1 and 2. Similarly, $CF1$ and $CF2$ denote the prices obtained using the method in [14] for the same case.

5.1.4. Basket spread options with default risk

For defaultable basket spread options, we consider the default risk modeled by the reduced-form model and the structural model. The cases for the underlying assets are specified as Case 1 ($n = 5, m = 0$) and Case 2 ($n = 10, m = 5$). Table 4 provides the price of the defaultable option under different moneyness. Compared to prices obtained from MC simulations, we observe that the prices obtained by the VBS method perform poorly, while those obtained through our pricing formula exhibit better performance, with errors generally around the third to fourth decimal place. Our pricing formulas demonstrate high accuracy.

We then investigate the impact of various parameters and dimensionality on the approximation error. Except for the parameters that are the subject of our investigation, the remaining parameters of the underlying assets are consistent with those presented in Tables 1 and 2, with the moneyness level fixed at 1. Figures 3 and 4 illustrate that, in various parameter sets, the relative approximation error of our formula remains nearly stable at 0. This stability indicates that our proposed approximation pricing method exhibits significant robustness to both parameter and dimensionality changes.

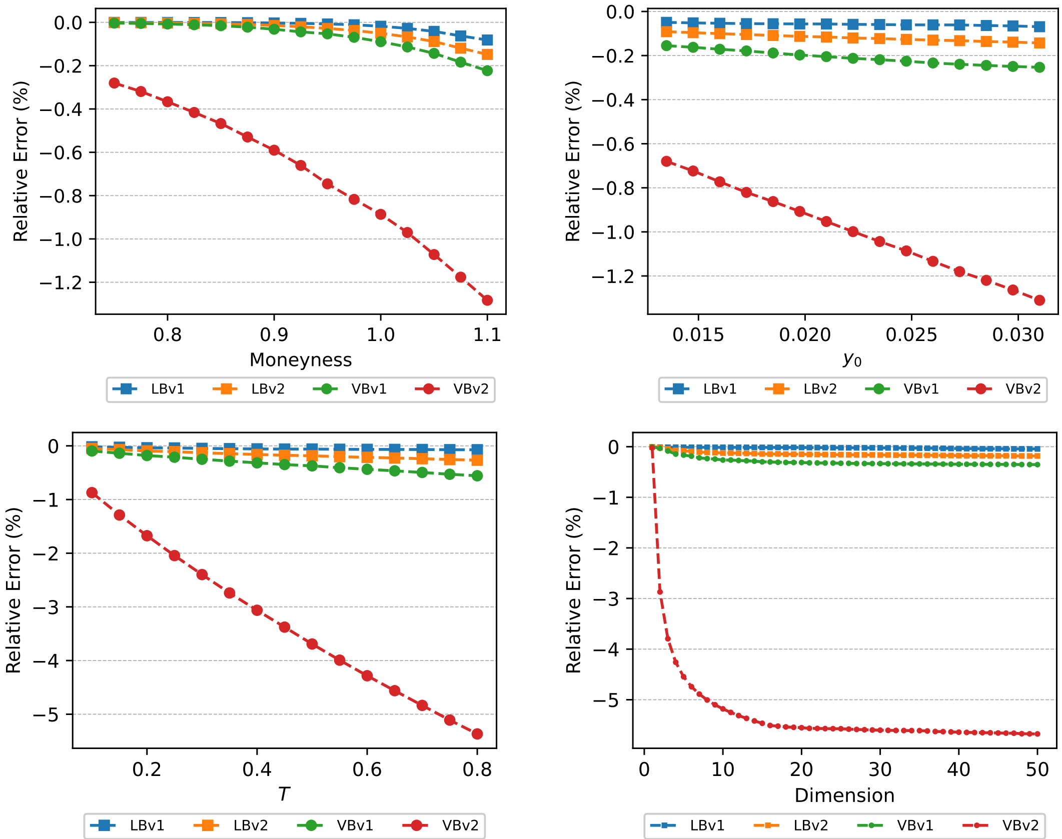

Figure 3. Comparison of approximation errors for defaultable options under the stochastic volatility model.

The symbols $LBv1$ and $LBv2$ represent the defaultable option price obtained through our pricing formula in Theorem 4.1 for the cases 1 and 2. Similarly, $VBv1$ and $Vbv2$ denote the prices obtained using the VBS method for the same case.

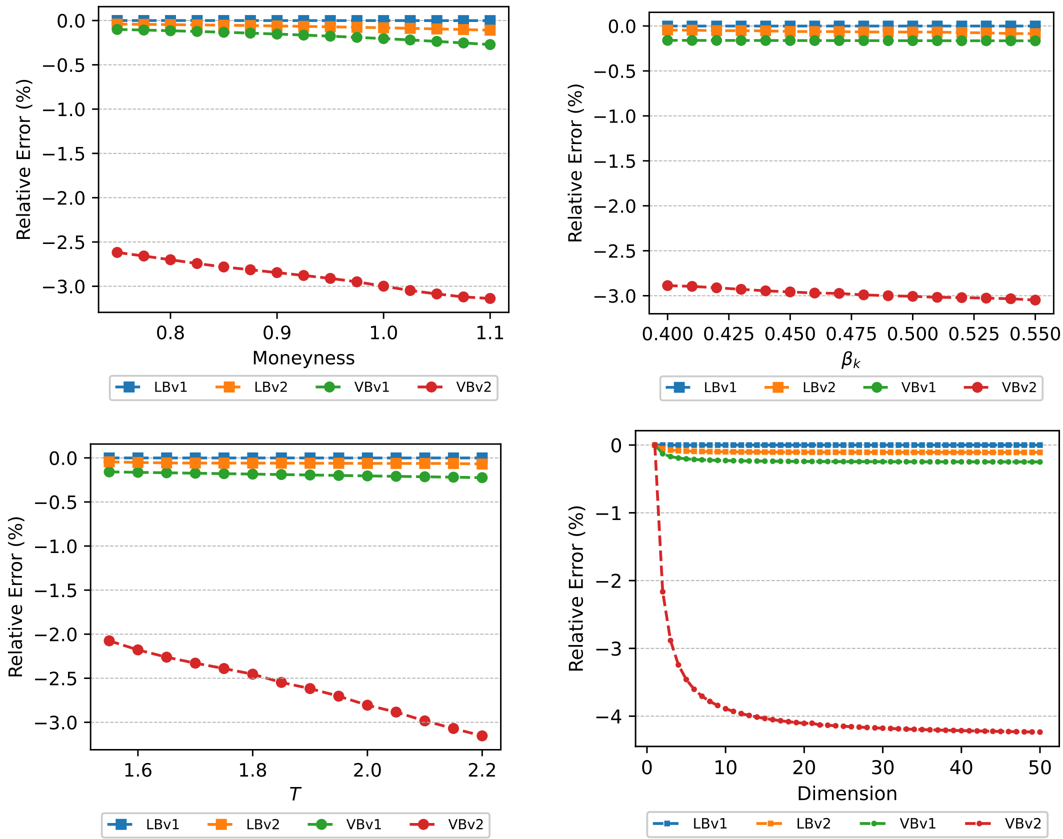

Figure 4. Comparison of approximation errors for defaultable options under stochastic liquidity model.

The symbols $LBv1$ and $LBv2$ represent the defaultable option price obtained through our pricing formula in Theorem 4.2 for the cases 1 and 2. Similarly, $VBv1$ and $Vbv2$ denote the prices obtained using the VBS method for the same case.

Table 4. Price of the basket spread options with default risk.

	Stochastic Volatility Model				Stochastic Liquidity Model			
Moneyness	0.8	0.9	1	1.1	0.8	0.9	1	1.1
Case 1: $n = 5, m = 0$								
MC	2.1403	1.2336	0.4874	0.0976	3.0729	2.4620	1.9100	1.4314
$Merror$	6.47E-06	2.42E-05	1.01E-04	4.75E-04	1.31E-06	2.48E-06	3.52E-06	5.12E-06
C	36.6354	36.6355	36.6320	36.6313	22.4260	22.4149	22.4136	22.4134
VB	2.1401	1.2332	0.4870	0.0974	3.0694	2.4582	1.9061	1.4275
C	0.5860	0.5930	0.5990	0.5710	0.8820	0.7660	0.6980	0.6720
C_0^{LB}	2.1403	1.2336	0.4873	0.0975	3.0729	2.4620	1.9100	1.4314
C	3.2460	3.2620	3.2690	3.3190	3.3800	3.0490	2.8380	2.8010
Case 2: $n = 10, m = 5$								
MC	0.5011	0.3631	0.2488	0.1603	1.1429	1.0613	0.9839	0.9107
$Merror$	6.48E-06	2.43E-05	1.07E-04	6.44E-04	2.65E-02	2.80E-02	2.96E-02	3.14E-02
C	213.8313	213.8625	213.8379	213.8336	67.9198	67.9183	67.9159	67.9353
VB	0.4993	0.3609	0.2466	0.1582	1.1120	1.0311	0.9544	0.8821
C	0.5420	0.5310	0.5170	0.5260	1.0820	1.0250	1.2440	1.1740
C_0^{LB}	0.5011	0.3630	0.2487	0.1600	1.1423	1.0606	0.9831	0.9097
C	2.8870	2.9310	2.9280	2.9290	7.5600	7.4620	8.7530	8.1830

Table 5. Parameter values for case in Section 5.2.

Initial Price	Volatility	Correlation Coefficient	Interest Rate	Maturity Date
$S_k(0) = 10$	$\sigma_0 = 0.1$	$\rho_{ij} = 0.75$ (or 0.9)	$r = 0.04$	$T = 1$

5.2. *Numerical illustration based on real data*

Due to space constraints, we only present the results of data-based for the basket spread options without default risk; however, similar constructions can be applied to scenarios with default risk. Specifically, let us consider from an omniscient perspective that the underlying asset of a fictitious basket option with $n = 5$ and $m = 0$, the price of the underlying asset is modeled by the following geometric Brownian motion:

$$\frac{dS_k(t)}{S_k(t)} = rdt + \sigma_k dB_k(t),$$

where $B_k(t)$ represents standard Brownian motion, with an instantaneous correlation ρ_{ij} between $B_i(t)$ and $B_j(t)$. The symbol σ_k denotes the instantaneous volatility of the k-th underlying asset. The value of the parameters is listed in Table 5. We utilize the price obtained from the MC simulations with 10^8 sample paths as the real price. To reflect the limitations of the availability of real market data, we assume access to only 1,000, 5,000, or 10,000 data points, corresponding to approximately 4, 20, or 40 years of trading data, respectively, derived from MC simulations. Based on the available data, we utilize the empirical joint moment-generating function to approximate the joint moment-generating function in Theorem 3.1, and subsequently derive the corresponding option price. We repeat the above procedure 1,000 times to generate the probability density plot of the option prices shown in Figure 5.

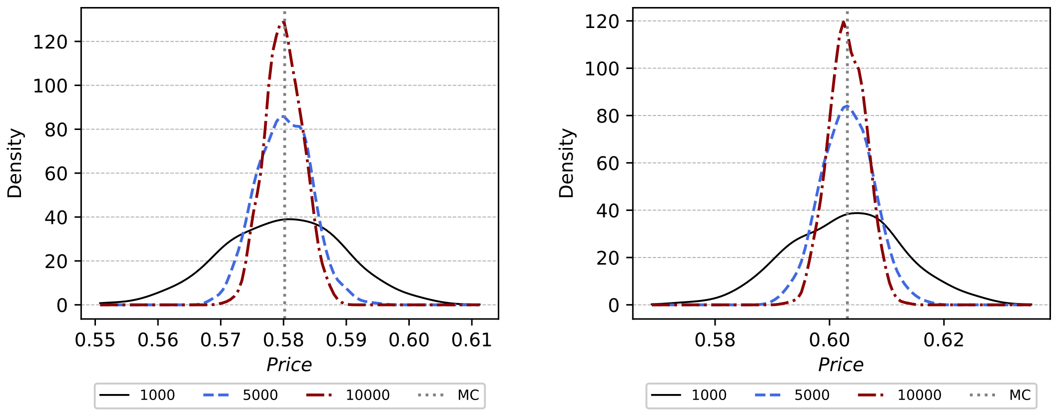

Figure 5. The probability density plot for option price.

The symbol *MC* represent the option price obtained through Monte Carlo simulation. The lines labeled "1000", "5000", and "10000" correspond to the option prices computed via Theorem 3.1 using 1,000, 5,000, and 10,000 data points of the underlying asset, respectively. The left (right) panel corresponds to $\rho_{ij} = 0.75$ ($\rho_{ij} = 0.9$).

In Figure 5, we find that regardless of the amount of data, the intensity of the benchmark price remains consistently the highest. Furthermore, our pricing formula exhibits considerable stability; when we can only obtain 1,000 data points for each underlying asset, the deviation from the benchmark price is 5%. However, as the number of data points increases to 5,000, the deviation from the benchmark price decreases to 2%.

6. Conclusion and discussion

In this paper, we develop a synthetic and general framework for pricing basket spread options with and without default risk. Our approach is model-free, requiring only the existence of the underlying assets' joint moment-generating function, making it applicable in both data-driven and model-based settings. By leveraging the LBS method to approximate the payoff structure and integrating it with the Fourier-sinc method, we first derive a computationally efficient pricing formula for basket spread options without default risk. Compared to existing methods, our approach significantly improves accuracy and efficiency while eliminating the need for optimization techniques. Furthermore, we extend this pricing framework to account for default risk by incorporating a measure-change technique. To the best of our knowledge, this is the first study to develop a model-free pricing framework for defaultable basket spread options, in contrast to previous research, which primarily relies on specific market models. Our method accommodates both reduced-form and structural default risk models, making it broadly applicable across different financial settings. The numerical results validate the effectiveness of our approach, demonstrating high accuracy and robustness across various parameter settings. Compared to the benchmark approach of [14], our pricing formula not only offers nearly 50% savings in computational time but also demonstrates strong reliability when estimated using limited market data. These findings highlight the practical relevance of our model-free framework in real-world pricing applications. Future research could further explore extensions of this framework to other exotic derivatives, incorporating additional risk factors, and generalize the approach to rely on characteristic function instead of moment-generating function to accommodate processes with heavy tails or infinite moments.

Author contributions

Qifeng Zhong: Methodology, conceptualization, software, formal analysis, writing – original draft. Zheng Liu: Conceptualization, formal analysis, writing – review and editing. Jing Yao: Conceptualization, supervision, funding acquisition, writing – review and editing. Xingye Yue: Conceptualization, supervision, funding acquisition, writing – review and editing.

Use of Generative-AI tools declaration

The authors declare that they did not utilize any artificial intelligence (AI) tools in the creation of this article.

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Conflict of interest

The authors have declared that no competing interests exist.

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